**Electron Density Measurement of an Electrostatically-
Confined, Spherically Symmetric, Helium Plasma Using
A Microwave Cavity Resonance Shift Technique**

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by

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ABSTRACT

Microwave resonant cavity frequency shift measurements are made to determine the electron density of an electrostatically confined, spherically symmetric helium plasma. Appropriate electron density distribution functions are assumed and then used to relate the frequency shift to the central electron density. Results from electron injection, with currents of up to 200. mA, accelerating voltages from 100. V to 450. V and pressures of 5.0, 1.0, and 0.5 mTorr, suggest that a potential well is formed and that the minimum value of perveance required for well formation depends on the accelerating voltage and the pressure. Maximum electron densities of $9.5 \times 10^9/\text{cm}^3$ are measured. Electron densities of $5.5 \times 10^9/\text{cm}^3$ are measured during ion injection with currents of up to 20. mA at accelerating voltages between 0.5 kV and 5.0 kV and pressures of 5.0, 1.0, and 0.5 mTorr. Electron densities measured during ion injection show remarkable agreement with those predicted by an earlier theoretical model.

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I. INTRODUCTION

Several experiments have been conducted since Elmore, Tuck, and Watson (1959) first suggested that a plasma could be confined electrostatically. In their proposed system, electrons would be accelerated radially through a positively biased, spherical grid. Upon nearing the center of the grid, they would be repelled by their own space charge and accelerated out through the grid. The resultant circulating electron current would form a potential well capable of trapping ions at the center of the grid. This mode of operation will be referred to as electron injection.

Hirsch (1967) generated considerable interest when he measured a copious neutron emission from a similar device operating in a deuterium-tritium background gas mixture. Rather than accelerate electrons, Hirsch reversed the polarity of the grid and injected ions, hence the name ion injection. By calculating from the fusion rate, he estimated a central ion density of $10^{12} - 10^{14}/\text{cm}^3$. Using an electron beam probe, Hirsch (1968) subsequently measured a potential well formed by electron injection.

A modified electron beam probe technique was used by Black (1971) to measure the shape of the potential profile. Careful analysis of his results shows that no potential well was formed in either the electron injection or ion injection mode. His theoretical model of the ion injection mode predicts electron and ion densities on the order of $10^9/\text{cm}^3$ in the center of the device.

In these experiments, particle densities were estimated from either experimentally measured fluxes or theoretical calculations.

It is the purpose of this work to develop a technique for measuring electron density directly. To do this, a microwave signal was coupled into the plasma-filled grid and the resonant frequency of the grid measured. The difference between this frequency and the free space resonant frequency is directly proportional to the electron density within the grid. In a comparison of density measurements of a mercury vapor discharge obtained by this technique to those found by probe measurements, Adler (1949) found excellent agreement. While the microwave measurements can be made with essentially no disturbance of the plasma, the presence of a probe in this device would be intolerable.

Chapter II contains a detailed description of the experimental apparatus and procedure. An explanation of the microwave resonant cavity density measurement technique is offered in Chapter III. Experimental results and analysis are presented in Chapter IV and the work summarized in Chapter V. Two appendices deal with the construction and microwave characteristics of parts of the experimental apparatus, while an estimate of the collision frequency is found in a third.

To facilitate understanding of the device, idealized models are presented in the following paragraphs. The electron injection mode will be treated first, followed by the somewhat more complicated ion injection case.

Electron injection mode operation of an idealized electrostatic confinement device of the type used in this work is illustrated in Fig. 1. Primary electrons are pulled away from the filaments by the low voltage outer grid. After passing through this grid, the electrons are strongly accelerated toward the high voltage inner grid. Ionizing collisions with background gas atoms in this region result in secondary

electrons, which are accelerated inwards, and ions which are repelled out of the system. The electrons reach their maximum kinetic energy as they pass through the inner grid.

Figure 1. Electron Injection - Idealized Model

Once inside, they are decelerated by their space charge until they stop and reverse direction. After being accelerated out through the inner grid, they again reverse direction due to the strong attraction of the high voltage. This reversal is expected to occur somewhere near the outer grid. The electrons continue to circulate in this manner until they are removed from the system, either by collision with the grid or recombination. Secondary electrons resulting from ionizations within the central region contribute to this circulating current. However, ions formed in this region are trapped by the potential well of the circulating electrons.

As illustrated in Fig. 2, ion injection mode operation is very similar to electron injection. The circulating electrons in this case are limited to the region near the outer grid while the inner grid area is traversed by a circulating current of ions. A potential well formed in this manner will therefore trap electrons. Note that background gas ionization near the outer grid is the principal source of ions in this device.

Figure 2. Ion Injection - Idealized Model

While the total reflection of injected particles described in these models is a useful concept for explaining the idealized operation of the device, it may never occur in the practical case. Most of the injected particles probably pass directly through the central region, since only a full potential well can reflect them all. However, even a shallow well is capable of trapping charged particles, and thus, particle trapping is not dependent upon total reflection. The electron density which is measured in the ion injection experiment is due to electrons which may be trapped in a shallow potential well. In contrast, the injected particles themselves are detected in the electron injection experiment.

