MEASUREMENTS OF HELIUM II FLOW THROUGH SMALL PORES

A THESIS

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

OF THE UNIVERSITY OF MINNESOTA

BY

BRIAN BERNARD SABO

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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MASTER

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CHAPTER I.

INTRODUCTION.

This work reports the measurement of variations of average superfluid density of liquid helium in small pores of well-defined geometry. In addition, the onset of dissipation of superfluid flow in these pores was investigated. The temperature range of the measurements was from about 1.1 K to within 12 mK of the lambda point at 2.1632 K (at one atmosphere).

The apparatus was designed to measure the ΔP vs. v_s curve (pressure drop vs. superfluid velocity) of a weak superfluid link between two reservoirs of helium in analogy to the ΔV vs. I curve of a superconducting weak link between two bulk superconductors. An ac source of mass current was used so that the resulting ΔP vs. v_s curve could be observed directly with an oscilloscope. It was hoped to investigate structure analogous to that observed in the superconducting case, structure which has been of great interest in relation to the Josephson effects. However, it became apparent during this work that it would be difficult with our apparatus to pursue possible Josephson effects in helium via this method in as direct a way as we had hoped, and we turned toward investigating somewhat different but related characteristics of fine pores connecting two baths of helium.

Three different superleaks were studied, a single 10 μm diameter orifice in a nickel substrate 38 μm thick, and 0.1 and

and 0.5 μm Nuclepore membranes 11 μm and 12 μm thick.⁽¹⁾ The latter membranes contained numerous pores of uniform shape and size, nominally 0.1 and 0.5 μm in diameter, respectively.

The experimental cell, shown schematically in Fig. 1, consisted of two chambers, separated by a wall into which was mounted the superleak under test. Piezoelectric ceramics in each of the two chambers of helium were driven with ac voltages out of phase, forcing superfluid flow in simple harmonic motion through the superleak between chambers. A differential pressure transducer mounted between the chambers measured the inertial and resistive contributions to the resulting pressure drop.

The geometry of the experiment suggests that Helmholtz oscillations should occur, and such oscillations are observed. Since the normal fluid is effectively clamped in the superleak and only the superfluid component flows, the temperature dependence of the resonant frequency is that of the fourth-sound Helmholtz resonator of M. Kriss and I. Rudnick.⁽²⁾ This dependence is given to a good approximation by the expression

$$\omega_0^2 = \epsilon \frac{\rho_s}{\rho} u_1^2,$$

where ϵ is a temperature-independent geometry-dependent factor and u_1 is the velocity of first-sound in helium. This expression allows measurement of the effective superfluid density ratio ρ_s/ρ in our superleak as a function of temperature.

The onset of dissipation in the flow can be explained in terms

of creation of vortex rings in two different regimes which we call the extrinsic and the intrinsic regimes. First, in a steady flow experiment, for some fluid velocity the kinetic energy of flow will be sufficient to allow creation of vortex rings, as suggested by R. P. Feynman.⁽³⁾ In this case the velocity threshold is quite sharp. Second, at sufficiently high temperatures thermal fluctuations can supply the energy to create some vortex rings, which then grow in energy at the expense of the kinetic energy of the fluid. In this regime energy loss can occur with a smaller velocity of flow than in the first case, and in fact the energy loss is very strongly velocity dependent.

In the first regime, with the energy loss mechanism in the flow of a fluid through a circular orifice assumed to be the formation of quantized vortex rings of the same diameter as the orifice, W. Zimmermann, Jr., has calculated⁽⁴⁾ that the onset of loss begins when the superfluid reaches a critical velocity

$$v_{s,c} = \frac{h}{\pi m d} \ln \left(\frac{4d}{a_0} - \frac{7}{4} \right), \quad (1)$$

an expression very similar to Feynman's earlier estimate. Here m is the mass of a helium atom, h is Planck's constant, d is the diameter of the uniform circular orifice, and a_0 is the radius of the vortex ring core. The critical velocity here is determined in part by the geometry of the flow channel, a factor extrinsic to the fluid. Hence we call this superfluid critical velocity the extrinsic critical velocity.

We have observed in each of our superleaks, over some range of temperature, temperature-independent critical velocities which are in good quantitative agreement with this prediction.

With regard to the second regime, the fluctuation theory of J. S. Langer and M. Fisher predicts that at higher temperatures in superfluid helium thermal fluctuations can nucleate vortex rings in the moving superfluid independent of geometry.⁽⁵⁾ The velocity for which this nucleation becomes macroscopically observable is called the intrinsic critical velocity and goes as

$$v_{s,c} = u_c \left(1 - \frac{T}{T_\lambda} \right)^{2/3},$$

where u_c is a constant of the order of 1500 cm/sec.

We have observed a region of behavior similar to this for our smallest pores, the 0.1 μm Nuclepore membrane. With other superleaks, extrinsic critical velocities mask this effect in the temperature regions explored.

The fluctuation theory has been extended by H. A. Notarys to predict the pressure drop resulting from this fluctuation-induced loss.⁽⁶⁾ The expression for this pressure drop is

$$\Delta P = P_0 e^{-v_0/v_s}, \quad (2)$$

where P_0 and v_0 are geometry and temperature dependent, and v_s is the superfluid velocity in the superleak. This behavior

has been seen experimentally by Notarys, but the behavior seen in our work does not seem to follow this law.

Our experiment resembles experiments of W. E. Keller and E. F. Hammel in which helium was forced through slits at constant velocity and the resulting pressure difference was measured.⁽⁷⁾

It contrasts to the more usual critical velocity measurements where helium flows under its own gravitational head, with the consequent flow velocity being measured.^(8,9) These two cases are close analogs of electrical high-impedance and low-impedance, respectively.

Since our experiment is done at low audio frequencies, hysteresis effects observed in some dc experiments should be less pronounced.^(7,10) At the same time, however, non-linear effects may be harder to interpret if they are in any way averaged over a cycle of drive.

Another problem in many-flow experiments, such as those using powders or Millipore filters to clamp the normal fluid, is the non-uniformity of the flow channels.⁽¹¹⁾ This non-uniformity of channel makes it difficult to analyze geometry-dependent effects. In our case the superfluid flow takes place in pores of well-defined geometry.

In our ac experiment the pressure drop consists of two terms. One component of the pressure drop is in phase with the fluid acceleration and is called the inertial pressure; it is that pressure which is needed to accelerate the superfluid in simple

harmonic motion. With increasing drive current a component of the ac pressure is seen that is 90° out of phase with the inertial signal. We call this the resistive pressure associated with superfluid energy losses. For the most part we observe a continuous, rapid increase in resistive pressure as the superfluid velocity is increased. However, for the single orifice superleak driven near the fourth-sound Helmholtz resonance the onset of loss was quite different, as discussed in Chapter V.

With the continuous onset of a resistive pressure the definition of a "critical" velocity becomes somewhat arbitrary. In this experiment it was defined as that fluid velocity for which the resistive pressure was detectable above the noise in the pressure sensing system (when operating in a wide-band mode, as described later). This definition of superfluid critical velocity is less arbitrary than it appears. Because the pressure drop is such a rapid function of drive velocity, a large change in the magnitude of this critical pressure results in only a small change in the superfluid critical velocity.

At fixed temperature, the superfluid critical velocities are for the most part independent of ac drive frequency in the range measured, from 2.5 Hz to 80 Hz. The exception to this behavior was the 0.1 μm Nuclepore superleak in which the measured critical velocities approached zero when the drive frequency approached the fourth-sound resonance frequency of the cell. The velocities could be fit to a phenomenological curve

$$V_{s,c} = \tilde{V}(T) \left| 1 - \frac{f}{f_0} \right|^{1/2},$$

for which no explanation has been found.

During the course of our work an additional piezoelectric ceramic driven at 200 kHz was placed near the superleak in order to search for the development of discrete static differences in chemical potential between the two chambers in analogy to one manifestation of the ac Josephson effect in superconductors. Although differences in chemical potential were seen to arise as a result of operating this transducer, no steps in the difference were observed. Although such steps have been reported by several authors^(12,13) in other geometries and interpreted as the ac Josephson effect in helium, it now appears that they were due instead to sound resonances which would not have appeared in our results in the same way.⁽¹⁴⁾

CHAPTER II.

APPARATUS HYDRODYNAMICS.

1. Introduction.

To treat the hydrodynamics of the experimental cell we use the two-fluid model for helium II. The linear dimensions of the two chambers of the cell were small in relation to the wavelengths of first- and second-sound at the frequencies used, so it is assumed that temperatures and pressures are uniform throughout each chamber and that only within the superleak do gradients in these quantities need to be considered. In particular the lowest first-sound mode of the cell appeared at about 650 Hz, a factor of ten above the highest fourth-sound resonances studied, so our neglect of pressure variations within a chamber in the following calculations is well justified. We also neglect temperature variations within a chamber, though the lowest second-sound resonance was calculated to be about 100 Hz. No resonance in this range was observed to couple to the pressure response, and since this frequency was still well above most drive frequencies used, the neglect of the second-sound resonance is considered justified.

A simple mechanical analog can explain the zero-order fluid dynamics of the cell at frequencies low compared to the lowest first- and second-sound resonant frequencies. One can look at the cell as a harmonic oscillator, in which the mass, the inertial mass of the superfluid in the superleak, is situated

between two anchored springs, the compressible fluid in each chamber, which alternately push and pull the mass. For a sufficiently small superleak the normal fluid motion is very small and can be considered to be a higher-order effect. However, this motion does give rise to energy loss which under some conditions is an important part of the damping of the oscillations.

Since the superfluid flow with clamped normal fluid gives rise to superfluid density oscillations in the cell, we expect there will be temperature oscillations. These are, however, too small to be seen directly with our apparatus. The temperature oscillations do induce small thermal currents in the wall between the chambers which try to reduce the temperature differences, and hence give rise to another damping mechanism.

We here analyze our system as a fourth-sound Helmholtz resonator in a way similar to Kriss and Rudnick.⁽²⁾ For simplicity treat the superleak as N parallel tubes of radius a and total cross section $A = N \pi a^2$, each of length l , connecting two chambers of volume Ω_1 and Ω_2 , respectively. The two-fluid hydrodynamic equations⁽¹⁵⁾ can then be reduced to one dimension, and the fluid motion between chambers can be described in terms of cross-section-averaged superfluid and normal fluid displacements in the superleak (positive towards chamber 1) $\xi_s(t)$ and $\xi_n(t)$, and velocities $v_s(t) = \dot{\xi}_s(t)$ and $v_n(t) = \dot{\xi}_n(t)$.

The linearized equations governing the motion of the two fluids are, in one-dimensional form,

Mass conservation

$$\Delta M_1(t) = A (\rho_s \xi_s(t) + \rho_n \xi_n(t)) = - \Delta M_2(t) \quad (3)$$

Entropy conservation

$$\Delta S_1(t) - \frac{\Delta Q_1(t)}{T} = s \rho A \xi_n(t) = - \Delta S_2(t) + \frac{\Delta Q_2(t)}{T} \quad (4)$$

Momentum conservation

$$- \frac{\Delta P(t)}{l} = \rho_s \xi_s(t) + \rho_n \xi_n(t) + R \dot{\xi}_n(t) \quad (5)$$

Superfluid flow

$$\ddot{\xi}_s(t) = - \frac{1}{\rho} \frac{\Delta P(t)}{l} + s \frac{\Delta T(t)}{l} \quad (6)$$

Here $\Delta M_i(t)$ and $\Delta S_i(t)$ are the mass and entropy changes in chambers 1 and 2, $\Delta Q_i(t)$ is the heat added by heaters and by thermal conduction between chambers and s is the entropy per unit mass of the fluid; $\Delta P(t) = P_1(t) - P_2(t)$ and $\Delta T(t) = T_1(t) - T_2(t)$ are the time-dependent pressure and temperature differences between chambers. We have added a damping term $R \dot{\xi}_n(t)$ to account for normal fluid viscosity, which in the case of laminar flow in a circular tube would take the form $\frac{8\eta_n}{a^2} \dot{\xi}_n(t)$. ⁽¹⁶⁾

2. Hydrodynamic Equations.

The mass density in each chamber can be related thermodynamically to entropy and pressure changes, as well as to mass influx to each chamber and to cell volume change,

$$\begin{aligned}\Delta M_1(t) &= A (\rho_s \epsilon_s(t) + \rho_n \epsilon_n(t)) \\ &= \Delta(\rho_1 \Omega_1) = \rho \Omega_1 \left[-\frac{\alpha T}{c} \Delta s_1(t) + K \Delta P_1(t) \right] + \rho \Delta \Omega_1(t).\end{aligned}\quad (7)$$

The term $\Delta \Omega_1(t)$ includes cell volume change attributable to cell wall flexibility as well as driver-transducer volume change. We consider s and P to be the independent variables, expressing other variables in terms of these, and using the definitions

$$\kappa \equiv \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_s$$

$$\alpha \equiv - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P$$

$$C \equiv T \left(\frac{\partial s}{\partial T} \right)_P.$$

Then using the chamber 2 analog of Eq. (7) the mass conservation equation becomes

$$\begin{aligned}
&= \frac{\alpha T}{C} \frac{\Delta s(t)}{L} + K \frac{\Delta P(t)}{L} + \frac{\Delta \Omega(t)}{L \Omega} \\
&= \frac{A}{L \Omega} \left[\frac{\rho_s}{\rho} \epsilon_s(t) + \frac{\rho_n}{\rho} \epsilon_n(t) \right] , \tag{8}
\end{aligned}$$

where

$$\Delta s(t) = \Delta s_1(t) - \Delta s_2(t)$$

$$\Omega = \frac{\Omega_1 \Omega_2}{\Omega_1 + \Omega_2}$$

$$\frac{\Delta \Omega(t)}{\Omega} = \frac{\Delta \Omega_1(t)}{\Omega_1} - \frac{\Delta \Omega_2(t)}{\Omega_2} .$$

The $\Delta \Omega(t)$ used here is not the change in the effective volume Ω ; $\Delta \Omega(t)/\Omega$ is an effective fractional volume change which is the difference between the fractional volume changes of each chamber.

