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I

INVESTIGATIONS ON CS¹³⁷

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II

Abstract

As a test of the R.P.I. beta ray spectrometer and in an attempt to gain information, some of the radiations of Cs^{137} and Au^{198} were studied. The internal conversion spectrum of Cs^{137} was investigated and its gamma ray energy was measured by comparison with the Hg^{198} K line. It is found, for the Ba^{137} spectrum, $K/L = 5.43 \pm 0.05$, $M/L = 0.27 \pm 0.01$, $L_I/L_{III} = 8 \pm 2$, the M line is predominantly M_I , and the gamma ray energy is 660.79 ± 0.27 kev. It is concluded that a systematic error enters into the instrument for measurements over large energy ranges (1000 gauss-cm) but that small range measurements (150 gauss-cm) and ratio measurements can be trusted. The Hg^{198} L line is thought to be predominantly L_I .

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I. Introduction

Up to this time, no serious attempts at precision measurements had been made with the R.P.I. beta ray spectrometer. The instrument had, in fact, been in operation for only a short period before the start of this research. In testing the instrument, then, it seemed wise to choose an isotope whose decay scheme was simple and whose characteristics were relatively well known. Such measurements as had been made indicated that reasonable results could be expected and so it was hoped some new information on the isotope could be obtained.

Cesium 137 was the isotope chosen. It has been studied by many investigators^{1,2,3,4,5,6} and displays the decay scheme shown in Figure 1. The gamma ray is internally converted with K conversion coefficient of 0.12¹, and is of order M^{45} . Because of its 37 year half life, simplicity of decay, and availability, Cs¹³⁷ comes immediately into interest as a calibration standard for

Figure 1. Cs¹³⁷ decay scheme

spectrometers. At the start of this investigation, however, the gamma energy was known to a precision of 0.1%⁶ and it was felt this could be improved with the R.P.I. instrument.

DuMond and his group have since increased the precision by a factor of five.⁷

The measurement of the ratio of K internal conversion to L and comparison with theory allows an assignment of multipole order to the gamma transition. With the completion of the L shell conversion coefficient calculations, knowing the predominant L sub-shell conversion will be of importance in confirming this assignment. The L_I , L_{II} , and L_{III} sub-shell ratios also give information on this⁸.

II. Apparatus

To clarify the following discussion, a block diagram of the apparatus is given in Figure 2.

A. Spectrometer

The spectrometer is of the double focussing type, a description of which may be found in the literature⁹. Resolutions ranged from 0.17% to 0.25%.

B. Current Regulator

The current regulator, utilizing a double cathode photocell circuit for regulation, supplies the spectrometer magnet coils. A description is given in reference 10.

C. Field Measuring Device

The field is measured by means of two coils mounted on a single shaft, rotating at a rate of 30 revolutions per second. One coil rotates in the spectrometer field, the other in the field

FIGURE 2. BLOCK DIAGRAM OF APPARTUS

FIGURE 3. BRIDGE CIRCUIT

of a permanent reference magnet. The signals from the coils are mixed through the bridge, the difference signal is fed to the 30 cycle amplifier, and the resulting wave form is displayed on the oscilloscope. Upon bridge balance, the scope signal goes to zero. This arrangement detects a field change of at least 1 part in 35,000, and very often 1 part in 70,000. A detailed description of the 30 cycle amplifier and the field monitor may be found in the literature¹⁰.

D. Counter

Electrons are received by the mica end window counter and are recorded on the scalar. The counter is permanently fixed in the spectrometer chamber, provisions for filling without removal having been made. The filling mixture is 9 cm argon and 1 cm alcohol vapor. Dead times were measured¹¹ as 10^{-4} to 10^{-3} seconds.

E. Field-Electron energy Relation

The relation among the spectrometer magnetic field, the electron path radius, and the electron kinetic energy is given by¹²

$$(1) \quad H\rho = \frac{10^{14}}{c} \left\{ W^2 + \frac{2m_0 c^3 W}{e \times 10^{14}} \right\}^{1/2}$$

where

H = magnetic field in gauss

ρ = radius of electron path in cm

c = velocity of light in cm

W = kinetic energy of electron in Mev

m_0 = rest mass of the electron

e = electronic charge in esu

It is $H\rho$ that is used as the variable of the spectrometer.

The reference magnet is therefore calibrated in $H\rho$ units since ρ is a constant.

F. Condition for bridge balance

The bridge is essentially as shown in Figure 3. Quantities with U subscripts are associated with the permanent magnet coil; those with L subscripts with the spectrometer magnet coil of the field monitor. C_U serves to bring E_U into phase with E_L , thus giving a satisfactory null scope pattern at bridge balance.

R. Liedtke* has shown the bridge balance condition to be to one part in 100,000

$$(2) \quad \frac{E_L}{E_U} = \frac{H_L}{H_U} = \frac{H\rho_L}{H\rho_U} = \frac{R_L + r_L}{R_U + r_U} = \frac{R_L'}{R_U'} \quad \text{since } \rho \text{ is constant}$$

So

$$H\rho)_x = H\rho)_{PM} \frac{R_L'}{R_U'}$$

where x indicates the unknown $H\rho$ and PM signifies the permanent magnet $H\rho$.

Appendix A considers the balance condition in some detail and shows the nature and validity of the approximation.

