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THE THREE DIMENSIONAL HEATED SURFACE JET IN A CROSS FLOW

H. H. Carter
R. Regier

MASTER

Reference 74-8 November 1974

Technical Report 88

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CHESAPEAKE BAY INSTITUTE

THE JOHNS HOPKINS UNIVERSITY

TECHNICAL REPORT 88

THE THREE DIMENSIONAL HEATED SURFACE JET
IN A CROSS FLOW

by

H.H. Carter

R. Regier

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November 1974

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Director

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List of Symbols

A	area of orifice, i.e., $b_o h_o$
A_ξ	area, i.e., $b_\theta(\xi)h_\theta(\xi) = b(\xi)h(\xi)$
A_r	aspect ratio, i.e., h_o/b_o
$b(\xi)$	width of jet
D_w	depth of water
F_r	initial densimetric Froude number $\equiv q_o / \sqrt{\frac{g\Delta\rho_o h_o}{\rho}}$
f	constant of proportionality for θ_p/θ_o and $\xi/\sqrt{A}$ in the stable region (eq. (25))
$f_2(\eta')$	lateral shape function
$g_2(z')$	vertical shape function
$h(\xi)$	depth of jet
k	trajectory constant of proportionality
L	perimeter of jet in ηz plane
m	trajectory slope
n	exponent in relation between A_ξ and ξ
P	non-dimensional distance $\equiv \frac{D_w}{r_o} \left(\frac{\xi_o}{\xi} \right)^n$
Q	volume flow rate of water in jet
q	jet velocity at centerline in ξ -direction
R	ratio of initial jet velocity, q_o , to ambient velocity, u_a
R_e	initial Reynolds number $\equiv q_o h_o / \nu$
r_ξ	radius of area equivalent to A_ξ
u	x-component of jet velocity at centerline
u_a	ambient velocity in x-direction
W	width of system

List of Symbols (continued)

$z_{1/2}$	half depth of plume (see Fig. 10)
x, y, z	fixed left-handed cartesian coordinate system with x parallel to shore, y perpendicular to shore and positive in offshore direction; z directed vertically downward
ξ, η, z	moving natural coordinate system with positive ξ directed along centerline of jet, η perpendicular to ξ and positive to left of ξ ; z directed vertically downward. Origin moves with the peak concentration and is located on the centerline trajectory
α	angle between jet centerline and x -axis; also a central angle in Figure 36
β	ratio of predicted to measured values of θ_p / θ_o in stable region when $z_{1/2} > D_w$
$2\eta_{1/2}$	half width of jet (see Fig. 10)
θ	excess temperature
ν	kinematic viscosity of water
ρ	fluid density

Subscripts

o	orifice
a	ambient
p	peak value
$1/2$	where value is one-half of peak value

Summary of Results and Recommendations

Various characteristics of 3-dimensional heated surface jets in a cross flow were measured in a large, low-velocity flume. The characteristics studied were *vertical growth*, *lateral spread*, *centerline trajectory*, and *dilution*. The results may be summarized as follows:

Vertical Growth

$$\text{Initial zone: } \frac{z_{1/2}}{\sqrt{A}} \cdot \frac{1}{A_r^{1/4}} = 0.87 \left(\frac{\xi}{\sqrt{A}} \right)^{1/2} \quad \text{Eq. (4)}$$

$$\text{Zone of maximum penetration: } \frac{z_{1/2, \max}}{\sqrt{A}} \cdot \frac{1}{A_r^{1/4}} = 0.342 F_r \quad \text{Eq. (3)}$$

$$\text{Final zone: } \frac{z_{1/2}}{\sqrt{A}} \cdot \frac{1}{A_r^{1/4}} = 0.80 F_r^{3/2} \left(\frac{\xi}{\sqrt{A}} \right)^{-1/2} \quad \text{Eq. (5)}$$

