SIGNATURES OF NEUTRAL CURRENTS AND HEAVY LEPTONS IN WEAK INELASTIC PROCESSES*

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ABSTRACT

With the aim of differentiating "muonless" reactions which proceed via neutral currents from those via an intermediate heavy lepton, various spectra of the hadronic continuum from the two processes are analyzed. Similarly, the spectra of the leptonic decay modes are examined. The width of the heavy lepton is evaluated using

$$R(s) = \frac{e^+ e^- \rightarrow \text{hadrons}}{e^+ e^- \rightarrow \mu^+ \mu^-} = a s + b$$

and a and b are related to the cross section for vanishing transverse momentum of the hadronic continuum.

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Of interest to us are reactions of the following type:

$$\nu_{\ell}(k) + N(p) \rightarrow \ell^{-}(K) + H(P) \quad (1)$$

$$\nu_{\ell}(k) + N(p) \rightarrow \nu_{\ell}(K) + H(P) \quad (2)$$

$$\begin{aligned} \nu_{\ell}(k) + N(p) &\rightarrow L_{\ell}^{+}(k') + H(p') \\ &\quad \searrow \\ &\quad \nu_{\ell}(K) + H(Q) \end{aligned} \quad (3)$$

$$\begin{aligned} \nu_{\ell}(k) + N(p) &\rightarrow L_{\ell}^{+}(k') + H(P) \\ &\quad \searrow \\ &\quad \begin{cases} \nu_{\ell}(K) + \ell'^{+}(Q) + \nu_{\ell'}(\bar{k}) & (4a) \\ \nu_{\ell}(K) + \ell^{+}(Q) + \nu_{\ell}(\bar{k}) & (4b) \end{cases} \end{aligned} \quad (4)$$

where H is any complex final hadronic state, L is a heavy lepton occurring in some models of unified gauge theories of weak and electromagnetic interactions.^{1,2}

Reaction (2) proceeds via a weak neutral current.³ Since in practice the incident ν beam is not 100% pure and has some $\bar{\nu}$ mixture, this results in providing a background to (4b) coming from

$$\bar{\nu}_{\ell}(k) + N(p) \rightarrow \ell^{+}(K) + H(P) \quad (5)$$

We will assume that these reactions can be described by effective four point couplings which is just the conventional Fermi coupling G for reactions (1), (3), (4) and (5) and is G_0 , say, for reaction (2) given by

$$G_0^2 = G^2 \sin^2 \theta_w \quad (6)$$

where θ_w is the so called Weinberg angle.

Let's consider reactions (2) and (3) which are the so called "muonless" reactions.⁴ The Feynman diagrams for the processes are shown in Figs. 1 and 2 respectively with momenta as labelled in reactions above. In Fig. 2, $P = p' + Q$ is the total momentum of the final hadronic state. What are the salient features of the spectrum of the hadronic continuum in the two reactions? Is it possible to use those features for a differentiation between the two "muonless" reactions and thereby choose between the two kinds of gauge theories involved? These are some of the questions we shall address ourselves to. Experimentally, such an analysis has become particularly accessible using a liquid neon bubble chamber.⁵

The (2×2) density matrix, ρ_0 , which describes the state of the produced heavy lepton in its own rest frame has the form:

$$\rho_0 = \frac{1}{2} \left(1 + \vec{\sigma} \cdot \frac{\vec{B}_0}{A_0} \right) \quad (7)$$

and has been derived elsewhere.⁶ From (7) the differential cross section for the production of heavy leptons results as

$$\begin{aligned} \frac{d^2 \sigma_p}{dk_f^2 k_\lambda^2} = & \frac{G^2}{4ME\pi} \frac{1}{k_o^2} \left\{ 2k' \cdot k W_1 + \frac{W_2}{M^2} (k' \cdot kp^2 - 2k' \cdot pk \cdot p) \right. \\ & + \frac{W_3}{M^2} (k' \cdot qk \cdot p - k' \cdot pk \cdot q) + \frac{W_4}{M^2} (k' \cdot kq^2 - 2k' \cdot qk \cdot q) \\ & \left. + \frac{W_5}{M^2} (k' \cdot kp \cdot q - k' \cdot pk \cdot q - k' \cdot qk \cdot p) \right\} \quad (8) \end{aligned}$$

where E is the incident energy in the lab frame and M is the nucleon mass. The

cross section σ_p for production of heavy leptons of various masses as a function of the incident ν energy is computed from (8) and is shown in Fig. 3.⁷

