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A STUDY OF ERROR EFFECTS IN MEASURING
CYCLIC-TEMPERATURE HEAT-TRANSFER COEFFICIENTS

by

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1

A STUDY OF ERROR EFFECTS IN MEASURING CYCLIC-TEMPERATURE HEAT-TRANSFER COEFFICIENTS

David A. Dingee and Joel W. Chastain

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An experimental investigation of the errors associated with the cyclic-temperature method of obtaining convective heat-transfer coefficients was made. The models studied were a large single tube, a bundle of 1/2-in.-ID tubes, and a bed of randomly packed spheres. In order to measure the error attenuation due to velocity mixing, cyclic dye and cyclic ion pulses were used.

It was found that most of the errors at the conditions under study could be explained by velocity mixing. The methods for measuring the attenuation errors were at best only qualitative and no real quantitative correction factor was found. The results do indicate that fluids of high density such as water are not suitable fluids for cyclic-temperature tests to obtain convective heat-transfer coefficients.

In future tests, gases are recommended as coolants. It is felt that reliable heat-transfer coefficients can be obtained if gases are used.

INTRODUCTION

The cyclic-temperature method of obtaining convective heat-transfer coefficients was developed at Battelle. This method has been applied with success on a variety of heat-transfer specimens using air as the coolant fluid.⁽¹⁾ However, a straightforward application of the cyclic-temperature method to some kinds of systems using water as the coolant fluid has not led to results which can be analyzed.

In order to understand the analysis of a cyclic-temperature test a brief description of the theory underlying the method is included here. A more detailed explanation was reported in BMI-747.⁽¹⁾

The cyclic-temperature method utilizes the fact that an attenuation of the amplitude (and also a phase shift) of a cyclic-temperature wave occurs in its passage through a test specimen. The equations* describing this attenuation for a simple Cartesian system are respectively:

- (1) A heat balance in a longitudinal differential element of the fluid,

$$\rho_f A_f C_p \left(\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} \right) dx = -h a (T - \theta_o) dx, \quad (1)$$

where $T = T(x, t)$.

(1) See references given at the end of the report.

* All the symbols appear under "Nomenclature" at the end of the report.

(2) Conduction into the specimen,

$$\frac{\partial \theta}{\partial t} = \frac{K_s}{\rho_s C_s} \frac{\partial^2 \theta}{\partial z^2} \quad (2)$$

with boundary conditions

$$\frac{\partial \theta}{\partial z} = \frac{h}{K_s} (T - \theta_0) \text{ at } z = 0, \quad (3)$$

and

$$\begin{aligned} \text{Lim } \theta &= 0; \theta = \theta(x, z, t) . \\ z &\rightarrow \infty \end{aligned} \quad (4)$$

(3) Cyclic temperature waves at the specimen inlet,

$$T(0, t) = A_0 \sin(\omega_0 t). \quad (5)$$

This problem has the following solutions for the fluid temperature T and specimen temperature θ :

$$\begin{aligned} T &= A_0 e^{-\alpha \xi} \sin(\omega \tau - \beta \xi) \\ \theta &= A_0 e^{-\alpha \xi - \epsilon \sigma} [\gamma \sin(\omega \tau - \beta \xi - \epsilon \sigma) + \delta \cos(\omega \tau - \beta \xi - \tau \sigma)] \end{aligned} \quad (6)$$

so that the ratio of inlet amplitude of the fluid to exit amplitude is

$$R_H = e^{-\alpha}; \alpha = nq \frac{1 + q}{1 + 2q + 2q^2} \quad (7)$$

from which α and hence h can be evaluated. Other solutions to the cyclic-temperature equations for systems in curvilinear coordinates are available, as are solutions to Cartesian systems with small dimensions where conduction effects in the specimen extend through its thickness.

A consideration of the general formulation of the analytical problem for any geometry will show the following implicit assumptions have been made:

- (1) Axial heat conduction is negligible.
- (2) The solid area and heat-transfer area are not dependent upon axial position.
- (3) The axial component of the fluid velocity is uniform and constant throughout the specimen.
- (4) The physical properties of the fluid and specimen are constant.
- (5) The heat-transfer coefficient is uniform and constant throughout the specimen.

Of these assumptions probably the least justified are (3) and (5). An investigation of the validity of these assumptions on the measured amplitude attenuation is the reason for the present program.

Theory

If the velocity distribution in a specimen is not uniform, then it may be expected that the portions of the fluid traveling at the greater velocities will carry the cyclic-temperature pulse faster than the slower moving portions of the flow. This would lead to an effective mixing of the warmer and colder fluids which would produce an attenuation of the heat wave. This effect is in general agreement with observed results reported herein. It is to be noted that with air as the coolant the velocity is generally high so that the heat wavelength will be long compared to the specimen and this mixing effect would be expected to be small. This would not necessarily be the case with water.