*Personal communication

G. Potentiometer

A Leeds and Northrup K-2 potentiometer with galvanometer was used in the resistance measurements

III. Procedure

The general procedure outline is as follows:

K:L:M ratios and sub-shell conversion prominences were found by plotting the Ba^{137} conversion spectrum as counts per minute vs $H\rho$ and making suitable measurements on the plots.

The crux of the Cs^{137} gamma ray energy measurement method lay in the precise energy value given the Au^{198} gamma ray by DuMond and his co-workers⁷. A composite source of Au^{198} and Cs^{137} was made and the $H\rho$ of the Ba K line was determined using the Hg K line as standard. Calculation of the Cs^{137} gamma ray energy followed readily.

A. Sources

1. Source backings

Thin mica frames, typical dimensions being shown in Figure 4, were used to give rigidity to the source backings. Thicknesses were estimated to be in the order of hundredths of a millimeter.

Formvar films on which the sources could be laid down were prepared by first dissolving a gram or less of formvar, depending on the areal density wanted, in 50 cm^3 of ethylene

dichloride. A glass plate was then dipped into the solution, quickly removed, and the ethylene dichloride allowed to evaporate. Immediately after evaporation, the film perimeter was freed from the glass, the film floated onto water, picked up with the mica frame, smoothed, and the combination dried. The formvar adhered sufficiently well to the mica so that no adhesive was needed. Typical film dimensions were 2 x 5 cm.

Figure 4. Source and source backing

So there would be no accumulation of charge on the sources, they were grounded to the spectrometer frame through a thin metal film evaporated onto the backings. Aluminum was used for the Cs sources, gold for the Au-Cs source.

As a precaution against charge buildup on the mica frames, these were also grounded through a coating of Aquadag.

Areal densities were measured simply with a sensitive balance and a centimeter scale. See Table I for density values.

2. Depositing of sources

An ordinary lettering pen attached to a screw-moved bed served to lay down the liquids onto the source backing. This gave a line width of about 5 mm.

Insulin was first laid down to act as a wetting agent between source solutions and backing.

3. Cs¹³⁷ Source

An aqueous solution of CsCl, obtained from Oak Ridge with a specific activity of approximately 1 mc/cm³, was laid down on the insulin line. The water evaporated, leaving the CsCl as a solid.

4. Au - Cs source

16 mg of Au, initial activity of 50 mc in Au¹⁹⁸, was obtained from Oak Ridge. This was dissolved in about 1 cm³ of aqua regia, then diluted to 2 cm³ with distilled water. 1 cm³ of this solution was removed and the volatile constituents driven off by heat, leaving solid AuCl₃. The AuCl₃ was next dissolved in 10 cm³ of distilled water. This procedure reduced the acidity of the AuCl₃ solution in order to prevent destruction of the source backing. Even so, the solution was sufficiently acidic to require the use of gold on the backing. A pen was also gold filmed to prevent the deposition of unwanted solids on the source through acid attack of the pen metal.

AuCl₃ solution was drawn from the 10 cm³ stock and mixed with the CsCl solution in a ratio approximately 3:1. It was calculated that such a ratio would give Hg and Ba K lines of equal intensity. Peak counts later showed a ratio of approximately 9:1 would have done this. The mixture was then laid down on the gold covered source backing and the water evaporated.

Data on the sources is found in Table I.

For detailed calculations on source areal densities, see Appendix B.

Table I. Data on sources

Source	Source backing areal density $\mu \text{ gm/cm}^2$	Source areal density $\mu \text{ gm/cm}^2$	Total areal density $\mu \text{ gm/cm}^2$
Cs^{137}	40 Formvar + Aluminum	70 - 280	100 - 320
$\text{Au}^{198}\text{-Cs}^{137}$	35 Formvar 15 Gold	15 AuCl_3 25 CsCl	90

B. Run Technique

In the early runs, little attention was paid to the direction of change in magnetic field. It was later found that the hysteresis effect in the magnet influenced rather markedly the results. Since this effect warrants separate discussion, it will be considered in a later section. Suffice it here to note the effect was recognized and an attempt was made to deal with it.

The final procedure followed in later runs was as follows:

The spectrometer magnet was demagnetized.

The magnet current, unregulated, was increased from zero to a value preset well below that eventually desired. This was done by bringing up the voltage on the power supply rectifier tubes.

Since the current regulator resistors had been set for photocell balance prior to degaussing, it was possible to switch in the cell on maximum sensitivity. In most cases, the current increased with the switching in. Had the cell been balanced after the current was flowing, a large variation in that current would

have resulted from manipulation of the resistors. One cannot say this switching on procedure leads to a reproducible initial current increase; it can only be said the current varies in the same and a known sense on each run*. The reason behind such a method lay in an attempt to follow the magnetization curve of the spectrometer magnet on each run.

Having switched in the photocell, the current was then slowly brought up to the region of interest and varied in the same direction throughout the run.

In general, the bridge variable resistance R_L was set on the value corresponding to the desired field increment, the oscilloscope indicating bridge unbalance. The field would then be increased to a point of rough balance. Fine balance was achieved by adjusting R_L for null scope signal.

At each field setting, counts were taken for one or two minutes, depending on the source activity and the region of operation on the line peaks. Temperature measurements of the permanent magnet were also made at each count.

For large field changes (2 gauss-cm) there would be a slow rise in field due to the magnet inductance. Drift due to the current regulator was checked by adjusting R_L for balance after a count as well as before. The average of these readings was taken

* The current seemed to increase or decrease with cell switching depending on the region of operation, i.e., the current itself.

