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NOZZLE CHAMBER TEMPERATURE
MEASUREMENT REQUIREMENTS
AND MEASUREMENT SYSTEM STUDIES

(NJD-0027)

NERVA Program, Contract SNP-1

31 December 1970

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NERVA Program

Contract SNP-1

December 1970

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R. D. Samuelson

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OBJECTIVE

To review problem areas associated with the calculation of specific impulse and the effective mixed mean nozzle chamber stagnation temperature. To establish preliminary static and dynamic performance requirements for those measurement system components which are used for the measurement of exhaust stream temperatures in the nozzle convergent section. To identify those measurement system studies that would be of assistance in designing system components and in formulating a data processing plan.

SUMMARY AND CONCLUSIONS

This report concerns itself with the calculation of the mixed mean nozzle chamber stagnation temperature and the measurement of engine exhaust stream temperatures in the nozzle convergent section. For both calculated and measured items, the most stringent accuracy requirements are those imposed by specific impulse computations while the most stringent bandwidth requirements are those established by the engine control system dynamics.

A variety of idealized nozzle flow equations have been proposed for calculating thrust and specific impulse values. They are summarized in Table I along with corresponding parametric sensitivity equations. Based upon the parametric sensitivity equations and estimated 3-sigma parametric uncertainty levels, none of the idealized nozzle flow equations can satisfy the $\pm 0.75\%$ specific impulse uncertainty requirement or the $\pm 2.00\%$ thrust uncertainty requirement even if the mixed mean nozzle chamber stagnation temperature uncertainty were zero. The estimated thrust and specific impulse computation uncertainties for the various nozzle flow equations of Table I are summarized in Table III.

The accuracy requirement imposed upon computed nozzle chamber stagnation temperature data depends on the equation used for calculating specific impulse. Since no rigorous NERVA nozzle flow equation has yet been presented for calculating specific impulse, it is impossible to obtain a valid sensitivity equation for determining the required uncertainty of computed nozzle chamber stagnation temperature data (the equations of Table I are largely inadequate for the purpose). As a result, no known valid accuracy requirement exists for

nozzle chamber stagnation temperature data. A $\pm 1.00\%$ uncertainty has been assumed in this report as a performance goal for the effective calculated mixed mean nozzle chamber stagnation temperature at the full power operating point. This uncertainty is a total error envelope (bias plus three sigma random error) and is based upon experience, judgement, and a knowledge of potential techniques that may be adopted to minimize instrumentation errors. As a result, the $\pm 1\%$ uncertainty assignment is a matter of personal opinion and is not based upon positive knowledge.

No firm bandwidth requirement can be assigned until a complete control system model has been implemented and verified. However, a tentative evaluation has been made of the anticipated bandwidth of measured nozzle convergent section temperature data and it appears to be marginal (0.3 Hz at full power) in that it is close to the expected bandwidth of actual nozzle chamber temperature variations. Since the bandwidth of measured data is effectively established by the thermocouple located in the convergent nozzle section, every effort, short of initiating an extensive thermocouple development program, should be made to reduce the thermocouple time constant. Estimated thermocouple time constant and corresponding bandwidth values are listed in Table IV. The thermocouple time constant is completely inadequate during pulse cooldown. As a result, either thermocouple data correction techniques or nozzle flow equations must be adopted if computed nozzle chamber stagnation temperature data is needed during pulse cooldown.

The measurement accuracy required of each individual convergent nozzle section thermocouple data channel depends upon the equations adopted to calculate the effective nozzle chamber stagnation temperature as well as the required nozzle chamber stagnation temperature computation uncertainty. Although the calculation uncertainty has been set at $\pm 1\%$, this is not a firm requirement. In addition, no equation has been presented for calculating

the mean nozzle chamber stagnation temperature from core exit and nozzle convergent section measurements. Thus, individual channel accuracy requirements are presently nonexistent. In lieu of actual requirements, it was decided to allocate the smallest uncertainty numbers that could be assigned with any reasonable chance of fulfillment. The resulting measurement system component uncertainty requirements for convergent nozzle section temperature channels are summarized in Table XI for the full power operating state point. The estimated overall measurement system uncertainty for nozzle convergent section temperature channels is 45.6°R or $\pm 1.07\%$ of the full power operating point ($\pm 1.07\%$ of 4250°R). The above overall channel uncertainty value only applies to that data channel which is associated with the thermocouple junction located at the tip of a multiple-junction thermocouple probe.

RECOMMENDATIONS

Many critical problem areas were uncovered during the course of this study. Although recommendations for their solution are presented in the text, it was felt that the more important ones should be summarized here.

1. It is recommended that the following sequence of events be implemented to define valid individual thermocouple data channel uncertainty requirements.

- (a) Implement a rigorous study to generate a comprehensive equation for calculating specific impulse. The equations of Table I are inadequate.

- (b) Derive a parametric sensitivity equation for the specific impulse equation developed in step (a) above.

- (c) Provide a maximum permissible mixed mean nozzle chamber stagnation temperature data uncertainty allocation from the parametric sensitivity equation derived in step (b) above. The sensitivity equations of Table I are inadequate.

(d) Implement a rigorous study to generate a technique for calculating the effective mixed mean nozzle chamber stagnation temperature from core exit and nozzle convergent section measurements.

(e) Establish individual thermocouple channel uncertainty tolerances based upon the nozzle chamber temperature uncertainty allocation made in step (c), the computation error associated with the nozzle chamber temperature equation developed in step (d), and the number and distribution of thermocouple sensors.

2. It is recommended that a thorough study be implemented at the earliest possible date to define thermal gradients and velocity vectors about each thermocouple junction for all operating state points of interest. This data should then be used to accurately define the thermal time constant and static uncertainty of each junction.

3. It is recommended that certain overall measurement system studies be initiated to formulate a data processing plan and to define system design constraints. In particular, the design of the data acquisition subsystem appears to be seriously impeded by a lack of certain end-to-end system studies. Some of the measurement system studies to be listed below are currently scheduled for completion by the measurement engineering group. The others should be scheduled as soon as practical. No attempt has been made to list the recommended studies in order of importance as priorities tend to change with time.

(a) A measurement system study is needed to determine the frequency spectrum of data presented to the input of the data acquisition subsystem. This study would have to consider the frequency spectrum generated by the signal source being measured and the manner by which the source spectrum is altered (distorted) by the use of transducers and cabling. This information is required to formulate channel filtering and data sample rate requirements.

(b) A comprehensive measurement system study is needed to identify, evaluate, and describe operational error mechanisms associated with flight sampled data systems and to assess the probable performance of various measurement system configurations. The study would, (1) evaluate the transient and steady state response characteristics of various types of presampling filters; (2) evaluate the causes and methods of suppressing common mode error signals, static and dynamic loading errors, channel crosstalk, and electromagnetic induced noise signals; (3) evaluate the dependency of aliasing errors upon sample rates and the envelope of the data spectrum with noise present; (4) evaluate analog-to-digital quantization errors and the sensitivity of various conversion techniques to the presence of noise; (5) define necessary signal conditioning requirements and determine which signal conditioning functions should be implemented by data acquisition system hardware and which should be performed by computer software; (6) assess in-flight calibration requirements and evaluate the feasibility of establishing a distinct subsystem to generate and distribute master time base and reference voltage signals and (7) evaluate those analog data bandwidth compression techniques that could be applied to vibration data, turbine shaft and bearing displacement data, and pulse rate neutronic data.

(c) A measurement system study is needed to define the data format, the system timing and the binary code scheme that will be used for communication between the data acquisition subsystem and the computer. This information is needed for interface design.

(d) A measurement system study is needed to define the signal processing intra-relationships that exist between the various parts of a measurement system. This study would employ sampled data information theory to establish a unified approach to the selection of filters, sample rates, encoding increments, and data compression and reconstruction techniques. This study is

required to identify those data processing restrictions that should be imposed upon one system element because of data processing operations occurring in other portions of the system. As a specific example, the adoption of specific data compression and reconstruction routines to satisfy flight qualification engine telemetry requirements may place a restriction on the selection of filters, sample rates, and minimum encoding increments if an overall data accuracy is to be maintained.

DISCUSSION

I. NOZZLE CHAMBER TEMPERATURE REQUIREMENTS

Nozzle chamber temperature data is required for engine control, specific impulse and thrust calculations, and for nozzle performance evaluation. The most stringent signal bandwidth requirements are those established by the engine control system dynamics (reference 9) while the most stringent accuracy requirements appear to be those imposed by the specific impulse computation equations.

A. ACCURACY REQUIREMENTS, EFFECTIVE MIXED MEAN NOZZLE CHAMBER STAGNATION TEMPERATURE

The NERVA Program Requirements Document, dated 13 November 1970, states that the engine control system shall provide sufficient precision and accuracy to achieve and maintain the following engine performance levels with the indicated three sigma limits.

1. Specific impulse - - - - - (825 sec) $\pm$ 0.75%
2. Thrust - - - - - (75,000 lb) $\pm$ 2.00%

Flight specific impulse and thrust values can be determined through the use of inertial guidance systems and nozzle flow equations. Ground test specific impulse and thrust values can be attained by direct thrust and weight flow rate measurements and by nozzle flow equations.

A variety of nozzle flow equations are available for calculating thrust (F) and specific impulse (I_{sp}) values. The accuracy requirement imposed upon nozzle chamber temperature data is dependent upon the I_{sp} equation adopted. To illustrate this point, a summary of ideal nozzle performance equations and corresponding parametric sensitivity equations is presented in Table I. The equations for method one, method two, method three - case one, and method three - case two were derived in enclosure two of reference 1. The equations

TABLE I

FULL POWER PERFORMANCE PARAMETER SENSITIVITY EQUATIONS

Method one: Measured; P_c and T_c Assigned; A_t and ϵ Calculated; F , I_{sp} , and $\dot{w}$

$$F = P_c A_t \left[\frac{P_e A_e}{P_c A_t} + \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]} \right]$$

$$\dot{w} = \frac{g P_c A_t}{\sqrt{RT_c}} \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$I_{sp} = F/\dot{w}$$

$$\frac{\Delta F}{F} = \left[0.99122 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.01436 \left(\frac{\Delta T_c}{T_c} \right)^2 + 1.00000 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.51642 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00004 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.46111 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00000 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 1.15036 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta \dot{w}}{\dot{w}} = \left[1.00434 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.31288 \left(\frac{\Delta T_c}{T_c} \right)^2 + 1.00000 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00000 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.12453 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

Method two: Measured; P_c and $\dot{w}$ Assigned; A_t and ϵ Calculated; F , I_{sp} , and T_c

$$F = P_c A_t \left[\frac{P_e A_e}{P_c A_t} + \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]} \right]$$

$$T_c = \frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{g P_c A_t}{\dot{w}} \right)^2$$

$$I_{sp} = F/\dot{w}$$

$$\frac{\Delta F}{F} = \left[1.46630 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.04676 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 1.47699 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.51642 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[1.46630 \left(\frac{\Delta P_c}{P_c} \right)^2 + 1.48499 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 1.47699 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.51642 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta T_c}{T_c} = \left[3.02296 \left(\frac{\Delta P_c}{P_c} \right)^2 + 3.20523 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 3.19629 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00000 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.49794 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

TABLE I (cont.)

FULL POWER PERFORMANCE PARAMETER SENSITIVITY EQUATIONS

Method three, case one: Measured; P_c , T_c , and $\dot{w}$. Assigned; A_t and ϵ . Calculated F and I_{sp} .

$$F = P_c A_t \left(\frac{P_e}{P_c} \frac{A_e}{A_t} \right) + \frac{\dot{w}}{g} \sqrt{\frac{2\gamma R T_c}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]}$$

$$I_{sp} = F/\dot{w}$$

$$\frac{\Delta F}{F} = \left[0.00026 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.44399 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.95505 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 0.00052 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 1.13273 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00026 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.44399 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00052 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 0.00052 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 1.13273 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

Method three, case two: Measured; P_c , T_c , and $\dot{w}$. Assigned; A_t and ϵ . Calculated F and I_{sp} .

$$F = P_c A_t \left[\frac{P_e}{P_c} \frac{A_e}{A_t} + \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]} \right] = P_c A_t C_f$$

$$I_{sp} = F/\dot{w} = \frac{P_c A_t C_f}{\dot{w}}$$

$$\frac{\Delta F}{F} = \left[0.99122 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.01436 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00000 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 1.00000 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.51642 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.99122 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.01436 \left(\frac{\Delta T_c}{T_c} \right)^2 + 1.00242 \left(\frac{\Delta \dot{w}}{\dot{w}} \right)^2 + 1.00000 \left(\frac{\Delta A_t}{A_t} \right)^2 + 0.00054 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.51642 \left(\frac{\Delta \gamma}{\gamma} \right)^2 \right]^{1/2}$$

TABLE I (cont.)

FULL POWER PERFORMANCE PARAMETER SENSITIVITY EQUATIONS

Method Three, case three: Measured P_c , T_c and $\dot{\omega}$. Assigned A_t and ϵ . Calculated F and I_{sp}

$$F = P_c A_t \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g)(H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\}$$

$$I_{sp} = \frac{P_c A_t}{\dot{\omega}} \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g)(H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\}$$

$$\frac{\Delta F}{F} = \left[0.00057 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.26490 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.95120 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 + 0.00039 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.29699 \left(\frac{\Delta Y}{Y} \right)^2 + 0.33444 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00821 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00057 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.26490 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00061 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 + 0.00039 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.29699 \left(\frac{\Delta Y}{Y} \right)^2 + 0.33444 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00821 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

Method Three, case four: Measured P_c , T_c and $\dot{\omega}$. Assigned A_t and ϵ . Calculated F and I_{sp} .

$$\left. \begin{aligned} F &= \dot{\omega} K \sqrt{H_c - H_e} \\ I_{sp} &= K \sqrt{H_c - H_e} \end{aligned} \right\} K = \sqrt{2J/g}$$

$$\frac{\Delta F}{F} = \left[0.00000 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.25184 \left(\frac{\Delta T_c}{T_c} \right)^2 + 1.00000 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 + 0.00085 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.50974 \left(\frac{\Delta Y}{Y} \right)^2 + 0.35160 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00863 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00000 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.25184 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00000 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 + 0.00085 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.50974 \left(\frac{\Delta Y}{Y} \right)^2 + 0.35160 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00863 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

for method three, case three and method three, case four are developed in Appendix A of this report. It should be noted that the nozzle equations of method three, case four are simplifications of the method three, case three equations. Also, it should be understood that the sensitivity constant for the $(\Delta P_c/P_c)^2$ term of the method three, case four sensitivity equations is 0.0000039 and not zero as implied by Table I.

Since the sensitivity equations pertain to idealized nozzle flow equations, a derating factor uncertainty must be incorporated into the sensitivity equations to account for our inability to predict, exactly, how the performance of the actual nozzle deviates from the ideal model. The various 3-sigma derating factor uncertainties are developed in reference 2 and summarized below for the full power operating point.

