BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

BEAM LOSS MECHANISM IN A COASTING BEAM
DUE TO A HIGH ORDER ISOLATED NONLINEAR RESONANCE

M. Month

December 26, 1973

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

Abstract

It is well known that for fixed tune high order (i.e. ≥ 5) isolated nonlinear resonances are intrinsically stable in the sense that sufficiently small amplitudes will not grow. On the other hand, for a system with changing tune, particles can "lock-into" a high order resonance and be carried to large amplitudes. In coasting beams such as the ISR beams, there is no obvious dynamic process to provide tune changes. However, it has been suggested that scattering processes, such as intrabeam proton-proton scattering, can induce momentum diffusion and thus, through the chromaticity, tune diffusion. In this way particles can "pass through resonances". A simple picture of this process is proposed here. Expressions for the fractional current loss rate are derived. Estimates for the loss rate are compared with ISR observations and are found to be in agreement. Only the classical theory of nonlinear resonances is used.

Introduction

It is well known that for fixed tune, high order isolated nonlinear resonances (order ≥ 5) are intrinsically stable. On the other hand, for a system with changing tune, particles can "lock-into" a high order resonance and be carried to large amplitudes.¹ In coasting beams, such as the ISR beams, there is no obvious dynamic process to provide tune changes. However, it has been suggested that scattering processes, such as intra-beam scattering of protons within a beam by Coulomb forces, can induce momentum diffusion^{2,3} and thus, through the chromaticity, tune diffusion. In this way particles can "pass through resonances".

Consider a beam of particles distributed in $[x, x' = dx/ds]$ phase space. Only one-dimensional motion is dealt with here. The variable x is a particle's position with respect to its equilibrium orbit, x' is its slope, s is the distance measured along the equilibrium orbit from some arbitrary reference point. A particle passes through resonance when its tune passes through some rational fraction. As this occurs the phase space is transformed. If the passage through resonance is in the proper direction, stable islands appear at the origin and begin to move outward to regions of larger betatron amplitude as the tune changes. If a particle is trapped in one of these stable islands, and furthermore, if the stable island reaches a physical aperture, then the particle is lost. Formally, the process can be described in terms of two probability functions, with the probability of particle loss being the product. By equating the probability with the fraction of a large number of particles, we can deduce the loss rate. First, we need the probability that a particle will be trapped in an outward moving island when the resonance is crossed. This will depend on the speed of resonance crossing, the excitation strength of the resonance and

also the nonlinear detuning, whose dominant contribution comes from a 0th harmonic octupole term. We denote this probability P_T . Secondly, we need the probability that the islands will reach the aperture. This is equivalent to finding the probability that a particle's tune will cross the resonant tune and reach that tune at which the island is at the aperture. This probability, P_D , is derived from the momentum or tune diffusion due to intrabeam scattering. The probability of loss, P_L , is then P_D , the probability of the moving islands reaching the particle sink at the aperture, multiplied by P_T , the probability that any given particle will be trapped as the islands move through the beam. That is,

$$P_L = P_D P_T \quad (1.1)$$

In section 2, we derive an expression for the loss rate, assuming a diffusion in particle tune and the excitation of a high order resonance. In section 3, we discuss the parameters of the betatron resonance in detail, while in section 4, the diffusion coefficient is related to the parameters of the intrabeam scattering. Finally, in section 5 we compare our predicted loss rates for the ISR with experimental observations made there.² We conclude that a combination of intrabeam proton-proton scattering and classical resonance theory¹ provides an adequate explanation of beam loss rates at the ISR.

2. Beam Loss Rate from Diffusion Through a High Order Nonlinear Resonance

As pointed out in the previous section, there are two processes to consider: the diffusion of tune across a resonance and the trapping of particles in outward moving islands. We assume that the two processes are uncoupled. This will be discussed again in section 4, but the essential point is that the time scale for the diffusion of tune is slow compared to the time scale for the movement of the stable islands through

the beam and to the aperture. In other words, the particle motion through the resonance is adiabatic. The model thus consists of an adiabatic shifting of the phase plane topology induced by slow tune changes which in turn are the result of intrabeam scattering momentum diffusion.