The intercept of the initial zone of vertical growth and the zone of maximum penetration is given by

$$(\xi/\sqrt{A})_{IM} = 0.155 F_r^2,$$

while the intercept of the zone of maximum penetration with the final zone is given by

$$(\xi/\sqrt{A})_{MF} = 5.472 F_r$$

Lateral Spread

$$\text{Small } \xi: \quad \frac{2\eta_{1/2} F_r}{R^2 b_o} = 0.58 \frac{\xi F_r}{R^2 b_o} \quad \text{Eq. (11)}$$

$$\text{Large } \xi: \quad \frac{2\eta_{1/2} F_r}{R^2 b_o} = 2.85 \sqrt{\frac{\xi F_r}{R^2 b_o}} \quad \text{Eq. (12)}$$

The intercept between the lateral spread zone for "small" ξ and that for "large" ξ is given by

$$(\xi/\sqrt{A})_{SM} = 24.145 (R^2/F_r \sqrt{A_r})$$

Centerline Trajectory

$$\frac{y}{\sqrt{A}} = R \left(\frac{x}{h_o} \right)^m \quad \text{Eq. (16c)}$$

where $m = 0.609 - 0.013 F_r$. Eq. (17)

Dilution

Length of core region: $\frac{\xi_o}{\sqrt{A}} = \frac{10.334}{\sqrt{F_r}}$ Eq. (19)

Entrainment region: $\frac{\theta_p}{\theta_o} = \frac{4.744}{F_r^{1/3}} \left(\frac{\xi}{\sqrt{A}} \right)^{-2/3}$ Eq. (18)

Stable region ($\frac{z_{1/2, \max}}{D_w} \leq 1$): $\frac{\theta_p}{\theta_o} = \frac{1.721}{R^{1/6} F_r^{1/3}} \left(\frac{\xi}{\sqrt{A}} \right)^{-1/3}$ Eq. (25)

Stable region ($\frac{z_{1/2, \max}}{D_w} > 1$): $\frac{\theta_o}{\theta_p} \approx \frac{Q(\xi)}{Q_o}$

$$= \left\{ \frac{1}{P} + \frac{2}{\pi} \left(\ln \frac{1+\sqrt{1-P^2}}{P} - \frac{\cos^{-1}P}{P} \right) \right\} \quad \text{Eq. (37)}$$

where $P \equiv \frac{D_w}{r_o} \left(\frac{\xi_o}{\xi} \right)^{2/3}$, and Eq. (36)

$$r_o = \sqrt{\frac{2A}{\pi}} \quad \text{Eq. (31)}$$

The intercept between the entrainment region and the stable region for the case ($z_{1/2, \max} / D_w \leq 1$) is given by

$$(\xi/\sqrt{A})_{ES} = 20.946 R^{1/2}$$

It was also demonstrated that Eq. (37), the Edinger intersection or geometric model, could be used with results comparable to Eq. (25) for predicting θ_o/θ_p when $z_{1/2,max} \leq D_w$ if $z_{1/2,max}$ were substituted for D_w and $\sqrt{\frac{2A}{\pi}}$ for r_o .

In the series of tests described herein, bottom slope was not a parameter. *Additional tests should be carried out at fixed values of R , F_p , and R but variable bottom slope, i.e., $\frac{dD_w}{dy}$.*

I. Introduction

The overall goals of our research program are to develop improved analytical and numerical models having general applicability for the prediction of the temperature distribution in the thermal plume from a condenser cooling water discharge; to develop design criteria for siting power plants on estuarine and coastal waters; and to develop field techniques for quantitating the distribution of excess heat¹ from an operating generating station. Although our research has emphasized problems associated with nuclear power plant siting on the Chesapeake Bay, the results of our program have application to other marine environments, as well as to large fresh water bodies such as the Great Lakes.