II. LABORATORY EQUIPMENT AND PROCEDURE

A detailed description of the laboratory equipment and experimental procedure follows. For convenience, the chapter has been divided into the following sections:

1. Vacuum System
2. Confinement Device
3. Power Supplies
4. Microwave System
5. Experimental Procedure

Vacuum System

The vacuum apparatus consisted of an 18 inch pyrex bell jar with steel shield mounted on a cabinet-based pumping station. The principal components of the pumping station were a 400 liter/sec diffusion pump and a 15 cfm mechanical pump. This system was capable of attaining pressures below 10^{-5} Torr. Repressurization with helium was accomplished by means of a micrometer valve connected through a port in the base plate. The inlet line was first purged using an auxiliary mechanical pump and then filled with gas. During the experiment, the line was maintained at a slightly positive pressure to insure a constant gas inlet rate of pure helium. Pressure was monitored via a cold cathode discharge gauge mounted at a port in the base plate. The range of this gauge was 10^{-2} to 10^{-5} Torr. An instability in the pressure, attributed to a non-uniform pumping rate of the diffusion pump, was detected early in the experiment. This condition was apparently due to intermittent hard boiling of the diffusion pump oil.

It was eliminated by installing a variac on the diffusion pump power line. Adjustments of the variac and micrometer valve resulted in a stable pressure.

The high voltage connection into the vacuum system was made via a standard high voltage feedthrough. An octal feedthrough was used for all other electrical connections except the microwave signal.

Confinement Device

The essential components of the device are the filaments and the two spherical grids. Stainless steel wires, spot-welded at the junctions, formed the grids. The 3.03 inch diameter inner sphere was made with 20 mil wire longitudinally and latitudinally except for a single 41 mil wire ring encircling each pole. The distance between the latitudinal rings was approximately 6 mm except at the equator where it was almost 12 mm. The longitudinal spacing at the equator was nearly 15 mm. The transparency v of the grid, defined as

$$v = \frac{\text{open surface area}}{\text{total surface area}}$$

can be calculated if one knows the total length of the wires in the grid. Two estimates of this length were obtained, the first by actually measuring the wires and the second by dividing the weight of the grid by the weight per unit length of the wire. With agreement between the resulting calculations within 1.0%, the grid was found to be 87.0% transparent.

Construction of the 6 inch diameter outer sphere was similar to that of the inner sphere except that 41 mil wires were used for latitude. With the exception of three support legs, it was open at

the equator to facilitate assembly of the device. The latitudinal rings were approximately 15 mm apart as were the longitudinal wires near the equator. The spheres were mounted concentrically within the bell jar, supported by concentric vertical tubes.

Four 1.5 inch tungsten filaments were mounted 1.2 inches below the outer grid equator in a horizontal ring of approximately 6.2 inch diameter. A second filament ring was mounted symmetrically above the equator, but preliminary observations showed that this second ring made no discernable contribution to the operation of the device. Therefore, it was turned off for the remainder of the experiment.

Power Supplies

The schematic diagram of the device given in Fig. 3 shows the configuration used for electron injection. Filament emission currents were measured by the ammeters shown on the filament circuits. An ammeter within the power supply ~~was~~ used to monitor the inner grid current. The values of ballast resistors R_1 and R_0 were 2000 and 1870 ohms, respectively.

Modifications of the system for ion injection required only minor changes in this configuration. Since a higher voltage was needed, the 200mA-500V power supply was removed. A 20mA-30kV high voltage supply was installed in its place with the polarity reversed to attract ions rather than electrons. Ballast resistors R_1 and R_0 were changed to 1000 ohms each. The voltmeter on the inner grid circuit was removed since the voltage drop across R_1 was negligible. Both current and voltage to the inner grid were therefore monitored on meters built into the power supply. The remainder

Figure 3. Schematic Diagram - Electron Injection

of the circuit in Fig. 3 was unchanged.

All power supplies, except the high voltage, were electronically regulated.

Microwave System

A block diagram of the microwave system is given in Fig. 4. One can best understand the system by following a hypothetical microwave signal as it travels through the various components. From its origin in the UHF signal generator, this signal passes through an isolator to eliminate any possible feedback effects on the generator. The microwave signal then proceeds to a directional coupler where a small part is diverted for frequency measurement. The frequency meter, crystal detector, and standing wave indicator are connected in series to the -20 db leg of the directional coupler and therefore receive only one percent of the microwave signal. The remaining signal continues into the bell jar via a microwave feedthrough constructed for that purpose. Once inside the vacuum system, the signal passes through a d.c. isolation coupler. This coupler was constructed to permit the application of high voltage to the inner sphere. The microwave signal then continues through a second directional coupler and enters the cavity through its mounting tube. If the frequency of the input signal does not coincide with the resonant frequency of the cavity, the signal will be reflected. However, at the resonant frequency, power is absorbed in the cavity and a relatively small signal is reflected back into the directional coupler. This directional coupler is oriented such that a part of the reflected microwave signal is sampled at the -20 db leg. This portion of the reflected signal

Figure 4. Microwave System Block Diagram

passes through a second d.c. isolation coupler and microwave feedthrough before entering a crystal detector. The output from this detector is finally displayed on an oscilloscope screen.

Within the UHF signal generator, the microwave signal was modulated at 1000 Hz as required by the standing wave indicator. All microwave connections were made with RG-8/U coaxial cable and series N coaxial connectors. The interested reader should consult Appendix A for details concerning the construction of the microwave feedthroughs, the d.c. isolation couplers, and the cavity mounting tube. The microwave characteristics of these components are reported in Appendix B.

