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THE EFFECTS OF RAYLEIGH TAYLOR INSTABILITY ON THE IMPLOSION OF LASER FUSION TARGETS

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In general, there are two sources for perturbations that can be amplified by Rayleigh Taylor instability. There can be surface perturbations due to imperfections during the manufacturing process. Also, the laser irradiation will not be uniform. Non-uniform illumination is essentially equivalent to a surface perturbation because the difference in intensity across the surface results in the imprinting of a surface perturbation. After the initial imprinting has occured, the exponential growth due to the Rayleigh-Taylor instability quickly dominates the effect of a non-uniform intensity. Use of a preheated, low density atmosphere can greatly reduce the effects of such a laser perturbation, because lateral heat conduction will smooth out the variation. Consider a 30-1 shell whose initial radius was 1.5 mm, had an atmosphere density of 3×10^{-4} gm/cc extending to 4.2 mm, and which was preheated to 1 keV by 7.5 kilojoule (kJ) prepulse of 4 μ light. A 4.5° per wavelength variation in intensity was equivalent to .0043 Å surface perturbation per percent variation when magnetic fields are not induced. When the production of magnetic fields by the non-uniform laser is included, and the transport co-efficients of Braginskii are used, the laser intensity variation is equivalent to .15 Å per percent at the same wavelength. For longer wavelengths, lateral heat conduction is not as effective at smoothing the intensity variation. However the growth rate is smaller so there is a tradeoff which varies from case to case and depends on the temperature, density distribution and radius of the atmosphere. Because the effects of non-uniform illumination depend on many parameters which are not relevant to Rayleigh-Taylor instability, we have concentrated most of our effort on targets with a given initial surface perturbation and uniform illumination.

The only ablation stabilization we have seen is a convective effect which appears to be similar to one predicted by Steve Bodner[1]. The effect occurs when the surface is being ablated faster than the perturbation can move into the shell. It has proved to require ablation rates which cannot be achieved in implosions which result in high compression and high gain. Since we have not found any stabilization mechanism strong enough to be of any use in target design, our efforts have been aimed at targets which can be compressed and burned with high efficiency even in the presence of Rayleigh-Taylor instability.

Our targets have been shells of pure DT with r/ r varying from 60 - 1 to 1 - 1 where r is the radius and r is the thickness of the shell with 1 - 1 being a solid sphere. We've also concentrated our efforts in the energy range of 100 kJ and targets which give a ratio of fusion energy out to laser energy in of 20 - 60.

Interest in the use of hollow shells stems from the fact that they can be imploded using a lower laser power and less severe pulse shaping. This arises from the fact that one must do a certain minimum of work on the DT fuel to compress and ignite it. This work is $W = \int PdV$ where P is the applied pressure and V is the volume. By increasing the volume, you decrease the required pressure and laser power. Lower power is important for two reasons:

a) Lower power means lower cost for the laser.

b) The existence of parametric plasma instabilities and resonance absorption processes lead to the production of very energetic electrons when a threshold laser intensity is reached. These energetic electrons result in preheat of the fuel and a drop in the driving pressure because of decoupling.

In most cases of interest to laser fusion, the atwood number is about one so that for classical Rayleigh-Taylor instability we have $\gamma \sim \sqrt{Ka}$. K is the wavenumber of the perturbation and a is the acceleration. Three ranges of wavelengths and physical effects are important.

a) $\lambda >> \Delta x_{min}$ where Δx_{min} is the minimum shell thickness: The effect of perturbations at these wavelengths is to reduce the overall symmetry of the implosion. With convergence ratios, given by the ratio of the initial radius to the final radius, on the order of 100, the symmetry and uniformity of implosion velocities must be maintained within a percent or so in order to get good spherical convergence and conversion of kinetic energy to thermal energy at the end of the implosion. Since long wavelengths have small growth rates, they generally do not cause a problem if the effects of shorter wavelengths can be tolerated.