Our investigations have supplied us with a certain fund of knowledge concerning: (a) the rate of initial mechanical dilution in a plume of heated water discharged as a surface or sub-surface horizontal jet into natural water bodies; (b) the physical processes of movement and dispersion in estuaries and coastal waters; and (c) the processes of loss of excess heat to the atmosphere across the air-sea boundary. A numerical model has been developed known as the Pritchard model. This model is essentially phenomenological or parametric and has been continuously refined by Pritchard as new field and/or flume data have become available. A recent comparison carried out by the Argonne National Laboratory (Paddock, R.A. et al. 1973) with respect to this model and others at two operating sites (Point Beach Unit 1 and Palisades on Lake Michigan) described this model as providing the best comparison with field data of those tested. The model needs broadening, however, to include the effects of cross currents, vertical expansion, and submergence of the discharge structure. We

¹Here excess heat means the difference between the heat that a given parcel of water would contain under conditions of a heated discharge and the heat it would contain under "natural" conditions.

have approached this goal by studying the processes or phenomena, which must be included in any model, experimentally through field studies at existing sites and in a low-velocity flume. Some of the more pertinent results have been reported in Carter (1968, 1969); Pritchard and Carter (1972); Carter, Schiemer, and Regier (1973); and Carter (1973).

This report is the third dealing with our low-velocity flume measurement program. It primarily covers work carried out between February 1973 and June 1974; some measurements previously reported in Carter et al. (1973) are also included for comparison. Some of the results contained herein modify our earlier conclusions. These modifications resulted partly from changes in our experimental procedures which were designed to reveal features of the jet which were poorly defined in previous tests, such as vertical growth; partly from adjustment of the flume to provide a more uniform velocity distribution laterally and vertically; and partly from a more comprehensive analysis of the data. We make no apology for whatever differences appear; our previous reports were clearly labelled preliminary. They were written and distributed in as short a time frame as possible in order to make the measurements available to other investigators. *Where earlier conclusions have been modified, they will be clearly identified.*

The phenomenon being studied is shown conceptually in Figure 1.

Figure 1. Schematic drawing of a buoyant surface three-dimensional bent-over jet (reprinted from Carter et al. (1973)).

Figure 2. Schematic of CBI low velocity flume (reprinted from Carter et al. (1973)).

II. The Experimental Setup and Test Program

Much of this section is essentially a repeat of material contained in Carter et al. (1973). It is included here for ready reference.

Our experiments were performed in a specially constructed low velocity water flume at Hydronautics, Inc., Laurel, Maryland. The flume has a free surface test section approximately 10 feet wide, 1 foot deep and 40 feet long. There are baffles in the upstream sump to distribute the flow evenly throughout the width of the flume before it enters the test section and a honeycomb at the upstream end to straighten the flow. The pump is a Worthington Axial Flow Type KLD discharging into a 12" pipe. It is driven by a 7.5 HP Boston Ratiotrol (variable speed drive) motor. The combination of pump and motor is capable of producing water velocities of up to $0.56 \text{ feet sec}^{-1}$ in the test section. The foregoing is shown schematically in Figure 2. Although not shown in the figures and not used in the test program described herein, the flume has an anodized aluminum false bottom capable of being tilted so that the effect of bottom slope can be studied.

For the heated discharge, a model thermal plant was constructed which consisted of a pump, a flow control system, a rotometer, electric immersion heaters to simulate the condensers, and a temperature sensing and control system to maintain a constant temperature rise between intake and discharge. Front and back views of the model plant are shown in Figures 3 and 4. The plant is capable of producing flows of 5 gpm at a temperature rise of 20°F .

The discharge arrangement is shown in Figures 5 and 6. The 14 gauge galvanized steel discharge canal is rectangular in cross section, $5 \frac{1}{2}$ inches wide and 5 inches deep with the discharge opening centered on one side of its 94 inch length. The canal is lined with $\frac{1}{8}$ inch closed cell neoprene foam lining to minimize heat losses. It was thought that considerable length would

Figures 3 and 4. Front and rear views of model thermal plant
(reprinted from Carter et al. (1973)).

Figures 5 and 6. Photographs of jet discharge canal and associated piping (not in place)(re-printed from Carter et al. (1973)).

be required to smooth the 5 gpm flow and that the double ended arrangement would ensure symmetry of the emergent jet. At low flows, however, the heat loss through the bottom and sides of the canal was excessive and the canal was shortened to 32 inches. The opening is adjustable to 5 inches in width and is faired on the inside to straighten and smooth the flow prior to discharge. Heated water from the model power plant divides at a tee and is distributed to opposite ends of the canal where it is discharged omni-directionally in the horizontal plane at low velocity via a flange and disc diffuser. Balanced flow was obtained by placing an identical orifice in each branch of a size dependent on flow rate. All piping was 5/8 inch garden hose.