Experimental Procedure

After pumpdown of the vacuum system was completed the micrometer valve was adjusted to obtain the desired helium pressure within the system. Voltages and currents on the filaments and grids were then adjusted. With the device in stable operation, the pressure, currents and voltages were recorded. The frequency of the microwave signal was then swept through the range of the expected resonance. When the signal frequency passed through the cavity resonant frequency, there was a sharp dip in the oscilloscope display. The signal frequency was then adjusted to coincide with the cavity resonance. The frequency was measured by adjusting the frequency meter to produce a null on the standing wave indicator. Next, the inner or outer grid power supply was turned off to interrupt the discharge. The microwave signal frequency was immediately readjusted to coincide with the new cavity resonance, measured, and recorded. The difference in the two frequencies was the shift due to the plasma within the cavity.

Duplicate measurements were made under a given set of operating conditions whenever possible.

During normal operation of the device, there was considerable heating of the cavity. The resulting thermal expansion and changes in the electrical properties of the grid wires would cause the effective cavity radius to increase. Since the resonant frequency is inversely proportional to the radius, there would be a drop in the resonant frequency, usually on the order of 15 MHz. When the discharge stopped the temperature would begin to drop and the resonant frequency would climb. However, due to the low pressure and the poor thermal conductivity of stainless steel, the cooling of the cavity was slow. After the power supply was turned off, nearly 5 sec would elapse before the frequency drift could be detected on the oscilloscope display. With some practice, the signal frequency could be readjusted within 2 sec and therefore any error caused by the cavity cooling was negligible. An error of ± 1.0 MHz due to the inability to adjust the null point precisely could be expected on any frequency measurement. The duplicate measurement procedure was used to insure against gross errors in the frequency measurements.

III. THEORETICAL BACKGROUND OF THE MICROWAVE CAVITY TECHNIQUE

The relationship between the microwave cavity resonant frequency shift and the electron density of the plasma is of central importance and will be derived in this chapter. Beginning with the equations for the electric field within the cavity, which can be found in any microwave engineering text, an equation relating frequency shift to average electron density is developed. Electron density distributions are then assumed and the final results are graphs relating the frequency shift to the central electron density.

The lowest order TM mode solution of Maxwell's equations in spherical coordinates yields the following equations for the electric field within the free space cavity (Ramo and Whinnery, 1960)

$$E_r(r, \theta) = \frac{-2j\eta C \cos \theta}{k^{5/2} r^2} \left[\frac{\sin kr}{kr} - \cos kr \right] \quad (3.1)$$

$$E_\theta(r, \theta) = \frac{j\eta C \sin \theta}{k^{5/2} r^2} \left[\frac{(kr)^2 - 1}{kr} \sin kr + \cos kr \right] \quad (3.2)$$

where $k = \omega(\mu_0 \epsilon_0)^{1/2}$ is the wave number, η is the intrinsic impedance of free space and C is an arbitrary constant. The resonance condition $E_\theta(r=A, \theta) = 0$ requires that $kA = 2.74$ for a perfectly conducting shell of radius A . This corresponds to a free space resonant frequency $f_0 = 13.1/A$, where f_0 is expressed in GHz and A is given in cm. Since the actual cavity walls are not perfect conductors but wire grids, A must be redefined to be the effective cavity radius. Using the experimentally determined resonant frequency $f_0 = 3.134$ GHz, one finds that

$A = 4.18$ cm. Note that this is almost 8.0% larger than the measured radius of 3.85 cm. The effective cavity radius will be used exclusively through the remainder of this report.

The presence of a plasma in the cavity changes the dielectric constant κ of the enclosed volume and consequently the resonant frequency of the cavity. The new dielectric constant can be expressed (Holt and Haskell, 1968) in terms of the plasma frequency ω_p as

$$\kappa = 1 - \frac{\omega_p^2}{\omega^2} \quad (3.3)$$

where $\omega_p = (N e^2 / m \epsilon_0)^{1/2}$, e and m are the electronic charge and mass, respectively, and N is the electron density. This equation is valid only when the effective collision frequency for momentum transfer between electrons and neutrals ν_{en} is much smaller than the plasma frequency. An estimate of ν_{en} and a comparison with ω_p are given in Appendix C.

The solution of Maxwell's equation for the new resonant frequency is severely complicated by the spacial dependence of the electron density. Using perturbation theory, Adler (1949) developed the following equation for the ratio of the resonant frequencies

$$\frac{f_r}{f_0} = \left[\frac{\int_V E^2 dV}{\int_V \kappa E^2 dV} \right]^{1/2} \quad (3.4)$$

where f_r is the resonant frequency with plasma in the cavity and V is the cavity volume. Squaring both sides and substituting from Eq. 3.3, one obtains

$$f_r = \left[\frac{e^2}{4\pi^2 m \epsilon_0} \frac{\int_V N E^2 dV}{\int_V E^2 dV} + f_o^2 \right]^{1/2}$$

Numerically, the resonant frequency can be expressed as

$$f_r = \left[.80595 \times 10^{-10} \frac{\int_V N E^2 dV}{\int_V E^2 dV} + f_o^2 \right]^{1/2} \quad (3.5)$$

where N is given in electrons/cm³ and the frequencies are in GHz. The shift in frequency Δf is simply $f_r - f_o$.