For slow passage through the resonance, we can approximate the trapping efficiency by simply taking the ratio of the total island area, A_I , at the beam edge to the total beam area, A_B ; that is,

$$P_T = \frac{A_I}{A_B} . \quad (2.1)$$

Now, it is clear that the trapping efficiency has in actuality a complicated dependence on, among other resonance characteristics, the speed of resonance crossing. In the adiabatic limit, (2.1) undoubtedly gives the correct value. In general, however, this expression is a limiting efficiency. We can formally include the effects of crossing speed, resonance strength, detuning, and particle phase space density distribution by defining a trapping factor, η_T , by

$$P_T = \eta_T \frac{A_I}{A_B} \quad (P_T < 1) . \quad (2.2)$$

An example where $P_T \approx 1$ is given in Ref. (4) where a numerical study of crossing a third integer resonance (with octupole detuning) can be found. Here we will not consider the complicated details of the trapping process. To estimate trapping efficiencies we will simply use (2.1), noting that it is probably an overestimate.

The second step in our model is to estimate the diffusion effects. Then, with the use of (1.1), we can obtain the loss rate. A simple picture, adequate for our purposes is that intrabeam scattering causes a random walk in tune, the entire process being determined by a diffusion constant,

$$D = \frac{d}{dt} (\Delta\nu)_{rms}^2 , \quad (2.3)$$

where t is the time, and $(\Delta\nu)_{\text{rms}}$ is the rms dispersion in the tune distribution.

The relationship of D to the intrabeam scattering process will be discussed in section 4. Here we simply consider that particles collide, with each collision causing the tune to change by $(\Delta\nu)_{\text{rms}}$ in magnitude. The process is then, as we have pointed out, completely described in terms of the diffusion coefficient, D .

Consider a given number of particles at $x = 0$ (x represents the tune) and at time $t = 0$. Let us further assume that at $x = x_1$ there exists an absorbing screen or sink. The sink of course represents the aperture. We have then, that given the diffusion constant D , the fraction of particles per unit time deposited at the aperture (i.e. at $x = x_1$) at time T is given by⁵

$$q_D(x_1, T) = \frac{x_1}{2T(nDT)^{1/2}} e^{-x_1^2/4DT} \quad (2.4)$$

Note that q_D is related to the probability of a particle reaching the aperture from $x = 0$ in time T by

$$P_D(x_1, T) = \int_0^T q_D(x_1, t) dt \quad (2.5)$$

The fraction of particles lost to the aperture depends not only on the diffusion but also on the trapping efficiency. To find the fraction lost in time T , we simply apply (1.1), using (2.5) for P_D and (2.2) for P_T . To obtain the loss rate, however, we use (2.4) instead of (2.5). If there are n particles at $x = 0$, $t = 0$, then the loss rate at time T is

$$\frac{\dot{n}}{n} = -P_T q_D(x_1, T) = -\frac{P_T x_1}{2T(nDT)^{1/2}} e^{-x_1^2/4DT} \quad (2.6)$$

Let us consider now a range of tunes

$$\nu_- \leq \nu \leq \nu_+$$

and a density distribution in tune, $\rho(\nu)$. Let the resonance tune be denoted ν_R and the tune corresponding to the stable islands reaching the aperture ν_A . Recalling that x_1 is the amount of tune change, we have

$$x_1 = x_1^- = \nu_A - \nu, \quad \text{if} \quad \nu_- \leq \nu \leq \nu_R,$$

and

(2.7)

$$x_1 = x_1^+ = \nu_A - \nu_R + \nu - \nu_R, \quad \text{if} \quad \nu_R \leq \nu \leq \nu_+.$$