Our primary temperature measuring system, shown in the foreground of Figure 7, measured the lateral distribution of temperature across the jet flow at a preselected depth. In this case, the thermistor was driven along an aluminum I-beam by a constant speed motor (72 rpm) through a binary gear box which produced linear thermistor speeds of $6.4 \text{ inches sec}^{-1}$. The thermistor travel at the ends of the beam was controlled by mechanical stops and switches and the direction of the travel changed by changing the direction of motor rotation. The beam, on which the thermistor traversed the flume, was mounted so that it could be rotated about a vertical axis which passed through the thermistor when it was at the end of the beam on the nearshore side, i.e., side of orifice or jet discharge. By rotating the beam, the thermistor could be driven at any angle with the x-axis from 0 to 90° . The thermistor beam support assembly was triangular and constructed from Nu-Rail fittings and aluminum pipe except for the hypotenuse which was another aluminum I-beam joined to the thermistor beam by a sliding block which was free to move along either beam. This entire assembly was mounted on a carriage so that it could be positioned longitudinally (x-direction) at any desired point in the test

Figure 7. The flume and jet flow temperature measuring system from the upstream end (reprinted from Carter et al. (1973)).

section. The result was a measuring system which could be positioned longitudinally and also rotated horizontally so that the direction of thermistor travel was approximately at right angles to the centerline of the jet flow.

Prior to this series of tests, the temperature of the oncoming flow, i.e., the background temperature, was sensed by a driven thermistor which continuously traversed the oncoming flow at constant depth from side to side while measurements were being made. It was found by experience, however, that for the range of parameters being studied, background temperature could easily be obtained from the record of the driven thermistor sampling the temperature distribution of the jet flow. That is, water at ambient or background temperature was always present between the offshore boundary of the jet and the far side of the flume and its temperature corresponded to that of the oncoming flow. Accordingly, the thermistor used in previous tests to measure background temperature was moved to a point in the discharge canal just upstream from the orifice as a check on θ_o .

All thermistors used were glass probe with measured time constants of 1/20 second in rapidly moving (3 feet per second) water. Temperatures were recorded on a 4 1/2" span Esterline Angus Speed Servo Recorder (discharge temperature) with a full scale response of 1/8 second and a 10" span Esterline Angus Speed Servo Recorder (jet flow measuring system) with a full scale response time of 2/10 seconds. The frequency response of these recorders is less than 10% attenuation at 4 Hz (4 1/2") and 3 Hz (10").

Figures 8 and 9 show the lateral distribution of downstream velocity, u_a , in the flume as measured by surface drogue (Fig. 8) and by deep drogue (Fig. 9) at the 10-foot section furthest upstream. It is readily apparent that lateral velocity gradients were considerably reduced between Series I and Series X and that phenomena that were heavily dependent on R, the ratio of the initial jet

FLUME VELOCITY, u , AS MEASURED BY SURFACE DROGUE VERSUS LATERAL DISTANCE, y , FOR THE UPSTREAM 10' SECTION AT $D_w = 0.83'$

Figure 8.

FLUME VELOCITY, u , AS MEASURED BY DEEP DROGUE VERSUS LATERAL DISTANCE, y , FOR THE UPSTREAM 10' SECTION AT $D_w = 0.83'$

Figure 9.

velocity, q_o , to the ambient velocity, u_a , such as offshore penetration would be modified. Drogue tests at sections further downstream were similar to the results shown in Figures 8 and 9.