The integrals in Eq. 3.5 can be simplified as follows. Equations 3.1 and 3.2 can be squared and added with the result:

$$E^2(r, \theta) = -\eta^2 C^2 \left[4 \cos^2 \theta B^2(r) + \sin^2 \theta D^2(r) \right]$$

where

$$B(r) = \frac{1}{k^{5/2} r^2} \left[\frac{\sin kr}{kr} - \cos kr \right]$$

and

$$D(r) = \frac{1}{k^{5/2} r^2} \left[\frac{(kr)^2 - 1}{kr} \sin kr + \cos kr \right]$$

The integral in the denominator of Eq. 3.5 then becomes:

$$\int_V E^2 dV = \int_0^A \int_0^\pi \int_0^{2\pi} -\eta^2 C^2 \left[4 \cos^2 \theta B^2(r) + \sin^2 \theta D^2(r) \right] r^2 \sin \theta d\phi d\theta dr$$

After performing the angle integrations, one has the equation

$$\int_V E^2 dV = \frac{8\pi}{3} n^2 c^2 \int_0^A \epsilon^2(r) r^2 dr$$

where $\epsilon^2(r) = 2 B^2(r) + D^2(r)$. In a spherically symmetric device such as this, the electron density will have no angular dependence. Therefore, the integral in the numerator of Eq. 3.5 can be similarly reduced with the final result:

$$f_r = \left[.80595 \times 10^{-10} \frac{\int_0^A N(r) \epsilon^2(r) r^2 dr}{\int_0^A \epsilon^2(r) r^2 dr} + f_o^2 \right]^{1/2} \quad (3.6)$$

From this, one can easily see that the frequency shift is proportional to a weighted average electron density. The weighting function $\epsilon^2(r)$ is plotted in Fig. 5. Note that the sensitivity increases with decreasing radius. If $N(r)$ is known, the integrals in Eq. 3.6 can be evaluated numerically.

Spectroscopic measurements during ion injection operation of a similar device by Robinson (1973) suggested an electron density distribution of the form

$$N(r) = \frac{a^2 N(0)}{r^2 + a^2} \quad (3.7)$$

where the constant a is the half width at half maximum and $N(0)$ is the density at the center. This distribution is illustrated in Fig. 6 with a representative value of a given as 1.2 cm (Robinson, 1973). Results from a theoretical model developed by Imel (1973) also indicate a radial dependence of this form.

Figure 5. Weighting Function

Figure 6. Normalized Density Distribution
Function $N(r) = N(0) \frac{a^2}{(r^2 + a^2)}$

With this electron density distribution, Eq. 3.7 has been evaluated numerically for various values of $N(0)$ and a . This was accomplished using a numerical quadrature integration subroutine (James, Smith and Wolford, 1967). Figure 7 shows the results for $a = 1.2$ cm with $N(0)$ plotted versus frequency shift. Also plotted are curves for $a = .5$ cm, a more sharply peaked distribution, and $a \rightarrow \infty$, a flat distribution. These represent extreme cases of nearly pure radial focusing and no radial focusing, respectively. Note that as a varies between these extremes, the density varies by little more than one order of magnitude.

This distribution function, although measured during an ion injection experiment, will be used arbitrarily for the case of electron injection when no potential well is present. However, if a potential well does develop, electrons will be reflected resulting in a drop in the density at the center. Clearly, a different distribution will be needed to depict this case. Imel's (1973) model suggests a distribution similar to that of Fig. 6 except that there is a depression at the center. The following "well distribution" was arbitrarily chosen to simulate this condition

$$N(r) = N_0 \left[\frac{a^2}{r_o^2 + a^2} \right] \left[\frac{r^2 + b^2}{r_o^2 + b^2} \right] h(r_o - r) + N_0 \left[\frac{a^2}{r^2 + a^2} \right] h(r - r_o) \quad (3.8)$$

where r_o is the radius at which the well begins, b is an arbitrary constant, and $h(x)$ is the step function. This distribution is shown

Figure 7. Density versus Frequency Shift
for $N(r) = N(0) \frac{a^2}{(r^2 + a^2)}$

in Fig. 8 with values of r_0 at .50 cm and 2.4 cm to illustrate shallow (narrow) and deep (wide) wells respectively. The constants a and b are held at 1.2 cm and 1.0 cm in that order. The corresponding density versus frequency shift curves are given in Fig. 9. These show both the density in the center $N(0)$ and the maximum density at the edge of the well $N_{\max}$ where

$$N(0) = N_0 a^2 b^2 / (r_0^2 + b^2) \quad (3.9)$$

and

$$N_{\max} = N_0 a^2 / (r_0^2 + a^2)$$

Comparison with the $a = 1.2$ cm curve on Fig. 7 shows that, at a given frequency shift, the densities for the shallow well distribution are slightly less than when no well was present. When a deep potential well is formed, the densities will be an order of magnitude or more lower than for the case with no well.

The electron density distribution functions detailed above are to some extent arbitrary since no experimental data of this nature is available for this particular device. It is felt, however, that they will give a reasonably good estimate of the electron density. Curves similar to those of Figs. 7 and 9 can be produced given any electron density distribution function.