The consequences of a nonuniform heat-transfer coefficient are somewhat more difficult to discover. Several possible nonuniformities must be considered. Both axial and radial nonuniformities as well as a frequency-dependent nonuniformity are possible. The case of an axial variation in h has been treated in BMI-747(1). For peripheral variations of h , it is necessary to consider fundamental heat-balance Equations 1 and 2 in the form

$$\rho_f A_f C_p \left[\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right] dx = -dx \int_y h'(y) [T - \theta'(x, y, 0, t)] dy, \quad (8)$$

$$\frac{\partial \theta'}{\partial t} = \frac{K_s}{\rho_s C_s} \nabla^2 \theta'; \quad \theta'(x, y, z, t), \quad T = T(x, t) \quad (9)$$

when the boundary conditions are

$$\frac{\partial \theta'}{\partial z} = \frac{1}{K} \int_y h'(y) [T - \theta'(x, y, 0, t)] dy \text{ at } z = 0 \quad (10)$$

$$T(0, t) = A_0 \sin \omega_0 t. \quad (11)$$

This then defines the computed value of h from the solution to the Equations (1) to (4) as

$$h = \frac{\int_y h' (T - \theta') dy}{\int_y (T - \theta') dy}, \quad (12)$$

providing conduction effects axially and parallel to the heat transferring surface are small and

$$\theta = \frac{\int_y \theta' dy}{\int_y dy} . \quad (13)$$

Both Equations (12) and (13) prove to be reasonable methods of averaging so that it is expected that radial nonuniformities in h will lead to reasonable measured attenuations.

Another factor which may contribute to a nonuniform h is one which arises from the cyclic-temperature frequencies. It is possible that the cycling frequency may be sufficiently high so that the heat-transfer barrier (laminar sublayer for axial flow) is itself an attenuating factor. For instance, a pulsating temperature damps out in passing into an infinite fluid medium as $\exp(-\omega_0 z / 2\bar{\alpha}_f)$. Accordingly, if ω_0 is sufficiently large a considerable damping is possible for even small distances into the surface so that effectively little heat will be transferred to the rest of the medium.

This latter problem was treated analytically as a two-region cyclic-heat-transfer problem. One region is the laminar sublayer adjacent to the specimen and the second region is the specimen itself. A straightforward analysis in the Cartesian system of Figure 1 using the differential equations

$$\frac{\partial T}{\partial t} = \bar{\alpha}_f \frac{\partial^2 T}{\partial z^2} , \quad 0 < z < \delta, \quad (14)$$

$$\frac{\partial \theta}{\partial t} = \bar{\alpha}_s \frac{\partial^2 \theta}{\partial z^2} , \quad z > \delta,$$

with boundary conditions

$$K_f \frac{\partial T}{\partial z} = K_s \frac{\partial \theta}{\partial z} , \quad z = \delta, \quad (15)$$

$$T = \theta, \quad z = \delta, \quad (16)$$

$$\rho_f A_f C_p \left[\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right] = K_f a \frac{\partial T}{\partial z}, \quad z = 0, \quad (17)$$

$$T = A_0 \sin \omega_0 t, \quad x = z = 0, \quad (18)$$

gives for the temperature-amplitude attenuation

$$R_H = e^{-P \gamma g(\gamma, \beta)}. \quad (19)$$

Where

$$g(\gamma, \beta) = 1/2 \frac{(1 + \beta^2) \sinh 2\gamma + 2\beta \cosh 2\gamma + (\beta^2 - 1) \sin 2\gamma}{\cos^2 \gamma (\cosh \gamma + \beta \sinh \gamma)^2 + \sin^2 \gamma (\sinh \gamma + \beta \cosh \gamma)^2} \quad (20)$$

It is possible to compare $g(\gamma, \beta)$ from Equation (19) and as developed from Equation (7). This is shown on Figure 2. It appears that the boundary-layer attenuation effect is confined to values of $\gamma > 0.6$, for reasonable β (say $\beta \sim 20$).

FIGURE 1. TWO-REGION CYCLIC-TEMPERATURE MODEL

The ratio of the amplitude attenuation including boundary layer effects to the calculated attenuation ignoring these effects for $\gamma > 2$ is

$$\frac{R_{(7)}}{R_{(19)}} = e^{p(\gamma - 1)}, \quad (21)$$

since $R_{(7)} \rightarrow e^{-P}$ and $R_{(19)} \rightarrow e^{-P\gamma}$, for large γ . It can be seen, therefore, whenever

$$\gamma > \frac{0.2}{p} + 1$$

some error is introduced by ignoring these effects since then $\frac{R_{(7)}}{R_{(19)}}$ is greater than

about 1.20. This correction will be referred to later in the report. It is worth mentioning that for water tests at moderately high frequencies the ratio $R_{(7)}/R_{(19)}$ may become significant.