The test program that was carried out is shown in Table 1. A chart speed of 2 inches min^{-1} was used for recording measurements in the jet. The beam thermistor was driven at various speeds from 0.4 inches sec^{-1} to 1.6 inches sec^{-1} depending on the location of the beam. Slower speeds were used, of course, as the beam was positioned closer to the discharge orifice. In general, the testing procedure followed was to position the pivot point of the beam approximately 3 feet downstream from the center of the orifice and traverse the near surface of the jet flow with the beam at angles of 0° , 10° , 20° , 30° , 60° , and 90° to the near shore. The beam was then left positioned at 90° to the near shore and moved at approximately 5 foot intervals until the excess temperature maximum in the jet was reduced to approximately 1°F . Five or six traverses of the thermistor were made at each beam position. The thermistor was then dropped in succession at 0.05 to 0.10 foot intervals and the entire procedure repeated at each depth. Additional depths were sampled if required to precisely define the half-depth, $z_{1/2}$, of the plume.

III. The Three Dimensional Heated Surface Jet in a Cross Flow

In Carter et al. (1973) both two and three dimensional heated surface jets were treated; in this report we shall confine our attention to only the three dimensional case. By three dimensional we mean that the water depth of the receiving waters, D_w , is greater than the discharge orifice depth, h_o . That is, by definition if $D_w = h_o$ the jet is two dimensional and if $D_w > h_o$, the jet is three dimensional. Depending on the initial densimetric Froude number, F_r , and aspect ratio, A_r , the three dimensional jet may still interact with

TABLE 1. The Test Program.

Test	Q,	θ_o ,	D_w ,	h_o ,	b_o ,	A_r	α_o ,	q_o ,	u_a ,	R	F_r	R_e
Desig.	GPM	°F	Ft	Ft	Ft		°	Ft sec ⁻¹	Ft sec ⁻¹			
SERIES V												
A-V	4	10	0.50	0.173	0.066	2.63	90	0.787	0.184	4.28	9.35	12,665
B-V	4	10	0.67	0.164	0.065	2.53	90	0.835	0.184	4.54	11.34	12,739
C-V	4	10	0.83	0.166	0.065	2.55	90	0.827	0.184	4.49	10.71	12,770
D-V	4	10	0.33	0.164	0.064	2.55	90	0.849	0.184	4.61	11.13	12,952
E-V	4	10	0.17	0.166	0.065	2.55	90	0.827	0.184	4.49	11.21	12,770
F-V	4	10	0.17	0.168	0.163	1.03	90	0.325	0.184	1.77	4.13	5,079
G-V	4	10	0.17	0.169	0.033	5.21	90	1.621	0.184	8.81	20.97	25,484
H-V	4	10	0.50	0.169	0.033	5.15	90	1.614	0.184	8.77	20.80	25,374
I-V	4	10	0.50	0.169	0.163	1.04	90	0.325	0.184	1.77	3.97	5,109
J-V	4	10	0.83	0.166	0.163	1.02	90	0.329	0.184	1.79	4.16	5,080
K-V	4	10	0.83	0.166	0.033	5.10	90	1.653	0.184	8.99	20.00	25,525
SERIES VI												
A-VI(1)	2	10	0.83	0.083	0.167	0.50	90	0.321	0.199	1.61	5.02	2,478
A-VI(2)	2	10	0.83	0.081	0.167	0.48	90	0.330	0.190	1.74	6.17	2,487
A-VI(3)	4	10	0.83	0.081	0.167	0.48	90	0.659	0.190	3.47	12.32	4,965
B-VI(1)	2	10	0.83	0.085	0.054	1.57	90	0.968	0.199	4.86	14.74	7,654
B-VI(2)	2	10	0.83	0.085	0.055	1.55	90	0.953	0.190	5.02	14.63	7,535
C-VI	2	10	0.83	0.087	0.033	2.62	90	1.554	0.199	7.81	25.24	12,577
SERIES X												
A-X	2	10	0.83	0.084	0.053	1.60	90	1.008	0.193	5.22	17.28	7,875
B-X	1	10	0.83	0.065	0.040	1.63	90	0.857	0.193	4.44	17.25	5,181
C-X	4	10	0.83	0.111	0.068	1.60	90	1.162	0.193	6.02	18.06	11,980
D-X	4	10	0.83	0.126	0.126	1.00	90	0.563	0.193	2.92	8.00	6,588
E-X	4	10	0.83	0.097	0.096	1.01	90	0.962	0.193	4.98	15.51	8,651
F-X	4	10	0.83	0.219	0.044	4.95	90	0.922	0.193	4.78	9.94	18,762
G-X	4	10	0.83	0.116	0.113	1.03	90	0.681	0.193	3.53	9.66	7,369