Figure 8. Normalized Density Distribution

$$\text{Function } N(r) = N_o \frac{a^2}{a^2 + r_o^2} \frac{b^2 + r^2}{b^2 + r_o^2} h(r_o - r) \\ + N_o \frac{a^2}{a^2 + r^2} h(r - r_o)$$

Figure 9. Density versus Frequency Shift for

$$\begin{aligned}
 N(r) = N_0 & \frac{a^2}{a^2 + r_0^2} \frac{b^2 + r^2}{b^2 + r_0^2} h(r_0 - r) \\
 & + N_0 \frac{a^2}{a^2 + r^2} h(r - r_0)
 \end{aligned}$$

IV. RESULTS AND ANALYSES

The experimental results and analyses for electron and ion injection will be presented in separate sections. Each will begin with a sample of the data taken, followed by plots of the complete results. These results are then explained in terms of the operating characteristics of the device.

Electron Injection

Data taken during the electron injection phase of the experiment was in the form of Table 1. This particular data was taken for the case of the inner grid voltage V_a at 300.V and the pressure P at 1.0 mTorr of helium. To obtain the data, the inner grid power supply voltage was set at a convenient level and the filament ring emission current then adjusted to obtain the desired 300.V on the inner grid. It was observed that when high current was being drawn, the outer grid was not needed to sustain the discharge. Although the outer grid remained connected to its power supply, this power supply was turned off, while the first four data points were obtained. At the 450.V level on the inner grid power supply, the discharge became somewhat unstable making it necessary to apply 10.V at the outer grid. Note that during operation, the actual voltage was higher than this due to coupling of the inner grid voltage through the discharge. The current in the outer grid circuit, while not actually measured because of the reversed polarity, was calculated and the result given in Table 1. The last four measurements, which were a repeat of the first four except that the outer grid power supply was turned on, serve as an

Table 1

Electron Injection Data

Pressure = 1.0 mTorr of He

 $V_a = 300.V$

Filament Ring Emission Current (mA)	Outer Grid Power Supply Voltage (V)	Outer Grid Current (mA)	Outer Grid Voltage (V)	Inner Grid Power Supply Voltage (V)	Inner Grid Current (mA)	Resonant Frequency with Plasma (GHz)	Resonant Frequency without Plasma (GHz)	Resonant Frequency Shift (MHz)
70	off	off	off	515	110	3.1655	3.1420	23.5
70	off	off	off	515	110	665	405	26.0
57	off	off	off	500	92	685	420	26.5
57	off	off	off	500	92	690	410	28.0
44	10	-6.4	22	450	63	550	390	16.0
44	10	-6.4	22	450	65	565	390	17.5
32	10	-3.2	16	400	44	560	410	15.0
32	10	-3.2	16	400	43	560	410	15.0
16	10	0.0	10	350	20	480	420	6.0
15	10	0.0	10	350	18	480	440	4.0
59	10	-77.5	155	500	93	690	420	27.0
58	10	-72.1	145	500	92	690	420	27.0
71	10	-97.8	193	515	110	650	410	24.0
70	10	-101.5	200	515	110	640	410	23.0

assurance that this had no appreciable effect on the frequency shift. It is therefore possible to compare this data to other cases in which the outer grid power supply remained off for the entire run. While Table 1 gives the data for the 300.V-1.0 mTorr case only, it is representative at all the electron injection data. Note that the grid currents are the collected particle currents, not the circulating currents mentioned in Chapter I.

The data points for the electron injection experiments are shown in Figs. 10 through 16. These plots show the frequency shift versus inner grid current I_a for a given voltage with pressure P as a parameter. A density scale taken from Fig. 7 with $a = 1.2$ cm has been included for convenience. The perveance K , a parameter used extensively in theoretical modeling is defined as

$$3.04 \times 10^4 I_a m_r^{1/2} \propto V_a^{-3/2}$$

where m_r is the ratio of the injected particle mass to the electron mass and the circulation factor α is the average number of trips an electron makes to the center. Since the mean free path for electrons at these pressures is much larger than the dimensions of the system, α is defined geometrically in terms of the grid transparency as $v/(1-v^2)$ (Hirsch, 1968) and given numerically as 3.58. A perveance scale has been added to these figures for easy reference. While the lines shown in the figures are the result of freehand curve fitting, they are more than adequate to show the trends evident from the experimental data points.

From Figs. 10 and 11, some very interesting characteristics are immediately apparent. As the inner grid current is increased in the

Figure 10. Electron Injection $V_a = 150.V$

Figure 11. Electron Injection $V_a = 200\text{V}$

Figure 12. Electron Injection $V_a = 250 \text{ V}$

Figure 13. Electron Injection $V_a = 300\text{V}$

Figure 14. Electron Injection $V_a = 350\text{V}$

Figure 15. Electron Injection $V_a = 400\text{V}$

Figure 16. Electron Injection $V_a = 450.V$ $P = 5.0 \text{ mTorr}$
 $V_a = 100.V$ $P = 1.0 \text{ mTorr}$

low range the density increases, almost linearly in some cases. As the current is increased further, the density peaks and then decreases at even higher currents. The effect of increasing the background gas pressure is to increase the initial slope and cause the peak density to occur at a lower current. The peak densities appear to be slightly greater at the higher background gas pressures than at the lower. While these trends continue in Figs. 12 and 13 for the 5.0 and 1.0 mTorr pressure curves, those for the 0.5 mTorr case appear to degenerate. Although the curves in Figs. 14 and 15 are somewhat truncated because it was not possible to draw high currents at these voltages, the general trends of the preceding plots are still readily apparent. Although no intercomparison of the curves in Fig. 16 is possible, they have been included for completeness.