b) $\lambda \sim \Delta x_{min}$: Wavelengths of this size result in a breakup of the shell and a gross mixing of high and low density matter. Perturbations of this size have high growth rates compared to the wavelengths which affect the overall symmetry and require much smaller surface perturbations. They are consequently much more difficult to live with.

c) $\lambda << \Delta x_{min}$: Short enough wavelengths are stabilized by viscosity and density gradient effects. But there is a range of wavelengths which have even higher growth rates than those for $\lambda \sim \Delta x_{min}$. These wavelengths reach the non-linear bubble and spike phase with an amplitude about equal to a wavelength and only grow linearly in time beyond this point. Perturbations at these wavelengths do not become as large as the shell thickness before being overtaken by perturbations at longer wavelengths which are still growing exponentially. The primary effect of these wavelengths is expected to be a modification of matter and energy transport at the ablation surface. We are not able to study this effect directly with Lasnex because a Lagrangian code cannot handle the non-linear turbulent stage of evolution.

Consider a velocity profile as shown in Fig. 1. In cases of interest, the atwood number is about 1, so that for classical Rayleigh-Taylor instability, we have

$$\gamma = \sqrt{Ka}$$

The number of generations of growth is given by

$$n = \int \gamma dt = 2\sqrt{Kx(1-\varepsilon)}$$

where ε is the fraction of the velocity reached in a fraction of the time $(1-\varepsilon)$ and x is the distance traveled by the shell and is assumed to be equal for all ε. This expression is crude because it does not include compression of the perturbation as the matter is compressed. This lowers the amplitude. Nor does it include a shift to shorter wavelengths or any other spherical affects[2], as the shell converges to small radius, which gives a higher average growth rate.

Using $K_{worst} = \frac{2\pi}{\Delta x_{min}}$

$$n = 2\sqrt{\frac{2\pi r}{\Delta x_{min}}(1-\varepsilon)}$$

Δx_{min} is some fraction of the initial shell thickness, $\Delta x_{min} = \Delta x_o/m$, so that

$$n = 2\sqrt{\frac{2\pi m r}{\Delta x_o}(1-\varepsilon)} \qquad (1)$$

n must be less than about 9 for typical cases in the 100 kJ range. This gives

$$2\sqrt{\frac{2\pi r}{\Delta x_o}(1-\varepsilon)} \lesssim 9$$

This formula gives a general idea of what can be done to decrease the number of generations of growth. Decreasing the $r/\Delta r$ decreasing m, and increasing ϵ will all help.

$\epsilon = 1/2$ corresponds to constant acceleration and is an implosion that makes maximal use of the levitation of a shell to decrease laser power. In this case

$$\frac{4\pi m r}{\Delta r_0} < 81 \quad \text{or} \quad \frac{r}{\Delta r_0} \lesssim \frac{20}{\pi m}$$

Since m is typically 10-30, this relation implies that no constantly accelerated shell target can work and this is born out in our 2-D calculations. Even shells with a surface finish of 1 Å do not survive for any ratio of $r/\Delta r$ if we try constant acceleration and assume no stabilization.

$\epsilon \to 0$ is an acceleration his ry for which the target is very slowly accelerated initially and then accelerated very rapidly. Such an acceleration history is required for a solid sphere in order to maintain an adiabatic compression and acceleration, and can be used with a shell. It is predicted to be even worse than constant acceleration for a shell and this turns out to be the case. For a solid sphere, a couple of spherical effects help you:

a) The shell thickness is kept up by spherical convergence, which occurs earlier in the pulse for a sphere than for a shell.

b) Compression of the matter results in compression of the perturbation and a lowering of the amplitude.

Because of these effects, our best estimate is that a solid sphere in the 100 kJ size range with a radius of about 570 μ, will work with surface perturbation of a few angstroms. Because of zoning dependent code problems involved in running short wavelengths with this target, we are not actually able to run the worst wavelength for this case. To make the above estimate, we run a longer wavelength perturbation that the code can handle. We then extrapolate this growth factor to the wavelength that would equal the minimum shell thickness at the time and radius this minimum is reached. Essentially, we multiply the number of generations by the square root of the wavenumbers involved and demand that the resultant amplitudes be less than the minimum shell thickness.