For the Cs runs, the current always increased while for the Au - Cs runs there was a current decrease on each run.

as the representative resistance. In all cases R_L was read to ± 2 ohms and in many runs, the sensitivity was up to ± 1 ohm. R_L values were in the order of 70,000 ohms for the Ba^{137} spectrum and 50,000 ohms for Hg^{198} . R_U was approximately 10^5 ohms for both.

So that the temperature variation of the system could be accounted for, at least two sweeps over a standard line were made on each run. For Cs runs, the Ba^{137} K line was used as standard; for the Au-Cs runs, the Hg^{198} K line served. The system variation appeared as a linear temperature dependence of $H\rho)_{PM}$.

IV. Data Analysis

A. Ratio and Predominance Determinations

The counting rate was first corrected for dead time by using the relation

$$N = \frac{n}{1 - nt}$$

where N = true counts

n = observed counts

t = dead time of tube

Corrected counting rate was next plotted against resistance for the standard line runs. By drawing a line through the successive mid-points of a curve and noting the intersection of that line with the curve, the line peak was located. A temperature could then be associated with the peak. $H\rho)_{PM}$ was computed for this temperature by equation 2. See Figure 5 for plots of $H\rho)_{PM}$ vs temperature.

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Since the $H_p)_{PM}$ temperature variation and each count temperature were known, the H_p for each count was readily calculated from equation 2. Plots were then made of counting rate vs H_p and the peaks were located as before. A plot of counting rate/ H_p vs H_p would be a more correct manner of plotting. However, dividing the results of the present plot by the peak H_p values turns out to be nearly the same. This is due to the symmetry of the lines and the small H_p range occupied by the lines.

K, L, and M lines were measured. The Ba^{137} K, L, and M lines are shown in Figures 6 and 7.

H_p differences were taken peak to peak and compared with calculated values of these differences to determine the predominant sub-shell conversions. The K/L and L/M ratios were found both by peak height and curve area measurements and the results were compared.

To find the L_I/L_{III} ratio, two lines of different height, each having the shape of the K line, were plotted with peak abscissa difference equal to $H_p)_{L_{III}} - L_I$. The widths of the curve resulting from the superposition of these two were matched to those of the experimental L line. L_{III} was varied in height until a close fit was obtained. Note that ratios involving L_{II} were not considered since theory predicts that conversion occurs here in less than 5% of the cases for Ba^{137} 13. See Figure 8 for these plots.

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FIGURE 8:
RESULTANT Ba^{137}
L LINE FOR
 $L_I/L_{III} = 8$

B. Cs^{137} Gamma Ray Energy Determination

The analysis proceeded as that for the Cs^{137} through the plotting of the temperature variation line for $H\rho)_{\text{FM}}$. Values for resistance and temperature for Hg K and the Ba K peaks were found from plots of counting rate vs resistance. More precise values of resistance ratios R_L/R_U than those calculated from resistance box readings were obtained by comparing voltage drops across the resistances as measured with a potentiometer. Finally, the Ba K $H\rho$ value was calculated using the relation which follows from equation 2:

$$H\rho)_{\text{Ba}} = \frac{H\rho)_{\text{Hg}}(R_U')_{\text{Hg}}(R_L')_{\text{Ba}}}{(R_L')_{\text{Hg}}(R_U')_{\text{Ba}}}$$

where the T subscript denotes temperature dependence.

Calculation of the Cs^{137} gamma ray energy proceeded directly out of equation 1.

The Hg^{198} L line was also investigated and its predominant conversion sub-shell considered.

See Figure 9 for plots of the Hg K and L lines and the Ba K line.

V. Errors

The sources of error will merely be mentioned here. A detailed error analysis may be found in Appendix C.

A. K:L:M Ratios

The principal sources of error in the K:L:M ratio measurements are the following:

1. Error in the counting tube dead time.
2. Statistics of counting.
3. H_p value for each point.
4. Location of the curve peaks.
5. Uncertainty in the curve area as measured with a planimeter.

Secondary errors may be listed as

1. Temperature correction influence on the shape of the curve.
2. Spectrometer magnet current drift errors.
3. Estimation of the extent of the peak "tails".

(This is of particular note in the L line since it merges with the M.)

A theoretically rigorous attack of the errors arising in the K:L:M ratios would indeed be difficult since each point on the plot of the conversion lines is subject to error and only one point, the peak, is of interest. First finding the equation of the curve, one could apply certain laborious methods to the problem. However, it seems reasonable to treat the totality of the errors by considering the result of each run as one measurement of the quantity K/L or L/M . Relatively simple methods of statistics may then be applied to evaluate the standard deviation.

B. Line Predominance

One may cope with the errors arising here in a direct manner, following simple error propagation rules.

As shown in Appendix C, error sources of concern are in R_L values for the line peaks. This uncertainty springs from

1. Error in setting R_L .
2. Limit of error of R_L .
3. Drawing of the curves and location of the peaks.
4. Temperature correction uncertainty.

C. Cs^{137} gamma ray energy

The errors involved are attributed to five sources:

1. Error in Au^{198} gamma energy
2. Uncertainty in the values of the physical constants.
3. Location of the line peaks.
4. Temperature correction uncertainty.
5. Resistance measurement errors.

VII. Results

Pertinent data leading to the following results are to be found in Appendix C.

The results and associated errors are:

Cs^{137} :

$$K/L = 5.43 \pm 0.05$$

$$M/L = 0.27 \pm 0.01$$

$$L_I/L_{III} = 8 \pm 2$$

M line is predominantly M_I .