1.	One-dimensional equilibrium performance	- - - - -	$\pm 0.14\%$
2.	Divergence loss	- - - - -	$\pm 0.17\%$
3.	Boundary layer loss	- - - - -	$\pm 0.50\%$
4.	Kinetic loss	- - - - -	$\pm 0.10\%$
5.	Nozzle contour imperfections	- - - - -	$\pm 0.05\%$
6.	Nozzle vibration	- - - - -	$\pm 0.05\%$
7.	Exhaust velocity and temperature gradients	- - - - -	$\pm 0.05\%$
	Total RSS statistical summation	- - - - -	$\pm 0.56\%$

Nozzle flow sensitivity equations similar to those presented in Table I can be used to calculate nozzle chamber accuracy requirements. As an example, assume that the I_{sp} equation of method one were adopted. Then the accuracy required of the T_c measurement could be approximated by

$$\left(\frac{\Delta T_c}{T_c}\right) \approx \pm 2.17 \left\{ \left(\frac{\Delta I_{sp}}{I_{sp}}\right)^2 - 1.15 \left(\frac{\Delta \gamma}{\gamma}\right)^2 - (0.56)^2 \right\}^{1/2}$$

where $(\Delta I_{sp}/I_{sp})$ is the required specific impulse computation uncertainty. However, the T_c uncertainty value $(\Delta T_c/T_c)$ obtained through the use of a nozzle

flow sensitivity equation is not the requirement to be assigned to individual measurement channels. The nozzle chamber temperature is actually the calculated effective mixed mean stagnation nozzle chamber temperature whose value is calculated from data taken at the core exit plane and at the nozzle convergent section. This report shall be limited to a discussion of measurement requirements imposed on the thermocouples located in the nozzle convergent section. The approximate location of these thermocouples is shown in Figure 1.

Some of the factors that must be accounted for in the computation of the effective mean nozzle chamber stagnation temperature are:

1. The thermocouples are not located in the nozzle chamber so that extrapolation techniques must be developed and applied.

2. The thermocouple is not a total temperature probe. The static temperature is the temperature which would be measured by a sensor moving with the exhaust stream. The total temperature is always higher, and is the result of compressive heating which appears whenever the exhaust stream is brought to rest adiabatically about a sensor. The thermocouple probe will actually attain a temperature that is between the static and total temperature. Thus, thermocouple data correction is necessary to interpret local stagnation temperatures by considering the construction and geometry of the probe and local thermal and velocity gradients within the gas stream near the nozzle wall.

3. The exhaust gas emanating from the core exit can be thought of as an assemblage of concentric tubes with each tube being characterized by individual temperature and velocity levels. Thus large radial and circumferential asymmetries may exist at the core exit. As the exhaust gas moves toward the nozzle throat, mixing will occur and the thermal and velocity gradients will tend to dissipate. However, the presence of gradients at the thermocouple station must be accounted for.

Figure 1. Location of Nozzle Chamber Thermocouple Relative to the Actual Nozzle Chamber Region

4. Core exit thermocouples measure the gas stream temperature of individual flow holes and are, therefore, distinct separate measurements. If a simple arithmetic average core exit gas stream temperature were to be calculated, the uncertainty of the average value would be the same as that of each individual channel, assuming that all channels possessed the same uncertainty. Gas stream mixing occurs as the gas leaves the core and moves toward the nozzle throat. If mixing were complete by the time the gas stream reached the thermocouples located in the convergent section (see Figure 1), then the statistical uncertainty of the average convergent section gas stream temperature would be $U/\sqrt{N}$ where U is the uncertainty of individual channels and N is the number of channels. Since mixing is not complete, the average convergent section gas stream temperature uncertainty (U_g) is located somewhere within the interval ($U/\sqrt{N} < U_g < U$). Stated differently, it may be possible to estimate the uncertainty of average convergent section temperature data by $U_g = UN^{-s}$ where s is a statistically determined variable whose value is bounded by $0 < s < 0.5$. In any event, this uncertainty must be accounted for along with all other error factors when computing the effective nozzle chamber temperature values to be supplied the control system as feedback data.

It could be argued that it is not necessary to provide an elaborate computation to provide T_c data as a simple arithmetic average was used for XE-P testing and engine control was maintained at the desired full power state point (4090°R). This is false logic as the engine control system was actually used to maintain an average thermocouple junction temperature of 4090°R. Since the relationship between the average junction temperature and the true mixed mean stagnation temperature was never established, the actual nozzle chamber temperature was probably higher than that indicated by the thermoelectric voltage level being maintained by the engine control system. This fact could well account for the -1.7% nozzle chamber temperature bias error that could not be accounted for when analyzing XE-P test data (Reference 3).

In summary, if the engine control system is to provide a $\pm 0.75\%$ specific impulse controllability tolerance, the following sequence of events must occur.

1. A rigorous study must be implemented to generate a valid technique for calculating I_{sp} . The equations of Table I are inadequate.

2. A parametric sensitivity equation must be developed for the I_{sp} equation developed in step one above.

3. A maximum permissible nozzle chamber temperature uncertainty allocation must be made from the parametric sensitivity equation derived in step two above. The sensitivity equations of Table I are inadequate.

4. A rigorous study must be implemented to generate a technique for calculating the effective nozzle chamber temperature from core exit and nozzle convergent section measurements.

5. Based upon the T_c uncertainty allocation made in step 3, the computation error associated with the T_c equation developed in step 4, and the number and distribution of thermocouple sensors, individual thermocouple channel uncertainty tolerances must be established.

6. Based upon the maximum allowed channel uncertainty tolerances derived in step 5 above, static and dynamic performance requirements must be allocated to individual data channel components and data processing routines.

Since steps 1 through 5 have not yet been completed, it is impossible to make a proper evaluation of nozzle chamber temperature accuracy requirements. All that can be done is to unilaterally establish a preliminary performance goal and hope that it will represent the best that can be achieved without initiating expensive component development programs. Therefore, the $\pm 1\%$ preliminary performance goal established in reference 1 for the calculated mixed mean nozzle chamber temperature uncertainty will be retained. It is

recognized, however, that the $\pm 1\%$ nozzle chamber temperature uncertainty allocation will not guarantee the attainment of a $\pm 0.75\%$ I_{sp} computation tolerance. To demonstrate this point, 3-sigma parametric uncertainty values are abstracted from reference 1, reference 2, and reference 4 and summarized in Table II.

Table II. Parametric Uncertainties

Nozzle Derating Factor Uncertainty - - - - -	$\pm 0.56\%$
Specific Heat Ratio Uncertainty at 4250°R ($\Delta\gamma/\gamma$) - - -	$\pm 4.16\%$
Throat Area Uncertainty ($\Delta A_t/A_t$) - - - - -	$\pm 0.30\%$
Expansion Ratio Uncertainty ($\Delta\epsilon/\epsilon$) - - - - -	$\pm 0.30\%$
Nozzle Chamber Temperature Uncertainty ($\Delta T_c/T_c$) - - -	$\pm 1.00\%$
Nozzle Chamber Pressure Uncertainty ($\Delta P_c/P_c$) - - - - -	$\pm 0.35\%$
Nozzle Chamber Enthalpy Uncertainty at 4250°R ($\Delta H_c/H_c$)	$\pm 0.47\%$
Nozzle Exit Enthalpy Uncertainty at 738°R ($\Delta H_e/H_e$) - -	$\pm 0.47\%$
Propellant Weight Flow Rate Uncertainty ($\Delta\dot{w}/\dot{w}$)	
Cryogenic Calibration with Line Sections - - - -	$\pm 0.60\%$
Water Calibration, Linear Range Only - - - - -	$\pm 3.75\%$

Note that the "specific heat ratio" uncertainty ($\pm 4.16\%$) exceeds the $\pm 2.77\%$ value reported in reference 1. This resulted from a misinterpretation of the NBS hydrogen property uncertainty plots contained in reference 4. It was assumed that the plots represented 3-sigma data, when in fact, the source error plots represent 2-sigma data as reported in reference 5. Thus, the 3-sigma error value is $1.5 \times 2.77\%$ or 4.16% . The same correction is used to obtain the 3-sigma source uncertainty values for enthalpy.

Inserting the parametric error values of Table II into the sensitivity equations of Table I yields an estimate of thrust and specific impulse computation uncertainties. These computation uncertainties are summarized in Table III.

Table III. Approximate Full Power Engine Performance Parameter Computation Uncertainties (3-sigma)

Equation Group	Uncertainty Parameter	Uncertainty Value	
		$\Delta\dot{w}/\dot{w} = \pm 0.6\%$	$\Delta\dot{w}/\dot{w} = \pm 3.75\%$
Method One	$\Delta F/F$	$\pm 3.078\%$	$\pm 3.078\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 4.548\%$	$\pm 4.548\%$
Method Two	$\Delta F/F$	$\pm 3.095\%$	$\pm 3.197\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 3.178\%$	$\pm 5.518\%$
Method Three, Case One	$\Delta F/F$	$\pm 4.550\%$	$\pm 5.813\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 4.512\%$	$\pm 4.513\%$
Method Three, Case Two	$\Delta F/F$	$\pm 3.078\%$	$\pm 3.078\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 3.136\%$	$\pm 4.855\%$
Method Three, Case Three	$\Delta F/F$	$\pm 2.477\%$	$\pm 4.378\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 2.407\%$	$\pm 2.409\%$
Method Three, Case Four	$\Delta F/F$	$\pm 5.605\%$	$\pm 7.321\%$
	$\Delta I_{sp}/I_{sp}$	$\pm 5.547\%$	$\pm 5.547\%$

The data of Table III confirms the conclusions reached in references 2 and 5 concerning the relative suitability of various nozzle flow equations. That is, the equations of method three, case three (the enthalpy equations) will yield the most accurate determination of specific impulse. However, even the enthalpy equations are not able to satisfy the $\pm 0.75\% I_{sp}$ tolerance required by the NPRD. It may be possible to satisfy the I_{sp} tolerance by combining inertial guidance system data and enthalpy equation data.

Another point that should be noted is that the thrust and specific impulse computation uncertainty values for the equations of method three, case four include a -2.471 bias error. This error term accounts for the fact that these equations fail to account for the pressure-area thrust component delivered at the nozzle exit plane (refer to Equation 1 of Appendix A).

In summary, a $\pm 1\%$ uncertainty shall be assigned as a preliminary performance goal for the effective mixed mean nozzle chamber stagnation temperature. This uncertainty is a total error envelope (bias plus three sigma random error) and is based upon experience, judgement, and a knowledge of potential techniques that may be adopted to minimize instrumentation errors. As a result, the $\pm 1\%$ uncertainty assignment is a matter of personal opinion and is not based upon positive knowledge.

B. BANDWIDTH REQUIREMENTS, EFFECTIVE MIXED MEAN NOZZLE CHAMBER STAGNATION TEMPERATURE

No firm bandwidth requirement can be assigned until after a complete control system engine model has been implemented and verified. However, a tentative evaluation has been made of the anticipated bandwidth and it appears to be extremely marginal. Since the bandwidth of the channel is, to a large extent, governed by the response of the thermocouple probe, every effort, short of initiating an extensive thermocouple development program, should be made to reduce the thermocouple time constant.

The equation presently employed to approximate the effective thermocouple first-order-system time constant is

$$\tau = 4.207 \dot{\omega}^{-0.5} \text{ seconds}$$

Due to the marginal nature of the probe time constant, a study was made to see if other equations might be more suitable. This study (enclosed as Appendix B of this report) revealed the existence of a discontinuity in the

weight-flow, time-constant relationship at the throttled state point (see Figure 3, Appendix B). Thus two new equations are recommended for describing the effective thermocouple time constant. These equations are derived in Appendix B and summarized below for convenience.

$$\text{Throttled to full power } \tau = 1.00 e^{-0.007025\dot{\omega}} \text{ sec} \quad (1)$$

$$\text{Start-up to throttled power . . . } \tau = 5.89 e^{-0.03874\dot{\omega}} \text{ sec} \quad (2)$$

An equation was also developed for pulse cooldown. However, many of the assumptions made to simplify the derivation are not completely valid. As a result, the equation only provides a gross approximation of the cooldown time constant. It should be useful, however, in estimating system response characteristics and is presented below.

$$\text{Pulse cooldown } \tau = 10.7 (\dot{\omega})^{-0.466} \text{ sec} \quad (3)$$

The approximate time constant, frequency response, and steady state ramp error values corresponding to various operating state points are summarized in Table IV.

TABLE IV

ESTIMATED TIME CONSTANT, BANDWIDTH,
AND STEADY STATE RAMP ERROR

Operating Realm	T _c (°R)	P _c (lb/in. ²)	$\dot{\omega}$ (lb/sec)	τ (sec)	f _{co} (Hz)	ΔT (°R)
Throttled to full power (T _c = 0 °R/sec)	4250	450	91.4	0.53	0.30	0
	4250	400	80.6	0.57	0.28	0
	4250	350	70.4	0.61	0.26	0
	4250	300	60.4	0.65	0.24	0
	4250	270	54.8	0.68	0.23	0
Start-up to throttled power (T _c = 150 °R/sec)	4250	270	54.8	0.68	0.23	102
	3200	200	46.5	1.03	0.15	154
	2000	120	36.0	1.51	0.11	226
	1400	80	28.0	1.99	0.08	298
	800	40	19.0	2.95	0.05	442
Pulse Cooldown	1290	1	0.4	16.40	0.01	N.A.

The time constant values listed in Table IV for start-up to full power operation will differ slightly from those predicted by Equations 1 and 2. The discrepancy exists because the time constant values presented in Table IV were calculated with a more comprehensive expression (Equation 6 of Appendix B) that accounted for gas stream viscosity (μ_f) and thermal conductivity (k_f) variations. It should be noted that Equation 2 will yield less accurate results if actual start-up and throttled power state points differ from those assumed in Table IV. If different state points are adopted, a new start-up to throttled power time constant equation should be derived. This is simply accomplished by using Equation 6 of Appendix B to calculate at least five new time constant values. Plot these new values as a function of the propellant flow rate on semi-log paper, draw a best-fit straight line through the points as shown in Figure 3 of Appendix B, take two points off the best-fit straight line and solve for the constants a and b of Equation 4 to obtain the desired expression.

$$\tau = a e^{-b\dot{\omega}} \quad (4)$$

The time constant values listed in Table IV differ slightly from those presented in Reference 6 (0.53 sec at full power versus 0.44 sec). This difference is not considered to be significant as both values were derived after making many idealized assumptions concerning the geometry of the probe, average material properties, and heat transfer mechanisms. More reliable values will be possible after the thermal boundary layer and velocity gradient adjacent to the nozzle wall have been more accurately defined.

The large probe time constant encountered during pulse cooldown presents a special problem. The cooldown pulse width narrows to approximately 40 seconds near the end of the pulse cooldown phase. This represents 2.44 thermocouple probe time constants. Thus, the measurement error due to the response characteristics of the probe is still -8.72% at the termination of the cooldown pulse.

It may be possible to apply data correction techniques to obtain a better estimate of nozzle chamber temperature data during pulse cooldown.

Figure 2. Transient Thermocouple Measurement Error (T_e)

From Figure 2, the difference between actual and indicated nozzle chamber temperatures is

$$T_e = (T_c - T)$$

where T_e represents the instantaneous measurement error. If the error term were known, the true nozzle chamber temperature could be determined from

$$T_c = T + T_e$$

As described in Appendix B, thermal balance equations for thermocouple probes can be adopted to describe the temperature difference ($T_c - T$) as shown below

$$T_c - T = \frac{Y}{h} \left(\frac{dT}{dt} \right) + \frac{P}{h} (T - T_o)$$

where: Y = Thermocouple mass factor. Its magnitude is dependent upon the surface area, specific heat, and density of the thermocouple's junction region. It is descriptive of the ability of the thermocouple junction to absorb and retain heat.

h = Unit effective thermal conductance between the environment and the thermocouple junction. Its magnitude is dependent upon gas velocity, density, temperature and turbulence. It is descriptive of the ability of the gas to transfer convective heat to the thermocouple's junction.

P = Thermocouple's mean thermal loss factor. Its magnitude is dependent upon the effective thermal conductance of the thermocouple probe assembly and is descriptive of the probe's ability to conduct heat away from its hot junction.

Thus, the actual gas stream temperature could be estimated from

$$T_c = T + [K_1(dT/dt) + K_2(T-T_o)]$$

where K_1 and K_2 are defined by (Y/h) and (P/h) , respectively. Although the magnitude of Y and P may be readily determined by calorimetric techniques, the value of h is extremely difficult to define. An approach that could be adopted would experimentally define the data correction coefficients K_1 and K_2 during nozzle flow calibration tests or from actual test firing data. The problems associated with the use of differentiated signals as feedback control data is appreciated. Thus, this technique is only offered for consideration and is not endorsed as a solution.