In general, whatever the cause of the error attenuation, be it velocity-mixing effect or boundary-layer damping, the measured attenuation by the cyclic-temperature method can be related to the attenuation due to heat-transfer effects as

$$R_M = R_E R_H. \quad (22)$$

This relation implies separable effects.

FIGURE 2. TEMPERATURE-AMPLITUDE ATTENUATION FACTOR, $g(\gamma, \beta)$ VERSUS γ

EXPERIMENTAL WORK

In order to experimentally investigate the existence of the two error attenuation factors mentioned above, a number of tests were performed. It was anticipated that the major attenuating effect in the cyclic-temperature pulses would arise from velocity-mixing effects in the test specimen. For this reason the experiments were designed to investigate this part of the error encountered in cyclic-temperature tests with water.

Two methods are available to evaluate the velocity-mixing error effect. One method consists in constructing two geometrically identical models having gross differences in physical properties. One of the models is selected to have poor thermal diffusivity so that very little heat is transferred to or from the cyclic heat waves. This model, in effect, provides the correction terms to be applied to a second model which has good thermal properties. This method has been used by Bettis Field.⁽²⁾ The second method consists in measuring the velocity-attenuation effect by a new technique using the identical heat-transfer specimen but cycling measurable waves into the flow which cannot exchange either energy or matter with the specimen. A record of the variation in intensity or concentration of the pulse before and after the specimen will then be a measure of the attenuation due to velocity mixing alone, since no other attenuating mechanism is present. This latter method was chosen for the present tests because of the difficulties in constructing two identical specimens from different materials, for cases of very irregular geometry. In the present tests, two types of pulses were used. One was a dye pulse which was injected cyclically over the cross section of the flow area; the other was an ion pulse (NaCl) injected similarly.

In both cases, cyclic-dye or cyclic-ion pulses, a pressure vessel was filled with a concentrated solution or mixture of the fluid to be cycled. Pressure was maintained on the container throughout the tests. The vessel was connected through a solenoid valve to the discharge line located in the flow stream. The discharge line consisted of a spiral-wound tube (1/8-in. copper tubing) of about five turns set in a 4-in. pipe transverse to the flow. About one hundred 15-mil holes were drilled uniformly throughout the spiral tube. The solenoid was tripped by a 90-deg on-off cam unit giving essentially square wave pulses of the material injected. A schematic of this system is shown in Figure 3.

In order to detect the concentration of dye at a given transverse section of the flow passage (immediately adjacent to the test section upstream and downstream), a photocell is mounted on a section of the pipe wall. This section is constructed of clear Lucite so that the photocell can be illuminated through the fluid stream, as in Figure 3. Precalibration of the photocell current sensitivity to equilibrium mixtures of dye and water in the tube allowed for transient analyses which will be described below.

The detection of ion concentration at transverse sections of the fluid stream is accomplished by measuring the current passing between closely spaced copper screens, maintained at a potential difference. As the ion-bearing stream passes the section the current rises because of increased electrical conductivity of the fluid. Again precalibration of the system, which showed essentially a linear relation between ion concentration and current, allows for the transient analysis.

Along with the tests utilizing these alternate measuring methods outlined above, a series of concurrent cyclic-heat tests was run. The method of detecting the

temperature-cycle amplitude depends on the change in resistivity of a spiral wound wire. A small change in the resistance of the wire causes unbalances of a Wheatstone-bridge circuit so that a current change may be recorded. This measuring technique is described in more detail in BMI-747⁽¹⁾.

FIGURE 3. SCHEMATIC DIAGRAM OF THE ION- AND DYE-PULSE APPARATUS

Test Procedure

The test procedure for the cyclic-dye tests was as follows. The loop was brought to an equilibrium temperature. Following this, the calibration reference on the oscillograph was made by recording the trace in the absence of a light source. The run was then made by cycling dye pulses into the flowing stream and recording the photocell output currents. After a number of cycles was recorded the dye was turned off and the system allowed to reach an equilibrium dye concentration, and this was recorded. Maintaining the system temperature and flow rate, a cyclic-heat run was made. For these runs a calibration was made at two equilibrium temperatures, before and after the runs for each of the two temperature pickups.