Gamma ray energy is 660.79 ± 0.27 kev

Au^{198} ,

L line is tentatively set as being predominantly L_I .

These results should be compared with those of other investigators. This may be done with the aid of Table II.

Table II

Collection of data on Cs^{137}

<u>K/L</u>	<u>M/L</u>	<u>K/L+M</u>	<u>Gamma energy(kev)</u>	<u>Reference</u>
4.8 ± 0.3				1
5.0				2
5.2				3
		4.57 ± 0.05		4
5.5	0.27			5
			661.4 ± 0.7	6
			661.60 ± 0.14	7
		4.52 ± 0.07		21
			661.65 ± 0.15	22

VII. Comparison with theory

The K/L ratio indicates, on comparison with theory and as found by other investigators, the gamma transition is of order M^4 .

Tralli and Lowen¹³, using non-relativistic theory, have plotted L_{III}/L_I for magnetic transitions as a function of

Z^2/E for $Z=35$ (E is energy). They note that L_{II} conversions are less than 5% of the L_I . Applying these curves to Ba, a value of L_{III}/L_I of approximately 0.25 is found. The authors, however, advise caution in the application of these curves to the values of Z greatly different from 35 (Ba $Z=56$).

Hebb and Nelson¹⁴ plot K/L against Z^2/E , $Z=35$, for electric transitions. They state that, for higher Z , these curves should be lowered somewhat. This may also be the case for L_{III}/L_I curves and magnetic transitions.

The value obtained for L_{III}/L_I of 0.12 seems, then, to be in reasonable agreement with theory.

The result that the Hg^{198} L line is predominantly L_I is in disagreement with theory¹⁵ and with results of Mihelich and Hill^{8,16}. The Au^{198} gamma transition is E^2 ¹⁷ and theory requires such transition to be predominantly L_{II} in our range of energy. The above investigators assign a value of 2.5 to the L_{II}/L_{III} ratio. This is based on theory, the L line not having been sufficiently well resolved and the original assignment having been L_I and L_{III} .

The L_I and L_{II} lines differ by about 60 ohms bridge resistance on the R.P.I. spectrometer and such a difference should be detectable. See Figure 10. The L_I assignment, it must be admitted, is the result of only one run. Two runs were taken but the first was discarded because of an hour stoppage in mid - run to repair the 30 cycle amplifier. It must also be noted that due to low counting rates, statistics were poor.

Nonetheless, the shape of the line appears to be clear and the temperature of the system did not vary more than 0.05°C while sweeping over the line. The current regulator was functioning properly during the run so that a hysteresis shift of peak seems unlikely.

It would be best to say, then, the predominance in the Hg^{198} L line is uncertain. Several runs with a stronger source should resolve this difficulty.

The L_{III} line was not detected because, no doubt, of the low intensity source.

VIII. Discussion

A. Hysteresis effect

A long unsuspected source of error, particularly in the K:L:M ratio measurements, was the effect of hysteresis in the spectrometer magnet. Suspicion was first aroused when it was noted the Ba K lines did not remain constant in height within statistics on successive runs. This was at a point in the investigation when the run procedure did not include the variation of current in one direction only.

One of the effects of hysteresis was evidently a change in the focussing properties of the instrument which caused a shift in peak height.

If there were no shift in the peaks energywise due to the hysteresis effect, the somewhat elaborate turn on procedure would seem unnecessary. However, the temperature variation of

the system must be accounted for and a hysteresis peak shift would be superimposed on the normal temperature shift. This indeed may be the reason for the scattering of some of the temperature correction points. Mittleman has shown a peak shift of $1/4$ line width is possible due to hysteresis while peak differences on the same sweep remain constant.*

Figure 11 shows the effect of hysteresis during a run. A rather violent current fluctuation took place at the curve break and the run was resumed.

Shull and Dennison¹⁸ have investigated the nature of the electron orbits in this type spectrometer and state conditions for best resolution. They call attention to the fact that the shape of the magnetic field must remain independent of field magnitude for best results. A field shape change may be present.

B. Reasons for discrepancies

One notes the difference between the value here obtained for the Cs^{137} gamma energy and those listed by DuMond and Siegbahn. In view of the consistency of the results in this investigation, it appears as though a systematic error is present.

The first object of suspicion is the hysteresis effect. It has been seen that it can cause an energy shift in peak. However, there was no current fluctuation of significance during the runs and the results seem to be consistent among themselves. The remaining possibility seems to be a change in field shape with

* Private communication.

magnitude of field, but this should be regarded as nothing more than a guess.

Siegbahn²² has reported a peak shift due to inelastic scattering of electrons in thick converters. For a converter of areal density 0.7 mg/cm^2 and an electron energy of 411 kev, 20% of the electrons were calculated to suffer inelastic collisions. At higher energies, thicker converters could be used with no increase in the number of such collisions. It was found the line peak suffered shift more than the foot of the high energy slope.

While the line peak has here been used for measurement, the source areal density is considerable less than that quoted above and the electron energy is greater.

For the Au - Cs runs the capacitor C_U was set to full value in order to have a decent scope null signal. There is a possibility this could have been a value such that the correction terms in Appendix A would not have been negligible.

The snapping in of the photo tube and the effect of temperature correction would seem to have little influence in view of the consistency of the results. These should be, at any rate, random not systematic effects.