An alternate means for calculation nozzle chamber temperatures during pulse cooldown would require the employment of a flow meter in the cooldown supply line to measure the mass flow rate of propellant through the nozzle ($\dot{w}$). Use would then be made of a nozzle flow equation such as that shown below:

$$T_c = \frac{\gamma}{R} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{g^P c A_t}{\dot{w}} \right)^2$$

In summary, the bandwidth of nozzle chamber temperature data is marginal in that it appears to be close to the bandwidth of actual nozzle chamber temperature variations. In addition, the channel bandwidth is established by the nozzle chamber thermocouple. Thus every effort must be made to improve the thermocouple time constant. The time constant equation presently used provides a good approximation of time constant values. However, more accurate equations are available and have been presented as Equations 1, 2, and 3 earlier in the section. The thermocouple time constant is completely inadequate during pulse cooldown. As a result, either thermocouple data correction techniques or nozzle flow equations must be adopted.

II. INDIVIDUAL NOZZLE CHAMBER TEMPERATURE CHANNEL REQUIREMENTS

The effective mean nozzle chamber stagnation temperature is calculated from individual thermocouple temperature measurements made at the core exit and within the nozzle convergent section. This discussion will only consider the performance of those data channels employed to measure gas stream temperatures in the nozzle convergent section. Although an extensive list of error mechanisms could be compiled, only the major areas of concern will be covered here. These major areas are summarized in Table V.

Table V. Major Channel Uncertainty Sources

System Components	Uncertainty Source	Source Description
Thermocouple	A-1 A-2	Thermocouple-environment coupling uncertainty Junction Temp.-emf. correlation uncertainty
Wiring Harness	B-1 B-2 B-3 B-4 B-5	Cable transition induced uncertainty Wire inhomogeneity induced uncertainty Electromagnetic noise signal uncertainty Reference junction thermal uncertainty Reference junction temp. sensor uncertainty
Data Acquisition Subsystem	C-1 C-2 C-3 C-4	Miscellaneous error sources Channel gain, linearity, and zero offset unc. Data sampling induced uncertainty Quantization induced uncertainty
Computer, Data Pre-processor	D-1 D-2 D-3 D-4	Reference junction compensation uncertainty Signal linearizing, scaling, and zero adjust uncertainty. Thermocouple-environment coupling correction uncertainty Nozzle chamber temperature computation unc.

A general discussion of the various components and their associated error sources shall now be presented along with specific performance requirement allocations.

A. THERMOCOUPLE ASSEMBLY REQUIREMENTS

The present nozzle chamber thermocouple design is analyzed in reference 6 and illustrated in Figure 3. The sheath is an arc-cast tungsten-rhenium alloy (W26Re) that is electro-polished to remove surface effects. The probe incorporates four sensory junctions that are fusion welded to the sheath to assure maximum speed of response. As a result, each junction is electrically grounded to the nozzle through the sheath. Each thermocouple wire pair will extend from the measurement junction to the reference junction zone box without a splice or connector. As presently planned, the eight thermocouple leads will leave the probe base as two four conductor, MgO insulated, steel sheathed cables which will be flame sprayed to the outside of the pressure vessel. Beryllia beads are used to insulate the thermocouple wires within the probe.

The four junctions shown in Figure 3 are primarily provided to establish the thickness of the thermal boundary layer which may extend anywhere from 0.5 to 3.0 in. from the inner nozzle wall. Once the thermal boundary layer is defined, it may be possible to reduce the number of sensing junctions. It may also be possible to shorten the probe and reduce its thickness which would reduce its thermal time constant (increase its signal bandwidth). As a result, the final configuration of the flight engine nozzle chamber thermocouple will probably resemble that of Figure 3 but may contain fewer junctions and possess improved response characteristics. However, the computation of the mixed mean nozzle chamber temperature may require the retention of all junctions to better define temperature gradients.

There are two primary steady-state error mechanisms associated with thermocouple probes. These are listed in Table V and summarized below:

1. Thermocouple-Environment Coupling Uncertainty

When a thermocouple is immersed in a fluid stream, the temperature attained by its junction is determined by: (a) the rate of heat transfer

FIGURE 3. Multijunction Nozzle Chamber Thermocouple

from the fluid to the thermocouple junction by forced convection; (b) the rate of heat accumulation within the thermocouple probe; and (c) the rate of heat transfer away from the measuring junction by heat conduction through the thermocouple support. Thus the magnitude of temperature differences existing between the thermocouple junction and the fluid medium depends upon the geometry and physical properties of the thermocouple probe as well as upon the properties and character of the fluid medium.

The magnitude of the thermo-coupling error is thought to be small for the junction closest to the probe tip when the engine is operating at or above the throttled power point. Larger errors are expected for thermocouple junctions located closer to the wall. To understand this point, note that the maximum gas-side nozzle wall temperature will be about 1960°R while the main gas stream temperature is 4250°R at the full power operating point. Thus, a large temperature gradient is developed along the probe that is accompanied by a large heat flow rate. Also, the size of the temperature gradient and the heat flow rate will increase as the nozzle wall is approached, and so will the accompanying heat flow induced temperature bias error at a thermocouple junction. Thus, stem conduction errors become larger as the nozzle wall is approached. Another problem contributing to larger coupling errors is the larger uncertainty associated with gas stream thermal gradients and velocity profiles near the nozzle wall.

2. Junction Temperature, Junction Emf Correlation Uncertainty

The object of calibrating thermocouples is to establish an emf-temperature relationship (calibration curve). The calibration of thermocouples may be divided into two general classes; calibration at fixed points such as the freezing or boiling points of various materials, or calibration by comparison with secondary standards such as thermocouples, resistance thermometers, or optical pyrometers. The nozzle chamber thermocouple will employ an optical pyrometer as a secondary standard for high temperature calibrations. Thus, the calibration uncertainty will be a function of the absolute accuracy of pyrometric measurements and the absolute accuracy of corresponding thermoelectric voltage readings. Thermocouple assembly requirements and recommendations are now presented.

3. Thermocouple Assembly Accuracy Requirements

Based upon past performance and anticipated boundary layer uncertainties, absolute full power measurement uncertainty requirements are assigned and summarized in Table VI.

Table VI. Absolute, Full Power, Steady-State Nozzle Chamber Thermocouple Uncertainty Allocations Expressed as a Percent of the Temperature Level

Distance of the Junction from the Nozzle Wall	1 inch	2 inches	3 inches	4 inches
Absolute Measurement Uncertainty Allocation	$\pm 3.0\%$	$\pm 2.0\%$	$\pm 1.5\%$	$\pm 1.0\%$

There is not enough information currently available to assign uncertainty allocations for lower engine operating levels. It must be understood that the uncertainty tolerances of Table VI express the three sigma deviation between the gas stream temperature existing at the indicated distance from the wall and the temperature listed for the thermoelectric voltage being generated by the corresponding thermocouple junction.

4. Thermocouple Assembly Bandwidth Requirements

The bandwidth requirement for the junction nearest the probe tip shall be that anticipated for the existing design until more firm requirements are generated by the controls analysis section. Thus, the probe bandwidth requirements shall be those listed in Table IV for throttled power to full power operating excursions.

5. Recommended Thermocouple Assembly Studies

It is recommended that a thorough study be implemented at the earliest possible date to define thermal gradients and velocity vectors about each thermocouple junction for all normal operating state points of interest. This data should then be used to define the thermal time constant and static accuracy of each junction under all normal operating conditions.

B. WIRING HARNESS REQUIREMENTS

As presently envisioned, each of the two cables leaving the base of each nozzle chamber thermocouple will contain two W5Re/W26Re thermocouple wire pairs that are insulated with MgO insulation and covered with a stainless steel sheath. The sheathed cables will terminate at the thermocouple reference junction unit. The flexible cables leaving the reference junction unit will be routed to the Data Acquisition Assembly.

The primary error mechanisms associated with the Wiring Harness are listed in Table V and summarized below.

1. Cable Transition Induced Uncertainties

A typical thermocouple circuit contains thermocouple cable splices between the measurement and reference junctions as well as several copper cable splices between the reference junction and the data system input. It should be noted that all wire samples having the same nominal composition do not possess identical emf-temperature characteristics. This should be obvious when one considers the impossibility of controlling, exactly, the chemical and physical properties of wire. Thus, a typical thermocouple circuit employs several discrete wire sections, each of which will usually differ thermoelectrically from all other sections. Because of these thermoelectric differences, a small voltage will be generated at each cable transition or splice. Approximate splice induced uncertainties are listed in Table VII.

Table VII. Approximate Splice Induced Microvolt Uncertainty for Thermocouple Cables

Splice Temperature	Thermocouple Cable Type			
	C/C	C/A	W/W26Re	W5Re/W26Re
-300°F	$\pm 37.91 \mu\text{V}$	$\pm 72.58 \mu\text{V}$		
-200°F	$\pm 35.29 \mu\text{V}$	$\pm 69.25 \mu\text{V}$		
-100°F	$\pm 18.41 \mu\text{V}$	$\pm 25.68 \mu\text{V}$		
0	$\pm 14.64 \mu\text{V}$	$\pm 41.11 \mu\text{V}$		
32°F	$\pm 16.01 \mu\text{V}$	$\pm 43.96 \mu\text{V}$	$\pm 11.20 \mu\text{V}$	$\pm 55.76 \mu\text{V}$
200°F	$\pm 22.00 \mu\text{V}$	$\pm 48.53 \mu\text{V}$	$\pm 33.24 \mu\text{V}$	$\pm 75.20 \mu\text{V}$
400°F	$\pm 60.94 \mu\text{V}$	$\pm 53.70 \mu\text{V}$	$\pm 45.60 \mu\text{V}$	$\pm 91.22 \mu\text{V}$
600°F	$\pm 102.82 \mu\text{V}$	$\pm 58.87 \mu\text{V}$	$\pm 76.69 \mu\text{V}$	$\pm 100.71 \mu\text{V}$
1000°F		$\pm 116.61 \mu\text{V}$	$\pm 133.17 \mu\text{V}$	$\pm 139.5 \mu\text{V}$
2000°F		$\pm 218.36 \mu\text{V}$	$\pm 332.15 \mu\text{V}$	$\pm 259.96 \mu\text{V}$

Table VII was abstracted from reference 7 and the thermocouple cable type designations are identified below:

C/C — — — — — Copper/Constantan

C/A — — — — — Chromel/Alumel

W/W26Re — — — — Tungsten/Tungsten - 26% Rhenium

W5Re/W26Re — — — — Tungsten - 5% Rhenium/Tungsten - 26% Rhenium

The splice induced uncertainty of copper wire cable is essentially constant at $\pm 4.6 \mu\text{V}$ over wide temperature ranges (reference 8). The microvolt uncertainty induced by a combination of cable splices can be arrived at by applying RSS summation techniques. As an example, if a channel incorporated two W5Re/W26Re cable splices at 200°F and three copper wire cable splices, the total probable splice induced uncertainty would be

$$UNC = \pm \sqrt{2 (75.20)^2 + 3 (4.6)^2} = \pm 106.65 \text{ microvolts}$$

Before the equivalent cable transition induced measurement uncertainty in degrees Rankine can be established, it is necessary to determine the thermoelectric sensitivity ($^{\circ}\text{R}/\mu\text{V}$) at the temperature level being monitored and multiply it by the splice induced microvolt uncertainty. As an example, assume that the temperature level being monitored is 4250°R where the thermoelectric sensitivity of W5Re/W26Re thermocouples is approximately $0.1626^{\circ}\text{R}/\mu\text{V}$. Then, if two W5Re/W26Re cable transitions and three copper cable transitions were incurred, the probable three sigma data uncertainty due to the effects of splicing would be

$$UNC = \pm (106.65) (0.1626) = \pm 17.34^{\circ}\text{R}$$

which is approximately $\pm 0.4\%$ of the temperature level being measured. If it can be assumed that the maximum splice temperature is 200°F and that the temperature level being monitored is 4250°R , then Table VIII may be used to estimate splice induced errors for nozzle chamber thermocouples when monitoring 4250°R .

Table VIII. Approximate Splice Induced Errors for Nozzle Chamber Thermocouples at 4250°R

Number of Copper Cable Transitions After the Reference Junction	Number of W5Re/W26Re Cable Transitions Between the Measurement and Reference Junction				
	0	1	2	3	4
0	$\pm 0.00^{\circ}\text{R}$	$\pm 12.23^{\circ}\text{R}$	$\pm 17.29^{\circ}\text{F}$	$\pm 21.18^{\circ}\text{R}$	$\pm 24.46^{\circ}\text{R}$
1	$\pm 0.75^{\circ}\text{R}$	$\pm 12.25^{\circ}\text{R}$	$\pm 17.31^{\circ}\text{R}$	$\pm 21.19^{\circ}\text{R}$	$\pm 24.47^{\circ}\text{R}$
2	$\pm 1.12^{\circ}\text{R}$	$\pm 12.27^{\circ}\text{R}$	$\pm 17.32^{\circ}\text{R}$	$\pm 21.21^{\circ}\text{R}$	$\pm 24.48^{\circ}\text{R}$
3	$\pm 1.30^{\circ}\text{R}$	$\pm 12.30^{\circ}\text{R}$	$\pm 17.34^{\circ}\text{R}$	$\pm 21.22^{\circ}\text{R}$	$\pm 24.49^{\circ}\text{R}$
4	$\pm 1.50^{\circ}\text{R}$	$\pm 12.32^{\circ}\text{R}$	$\pm 17.36^{\circ}\text{R}$	$\pm 21.23^{\circ}\text{R}$	$\pm 24.50^{\circ}\text{R}$
5	$\pm 1.67^{\circ}\text{R}$	$\pm 12.34^{\circ}\text{R}$	$\pm 17.37^{\circ}\text{R}$	$\pm 21.24^{\circ}\text{R}$	$\pm 24.51^{\circ}\text{R}$
6	$\pm 1.83^{\circ}\text{R}$	$\pm 12.36^{\circ}\text{R}$	$\pm 17.39^{\circ}\text{R}$	$\pm 21.26^{\circ}\text{R}$	$\pm 24.52^{\circ}\text{R}$
7	$\pm 1.98^{\circ}\text{R}$	$\pm 12.39^{\circ}\text{R}$	$\pm 17.41^{\circ}\text{R}$	$\pm 21.27^{\circ}\text{R}$	$\pm 24.53^{\circ}\text{R}$
8	$\pm 2.12^{\circ}\text{R}$	$\pm 12.41^{\circ}\text{R}$	$\pm 17.42^{\circ}\text{R}$	$\pm 21.28^{\circ}\text{R}$	$\pm 24.55^{\circ}\text{R}$
9	$\pm 2.24^{\circ}\text{R}$	$\pm 12.43^{\circ}\text{R}$	$\pm 17.44^{\circ}\text{R}$	$\pm 21.30^{\circ}\text{R}$	$\pm 24.56^{\circ}\text{R}$
10	$\pm 2.37^{\circ}\text{R}$	$\pm 12.45^{\circ}\text{R}$	$\pm 17.45^{\circ}\text{F}$	$\pm 21.31^{\circ}\text{R}$	$\pm 24.57^{\circ}\text{R}$

The number of copper wire cable transitions includes all transitions between the reference junction and the first signal conditioning element (amplifier, analog-to-digital converter, etc.) in the Data Acquisition Subsystem.