The total entropy in each chamber can also be related to thermodynamic variables, and to the entropy introduced by normal fluid flow, by a possible heater-driver source, and by wall conduction between chambers. Equation (4) can be expanded to

$$\begin{aligned}
\Delta S_1(t) &= \Delta(s_1 M_1) = s \rho A \epsilon_n(t) + \frac{\Delta Q_1(t)}{T} \\
&= M_1 \Delta s_1(t) + s A \left[\rho_n \epsilon_n(t) + \rho_s \epsilon_s(t) \right] ,
\end{aligned}$$

with $\Delta Q_1(t)$ being the heat added to chamber 1. We can combine

this equation with a similar one for chamber 2 to get

$$\frac{\Delta s(t)}{l} + \frac{A}{2\Omega} s \frac{\rho_s}{\rho} [\xi_s(t) - \xi_n(t)] = \frac{\Delta Q(t)}{\rho l \Omega T}, \quad (9)$$

where $\Delta Q(t)$ is an effective heat input, defined by

$$\frac{\Delta Q(t)}{\Omega} = \frac{\Delta Q_1(t)}{\Omega_1} - \frac{\Delta Q_2(t)}{\Omega_2}.$$

Next we combine Eqs. (5) and (6) to get the expressions

$$\ddot{\xi}_s(t) = -\frac{1}{\rho} \frac{\Delta P(t)}{l} + s \frac{\Delta T(t)}{l} \quad (10)$$

$$\ddot{\xi}_n(t) = -\frac{1}{\rho} \frac{\Delta P(t)}{l} - s \frac{\rho_s}{\rho_n} \frac{\Delta T(t)}{l} - \frac{R}{\rho_n} \dot{\xi}_n(t). \quad (11)$$

Because we consider T to be a function of s and of P , $\Delta T(t)$ can be written as

$$\Delta T(t) = \frac{T}{C} [\Delta s(t) + \frac{\alpha}{\rho} \Delta P(t)] \quad (12)$$

If we assume a time dependence of $e^{i\omega t}$ for all the time varying quantities in Eqs. (10) and (11), and if we define ΔP , ΔT , ξ_s , and ξ_n as the amplitudes of the variables $\Delta P(t)$, $\Delta T(t)$, $\xi_s(t)$, and $\xi_n(t)$, we get the angular-frequency-dependent equations

$$\frac{1}{\rho} \left(1 - \frac{\alpha s T}{C} \right) \frac{\Delta P}{\ell} - \frac{s T}{C} \frac{\Delta s}{\ell} - \omega^2 \epsilon_s = 0 \quad (13)$$

$$\frac{1}{\rho} \left(1 + \frac{\rho_s}{\rho_n} \frac{\alpha s T}{C} \right) \frac{\Delta P}{\ell} + \frac{\rho_s}{\rho_n} \frac{s T}{C} \frac{\Delta s}{\ell} + \left(\frac{i \omega R}{\rho_n} - \omega^2 \right) \epsilon_n = 0 \quad (14)$$

3. Non-Driven Solution to Hydrodynamic Equations.

We first wish to solve these equations for the natural resonant frequency. It will be convenient to define $\epsilon = A/\ell \Omega$, which is a function of the system geometry, and

$$u_1^2 = 1/K \rho$$

$$u_2^2 = \frac{\rho_s}{\rho_n} \frac{T s^2}{C},$$

which are the squares of the first- and second-sound velocities in helium II. Our definition of the second-sound velocity uses the specific heat at constant pressure rather than the more conventional specific heat at constant volume C_V . With our definition the calculated second-sound velocity coincides more closely with the experimentally observed velocity, as corrections for thermal expansion (and consequent coupling of the first- and second-sound velocities) are, to first order, taken into account.

Then, if we assume the driving functions are zero ($\Delta\Omega = 0$ and $\Delta Q = 0$), we can solve the set of equations

$$\frac{\Delta s}{\ell} + s\epsilon \frac{\rho_s}{\rho} \epsilon_s - s\epsilon \frac{\rho_s}{\rho} \epsilon_n = \frac{\Delta Q \Omega}{\rho \ell \Omega T} \quad (15)$$

$$K \frac{\Delta P}{\ell} - \frac{\alpha T}{C} \frac{\Delta s}{\ell} - \epsilon \frac{\rho_s}{\rho} \epsilon_s - \epsilon \frac{\rho_n}{\rho} \epsilon_n = - \frac{\Delta \Omega}{\ell \Omega} \quad (16)$$

$$\frac{1}{\rho} \left(1 - \frac{\alpha s T}{C}\right) \frac{\Delta P}{\ell} - \frac{s T}{C} \frac{\Delta s}{\ell} - \omega^2 \epsilon_s = 0 \quad (17)$$

$$\frac{1}{\rho} \left(1 + \frac{\rho_s}{\rho_n} \frac{\alpha s T}{C}\right) \frac{\Delta P}{\ell} + \frac{\rho_s}{\rho_n} \frac{s T}{C} \frac{\Delta s}{\ell} + \left(\frac{i\omega R}{\rho_n} - \omega^2\right) \epsilon_n = 0 \quad (18)$$

for the resonant frequency and damping time by setting the determinant of the coefficients equal to zero. This operation gives us the following equation which determines the resonant frequency

$$\begin{aligned} \omega^4 - \epsilon [u_1^2 + u_2^2 + \frac{\rho_s}{\rho_n} u_1^2 \left(\frac{\alpha s T}{C}\right)^2] \omega^2 + \epsilon^2 u_1^2 u_2^2 \\ - \frac{i\omega R}{\rho_n} \left\{ \omega^2 - \epsilon \left[\frac{\rho_s}{\rho} u_1^2 \left(1 - \frac{\alpha s T}{C}\right)^2 + \frac{\rho_n}{\rho} u_2^2 \right] \right\} = 0. \end{aligned} \quad (19)$$

In the limit $R \rightarrow \infty$ (that is, totally clamped normal fluid) Eq. (19) has the solution

$$\omega_0^2 = \epsilon \left\{ \frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2 + \frac{\rho_s}{\rho} u_1^2 \left[\left(1 - \frac{\alpha s T}{C} \right)^2 - 1 \right] \right\} .$$

The quantity in curly brackets we will call the square of the fourth-sound velocity u_4 . It can be shown to be identical to the fourth-sound velocity defined by I. Rudnick and K. A. Shapiro.⁽¹⁷⁾ Even for our worst-case conditions (highest temperature at 2.15 K), dropping the term involving α introduces only a 1/2% error. Hence we neglect it and are left with the simple relation

$$\omega_0^2 = \epsilon u_4^2 \approx \epsilon \left(\frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2 \right) . \quad (20)$$

In order to take damping into account for small values of $\omega \rho_n / R$ (large but finite R) we can write Eq. (19) as

$$\omega^2 - \epsilon u_4^2 = \frac{\rho_n}{i\omega R} \left[\omega^4 - \epsilon (u_1^2 + u_2^2) \omega^2 + \epsilon^2 u_1^2 u_2^2 \right] .$$

Let ω equal $\omega_0 + i\delta$, where $i\delta$ represents the departure of from ω_0 as $\omega \rho_n / R$ increases from zero. $\text{Re } \delta$ thus measures the damping rate and $\text{Im } \delta$ the frequency shift. For small $\omega \rho_n / R$ δ will be small compared to ω_0 and to first order we may replace ω on the right by ω_0 and solve for δ .

After some algebra the expression for δ can be reduced to

$$\delta = \frac{\rho_n \epsilon}{2R} (u_1^2 + u_2^2 - u_4^2 - \frac{u_1^2 u_2^2}{u_4^2}) \quad (21)$$

We see that to first order δ is real, representing damping without frequency shift. At 2.15 K and for the 10 μm orifice, the ratio δ/ω_0 is less than 0.04. For lower temperatures and other superleaks the ratio is smaller. We will later use Eq. (21) to calculate the normal-fluid-viscosity contribution to the damping of the fourth-sound oscillations.

4. Response to a Drive.

From the set of Eqs. (15)-(18) we can solve for the temperature and pressure response to cell volume change (resulting from transducer expansions and contractions), and find

$$\Delta P = -\frac{1}{K} \frac{\Delta \Omega}{\Omega} \frac{\omega^2 - \epsilon \frac{\rho_n}{\rho} u_2^2}{\omega^2 - \epsilon u_4^2} \quad (22)$$

where terms of the order $\rho_n \omega/R$ have been neglected compared to unity.

Similarly we can get

$$\Delta T = \frac{sT}{C} \frac{\Delta \Omega}{\Omega} \frac{\epsilon \frac{\rho_s}{\rho} u_1^2}{\omega^2 - \epsilon u_4^2} \left(1 - \omega^2 \frac{\alpha}{\epsilon \frac{\rho_s}{\rho} s} \right) \quad (23)$$

and

$$\xi_s = -\frac{1}{l} \frac{\Delta\Omega}{\Omega} \frac{u_1^2}{\omega^2 - \epsilon u_1^2} \quad (24)$$

In Eq. (22) we have $\epsilon \frac{\rho_n}{\rho} u_2^2 < 40 \text{ sec}^{-2}$ at all temperatures. Thus for $f > 3 \text{ Hz}$ we can neglect $\epsilon \frac{\rho_n}{\rho} u_2^2$ compared to ω^2 . Then combining Eqs. (22) and (24) we have the relation

$$\Delta P = l \rho \omega^2 \xi_s = -i \rho \omega l v_s \quad (25)$$

As will be shown later, the superfluid critical velocity can be calculated by measuring $\Delta\Omega/\Omega$ and applying Eq. (24). Near the fourth-sound resonance, however, the denominator is small enough that terms of the order $\rho_n \omega/R$ cannot be neglected if this equation is to be applied. However, another technique for measuring the superfluid critical velocity is to measure the inertial pressure at the onset of dissipation. Eq. (25) allows us to calculate v_s directly, even at frequencies near the fourth-sound resonance.

We can calculate the temperature variations associated with the observed pressure variations and show that these temperature variations are too small to be observed with our bridge. The ratio of Eqs. (22) and (23) gives the expression

$$\frac{\Delta P}{\Delta T} = - \frac{C_P}{S_T} \frac{\omega^2}{\epsilon \frac{\rho_s}{\rho} (1 - \omega^2 \frac{\alpha}{\epsilon \frac{\rho_s}{\rho} s})}$$

Assuming a drive frequency of 3 Hz, an ϵ of $3.69 \times 10^{-5} \text{ cm}^{-2}$, and a ΔP of 10 dynes/cm^2 (the largest pressure difference observed) we would expect temperature differences of

$$\Delta T = \begin{cases} 1.7 \text{ } \mu\text{K} & \text{at } 1.1 \text{ K} \\ 0.14 \text{ } \mu\text{K} & \text{at } 2.15 \text{ K,} \end{cases}$$

corresponding to resistance changes of

$$\Delta R = \begin{cases} 1.7 \text{ ohms out of } 20 \text{ kohms at } 1.1 \text{ K} \\ 0.0021 \text{ ohms out of } 1.5 \text{ kohms at } 2.15 \text{ K,} \end{cases}$$

which are somewhat below our detection capability. With lock-in detection one might expect to see temperature variations of this magnitude; however, our commonly observed pressure amplitudes were much less than 10 dynes/cm^2 . Pressure amplitudes as large as 10 dynes/cm^2 would under most circumstances result in flow velocities well over the critical velocity.