C. Evaluation of the instrument

It appears that ratios obtained from area measurements can be trusted. Counts measurements may also be considered valid provided that hysteresis precautions are taken.

Energy differences taken over small intervals (100 -

300 gauss-cm) evidently are without serious systematic error. Large energy intervals (1200 gauss-cm) seem to introduce significant error.

D. Suggestions for future work

As has been evident, little is known of the hysteresis effect. A thorough investigation of this effect would indeed be of value. Perhaps an isotope having a number of gamma transitions giving rise to a number of conversion lines would serve to check the effect.

Several runs on the Hg^{198} spectrum with a stronger source than that used here should decide the prominence of the L line.

IX. Acknowledgements

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Appendix A

The following is a condensed reproduction of a derivation by R. Liedtke. Reference is made to Figure 3.

The bridge balance condition without approximation may be shown to be

$$(a) \quad E_U e^{j\omega t} - \left\{ \frac{E_U e^{j\omega t} + E_L e^{j(\omega t + \phi)}}{A + jB} \right\} (A_U + jB_U) = 0$$

where

$$A = r_U + r_L + R_L + \frac{R_U}{D}$$

$$B = \omega L_U + \omega L_L - \frac{\omega R_U^2 C_U}{D}$$

$$A_U = r_U + \frac{R_U}{D}$$

$$B_U = \omega L_U - \frac{\omega R_U^2 C_U}{D}$$

$$D = 1 + \omega^2 R_U^2 C_U^2$$

Expanding (a), setting the real and imaginary parts equal to zero, saying ϕ is small and therefore that $\sin \phi \sim \phi$, $\cos \phi \sim 1$ leads to the condition

$$(b) \quad \frac{E_L}{E_U} = \frac{A_U(A - A_U) + B_U(B - B_U)}{A_U^2 + B_U^2}$$

Now, saying that $\omega^2 R_U^2 C_U^2 \ll 1$, making suitable expansions, and neglecting higher orders of this quantity yields the final approximate condition

$$\frac{E_L}{E_U} = \frac{R_L + r_L}{R_U + r_U} \left\{ 1 + \frac{\omega^2 R_U^3 C_U^2}{R_U + r_U} + \frac{\omega^2 L_U L_L - \omega^2 R_U^2 L_L C_U}{(R_U + r_U)(R_L + r_L)} \right. \\ \left. - \frac{\omega^2 L_U^2 - 2\omega^2 R_U^2 L_U C_U + \omega^2 R_U^4 C_U^2}{(R_U + r_U)^2} \right\}$$

Nominal values for the above quantities are

$$\omega = 200$$

$$C_U = 2 \times 10^{-10} \text{ f}$$

$$R_U + r_U \sim R_U \sim 10^5 \text{ ohms}$$

$$R_L + r_L \sim R_L \sim 7 \times 10^4 \text{ ohms}$$

$$L_U \sim L_L \sim 10^{-4} \text{ henries}$$

These values give for the terms in brackets

$$\text{the second term: } \sim 2 \times 10^{-5}$$

$$\text{the third term: } \sim 10^{-9}$$

$$\text{the fourth term: } \sim -2 \times 10^{-5}$$

So the correction is in the order of 10^{-5} assuming the second and fourth terms do not cancel exactly. Other errors of the experiment make this factor negligible.

It may then be said

$$\frac{E_L}{E_U} = \frac{R_L'}{R_U'}$$

Appendix B.

Source Calculations

Cs^{137} sources:

Typical formvar film areal densities amounted to
40 $\mu\text{gm}/\text{cm}^2$ *

The CsCl solution had a specific activity $\sim 1 \text{ mc}/\text{cm}^3$.

$$1 \text{ mc}/\text{cm}^3 = 3 \times 10^7 \text{ disintegrations}/\text{sec cm}^3$$

Now

$$N_0 \lambda = 3 \times 10^7 \text{ disintegrations}/\text{sec cm}^3$$

where

$$N_0 = \text{the number of Cs nuclei}/\text{cm}^3 \sim 5 \times 10^{18} \text{ nuclei}/\text{cm}^3$$

$$\lambda = \text{Cs decay constant} \sim 6 \times 10^{-10} \text{ sec}^{-1}$$

CsCl has a molecular weight of 168. Converting to grams gives a density of $1.4 \text{ mg}/\text{cm}^3$. The specific activity, then, is $715 \mu\text{c}/\text{mg}$. Now

$$1 \mu\text{c} \equiv 1.8 \times 10^6 \text{ disintegrations}/\text{min}$$

Approximately 0.1 of the Cs disintegrations will result in K electrons. Then, assuming a solid angle factor of 0.001*

$$1 \mu\text{c} \sim 200 \text{ K electrons}/\text{min}$$

At a counting rate at the Ba K peak of 8000 counts/min**

$$\frac{8000 \text{ \#}/\text{min}}{200 \text{ \#}/\text{min}/\mu\text{c}} \sim 40 \mu\text{c}$$

$$\frac{40 \mu\text{c}}{715 \mu\text{c}/\text{mg}} \sim 56 \mu\text{gm}$$

Assuming a source area of 0.2 cm^2 , the areal density is $280 \mu\text{gm}/\text{cm}^2$.

* B. Hildebrand, private communication

** One more properly should use integrated counts on N/Hp vs Hp plot.

In a like manner, a 2000 count/min K peak can be shown to correspond to an areal density of $70 \mu\text{gm}/\text{cm}^2$. These values represent extremes of source densities.