For the decay process

$$L(k') \rightarrow \nu(K) + H(Q) \quad (9)$$

using the spectral decomposition of the hadronic weak current⁷, J_μ^W ,

$$\begin{aligned} \sum_H \langle 0 | J_\mu^W | H(Q) \rangle \langle H(Q) | J_\nu^W | 0 \rangle (2\pi)^3 \delta^4(k' - K - Q) = \\ = f_1(Q^2) (Q_\mu Q_\nu - Q^2 \delta_{\mu\nu}) + f_2(Q^2) Q_\mu Q_\nu \end{aligned} \quad (10)$$

the decay matrix, D_0 , in the rest frame of L is obtained as

$$D_0 = 2m_L G^2 \left\{ (K_0 + \vec{\sigma} \cdot \vec{K}) \left[2(f_2 - f_1) Q^2 + 2(f_2 + f_1) k' \cdot K \right] + 2(f_2 + f_1) m_L^2 K_0 \right\} \quad (11)$$

Since,

$$\lim_{Q^2 \rightarrow \infty} f_1(Q^2) = \frac{1}{4\pi^2} \lim_{s \rightarrow \infty} R(s) = \frac{1}{4\pi^2} \lim_{s \rightarrow \infty} \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} \quad (12)$$

to accomodate the recent experimental result on e^+e^- annihilation⁸, we substitute:

$$f_1(Q^2) = \frac{1}{4\pi^2} (a|Q^2| + b) \quad (13)$$

hence the width for decay into hadrons is given by

$$\Gamma_H = \Gamma_0 \left[\frac{9}{20} a m_L^2 + \frac{3}{2} b + 4\pi^2 f_2 \right] \quad (14)$$

where

$$\Gamma_0 = \frac{G^2 m_L^5}{192 \pi^3} \quad (15)$$

Fig. 4 shows the variation of the decay width and the branching ratio with the mass of the heavy lepton.

Now using ρ_o , and D_o from (7) and (11) the cross section for reaction (3) via Fig. 2 is obtained as:

$$\sigma = \frac{G^4}{8m_L ME\pi^4 \Gamma} \int \frac{d^3 k'}{2k'_o} \theta \left[-(k' - K)^2 \right] \left[2m_L^2 f_1 + 4k' \cdot K f_1 - m_L^2 (f_2 + f_1) \right] \\ \left[A_{k'} k' \cdot K + A_k k \cdot K + A_p p \cdot K + A_q q \cdot K \right] + (f_2 + f_1) m_L^2 k' \cdot K A_{k'} \quad (16)$$

where,

$$A_{k'} = 2 \left[2W_1 k' \cdot k + \frac{W_2}{M^2} (k' \cdot k p^2 - 2k' \cdot p k \cdot p) - \frac{W_3}{M^2} (k' \cdot p k \cdot q - k' \cdot q k \cdot p) \right. \\ \left. + \frac{W_4}{M^2} (k' \cdot k q^2 - 2k' \cdot q k \cdot q) + \frac{W_5}{M^2} (k' \cdot k q \cdot p - k' \cdot p k \cdot q - k' \cdot q k \cdot p) \right] \\ A_p = -m_L^2 \left[\frac{W_3}{M^2} k \cdot q + \frac{W_5}{M^2} k \cdot q + 2 \frac{W_2}{M^2} k \cdot p \right] \\ A_q = -m_L^2 \left[-\frac{W_3}{M^2} p \cdot k + \frac{W_5}{M^2} k \cdot p + 2 \frac{W_4}{M^2} k \cdot q \right] \\ A_k = m_L^2 \left[2W_1 + \frac{W_5}{M^2} p \cdot q + \frac{W_2}{M^2} p^2 + \frac{W_4}{M^2} q^2 \right] \quad (17)$$

The θ function provides the upper limit, Φ , on the azimuthal angle ϕ for the vector $\vec{K}$ yielding

$$\frac{d^4 \sigma}{dq^2 dv dK_\perp^2 dK_\parallel} = \frac{G^4}{32 \pi^3 m_L ME^2 \Gamma K_o} \left\{ \left(\frac{\Psi_0 + \Psi_2}{2} \right) \Phi + (\Psi_1 + \Psi_3) \sin \Phi \right. \\ \left. + \left(\frac{\Psi_2}{4} \right) \sin^2 \Phi - \left(\frac{\Psi_3}{3} \right) \sin^3 \Phi \right\} \quad (18)$$

where Ψ_i is the coefficient of $(\cos \phi)^i$ in (16).