In all of the cyclic runs, dye, heat, and ion, the pulses have essentially a square wave shape for the low-frequency runs. This comes about from the method of cycling. The dye and ion cycles are introduced by the opening and closing of a solenoid valve. The heat pulse is introduced by the sharp opening and closing of an electronic circuit. Although the theory is specifically designed for sinusoidal waves it has been found to work satisfactorily with this square wave pulse.⁽³⁾ In all cases at relatively high frequencies, considerable rounding of the pulses occurred which led to nearly sinusoidal waves.

The test procedure in the case of the cyclic-ion tests was as follows. The loop was brought up to its equilibrium temperature and a cyclic-heat run was made. Then a reference was taken with the equilibrium current flowing between the spaced screens (tap water was used for the tests, so a small conductivity is present even without ion injection). A cyclic-ion run was then made and a reference taken for calibration purposes after the system had reached equilibrium with a low ion concentration. The system was then drained and the operation repeated.

During the course of the cyclic-ion runs it was noted that there was a build-up of electrochemical products (CuCl in this case) on the measuring screens. This tended to change both the sensitivity of the system and the individual screen properties. Both these factors are accounted for in the constants k_u , k_D , M_u , M_D used in the analysis described below.

Test Models

Two basic models were selected for study. One is a packed bed of spheres. It consists of a random packing of 504 Type 347 stainless steel balls 1/2 in. in diameter in a 4-in. -ID by 4-1/4-in. -long Lucite cylinder. This model represents an approach to a porous medium of irregular geometry for which the cyclic-temperature technique was originally developed.

The second basic model is a tube. Two tube models were used. One is an 11-in. length of 4-in. -ID Lucite tube stock. The other model is a bundle of 31 parallel tubes. Each tube is 0.462 in. in ID by 23 in. long. These tube models were selected as reference specimens because considerable information on heat transfer and flow characteristics is known.

Scope of Test Work

Since each test using either cyclic-ion or dye must be coupled with a corresponding cyclic-heat test in the analysis, the frequency-cycling conditions were limited by the cyclic-heat apparatus. One feature of this apparatus which limits the cycling frequency is the heating element. This element is a laminated series of resistance-heating sections. The over-all length is about 3-1/2 in. Consequently, for each flow a limiting frequency is attained above which the generation of cyclic waves is not possible because the waves overlap within the heater element. For the two flows used, 10 and 20 lb per sec in 4-in. pipe, these limiting frequencies are, respectively, 6 and 12 cps. Tests were performed up to 5 cps at 10 lb per sec and to 7.5 cps at 20 lb per sec.

A summary of the conditions tested is shown in Table 1.

TABLE 1. CONDITIONS FOR EXPERIMENTAL RUNS

Model	Frequency, cps	Re ^(a)	Pr ^(b)	Type Test
Packed spheres	1, 3, 5,	1.3×10^4	3.6	Heat, dye, ion
	1, 3, 5, 7.5	2.6×10^4	3.6	Heat, dye, ion
Tube bank	1, 3, 5	2.4×10^4	3.6	Heat, dye
	1, 3, 5, 7.5	4.8×10^4	3.6	Heat, dye
4-in. -ID tube	1, 3, 5	8.4×10^4	3.6	Heat, ion
	1, 3, 5, 7.5	1.7×10^5	3.6	Heat, ion

(a) Re - Reynolds number based on the characteristic dimensions of the test.

(b) Pr - Prandtl number.

METHOD OF ANALYSIS

Dye-Pulse Tests

The quantity to be investigated is the ratio of the amplitude of the dye-concentration variations downstream to the variations upstream of the test specimen.

The light transmitted through a dye solution can be related to the dye concentration, i. e., dye volume/total volume by

$$I = I_0 e^{-kc}, \quad (23)$$

where k is a constant depending on the nature of the dye and fluid used. Furthermore, since the light intensity is linearly related to the current output from a photocell (true for external load resistances of less than 100 ohms) then the current is

$$i = i_0 e^{-kc} . \quad (24)$$

Then for a variation in dye concentration at some position

$$\ln i^1/i^2 = -k (c^1 - c^2). \quad (25)$$

The ratio of a concentration variation downstream to a variation upstream is

$$\Delta c_D/\Delta c_u = \ln i_D^1/i_D^2 / \ln i_u^1/i_u^2 . \quad (26)$$

On the recording oscillograph the ratio of currents is the same as the ratio of deflection distances of the galvanometers, so the final equation to be used in analysis is

$$\Delta c_D/\Delta c_u = \ln d_D^1/d_D^2 / \ln d_u^1/d_u^2 . \quad (27)$$

The use of Equation (27) is dependent on a reference concentration level. That is, the distances d_D^1, d_D^2 , etc., must be measured from some base level. This is determined from Equation (23) for $c = \infty$ (i. e., no light).