When $\epsilon \to 1$, acceleration is very large early in time and then the shell coasts. Such an acceleration is possible with a shell but not a solid sphere. If you want to maintain an adiabatic compression and acceleration, then the acceleration cannot occur in a time less than $\tau \sim \Delta x/v_s$. Δx is the shell thickness and v_s is the sound velocity behind the initial weak shock that sets the adiabat. For a bare drop, this time is the entire implosion time since $\Delta x = r$. But for a shell, $\Delta x < r$ and the above time can be a small fraction of the total implosion time. The thinner the shell, the smaller the fraction of the implosion time over which you can accelerate it. Putting this relation for the minimum acceleration time into Equation (1) gives

$$n = 2\sqrt{2\pi \frac{\Delta xo}{\Delta x_{min}} \frac{Vo}{Vs}} \qquad Vo = \text{final shell velocity.} \qquad (2)$$

This is independent of r/ r. Such a state is not actually realized, however, because it is not possible to get all the acceleration in time . Even if all the energy is put into the target in time , the hot atmosphere remains around for a considerable time after this and continues to push on the shell. The net result is that the number of generations continues to increase as the r/ r is increased. But for a given r/ r, the more rapid the acceleration, the fewer the generations.

Figure 2 indicates the ratio of the number of e-foldings expected at the worst wavelength to the maximum tolerable number of e-foldings as a function of r/ r. This ratio is estimated by taking the square root of the ratio of the worst wavenumber to the largest wavenumber at which success was achieved. The maximum tolerable number of e-foldings is calculated on the basis of a 10 Å initial perturbation. Each of the numbers indicated is the best case at that r/ r, which in each case was achieved with the most rapid acceleration possible, consistent with high gain. The last entry, for a 2 1/3 - 1 shell is for a shell that successfully imploded with a 1 Å surface finish at the worst wavelength.

Instead of applying a continuous power source, one can impulsively accelerate the target by turning the laser on and off. In this way, the target is subjected to bursts of very rapid acceleration followed by near coasting. After the passage of each impulse, the perturbations do not grow exponentially but they do grow linearly. As given by Richtmeyer[3].

$$\dot{a} = K \Delta v \alpha a_0 \qquad (3)$$

Δv is the velocity of the material behind the shock relative to that in front of the shock. a_0 is the initial amplitude, α is the atwood number and $\dot{a}$ is the time rate of change of the amplitude. This growth arises because of shock focusing as the shock passes a perturbed surface. The smallest growth possible occurs when the shell receives its entire velocity from a single shock. In this case, the growth factor is given approximately by

$$\frac{a}{a_0} = KR \qquad (4)$$

This growth factor is a lower limit to what one can achieve with implosions subject to Rayleigh-Taylor instability. For the 2 1/3 - 1 shell considered here, this factor is 160 or 5 e-foldings. With such growth, one could tolerate an initial perturbation of a couple hundred angstroms. However, the growth factor increases as more shocks are used, and several shocks are necessary to maintain near adiabatic compression. In the limit of a large number of weak shocks, the growth factor goes over to the Rayleigh-Taylor value.

By suitably timing the several pulses and keeping the ratio of magnitudes of succeeding shocks within a factor of 2-3, one can decrease the number of generations and maintain high gain. Lasnex has trouble correctly calculating such an acceleration because of the way it treats shocks. In order to solve the zero order hydrodynamics, shocks are spread over 3 zones with a von Neuman Q. This means that the maximum acceleration that matter can feel when a shock passes is $a_{max} \sim \frac{(\Delta v)^2}{\Delta x}$ where Δv is the matter velocity across the shock and Δx is the shock thickness. In this situation, we are zone limited in the acceleration and expect that Lasnex will give somewhat pessimistic answers. Nonetheless, we are able to lower the power to 10^{14} watts, an order of magnitude lower than for a typical solid sphere and survive with a 5 Å surface finish using a 2 1/3 - 1 shell. The velocity profile from Lasnex, shown in Fig. 3, comes from 4 shocks of increasing strength, followed by a constant power of 10^{14} watts. Also shown in Fig. 3 is the power versus time history. The geometry of the target is shown in Fig. 4. Perturbation amplitudes versus time are shown on Fig. 5. The peak laser intensity is about 2 X 10^{15} w/cm^2 at a peak temperature of 5 keV. This intensity is about an order of magnitude above threshold for the parametric decay instability at 1/4 ω, although about 85% of the light is absorbed by inverse Bremsstrahlung.