A careful review of these figures will show that when the background gas pressure was 5.0 mTorr, the peak densities all occurred at approximately 65.mA, independent of the accelerating voltages. Similarly, the peak densities for the various voltages in the 1.0 mTorr pressure case fell in the 85.mA to 95.mA current range. An adequate explanation for this effect has not been forwarded as yet. However, over a wider range of voltages and pressures, one would expect the voltage effects to be more evident. To more easily compare the results at constant pressure, the lines in Figs. 10 through 16 have been reproduced without the data points in Figs. 17 through 19. These figures show density versus perveance with accelerating voltage as a parameter.

Figures 17 and 18 clearly show that the peak density increases with the accelerating voltage. Note that the spacing of the peaks

Figure 17. Electron Injection $P = 5.0\text{ mTorr}$

Figure 18. Electron Injection $P = 1.0$ mTorr

Figure 19. Electron Injection $P = 0.5 \text{ mTorr}$

on this permeance scale is due primarily to the $V^{-3/2}$ dependence since the currents at the peaks are nearly equal. The 150.V curve on Fig. 17 suggests that the density may again increase at high current. The degeneration of the 0.5 mTorr case is especially apparent in Fig. 19. Although the peaking characteristic is still discernible, the peak density appears to decrease as the voltage increases.

Recalling that the frequency shift is proportional to the weighted average density, one can see from Figs. 10 through 19 that, as the injected electron currents are increased above some threshold, the weighted average electron densities inside the cavity decrease. The obvious, albeit speculative, conclusion is that a potential well is formed and reflects some of the injected particles. A comparison of the $a = 1.2$ cm curve on Fig. 7 to the $N(0)$ curves on Fig. 9 showed that the density corresponding to a measured frequency shift was lower when a well distribution rather than a peaked distribution was assumed. If a potential well does indeed form, the densities at the center of the sphere will be lower than those shown on these figures which were calculated with the aid of Fig. 7. The true central densities would be expected to follow the curves shown until the maxima are reached. At this point the potential well forms and the densities beyond the maximum therefore decrease more rapidly than the figures indicate. More precise information concerning the shape of the potential wells would be needed to determine how much the densities would drop.

The peaking characteristic observed in this work has also occurred in the single species theoretical model by Imel (1973). In his model, the peaking has been coincident with potential well

formation.

It is interesting to note that the assumed potential wells of this experiment were formed at the relatively high perveances of unity or greater. The deep potential wells produced at pressures near 10^{-7} Torr and accelerating voltages at 600.V and 900.V by Hirsch (1968) occurred at perveances of approximately 0.5. The perveance was an order of magnitude lower than this for the electron injection experiment in which Black (1971) found no potential wells. The implication is that for a given pressure and accelerating voltage, there is some minimum perveance requirement $K_{\min}(P, V_a)$ that must be satisfied before a potential well can form. Additional research will be needed to determine the relationship between P , V_a , and $K_{\min}$, if indeed one exists. Extrapolation from Fig. 18 to the conditions of Hirsch's work is extremely difficult because it involves both voltage and pressure variation far beyond the range of this experiment. However, the data indicate that the effects of decreased pressure and increased voltage will tend to cancel one another, resulting in a predicted value between 0.2 and 1.0 for the perveance requirement.

Ion Injection

Table 2 is a sample of the ion injection data. This data was taken with the pressure at 0.5 mTorr of helium and $V_a = 3.0$ kV. The outer grid power supply voltage was set at its maximum value of 440.V and remained there throughout this part of the experiment. The filament emission current was then adjusted to maximize the inner grid current and two shift measurements were made. The filament emission current was then readjusted both above and below this optimum

Table 2

Ion Injection Data

Pressure = 0.5 mTorr of He

 $V_a = 3.0$ kV

Outer Grid Power Supply Voltage = 440.V

Filament Ring Emission Current (mA)	Outer Grid Current (mA)	Outer Grid Voltage (V)	Inner Grid Current (mA)	Resonant Frequency with Plasma (GHz)	Resonant Frequency without Plasma (GHz)	Resonant Frequency Shift (MHz)
250	305	115	13.6	3.1485	3.1400	8.5
250	304	135	13.5	480	400	8.0
300	333	101	8.8	460	405	5.5
300	333	100	8.8	470	410	6.0
236	284	150	12.0	490	405	8.5
235	280	153	11.5	480	410	7.0
230	272	165	10.0	460	410	5.0
230	272	164	10.2	470	410	6.0
220	270	163	10.0	470	415	5.5
218	252	182	8.0	480	435	4.5
220	255	179	8.3	480	425	5.5
220	250	185	7.5	475	435	4.0
210	240	197	6.2	475	445	3.0
208	238	198	6.2	480	450	3.0

and pairs of data points taken at arbitrary intervals.