As seen in Table VIII, the existence of even one W5Re/W26Re cable transition can produce a significant measurement uncertainty ($\pm 0.3\%$ of 4250°R). Thus, it is recommended that the thermocouple leads extend from the measurement junction to the reference junction without a transition even though this approach will complicate the design of the reference junction unit. This recommendation is necessitated by the extreme channel accuracy requirements imposed by specific impulse computation tolerances. Therefore, this report will assume that no W5Re/W26Re cable transitions will exist before the reference junction but that ten copper cable transitions will be encountered to yield an estimated total cable transition induced uncertainty of 2.5°R when full power nozzle chamber temperatures are being measured (a $\pm 0.06\%$ transition uncertainty is assumed).

2. Wire Inhomogeneity Induced Uncertainties

Wire materials are not completely uniform in both chemical composition and physical condition throughout their length and the effects of this inhomogeneity must be considered when designing thermocouple circuits. When a length of inhomogeneous wire is located in regions characterized by large temperature gradients, spurious emf's are generated that can be significant when compared to normal thermocouple signal levels. The effects of inhomogeneity in both wires of a thermocouple circuit may be either additive or subtractive and, as the emf developed along an inhomogeneous wire depends upon local temperature distributions and wire characteristics, attempts to predict inhomogeneity effects are impractical if not impossible.

In general, thermocouple wire produced by manufacturers in this country is sufficiently homogenous in chemical composition for most

purposes. More often, physical inhomogeneity is introduced by inadequate post fabrication annealing and improper installation handling. Although the use of stranded wire should minimize induced inhomogeneity, consideration should be given to checking the homogeneity of thermocouple cable samples by inducing temperature gradients similar to those expected in service. No uncertainty assignment can be made to account for wire inhomogeneity until actual sample testing has been accomplished.

3. Electromagnetic Noise Signal Coupling Uncertainties

Noise in an instrumentation system is defined as a response in the system that is not data. For practical purposes, the types of noise that are of concern for the nozzle chamber thermocouple cables originate from adjacent cables that are carrying power or high frequency carrier signals and from the various actuator assemblies. Thermal noise is always present but will be ignored. Anyone who has a system which is limited in performance by cable thermal noise is either very fortunate or is making unusual measurements. The effects of charge generation by ionizing radiation fields and triboelectric phenomenon shall also be ignored as noise signals resulting from these effects are usually small when dealing with the low source impedance of thermocouple circuits.

Thermocouple circuits are particularly sensitive to electromagnetic signal coupling for several reasons. First, a thermocouple circuit consists of two wires that differ in their respective resistivity values. As a result, this inherent resistive unbalance reduces the effectiveness of common mode rejection techniques. This problem could be minimized by equalizing distributed wire resistance values. This could be accomplished by increasing the diameter of the more resistive W26Re wire relative to that of the W5Re wire. Secondly, sheathed thermocouple cable is very sensitive to magnetic coupling as the wires are not normally twisted. The effects of magnetic coupling can be reduced by using sheath materials possessing high magnetic permeability values. Nickel base alloys fall under this category. The third, and most obvious

reason, results from the low signal levels associated with thermocouple circuits. Even a small amount of coupling can significantly degrade the channels signal to noise ratio.

A magnetic field source is a loop of wire carrying a large current. Thus the cables delivering power to the various actuator assemblies will be a source of potentially strong magnetic fields if the cable wires are not tightly twisted.

An electric field source is a straight wire excited with high frequency voltage signals. Thus the cables carrying high frequency data from the eddy current transducers can be a source of strong electric fields within connectors if inter-connector shields are not employed.

It is, therefore, important to provide the greatest possible separation between the nozzle chamber thermocouple cables and the cables used to transmit power or high frequency data signals. In particular, thermocouple leads must never share a connector with power or high frequency data channels. In any event, the amount of electromagnetic noise coupled into nozzle chamber thermocouple cables can not be predicted until all electromagnetic field generators are identified and the orientation of thermocouple cables relative to these generators known.

4. Reference Junction Thermal Uncertainty

It shall be assumed that the thermocouple reference junction unit will incorporate a metal block that will establish the reference junction temperature for one or more channels. It is further assumed that a platinum resistance sensor will be used to monitor block temperature variations.

There are three primary thermal uncertainty sources. The first results from heat being delivered to or being removed from the reference

junction by heat conduction along individual lead wires. The second results from temperature gradients within the block between individual reference junctions. The third is the temperature difference existing between the platinum resistance grid used to monitor block temperatures and the actual mean block temperature. None of these error sources can be described quantitatively as a reference junction design does not exist. However, a $\pm 0.05^{\circ}\text{R}$ reference junction thermal uncertainty does not seem unreasonable and shall be assumed for this report.

5. Reference Junction Temperature Sensor Uncertainty

The reference junction temperature sensor uncertainty arises from several factors. The first would normally be the temperature difference existing between the platinum grid and the mean block temperature. However, this error was considered earlier as a thermal error mode and will not be included here under sensor uncertainty. Thus, the first sensor error mode will be induced by mechanical strain inadvertently coupled across the grid-block interface. The second is the uncertainty in the assumed grid resistance-temperature correlation used to determine block temperatures from measured grid resistance values. Sensor uncertainty values are not known as the sensor has not yet been designed. However, it shall be assumed that a $\pm 0.25^{\circ}\text{R}$ sensor uncertainty can be achieved through the use of individual sensor calibration curves.

The total temperature uncertainty of the reference junction unit is the RSS summation of the thermal error ($\pm 0.05^{\circ}\text{R}$) and the sensor error ($\pm 0.25^{\circ}\text{R}$) which is $\pm 0.255^{\circ}\text{R}$. The microvolt sensitivity of the reference junction relative to that of the measurement junction must be known before the effect of a $\pm 0.255^{\circ}\text{R}$ reference junction error is established. As an example, if the reference junction temperature were 200°F (660°R) where the thermocouple

sensitivity is $8.275 \mu\text{V}/^\circ\text{R}$, a $\pm 0.255^\circ\text{R}$ reference junction temperature uncertainty would correspond to a $\pm 2.110 \mu\text{V}$ uncertainty. But the thermoelectric sensitivity of a W5Re/W26Re thermocouple junction at 4250°R is $0.1626^\circ\text{R}/\mu\text{V}$ so that a $\pm 2.110 \mu\text{V}$ signal uncertainty yields a $\pm 0.343^\circ\text{R}$ data uncertainty. Thus, a $\pm 0.255^\circ\text{R}$ uncertainty in the temperature of a reference junction at 200°F appears as a $\pm 0.343^\circ\text{R}$ uncertainty in the temperature of a measurement junction at 4250°R .

The measurement junction error corresponding to various reference junction temperatures is presented in Table IX.

Table IX. Measurement Errors for a 4250°R Measurement When a $\pm 0.255^\circ\text{R}$ Reference Junction Temperature Uncertainty Exists and Various Reference Junction Temperatures are Assumed

Ref. Junction Temperature	100°R	200°R	300°R	400°R	500°R	660°R
Induced Data Uncertainty	$\pm 0.075^\circ\text{R}$	$\pm 0.137^\circ\text{R}$	$\pm 0.196^\circ\text{R}$	$\pm 0.247^\circ\text{R}$	$\pm 0.289^\circ\text{F}$	$\pm 0.343^\circ\text{R}$

From Table IX, it is seen that reference junction errors become larger as the reference junction temperature increases. Assuming that 660°R is the highest temperature attained by a passive reference junction in a space environment, the allotted reference junction uncertainty shall be $\pm 0.35^\circ\text{R}$ when measuring 4250°R gas temperatures in the nozzle chamber.

6. Wiring Harness Requirements

Cable Harness Assembly requirements and recommendations are now summarized. The requirements only pertain to full power measurements where the nozzle chamber temperature is 4250°R and the source signal bandwidth is 0.3 Hz.

(a) Wiring Harness Uncertainty Allocation

(1) The cable transition induced uncertainty shall not exceed $\pm 2.5^{\circ}\text{R}$.

(2) The reference junction induced uncertainty resulting from temperature differentials within the unit and sensor resistance-temperature correlation discrepancies shall not exceed $\pm 0.35^{\circ}\text{R}$.

(b) Wiring Harness Bandwidth Requirement

Due to the narrowness of the source signal bandwidth (0.3 Hz), no frequency response requirements shall be imposed as the effects of propagation phase delay and signal transport attenuation should be insignificant.

7. Recommended Wiring Harness Studies

(a) Thermocouple cable design and fabrication studies dealing with the control of wire inhomogeneity should be continued. A check for wire inhomogeneity should be included in future thermocouple cable specifications.

(b) It is recommended that the use of high magnetic permeability sheath materials be evaluated even though a requirement for these materials has not yet been established.

(c) Design studies dealing with the feasibility of running all nozzle chamber thermocouple wires from the measurement junction to the reference junction without a single transition should be continued. This configuration was assumed when making transition induced measurement uncertainty allocations.

(d) The electromagnetic fields generated by power cables, actuator assemblies, pulse counter neutronic channels, and eddy current channels should be evaluated.

(e) The assumption that charge induced noise signals resulting from ionizing radiation fields and triboelectric phenomenon are small enough to be ignored in thermocouple circuits should be verified.

C. DATA ACQUISITION SUBSYSTEM REQUIREMENTS

The data processing uncertainty of a data acquisition system or assembly depends upon the particular configuration or arrangement of system elements (placement of filters, amplifiers, etc.), the types of elements selected (active versus passive filters, differential versus single ended amplifiers, etc.), the components used within the system elements (bipolar junction transistors versus field-effect transistors, etc.), and the service environment to which the various system elements are exposed. As a result, any realistic assignment of performance requirements must recognize the inherent limitations of various system arrangements and elements that can be employed in the various vehicle environments.

The location, arrangement, and type of all system elements has not yet been firmly established due to inadequate information concerning the effects of nuclear radiation on various system elements and components and inadequate information relative to specific channel requirements. To be more specific, the following factors must be considered before a proper data system configuration can be selected.

1. The reliability-life characteristics required of the data system.
2. The environmental conditions (temperature, vibration, nuclear radiation, etc.,) that the data system components will be subjected to.
3. The maximum and minimum full scale signal voltages presented to the data system by each channel.
4. The signal source impedance and the impedance to ground for each channel.
5. The size, weight and power restrictions imposed by space flight system requirements.

6. The classification of each data channel as originating with a passive or active source and wired as a; grounded single-ended channel; an ungrounded single-ended channel, a grounded differential channel, or an ungrounded differential channel.

7. The number of each channel type.

8. The accuracy to which each data channel signal must be processed.

9. The bandwidth and roll-off characteristics of individual data channel signals.

10. The maximum data system induced amplitude and phase distortions allowed for each channel as a function of frequency.

11. The maximum common mode noise signal amplitude and bandwidth present on each channel.

Thus, the initial selection of a system configuration depends on tentative system requirements. However, some knowledge of the systems capabilities must be known if the assignment of unreasonable requirements is to be avoided. If the selected system can not satisfy all requirements, then either the system or the requirements must be changed. Since the Data Acquisition Subsystem has not yet been fully defined, all that can be done at this time is to consider the kinds of error mechanisms that generally occur in most sampled data systems and then provide a list of requirements which will, admittedly, be a compromise between what the data requestor asks for and what is possible. The various error mechanisms commonly encountered in most sampled data systems are listed in Table V and described in the following discussion.

1. Miscellaneous Error Sources

There are four primary types of electrical noise or error signals and they are: bias voltages, stationary or cyclic voltage signals, transient voltage spikes or pulses, and random or white noise. Some of the ways by which these various noise signals are generated will now be considered.

(a) Static Loading Errors

The signal source resistance, the series lead wire resistance, the series resistance of the closed switches and the load resistance presented to the output of the multiplexer switch assembly will form a resistance divider network that will introduce bias errors. Because source and series resistance values differ for each channel, the bias error will vary as the channels are sequentially sampled. The bias error can also vary with time as when working with platinum resistance temperature sensors whose sensing element resistance value can vary from 6 ohms to 1500 ohms as the sensor cycles from liquid hydrogen to room temperatures. This type of problem also exists when working with resistance potentiometer transducers whose resistance can vary from zero to 2000 ohms.

Static loading bias errors will be insignificant if the source and series resistance values are very small relative to the load impedance presented to the output of the multiplexer switch assembly. The use of amplifiers or active pre-sampling filters in front of the multiplexer can reduce the size of the bias error for high impedance source circuits as these devices can provide extremely low output impedances. Voltage calibration techniques could effectively eliminate these bias errors if the source and series resistance values were to remain essentially constant. Obviously, calibration techniques are less effective when working with platinum resistance or potentiometric transducers and active elements are not placed between the transducer and multiplexer. However, if calibration techniques are combined with data correction techniques, the effects of large source impedance variations can, in part, be accounted for.

Static load induced bias errors for thermocouple circuits should be insignificant unless passive pre-sampling filters are used that possess large series resistance values or a multiplexer switch possessing a large "on" impedance is used. A series resistance is usually considered to be large if it exceeds 1000 ohms. The on resistance of J-FET semiconductor switches can be as low as 10 ohms for hybrid microelectronic IC multiplexers at as high as 1200 ohms for monolithic IC multiplexers. Assuming that the monolithic IC is used to provide the smallest possible system package and that a differential input is desired, the total switch induced series impedance will be 2400 ohms for every level of sub-multiplexing adopted. If a 100 sample-per-second sample rate were adopted, a 3-level sub-multiplexing scheme adopted ($R_{\text{series}} = 7200$ ohms), and an effective system input impedance of one megohm exists; then the loading error would be -0.7% of the signal level being monitored. Fortunately, static loading induced bias errors can be largely eliminated through the application of voltage calibration techniques for the nozzle chamber thermocouple circuits.

(b) Dynamic Loading Errors

A multiplexer, like any other sampling device, requires the source to supply a certain amount of energy every time it is sampled. Most of this energy is used to charge the input capacitance of the multiplexer assembly. The resultant charging current flows through the source and series impedances to yield a loading error. If the sample period exceeds the time required for the charging current to dissipate, the loading error would be equivalent to that produced by a large load impedance. When a high sample rate is applied, the charging current will not have time to dissipate, and the loading error will be equivalent to that produced by a smaller impedance load. Thus, the loading error increases with the sample rate. That is, the effective multiplexer input impedance, as viewed from the signal source, appears to become smaller as the sample rate increases. Dynamic loading induced bias errors can normally be reduced by reducing the effective multiplexer input capacitance through the adoption of sub-multiplexer assemblies. Dynamic

loading induced bias errors can also be reduced or eliminated by applying those techniques that are effective in reducing static loading errors.

Dynamic loading errors should not be a problem for the nozzle chamber thermocouple channels unless the output of the multiplexer is routed to the top of the propellant tank as an amplitude modulated pulse train where the analog-to-digital conversion process is accomplished. If a 120 ft line section, a 29×10^{-12} farad per foot distributed line capacity, and a 7200 ohm series impedance (three level sub-multiplexing) is assumed, the effective RC time constant will be $120 \times 29 \times 7200 \times 10^{-12}$ or 25.1 microseconds. Assuming that nine time constants are required to reduce the dynamic bias error to insignificance (to -0.01%), the required signal sample duration will be 0.23 milliseconds.

Although 0.23 milliseconds may be required for a data sample to settle to within $\pm 0.01\%$ of its full scale amplitude, the time needed to discharge (drain) the signal from the cable is considerably less. This results from the fact that the sending end of the cable can be shorted between samples so that the time required for signal discharge is only limited by the distributed resistance and capacitance of the cable. If 20 microseconds is taken as a reasonable discharge time, the total time required to impress and then remove a sampled data signal is approximately 0.25 milliseconds. This corresponds to a sampled data through-put rate of 4000 samples per second. If the average data channel sample rate were 25 samples per second, the multiplexer assembly could only process data from 160 channels.

Based upon the results of the example just presented and the knowledge that more than 160 engine data channels can be expected, it appears that it would not be possible to serially transmit all engine system data to the top of the propellant tank if the multiplexer switch were coupled directly to the transmission cable.