Au^{198} - Cs^{137} source:

The Au^{198} foil, mass 0.016 gm, was measured as 50 mc on 8/18/52 at 9 A.M.

The first Hg K peak was run over $\sim 4:30$ P.M., 8/20/52, the activity having dropped by this time to 28 mc.

In aqua regia, the gold goes into AuCl_3^*

The mass of AuCl_3 is

$$16 \text{ mg} \times 304/198 \approx 24.6 \text{ mg}$$

At the time of the first peak, then, the specific activity was $1.13 \times 10^3 \mu\text{c}/\text{mg}$.

Now, for Au^{198} , $\alpha_K \approx 0.028$ and considering 0.1% solid angle factor

$$1 \mu\text{c} \sim 50 \text{ K electrons}/\text{min}$$

170 counts/min were received at the Hg K peak, first

* The reaction is

with drying

in aqueous solution

The last equation plus the fact that Ag and Al lie above Au in the electrochemical series explain why the former films were attacked by the AuCl_3 solution.

traverse. This corresponds to $3.3 \mu\text{c}$.

So

$$\frac{3.3 \mu\text{c}}{1.13 \times 10^3 \mu\text{c/mg}} \sim 3 \mu\text{gm AuCl}_3$$

Assuming a 0.2 cm^2 area source, the areal density is

$$\sim 15 \mu\text{gm/cm}^2$$

The Ba K peak gave a counting rate of ~ 700 counts/min.

Analysis similar to that above results in an areal density for

CsCl of $25 \mu\text{gm/cm}^2$.

Appendix C. Results and error analysis

A. Values of errors

1. R_L

R_L could always be set for bridge balance to ± 2 ohms. The limit of error of the resistance boxes was listed as $\pm 0.1\%$. However, comparing resistance readings with readings of the K-2 potentiometer, one sees agreement within the error of the latter instrument ($\pm 0.01\%$). See Table III. $\pm 0.01\%$ will be used as the limit of error of the resistances. The error in measuring r_U and r_L was neglected since it would have negligible effect on R_U' or R_L' .

Table III

Comparison of resistance and potentiometer readings

<u>R_L' (ohms)</u>	<u>$(R_U'/R_L')_{\text{pot}}$</u>	<u>$(R_U'/R_L')_{\text{resistance}}$</u>
51073	2.11462	2.11460
51126	2.11250	2.11243
51109	2.11319	2.11313

2. Plotting

Run 11 data was plotted six times and the peak located. The peak was found to have a standard deviation in resistance of $\pm 0.003\%$ and this will be used as the error in R_L values. One would expect the errors in setting R_L to balance out, being as often negative as positive. It would be of interest to list

the results of these six plots. The standard deviation is given.

$$H\rho)_K = 3380.54 \pm 0.09 \text{ gauss-cm}$$

$$H\rho)_L = 3497.72 \pm 0.05 \text{ gauss-cm}$$

$$H\rho)_{L-K} = 117.18 \pm 0.08 \text{ gauss-cm}$$

$$K/L = 5.21 \pm 0.05$$

3. Temperature correction

By noting the extreme lines which could be drawn through a set of data, an estimation of temperature correction error of $\pm 0.01\%$ in the value of $H\rho)_{PM}$ was made. See Figure 5.

4. Au^{198} gamma energy

The DuMond group lists the error in the measurement of the Au^{198} gamma energy as ± 0.14 kev.

5. Physical constants

The values of DuMond were used.¹⁹

6. Potentiometer

Leeds and Northrup lists the limit of error of the K-2 potentiometer as $\pm 0.01\%$ for the range here used.

B. Method

Statistical methods as described by Worthing and Geffner²⁰ will be applied.

C. $K:L:M$ ratios

Table V lists the results of runs, averages, and standard deviations. Counts measurements are included for interest but only area measurements were used to determine ratios. More will

be said on this matter in Appendix D. On runs 8a to 10b an unusually high background, due possibly to contaminated source holders, prevented area measurements. The M line was not covered in most runs.

Multiplying the area measurements by $H\rho)_L$ or $M/H\rho)_K$ as explained in Appendix D yields final results

$$K/L = 5.43 \pm 0.05$$

$$M/L = 0.27 \pm 0.01$$

In the L_I/L_{III} determination, a large change in the L_{III} line produces a small change in the composite line. Assuming the L line widths are good to ± 0.1 gauss-cm, it seems one cannot say the ratio is good to better than ± 2 . Even this figure may be optimistic. Table IV lists observed L line widths for comparison with Figure 8.

Table IV

Observable widths of L line, Run 11

<u>Height, %</u>	<u>Width, $H\rho$</u>
10	13.0 ± 0.1
20	11.0
30	9.6
40	8.4
50	7.2
60	6.0
70	5.2
80	4.2
90	3.0

Table V

Summary of results on ratios

Run	<u>K/L) counts</u>	<u>K/L) area</u>	<u>M/L) counts</u>	<u>M/L) area</u>
1	5.82	5.24	0.243	0.276
2a	5.64	5.55*		
b	6.01	5.16		
3a	5.42	5.25	0.259	0.253
b	6.10	5.30		
4	6.00	5.26	0.226	0.253
5	5.34	5.25		
6	5.86	5.45**		
7	5.59	5.35		
8a	5.43			
b	5.51			
9a	5.24			
b	5.32			
10a	5.30			
b	5.22			
11	5.30	5.21	0.248	0.248
Average	5.28	5.25	0.244	0.258
σ	0.04	0.05	0.012	0.010

* Eliminated through Chauvenet's Criterion

**Misaligned source, eliminated

D. Line predominance

Table VI gives data, averages, and the standard deviation obtained by statistical treatment of that data.