We expect to be able to live with this intensity be seeding with a higher Z material. Improvements in the impulsive acceleration technique may allow us to further lower the intensity.

CONCLUSIONS

Lasnex calculations indicate that targets of pure DT with $r/\Delta r$ ratios of 3-1 or less can be successfully imploded in the presence of Rayleigh-Taylor instability with an initial surface perturbation of at least 5 Å. Further calculations are being carried out using an impulsive acceleration technique which we expect will increase the tolerable surface perturbation. However, the peak amplitude is expected to be considerably less than 100 Å for a 3 - 1 shell. These calculations have been done at the 100 kJ level, however, the number of generations of growth

$$n \sim 2\sqrt{\frac{2\pi mr}{\Delta xo}\,(1-\epsilon)}$$

is almost independent of the size of the target. Thus one can tolerate a surface perturbation which increases linearly with radius. Actually, the tolerable surface perturbation would increase somewhat faster than linearly. Larger targets require somewhat lower implosion velocities, and hence, somewhat lower driving pressures. The lower driving pressure results in somewhat thicker shells and hence, longer worst unstable wavelengths.

We see no evidence of any ablation stabilization mechanism that is strong enough to be of any use to pellet design. Unless Lasnex is making some systematic error, our conclusion is that targets designed for high compression and gain will have to be manufactured very carefully if they are to survive this instability.

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APPENDIX

NARRATIVE FOR A MOVIE OF RAYLEIGH-TAYLOR INSTABILITY CALCULATIONS

The movie shows the implosion of two shells. The first half shows the implosion of a uniform 30 - 1 shell with a 7% RMS variation in intensity for an ℓ = 80 or 118 μ wavelength. This wavelength is almost a factor of 50 longer than the worst instable wavelength so the shell would actually break up much sooner than seen in the movie. Nonetheless, this shell is destroyed well before reaching the origin. The calculation shows the existence of a low density atmosphere extending to 4.2 mm at a density of 2×10^{-4} gm/cc and preheated to 1 keV by 6.0 kJ of 4 μ light. At this particular wavelength, a 7% RMS variation in intensity was equivalent to a surface perturbation of .5 Å RMS. The second calculation has no atmosphere since we decided to concentrate on surface perturbation as the more serious problem.

There are a number of counters at the bottom of each frame of the movie. In the left column are the full scale R and Z coordinates in cm, and Z_{mult} which is the ratio of R_{max} to Z_{max}. The horizinal Z axis is the axis of symmetry for LASNEX. The center column indicates the energy input and yield in kJ. The right column indicates the time in nanoseconds, velocity in cm-N sec and the peak density in gm/cm^3. The movie shows the entire calculation once with just the grid lines to indicate the shell deformation. Then we decrease the scale and show the isodensity contours for the end of the implosion. The breakup of the matter into islands is very dramatic in the 30 - 1 shell.

The second half of the movie shows a shell initially 2 - 1. The initial perturbation is 5 Å, peak to peak at a wavelength of $2.25^{o}/\lambda$ or 27.5 μ which is the worst wavelength. This shell receives its velocity from a series of 9 shocks which impulsively accelerate it. Several of these can be seen distinctly in the movie. Notice that before the shell even moves, it compresses to a shell with $r/\Delta r$ about 20. It is this compression to a very small fraction of its initial thickness which makes the implosion of shells so difficult. The implosion of this shell was successful and gives a yield very close to the 1 - D result.