5. Heat Conduction as a Loss Mechanism.

If we include in the heat-added term of Eq. (15) the heat

transferred from chamber 1 to chamber 2 by thermal conduction, the added contribution takes the form

$$\frac{d}{dt} \Delta Q_1(t) = - \frac{1}{R_T} \Delta T(t) = - \frac{d}{dt} \Delta Q_2(t) ,$$

where R_T is the thermal resistance of the wall. For a uniform wall of thickness L , area A_T , and thermal conductivity k , R_T would equal $L/A_T k$. This inclusion makes the entropy equation, Eq. (15), in the angular-frequency domain, become

$$\frac{\Delta S}{\ell} + \epsilon s \frac{\rho_s}{\rho} (\xi_s - \xi_n) = - \frac{k A_T}{i \omega \rho \ell \Omega T L} \Delta T = - \frac{1}{i \omega \rho \ell \Omega T R_T} \Delta T .$$

Expressing ΔT in terms of Δs and ΔP , the first equation of the set of Eqs. (15)-(18) becomes

$$(1 - iJ) \frac{\Delta S}{\ell} - i \frac{\alpha}{\rho} \frac{\Delta P}{\ell} + s \epsilon \frac{\rho_s}{\rho} \xi_s - s \epsilon \frac{\rho_s}{\rho} \xi_n = 0$$

with $J = 1 / \omega \ell \rho \Omega R_T$. Over our entire temperature range we have $J < 0.02$. The resonant frequency condition derived by setting the determinant of the coefficients equal to zero now becomes

$$\omega^4 - \epsilon [u_1^2 + u_2^2 + \frac{\rho_s}{\rho_n} u_1^2 (\frac{\alpha T}{C})^2] \omega^2 + \epsilon^2 u_1^2 u_2^2 - \frac{i\omega R}{\rho_n} [\omega^2 - \epsilon u_4^2]$$

$$-iJ((1 + \frac{\alpha^2 T}{C} u_1^2) \omega^4 - \epsilon u_1^2 \omega^2 - \frac{i\omega R}{\rho_n} [(1 + \frac{\alpha^2 T}{C} u_1^2) \omega^2 - \epsilon \frac{\rho_s}{\rho} u_1^2]) = 0 \quad (26)$$

This equation can be solved to zeroth-order, as above, to get

$\omega_0^2 = \epsilon u_4^2$. For an approximation beyond this, we assume $\omega = \omega_0 + i\delta$ and that $\rho_n \omega/R$ and J are small. This assumption gives us to first order

$$\frac{1}{\tau} \doteq \delta = \delta_1 + \delta_2, \quad (27)$$

where

$$\delta_1 = \frac{\rho_n \epsilon}{2R} (u_1^2 + u_2^2 - \frac{u_1^2 u_2^2}{u_4^2}) \quad (27a)$$

is the fluid viscosity damping term and

$$\delta_2 = \frac{1}{2R_T \rho_n C} [(1 + \frac{\alpha^2 T}{C} u_1^2) - \frac{\rho_s}{\rho} \frac{u_1^2}{u_4^2}] \quad (27b)$$

is the thermal conductivity damping term. The thermal resistance R_T in our case includes the parallel contribution of wall

conduction plus orifice substrate conduction, and each of these two terms includes a boundary resistance. We can write the expression as

$$\frac{1}{R_T} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4},$$

with each of the terms derived in an obvious manner from the following:

R_1) Brass wall resistance. The low-temperature thermal conductivity of brass is about $k = k_1 T$, with $k_1 = 7.5 \times 10^4$ erg/cm-sec-deg.⁽¹⁸⁾ Our cell wall dimensions are 0.635 cm thickness and 44 cm² area.

R_2) Kapitza resistance at the chamber wall. In the absence of known values of the boundary resistance for the He⁴-brass boundary, we use the Fairbank and Wilks value for He⁴-copper, $r_K = 4.5 \times 10^{-6} / T^2$ deg-cm²/(erg/sec).⁽¹⁹⁾

R_3) Nickel substrate resistance. The substrate area exposed to helium is 0.317 cm² and the thickness is 3.8×10^{-3} cm. The low-temperature conductivity of nickel is roughly $k = k_3 T$, where $k_3 = 4.3 \times 10^{-6}$ erg/cm-sec-deg.⁽²⁰⁾

R_4) Kapitza resistance of the substrate. As in the He⁴-brass case, the resistance for a He⁴-Ni boundary is unknown, and we assume the same coefficient as for a He⁴-Cu boundary. This term then dominates the internal resistance of the nickel

foil R_3 by four orders of magnitude, and thus R_3 can be neglected.

The calculated curve for the decay time derived from these damping terms for the 10 μm orifice is plotted in Fig. 16 along with the experimental results, and is discussed in Chapter V. For the 0.5 μm Nuclepore superleak the viscous contribution to the damping time should be two orders of magnitude smaller than for the 10 μm orifice; for the 0.1 μm Nuclepore, several orders of magnitude smaller. If there is no significant change in the thermal damping, the decay times should thus be much longer for the Nuclepore superleaks than for the orifice. However, no decay-time measurements were made using the Nuclepore superleaks.

Having discussed the hydrodynamic behavior of the system we now turn to a detailed description of the apparatus.

CHAPTER III.

APPARATUS DESCRIPTION.

1. Introduction.

The experimental cell consisted of two chambers separated by a wall into which was mounted the superleak under test--an orifice or a multipore membrane. Fluid was driven through the superleak by a pair of piezoelectric ceramic transducers, one in each chamber, driven out of phase in a volume-expansion mode.

Also in the wall was a pressure-sensing membrane, used as one plate of a parallel-plate capacitor. The membrane and an rf inductor, together with a back diode, were used to form an rf oscillator. Changes in the differential pressure between chambers resulting from fluid flow and volume change were manifest as changes in the frequency of the oscillator. The resulting fm signal was amplified atop the dewar at room temperature, then demodulated and displayed on an oscilloscope.

2. Mechanical Description--Cell.

The experimental cell, shown in Figs. 2 and 3, was rectangular in shape, 2.00 inches wide, 5.25 inches long, and 1.25 inches thick. It was machined from brass, with a 0.25 inch wall between the two chambers. Brass cover plates of minimum thickness 0.25 inches were sealed with indium wire gaskets to form a superleak-tight assembly. The chamber volumes were about 11 cm³ for the front chamber (chamber 1) and about 16 cm³ for the back chamber (chamber 2). The

central wall between chambers contained three holes into which were mounted three plugs containing the differential pressure transducer, the superleak under test, and the piezoelectric drivers. The latter plug, unlike the first two, was solid and merely permitted easy replacement of drivers. The pressure transducer and the piezoelectric drivers are discussed in the electrical section. The plugs were sealed into the walls with indium wire gaskets so that the only fluid connection between chambers was provided by the mounted superleak.

The superleak mounting was fabricated differently for the single orifice than for the Nuclepore membrane. The single orifice was an electroformed hole in a 0.375 inch diameter nickel disc 0.0015 inch thick.⁽²¹⁾ The disc was soft-soldered onto a brass mounting plug as shown in Fig. 4. The larger central hole in the brass piece constituted a hydrodynamic inertiance which was in series with the superleak inertiance but about two orders of magnitude smaller and therefore negligible compared to that of the superleak.

The Nuclepore membrane was pliant enough that a large unsupported area would have been undesirable, for if it flexed under pressure, part of the mass current would flow into this volume rather than through the superleak. To stiffen the membrane mount, a backing disc of 0.005 inch thick phosphor bronze was used, into which a central hole of 0.05 inch diameter was drilled. This disc was soft-soldered to a plug identical to that used for the single orifice. A one cm diameter Nuclepore filter was bonded to the disc

with household epoxy⁽²²⁾ using care that the 0.05 inch diameter central region was not covered with epoxy. Then with a fine pointed probe, epoxy was spread to cover the remaining area except for a small pre-determined area, chosen to yield a fourth-sound resonant frequency in the tens of hertz.

3. Mechanical Description--Cryogenics and Gas Handling.

An overall view of the lower end of the cryostat is shown in Fig. 3. The cell was supported below a He^4 pot which could be pumped to cool the cell to as low as 1.1 K. An OFHC copper thermal shield at the pot temperature surrounded the cell, and the pot and cell were thermally isolated from the 4.2 K bath of helium by being situated in a vacuum can. The heat leak to the pumped pot was small enough that the pot could be kept at the lowest temperature for more than three days before refilling was necessary.

The cell was filled by condensation of helium gas from a room-temperature bottle. A liquid-nitrogen-cooled trap of Linde 13X Molecular Sieve was used to reduce the chances of the fill capillaries becoming plugged by contamination. Separate fill lines existed for front and back chambers, with 0.219 inch o.d. thinwall stainless steel tubes passing from room temperature through the helium bath to the vacuum-can top flange. The tubes contained 0.125 inch diameter valve shafts. At the flange a needle valve was located in each line for sealing off the line, and below each valve a 0.005 inch i.d. stainless steel capillary continued 70 cm down to the cell. Each needle valve (Hoke stem type 1325M4B) consisted of a stainless

steel shaft with a $2\text{-}1/2^\circ$ taper, advanced by a 10-56 thread and sealed against a copper seat. Thirty-five cm below the flange each capillary was thermally grounded by soft-soldering 20 cm of its remaining length to an OFHC copper post which in turn was bolted and greased to the OFHC copper bottom of the helium pot. This left 15 cm of capillary between post and cell. In latter runs a 0.004 inch diameter tungsten wire was inserted in each capillary 12 cm above the post and 15 cm below the post. This modification reduced the heat leaks down the capillaries to the point where the pot and cell temperatures differed by only 15 mK.

The cell was usually filled by keeping the pot and cell below the lambda point. Pressure at the gas supply was kept above one atmosphere and the filling was interrupted periodically. If the cell were only partially filled, the pressure in the fill tube relaxed towards the vapor pressure of the liquid below the lambda point. When the cell was filled and the liquid level rose in the fill tubes to some point in the 4.2 K helium bath, the pressure relaxation stopped at atmospheric pressure.

Care was taken during filling and emptying the cell to keep the differential pressures small, to protect the transducer membrane. Front-to-back pressures of one atmosphere could probably easily be tolerated, but it is not known whether back-to-front pressures of this magnitude would leave the membrane undamaged. One membrane burst under excessive pressure.

Since our data were not taken at saturated vapor pressure, the lambda temperature needed to be corrected for pressure. With the helium level in the fill tubes at some point in the 4.2 K bath above the vacuum flange, the cell pressure was equal to atmospheric pressure plus the hydrostatic pressure of the helium in the fill tubes. The hydrostatic pressure of the helium standing in the fill tubes varied from run to run because the tubes were not always filled to the same level, but even for the worst case the total contribution to the pressure was much less than 1% of one atmosphere. Changes in atmospheric pressure, however, were $\pm 3\%$ over a period of time. The cell needle valves were closed only after reaching thermal equilibrium at whatever temperature data were being taken, so the bulk lambda temperature needed to be corrected for the atmospheric correction. From recent measurements of G. Ahlers the slope of the lambda line $\frac{\partial P}{\partial T}_\lambda$ is about 9 mK/atm,⁽²³⁾ so that our lambda-point temperature is taken to be $T_\lambda = 2.1632$ K, with an uncertainty of ± 0.3 mK, in the absence of any monitoring of atmospheric pressure.

4. Electrical Description--Pressure Transducer.

Detection of a pressure differential between the two cell chambers was accomplished by means of a capacitor microphone.

A 0.0005 inch thick Kapton⁽²⁴⁾ film, gold plated on one side, was bonded to a brass plug with household epoxy as shown in Fig. 5, with the gold layer out. The gold layer was electrically grounded

to the plug with a silver conducting cement⁽²⁵⁾ and constituted one plate of the parallel plate capacitor. Tests with a helium leak detector indicated that this system was leak-tight to helium gas at liquid helium temperatures. At room temperature there was some helium gas diffusion through the membrane.

The back capacitor plate was a brass piece mounted in threaded Teflon⁽²⁶⁾ and perforated for liquid communication with the back chamber. A 0.001 inch depression over most of the back plate allowed the membrane to flex in both directions with pressure changes. Membrane tension could be adjusted at room temperature by advancing the threaded Teflon mount in the brass plug threads.

The back capacitor plate was connected to the rest of the oscillator circuit in the vacuum space outside the cell by means of a 0.020 inch o.d. coaxial cable 10 cm long.⁽²⁷⁾ Superleak-tight seals were made with chalk⁽²⁸⁾-impregnated household epoxy, taking care to seal the dielectric so that no leaks could take place from the cell to the vacuum can down the space between inner and outer conductor.