Ion-Pulse Tests

The quantity to be measured here, like the cyclic-dye tests, is a variation in amplitude of the ion concentrations across the test specimen. The concentration is observed as a current passing between two closely spaced screens, as stated above. The circuit diagram for the measuring system is shown in Figure 3. A resistor is included in one leg of the circuit in order to compensate in part for different galvanometer sensitivities.

The method of measuring depends on the fact that the concentration of salt in the water is linearly related to the electrical resistance between the screens (for small concentrations). Then

$$\Delta c_D = k_D (R_{SD}^1 - R_{SD}^2), \quad \Delta c_u = k_u (R_{Su}^1 - R_{Su}^2).$$

But from the current diagram, changes in the ion screen resistances are related to the impressed voltage and observed current as

$$\Delta R_{SD} = E \left[\frac{1}{i_D^1} - \frac{1}{i_D^2} \right],$$

$$\Delta R_{Su} = E \left[\frac{1}{i_u^1} - \frac{1}{i_u^2} \right],$$

then

$$\Delta c_D / \Delta c_u = \frac{k_D}{k_u} \frac{i_u^1 i_u^2}{i_D^1 i_D^2} \left[\frac{i_D^2 - i_D^1}{i_u^2 - i_u^1} \right]. \quad (28)$$

In terms of measured displacements on the oscillograph tapes where

$$i_u = M_u d_u; i_D = M_D d_D, \quad (29)$$

Equation (29) becomes

$$\frac{\Delta c_D}{\Delta c_u} = \frac{k_D}{k_u} \frac{M_u}{M_D} \frac{d_u^1 d_u^2}{d_D^1 d_D^2} \left[\frac{\Delta d_D}{\Delta d_u} \right]. \quad (30)$$

As in the dye tests, Equation (30) implies the use of a reference level for current. This is taken as the true zero-current position of the galvanometer. The constants k_D , M_u , etc., are precalibrated and checked along with the actual runs.

Cyclic-Temperature Tests

The method of analysis of cyclic-temperature results is the same as given in BMI-747⁽¹⁾.

EXPERIMENTAL RESULTS AND DISCUSSION

Velocity-Mixing Effects

The results of the velocity-mixing experiments are presented in Figures 4 through 8 as graphs of measured attenuation versus cyclic frequency. These graphs are presented for each of the test models.

FIGURE 4. LOW-FLOW RESULTS ON PARALLEL-TUBE MODEL

31 tubes 1/2 in. in ID by 2 ft long.

$$Re = 2.4 \times 10^4.$$

FIGURE 5. HIGH-FLOW RESULTS ON PARALLEL-TUBE MODEL

31 tubes 1/2 in. in ID by 2 ft long.

$$Re = 4.8 \times 10^4.$$

FIGURE 6. RESULTS ON SINGLE-TUBE MODEL

4 in. in ID by 10 in. long.

FIGURE 7. LOW-FLOW RESULTS ON PACKED-SPHERE MODEL

Random packing, 504 1/2-in. spheres.

$Re = 1.3 \times 10^4$.

FIGURE 8. HIGH-FLOW RESULTS ON PACKED-SPHERE MODEL

Random packing, 504 1/2-in. spheres.

$$Re = 2.6 \times 10^4.$$

The tube models used to obtain the results of Figures 4 through 6 have properties which make α of Equation (7) essentially zero. This is borne out to a large extent in the attenuation measured by the other cyclic tests, shown also in these figures.

In the case of the packed-sphere model it is not possible to get a really good estimate of what the expected attenuation due to heat transfer should be. It would seem from previous (unpublished) data obtained at Battelle that the effective heat-transfer coefficient will vary considerably with the solidity ratio (ratio of sphere volume to total volume). This follows since the measured friction factor appears to depend strongly on solidity ratio and the fact that heat transfer and friction pressure drop are related. As a consequence of this uncertainty on actual heat-transfer effects, the curves drawn on Figures 7 and 8 which are labeled "... Attenuation With Heat Transfer h ", on the basis of published work⁽⁴⁾, are somewhat in question. Curves which represent attenuation with an infinite heat-transfer coefficient are also included in Figures 7 and 8. It is seen that the actual experimental points do fall below this limiting line, which indicates error-attenuating factors coming into play. That these error terms are present is again borne out by the other cyclic tests. These other tests (ion and dye tests) actually indicate that a large part of the measured attenuation comes from error effects. At least to a rough approximation the product of error attenuation and expected attenuation (based on h) is close to the measured attenuation; see Equation (22).