MODEL ASSUMED FOR VELOCITY VERSUS TIME OF IMPLODED SHELLS

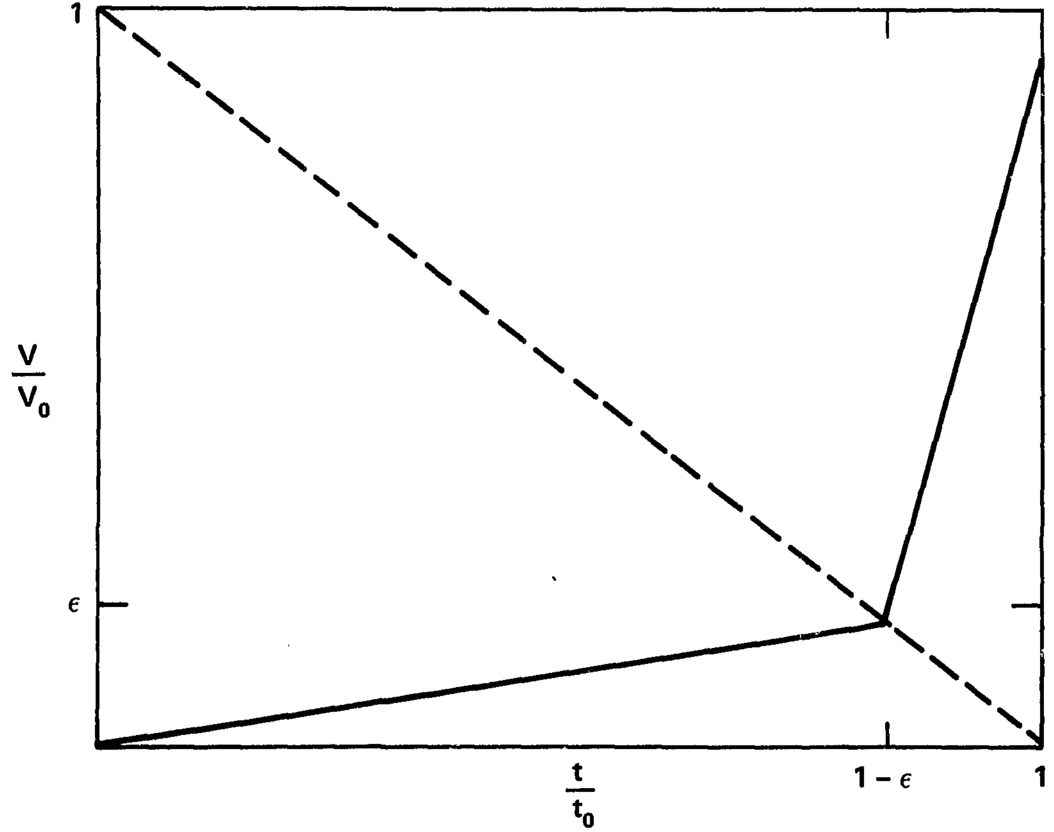

FIGURE 1

RATIO OF THE NUMBER OF E-FOLDINGS TO THE TOLERABLE NUMBER VERSUS R / ΔR