The inclusion of a cable between the multiplexer switch assembly and an instrumentation package containing amplifiers, A/D converters, etc., would require the adoption of one of at least two system configurations.

The first configuration would employ a line driver amplifier whose output impedance matched the characteristic impedance of the cable and whose input impedance would be made as large as possible. The use of a line driver amplifier would allow all engine data to be transmitted over a single cable and would greatly reduce the susceptibility of the measurement system to dynamic loading errors.

The second configuration would use several parallel multiplexer-cable-electronic data processing paths with each accepting a limited number of data channels. Although this configuration has the advantage of not requiring the use of line driver amplifiers, it does require the adoption of a more complex data processing control scheme. As a result, the configuration incorporating a line driver amplifier and a single transmission cable is preferred over a multiple path configuration because it is simpler (probably more reliable), and would provide a faster pulse signal settling time which may allow the use of dual slope integrating encoders. Operationally, the best system configuration would be that which consolidated the data acquisition subsystem so that all components were located in one location and not dispersed with long lengths of interconnecting cables. Regardless of the system configuration adopted, residual dynamic loading errors encountered by nozzle chamber thermocouple channels can be suppressed by employing channel calibration techniques.

(c) Charge Storage Induced Channel Cross-Talk

Channel cross-talk appears in a sampled data system whenever the signal level of one channel modifies the sampled level of the following channel. As an example, assume that a channel is being sampled that possesses a large negative signal. Also assume that the next channel to be sampled has a large positive signal level. As the multiplexer switches from the channel with the large negative signal to that with the large positive signal, there may not be time for the multiplexer input capacitance to charge up to the new positive signal value. As a result, the sampled data level will be smaller than true. The amount of charge storage induced cross-talk produced is proportional to the sample rate, the source and series resistance values, and the capacitance load presented by the multiplexer. This type of cross talk error

can be eliminated by shorting the multiplexer input during the interval used to switch from one channel to another. This solution must be approached with caution, however, as all data routed to the sub-multiplexer could be lost if the shorting switch failed in the closed position. Channel calibration is completely ineffective as a means for eliminating charge storage induced cross-talk. The best method of reducing this type of cross-talk for the nozzle chamber thermocouple channels is to employ sub-multiplexing techniques to reduce input capacity, to employ switches with a low on resistance, to avoid the use of passive filters, and to avoid sampling a high level voltage signal just prior to sampling a nozzle chamber thermocouple signal.

(d) Signal Feedthrough Induced Cross-Talk

A signal can feed capacitively through the open switch of an adjacent channel. Signals can also feed resistively through an adjacent switch. Since the impedance of an open switch is normally very large, each open switch in a common multiplexer array will appear as a constant current source. If enough switches are paralleled, the flow of all leakage currents through the source and series resistances of the channel being sampled can cause significant bias voltage errors. The effect of signal feedthrough induced cross-talk cannot be eliminated by calibration techniques, but can be minimized by grouping channels that possess similar common mode and full-scale signal voltages and by adopting sub-multiplexing techniques.

(e) Common Mode Rejection

Common mode voltages represent signals that are present on both differential channel conductors but do not represent pertinent data information. A system with good common mode rejection responds to the voltages present across the two leads but not to any voltages common to both lines. Common mode voltages can result from dynamic electromagnetic coupling, ground potential variations, and off ground measurements originating within electrical systems or from full bridge transducers.

Common mode voltages can be eliminated when employing a data system element that is designed to determine the algebraic difference between two signals and reproduce the difference voltage (the desired data signal) at its output. However, the common mode rejection process just described

will only be effective when electrical isolation is maintained between the signal source ground and the ground of the data acquisition subsystem. If electrical isolation is not maintained between channels or between the signal and data system ground, common mode leakage currents will appear. If these leakage currents flow through unbalanced circuit impedances, the resulting current induced potentials will appear as normal mode voltages that are indistinguishable from valid data signals.

The nozzle chamber thermocouple channels are particularly sensitive to common mode errors as the channels contain relatively long runs of wires that differ resistively. Also, the adoption of passive filters could further add to the unbalance as could unmatched multiplexer switches. When field-effect transistor switches are used, care must be taken to match the gating voltages as the "on" resistance is dependent upon the level of the switch drive potential.

The adoption of guard shielding techniques is recommended as the most effective means for minimizing common mode error signals. A guard shield is a shield that completely envelopes the signal while maintaining only one ground reference. The signal is normally transferred across the guard shield with an isolation transformer that also functions as a differential to single-ended signal converter.

Common mode rejection is normally optimized by using a guarded differential amplifier for each channel. The guard shield would terminate in the amplifier and the output of the amplifier would be single-ended which would simplify the design of the multiplexer. Another acceptable approach would route both the signal and shield into the multiplexer where the signal would be chopped to generate a symmetric, bilevel square wave that is then transformer coupled across the shield. Unfortunately, this technique does not lend itself to IC circuitry where it is difficult to form a complete shield around all components that are associated with a particular channel. There are two techniques that have been successfully employed to solve the problem posed

by the adoption of IC multiplexers. The first consists of grouping those data channels that are likely to possess similar common mode voltage levels and then routing them to a multiplexer module that is surrounded with a guard shield that is maintained at a mean common mode level. The mean common mode level is achieved by connecting all of the channel shields together or by electronic averaging. The second technique incorporates a guard shield driver that consists of a high input impedance, low output impedance, unity gain amplifier that senses the common mode voltage existing on the channel being sampled and then drives the guard shield envelope enclosing the sub-multiplexer assembly.

(f) Electromagnetic Coupling

Multiplexers are often operated by having their gate terminals actuated by a decode matrix, which is, in turn, driven by a binary counter. Thus an analog multiplexer is not just a simple array of analog switches, but includes buffers, drivers, and logic. Great care must be taken to prevent digital logic signals from being coupled into the analog data stream.

(g) Miscellaneous Error Signals

It was mentioned earlier that sub-multiplexing can be effective in reducing dynamic loading errors, charge storage induced cross-talk, signal feedthrough induced cross talk, and signal lag error signals. However, the use of sub-multiplexing will require each analog signal to pass through more active switches and each switch will add to the random noise level. In addition, each transistor switch will induce switching transients. The sharp rise and fall times of the leading and trailing edges of the square wave drive waveform will produce voltage spikes at the output switch terminals as a result of capacitive coupling between terminals.

Thus far, this discussion dealt in a very general way with the generation and effect of some of the more common error signal mechanisms. It is recommended that a more complete study be implemented in the near

future to evaluate the cause and effect of common mode error signals, channel cross talk error signals, and induced random noise signals and the effectiveness of error signal suppression techniques.

2. Channel Gain, Linearity, and Zero Off-Set Errors

This section shall briefly discuss those operating characteristics that must be considered and accounted for in the design of high speed amplifiers. By assuming that we are dealing with a differential amplifier that is designed to process sampled data, the more important characteristics are: input bias current, gain-step accuracy, linearity, gain stability, bandwidth, slewing rate, reliability, environment sensitivity, zero drift, supply voltage sensitivity, differential input impedance, common mode impedance, maximum differential voltage, maximum common mode voltage, overload voltage, overload recovery time, common mode rejection, amplifier noise, maximum output voltage and current, output settling time, output impedance and input-output isolation.

Although all of the above characteristics are important, most of them are well understood and will not be considered here. Those characteristics that are often less well understood are the input bias current, the slewing rate and the settling time.

The input bias current, which is frequently referred to as the pump-out-current or current-feed-to-source, is the current flowing into or out of each amplifier input lead with zero signal and zero common mode voltage applied. These currents are generated by the amplifier input stage and occur when either junction or field effect transistors are used. As the currents flow out of the amplifier into the input leads, potentials are developed across the various series resistance elements. If the bias currents are equal and the channel is balanced resistively, then common mode voltages are developed. If the channel is unbalanced or if unequal bias currents are generated, then normal mode voltages are developed that can not be distinguished from valid data.

Bias current can range up to one microampere while the sensitivity of bias currents to temperature variations can be both large and highly nonlinear. As a result, bias currents can produce significant error signals in the nozzle chamber thermocouple channels if passive filters are employed or if large multiplexer switch "on" impedances are encountered.

The slewing rate is usually expressed in volts per microsecond and is most frequently defined as the maximum rate at which the output voltage can change when the input is subjected to a step voltage function. The usual source of slew rate limiting is current limiting. Externally, slew rate limiting will occur when operating into a capacitive load where the required charging current exceeds the output capabilities of the amplifier. Internally, stabilization networks and parasitic capacitances impose current requirements. When these requirements can not be satisfied, the amplifier is again slew rate limited.

When minor slew rate limiting is incurred, the output signal may appear to be undisturbed but a dc shift can appear in the output. In addition, repetitive input waveforms, such as those leaving the multiplexer, whose rise time exceeds the amplifier slewing rate will generate voltage spikes at the feedback summing junction. These spikes are usually unsymmetrical and may produce dc voltage bias errors at the output. It is the slewing rate that often determines the bandwidth of wide-band amplifiers. For a sinusoidal signal, the maximum slope or rate of voltage change occurs at the zero crossing and is proportional to the signals peak amplitude and frequency. Therefore, the bandwidth is established by that frequency which produces a zero crossing rate of change that exceeds the slew rate. As the amplitude of the input signal is reduced, the frequency can be proportionally increased without exceeding the slew rate. This accounts for the fact that the small signal amplifier bandwidth will exceed the large signal bandwidth. As will now be shown, the amplifier settling time is also dependent upon the slew rate.

The settling time is the time lapse measured from the application of an ideal instantaneous step input to the time at which the amplifier output has entered and remained within a specified error band, usually symmetrically positioned about the final value. The settling time includes a very brief propagation delay, plus the time required for the output to slew to the vicinity of the final value, recover from the overload condition associated with slew limiting (current saturation), and finally settle to within the specified error. Amplifiers sensing the output of a multiplexer are exposed to rapidly changing signal levels and must faithfully reproduce an amplified version of the signal to a high degree of accuracy within a period that is measured in microseconds. This requires an amplifier that has been designed to optimize the settling time. Settling time is important in that it is the principal factor which determines the maximum data transfer rate for a given accuracy.

3. Data Sampling Induced Uncertainties

Based upon the Sampling Theorem, it is theoretically possible to completely reconstruct analog data that has been sampled if the following ideal constraints are applied to the data acquisition system.

(a) The signal to be sampled is ideally band limited in that its power density spectrum is identically zero for all frequencies above ω_s .

(b) The sample duration (aperture time, k) is extremely short relative to the period of the highest frequency component contained in the signal. That is, $k \ll 2\pi/\omega_s$.

(c) The filter used to recover the signal is ideal in that it possesses a flat "unity gain" response up to the signal band limit (ω_s), infinite attenuation beyond that frequency, and zero phase delay over the pass-band.

(d) The sample rate (ω_p) is greater than twice the signal bandwidth. That is, $\omega_p > 2 \omega_s$.

All four operating conditions, taken together, define the Sample Theorem even though the fourth condition is often the only one specified. Since the four idealized operating conditions can never be satisfied, perfect signal recovery is not possible and some data information is always irretrievably lost. All that can be done is to determine the bandwidth and roll-off rate of the signal to be sampled and then intelligently select sample rates, pre-sampling filters, and data reconstruction techniques that will satisfy data processing uncertainty requirements.

The data processing uncertainty of greatest concern is data aliasing. It is not possible to reconstruct signal components whose frequencies exceed half of the sample rate. That is, all data occurring at frequencies above the half sample rate are "folded" back into the low frequency realm where they can not be distinguished from authentic low frequency data. Aliasing is a particularly insidious property of all sampled data systems.

Since continuous signals of real physical origin are being sampled, some signal components will always exist at frequencies above half the sample rate irrespective of how high the sample rate is set, and an error of corresponding magnitude will always appear in the recovered data. Further, the error resulting from the appearance of aliased data can not be eliminated as the aliased data appears in the frequency band of the desired signal and is indistinguishable from valid (real) data. However, the size of the error can be reduced to an acceptable level (1% rms) by increasing the sample rate or by increasing the signal roll-off rate through the judicious use of filters. This point can be clarified with the aid of Figure 4. The curves of Figure 4 assume that the data roll-off rate is identical to that described by the transfer function of a butterworth filter and is optimistic in that reconstruction filtering errors and pre-sample filter phase shift

Figure 4 - Aliasing Error

errors are not accounted for. The curves are, however, extremely useful when estimating the sample rate and roll-off characteristics needed to obtain a 1% rms aliasing error.

Based upon Reference 9, the bandwidth of nozzle chamber temperature data should not exceed 0.2 Hz and the expected roll-off rate is -6 db/octave. Also, the bandwidth of the nozzle chamber thermocouple, as inferred from time constant estimates, is approximately 0.3 Hz. In addition, the thermocouple transfer function resembles that of a first order system so that its roll-off rate is also -6 db/octave. Thus, it shall be assumed that the source signal generated by the nozzle chamber thermocouple has a maximum bandwidth of 0.3 Hz and has a -12 db/octave signal roll-off rate.

Reference to Figure 4 indicates that the sample rate should be at least 30 times as large as the signal cut-off frequency or 9 sps if aliasing errors are not to exceed 1% rms. Therefore, the sample rate requirement for the nozzle chamber thermocouple channels shall be set at 10 samples per second, assuming that the channel noise levels are low enough to preclude the need for pre-sampling filters. Should subsequent studies indicate the probable existence of excessive noise levels, then a 2-pole, 3 Hz pre-sampling filter is recommended. This will yield a signal whose effective bandwidth is 0.95 Hz and whose effective roll-off rate is -24 db/octave. Thus, a 6 sps sample rate will provide a 1% rms aliasing error. However, the recommended sample rate is still conservatively set at 10 sps to allow for errors in the determination of the thermocouple time constant or in the natural bandwidth of the engine nozzle chamber temperature environment.

4. Quantization Induced Uncertainties

Quantization induced uncertainties are produced when the analog data sample is routed to an analog-to-digital converter that encodes (converts) the analog signal into a binary digital signal which is then transmitted to the computer for storage or processing.

Representing a signal by a finite number of discrete voltage levels is called quantizing or encoding. This process introduces an error in the amplitude of the data sample, giving rise to quantization noise. The quantization error can never exceed $\pm 1/2$ of a single discrete encoding level interval. All values of error up to this maximum value are equally likely. When the sampled data is decoded and reconstructed, what is obtained is the original signal plus quantization noise, assuming that aliasing and reconstruction errors are insignificant. The quantization noise spectrum is uniform out to the half sample frequency with an rms amplitude of $1/\sqrt{12}$ times the height of a single discrete encoding level interval. Thus, the ratio of the peak-to-peak signal level to the rms quantization noise level is

$$S/N = \sqrt{12} \ b^n$$

or
$$S/N \approx 10.79 + 6.02 \ n \text{ decibels}$$

where $b = 2$, b^n is the number of discrete encoding levels, and n is the number of binary digits used to digitize the signal.

The minimum discrete encoding level increment adopted depends upon the measurement resolution required and the noise level present. Every system is characterized by random noise signals that limit the number of discrete voltage levels by which a signal may be partitioned. If the inherent noise fluctuations were to vary within $\pm q$ most of the time, then q would be the minimum signal voltage variation that could be reliably detected by an encoder. Most wide-band noise has a Gaussian distribution so that it is necessary to consider the probability that the peak noise level will exceed some assigned noise level. To clarify this point, Table X is presented to illustrate the probability of having the peak noise value exceed an assigned noise level where the assigned noise level is some multiple of the rms noise level (p).