The measured capacitance of the transducer C_m was about 50 pf, so that a parallel rf inductor of inductance $L = 4.7 \mu\text{H}$ gave an anti-resonant frequency f_0 of about 10 MHz.

A General Electric BN-4 back diode biased by a dc current of about 20 μA from atop the dewar provided a negative conductance for the oscillator circuit, shown in Fig. 7. The average power dissipated by the diode was about 3.5 μW . A capacitor $C_0 = 560 \text{ pf}$

shunted a 0.020 inch coaxial cable to room temperature and reduced the effect of possible variations in the capacitance of the output cable. It did, however, also reduce the rf signal strength. A trimmer capacitor C_1 of 1 to 3 pf adjusted the resonant frequency to near 10 MHz.

An rf preamp at the room-temperature end of the circuit amplified the millivolt-level signal to about 200 mV. A block diagram of the room-temperature electronics is shown in Fig. 8.

The measured sensitivity of the transducer $\Delta f / \Delta P$ was 60 Hz/(dyne/cm²) at liquid-helium temperature. The detection system was a Hewlett-Packard 5210A Frequency Meter whose output sensitivity to fm signals $V_d / \Delta f$ was equal to V_d^* / f^* , where V_d is the output voltage deviation from zero, Δf is the frequency deviation from the average rf frequency, f^* is the frequency range setting, and V_d^* is 1.0 volt. Since our pressure amplitudes were of the order of at most a few dynes/cm², the output voltages were of the order of 10 μ V with $f^* = 10$ MHz. Because the inherent noise of the discriminator circuit was of this order of magnitude, it was found necessary to down-convert the fm signal before demodulation in order to increase the discriminator output signal in relation to noise. For the down-conversion, capacitor C_1 was adjusted so that the oscillator frequency f_0 was within tens of kilohertz of 10 MHz. The 10 MHz standard frequency signal from a Hewlett-Packard 5243L Electronic Counter was then mixed with f_0 in a non-linear circuit, yielding a spectrum of signals, out of which all signals other

than the one at a few tens of kilohertz were easily filtered.

The frequency deviation with a signal present, $\Delta f(t)$, was unchanged by the introduction of the down-converting circuit, but since the carrier frequency was then a few tens of kilohertz rather than 10 MHz the discriminator output voltage was $\Delta f(t) \times 1.0 \text{ V/100 kHz}$ rather than $\Delta f(t) \times 1.0 \text{ V/10 MHz}$, an increase of two orders of magnitude, while the discriminator output noise was left unchanged.

5. Electrical Description--Driver.

Liquid helium was driven through the superleak by the volume expansion of piezoelectric ceramics in each cell. In the earliest run (10 μm orifice run A) a disc of Clevite PZT-4 lead zirconium titanate ceramic 0.5 inch diameter and 0.1 inch thick was used in each cell.⁽²⁹⁾ However, the fluid current was inadequate, and later a sandwich arrangement was made as follows. Four PZT-4 ceramic discs 0.75 inch in diameter and 0.025 inch thick, as shown in Fig. 6, were soldered together with 0.005 inch thick electrode tabs between layers to provide easy contact for connecting wires. The sandwiches were cemented to the mounting plug by Duco⁽²⁶⁾ cement with Teflon tape insulation. The drivers were wired so that when the same voltage was applied to each driver, one sandwich contracted and the other expanded.

Electrical leads into the cell from the vacuum can were #36 manganin wires through 0.0938 inch o.d. stainless steel tubes.

One end of the stainless tube was sealed with chalk-epoxy; the other was soft-soldered into the chamber. Driver connections between chambers were made by epoxying manganin wires into a 0.050 inch hole through the driver mounting plug with the same chalk-epoxy.

All electrical leads from the vacuum can to room temperature (except for an rf coaxial cable) were similar manganin wires. They were run in groups of ten through six thinwall stainless steel tubes, sealed as they entered the vacuum can by chalk-epoxy and at the room-temperature end by household epoxy. To anchor them thermally to the helium bath (and reduce the heat leak to the pot and cell) slots were cut in the stainless steel tubes to allow contact with the 4.2 K liquid helium above the vacuum can.

The ceramics were driven by a Hewlett-Packard 200CDR Wide Range Oscillator which was capable of delivering 60 volts peak-to-peak. When necessary for higher drive voltage, a transistor amplifier was used to increase the drive signal to 200 volts peak-to-peak.

6. Thermometry.

The temperature sensors in the apparatus were five Allen-Bradley Type TR 1/10 watt 33 ohm carbon resistors and one CryoCal germanium resistor. The germanium resistor and one carbon resistor were mounted in the back chamber of the cell, and one carbon resistor each was mounted in the front chamber, on the

outside of the cell, the outside of the helium pot, and in the 4.2 K helium bath. Resistances were measured with a three-wire ac Wheatstone bridge, usually at a frequency of 300 Hz.

Near each carbon resistor was a 500 ohm Evanohm wire resistor used as a heater. The pot and cell heaters were wound on OFHC copper posts which were bolted and greased to the pot and cell exteriors. The front and back chamber heaters were wound on OFHC copper tubes and placed in the chambers so that they had minimal contact with the cell walls. When good temperature stability was needed the off-balance signal from the bridge was fed to a phase-sensitive detector, amplified, and fed to one of the heater resistors. Feedback was such that a warming (or cooling) of the carbon resistor reduced (or increased) the quiescent voltage on the adjacent heater resistor and temperature excursions were considerably reduced. Typical quiescent heater powers for cell regulation were 0.1-0.25 mW.

Both carbon and germanium resistors were calibrated against He^4 vapor pressure several times by the following procedure. Helium exchange gas was added to the vacuum can (though only in the last calibration was enough added to form liquid in the can). The dewar bath was pumped with a high speed mechanical pump while its temperature was electronically regulated. When thermal equilibrium was established at a given temperature, as evidenced by unchanging resistance bridge readings, the vapor pressure of He^4 in the partially filled helium pot was measured with an oil and a

mercury manometer to 0.01 mm liquid height. Corrections were made for non-zero pressure on the high-vacuum arm of the manometer, for ambient temperature, and for the local value of g . Resistance and temperature data (1958 He⁴ scale) were fitted by least-squares to the expression

$$\ln (R/R_0) = A (T_0/T) + B + C (T/T_0) .$$

Values of A , B , and C are listed in Table 1 for $R_0 \equiv 1$ ohm and $T_0 \equiv 1$ K.

For calibration number 5, lambda-point measurements were also included in the data, with the lambda point being determined by the heat capacity maximum during slow heating of the cell exterior. Because all our measurements were made near one atmosphere (an experimental convenience) the temperature of the lambda point was corrected to $T_\lambda = 2.1632$ K.

CHAPTER IV.

MEASUREMENTS

1. Pressure Transducer and Piezoelectric Driver Calibration.

The magnitude of the superfluid velocity v_s in the superleak can be related to the magnitude of the piezoelectric drive voltage by the expression, derived from Eq. (24),

$$v_s = \frac{1}{\ell} \frac{\omega u_1^2}{\omega^2 - \omega_0^2} \left(\frac{1}{\Omega} \frac{\Delta\Omega}{V} \right) V = \frac{1}{\ell} \frac{\omega u_1^2}{\omega^2 - \omega_0^2} ZV, \quad (28)$$

where Eq. (28) defines Z , the effective fractional change in cell volume per unit voltage applied to the piezoelectric transducers, and where V is the amplitude of the drive voltage at angular frequency ω . (From here on we consider only the magnitudes of the amplitudes and omit any reference to phase relationships.) In order to measure Z , a property of the piezoelectric transducers and a quantity which must be known in order to use Eq. (28), it is useful to relate it to other quantities by the expression

$$Z = \frac{1}{\Omega} \frac{\Delta\Omega}{V} = \frac{(V_0/V)}{(V_0/\Delta f)(\Delta f/\Delta P)(\Omega \Delta P/\Delta\Omega)}, \quad (29)$$

where V_0 is the amplitude of the amplified discriminator output voltage. Z can then be determined by measuring or calculating each of the quantities in Eq. (29) under consistent conditions.

First, V_0/f is determined from the frequency meter setting and the gain of the following amplifiers and was equal to 1.00×10^{-3} volts/Hz. Second, the pressure sensitivity $\Delta f/\Delta P$ of the differential pressure transducer is obtained from a measurement of the shift of the rf oscillator frequency with differential applied pressure. Third, the quantity V_0/V , which we call the ac transfer ratio, must be measured directly. Last, the quantity $\Omega \Delta P/\Delta \Omega$ must be calculated under the same conditions for which V_0/V is measured.

Initial calibration of the pressure transducer sensitivity, using helium gas at 4.2 K, was made by replacing the superleak with a blank plug to isolate the two chambers so that steady differential pressures could be maintained.

Although the sensitivity calibration was done at zero frequency in contrast to the ac measurements of the other quantities used in Eqs. (28) and (29), the pressure sensitivity should be independent of frequency as long as the frequencies used were well under the lowest resonant frequency of the transducer membrane. With liquid helium in the cell the liquid and membrane formed a strongly coupled system whose lowest resonant frequency was about 650 Hz, so that by measuring the static pressure sensitivity we obtained a value valid throughout the range of frequencies at which measurements were made.

Using cell fill tubes large enough (0.031 inch o.d. thinwall tubing) that pressure equilibrium was obtained in a few seconds,

helium gas was admitted to one chamber of the cell and the rf oscillator frequency was measured as a function of cell differential pressure. Differential pressures of about 1000 dynes/cm² (a few torr) could be applied in both directions and still produce linear response. At these absolute pressures and temperatures of about 4.2 K we were assured that only helium gas, not liquid, was present in the cell. While absolute pressure was not carefully monitored, it appears that thermomolecular corrections to the cell pressure were well under 2% if the cell absolute pressure exceeded 1000 dynes/cm², as we believe to have been the case.⁽³⁰⁾

The pressure sensitivity $\Delta f/\Delta P$ at liquid helium temperatures stabilized to 60 Hz/(dynes/cm²) after several temperature cycles between room temperature and liquid helium temperature. In the last four thermal cycles the values of $\Delta f/\Delta P$ were within a 5% range. These measurements were completed in the early stages of the experiment, but indirect checks on the long-term stability of $\Delta f/\Delta P$ could be made throughout the experiment as discussed further on below and indicated a stable value through much of the work. However, late in the work the sensitivity jumped unexpectedly to 73 Hz/(dyne/cm²).

The ac transfer ratio V_o/V was measured in three different limits and $\Omega \Delta P/\Delta \Omega$ calculated for each of these limits in turn. In fact, each limit described below corresponds to no flow of either normal or superfluid between chambers. First, if the

superleak is replaced by a blank plug then ξ_s and ξ_n are identically zero, and Eq. (8) gives us

$$K \Delta P + \frac{\Delta \Omega}{\Omega} = 0 ,$$

so that Z becomes (again, neglecting phase factors)

$$Z = K \frac{\Delta P}{V} = K \frac{(V_0/V)}{(V_0/\Delta f)(\Delta f/\Delta P)} \quad (30)$$

Second, with the superleak present but with the temperature above the lambda point, where $\rho_s = 0$, the solution to Eqs. (15) to (18) is

$$\Delta P = \frac{1}{K} \left(\frac{1}{1 + \frac{\epsilon u^2}{\omega^2 - i\omega R/\rho}} \right) \frac{\Delta \Omega}{\Omega} .$$

Here the velocity of ordinary sound u has replaced $\tilde{u}_T$. For $f \gg 0.3$ Hz (which is the case for all our measurements) this expression reduces to

$$\Delta P = \frac{1}{K} \frac{\Delta \Omega}{\Omega} ,$$

so that the same expression for Z is obtained as for the case where $\xi_s = \xi_n = 0$.

Third, with the temperature below the lambda point but for

driving frequencies well above the fourth-sound resonance, $f \gg f_0$, Eq. (22) reduces to the same expression for ΔP as above, and we get the same expression for Z as in the other two limits, Eq. (30). Thus when the ac transfer ratio is measured in one of the three limits discussed above, we may use to determine the relation obtained by combining Eqs. (28) and (30)

$$V_s = \frac{(V_0/V)}{(V_0/\Delta f)(\Delta f/\Delta P)} \frac{1}{\rho l} \frac{\omega}{\omega^2 - \omega_0^2} V. \quad (31)$$

In Eq. (30) Z is not expected to change with time, since it is determined by the piezoelectric coefficients. Nor was $V_0/\Delta f$ subject to change. Hence any changes in $\Delta f/\Delta P$ should be reflected in V_0/V , and thus regular checks of V_0/V should provide us an easy method of monitoring the long term stability of $\Delta f/\Delta P$ in the absence of direct measurements.