The data from the ion injection experiment is plotted in Figs. 20 through 25 as shift versus current at a given inner grid voltage V_a with the pressure as a parameter. A density scale based on Fig. 7 with $a = 1.2$ cm along with a perveance scale as defined earlier except that a dimensionless correction factor β has been added. This was necessary because the geometric circulation factor α is not valid for ions where charge exchange collisions are important. Calculating from the charge exchange cross section for ions in helium 10^{-15} cm² (Black, 1971), the mean free path for ions at a pressure of 0.5 mTorr is nearly 62.cm. Compare this to the average distance that an ion would travel, neglecting particle interactions, before it collided with the grid. This distance, merely the geometric circulation factor α times the round trip distance to the center, is 54.cm. Thus, the circulation factor for ions at 0.5 mTorr will be slightly more than half of its geometric counterpart. A value of .54 for β was calculated for this case. Mean free paths of approximately 31.cm for the 1.0 mTorr pressure case and .6 cm for the 5.0 mTorr case result in β 's of .37 and .10, respectively. The lines shown in the figures are the result of a least squares linear fit technique. These lines have been reproduced without the data points in Figs. 26 through 28. Frequency shift and density versus current at constant pressure are shown on these graphs with voltage V_a as a parameter.

This data exhibits characteristics which one would expect intuitively. In all cases, an increase in the injected ion current results in a higher electron density due to increased ionization within the central region. At a fixed injection current, an increase in the

Figure 20. Ion Injection $V_a = 5.0$ kV

Figure 21. Ion Injection $V_a = 4.0 \text{ kV}$

Figure 22. Ion Injection $V_a = 3.0 \text{ kv}$

Figure 23. Ion Injection $V_a = 2.0 \text{ kV}$

Figure 24. Ion Injection $V_a = 1.0 \text{ kV}$

Figure 25. Ion Injection $V_a = 0.5 \text{ kV}$

Figure 26. Ion Injection $P = 5.0$ mTorr

Figure 27. Ion Injection $P = 1.0$ mTorr

Figure 28. Ion Injection $p = 0.5 \text{ mTorr}$

background gas pressure results in an increased density, again due to greater ionization. The only exception, the 5.0 mTorr line on Fig. 23, appears from Fig. 26 to be an experimental quirk of unknown cause. From Figs. 26 through 28, one can see that a decrease in the voltage at a fixed current, and therefore an increase in the perveance, will cause an increase in the density as expected. It is not known from this data whether the electrons are trapped in a potential well or are merely the products of continuous ionization within the grid. In either case, the electron density would increase with current. Additional data would be needed before one can draw any conclusions about potential well formation. Extrapolation from Fig. 25 to the conditions explored in the theoretical model by Black (1971) results in a predicted central electron density of approximately $1.0 \times 10^9/\text{cm}^3$. This is almost exactly Black's result. The extremely high voltages used by Hirsch (1967) make meaningful extrapolation impossible.

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V. SUMMARY

The purpose of this work was to develop a technique for directly measuring the electron density of a plasma within an electrostatic confinement device. To do this, the microwave cavity resonance shift method, originally demonstrated by Adler (1949), was adapted to the spherical geometry of the device. The frequency shift measured experimentally was proportional to a weighted average electron density. It was converted to central electron density using a selected density distribution function.

Electron injection experiments were conducted with collected currents of up to 200 mA, accelerating voltages between 100 V and 450 V, and pressures of 5.0, 1.0, and 0.5 mTorr of helium. The results strongly suggest that a potential well was formed. Electron densities just before well formation would reach $9.5 \times 10^9/\text{cm}^3$ and then drop sharply with increasing collected current to less than $7.5 \times 10^9/\text{cm}^3$ when the well was present. This general behavior was observed by Imel (1973) in his theoretical model. Comparison with the experimental results of Hirsch (1968) and Black (1971) implied the presence of a minimum perveance $K_{\min}(P, V_a)$ below which potential well formation is impossible. Further study will be needed to determine the exact functional dependence.

Working at the same background helium pressures as above, ion injection experiments were conducted with accelerating voltages between 0.5 kV and 5.0 kV and collected currents of up to 20 mA. Conclusions concerning potential well formation could not be drawn from this data alone. Electron densities measured in this case

showed steady increase with collected current, terminating at a maximum near $5.5 \times 10^9/\text{cm}^3$. Extrapolation from these results showed almost perfect agreement with the theoretical model by Black (1971).

The only conspicuous deficiency of this work is the need to arbitrarily assume an electron density distribution function. Although the resultant central electron measurement is relatively insensitive to small changes in the assumed distribution, the effect of assuming a radically different distribution cannot be overlooked. This inadequacy could be eliminated in future work by including a modification of Black's (1971) electron beam probe technique. From the potential profiles obtained from these measurements, the electron density distribution functions for the various experimental conditions could be accurately calculated. An alternative would be the inclusion of spectroscopic studies for electron injection such as those conducted by Robinson (1973). It would also be interesting to explore the possibility of exciting more than one mode of resonance in the cavity. The frequency shift measured in each mode would again be a weighted average electron density, however, the weighting function for each mode would be different. Comparison of these results would serve as a check on the validity of the density distribution function.

APPENDIX A

Microwave Component Construction

Of the microwave components mentioned in Chapter II, all were standard commercially available items except for the following:

1. Microwave Feedthrough
2. D.C. Isolation Coupler
3. Cavity Mounting Tube

The construction details of each will now be presented.