Table X. The Probability That the Noise Level Will or Will Not Exceed Some Assigned Level*

rms Noise Level	Assigned Noise Level (q)	Probability of Exceeding the Assigned Level	Probability of Not Exceeding the Assigned Level
p	1.000 p	0.317311	0.682689
p	1.645 p	0.100000	0.900000
p	2.576 p	0.010000	0.990000
p	3.291 p	0.001000	0.999000
p	3.891 p	0.000100	0.999900
p	4.417 p	0.000010	0.999990
p	4.892 p	0.000001	0.999999

* Tables of Normal Probability Functions, National Bureau of Standards Applied Mathematics Series, Table AMS23, 5 June 1953, United States Government Printing Office, Washington 25, D.C.

If the encoding increment is set equal to the assigned noise level (q), then the probability of not exceeding the assigned noise level is synonymous with the probability of correctly assigning the nearest binary number to represent the signal voltage being digitized. Thus, the probability of not exceeding the assigned noise level is actually the probability of successful signal encoding and is, therefore, a reliability number. As an example of how Table X should be used, assume the following:

(1) The functional reliability level assigned to proper signal encoding is 0.999990.

(2) The rms channel noise level at the analog-to-digital converter input is ± 3 millivolts.

(3) The full scale voltage level applied at the analog-to-digital converter input is 10 volts.

Thus, the assigned noise level would be 4.417×3 or 13.251 millivolts which is 0.13% of the full scale signal range. The encoding error of a 8 bit binary signal encoder is $\pm 0.19\%$ of full scale while that of a 9 bit binary signal encoder is $\pm 0.10\%$ of full scale. Thus, an 8 bit analog-to-digital converter would be selected for the particular example being considered here.

Another type of encoding error is encountered when the level of the signal being digitized changes during the conversion process. When this occurs, the digital output will correspond to an input signal level that existed at some undefined time during the conversion period. This problem can usually be reduced to insignificance by adopting sample-and-hold circuits which sample and then retain the signal level existing at a predetermined moment in time. The signal level being maintained is then presented to the analog-to-digital converter for encoding.

A dual-slope integrating encoder is highly recommended for digitizing signals that possess very low cut off frequencies and when large multiplexer load capacities are not encountered. The attenuation characteristic of a dual-slope integrating encoder is described by

$$\text{Signal attenuation} = 20 \log \sin^2 \pi T f - 20 \log \pi T f \text{ decibels}$$

where T is the signal integration period and f is the signal frequency. The attenuation curve for a dual-slope integrating encoder is illustrated in Figure 5

Figure 5. Signal Attenuation Induced by a Dual-Slope Integrating Encoder

Thus, the dual-slope integrating encoder generates an attenuation function that is sine-periodic as seen by reviewing the signal attenuation equation and Figure 5. The first term of the attenuation equation describes infinite attenuation cusps that occur at frequencies that are multiples of that frequency component which has a period equal to the signal integration period. The second equation term acts essentially as a single pole, -6 db/octave filter with a corner frequency of $1/\pi T$ Hz.

The attenuation curve of Figure 5 clearly illustrates how an integrating encoder can attenuate noise signals. In fact, the effective rms noise level could, for all practical purposes, be completely attenuated if it were possible to employ long enough signal integration periods. Since the effective noise level is reduced, it is possible to employ a smaller analog-to-digital encoding increment and increase the resolution of the measurement.

Because the time required for a sample-and-hold circuit to sample an analog voltage level is very short (50 nanosecond aperture times are

common), analog-to-digital converters that employ a sample-and-hold circuit are particularly sensitive to transient voltage spikes. It is possible for the sample-and-hold circuit to sample just as a noise spike appears and select a voltage level that is not truly representative of the signal being sampled. The dual-slope integrating encoder will, however, smooth out infrequent noise perturbations.

The dual-slope integrating encoder should be inherently less sensitive to the effects of nuclear radiation. The encoder contains amplifiers, resistors, and capacitors. As the gain of the amplifiers change and resistance and capacitance values drift due to nuclear radiation damage, both the positive and negative integration ramp rates should be affected to very nearly the same extent so that the encoder performance should not be degraded. As a result, a dual-slope integrating encoder can tolerate much larger component variations than any other encoder type.

The location of the infinite attenuation cusps shown in Figure 5 could be shifted along the frequency axis until one of the cusps coincided with a stationary noise signal that has been coupled into the nozzle chamber thermocouple channel. As a result, the encoder can be used as both a notch filter and a single pole low pass filter. The low pass filter behavior of a dual-slope integrating encoder is probably its most important attribute as noise signals can be attenuated without the use of pre-sampling filters. However, the filtering action occurs after the signal is sampled so that its use can not possibly reduce aliasing errors or reduce the required sample rate.

Finally, the adoption of a dual-slope integrating encoder could alleviate the need for pre-sample filters for the nozzle chamber thermocouple channels. It was demonstrated in the discussion on aliasing that the cut-off frequency of the thermocouple signal is approximately 0.3 Hz and a 9 sps sample rate is sufficient to reduce the aliasing error to 1% rms. Thus, filters are not required to lower the sample rate as a 9 sps is not an

excessive rate. The only reason filters may be required would be for noise attenuation and this can be accomplished with the dual-slope integrater, assuming that the frequency of coupled stationary noise signals exceeds several hundred Hertz. If pre-sampling filters can be eliminated, it may then prove feasible to calibrate the nozzle chamber thermocouple channels by feeding calibrated voltage steps into the data system through the front end of a single dedicated multiplexer channel that is not carrying data. Although this technique would not account for channel to channel switch impedance variations, it would provide a means for eliminating bias, linearity, and gain variations associated with the sub-multiplexer assembly amplifiers and should be useful in eliminating static and dynamic loading errors. Also, since the calibration signal is not superimposed on a data channel, the adoption of signal calibration would not degrade the reliability of nozzle chamber temperature channels. Although channel calibration is desirable, it is absolutely essential to provide accurate reference voltages to the analog-to-digital converters.

5. Requirements

The data acquisition subsystem is unique in that it may be the only measurement system assembly that contains elements whose proper operation in the service environment has not been demonstrated, assuming that some portions of the data acquisition assembly will be located in a nuclear radiation environment. The problems associated with the use of transducers and cables in a radiation environment are fairly well known as a result of past test experience. The computer, although a very complex component, will be located in a shielded area that will allow the adoption of most conventional design and fabrication techniques. The data acquisition assembly, however, presents a special problem.

Taken individually, active filters, multiplexers, amplifiers, analog-to-digital converters, etc., are very complex devices that can pose serious engineering design problems. It could be argued that the preceding

statement is an exaggeration as many designs for the above mentioned devices have evolved over the years so that a large reservoir of practical design experience is available. In addition, radiation effects data on components is available. The argument is not valid, however, as many of the standard proven designs incorporate components and materials that are not compatible with the engine environment. Also, the performance of many devices working together in a circuit that is exposed to nuclear radiation cannot be reliably predicted from radiation effects data acquired on each device taken separately. This is especially true with regard to high speed data systems as most components are evaluated under static conditions but would be expected to perform under highly dynamic conditions. Since new design approaches will be required, the performance of the data acquisition system in a nuclear radiation environment can not be properly assessed by reference to the capabilities of data systems designed for conventional operating environments. Thus, there is no way of knowing whether or not the assigned accuracy requirement can be met, or even approached. As a result, the accuracy requirement imposed by this report should be viewed as a performance goal whose achievement should be evaluated at the earliest possible date since the inability of one system element to meet a performance requirement often necessitates a reassignment of performance requirements for all system elements in order to retain an overall system capability.

(a) Accuracy

$\pm 0.25\%$ of the full scale millivolt range.

(b) Source Signal Range

The subsystem shall be capable of accepting and processing a signal that will range from -3 to +37 mv, depending upon the relative instantaneous temperatures assumed by the measurement and reference junctions.

(c) Filter Bandwidth

The bandwidth of any low pass filtering action shall equal or exceed 3.0 Hz. The filtering action can be implemented with pre-sampling filters, dual-slope integrating encoders, or digital filtering.

(d) Sample Rate

The nozzle chamber thermocouple channel sample rate shall equal or exceed 10 samples per second.

6. Recommended Data Acquisition Subsystem Studies

(a) The possible use of long cable lengths between the multiplexers and other system elements such as amplifiers or analog-to-digital converters may impose unusual loading problems due to the distributed capacitance of the cable. Thus, the effect of this capacitance should be evaluated for different system configurations. The effects of propagation phase delay, transport attenuation and signal reflection should also be considered.

(b) The superposition of electrical noise upon the data signal will degrade the conversion resolution of analog-to-digital converters not employing dual-slope integration techniques and increase aliasing errors. There are four obvious noise suppression techniques that should be given consideration: active pre-sampling filtering, passive pre-sampling filtering, the combined adoption of a wide sample aperture and dual-slope integrating encoders, or the combined adoption of high speed channel sampling rates and digital filtering. The selection of noise suppression techniques and components depends upon the data signal bandwidth, the spectral content of the noise signal and the environment (nuclear radiation, temperature, etc.) to which the various noise suppression components would be subjected to. In any event, it is recommended that the noise suppression problem be resolved.

(c) The adoption of a passive thermocouple reference may pose a problem with narrow range thermocouple channels as large reference junction temperature variations would increase the signal range presented to the data system. This would, in turn, reduce the measurement resolution unless the reference junction induced off-set voltage is eliminated before signal encoding. The offset signal could be cancelled with a digital-to-analog

converter under computer control. Thus, the adoption of this and other reference junction compensation techniques should be evaluated. In addition, the added system complexity resulting from the adoption of reference junction compensation must be weighed against the degree of measurement resolution improvement obtained.

(d) It is recommended that a study be implemented to determine whether or not the $\pm 0.25\%$ data processing uncertainty allotment will require the adoption of voltage calibration techniques.

D. COMPUTER, DATA PRE-PROCESSOR REQUIREMENTS

The pre-processor accepts individual nozzle chamber thermocouple signals, employs channel calibration data to correct the voltage signals, employs reference junction block temperature data to eliminate the effects of a drifting reference junction, notes the engine operating state point and then refers to stored nozzle chamber thermocouple calibration data or equations and performs an engineering unit conversion, and then calculates the nozzle chamber stagnation temperature from core exit and convergent section temperature data. The major error mechanisms associated with this sequence of events are summarized in Table V.

The use of calibration data for zero offset, linearity and gain corrections is well known and can usually be implemented without incurring significant errors. There are several approaches that can be adopted to account for a drifting thermocouple reference junction. One approach would have the computer refer to a thermocouple channel identification matrix in memory to determine the particular thermocouple that is associated with the particular channel being sampled. Having identified the thermocouple, the computer could then refer to a stored thermocouple signal correction table to determine the polarity and magnitude of the required reference junction compensation voltage. The thermocouple signal correction table could be

updated once a second by determining the temperature of the passive reference block and then calculating the millivolt correction signal required by each thermocouple channel.

Once the computer has identified the thermocouple channel being sampled and has determined the required correction signal, either analog or digital correction could be implemented. Analog correction could be implemented by having the computer drive a digital-to-analog converter which would be located in the data acquisition subsystem. The digital-to-analog converter would generate the required millivolt correction and insert it in series with one leg of the thermocouple channel before signal encoding. Digital correction would be implemented in the computer and is recommended for the nozzle chamber thermocouple channels. The accuracy of digital correction is controllable and it will be assumed that the correction uncertainty will remain insignificant compared to the block temperature measurement errors listed under the section dealing with cable harness requirements.

It shall be assumed that either empirical equations or stored tables will be adopted to determine what the gas temperature, thermoelectric voltage relationship is for different engine operating state points. The difference existing between the gas stream temperature and the corresponding thermocouple junction temperature is defined as the thermal-coupling factor. The other relationship that must be accounted for is the thermocouple junction emf-temperature conversion factor. Both factors taken together establish the gas temperature, thermoelectric voltage relationship which must be accounted for when performing an engineering unit conversion. If the engine were to always follow a finite number of operating paths when experiencing a transition between discrete operating state points, it might be possible to adopt a finite number of relatively simple empirical equations. If compensation must be provided for all operating points, then either a comprehensive data correction matrix or a combination of empirical and theoretical equations must be employed. In any event, the uncertainty associated with an engineering unit conversion will depend upon the thoroughness with which all of the factors affecting the gas temperature, thermoelectric voltage relationship are accounted for. The

calculation complexity or computer storage requirements needed to reduce the engineering unit conversion uncertainty to insignificance may be difficult to justify. Thus, it may be necessary to purposely introduce a small process error just to place a reasonable limit on the computer burden imposed by the engineering unit conversion process. It will be assumed that the intentional error term will not exceed $\pm 0.1^{\circ}\text{R}$ at the full power operating point.

The nozzle chamber temperature is actually the effective mixed mean stagnation nozzle chamber temperature whose value is calculated from data taken at the core exit plane and within the nozzle convergent section. Although the effective mixed mean stagnation nozzle chamber temperature equations have not yet been developed, it is assumed that they will be extremely complex. Therefore, it may again be necessary to intentionally introduce a small data processing error just to place a practical limit on the required computer burden. It will be assumed that the intentional error term will not exceed $\pm 0.1^{\circ}\text{R}$ at the full power operating point.

There are three other computer computations that may be required. The first has to do with correcting thermocouple data to eliminate transient response errors. The second has to do with possible data compression for stage telemetry while the third may be needed to provide analog data reconstruction for astronaut display.

1. Data Pre-processor Requirements

The data pre-processor shall accept individual nozzle convergent section thermocouple channel data, employ channel calibration voltages and measured reference junction block temperatures to correct thermocouple signals, perform an engineering unit conversion, and then calculate the effective mixed mean nozzle chamber stagnation temperature by combining all authenticated core exit and nozzle convergent section data. The known data processing uncertainty shall not exceed $\pm 0.14^{\circ}\text{R}$ at the full power operating point.

2. Recommended Studies

The computer burden required for data processing may prove to be larger than expected because of unknown data correction requirements. So that some unexpected data sizing problems will not appear, it is recommended that each type of channel be evaluated to identify all of the more probable data correction requirements.

E. ESTIMATED UNCERTAINTY AND BANDWIDTH OF NOZZLE CONVERGENT SECTION TEMPERATURE MEASUREMENTS

1. Estimated Uncertainty

The data processing uncertainty allocated to the data acquisition subsystem was $\pm 0.25\%$ of 40 millivolts which corresponds to ± 0.1 mv. The thermoelectric sensitivity of W5Re/W26Re thermocouples at 4250°R is approximately $0.1626^{\circ}\text{R}/\mu\text{V}$. Then an ± 0.1 mv data processing uncertainty is equivalent to $\pm 16.26^{\circ}\text{R}$ when measuring full power temperatures. These and other full power uncertainty estimates are now summarized in Table XI.

TABLE XI

TENTATIVE MEASUREMENT SYSTEM UNCERTAINTY REQUIREMENTS
FOR CONVERGENT NOZZLE SECTION TEMPERATURE CHANNELS
WHEN MEASURING FULL POWER TEMPERATURE LEVELS
(WHEN MEASURING 4250°R)

Measurement System Component	Uncertainty Allotment	
	Degrees Rankine	% of Point
Thermocouple assembly	$\pm 42.50^{\circ}\text{R}$	$\pm 1.0000\%$
Combined reference junction module and cable harness	$\pm 3.00^{\circ}\text{R}$	$\pm 0.0706\%$
Data acquisition subsystem	$\pm 16.26^{\circ}\text{R}$	$\pm 0.3826\%$
Computer, data pre-processor	$\pm 0.14^{\circ}\text{R}$	$\pm 0.0033\%$
Statistical RSS total	$\pm 45.60^{\circ}\text{R}$	$\pm 1.0730\%$

Thus, the estimated overall measurement system channel uncertainty is $\pm 45.6^{\circ}\text{R}$ or $\pm 1.07\%$ of the full power operating point ($\pm 1.07\%$ of 4250°R). Note that the uncertainty allotment for the thermocouple assembly only applies to the thermocouple junction located at the tip of the probe. It should also be mentioned that these are optimistic assessments that may prove to be unrealistic once more is known about each component. As a result, the allocated uncertainty numbers should be viewed as preliminary performance goals in that they are the smallest numbers that can be assigned with any reasonable chance of fulfillment.