We thus conceive of particles as being fed into the resonance by diffusion, transferred to the aperture by the resonance and thereby lost. The total beam loss rate per unit tune

$$\frac{d^2N}{dTd\nu} = - \frac{\rho(\nu) P_T x_1}{2T(\pi DT)^{\frac{1}{2}}} e^{-x_1^2/4DT}, \quad (2.8)$$

where N is the total number of particles in the beam. If we assume ρ is constant, we can integrate over tune to obtain the total beam loss rate at time T ,

$$\dot{N} = - 2 P_T \rho \sqrt{\frac{D}{\pi T}} e^{-(\nu_A - \nu_R)^2/4DT}, \quad (2.9)$$

where we have assumed that the distance from resonance to aperture is significantly smaller than the distance from resonance to ν_+ or ν_- . This function has a maximum at

$$T = \frac{\delta^2}{2D}, \quad (2.10)$$

where

$$\delta = \nu_A - \nu_R. \quad (2.11)$$

If we further write the tune width of the beam as

$$\Delta = \nu_+ - \nu_-, \quad (2.12)$$

then

$$N = \rho \Delta, \quad (2.13)$$

and the maximum percentage loss rate is

$$\left(\frac{\dot{N}}{N} \right)_{\max} = -0.97 \frac{P_T D}{\Delta \delta} \approx - \frac{P_T D}{\Delta \delta} \quad (2.14)$$

Note that the falling off of the loss rate in time is slow, like $T^{-\frac{1}{2}}$ and we can use (2.14) as a measure of the average loss rate. We will use expression (2.14) in section 5 to estimate loss rates for the ISR.

3. Characteristics of a High Order Isolated Nonlinear Resonance

Invariant for a Nonlinear Resonance with Octupole Detuning

Using either phase amplitude or Hamiltonian methods, we can derive the one-dimensional invariant for a particle when its motion is dominated by an isolated resonance. Including octupole detuning, and in terms of the amplitude and phase variables γ and φ , we have for the invariant¹

$$I = \gamma \Delta_L + \frac{1}{2} \gamma^2 \Delta_{NL} + \frac{2}{p} \gamma^{p/2} \Delta_e \cos p \varphi \quad (3.1)$$

The amplitude variable is defined by

$$\gamma = \frac{\epsilon}{\epsilon_0} \quad , \quad (3.2)$$

where ϵ is the square of the phase space amplitude, or the usual emittance of the particle motion (which may, of course, change in the course of its trajectory when nonlinearities are present); ϵ_0 is a reference emittance, which may be taken to be the beam emittance, ϵ_B — i.e. the emittance boundary for the beam. The phase variable φ is a phase space angle related to the ratio of slope and position of a particle with respect to its equilibrium orbit.

We can define the quantities Δ_L , Δ_{NL} and Δ_e in terms of the nonlinear field and betatron parameters: Δ_L is the linear tune shift from resonance,

$$\Delta_L = \nu - \frac{n}{p} \quad , \quad (3.3)$$

where ν is the linear tune, which is a function of the particle momentum through the chromaticity of the ring,

n is the harmonic number of the exciting resonance, and

p is the order of the exciting resonance;

Δ_{NL} is the nonlinear tune shift (i.e. detuning) at the reference emittance,

$$\Delta_{NL} = -\frac{3}{8} \langle \beta \rangle^3 \nu \bar{O} \epsilon_0, \quad (3.4)$$

where $\langle \beta \rangle = R/\nu$ is the average β -function, R is the average ring radius, and $\bar{O}$ is the strength of the octupole field component, defined below; Δ_e is the resonance excitation width at the reference emittance,

$$\Delta_e = -\frac{1}{2^p} \langle \beta \rangle^{(p/2+1)} \nu g_n \epsilon_0^{(p/2-1)}, \quad (3.5)$$

where g_n is the n^{th} azimuthal harmonic of the resonance component of the exciting field. We can write the exciting field, using the nominal median plane magnetic field terminology, as

$$\frac{1}{(B\rho)} B(\theta, x) = \dots + O(\theta)x^3 + \dots + g(\theta)x^{p-1} + \dots; \quad (3.6)$$

where $(B\rho)$ is the particle magnetic rigidity (i.e. momentum);

x is the transverse position;