The ac transfer ratio was measured periodically during experimental runs in one or another of the three limiting cases just described. Except for the last run, the value averaged to 10.8 mV/V and was reproducible to $\pm 15\%$. The temperature dependence was not systematically analyzed and the temperature variation of the compressibility K could account for a large part of the variation of the ac transfer ratio. In the last run the value of the ac transfer ratio increased to 15.2 mV/V. The cause of the change is unknown: the cell was not opened between the last

two runs so that there was no chance of damage by mechanical disturbance of the membrane. Nevertheless, the increase is believed to be the result of an increase in pressure transducer sensitivity, since a transducer calibration after the last run showed a pressure sensitivity of $73 \text{ Hz}/(\text{dynes}/\text{cm}^2)$ a shift from its earlier value of $60 \text{ Hz}/(\text{dynes}/\text{cm}^2)$. While this change in $\Delta f/\Delta P$ does not entirely account for the change in V_0/V , the last pressure sensitivity calibration was made several months after the last run and it cannot be ruled out that some sort of aging occurred in this time.

From the above measurements we may obtain an experimental value for Z . The value of $Z = 2.25 \times 10^{-9}/\text{volt}$ calculated from Eq. (29) was roughly the value predicted from extrapolations of the manufacturer's incomplete low temperature data on the piezoelectric coefficients.

The derivations made from Section 3 of Chapter II up to this point assume the fluid is the only compliant part of the system and that the cell walls are rigid. In practice one should be concerned with the compliance of the pressure transducer, the piezoelectric driver, the orifice substrate, and the remaining portions of the cell walls. The latter three have been calculated and are small enough to be neglected.

The microphone compliance is harder to estimate because the restoring force is partly due to membrane tension and partly to the shear modulus of the cold Kapton. Experimental evidence

concerning whether the membrane should be considered "stiff" or "flexible" is conflicting.

For a flexible membrane we would need to introduce differential-pressure-dependent chamber volume changes $\Delta\Omega_1 = \Delta\Omega^m$ and $\Delta\Omega_2 = -\Delta\Omega^m$ which are proportional to ΔP and are conveniently written $\Delta\Omega_1 = K \Omega^m \Delta P$ and $\Delta\Omega_2 = -K \Omega^m \Delta P$ where Ω^m is a constant which we call the effective volume of the microphone-membrane. (Note that with this definition the microphone effective volume is not the volume of fluid displaced as the membrane flexes, $\Delta\Omega^m$, but is the volume of fluid which experiences the change $\Delta\Omega^m$ when pressure ΔP is applied.) Introducing these terms explicitly into Eq. (7) and its chamber 2 counterpart, and ignoring the temperature dependence of Ω^m , we can reduce the two-fluid equations to a set of equations whose only change from Eqs. (15)-(18) is the replacement of K with $K \frac{\Omega + \Omega^m}{\Omega}$ and thus u_1^2 with $\frac{\Omega}{\Omega + \Omega^m} u_1^2$.

We can then write the equations for pressure and displacement response

$$\Delta P \approx \frac{1}{K} \frac{\Delta\Omega}{\Omega + \Omega^m} \frac{\omega^2}{\omega^2 - \omega_0'^2}$$

and

$$\xi_s \approx \frac{1}{\ell} \frac{\Delta\Omega}{\Omega + \Omega^m} \frac{u_1^2}{\omega^2 - \omega_0'^2}.$$

Here we have defined $\omega_0'^2 = \epsilon' \frac{\rho_s}{\rho} u_1^2 + \epsilon \frac{\rho_n}{\rho} u_2^2$, where $\epsilon' = A/\ell(\Omega + \Omega^m)$, so that if $\Omega^m \leq \Omega$ holds, we can use the

approximation $\omega_0'^2 = \epsilon' \frac{\rho_s}{\rho} u_1^2 = \left(\frac{\Omega}{\Omega + \Omega^m} \right) \omega_0^2$, except very near the lambda point. The effect, then, is to change the pressure response by a factor of $\Omega/(\Omega + \Omega^m)$, and to change the fourth-sound resonance frequency, if $\Omega^m \lesssim \Omega$, by a factor $\Omega/(\Omega + \Omega^m)$.

With regard to the determination of v_s it is not hard to see that Eq. (31) is unchanged by the presence of membrane flexibility save for the replacement of ω_0^2 by $\omega_0'^2$. For with non-zero Ω^m Eq. (28) becomes

$$v_s = \frac{1}{\ell} \frac{\Omega}{\Omega + \Omega^m} \frac{\omega u_1^2}{\omega^2 - \omega_0'^2} \quad ZV \quad (32)$$

and Eq. (30) becomes

$$Z = K \frac{\Omega + \Omega^m}{\Omega} \frac{(V_0/V)}{(V_0/\Delta f)(\Delta f/\Delta P)} \quad (33)$$

When these two equations are combined, the terms containing Ω^m explicitly cancel each other. (The identical cancellation also occurs for the ΔP and ΔT calculations.) Since we measure the resonant frequency as part of every superfluid critical velocity measurement, the effect of the non-zero Ω^m on the critical velocity calculation need not concern us further.

In the interest of a more complete understanding of the apparatus behavior, however, let us try to estimate the size of Ω^m . We examine three different measurements and find that the evidence is conflicting.

One expects that for a stiff membrane ($\Omega^m \ll \Omega$) the approximately degenerate lowest two first-sound modes would occur roughly at the frequency for which the half-wavelength is the length of the cell, as indicated in Fig. 10(a). For our cell dimensions this would be for $f \approx 1100$ Hz. However, the lowest first-sound mode measured was $f \approx 650$ Hz, as if the longitudinal wave were as shown in Fig. 10(b). This mode would appear for a membrane flexible enough that it contributed negligible restoring force ($\Omega^m \gg \Omega$).

On the other hand, the experimental values of ϵ' (discussed in Chapter V and tabulated in Table 3) agree with calculated values of ϵ to within constants of order of magnitude one. The values of ϵ , calculated assuming no microphone effective volume contribution, are in fact all somewhat larger than the experimental values. This discrepancy, at least for the two Nuclepore membranes and the $10\text{ }\mu\text{m}$ orifice run A, could be reconciled by invoking a microphone volume which is somewhat smaller than Ω , but not negligible. For the $10\text{ }\mu\text{m}$ orifice runs B and C, other geometry changes (discussed in Chapter V, section 2.D) have complicated this analysis. It must be pointed out, however, that the areas open to fluid flow that we have used for the Nuclepore superleaks are uncertain to within a factor possibly as high as 2, and that this area uncertainty could thus explain the discrepancy between the measured ϵ' and the calculated ϵ . Thus, from the resonant frequency data we see that the value of Ω^m , if not negligible, is at most comparable to Ω .

Finally the substantial agreement of the value of Z calculated from Eq. (30) with the Z calculated from the manufacturer's data implies that we have $\Omega^m \ll \Omega$.

The question of membrane compliance is one which needs further investigation.

Up to now we have been discussing the determination of v_s from measurements of V . It is also possible to determine v_s from measurements of ΔP making use of Eq. (25). Since in actuality we measure V_0 rather than ΔP itself the relevant formula connecting the magnitudes of the amplitudes is

$$v_s = \frac{1}{\rho l \omega (V_0 / \Delta f) (\Delta f / \Delta P)} V_0 \quad (34)$$

There is a theoretical advantage to this approach in that neither the ac transfer ratio nor $\Omega \Delta P / \Delta \Omega$ enter the formula.

In practice, however, because of the noise present in the signal V_0 , the determinations of v_s by this approach are not as reliable as those obtained from measurements of V . Nevertheless, on one occasion superfluid critical velocities calculated from drive voltages and those calculated from the pressure response were found to agree to within 15% despite noise limitations.

2. Noise in the Pressure Sensing System.

Noise in the pressure sensing system arose from several

sources. The principal source was the electronic noise generated in the first stage of the active filter in the frequency discriminator output. This band-pass filter was necessary to remove the i.f. carrier signal. The noise generated in this stage could be reduced by lowering the upper cutoff frequency. Cutoffs of 100 Hz to 1000 Hz were used, most commonly 500 Hz. With this bandwidth, the amplified discriminator output had about a 10 mV rms noise level. The lower cutoff frequency of 3 Hz was fixed by the discriminator and following amplifiers.

The pressure sensing system is referred to as being used in the wide-band mode when it is used with the 500 Hz bandwidth. At certain other times the discriminator output was measured with a tuned amplifier or a lock-in amplifier.

A second source of noise was the acoustic pickup from boiling in the liquid nitrogen outer dewar bath. The frequency spectrum of the noise from the bursts of bubbles was rich enough in the high frequencies to excite the lowest first-sound mode of the coupled system of cell and membrane. The noise contributed by this 650 Hz resonance to the pressure signal could be reduced by lowering the cutoff frequency of the low-pass filter, but this did not reduce its effect in accelerating the superfluid in the superleak, an effect which may have seriously limited our ability to make observations near the lambda transition, where critical velocities were low.

A technique was tried which reduced the large bursts of

liquid nitrogen boiling in a test dewar. A 0.125 inch diameter tube ending in a 0.020 inch i.d. capillary about 5 cm long was placed in the dewar with the capillary near the bottom. When the tube was pumped with a mechanical pump the bath was significantly quieted. The quieting occurred for the following reason. Heat incident from room temperature warms the nitrogen in the lower part of the dewar. For large enough heat influx, convection and conduction are insufficient to transport the heat to the free surface to permit evaporation to cool the liquid. Instead the temperature of the liquid in the lower part of the dewar tends to rise above that of the surface until the nitrogen is vaporized in the lower part and boiling results. By letting a controlled quantity of fluid in this region evaporate into an evacuated tube the heat is removed from the dewar without violence. However, the same technique used in our research dewar with several capillary lengths did not noticeably change the boiling. We were unable to determine what went wrong.

The pressure transducer, being a sensitive microphone, also picked up acoustic room noises. These were sometimes shielded out by assembling an acoustic quiet room around the dewar stand.

3. Frequency Measurements.

The resonant frequency of the fourth-sound Helmholtz resonance was measured for each of the three superleaks at various temperatures. The resonant frequency of the system was important

in verifying the fourth-sound frequency relation, $\omega_0'^2 = \frac{\rho_s}{\rho} \varepsilon' u_1^2$, in attempting to study ρ_s suppression in small pores, and also in determining the fluid velocity in the superleak using the resonant-frequency-dependent Eq. (31).

It was found that the most convenient measurement of resonant frequency was made during a period of natural decay after excitation. The output of the signal generator which drove the ceramic transducer was also fed to the horizontal input of an oscilloscope. The vertical input was the usual pressure signal from the frequency discriminator. The drivers were pulsed on momentarily (sometimes above the critical velocity, sometimes below, with no distinguishable results), and the decaying pressure oscillations were observed. With the driver frequency near the natural resonance of the cell a slowly rotating and decaying Lissajous figure was seen, and the generator frequency was adjusted until the pattern appeared stationary except for the decaying vertical amplitude. The period of the driver signal was then measured with a digital timer. In practice there was always noise on the pressure signal which would often instantaneously disrupt the phase. This made it hard to determine when the pattern should be considered stationary. The error in the frequency was defined as the change necessary to produce a perceptible tumbling of the Lissajous figure. For lower temperatures this was usually less than $\pm 1\%$ error, often as good as $\pm 0.2\%$. At higher temperatures and lower critical velocities the noise was a larger fraction of the observed pressure

signal, and errors of $\pm 10\%$ were recorded.

Near the lambda point, pressure amplitudes for which one could observe natural decay became very small, and for sufficiently small $T_\lambda - T$ one could not follow the oscillation above the noise.

Another effect which limited our ability to make resonant frequency measurements on the single orifice near the lambda point was what appeared to be "plugging" of the orifice. This is discussed further in Section 2.8 of the next chapter.

Measurements of amplitude and phase response were made using a lock-in amplifier by applying a small steady drive to the system. The resonant frequency of the system was determined from the frequency of maximum amplitude and from the 90° phase shift point. This procedure for determining ω_0' turned out somewhat less satisfactory than the natural-decay method for the following reasons. For $v_s \ll v_{s,c}$ there is a change of phase of 180° in the pressure response between $f \ll f_0$ and $f \gg f_0$ as predicted by Eq. (24). At $f = f_0$ the phase shift for $v_s \ll v_{s,c}$ should be 90° , in accord with Eq. (24) generalized to include normal fluid losses. Because an additional phase shift is introduced as v_s is increased to the point where superfluid dissipation sets in, the apparent resonant frequency is strongly drive-dependent and measurements must be made at very small drive amplitudes.

To reduce the effect of random noise at these low drives we recorded our signals on a Nuclear Data Enhancetron 1024 digital

signal averager, but even with averages of several minutes per point on the resonance curve the accuracy was not an obvious improvement over the direct measurement above.