Admittedly, the channel uncertainty allotments should be based upon measurement system requirements rather than probable performance estimates. Unfortunately, measurement uncertainty requirements for individual nozzle chamber thermocouple channels have not yet been generated. The uncertainty allocations listed in this report will be re-evaluated once firm requirements are available.

2. Estimated Bandwidth

The estimated channel bandwidth is 0.3 Hz which is identical to that assumed for the thermocouple assembly.

LIST OF SYMBOLS

A_e	=	Nozzle exit area, ft^2
A_t	=	Effective throat area, ft^2
c_p	=	Specific heat at constant pressure, $\text{BTU/lb-}^\circ\text{R}$
c_v	=	Specific heat at constant volume, $\text{BTU/lb-}^\circ\text{R}$
f_{co}	=	Cut-Off frequency, Hz
F	=	Thrust, lbs
g	=	Gravitational acceleration, 32.1740 ft/sec^2
H_c	=	Nozzle chamber enthalpy, BTU/lb
H_e	=	Nozzle exit enthalpy, BTU/lb
I_{sp}	=	Specific impulse, sec
J	=	Mechanical equivalent of heat, 778.28 ft-lb/BTU
$\dot{m}$	=	Propellant mass flow rate, slugs/sec
M	=	Molecular weight, 2.01565 lb/mole for hydrogen
P_a	=	Atmospheric pressure, 2116.22 lb/ft^2 standard sea level
P_c	=	mixed mean nozzle chamber pressure, lb/ft^2
P_e	=	Mixed mean nozzle exit pressure, lb/ft^2
q	=	Minimum detectable signal increment
R_c	=	Universal gas constant, $1545.32 \text{ lb-ft/mole-}^\circ\text{R}$
R	=	Normalized gas constant for hydrogen = $gR_c/M = 24666.5 \text{ ft}^2/^\circ\text{R-sec}^2$
T_c	=	Mixed mean nozzle chamber temperature, $^\circ\text{R}$
T_e	=	Mixed mean nozzle exit temperature, $^\circ\text{R}$
ΔT	=	Steady state ramp error, $^\circ\text{R}$
v_e	=	Nozzle exit velocity, ft/sec
$\dot{w}$	=	Propellant weight flow, lb/sec
γ	=	c_p/c_v = Specific heat ratio
ϵ	=	A_e/A_t = Nozzle area expansion ratio
π	=	3.1415927
ω_p	=	Signal sample rate, samples per second
ω_s	=	Bandwidth of an ideally band limited signal, radians/sec

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APPENDIX A

PARAMETRIC SENSITIVITY
EQUATION DEVELOPMENT

A. INTRODUCTION

In determining the performance of rocket engines from static-test-firing and flight data, the influence of measurement system errors upon ballistic-parameter calculations must be considered. That is, the accuracy to which engine performance parameters may be calculated is dependent upon the error incurred during the measurement of such fundamental physical quantities as pressure, temperature, and mass flow. Performance computation errors resulting from invalid test data will be examined to gain an understanding of the effect of measurement error upon ballistic-parameter calculations.

Total thrust (F) is a combination of exhaust momentum and pressure differentials. That is

$$\text{Thrust} = (\text{Momentum Thrust}) + (\text{Pressure Thrust})$$

$$F = \dot{m}v_e + (P_e - P_a) A_e \quad (1)$$

where:

$$\begin{aligned} A_e &= \text{nozzle exit area, ft}^2 \\ F &= \text{engine thrust, lb} \\ \dot{m} &= \text{propellant mass flow, slugs/sec} \\ P_e &= \text{nozzle exhaust exit pressure, lb/ft}^2 \\ P_a &= \text{atmospheric pressure, lb/ft}^2 \\ v_e &= \text{nozzle exhaust exit velocity, ft/sec} \end{aligned}$$

An expression for the nozzle exhaust exit velocity can be derived from one-dimensional, steady-state, isentropic flow equations [1]. That is,

$$H + \frac{v^2}{2gJ} = \text{constant} \quad (2)$$

where: q = acceleration due to gravity, 32.174 ft/sec^2
 H = enthalpy, Btu/lb
 J = mechanical equivalent of heat, 778.28 ft-lb/Btu
 v = gas stream velocity, ft/sec

By employing equation 2, we obtain

$$H_c + \frac{v_c^2}{2gJ} = H_e + \frac{v_e^2}{2gJ} \quad (3)$$

where: H_c = nozzle chamber enthalpy, Btu/lb
 H_e = nozzle exit enthalpy, Btu/lb
 v_c = nozzle chamber velocity, ft/sec
 v_e = nozzle exhaust exit velocity, ft/sec

Equation 3 is now rearranged to yield

$$v_e = \sqrt{2gJ (H_c - H_e) + v_c^2} \quad (4)$$

Since we are assuming space flight conditions, $P_a = 0$. The term v_c^2 appearing in equation 4 can also be eliminated as $v_e^2 \gg v_c^2$. Thus, after P_a and v_e^2 have been equated to zero, equations 1 and 4 are combined to obtain

$$F = P_c A_t \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g) (H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\} \text{ lb} \quad (5)$$

where $\dot{\omega}$ equals $\dot{g}\dot{m}$ and is the propellant weight flow rate. Since specific impulse (I_{sp}) equals $F/\dot{\omega}$, the specific impulse equation becomes:

$$I_{sp} = \frac{P_c A_t}{\dot{\omega}} \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g) (H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\} \text{ sec} \quad (6)$$

If one is interested in a less accurate estimate of thrust and specific impulse, the pressure thrust component of equation 1 may be eliminated and equations 5 and 6 simplified to yield

$$\left. \begin{aligned} F &\approx \dot{\omega} K \sqrt{H_c - H_e} \\ I_{sp} &\approx K \sqrt{H_c - H_e} \end{aligned} \right\} K = \sqrt{2J/g} \quad (7)$$

(8)

Note that an additional 2.47% full power thrust and specific impulse computation error is incurred when adopting equations 7 and 8 instead of equations 5 and 6. In order to determine the sensitivity of calculated values upon measured and assigned parameter uncertainties, known measured parameter variations were sequentially assigned and the variation in calculated engine performance parameters noted. The following nominal, full power state points were assumed:

Nozzle chamber pressure, (P_c)	----	64800 lb/ft ²
Nozzle chamber temperature, (T_c)	----	4250°R
Propellant weight flow rate, ($\dot{\omega}$)	----	91.24 lb/sec
Thrust, (F)	-----	75000 lb
Specific impulse, (I_{sp})	-----	822 seconds
Effective throat area, (A_t)	-----	0.68533 ft ²
Nozzle expansion ratio, (A_e/A_t)	----	100
Nozzle chamber enthalpy, (H_c)	----	15916.655 Btu/lb
Nozzle exit enthalpy, (H_e)	-----	2494.280 Btu/lb

When the nominal full power values are inserted into the idealized nozzle equations, the calculated values differ from the assumed nominal values. As a result, correction constants were inserted into the thrust equation. The I_{sp} equation is not significantly effected.

$$F = 0.994869 P_c A_t \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g)(H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\} \text{ lb} \quad (8)$$

$$I_{sp} = \frac{P_c A_t}{\dot{\omega}} \left\{ \frac{\dot{\omega}}{P_c A_t} \sqrt{(2J/g)(H_c - H_e)} + \frac{P_e}{P_c} \cdot \frac{A_e}{A_t} \right\} \text{ sec} \quad (6)$$

Computation now proceeds by determining which values of γ correspond to selected temperature and pressure values. Once γ values are obtained and A_e/A_t selected, values of (P_e/P_c) and (T_e/T_c) are defined. Nozzle exit pressure (P_e) and temperature (T_e) values are then found from $P_c(P_e/P_c)$ and $T_c(T_e/T_c)$, respectively. Enthalpy values correspond to applicable temperature and pressure values.

Values for γ , H_c , and H_e were obtained from the NBS hydrogen property generating codes while (P_e/P_c) values are obtained by applying polynomial extrapolation techniques to the Solid Engine Design Handbook, Thrust Coefficient and Expansion Ratio Tables. (T_e/T_c) values were calculated from

$$\frac{T_e}{T_c} = \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma}$$

Applicable values for γ , (P_e/P_c) , (T_e/T_c) , P_e , T_e , H_c , and H_e are listed in Table I.

TABLE I Full Power Parameter Values for Calculating F and I_{sp}

Temperature (°R)	Pressure (lb/ft ²)	γ	ϵ	(P_e/P_c)	(T_e/T_c)	P_e (lb/ft ²)	T_e (°R)	H_c (Btu/lb)	H_e (Btu/lb)
4250	64800	1.29044	98	0.000430993	0.174791	27.928	742.86	15916.655	2510.116
4250	64800	1.29044	100	0.000419416	0.173723	27.178	738.32	15916.655	2494.280
4250	64800	1.29044	102	0.000408368	0.172682	26.462	733.90	15916.655	2478.863
4165	64800	1.29467	100	0.000411109	0.169562	26.640	706.23	15531.889	2382.350
4250	64800	1.29044	100	0.000419416	0.173723	27.178	738.32	15916.655	2494.280
4335	64800	1.28603	100	0.000428254	0.178175	27.751	772.39	16301.424	2613.116
4250	63504	1.29028	100	0.000419734	0.173882	26.655	739.00	15918.754	2496.652
4250	64800	1.29044	100	0.000419416	0.173723	27.178	738.32	15916.655	2494.280
4250	66096	1.29060	100	0.000419099	0.173564	27.701	737.65	15914.256	2491.943
4250	64800	1.26463	100	0.000473889	0.201544	30.708	856.56	15916.655	2906.701
4250	64800	1.29044	100	0.000419416	0.173723	27.178	738.32	15916.655	2494.280
4250	64800	1.31625	100	0.000371118	0.149889	24.048	637.03	15916.655	2140.981

Thrust and specific impulse values now calculated as discrete parameter values are inserted into equations 6 and 8 and the results tabulated in Table II.

TABLE II Thrust and Specific Impulse Computation Errors Corresponding to Discrete Parameter Errors

Input Parameter Variations			Thrust Calculations		Specific Impulse Calculations	
Parameter	% Error	Value	Value	% Error	Value	% Error
A_e/A_t	-2%	98	74969.90	-0.0401304	821.6780	-0.0401304
	0%	100	75000.00	0.0000000	822.0079	0.0000000
	2%	102	75029.27	0.0390258	822.3287	0.0390258
T_c (°R)	-2%	4165	74216.05	-1.0452611	813.4158	-1.0452611
	0%	4250	75000.00	0.0000000	822.0079	0.0000000
	2%	4335	75760.11	1.0134822	830.3388	1.0134822
P_c (lb/ft ²)	-2%	63504	74963.57	-0.0485705	821.6086	-0.0485705
	0	64800	75000.00	0.0000000	822.0079	0.0000000
	2%	66096	75035.46	0.0472845	822.3966	0.0472845
γ	-2%	1.26463	74108.13	-1.1891576	812.2329	-1.1891576
	0	1.29044	75000.00	0.0000000	822.0079	0.0000000
	2%	1.31625	75743.03	0.9907113	830.1516	0.9907113
$\dot{w}$ (lb/sec)	-2%	89.4152	73537.06	-1.9505857	822.4224	0.0504227
	0%	91.2400	75000.00	0.0000000	822.0079	0.0000000
	2%	93.0648	76462.94	1.9505853	821.6097	-0.0484458
H_c (Btu/lb)	-2%	15598.322	74127.40	-1.1634708	812.4441	-1.1634708
	0%	15916.655	75000.00	0.0000000	822.0079	0.0000000
	2%	16234.988	75862.32	1.1497540	831.4590	1.1497540
H_e (Btu/lb)	-2%	2444.394	75135.80	0.1810721	823.4963	0.1810721
	0%	2494.280	75000.00	0.0000000	822.0079	0.0000000
	2%	2544.166	74863.94	-0.1814088	820.5167	-0.1814088

First-order, best fit, straight line equations are now derived to determine the sensitivity of F and I_{sp} calculations upon P_c , T_c , $\dot{\omega}$, γ , H_c , H_e , and A_e/A_t uncertainties. These sensitivity relationships were then combined using RSS techniques to yield the following parametric sensitivity equations for equations 5 and 6.

$$\frac{\Delta F}{F} = \left[0.00057 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.26490 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.95120 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 \right. \\ \left. + 0.00039 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 + 0.29699 \left(\frac{\Delta \gamma}{\gamma} \right)^2 + 0.33444 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00821 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00057 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.26490 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00061 \left(\frac{\Delta \dot{\omega}}{\dot{\omega}} \right)^2 + 0.0039 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 \right. \\ \left. + 0.29699 \left(\frac{\Delta \gamma}{\gamma} \right)^2 + 0.33444 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00821 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

Thrust and specific impulse values are now calculated for the simplified equations (equations 7 and 8) for discrete parameters values and the results tabulated in Table III.

TABLE III Thrust and Specific Impulse Computation Errors Corresponding to Discreet Parameter Errors When Using Equations 7 and 8

Input Parameter Variations			Thrust Calculations		Specific Impulse Calculations	
Parameter	% Error	Value	Value	% Error	Value	% Error
A_e/A_t	-2%	98	74955.74	-0.0590084	821.5150	-0.0590084
	0	100	75000.00	0.0000000	822.0000	0.0000000
	2%	102	75043.06	0.0574137	822.4719	0.0574137
T_c (°R)	-2%	4165	74233.83	-1.0215658	813.6027	-1.0215658
	0	4250	75000.00	0.0000000	822.0000	0.0000000
	2%	4335	75739.33	0.9857744	830.1031	0.9857744
P_c (lb/ft ²)	-2%	63504	74999.71	-0.0003930	821.9968	-0.0003930
	0	64800	75000.00	0.0000000	822.0000	0.0000000
	2%	66096	75000.29	0.0003930	822.0032	0.0003930
γ	-2%	1.26463	73838.77	-1.5483053	809.2729	-1.5483053
	0	1.29044	75000.00	0.0000000	822.0000	0.0000000
	2%	1.31625	75980.65	1.3075340	832.7479	1.3075340
$\dot{w}$ (lb/sec)	-2%	89.4152	73500.00	-2.0000000	822.0000	0.0000000
	0	91.2400	75000.00	0.0000000	822.0000	0.0000000
	2%	93.0648	76500.00	2.0000000	822.0000	0.0000000
H_c (Btu/lb)	-2%	15598.322	74105.29	-1.1929453	812.1940	-1.1929453
	0	15916.655	75000.00	0.0000000	822.0000	0.0000000
	2%	16234.988	75884.16	1.1788808	831.6904	1.1788808
H_e (Btu/lb)	-2%	2444.394	75139.24	0.1856591	823.5261	0.1856591
	0	2494.280	75000.00	0.0000000	822.0000	0.0000000
	2%	2544.166	74860.50	-0.1860044	820.4710	-0.1860044

First order, best fit, straight line equations are again employed to derive the following parametric sensitivity relationship for equations 7 and 8.

$$\frac{\Delta F}{F} = \left[0.00000 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.25184 \left(\frac{\Delta T_c}{T_c} \right)^2 + 1.00000 \left(\frac{\Delta \omega}{\omega} \right)^2 + 0.00085 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 \right. \\ \left. + 0.50974 \left(\frac{\Delta \gamma}{\gamma} \right)^2 + 0.35160 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00863 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

$$\frac{\Delta I_{sp}}{I_{sp}} = \left[0.00000 \left(\frac{\Delta P_c}{P_c} \right)^2 + 0.25184 \left(\frac{\Delta T_c}{T_c} \right)^2 + 0.00000 \left(\frac{\Delta \omega}{\omega} \right)^2 + 0.00085 \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 \right. \\ \left. + 0.50974 \left(\frac{\Delta \gamma}{\gamma} \right)^2 + 0.35160 \left(\frac{\Delta H_c}{H_c} \right)^2 + 0.00863 \left(\frac{\Delta H_e}{H_e} \right)^2 \right]^{1/2}$$

REFERENCES

- (1) C. Y. F. Pai Fluid Dynamics of Jets D. Van Nostrand Co., Inc.,
New York, 1954, pp 25.