θ is the azimuth, chosen so that the n^{th} harmonic of $g(\theta)$ is simply

$$g_n = \frac{1}{\pi} \int_0^{2\pi} g(\theta) \cos n\theta d\theta; \quad \text{and} \quad (3.7)$$

$\bar{O}$ is related to $O(\theta)$ by

$$\bar{O} = \frac{1}{2\pi} \int_0^{2\pi} O(\theta) d\theta. \quad (3.8)$$

Fixed Points

The fixed points influencing the phase plane topology are obtained from

$$\frac{\partial I}{\partial \gamma} = \frac{\partial I}{\partial \varphi} = 0 \quad ; \quad (3.9)$$

or,

$$\Delta_L + \Delta_{NL} \gamma \pm \Delta_e \gamma^{(p/2-1)} = 0 \quad . \quad (3.10)$$

It can also be shown that the fixed points are stable (sfp) or unstable (ufp) according to the prescription:

$$\begin{aligned} (p/2 - 1) \Delta_e \gamma^{(p/2-2)} \pm \Delta_{NL} &> 0 \text{ (ufp)} \\ &< 0 \text{ (sfp)} \end{aligned} \quad . \quad (3.11)$$

Normal Stability Limit ($p > 4$)

We can find the usual stability limit roughly by taking $\Delta_L = 0$. This leads to

$$\gamma_{\text{st.limit}} \approx \left| \frac{\Delta_{NL}}{\Delta_e} \right|^{[2/(p-4)]} \quad . \quad (3.12)$$

At this amplitude, we have a series of p unstable fixed points (really unstable in that detuning will result only from terms of higher order than p). At such amplitudes, particles can be drawn to much larger amplitudes and in this sense become unstable. Since $\gamma_{\text{st.limit}}$ is generally well outside the machine aperture, it provides no particle sink. That is, if the betatron amplitude of a particle is increasing, it will reach a physical aperture before it reaches the resonance stability limit. If $\gamma_{\text{st.limit}}$ enters the machine physical aperture, then of course this amplitude becomes a particle sink, i.e. a channel for drawing particles swiftly to the chamber wall. Since for the cases of concern to us, this does not occur, we will here neglect this case and consider that only the physical aperture itself is the particle sink.

Stable Islands ($p > 4$)

Equation (3.10) also gives a set of $2p$ fixed points nearer to the origin, p of them are stable fixed points, p unstable. The topology in the phase space

is a set of p stable islands wrapped around the origin like a string of pearls. The position of the fixed points defining the islands can be evaluated in a series in the variables Δ_L/Δ_{NL} and Δ_e/Δ_{NL} . Keeping the first two terms in this expansion, we have for the stable and unstable fixed points, γ_s and γ_u respectively,

$$\gamma_s = \left| \frac{\Delta_L}{\Delta_{NL}} \right| + \left| \frac{\Delta_e}{\Delta_{NL}} \right| \left| \frac{\Delta_L}{\Delta_{NL}} \right|^{(p/2-1)}, \quad (3.13)$$

$$\gamma_u = \left| \frac{\Delta_L}{\Delta_{NL}} \right| - \left| \frac{\Delta_e}{\Delta_{NL}} \right| \left| \frac{\Delta_L}{\Delta_{NL}} \right|^{(p/2-1)}. \quad (3.14)$$

Note that these islands only appear on "one side of the resonance"; i.e. given a sign of the detuning, Δ_{NL} , the side of the resonance where they appear is that side defined by Δ_L being of the opposite sign. Thus,

$$\frac{\Delta_L}{\Delta_{NL}} < 0. \quad (3.15)$$

For our purposes, we can neglect the second term in the expansion and define the island position by

$$\gamma_{IS} = \left| \frac{\Delta_L}{\Delta_{NL}} \right|, \quad (3.16)$$

which is independent of the resonance excitation width, Δ_e .