Letting $m_L = 0$ and using (6) in (8) the transverse momentum spectrum of the hadronic continuum in reaction (2) is computed. In Fig. 5a the spectra resulting from reactions (2) and (3) are shown. Further, for $E = 100$ GeV and $m_L = 5$ GeV the distribution of events in various transverse momentum bins for the two reactions is shown in Fig. 5b.

For leptonic decay mode (4) a similar analysis is conducted using the expression for the differential cross section (K is here the momentum of the charged lepton):

$$\frac{d^4\sigma}{dq^2 dv dK_\ell^2 dK_\ell} = \frac{G^4}{48(2\pi)^5 m_L \int E^2 MK_o} \left[(A_{0K} + A_{2K}/2) \Phi_K + \right. \\ \left. + A_{1K} \sin \Phi_K + (A_{2K}/4) \sin^2 \Phi_K \right] \quad (19)$$

where the coefficients A_{0K} , A_{1K} , A_{2K} are given in reference 6. For the competing reaction (5) we use equation (8). The corresponding results for the leptonic decay mode are also shown on Figs. 5a and 5b.

Non-Locality Tests⁹

For the local interactions (2) and (5) the differential cross sections with respect to q^2 , v are obtained from (8). For reactions (3) and (4) one has to use (18) and (19) to obtain the cross sections with respect to q'^2 , v' where

$$q'^2 = (k - K)^2, \quad v' = (E - K_o) \quad (20)$$

Again for $E = 100$ GeV, $m_L = 5$ GeV Figs. 6a and 6b illustrate the various distributions. Evidently, the distributions resulting from (2) are significantly different from (3) and could therefore constitute tests for differentiating between neutral currents and heavy leptons in "muonless" reactions. Similarly, reactions (4) and (5) result in strikingly different distributions yielding a test for heavy lepton production.

The Limit $K_t \rightarrow 0$

Let K_ℓ , K_t denote the components of the vector $\vec{K}$ longitudinal and transverse to $\vec{k}$ in the lab frame. Then we find from (18) for reaction (3)

$$\left. \frac{d^3\sigma}{dq^2 dv dK_t^2} \right|_{K_t=0}^{K_\ell \geq 0} = \frac{G^4 m_L^3}{128 ME^2 \pi^4 \Gamma} \left\{ \frac{a m_L^2}{12} \left[A_q - \frac{MA_p}{(k_o' - k_\ell')} - 4A_{k'} \right] - \frac{b}{2} A_{k'} - \frac{f_2 m_L^2}{2} \left[A_q - \frac{MA_p}{(k_o' - k_\ell')} \right] \right\} \quad (21)$$

$$\left. \frac{d^3\sigma}{dq^2 dv dK_t^2} \right|_{K_t=0}^{K_\ell \geq 0} = \frac{G^4 m_L^3}{128 ME^2 \pi^4 \Gamma} \left\{ \frac{a m_L^2}{12} \left[\frac{2A_k E + MA_p + 2A_q E}{(k_o' + k_\ell')} - A_q - 4A_{k'} \right] - \frac{b}{2} A_{k'} - f_2 \frac{m_L^2}{2} \left[A_q - \frac{2A_k E + MA_p + 2A_q E}{(k_o' + k_\ell')} \right] \right\} \quad (22)$$

The two cross sections above involve three unknown constants a , b , and f_2 . One may, without being too unreasonable, assume $f_2 = 0$, then the sum and difference of (21) and (22) would directly yield a and b and thereby constitute a test of unified gauge theories of electromagnetic and weak interactions by comparing their value with that from $e^+e^- \rightarrow \text{hadrons}$. Alternatively, using a and b from $e^+e^- \rightarrow \text{hadrons}$ one could interpret (21) and (22) as a test for heavy lepton production.