APPENDIX B

EFFECTIVE THERMOCOUPLE TIME CONSTANT APPROXIMATION

Assume that a thermocouple probe is exposed to an isothermal gas stream where the probe extension has the same cross section as the junction region as shown in Figure 1.

Figure 1. A Cylindrical Thermocouple Probe Measuring Gas Stream Temperatures

The following thermal balance equation can be derived from Figure 1.

$$\left[\begin{array}{l} \text{Thermal energy} \\ \text{flowing into the} \\ \text{probe tip, } Q_c \end{array} \right] = \left[\begin{array}{l} \text{Rate of thermal energy} \\ \text{storage within the tip} \\ \text{of the probe, } Q \end{array} \right] + \left[\begin{array}{l} \text{Thermal energy} \\ \text{flowing out of} \\ \text{the probe tip,} \\ Q_L \end{array} \right]$$

$$Q_c = Q + Q_L$$

where:

$$\left. \begin{array}{l} \text{Convective heat being transferred} \\ \text{to the thermocouple probe tip} \end{array} \right\} \longrightarrow Q_c = A_j h (T_e - T) \text{ Btu/sec}$$

$$\left. \begin{array}{l} \text{Rate of heat storage within the} \\ \text{thermocouple probe tip} \end{array} \right\} \longrightarrow Q = mc \left(\frac{dT}{dt} \right) \text{ Btu/sec}$$

$$\left. \begin{array}{l} \text{Conductive heat being transferred} \\ \text{out of the thermocouple probe tip} \end{array} \right\} \longrightarrow Q_L = \frac{k_p A_p}{L_p} (T - T_o) \text{ Btu/sec}$$

A_j = Effective "gas stream-thermocouple" interface area, in.²;

A_p = cross sectional area of the thermocouple probe extension, in.²;

c = mean specific heat capacity of the thermocouple probe cross-section, Btu/lb-°F;

h = convective conductance of the gas stream acting against the probe, Btu/in.²-sec-°F;

k_p = cross sectional thermal conductivity of the thermocouple probe extension, Btu/sec-in.-°F;

L_p = effective probe support stem length, in.;

L_t = effective probe tip length, in.;

m = effective mass of the thermocouple probe tip, lb;

T = instantaneous thermocouple probe tip temperature, °F;

T_f = free gas stream temperature, °F;

T_o = thermocouple probe support temperature, °F.

By assuming that the temperature of the gas stream steps from T_o to T_f , the energy balance equation becomes

$$h (T_f - T) = \frac{mc}{A_j} \left(\frac{dT}{dt} \right) + \frac{A_p k_p}{A_j L_p} (T - T_o)$$

which may be simplified to yield

$$h (T_f - T) = Y \left(\frac{dT}{dt} \right) + P (T - T_o)$$

Solving the above differential equation yields:

$$T = T_o + \frac{T_f - T_o}{1 + (P/h)} \left[1 - e^{-(h+P)t/Y} \right] \text{ } ^\circ\text{F}$$

which, by assuming that P is normally much smaller than h, reduces to

$$T = T_o + (T_f - T_o) [1 - e^{-t/\tau}] \text{ } ^\circ\text{F} ; \tau = Y/h \quad (1)$$

where τ is the effective first-order system time constant for the probe assembly.

If the probe cross-section is nearly circular, a circular cross section can be assumed where D_o is the effective cross section diameter. Then;

$$\tau = \frac{Y}{h} = \frac{mc}{A_j h} = \frac{\rho L_t (0.25 \pi D_o^2) c}{L_t (\pi D_o) h} = \frac{\rho c D_o}{4h} \text{ sec} \quad (2)$$

where ρ is the mean density of the probe cross section.

For many practical applications, the mean convective conductance of a gas stream flowing perpendicular to the center line axis of a cylindrical probe is

$$h = \frac{k_f C}{D_o} \left(\frac{\rho_f V D_o}{\mu_f} \right)^n \text{ Btu/in.}^2\text{-sec-}^\circ\text{F}$$

where: k_f = free-stream gas thermal conductivity, Btu/sec-in.- $^\circ\text{F}$;
 ρ_f = free-stream gas density, lb/in.³;
 μ_f = free-stream gas viscosity, lb/in.-sec;
 V = free-stream velocity, in./sec

The coefficients C and n vary with the Reynolds Number as shown in Table I*.

TABLE I Coefficient for Calculating the Average Heat Transfer Coefficient of a Circular Cylinder in a Gas Flowing Normal to its Axis

$Re = \rho V D_o / \mu$	C	n
0.4 - 4	0.891	0.330
4 - 40	0.821	0.385
40 - 4000	0.615	0.466
4000 - 40000	0.174	0.618
40000 - 400000	0.0239	0.805

The free stream velocity can be approximated from

$$V = \frac{\dot{\omega}}{\rho_f A_n} \quad \text{in./sec}$$

where $\dot{\omega}$ is the propellant weight flow rate through the nozzle and A_n is the cross sectional area of the flow stream at the plane of the nozzle chamber thermocouples ($A_n \approx 812.6 \text{ in.}^2$).

The following start up, full power, and pulse cooldown state points are assumed and listed in Table II.

* F. Kreith, Principles of Heat Transfer, 2nd Ed., International Textbook Co., 1965, pp 41.

TABLE II Operating State Point and Operating Values

Parameter	Start Up	Full Power	Pulse Cooldown Power
T_c	800°R	4250°R	1290°R
P_c	40 lb/in. ²	450 lb/in. ²	1.0 lb/in. ²
$\dot{w}$	19 lb/sec	91.4 lb/sec	0.4 lb/sec
V	4330 in./sec	10203 in./sec	5860 in./sec
ρ_f	5.4×10^{-6} lb/in. ³	1.1×10^{-5} lb/in. ³	8.4×10^{-8} lb/in. ³
μ_f	6.4×10^{-7} lb/in.-sec	2.0×10^{-6} lb/in.-sec	8.9×10^{-7} lb/in.-sec
Re^*	7307	11223	111
C	0.174	0.174	0.615
n	0.618	0.618	0.466

* D_o assumed to be 0.2 inches.

Therefore, the approximate probe time constant for start up, full power, and shut-down may be described by:

$$\begin{aligned}
 \tau &\approx \frac{\rho c D_o}{4 h} \\
 \tau &\approx \frac{\rho c D_o^{1.382}}{0.696 k_f (\rho_f V / \mu_f)^{0.618}} \quad \text{seconds} \\
 \tau &\approx \frac{90.30 \rho c D_o^{1.382}}{k_f (\dot{w} / \mu_f)^{0.618}} \quad \text{seconds} \quad (4)
 \end{aligned}$$

Similarly, the approximate probe time constant for pulse cooldown is

$$\tau \approx \frac{9.227 \rho c D_o^{1.534}}{k_f (\dot{w}/\mu_f)^{0.466}} \quad (5)$$

The time constant equations can be simplified by inserting the $\rho c D_o^{(2-n)}$ value for the probe tip region. The effective ρc value is

$$\rho c = \frac{(\rho c A)_w + (\rho c A)_I + (\rho c A)_S}{A_T}$$

where the subscripts W,I,S, and T stand for wire, insulation, sheath, and total, respectively.

The various cross section values are obtained from Figure 2b and summarized in Table III along with appropriate c and ρ values.

TABLE III Applicable c, ρ , and A Values for the Probe Tip Region

Section	Specific Heat (c) Btu/lb-°R	Density (ρ) lb/in. ³	Area (A) in. ²	c ρ A Btu/in.-°R
W26% Re wire	0.04*	0.714	0.0000077684	0.0000002219
W5% Re wire	0.04*	0.701	0.0000077684	0.0000002178
BeO insulation	0.54*	0.108	0.0128524269	0.0007495535
W26% Re sheath	0.04*	0.714	0.0197988553	0.0005654553
Summations			0.0326668190	0.0013154485

* At 3000°R

(a) Exact probe tip region cross section

(b) Approximate probe tip region cross section

Figure 2. Cross Section Probe Area

Thus; $c_p \approx \frac{\Sigma c_p A}{A} \approx 0.0403 \text{ Btu/in.}^3\text{-}^\circ\text{R}$

Also; $D_o \approx \left[\frac{4(0.032666819)}{\pi} \right]^{0.5} \approx 0.204 \text{ in.}$

Then, the approximate probe time constant for start up, full power, and shut down is

$$\tau = \frac{0.4045}{k_f (\dot{\omega}/\mu_f)^{0.618}} \quad (6)$$

while that for pulse cooldown becomes;

$$\tau = \frac{0.0325}{k_f (\dot{\omega}/\mu_f)^{0.466}} \quad (7)$$

The approximate time constant values predicted by Equations 6 and 7 are now obtained for various state points and the results listed in Table IV and plotted in Figure 3. The τ' time constant value listed in Table IV was calculated from

$$\tau' = 5.0985 (\dot{\omega})^{-0.5}$$

which is the equation presently employed to estimate the time constant of nozzle chamber thermocouples. Reference to Figure 3 indicates a discontinuity in the time-constant, flow-rate relationship at the throttle point. Thus it would appear that two equations should be used.

Figure 3 contains two graphs, one having a log-log scale while the other has a semi-log scale. Note that data which gives a straight-line graph when plotted on log-log paper has the form

$$y = a(x)^b$$

while data giving a straight-line plot on semi-log paper has the form

$$y = a(b)^x$$

Therefore, when using a dashed straight-line on the log-log plot to represent state points below the throttled level and Equation 6 with average μ_f and k_f values to represent state points between the throttled and full power condition, the following equations are obtained.

$$\text{Throttled to full power} \dots \tau_1 = 8.39 (\dot{\omega})^{-0.618} \quad (8)$$

$$\text{Start up to throttled power} \dots \tau_2 = 194 (\dot{\omega})^{-1.385} \quad (9)$$

Using the two best-fit straight-line plots shown on the semi-log graph yields the following time constant relationships

TABLE IV

TIME CONSTANT VALUES FOR NOZZLE CHAMBER THERMOCOUPLES

State Point	T_c °R	P_c psia	$\dot{w}$ lb/sec	μ_f lb/in.-sec	k_f Btu/in.-sec-°R	τ sec	τ' sec
Full Power →	4250	450	91.4	2.00×10^{-6}	1.400×10^{-5}	0.5333	0.533
	4250	400	80.6	2.00×10^{-6}	1.425×10^{-5}	0.5663	0.568
	4250	350	70.4	2.00×10^{-6}	1.450×10^{-5}	0.6050	0.608
	4250	300	60.4	2.00×10^{-6}	1.475×10^{-5}	0.6539	0.656
Throttled Power →	4250	270	54.8	1.99×10^{-6}	1.484×10^{-5}	0.6828	0.688
	3200	200	46.5	1.64×10^{-6}	9.737×10^{-6}	1.0299	0.748
	2000	120	36.0	1.20×10^{-6}	6.428×10^{-6}	1.5065	0.850
	1400	80	28.0	9.38×10^{-7}	4.877×10^{-6}	1.9918	0.964
Start Up →	800	40	19.0	6.38×10^{-7}	3.297×10^{-6}	2.9506	1.170
Pulse Cooldown →	1290	1	0.4	8.90×10^{-7}	4.600×10^{-6}	16.4100	8.061

Figure 3. Thermocouple time constant as a function of flow rate.

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$$\text{Throttled to full power } \tau_3 = 1.00 (0.993)^{\dot{\omega}} \quad (10)$$

$$\text{Start up to throttled power . . . } \tau_4 = 5.89 (0.962)^{\dot{\omega}} \quad (11)$$

Since large stem conduction errors and large flow velocity gradient variations are expected during pulse cooldown, the assumptions adopted for this discussion are not applicable. Thus, separate equations will not be developed. Instead, Equation 7 will be used with the understanding that the resulting time constant error may be large. Thus

$$\text{Pulse cooldown } \tau_5 = 10.7 (\dot{\omega})^{-0.466} \quad (12)$$

Time constant values predicted by Equations 8 through 12 are compared to actual state point values in Table V. Based upon Table V, Equations 10 and 11 provide the best data fit. This same conclusion could have been reached by reviewing Figure 3 and noting that there is a better straight-line data fit when using the semi-log scale rather than the log-log plot. Thus, Equations 10 and 11 are recommended for describing time constant values. However, since most computers use standard utility routines for obtaining exponential values, it may be useful to rearrange Equations 10 and 11 to obtain the following alternate expressions

$$\text{Throttled to full power } \tau_3' = 1.00e^{-0.007025\dot{\omega}} \quad (13)$$

$$\text{Start up to throttled power } \tau_4' = 5.89e^{-0.03874\dot{\omega}} \quad (14)$$

The selection of Equations 13 and 14 over Equations 10 and 11 is a matter of personal preference as the two equation groups yield nearly identical results.

TABLE V
COMPARISON OF TIME CONSTANT EQUATION

Time Constant	T_c	P_c	$\dot{\omega}$	τ (Exact)	Equation Set	Equation Set
					$(\tau_1), (\tau_2), (\tau_5)$	$(\tau_3), (\tau_4), (\tau_5)$
Throttled to Full Power	4250°R	450 lb/in. ²	91.4 lb/sec	0.5333 sec	0.5151 sec	0.5262 sec
	4250°R	400 lb/in. ²	80.6 lb/sec	0.5663 sec	0.5567 sec	0.5677 sec
	4250°R	350 lb/in. ²	70.4 lb/sec	0.6050 sec	0.6053 sec	0.6099 sec
	4250°R	300 lb/in. ²	60.4 lb/sec	0.6539 sec	0.6654 sec	0.6542 sec
	4250°R	270 lb/in. ²	54.8 lb/sec	0.6828 sec	0.7066 sec	0.6805 sec
Start Up to Throttled Power	4250°R	270 lb/in. ²	54.8 lb/sec	0.6828 sec	0.7579 sec	0.7049 sec
	3200°R	200 lb/in. ²	46.5 lb/sec	1.0299 sec	0.9514 sec	0.9722 sec
	2000°R	120 lb/in. ²	36.0 lb/sec	1.5065 sec	1.3562 sec	1.4602 sec
	1400°R	80 lb/in. ²	28.0 lb/sec	1.9918 sec	1.9208 sec	1.9907 sec
	800°R	40 lb/in. ²	19.0 lb/sec	2.9506 sec	3.2864 sec	2.8213 sec
Pulse Cooldown	1290°R	1 lb/in. ²	0.4 lb/sec	16.41 sec	16.399 sec	16.399 sec

$$\begin{aligned} \tau_1 &= 8.39 (\dot{\omega})^{-0.618} \text{ sec} & (8) \\ \text{Throttled to full power} & \dots \dots \dots \tau_3 &= 1.00 (0.993)^{\dot{\omega}} \text{ sec} & (10) \\ \tau_2 &= 194 (\dot{\omega})^{-1.385} \text{ sec} & (9) \\ \text{Start Up to throttled power} & \dots \dots \dots \tau_4 &= 5.89 (0.962)^{\dot{\omega}} \text{ sec} & (11) \\ \tau_5 &= 10.7 (\dot{\omega})^{-0.466} \text{ sec} & (12) \\ \text{Pulse cooldown} & \dots \dots \dots \end{aligned}$$