Particle Loss by Transport in the Stable Islands

If the tune is fixed, the stable islands do not grossly affect the particle motion. However under conditions when the tune is time dependent, particles can "lock-onto" a resonance, leading to amplitude growth.¹ In our picture, if a particle tune crosses the resonance value in the appropriate direction [see (3.15)], the stable islands are created and move to large amplitudes. The island motion is governed by (3.16). If the tune variation is sufficiently slow,¹ the motion is adiabatic. Under these

circumstances, as the islands pass out of the beam they trap particles and carry them to larger amplitudes. If the islands reach the machine aperture, the particles within them will be lost. The tune at which this occurs is given by setting

$$\gamma_{IS} = \gamma_{AP} \quad , \quad (3.17)$$

or when

$$|\Delta_L| = |\Delta_{NL}| \gamma_{AP} \quad . \quad (3.18)$$

This is just the quantity δ defined in (2.11).

Thus, if a particle's tune passes through resonance and reaches the value given in (3.18), then the particle can be trapped by the outward moving islands and transported to the aperture. Given such a tune variation, we can compute the probability of a particle being trapped and transported. This probability corresponds to the quantity P_T discussed in section 2. From (2.1), we can see that P_T is simply the ratio of the total stable island area at the beam edge to the beam area. As was pointed out, this is valid in the adiabatic limit. Taking the lowest order in Δ_e/Δ_{NL} and taking ϵ_0 , the reference emittance, to be the beam emittance (i.e. $\gamma_{BEAM} = 1$), then the trapping probability as the islands pass through the beam is given by

$$P_T = \frac{A_I}{A_B} = \frac{2\sqrt{2}\pi}{\sqrt{p}} \left| \frac{\Delta_e}{\Delta_{NL}} \right|^{\frac{1}{2}} \quad . \quad (3.19)$$

The condition that the island area at the beam edge be less than the beam area corresponds to the physical constraint that the trapping probability be less than one and leads to the limit on the excitation strength of the resonance

$$\left| \frac{\Delta_e}{\Delta_{NL}} \right| < \frac{p}{8\pi^2} \quad . \quad (3.20)$$

This is, in effect, a necessary condition for the validity of the approximations used above.

We will use in section 5 the expressions given here for δ and P_T [Eqs. (3.18) and (3.19) respectively] in order to estimate loss rates with (2.14).

4. Diffusion from Intrabeam Scattering

The multiple scattering of protons in a beam due to Coulomb forces, that is, intrabeam scattering, causes a diffusion in longitudinal momentum.⁶ The diffusion coefficient in the lab frame of reference is derived in Ref. 6 and can be written

$$D_p = \frac{d}{dt} \left(\frac{\delta p_{rms}}{p} \right)^2 = \frac{\sqrt{\pi} c r^2 \rho}{\beta^3 \gamma^3 \varphi_{rms}} f(x_0) \quad , \quad (4.1)$$

where δp_{rms} is the rms momentum spread,

p is the longitudinal momentum,

ρ is the average particle density,

β is the velocity in units of c ,

γ is the energy in proton mass units,

r is the classical proton radius ($= 1.54 \times 10^{-18}$ m),

φ_{rms} is the rms dispersion in the betatron angle.

The quantities δp_{rms} , p and ρ are to be evaluated in the lab frame of reference.

The definitions of ρ , φ_{rms} and $f(x_0)$ are:

$$\rho = N/V \quad , \quad (4.2)$$

where N is the number of particles in the volume V ,

$$V = Cdh \quad , \quad (4.3)$$

$C = 2\pi R$ is the circumference,

d is the total beam width,

$$d = \sqrt{4\epsilon_x \beta_x + \left(x_p \frac{\delta p}{p} \right)^2} \quad , \quad (4.4)$$

h is the total beam height,

$$h = \sqrt{4\epsilon_y \beta_y} \quad (4.5)$$

with β_x , β_y the average horizontal and vertical β -function values,

$$\beta_x = R/v_x, \quad \beta_y = R/v_y, \quad (4.6)$$

ϵ_x , ϵ_y the 95% horizontal and vertical emittances (Area = $\pi\epsilon$),

ν_x , ν_y the horizontal and vertical tunes;