Before going into any detailed discussion of these experimental results, several factors should be brought out. In the first place the results of cyclic-ion and -dye tests are not considered to be sufficiently accurate for good quantitative results. In the case of the dye tests, the detection system will introduce errors because of preferential sampling of the cross section of the fluid stream. In addition, as the frequency of waves was increased, the amplitude became vanishingly small so that the analysis was difficult and considerable scatter resulted. Similarly, because the ion-pulse tests had low amplitude waves, they were difficult to analyze. The two tests are, however, in general agreement regarding all qualitative results on the error attenuation.

As stated, the results on parallel-tube models show that all the attenuation noted in cyclic-heat tests appears to be due to error terms. For the particular case of turbulent flow in tubes it is possible to set up a relation between the error attenuation and flow and cycle-frequency parameters. The analytical problem is set up as follows:

- (1) The passage is circular in cross section.
- (2) There is no mixing or thermal conduction in the fluid.
- (3) The fluid maintains a $1/7$ -power velocity profile.⁽⁵⁾
- (4) The fluid enters with a uniform sinusoidal temperature.

The problem is then to evaluate the mixed mean temperature:

$$T_m(x, t) = \frac{\int_A TV dA}{\int_A V dA},$$

where

$$V(x, r) = V_o \left(\frac{r}{s} \right)^{1/7}$$

$$T(x, r, t) = T_o \sin \left(\omega_o t - \frac{r\omega_o}{V(x, r)} \right)$$

$$T(0, r, t) = T_o \sin \omega_o t .$$

The problem does not lend itself to a straightforward computational solution since integrals of the form

$$\int_{-\infty}^{\infty} \frac{\cos U}{U^{1/6}} dU, \quad \int_{-\infty}^{\infty} \frac{\sin U}{U^{1/6}} dU,$$

must be evaluated. In the range $0 < \omega < 5$ this is possible but for large ω these integrals are very slowly convergent. A curve of the apparent attenuation

$$R_E = \frac{|T_m(x, t)|}{|T_m(0, t)|}$$

as a function of ω appears in Figure 9. It is to be noted that the theory in this case predicts mixing errors to be a function purely of cyclic wavelength to specimen wavelength. In order to check this out for the specimens tested, the high-flow data have been replotted onto the low-flow graphs at half-frequency values. The high-flow velocity is just double the low-flow velocity. This method of plotting should lead to a single curve since it is tantamount to plotting R versus ω . The cyclic-heat data in Figure 4 show this to be the case for the tube data. Figure 7 shows approximate verification for the sphere data.

Some points are included on the graph of Figure 9 to indicate the extent of agreement between the cyclic-heat data and the pure velocity-attenuation curve. It would be anticipated that the points would fall above the theoretical curve due to transverse mixing effects, since perfect transverse mixing leads to no attenuation.* This is the case for the single 4-in. tube data. However, the tube bundles fall somewhat below this curve. This may be due to a redistribution of flow upon entering and leaving the bundle from the 4-in. pipe between the temperature-sensing screens.

*The case of no axial heat conduction and perfect transverse heat transport is considered. A heat balance across a differential axial element without external heat transfer and with arbitrary velocity profile is by Equation (1)

$$\rho_f C_P \int_A V(A) \frac{\partial T}{\partial x} dA = -\rho_f C_P \int_A \frac{\partial T}{\partial t} dA .$$

But since T is independent of A , this becomes

$$\int_A V(A) dA \frac{1}{A} \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = 0 ,$$

or, since

$$\int_A V(A) dA \frac{1}{A} = V ,$$

the heat balance becomes

$$V \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = 0 .$$

If the condition that $T(0, t) = \Delta \sin \omega_o t$ is imposed, then the solution

$$T = \Delta \sin \left(\omega_o t - \frac{\omega_o x}{V} \right)$$

results. This solution shows the temperature amplitude is independent of axial position x . Consequently, no attenuation of the pulses would be noted at any downstream position.

FIGURE 9. COMPARISON OF MEASURED AND CALCULATED ERROR ATTENUATION

In the case of packed spheres no estimate of the velocity attenuation can be made. The data indicate that error attenuations as measured by the ion and dye techniques are of the same order of magnitude as the measured cyclic-heat attenuation. This would preclude any real separable analysis of cyclic heat without extremely accurate measurements of the error effect.

Other Effects

Earlier in the report some discussion was devoted to the effect of the boundary layer, or laminar sublayer, on measured attenuations. It is possible to evaluate the magnitude of the sublayer attenuation in the case of the parallel tubes. In general, these values are completely negligible, that is, the attenuation including the boundary layer phenomenon is about 0.993 to 1.00 times the attenuation calculated ignoring the conduction in the sublayer. This might have been anticipated since the velocity-mixing attenuation was already quite close to the measured error attenuation.