Microwave Feedthrough

A sectional view of the feedthrough is given in Fig. 29. A blank feedthrough plug was first drilled and tapped to accept series N connectors. The vacuum seal at the bottom was insured by a ring of epoxy. An aluminum sleeve and center conductor provided the necessary coupling while maintaining an approximate impedance match within the feedthrough.

D.C. Isolation Coupler

The d.c. isolation coupler consisted of two identical aluminum cylinders with their open ends fitted into a plexiglass disc as in Fig. 30. Holes were drilled and tapped to accept series N connectors, as shown. The aluminum stubs provided the microwave coupling into and out of the device which was excited in the TE_{11} mode. Having no axial electric field components, this mode was able to propagate through the dielectric disc and excite an outgoing signal in the other coaxial line.

Figure 29. Microwave Feedthrough

Figure 30. D.C. Isolation Coupler

The dielectric disc allowed one-half of the coupler to be at ground potential while the other was at several kilovolts.

Cavity Mounting Tube

A .500 inch diameter stainless steel tube with a .375 inch inside diameter was used to mount the cavity. A series N connector was silver soldered to the bottom of the tube, as shown in Fig.31, for connection to the directional coupler. A section of RG-8/U coaxial cable with the outer insulator removed was inserted into the tube and the braid silver soldered to the top of the tube. Stainless steel wires were then spot-welded to the top of the tube and to the cavity. One strand of the center conductor of the coaxial cable was extended into the cavity and attached forming a loop which provided the microwave coupling for the TM_{101} mode of oscillation. To best accommodate this mode, the cavity was mounted so that its longitudinal grid wires were aligned with the high frequency currents flowing in the cavity wall.

Figure 31. Cavity Mounting Tube

APPENDIX B

Characteristics of Microwave Components

The effects on the signal of the non-standard microwave components were determined experimentally using basically the two equipment configurations shown in Fig. 32. Case I with the cavity removed was used to determine the output characteristics of the UHF signal generator in the frequency range of the experiment. The reflection characteristics of the cavity were measured using this configuration with the cavity in place. The configuration of Case II without the cavity was used to determine the characteristics of the generator and the intervening components. Finally, the entire system characteristics were measured after connecting the cavity to this configuration. These four characteristic curves are plotted as frequency versus attenuation in Fig. 33. While the dramatic effect on the signal amplitude is clearly evident, note that the resonant frequency of the cavity was not affected.

Figure 32. Test Configurations
 Case I. Solid Line
 Case II. Dashed Line

Figure 33. Microwave Characteristics of Components

APPENDIX C

Collision Frequency Estimate

The effective collision frequency for momentum transfer between electrons and neutrals ν_{en} is defined by the equation $\nu_{en} = \sigma_{en} v n$ where σ_{en} is the effective cross section, v is the electron velocity and n is the neutral particle density. Brown (1967) gives σ_{en} as a function of electron energy in the form of Fig. 34. At a background gas pressure of 5 mTorr, the neutral particle density will be $1.63 \times 10^{14}/\text{cm}^3$. By using a simple relationship between electron velocity and energy, one can write

$$\nu_{en} = 8.48 \times 10^5 \sigma_{en} E^{1/2}$$

where the energy E is in electron volts. Using Fig. 34 one finds that the collision frequency maximum of $1.19 \times 10^7/\text{sec}$ occurs when $E \approx 11.0$ eV and $\sigma_{en} \approx 4.20 \times 10^{-16} \text{ cm}^2$. On the other hand, an electron density of $10^9/\text{cm}^3$ corresponds to a plasma frequency of $1.7 \times 10^9/\text{sec}$. Thus, ν_{en} is indeed much less than ω_p .

Figure 34. Effective Cross Section

BIBLIOGRAPHY

1. Adler, F. P. "Measurement of the Complex Conductivity of an Ionized Gas at Microwave Frequencies." Journal of Applied Physics, Vol. 20, p. 1125 (November 1949).
2. Black, W. M. Potential Well Formation in an Electrostatic Confinement Device. A Thesis in Nuclear Engineering, The Pennsylvania State University (1971).
3. Brown, S. C. Basic Data of Plasma Physics, 1966. M.I.T. Press, Cambridge, Massachusetts (1967).
4. Elmore, W. C., J. L. Tuck and K. M. Watson. "On the Inertial-Electrostatic Confinement of a Plasma." The Physics of Fluids, Vol. 2, p. 239 (May-June 1959).
5. Hirsch, R. L. "Inertial-Electrostatic Confinement of Ionized Fusion Gases." Journal of Applied Physics, Vol. 38, p. 4522 (October 1967).
6. Hirsch, R. L. "Experimental Studies of a Deep, Negative, Electrostatic Potential Well in Spherical Geometry." The Physics of Fluids, Vol. 11, p. 2486 (November 1968).
7. Holt, E. H. and R. E. Haskell. Foundations of Plasma Dynamics. Macmillan Company, New York (1968).
8. Imel, G. R. A Theoretical Model of a Spherical Electrostatic Confinement Device. A Thesis in Nuclear Engineering, The Pennsylvania State University (1973).
9. James, M. L., G. M. Smith and J. C. Wolford. Applied Numerical Methods for Digital Computation with Fortran. International Textbook Company, Scranton, Pennsylvania (1967).
10. Ramo, S. and J. R. Whinnery. Fields and Waves in Modern Radio, Second Edition. John Wiley & Sons, New York (1960).
11. Robinson, J. W. Private Communication (1973).