$$\varphi_{rms} = \frac{1}{2} \sqrt{\frac{\epsilon_x}{\beta_x} + \frac{\epsilon_y}{\beta_y}}; \quad (4.7)$$

and

$$f(x_0) = \int_{x_0}^{\infty} \frac{1}{x} e^{-x} \ln \frac{x}{x_0} dx, \quad (4.8)$$

where

$$x_0 = \frac{2\pi}{\beta_y^2 \varphi_{rms}^2} \left(\frac{\rho}{\beta_y} \right)^{1/3}. \quad (4.9)$$

The number of particles N is related to the current I by

$$I = \frac{e N c \beta}{C}. \quad (4.10)$$

The function f can be estimated from a graph given in Ref. (6).

By way of the chromaticity, ξ , defined as tune change per unit fractional momentum change, momentum is related to betatron tune. The distributions can be connected through the relation

$$\delta \nu_{rms} = \xi \frac{\delta p_{rms}}{p}. \quad (4.11)$$

Thus, the diffusion coefficient for tune

$$D_\nu = \xi^2 D_p. \quad (4.12)$$

The diffusion coefficient describes the macroscopic behavior of a system. Although the microscopic effects are discrete shocks (from scattering events), the sum of all these effects, over "long" time periods are manifest as a

continuous process. The meaning of "long" time is simply a time interval over which many individual events have occurred. The averaging over this time interval then leads to a pseudo-continuous process.⁵

In our model, we have assumed that the tune of a given particle is changing continuously. In fact, of course, it is not. The shocks from the scattering events change the particle phase advance. However, if there are a large number of such shocks in the period of a revolution, and if each shock produces a sufficiently small effect, then the averaging process over this time period becomes valid. The change in the average phase advance, i.e. averaged over a revolution, will then appear as a slow, essentially continuous process. Now, this is exactly what our model requires, i.e. that the change in the tune, the phase advance per revolution, be a slow, pseudo-continuous process.

Quantitatively, we require (1) that the number of collisions per revolution be large: this ensures a "continuous" tune change process; and (2) that the average change in tune per revolution be sufficiently small for the process to be considered adiabatic.

The second condition can be tested simply from knowledge of the diffusion coefficient — that is, the average change in tune is essentially given by⁵

$$\delta\nu_{\text{rev}} = (2 D_{\nu} T_{\text{rev}})^{1/2} = \left(\frac{2 D_{\nu} C}{c} \right)^{1/2} . \quad (4.13)$$

The limit on $\delta\nu_{\text{rev}}$, or D_{ν} , can be obtained from the adiabatic requirement for motion near a nonlinear resonance¹:

$$\delta\nu_{\text{rev}} \leq 4\pi |\Delta\nu_c \Delta\nu_{\text{NL}}| , \quad (4.14)$$

where $\Delta\nu_c$ and $\Delta\nu_{\text{NL}}$ are defined in section 3. It should be recalled that both $\Delta\nu_c$ and $\Delta\nu_{\text{NL}}$ are amplitude dependent.

The first condition, that dealing with the minimum number of events that would allow time averaging is difficult to state quantitatively. A simple criterion which might suffice for our purposes is simply that the mean tune change per collision, $\langle \epsilon \rangle_{\text{rms}}$, be much smaller than the average tune change per revolution:

$$\langle \epsilon \rangle_{\text{rms}} \ll \delta v_{\text{rev}} \quad . \quad (4.15)$$

This corresponds roughly to the criterion that the number of collisions per revolution, n' , be much greater than one:

$$n' \gg 1 \quad . \quad (4.16)$$

An expression for n' can be obtained using Ref. (6):

$$n' = \sqrt{\pi} C \left(\frac{\rho}{\beta \gamma} \right)^{1/3} \varphi_{\text{rms}} \quad . \quad (4.17)$$

To recapitulate, we have that under the conditions expressed in (4.14) and (4.16), we expect that the model given in section 2 is applicable. Particles are fed into a resonance by the tune diffusion process and transported to the aperture by stable islands moving outward in betatron amplitude. The loss rate can be estimated from (2.14).