First note the most fundamental relation for $H\rho$ is*

$$H\rho)_x = H\rho)_s \cdot R_{L_x}' / (R_{L_s}')_T$$

* It is seen here that a consideration of $H\rho)_{PM}$ need never be made. Instead of plotting $H\rho)_{PM}$ vs temperature, R_{L_s}' could be plotted with a considerable saving in computation.

where

$H\rho)_x$ = unknown $H\rho$ value

$H\rho)_s$ = standard $H\rho$ value

R_{Lx}' = unknown bridge resistance

$(R_{Ls}')_T$ = temperature corrected standard bridge resistance

Since $\Delta H\rho = H\rho)_x - H\rho)_s$ is considered and $R_{Lx}'/(R_{Ls}')_T \sim 1$, the error in $H\rho)_s$ need not be included.

Table VI

Results of $\Delta H\rho$ measurements

Run	$\Delta H\rho)_{L-K}$	$\Delta H\rho)_{M-K}$
1	117.00 gauss-cm	134.44 gauss-cm
2a	116.82	
b	116.69	
3a	117.12	134.45
b	117.03	
4	116.69	134.22
5	116.85	
7	117.12	
8b	116.80	
9b	116.25	
10a	116.52	
b	116.90	
11	<u>117.18</u>	<u>134.40</u>
Average	116.85 gauss-cm	134.38 gauss-cm
σ	0.26	0.10

Note: Runs 6, 8a, and 9a were omitted due to, in 6 a misaligned source, in 8a and 9a unstable magnet current. The 9a current shift took place prior to reaching the peak so that a decent value of peak counts was obtainable.

Now, the error in $(R_{L_s}')_T$ due to temperature and plotting is $\pm 0.013\%$. The error in R_{L_x}' is $\pm 0.003\%$ since only plotting is involved.

A sample computation will be made:

$$H\rho)_L \pm \sigma = H\rho)_K \frac{(R_{L_L}') \pm \sigma_1}{(R_{L_K}')_T \pm \sigma_2}$$

So

$$\begin{aligned} (\sigma/H\rho)_L^2 &= (\sigma_1/R_{L_L}')^2 + (\sigma_2/R_{L_K}')^2 \\ &= 9 \times 10^{-10} + 169 \times 10^{-10} \\ &= 178 \times 10^{-10} \end{aligned}$$

$$\sigma/H\rho)_L = \pm 0.013\%$$

$$\sigma = \pm 0.44 \text{ gauss-cm}$$

and this is the error in $\Delta H\rho)_{L-K}$ due to statistics.

13 measurements were taken, so the final statistical standard deviation is

$$\sigma = \pm 0.44/\sqrt{13} = \pm 0.12 \text{ gauss-cm}$$

Considering the limit of error (L.E.) similarly yields

$$\text{L.E.})_{H\rho)_L} = \pm 0.48 \text{ gauss-cm}$$

And we see the total error in $\Delta H\rho)_{L-K}$ is

$$\pm (0.12 + 0.48) = \pm 0.60 \text{ gauss-cm}$$

For the $\Delta H\rho)_{M-K}$ measurement, the total error turns out to be ± 0.70 gauss-cm.

The result of one run over the Hg^{198} L line gave a value of 2502.68 gauss-cm. Therefore

$$\Delta H\rho)_{L-K} = 280.17 \text{ gauss-cm}$$

Calculations following those above gave a total error of ± 0.70 gauss-cm.

The following results may then be listed:

Ba¹³⁷:

$$\Delta H\rho)_{L-K} = 116.8 \pm 0.6 \text{ gauss-cm}$$

$$\Delta H\rho)_{M-K} = 134.4 \pm 0.7 \text{ gauss-cm}$$

Hg¹⁹⁸:

$$\Delta H\rho)_{L-K} = 280.2 \pm 0.7 \text{ gauss-cm}$$

Comparing these results with values calculated with the use of Mihelich and Church's x-ray critical absorption tables shows

Ba¹³⁷:

L line is predominantly L_I

M line is predominantly M_I

Hg¹⁹⁸:

L line is predominantly L_I

See Table VII for the calculated values.

Table VII

Calculated values of $\Delta H\rho$

<u>Line differences</u>	<u>$\Delta H\rho$ in gauss-cm</u>
L _I - K	116.90
L _{II} - K	118.39
L _{III} - K	119.87
M _I - K	134.33
M _{II} - K	134.93
M _{III} - K	135.19
M _{IV} - K	136.19
M _V - K	136.26

Note the difference between the statistical standard deviation given in Table VI and the error which was calculated. The limiting factor is the limit of error on the resistances in the latter. This limit seems to be a fair choice in view of these results. Also note the larger standard deviation for the L - K values. One would expect the reverse to be true because of the lower counting rate of the M line. The presence of two lines in the L line with significant intensity may make for large fluctuations in the composite line.

E. Cs¹³⁷ gamma ray energy

Four runs were made with four sweeps over the Ba K line and six over the Hg K. By cross checking, 24 values of the Ba K line $H\rho$ were obtained. These are to be found in Table VIII.