The case $a = f_2 = 0$ corresponds to the "conventional" one of $f_1 = \text{constant}$, and so from (21) and (22) for reaction (3):

$$\left. \frac{d\sigma}{dK_t^2} \right|_{K_t=0}^{K_t \geq 0} = \left. \frac{d\sigma}{dK_t^2} \right|_{K_t=0}^{K_t \geq 0} = \frac{4}{m_L^2} \sigma_p R_H \quad (23)$$

reminiscent of relation (1) of reference 6 for reaction (4) due to Fierz symmetry.¹⁰

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References and Footnotes

1. H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1949 (1972), Phys. Rev. D7, 561 (1973). B. W. Lee, Phys. Rev. D6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B 47, 99 (1972).
2. J. D. Bjorken and C.H. Llewellyn Smith, Phys. Rev. D7, 887 (1973).
3. A. Salam and J. E. Ward, Phys. Letters 13, 168 (1964). S. Weinberg, Phys. Rev. Letters 19, 1264 (1967) and 27, 1688 (1971).
4. A. Benvenuti, et al., preprint, "Observation of Muonless Neutrino-Induced Inelastic Interaction", NAL (1973) and Phys. Rev. Letters 32, 800 (1974).
5. C. Baltay, et al. - Proposal dated 1/15/74, submitted to the Brookhaven National Laboratory, and NAL Proposal 53a .
6. A. Soni, Phys. Rev. D1 - to appear in the issue of 4/1/74 . In this reference the matrix elements and the differential cross sections are expressed for any

general coupling of the heavy lepton. For the present work, we are assuming (V-A) current for the heavy lepton. In the notation of the previous paper (V-A) case corresponds to $g = 1$.

7. For numerical work we use the parameterization: $F_2(\omega') = 1.1 \left(1 - \left(\frac{1}{\omega'}\right)^2\right)^3$,

$$F_1(\omega') = \frac{\omega'}{2} F_2(\omega') \quad , \quad F_3(\omega') = -|B| \omega' F_2(\omega') \quad , \quad F_4(\omega') = 0 \quad ,$$

$$F_5(\omega') = 2F_1(\omega') \quad \text{where} \quad F_1 = W_1 \quad , \quad F_i = \frac{v}{M} W_i \quad \text{with } i = 2, 3, 4 \text{ and } 5 \quad , \quad \omega' = (2Mv + M^2)/q^2 \quad , \quad B = 1 \quad .$$

8. A. Litke, et al., Phys. Rev. Letters 30, 1189 (1973). B. Richter, invited talk at the Irvine Conference, December 1973 (unpublished).

9. C. H. Albright, Phys. Rev. Letters 28, 1150 (1972).

10. Such relations actually follow from an elegant property of the weak vertex involving polarized spin $\frac{1}{2}$ particles discussed in "Weak Inelastic Production and Leptonic Decays of Heavy Leptons", Columbia University Thesis (1974)- by A. Soni (unpublished).

Figure Captions

Figure 1: Feynman diagram for reaction (2). Here k and K are the initial and final neutrino momenta and p and P are the corresponding momenta of the nucleon.

Figure 2: Feynman diagram for reaction (3). Here k and p are the momenta of the initial neutrino and nucleon, k' and p' are the momenta of the heavy lepton and the nucleon produced at the first vertex, K and Q are the momenta of the neutrino and the hadron which are

the decay products. $P = p' + Q$ is the total momenta of the final complex hadronic continuum state.

Figure 3: The production cross section as a function of the incident energy for heavy lepton masses. $m_L = m_\mu$ and $m_L = 2, 5, 8$ and 11 GeV.

Figure 4: The figure shows the variation of the decay width and the branching ratio into hadrons of the heavy lepton as a function of m_L^2 . In the width for decay into leptons $\Gamma_L = 2 \Gamma_{L \rightarrow l + \bar{\nu}_l} + \Gamma_{L \rightarrow l' + \bar{\nu}_{l'}}$, the factor of 2 comes from the identity of the neutrinos and

$$\frac{R(L_\mu^+ \rightarrow \mu^+ + \nu_\mu + \bar{\nu}_\mu)}{R(L_\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu)} = 2,$$

an elegant test for heavy leptons.

Figure 5: In Figure 5a the differential cross section with respect to the transverse momenta for the five reactions are shown. In Fig. 5b the percentage distribution of the cross section in 1 GeV bins of transverse momenta are presented for $E = 100$ GeV and $m_L = 5$ GeV.

Figure 6: In Figure 6a the differential cross sections with respect to q^2 , ν for reactions (1), (2), and (5) and with respect to q'^2 , ν' for reactions (3) and (4) are compared. In Fig. 6b the percentage distribution in 10 GeV $\nu(\nu')$ bins are illustrated for the five reactions for $E = 100$ GeV and $m_L = 5$ GeV.

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5a.

Figure 5b.

Figure 6a.

Figure 6b.