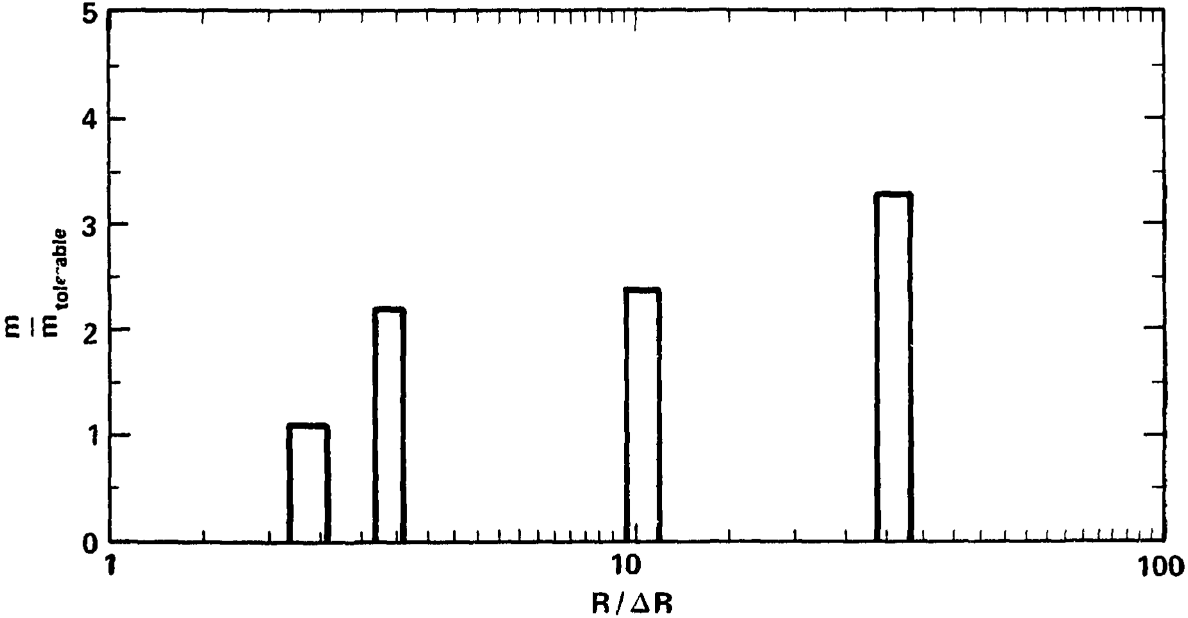

FIGURE 2

VELOCITY AND POWER VERSUS TIME FOR IMPULSIVELY ACCELERATED SHELL

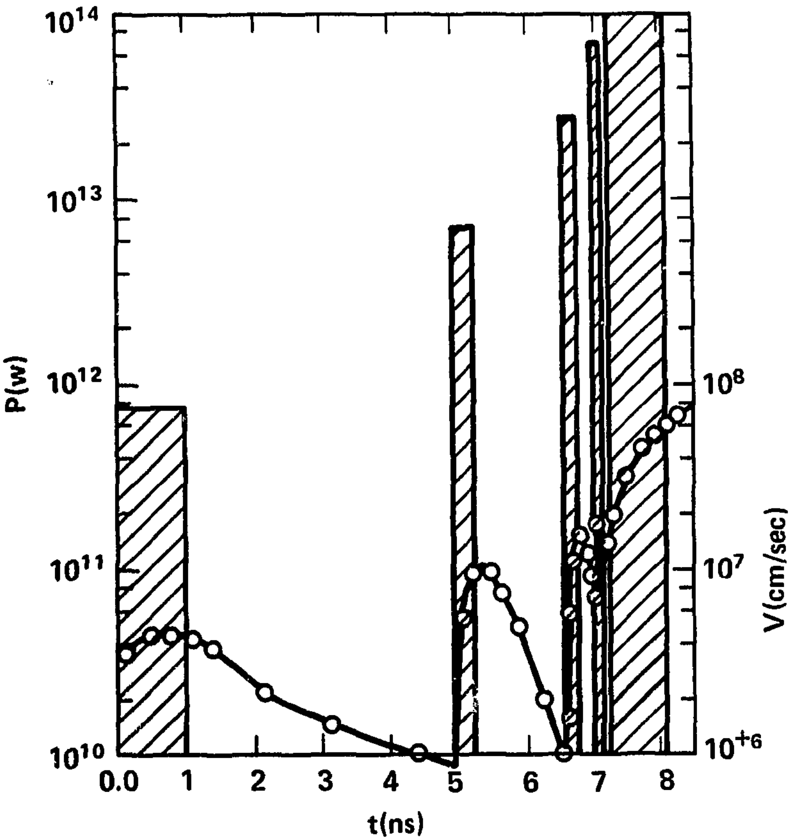

FIGURE 3