It is not possible to get an estimate of the order of magnitude of the boundary-layer effect in the packed-sphere bed. The results of error-attenuation tests using the cyclic-ion and -dye pulses indicate that practically all the observed error attenuation in cyclic-heat tests may be attributed to velocity effects. This is consistent with the apparently large heat-transfer coefficients observed in air tests on packed spheres⁽⁴⁾ inasmuch as large heat-transfer coefficients may be thought of as small laminar sublayer thicknesses.

Although the boundary-layer effect is apparently not appreciable in any of the presently reported tests, it can become important for other cases. At low flow velocities and high cyclic frequencies in long models it becomes important. The effect is included in the report in order that all the problems pertinent to the use of the cyclic-heat method are understood.

Discussion

In both the present tests, utilizing attenuations of other than heat pulses, and in tests performed at Bettis Plant⁽²⁾ utilizing geometrically similar models, the error effects have been found to be large. In fact, in both cases the experimental techniques have not proved quantitatively successful since the accuracy in determining the error terms has been on the order of or larger than the actual heat-transfer terms. This gives a discouraging outlook for the future use of the cyclic method with water as the coolant.

A further point which is worth considering relates to the amplitude attenuation that is expected under given operating conditions of flow and frequency. Associated with this is the resolution of heat-transfer coefficients from the measured attenuation. Figure 10 is illustrative on these points. On Figure 10 the flow and frequency parameter n is plotted against the expected attenuation with the dimensionless heat-transfer coefficient quantity q as parameter. It is clear that for small values of n very little attenuation of the cyclic-temperature wave is expected. Moreover, for low n values a small error in the measured attenuation R will lead to very large errors in calculated q values. For example, at $n = 0.1$ a variation in measured attenuation from 0.95 to 1.0 would lead to a change in computed q values from 0 to infinity. It then becomes important to perform the tests at as large values of n as feasible from the standpoint of resolving the measured attenuation into heat-transfer values. Large values of n are gained by large models (through product aL), low mass flows (through small $\rho_f A_f V$ product), and high frequencies (through ω_0), and, finally, by using models having good thermal properties (through the $C_s \rho_s K_s$ product).

FIGURE 10. VARIATION OF TEMPERATURE-AMPLITUDE RATIO, R , WITH n AND q , LOW-CONDUCTIVITY THEORY

$$n \equiv \frac{aL}{\rho_f A_f V C_p} \sqrt{2 K_s C_s \rho_s \omega_o} ; \quad q \equiv \frac{h}{\sqrt{2 K_s C_s \rho_s \omega_o}}$$

$$R = e^{-nq} \left(\frac{1+q}{1+2q+2q^2} \right)$$

Considering the above points, it is reasonable to inquire as to the over-all picture regarding the utility of the cyclic-temperature method of obtaining heat-transfer coefficients. This question can be answered by referring again to the general assumptions made in setting up the equation describing the attenuation of a cyclic-temperature wave. It is required that the fluid flow approximates the condition of uniform and constant velocity as well as lead to refined heat-transfer coefficients which are independent of the frequency. In order for the fluid to have an apparent "flat" velocity profile, ω must be small. This is accomplished by having either high velocities, low-frequency temperature waves, or short test specimens. These conditions are generally incompatible with the conditions described above leading to large values of n unless a fluid of low density (leading to low mass flows) is used. This probably explains why consistently good data have been obtained using air as the coolant.

The second condition to be imposed on the fluid is that computed values of γ be small (see Figure 2). This can be done by low flow velocities or low cycling frequencies. These are again in the direction of less attenuation of the cyclic wave and poor resolution of heat-transfer coefficients; i.e., small n . However, a fluid of low density will also generally have low γ values. Accordingly, gases are given a good chance for accurate heat-transfer measurements.

CONCLUSIONS

A survey of the sources of error associated with the cyclic-temperature method of obtaining convective heat-transfer coefficients has been made. Based on the available information it would appear that fluids of high density such as water are not suitable. The main disadvantage has been demonstrated to be the inability to achieve measurable heat-transfer-dependent temperature-amplitude attenuations without introducing sizable error attenuations from other sources.

The experimental methods employed in an attempt to measure the error attenuations due to nonuniformity of the velocity profile gave at best only an approximate quantitative picture. No real correction-factor method was found. The results do, however, indicate that most of the error encountered in the cyclic-temperature tests may be attributed to this source.