For the 10 μm orifice run A the decay time of the fourth-sound resonance was measured as a function of temperature. This measurement was made by driving the system at resonance for a fixed number of cycles, then recording the decaying envelope of oscillations on the signal averager. Photographs were made of the oscilloscope display and the logarithmic decrement calculated from these.

4. Critical Velocity Measurements.

A. Continuous Onset of Loss.

Before discussing the procedure used in the bulk of the superfluid critical velocity measurements, we will mention the measurements on the 0.1 μm Nuclepore that demonstrate the continuous onset of loss as a function of superfluid velocity associated with intrinsic critical velocity effects.

Equation (25) shows the 90° phase difference between differential pressure and fluid velocity, for velocities sufficiently small that we can neglect superfluid losses. With increasing drive voltages we expect that losses eventually set in, and necessarily a component of pressure appears that is in phase with the drive velocity. Measurements of this resistive pressure were

made in three ways.

First, measurements were made of the resistive pressure by oscilloscope observations of the pressure at the $V = 0$ intercept. Second, pressure signals were recorded on a signal averager, the inertial signal was subtracted electronically, and the amplitude of the remaining signal measured. Both of these procedures were not able to give data over a wide range of pressure amplitude because of noise in the wide-band pressure detection system. We were, however, able to get some data concerning the harmonic content of the pressure response by analyzing photographs (such as are shown in Fig. 19) of the response for two values of drive amplitude. With the exception of components at $2f$ and $3f$, both of which had amplitudes which were 6% of the amplitude of the fundamental frequency f , all harmonics had amplitudes less than 2% of the fundamental. While the presence of an even harmonic signal at $2f$ was unexpected, it can be explained as an effect of superleak asymmetry which results in slightly differing superfluid critical velocities for the two flow directions. Because of the low harmonic content of the pressure signal, it was thought that an adequate measure of the pressure response could be obtained by measuring the in-phase and quadrature fundamental-frequency signals with a lock-in amplifier. The use of a lock-in amplifier allowed response measurements to be made over a much wider range of pressure than could be made with direct oscilloscope observations. At low drive voltages the phase of the amplifier was set so that the

in-phase output was maximized and there was no output in the quadrature channel. Then both in-phase and quadrature pressure responses were measured as functions of the drive voltage.

B. Defining a Critical Velocity.

As previously mentioned, the continuous onset of a resistive pressure across a superleak makes the concept of a critical velocity imprecise, at least for our intrinsic critical velocity measurements. We have made measurements on three different superleaks, defining the "critical" velocity for the 0.1 μm Nuclepore, and for the 10 μm orifice runs B and C as the velocity above which resistive pressure is detectable above the noise of the pressure sensing system operating in the wide-band mode. This noise, measured in pressure units, is about 0.3 dynes/cm² rms.

For the 0.5 μm Nuclepore this same criterion for defining the critical velocity was used for some of the measurements, but more frequently the response was observed with a tuned amplifier, and the critical velocity was defined as the velocity at which a 5-15° phase shift occurred between the resultant pressure and the drive voltage. Fifteen degrees corresponds to a resistive pressure one-fourth of the magnitude of the inertial pressure. One would think that such a definition would result in a measured critical velocity which was frequency dependent (since the inertial pressure itself varies with frequency), but such a frequency dependence was

not found. Furthermore, there was no obvious discrepancy between the critical velocities of the 0.5 μm Nuclepore determined with the two criteria. This agreement is apparently a consequence of the very rapid rise of resistive pressure with superfluid velocity.

Errors in the critical voltage (the voltage which generates the critical velocity) were assigned as follows. A minimum value of the critical voltage was recorded at which there was no clear evidence of a phase shift, or alternatively of a pressure signal above noise. A maximum value of the critical voltage was recorded at which there was clearly a phase shift (of the order of 15°), or alternatively a significant signal above noise. The average of these extremes was considered the critical voltage, with half the difference considered as the error.

CHAPTER V.

RESULTS AND CONCLUSIONS.

1. Fourth-Sound Resonant Frequency.

The most direct and straight-forward measurement with our apparatus is the fourth-sound resonant frequency determination. Equation (20) corrected for effective microphone volume predicts that the frequency should vary with temperature as the fourth-sound velocity does, with $f_0'^2$ replaced by f_0^2 hereafter

$$f_0^2 = \frac{\epsilon'}{(2\pi)^2} u_4^2 .$$

Rearranging terms in this expression we have

$$\frac{f_0^2}{u_1^2} = \left(\frac{\epsilon'}{(2\pi)^2} \right) \left(\frac{\rho_s}{\rho} \right) \left(1 + \frac{\rho_n}{\rho_s} \frac{u_2^2}{u_1^2} \right) .$$

On the right hand side the three factors are, respectively, 1) temperature independent, 2) strongly temperature dependent, and 3) very weakly temperature dependent. The last factor attains its highest value of 1.015 at the highest temperature for which we have data, and drops rapidly to unity as the temperature is lowered. Since in this higher temperature region our frequency measurements had errors of perhaps 10%, the resulting 20% error in the square of the frequency makes it reasonable to approximate the last factor by unity. To a good approximation we have then

$$\frac{f_0^2}{u_1^2} = \frac{\epsilon'}{(2\pi)^2} \frac{\rho_s}{\rho}$$

The results of the resonance measurements are displayed in the form of plots of f_0^2/u_1^2 vs. ρ_s/ρ in Figs. 11 through 15. Superfluid density values were interpolated from the data of Tough, McCormick and Dash for $\rho_s/\rho > 0.5$.⁽³¹⁾ At higher temperature, that is, for $\rho_s/\rho < 0.5$, it is necessary to take account of the lambda-point shift (since our measurements were made at one atmosphere rather than saturated vapor pressure). Here we used the expression

$$\frac{\rho_s}{\rho} = 1.438 (2.1632 \text{ K} - T)^{2/3}$$

The temperature dependence assumed here is the temperature dependence found by Clow and Reppy with the lambda temperature changed to that at atmospheric pressure.⁽³²⁾ Interpolation of more recent data of R. H. Romer and R. J. Duffy for one atmosphere pressure indicates that the coefficient 1.438 is still valid.⁽³³⁾ The first-sound velocity u_1 is calculated for each temperature from an analytic interpolation of data of Vignos and Fairbank⁽³⁴⁾ which is

$$u_1(T, P=1 \text{ atm}) = 24402 \text{ cm/sec} - (12.0845 \text{ cm/sec}) \times \exp(4.4242 \text{ K}^{-1}(T - 1 \text{ K})).$$

Assuming Mamaladze's description of ρ_s suppression in small pores, and the consequent lambda-point depression,

$$\Delta T_\lambda = 1.7 \times 10^{-11} d^{-3/2} K \cdot \text{cm}^{3/2},$$

our smallest pores should exhibit an apparent lambda-point shift of 1.5 mK. (35)

In each plot of f_0^2/u_1^2 vs. ρ_s/ρ the data fit a straight line well, except for a slight bend for small values of ρ_s/ρ , indicating a possible lambda-point shift. In order to determine the shift we fit the data for f_0^2/u_1^2 to $B \rho_s^P/\rho$ using the function

$$\frac{\rho_s^P}{\rho} = 1.438 (T_\lambda^P - T)^{2/3} \quad (35)$$

for $\rho_s < 0.5$. Here the effective average superfluid density in the superleak pores ρ_s^P/ρ is distinguished from the bulk superfluid density. Values of B and T_λ^P which gave the best fits are shown in Table 3. In the 0.1 μm Nuclepore membrane, where we would expect the strongest ρ_s suppression, the lambda-point depression is only 0.4 mK. This depression is outside the error limits of the expected 1.5 mK depression. The shift for the 0.5 μm Nuclepore, 0.1 mK, is within experimental error of the expected shift. The 10 μm run A data did not extend close enough to the lambda point to determine the shift in this way. The source of the large shift indicated by the 10 μm run C data is unknown.

Extrapolations of the plots of Figs. 11 through 15 were made

to $T = 0$ and the values of $\epsilon' = \omega_0^2 / u_1^2$ were determined.

Calculated values of $\epsilon = A / \lambda \Omega$ for comparison with the experimental values of ϵ' were determined in the following way. Chamber volumes were estimated by graphical integration and by geometric calculations. Superleak lengths were obtained from manufacturer's data and substantiated by substrate thickness measurements. The effective length of the pores was used, in which the actual length is corrected for hydrodynamic end effects according to the relation

$$\lambda = \lambda_{\text{actual}} + \frac{16a}{3\pi} ,$$

where a is the pore radius.⁽³⁶⁾ The flow area for the 10 μm orifice was calculated from the orifice diameter determined by optical diffraction using a He-Ne laser. This measured diameter was 13.0 μm , significantly larger than the nominal diameter. Flow areas for the Nuclepore membranes were found by assuming the pore size given by the manufacturer and then multiplying by the number of pores exposed. The latter was found from the manufacturer's stated pore density and a crude microscopic determination of the open area not covered by epoxy.

For the two Nuclepore membranes the differences between ϵ and ϵ' are nearly identical, the experimental values of ϵ' being 1.86 and 1.89 times smaller than the calculated values of ϵ . For the 10 μm orifice run A the experimental ϵ' is 1.11 times smaller than the calculated ϵ . In runs B and C the experimental values

of ϵ' were 2.38 and 5.22 times smaller. In both these runs a piezoelectric ceramic was placed in close proximity to the orifice, and this placement increased the effective path length of the fluid flow. From run B to run C no changes were made in the apparatus and the source of the resonant frequency change is unknown, but might be attributable to partial plugging of the orifice area by nitrogen ice. The experimental values of ϵ' will be used in the next section to calculate superfluid critical velocities.

For run A the time constant for the decay of the fourth-sound resonance was measured as a function of temperature. The results are plotted in Fig. 16 along with the values expected from Eq. (27). The measured decay times at lower temperatures were considerably shorter than the calculated ones, while the high temperature values are longer by as much as a factor of two. The contribution of δ_2 to the damping at any of the temperatures in our range was nearly negligible. At low temperatures the more rapid decay than we predict may be the result of some other loss mechanism than we have considered. No set of plausible deviations of λ , A , and Ω from their assumed values was consistent with the error in τ for the high temperature region and at the same time consistent with other measured parameters such as ϵ' . Hence we are unable to explain the discrepancy in τ of a factor of two in this region.

2. Critical Velocities.

A. Resistive Pressure Onset.

Inertial and resistive pressures across the superleak were measured with a lock-in amplifier for the 0.1 μm Nuclepore at several temperatures. From evidence mentioned in Chapter IV, Section 4.A we feel that this is a good measure of the fundamental fourier component for the resistive pressure. Figures 17 and 18 show the inertial pressure and resistive pressures for 1.167 K and 20 Hz drive, and for 2.104 K and 9 Hz drive, both of which frequencies were well below the fourth-sound resonance.

The inertial pressures for each case are linearly proportional to drive voltages. The rapid increase of the resistive pressure with increasing drive follows a curve $P_r = A V^\alpha$ for the region where $P_r \ll P_f$. The exponents in the two cases are $\alpha = 5.3$ for the low-temperature example and $\alpha = 3.9$ for the high temperature one. This dependence was not exhaustively investigated. Similar curves at other temperatures and drive frequencies often gave much smaller exponential dependences.

The low-temperature and high-temperature cases illustrated are in the temperature ranges which are shown below to be in the extrinsic and intrinsic critical velocity regions, respectively. There is no clear difference in the behavior except possibly for the differing exponents.

Since the superfluid drive velocity varies sinusoidally with time, we cannot a priori apply the formula for the static case,

$$\Delta P = P_0 e^{-v_0/v_s},$$

directly to our ac experiment. Let us assume, however, a drive $v_s = v_1 \sin \omega t$ and, further, assume that the pressure is instantaneously given by Eq. (2), where v_s is then the instantaneous superfluid velocity. We make use of Notarys' experimental data (although his pore size was 0.08 μm , slightly smaller than our 0.1 μm) to get values of P_0 and v_0 which gave us the instantaneous pressure drop

$$\Delta P(t) = 2 \times 10^8 \text{ (dynes/cm}^2\text{)} \exp[-930 \text{ (cm/sec)}/v_s(t)]$$

at $T = 2.104 \text{ K}$.

Then by numerical integration we calculate that the amplitude of the fourier component at angular-frequency ω is

$$\begin{aligned} P_1 &= \frac{2}{T} \int_0^T \Delta P(t) \sin \omega t \, dt \\ &= \frac{\omega}{\pi} P_0 \int_0^{2\pi/\omega} \exp(-v_0/v_1 \sin \omega t) \sin \omega t \, dt. \end{aligned}$$

Since by experimental observation we found that the fundamental component was much larger than the other components, and since the fourier analysis gives us amplitudes of the higher harmonics which are also considerably smaller than the fundamental, we neglect all but the fundamental component in this analysis.