The errors in the Au¹⁹⁸ gamma energy and in the physical constants may be handled through equation 1:

$$H\rho = \frac{10^{14}}{o} \left\{ W^2 + \frac{2m_0 c^3 W}{e \times 10^{14}} \right\}^{1/2}$$

The standard deviation in $H\rho$ may be found by

$$\sigma_{H\rho}^2 = \left(\frac{\partial H\rho}{\partial o} \right)^2 \sigma_o^2 + \left(\frac{\partial H\rho}{\partial W} \right)^2 \sigma_W^2 + \left(\frac{\partial H\rho}{\partial m_0} \right)^2 \sigma_{m_0}^2 + \left(\frac{\partial H\rho}{\partial e} \right)^2 \sigma_e^2$$

Now

$$\partial H\rho / \partial c = 9.85 \times 10^{-9}$$

$$\partial H\rho / \partial W = 4.2 \times 10^3$$

$$\partial H\rho / \partial m_0 = 9.2 \times 10^{29}$$

$$\partial H\rho / \partial e = 1.75 \times 10^{12}$$

$$\sigma_c = 9 \times 10^4 \text{ cm/sec}$$

$$\sigma_W = 4 \times 10^{-5} \text{ Mev}$$

$$\sigma_{m_0} = 2.5 \times 10^{-32} \text{ gm}$$

$$\sigma_e = 7 \times 10^{-15} \text{ esu}$$

And it turns out

$$\sigma_{H\rho} = \pm 0.17 \text{ gauss-cm}$$

This is the error in $H\rho)_K$ of Hg^{198} . It is fixed and cannot be reduced by a series of measurements.

Now, realizing the relation

$$H\rho)_{\text{Ba}} = \frac{H\rho)_{\text{Hg}}(R_U')_{\text{Hg}}(R_L')_{\text{Ba}}}{(R_L')_{\text{HgT}}(R_U')_{\text{Ba}}}$$

and recalling that R_U' is not subject to statistical error, the statistical error in $H\rho)_{\text{Ba}}$ may be shown to be ± 0.22 gauss-cm.

The limit of error amounts to ± 0.73 gauss-cm.

The net error is therefore ± 0.95 gauss-cm.

By a similar computation to that above for the standard deviation for $H\rho$, one may find the standard deviation of the gamma energy. The error turns out to be ± 0.27 kev.

A summary of data is given in Table VIII.

Table VIII

 $H\rho$ values for the Ba^{137} K line

<u>Ba K point used</u>	<u>Hg K point used</u>	<u>$H\rho$)_{Ba K} (gauss-cm)</u>
1, 8/20 run	1, 8/20 run	3378.20*
2, 8/20	"	3378.10
1, 8/21	"	3378.29
2, 8/21	"	3377.58
1, 8/30	2, 8/20 run	3377.87
2, 8/20	"	3377.74*
1, 8/21	"	3377.93
2, 8/21	"	3377.39
1, 8/20	3, 8/20 run	3378.13
2, 8/20	"	3378.03
1, 8/21	"	3378.22
2, 8/21	"	3377.68
1, 8/20	1, 8/21 run	3378.15
2, 8/20	"	3378.07
1, 8/21	"	3378.60
2, 8/21	"	3377.72
1, 8/20	2, 8/21 run	3378.73
2, 8/20	"	3378.65
1, 8/21	"	3378.82*
2, 8/21	"	3378.30
1, 8/20	3, 8/21 run	3378.35
2, 8/20	"	3378.26
1, 8/21	"	3378.45
2, 8/21	"	3377.91*

* Results of the same run

Average $H\rho$ = 3378.13 gauss-cm

Appendix D. A consideration of data treatment

Due to a finite "slit" width of the spectrometer, there will be a Δp about p radius for accepted electrons. This will be a constant for the instrument.

For a field setting H , electrons in a momentum group rather than those having a single momentum will be accepted for counting.

The resolution of the instrument may then be defined as

$$\frac{H \Delta p}{H p} = \frac{\Delta H p}{H p}$$

This is a measure of the momentum spread since $H p$ is proportional to momentum. The resolution is a constant.

Now it may be said

$$C = KN(Hp) d(Hp)$$

where

C = counting rate

K = constant

$N(Hp) d(Hp)$ = the number of electrons with momenta between Hp and $Hp + d(Hp)$

Multiplying the right hand side by Hp/Hp

$$C = KN(Hp) \frac{d(Hp)}{Hp} Hp$$

$$C = K'N(Hp) Hp$$

$$N(Hp) = K''C/Hp$$

Now

$$\int_{\text{allowable}} N(H\rho) d(H\rho) = \text{total number of electrons in the conversion line}$$

and it becomes clear why a plot of $C/H\rho$ is desirable.

Each conversion line, it would seem, should have similar electron distributions, differing only by a constant factor. A ratio of corresponding heights on two distributions should then be equal to the ratio of the total areas. It would then appear, choosing the peaks as the most accessible point,

$$\frac{N_1(H\rho_{\text{peak}})}{N_2(H\rho_{\text{peak}})} = (C/H\rho)_{1\text{peak}} (H\rho/C)_{2\text{peak}}$$

It is said to be incorrect to make the $H\rho$ division when peak height ratios are taken; only counting rate should be used.*

In view of the preceding development and the consistency of count and area measurements (and inconsistency when area measurements alone are subject to $H\rho$ division) it is not clear to the author why this should be so.

This dilemma led to the using of area measurements only for ratio results.

* G.N. Glasoe, private communication

I-III

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