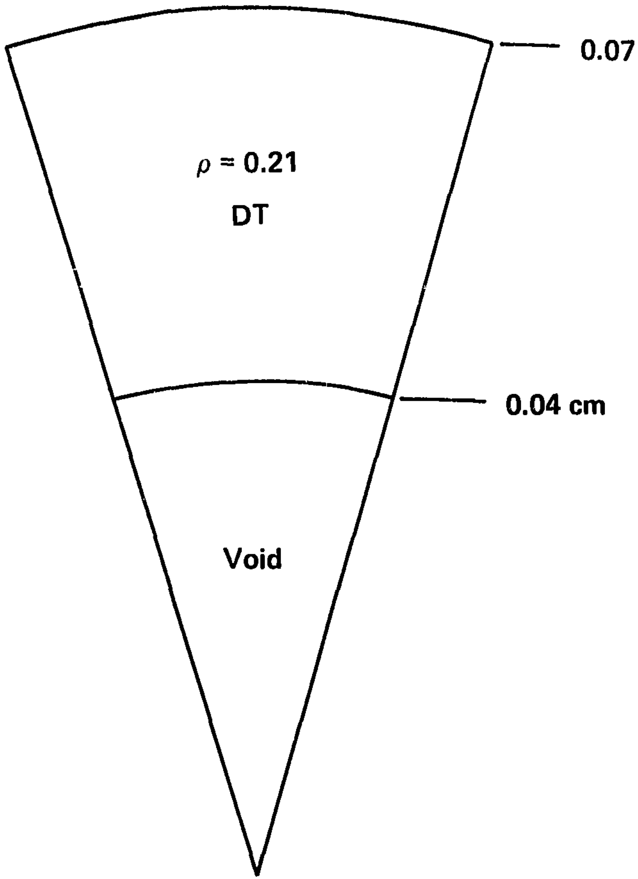

FIGURE 4

ROOT MEAN SQUARE AMPLITUDE VERSUS TIME
FOR IMPULSIVELY ACCELERATED 2 - 1 SHELL

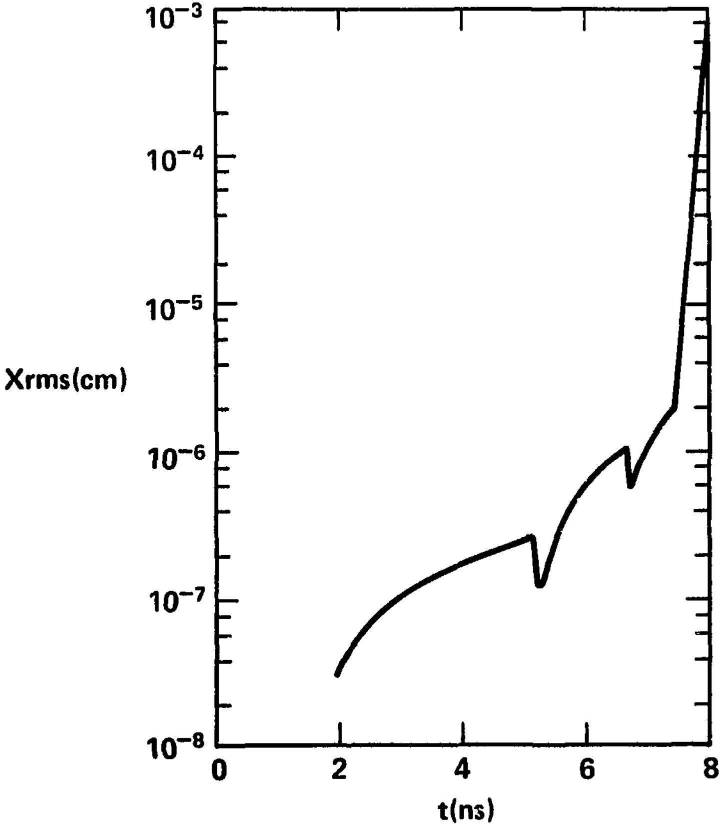

FIGURE 5