5. Application to the ISR

We can use ISR parameters in our model to predict loss rates for the ISR. Using the values $\gamma = 26$, $R = 150$ m, $I = 15$ A ($N = 2.94 \times 10^{14}$), $v = 8.6$, $\epsilon_x = 1.5 \times 10^{-6}$ rad-m, $\epsilon_y = 0.75 \times 10^{-6}$ rad-m, $\delta p/p = 0.03$, and $\xi = 1$, we obtain for the diffusion coefficient [from (4.1) and (4.12)]

$$D_v = 4.9 \times 10^{-11} \text{ sec}^{-1} \quad .$$

We can get a rough estimate of resonance parameters using the computations in Ref. (7) for the beam-beam interaction. We will evaluate nonlinear parameters at an amplitude corresponding to the rms beam size. We then obtain

$$\Delta_{\text{NL}} = \frac{3}{8} \Delta_{\text{bb}} \quad , \quad (5.1)$$

where Δ_{bb} is the linear beam-beam tune shift, which for the ISR has the value

$$\Delta_{bb} = 3.5 \times 10^{-4} \quad .$$

Since the ISR has an eight-fold symmetry, resonances of the order of 5 or 6 are not excited for identical beam-beam interactions. These resonances will however be excited by differences in the interactions. Let us then take a typical resonance excitation width, say the average of the 6th order resonances, and write

$$\Delta_e = 0.075 \, \eta \, \Delta_{bb} \quad , \quad (5.2)$$

where η is a factor which takes account of the fact that the intersections are non-identical. We arbitrarily take η to be a few percent. For a 5th order resonance, $p = 5$, and $\eta = 2\%$, we have for the trapping probability, from (3.19),

$$P_T = 0.25 \quad .$$

The tune change required for the stable islands to reach the aperture is obtained from (3.18). Assuming the aperture to be 3 times the beam size,

$$\delta = 1.2 \times 10^{-3} \quad .$$

The tune width of the beam is determined by the momentum spread and the chromaticity. With the values assumed above,

$$\Delta = \xi \frac{\Delta p}{p} = 0.03 \quad .$$

Using $D_v (= D)$, P_T , δ and Δ in (2.14) we obtain for the loss rate

$$\frac{\dot{N}}{N} = \frac{\dot{I}}{I} = - 2.0 \times 10^{-5} \text{ min}^{-1} \quad .$$

This loss rate is consistent with observations made at the ISR.⁸

With the parameters used here, the number of collisions per revolution [from (4.17)] is $n' = 9000$, thus satisfying the criterion (4.16). However condition (4.14) is not satisfied, which means that the trapping probability we assumed is perhaps an overestimate.

6. Conclusions

We have proposed a model, based on ideas given in Refs. (2, 3), which accounts for the beam loss rate observed at the ISR. The model describes a mechanism for beam loss due to a high-order isolated nonlinear resonance when fed by a diffusion process, which was taken to be intrabeam scattering. It is perhaps important to emphasize that only the classical theory of nonlinear resonances has been used.

References

1. A Schoch, CERN Report, CERN 57-23 (1958).
2. E. Keil, III All-Union National Conf. on Particle Accelerators, October, 1972.
3. H.G. Hareward, CERN Report, CERN/ISR-DI/72-26 (1972).
4. A van Steenbergen and M. Month, 7th International Conf. on High Energy Accelerators, Yerevan, USSR, September 1969.
5. S. Chandrasekhar, Rev. of Mod. Phys., 15, 1, p. 1 (1943).
6. H. Bruck, Accélérateurs Circulaires De Particules, Presses Universitaires De France, Paris (1966).
7. E. Keil, CERN Reports, CERN/ISR-TH/72-7 and 72-25 (1972).
8. E. Keil, CERN Report, CERN/ISR-TH/73-38 (1973).