The results of this calculation are plotted in Fig. 18. The resistive pressure found from this calculation increases much faster with increasing superfluid velocity than that found from our experimental results, which are also shown in Fig. 18. The reasons for this discrepancy are not understood, although with the strongly non-linear relationship between pressure and superfluid velocity any departure from the assumed instantaneous response of $\Delta P(t)$ to $v_s(t)$ would result in a very complex convolution integral.

It is interesting to note that in doing the fourier analysis of $\Delta P(t)$ the amplitude of the fundamental component of the pressure response has been found to have very closely the same functional dependence on the amplitude of the drive as in the static case. This behavior is apparently the result of the extremely rapid dependence of ΔP on v_s and the consequent weighting in the integral of only the very largest values of superfluid velocity. As a result, the amplitude of the fundamental component of the pressure response as calculated from the Notarys theory goes as

$$P_r \sim e^{-v_0/v_1}$$

Although the plot of this function appears to be a straight line on our log-log graph using the Notarys parameters, it is impossible to use this functional form, varying the v_0 on the P_0 , to fit our data.

B. Drive-Frequency Dependence.

Before discussing the temperature and pore-size dependence of the critical velocity we will present the results of frequency dependence studies. The primary interest of this discussion concerns the validity of our hydrodynamic calculations of Chapter II.

For all the superleaks except the 10 μm orifice run A the critical velocity was measured for a range of drive frequencies, often above as well as below the fourth-sound resonance. (Run A was not included because of the inadequate fluid drive current away from the fourth-sound resonance.) The range of frequencies for a given temperature was as great as a factor of five from lowest to highest, but more often was a factor of two or three. The extremes of all the data were 2.5 to 80 Hz. Because of the highly frequency-dependent response of this resonant system, drive voltages needed for critical velocities ranged over three orders of magnitude.

For cases of high resonant frequency, and for sufficiently low drive frequencies, voltages were required in excess of the 200 volts available. Frequency measurements above resonance were less accurate because resistive pressure drops in that region were smaller in comparison to inertial pressures. These two factors restricted the usable frequency range in which measurements could be made.

The photographs reproduced in Fig. 19 show the oscilloscope patterns typically observed when differing drive voltages and drive

frequencies were used. The drive frequencies in the three illustrations are 40 Hz, 55 Hz, and 75 Hz. The more distinct change of oscilloscope pattern of the 40 Hz example as drive voltage is increased indicates the desirability of making measurements far below the fourth-sound resonant frequency (about 65 Hz in this case).

Superfluid critical velocities could also be found by gradually increasing the piezoelectric drive voltage and measuring the pressure at which an out-of-phase signal first appears. One would then use Eq. (25) to get the fluid velocity equation

$$v_{s,c} = \frac{(\Delta P / \Delta f)(\Delta f / \Delta V_0)}{\omega \rho l} V_{0,c} \quad (36)$$

In practice, because of the noise signal, the results are not as reliable as those from measuring voltages where the resistive pressure first appears. Nevertheless the velocities calculated from drive voltages and those calculated from the pressure response for these photographs agree to within 15% despite noise limitations.

There is a theoretical advantage to making critical pressure response rather than critical voltage measurements. The critical velocity for each drive frequency is related to the critical drive voltage V_c by the equation

$$v_{s,c} = \frac{\Delta V_0 / \Delta V}{\left(\frac{\Delta V_0}{\Delta f} \right) \left(\frac{\Delta f}{\Delta P} \right) \rho l} \frac{\omega}{\omega^2 - \omega_0^2} V_c \quad (37)$$

But when measuring critical pressure by measuring the output voltages at which the out-of-phase pressures first appear, using Eq. (36), we do not need to use the ac transfer ratio $\Delta V_o / \Delta V$. In practice, however, the ac transfer ratio would be measured anyway to monitor possible long term changes of the pressure sensitivity.

For the 10 μm orifice runs B and C, and for the 0.5 μm Nuclepore membrane the superfluid critical velocity showed no distinct frequency dependence. Figures 20 and 21 show the critical velocity of the 0.5 μm Nuclepore and the 10 μm run B as a function of frequency for several temperatures.

This independence of frequency seems to imply that the time for nucleation of the loss process is much less than tens of milliseconds, which was the period of the drive cycle. The result that the critical velocities calculated from Eq.(37) are independent of frequency gives us a degree of confidence in our hydrodynamic derivation.

However, the critical velocity measured for the 0.1 μm Nuclepore membrane appeared to depend on frequency and approached zero as the drive frequency approached the fourth-sound resonant frequency. The critical velocity could be fit well to the equation

$$v_{s,c} = \tilde{v}(T) \left| 1 - f/f_0 \right|^{1/2} \quad (38)$$

Figure 22 shows the measured $v_{s,c}$ at $T = 1.951$ K for various drive frequencies. Also plotted are the values for the normalized

critical velocity v , derived from Eq. (38), showing the wide range over which this normalization yields a frequency-independent velocity. Figure 23 shows how data from several temperatures and drive frequencies can be normalized to a universal curve.

Measured critical velocities have been divided by the critical velocity measured at half the resonant frequency to get the normalized velocities that are plotted. The solid line in the figure is the curve

$$|1 - f/f_0|^{1/2}.$$

It must be emphasized that the form is purely phenomenological.

Generally there was difficulty in obtaining reproducible values of critical velocity. The most troublesome examples occurred for higher temperatures, where frequently the orifice would appear to plug up spontaneously. When this occurred no fourth-sound resonance could be found, and the response to a drive was as if no superfluid were flowing. Occasionally one could return it to its superfluid-conducting state by momentary pulses of heat to one chamber or the other, as if the resulting pressure or fluid flow blew away some contaminant.

G. B. Hess has had success in obtaining reproducible critical velocities of superfluid flow through relatively large orifices.⁽⁹⁾ By adding packed rouge superfluid filters on either side of an orifice plate he has not only reduced day-to-day variations in the measured critical velocities, but he has obtained larger values for

the velocities than for unfiltered orifices. His varying results using unfiltered orifices are interpreted as being caused by pre-existing superfluid vorticity or by some contamination such as dust. The rouge filters screen out vorticity or contamination so that the intrinsic rather than extrinsic critical velocities can be measured.

C. Temperature Dependence.

We next wish to show the temperature dependence of the critical velocity.

Since the measured critical velocity was essentially frequency independent, except for the 0.1 μm Nuclepore case, we averaged, for a given pore size and temperature, the velocities over the various frequencies at which readings were made. Data for $f > 0.7 f_0$ were discarded, as there was often much more scatter near the fourth-sound resonance than at lower frequencies. The averaged critical velocities are shown in Fig. 24.

For the 0.1 μm Nuclepore the variation of $v_{s,c}$ with frequency did not permit us to average in this way. Instead we took the highest critical velocity recorded at a particular temperature and plotted these against temperature. Usually this highest velocity occurred at the lowest drive frequency. These critical velocities are also plotted in Fig. 24. (Plotting $\tilde{v}(T)$ against temperature produced a curve of similar shape, with a normalization which increased the velocity by about 20%. In the absence of any

argument why the velocity should be normalized by $|1 - f/f_0|^{1/2}$, the highest observed $v_{s,c}$ was used.)

Except for the 0.1 μm Nuclepore the critical velocities as shown in Fig. 24 are temperature independent.

The critical velocity for the 0.1 μm Nuclepore exhibits both temperature independent and temperature dependent behavior. At temperatures closer than 200 mK to the lambda-point the data fall roughly on the curve

$$v_{s,c} = u_c \left(1 - \frac{T}{T_\lambda}\right)^{2/3},$$

where $u_c = 450 \pm 100$ cm/sec. This behavior illustrates the intrinsic critical velocity regime of the Langer-Fisher theory. The coefficient u_c is somewhat lower than Clow and Reppy's value of $u_c = 690$ cm/sec⁽³²⁾ and Langer and Fisher's theoretical $u_c = 1500$ cm/sec.

Within about 30 mK of the lambda point the critical velocity for this pore size changes more rapidly than $(1 - T/T_\lambda)^{2/3}$. Though this behavior could be accounted for by errors in temperature measurement or lambda-point placement, one can plot $v_{s,c}$ vs. f_0^2 and find the same behavior. One would expect that f_0^2 , being proportional to ρ_s/ρ , should be a function of the actual $T_\lambda - T$ and that plotting $v_{s,c}$ vs. f_0^2 should yield a result independent of any error in the value of $T_\lambda - T$ used in Fig. 24. The behavior observed may be an artifact of our experimental technique.

D. Pore Size Dependence.

After demonstrating the existence of the extrinsic critical velocity for several pore sizes, we wish to show how that critical velocity depends on pore size.

For each superleak the critical velocities in the extrinsic regime were averaged. These values are tabulated in Table 4. Also tabulated are the critical velocities based on vortex ring creation, calculated from Eq. (1).

For the two Nuclepore membranes, the agreement with the predicted velocities is excellent. The experimental velocities of 321 cm/sec and 60.0 cm/sec for the 0.1 μm and the 0.5 μm Nuclepore are in each case only 3% from the velocities predicted by Eq. (1).

For the 10 μm orifice run B the experimental critical velocity based on Eq. (25) is 9.8 cm/sec, a factor of 2.38 higher than predicted by Eq. (1) simply using the orifice parameter alone.

In our geometry for runs B and C the flow path is sufficiently complex that we will use an experimental determination of the path length for run B, calculated from

$$\ell_B = \ell_A \left(\frac{\epsilon'_A}{\epsilon'_B} \right)_{\text{meas}} \frac{(\Omega_A + \Omega^m)}{(\Omega_B + \Omega^m)} .$$

In this expression we also corrected for the change in volume of the cell between runs A and B. Hence we replaced the ℓ in Eq. (25) by 2.14ℓ , assuming we can neglect Ω^m . Even with the worst case of Ω^m (that is, $\Omega^m \gg \Omega_A$ and $\Omega^m \gg \Omega_B$) the error

introduced is only about 10%, and probably not more than 5%. With this substitution the experimental critical velocity becomes 4.58 cm/sec, 19% higher than the predicted velocity, but in much more reasonable agreement with it than before.

In run C the fourth-sound resonant frequencies, and hence the measured c' , were unexpectedly lower than in run B with no change in the apparatus configuration. Attributing this to a change in orifice area (possibly icing, as suggested above) rather than effective path length, the critical velocity determined by measurement is 4.48 cm/sec, 16% larger than the predicted velocity and very close to the velocity as determined in run B.

For the 10 μ m orifice run A the results did not follow expectations in several respects. First, the critical velocity was temperature dependent in the entire temperature range investigated (1.33 K to 1.94 K). Second, the temperature dependence was not that of the superfluid density. The data were not entirely consistent, but it appeared that $v_{s,c}$ varied more rapidly than with a linear dependence on ρ_s/ρ . Finally the low temperature limit of superfluid critical velocity was near 100 cm/sec, a factor of 25 more than the figure predicted for a 10 μ m orifice, or subsequently measured in runs B and C. A graph of superfluid critical velocity is shown in Fig. 25 for run A.

The measurements made in run A to determine the critical velocities were different from those made in other runs. Because of the insufficient mass-current drive mentioned above a critical

pressure was measured rather than a critical voltage.

To obtain the necessary mass current the system had to be driven at or near the fourth-sound resonance, and Eq. (25) was used to determine the superfluid critical velocity. The way in which the critical pressure manifested itself in this run is quite unlike the occurrence of the critical pressure in other runs. It is discussed in the next section.

The reasons why the measured critical velocity in run A is so much higher than predicted by Eq. (1) and measured in runs B and C is not known. Possibly run A had a much cleaner chamber than subsequent runs, or the orifice itself was cleaner. The presence of a significant nearby obstruction (the added piezoelectric ceramic) in runs B and C probably was a factor in the large decrease in critical velocity for those runs, but there is no reason to expect the latter two runs to have had velocities agreeing with those derived from the vortex ring creation theory and the first run to have had velocities disagreeing.

3. Unusual Single Orifice Behavior.

As mentioned above, during run A the mass current available from the piezoelectric driver was insufficient to exceed the critical velocity except very near the fourth-sound resonance. This was partly because the critical velocity was much higher than expected, about 100 cm/sec compared to about 4 cm/sec measured for runs B and C.