The use of gas in future tests is recommended. This recommendation is based on the fact that reliable information has been gained in past tests and the theoretical demonstration that error effects can be expected to be small.

REFERENCES

- (1) Dayton, R. W., Fawcett, S. L., Grimble, R. E., and Sealander, C. E., "Improved Measurements of Surface Heat Transfer by the Method of Cyclic Temperature Variations", BMI-747 (May 28, 1952).

- (2) LeTourneau, B. W., and Grimble, R. E., "Amplitude Attenuation of a Sinusoidal Temperature Wave in a Rectangular Channel Without Heat Transfer", WAPD-TH-179.
- (3) Dingee, D. A., Bell, W. A., Long, J. K., Chastain, J. W., and Fawcett, S. L., "Heat-Transfer Studies on a Twisted-Ribbon Reactor Fuel Element", BMI-974 (January 17, 1955). Classified.
- (4) Denton, W. H., Robinson, C. H., and Tibbs, R. S., "The Heat Transfer and Pressure Loss in Fluid Flow Through Randomly Packed Spheres", United Kingdom Report HPC-35 (June 28, 1949).
- (5) Prandtl, L., Essentials of Fluid Dynamics, Hafner Publishing Company, New York.

NOMENCLATURE

Roman Letter

a	- Heat transfer surface area per unit length
A	- Cross-sectional area tranverse to the flow
A_o	- Amplitude of wave pulse
A_f	- Cross-section area of the flow passage
c	- Concentration of dye or ion pulses, c_1 and c_2 are maximum and minimum values, respectively
C_p	- Fluid heat capacity
C_s	- Specimen heat capacity
Δc	- Change in concentrations, either ion or dye

$$\Delta c_u = c_u^1 - c_u^2, \text{ etc.}$$

d	- Galvanometer deflection, d_D^1 is maximum deflection for the downstream position, etc.
D_e	- Characteristic model dimension used in the Reynolds number
E	- Voltage
$g(\gamma, \beta)$	- A function of γ, β defined in the text
h'	- Local heat transfer coefficient
h	- Average heat transfer coefficient
i	- Current generated in the recording circuit, i_u^1 is the maximum current at the upstream position, etc.
I_o, I	- Incident and transmitted light intensities
k	- Constant of proportionality, sometimes subscripted
K_s, K_f	- Thermal conductivity of the specimen and fluid, respectively
L	- Specimen length
M	- Constant of proportionality, subscripted
r	- Radial distance
R_E	- Error attenuation, ratio of downstream amplitude to upstream amplitude of pulses.
R_M	- Measured attenuation using cyclic temperature waves
R_H	- Attenuation directly attributable to heat transfer effects

$R_{(7)}, R_{(19)}$	- Attenuations defined by the subscripted equations
R_S	- Resistance between measuring screens, subscripted and superscripted (i. e., $R_{S_u}^1$ - maximum resistance at the upstream position)
s	- Distance measured from the tube wall
t	- Time
T	- Temperature of the fluid
T_m	- Mixed mean temperature
U	- Dummy integration variable
V	- Fluid velocity
(x, y, z)	- Cartesian coordinates, x - in the axial direction, y - perpendicular to x and z - into the specimen perpendicular to the surface and measured from the surface.

Greek Letter Symbols

δ	- Boundary layer thickness
θ	- Average specimen temperature
θ'	- Local specimen temperature
μ_f	- Fluid viscosity
ρ_f, ρ_s	- Densities of the fluid and specimen, respectively
ω_o	- Cyclic frequency.

Dimensionless Groupings

n	- $\frac{aL}{C_p \rho_f A_f V} \sqrt{2K_s C_s \rho_s \omega_o}$
p	- $\frac{K_f a L}{\rho_f C_p V A_f \delta}$
Pr	- $\frac{C_p \mu_f}{K_f}$
q	- $\frac{h}{\sqrt{2K_s C_s \rho_s \omega_o}}$
Re	- $\frac{\rho_f V D}{\mu_f}$
α	- Log ratio of temperature amplitudes
$\bar{\alpha}_f, \bar{\alpha}_s$	- Thermal diffusivity of the fluid and specimen, respectively
β	- $\sqrt{\frac{\rho_s C_s K_s}{\rho_f C_p K_f}}$
γ	- $\sqrt{\frac{\omega_o \rho_f C_p}{2K_f}}$
ξ	- $\frac{x}{L}$, dimensionless distance

25

25 and 26

σ - $\frac{z}{L}$, dimensionless distance

τ - $\frac{tV}{L}$, dimensionless time

ω - $\frac{\omega_0 L}{V}$, ratio of specimen length to cyclic wave length.

DAD:JWC/cm