The critical velocity for the single orifice driven near the fourth-sound resonance was manifest in a striking way. When driven at this frequency the cell oscillations built up as they would in any driven system of moderately high Q . For small drives the envelope of observed pressure oscillations exhibited an exponential shape with the limiting value probably determined by the classical loss mechanisms (normal fluid viscosity and thermal conduction). At sufficiently high drive the pressure oscillations would increase to a value we call the breaking pressure, at which point the oscillation envelope would collapse. Oscilloscope traces showing this behavior are displayed in Fig. 26. The superfluid velocity is 90° out-of-phase with the pressure signal, so the break in pressure near zero pressure represents a discontinuity in the superfluid velocity near its maximum.

The collapse of the envelope to zero in a small fraction of the period of oscillation means that the entire energy of oscillation has been dumped in a few milliseconds or less. Calculations indicate that a complete collapse represented an energy loss enough to create about 8000 vortex rings of $10\ \mu\text{m}$ diameter.

The envelope amplitude after collapse did not always go to zero. Drops in amplitude as little as 10% were seen, after which the build-up continued until again the pressure reached a breaking value. Often after collapsing, the phase of the pressure signal was seen to be reversed by 180° so that the process of "building up" of the amplitude resulted in the envelope first going to zero,

then growing as before. This phenomenon is clearly shown in Figure 26(a), where A marks the envelope zero.

Figure 26(b) shows a two-step collapse of the oscillation, with a significant drop in amplitude at B, and an even larger drop at C. Close inspection will show that the breaks in the slope of $\Delta P(t)$ vs. t occur at small values of pressure and hence at relatively large superfluid velocities.

While the envelope collapse could occur over a wide range of pressures, there was a fairly consistent upper limit at a given temperature, with higher upper limits occurring farther from the lambda point. These upper limits are the values of pressure we call the critical pressure, the ones used to calculate the superfluid critical velocities for the 10 μm orifice run A which are shown in Fig. 25.

Attempts were made in several ways to trigger the collapse of the oscillation. Using one piezoelectric driver to build the oscillation envelope the other driver was pulsed with large voltages as the envelope pressure approached the breaking pressure. Heater resistors were also used to generate superfluid currents. The dewar stand was struck with hammer blows in various directions. None of these techniques were successful in collapsing the oscillations.

This phenomenon was also observed in runs B and C at frequencies near resonance, but the critical velocities, and hence the breaking pressures, were much lower, so that noise tended to obscure the behavior. The collapsing behavior was never observed with the

multipore superleaks, the effect possibly being averaged out over the thousands of pores available for flow.

For drive frequencies well away from resonance, runs B and C showed responses similar to the multipore superleaks, a fairly fast onset of resistive pressure drop when the drive voltage was sufficiently high.

4. Conclusions and Suggested Improvements.

In the work described here several accomplishments have been achieved. First of all, we have demonstrated a new and convenient method of measuring the effective superfluid density ratio ρ_s/ρ as a function of temperature in small pores of well-defined geometry.

We have measured the superfluid density in three sets of superleaks. We have looked for evidence of superfluid density suppression in these superleaks, and while a suggestion of the expected effect has been seen, our experimental error was too large for us to obtain a definite indication. However, with the experience developed in the runs presented here, we believe that we would be able to measure temperature relative to the lambda temperature with better accuracy in future runs and thereby improve the visibility of density suppression effects.

We have studied the onset of superfluid flow dissipation in these superleaks. By use of an ac technique with lock-in detection we were able to measure the resistive pressure drop to as small an order of magnitude as 10^{-3} dynes/cm², comparable to the best

sensitivities that have been achieved by dc pressure detectors which measure helium heads by measuring the capacitance of coaxial capacitor standpipes containing the helium.

We have observed a pressure drop across our superleaks which is a fast-varying function of superfluid velocity, but whose velocity dependence is different from the dependence observed by Notarys in a dc experiment.

We have observed superfluid critical velocities which are in good agreement with those predicted by a vortex-ring argument in the extrinsic regime. We have also observed a temperature-dependent critical velocity whose temperature dependence varies in a way similar to that predicted by Langer and Fisher, and seen by Clow and Reppy in a fibrous material with less well-defined channels.

Finally, by the use of a pressure detection system with a fast response time we have been able to observe an unexpected and rapid energy dissipation in a single orifice.

A number of improvements in the apparatus and in the data-taking might result in more reliable results.

A systematic study of the ac transfer ratio as a function of temperature should be made, with a blank plug replacing the superleak for the purpose of this calibration. The temperature-dependent ac transfer ratio should then be used in Eq. (31). With this data a more frequent monitoring of the ac transfer ratio during the runs with superleaks would make the detection of any possible change in the pressure sensitivity more readily discernible.

The success of the filtered superleak of Hess warrants a trial

in our apparatus. If we interpret the erratic on-again off-again behavior of our superleaks near the lambda point as resulting from dust or primordial vortex lines, we might, by filtering these out, experience improvement in our ability to reach temperatures nearer the lambda point. Most interesting would be the effect of the superleak filter on the collapsing oscillations that were seen in the 10 μm orifice run A and on the low critical velocities seen with this same orifice in runs B and C.

Nuclepore filters are now available with 0.03 μm diameter pores, and these should be investigated. These 0.03 μm pores should have a 9 mK lambda-point depression, which would be easy to observe with our apparatus even with no improvement in thermometry.

The reduction of both acoustic and electrical noise would be especially desirable, since measurements near the lambda point were frequently noise-limited. Reduction of noise might also make it possible to define a critical velocity more precisely, since it might be defined with less ambiguity in terms of some definite pressure.

One source of noise, the discriminator noise, can be reduced with a different electronic filtering system. Another, perhaps more fundamentally important source, is the noise induced by boiling liquids in the dewars. With some effort the pumping technique discussed in Chapter III should be workable.

If one wanted to modify the apparatus further, two changes might make the measurements or their interpretation more easy. First, one could design the fill lines so that experiments could be more easily run at saturated vapor pressure rather than atmospheric pressure. Reference data is more generally available for helium at saturated vapor pressure than for helium under pressure, and corrections and interpolations are not so frequently necessary.

Second, one could shorten the cell to increase the first-sound resonant frequencies. Since the first-sound resonant frequencies go as $(\text{length})^{-1}$ and the fourth-sound resonant frequencies as $(\text{volume})^{-1/2} \propto (\text{length})^{-1/2}$ (with unchanged cell thickness and width), there would be an increased separation of the two resonant modes for a shorter cell.

With modest apparatus change, our technique for measuring the superfluid density ratio would work at various pressures throughout the liquid region, and, with a He^3 refrigeration stage installed, to temperatures near 0.3 K. It could easily be extended to He^3 - He^4 mixtures to measure superfluid density and superfluid density suppression in small pores.

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SCHEMATIC DIAGRAM OF CELL WITH
SINGLE ORIFICE SUPERLEAK

Figure 1

OVERALL VIEW OF LOWER END OF CRYOSTAT

Figure 2

VIEW OF FRONT CHAMBER WITHOUT COVER

Figure 3

SUPERLEAK MOUNTING PLUG

Figure 4

PRESSURE TRANSDUCER PLUG

Figure 5

SANDWICH OF PIEZOELECTRIC DRIVERS (EXPANDED)

Figure 6

LOW-TEMPERATURE PRESSURE SENSING CIRCUIT

Figure 7

BACK DIODE CURRENT SUPPLY AND RF PREAMP

Figure 8

ROOM TEMPERATURE PRESSURE-SENSING ELECTRONICS

Figure 9

LOWEST FIRST-SOUND RESONANCES

(a) STIFF MEMBRANE

(b) FLEXIBLE MEMBRANE

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

(a)

 $f = 40 \text{ Hz}$ $V = 20, 40, 60,$
and 80 volts

(b)

 $f = 55 \text{ Hz}$ $V = 5, 10, 25,$
and 50 volts

(c)

 $f = 75 \text{ Hz}$ $V = 2.5, 10,$
and 25 volts

Figure 19.

Photographs of Pressure Response to Drive Voltage,
 $0.5 \text{ } \mu\text{m}$ Nuclepore, $T = 1.32 \text{ K}$.

Figure 20

SUPERFLUID CRITICAL VELOCITY vs DRIVE FREQUENCY
10 μm ORIFICE RUN B

Figure 21

Figure 22

Figure 23

Figure 24

Figure 25

(a) Horizontal Scale 1 sec/div.

(b) Horizontal Scale 50 msec/div.

Figure 26.

Photographs of Collapsing Oscillations, 10 μ m Orifice Run A. (a) Growing Envelope, Showing One Example of Velocity Reversal. (b) Expanded Scale, Showing a Two-Step Collapse at Times Marked A and B.

Table 1. Front Chamber Carbon Resistor Thermometer Calibrations.

$$\ln (R/R_0) = A(T_0/T) + B + C (T/T_0),$$

$$R_0 = 1 \text{ ohm}, \quad T_0 = 1 \text{ K}.$$

Calibration Number	Used for Runs	A	B	C
1	10 μm A	4.699 ± 0.046	5.847 ± 0.059	-0.317 ± 0.018
2	0.1 μm , 0.5 μm	4.861 ± 0.014	5.631 ± 0.016	-0.259 ± 0.005
5	10 μm B, 10 μm C	5.083 ± 0.027	5.382 ± 0.031	-0.194 ± 0.009

Table 2. Geometric Parameters for Various Superleaks.

Superleak	0.1 μm Nuclepore	0.5 μm Nuclepore	10 μm A Orifice	10 μm B Orifice	10 μm C Orifice
Pore Diameter, $d = 2a$, μm	0.1	0.5	13	13	13
Number of Pores, N	7700	2500	1	1	1
Substrate Thickness, l_s , cm	1.10×10^{-3}	1.20×10^{-3}	3.81×10^{-3}	3.81×10^{-3}	3.81×10^{-3}
Effective Orifice Length, $l = l_s + 16a/3\pi$, cm	1.13×10^{-3}	1.33×10^{-3}	4.66×10^{-3}	4.66×10^{-3}	4.66×10^{-3}
Front Chamber Volume, Ω_1 , cm^3	11.4	11.4	11.8	10.6	10.6
Back Chamber Volume, Ω_2 , cm^3	16.4	16.4	16.8	16.8	16.8
Effective Volume, $\Omega = \Omega_1 \Omega_2 / (\Omega_1 + \Omega_2)$, cm^3	5.72	6.72	6.94	6.50	6.50
Calculated Flow Area, $A = N \pi a^2$, cm^2	6.0×10^{-7}	4.9×10^{-6}	1.33×10^{-6}	1.33×10^{-6}	1.33×10^{-6}

Table 3. Experimental Parameters for Various Superleaks.

Superleak	0.1 μm Nuclepore	0.5 μm Nuclepore	10 μm A Orifice	10 μm B Orifice	10 μm C Orifice
Fourth-Sound Resonance f_0 at $T = 0$, Hz	25.44	66.25	23.60	16.67	11.25
Experimental $\epsilon' = \omega_0^2(0)/u_1^2(0)$, cm^{-2}	4.29×10^{-5}	2.91×10^{-4}	3.69×10^{-5}	1.84×10^{-5}	8.39×10^{-6}
Calculated $\epsilon = A/\lambda\Omega$, cm^{-2}	7.69×10^{-5}	5.5×10^{-4}	4.11×10^{-5}	4.38×10^{-5}	4.38×10^{-5}
Ratio, $\epsilon_{\text{calc}}/\epsilon'_{\text{exp}}$	1.86	1.89	1.11	2.38	5.22
Fit to $f_0^2/u_1^2 = 1.438 B (T_\lambda^P - T)^{2/3}$: B, $\text{cm}^{-2}\text{K}^{-2/3}$	1.0867×10^{-6}	7.370×10^{-6}	9.352×10^{-7}	4.669×10^{-7}	2.1247×10^{-7}
T_λ^P , K	2.1628	2.1631	---	2.1628	2.1714
Measured Lambda Point Shift, $T_\lambda^P - T_\lambda$, mK	-0.4 ± 0.4	-0.1 ± 0.4	---	-0.4 ± 1.1	8.2 ± 1.1
Predicted Lambda Point Shift (Mamaladze), mK	-1.5	-0.14	-.0015	-.0015	-.0015

Table 4. Extrinsic Superfluid Critical Velocities.

Superleak	0.1 μm Nuclepore	0.5 μm Nuclepore	10 μm A Orifice	10 μm B Orifice	10 μm C Orifice
Assumed Pore Length, l , cm	1.13×10^{-3}	1.33×10^{-3}	4.66×10^{-3}	9.97×10^{-3}	9.97×10^{-3}
Measured Superfluid Critical Velocity, $v_{s,c}$, cm/sec	231	60.0	100	4.58	4.48
Predicted Superfluid Critical Velocity, $v_{s,c}$, cm/sec	238	57.9	3.85	3.85	3.85
Ratio of Measured to Predicted Superfluid Critical Velocity	0.97	1.03	26	1.19	1.16

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