TABLE 1. (continued)

Test	Q,	θ_o ,	D _w ,	h _o ,	b _o ,	A _r	α_o ,	q _o ,	u _a ,	R	F _r	R _e
Desig.	GPM	°F	Ft	Ft	Ft		°	Ft sec ⁻¹	Ft sec ⁻¹			
H-X	4	10	0.83	0.088	0.171	0.51	90	0.596	0.193	3.09	9.90	4,853
J-X	4	10	0.83	0.116	0.113	1.03	90	0.684	0.093	7.35	10.31	8.515
K-X	4	10	0.83	0.115	0.113	1.02	90	0.689	0.366	1.88	10.34	7,371

$$R_e \equiv \frac{q_o h_o}{\nu} \quad \text{where } \nu \text{ is kinematic viscosity of water}$$

$$F_r \equiv \frac{q_o}{\sqrt{\frac{g \Delta \rho_o h_o}{\rho}}} \quad \text{where } g \text{ is the acceleration of gravity}$$

$$A_r \equiv h_o / b_o$$

the bottom at some distance from the orifice. If so, the jet will be prohibited from achieving its maximum vertical growth by the bottom and not due to any limitation arising because of F_r or A_r . A definition sketch, supplementary to Fig. 1, is shown in Figure 10.

Our previous investigations and those of other investigators such as Jen et al. (1966), Hayashi et al. (1967), Tamai et al. (1969), and Stolzenback and Harleman (1971) of heated surface jets have shown that the dimensionless numbers of greatest import are the Reynolds number, R_e , the densimetric Froude number, F_r , the ratio of initial jet velocity, q_o , to ambient velocity, u_a , and the aspect ratio of the orifice, A_r . They are for the origin or point of discharge

$$R_e = \frac{q_o h_o}{\nu}$$

where q_o is the initial jet velocity, h_o is the vertical dimension of the orifice and ν is the kinematic coefficient of viscosity,

$$F_r = \frac{q_o}{\sqrt{g \frac{\Delta\rho}{\rho_o} h_o}}$$

where g is the acceleration of gravity, and $\Delta\rho$ is the difference between the density of discharge ρ_o and the density of the receiving waters, ρ_a ,

$$R = \frac{q_o}{u_a}$$

where u_a is the velocity of the receiving waters, and

$$A_r = \frac{h_o}{b_o}$$

where b_o is the horizontal dimension of the orifice. Other parameters of importance are, α_o , the angle of discharge measured relative to the x-axis

Figure 10. Definition sketches of the three-dimensional heated surface jet in a cross flow.

(see Figure 10), the bottom slope, and the surface cooling coefficient. In the program described herein, α_o was maintained at 90° , the depth of water, D_w , was everywhere constant, and surface cooling was neglected. The effect of neglecting surface cooling is negligible as only the *near field* was considered. See Appendix A of Carter (1969).

In all except two tests, A-VI(1) and A-VI(2), the Reynolds number was in excess of 3000 such that the jet was fully turbulent and the results are therefore independent of R_e . Our program, therefore, was designed to reveal the dependence of certain characteristics of the heated surface jet on F_r , R , and A_r . The characteristics chosen were (1) the centerline trajectory as defined by the locus of peak temperatures on the xy plane at $z = 0$, (2) the depth of the jet as a function of downstream distance, ξ , as defined by $z_{1/2}$, the value of z at which $\theta = \frac{1}{2} \theta_p$ measured on the vertical surface passing through the centerline trajectory, (3) the lateral spread of the jet as a function of downstream distance, ξ , as defined by $2\eta_{1/2}$, the distance measured on the free surface, i.e., $z = 0$, between the point on the $- \eta$ axis where $\theta = \frac{1}{2} \theta_p$ and the point on the $+ \eta$ axis where $\theta = \frac{1}{2} \theta_p^2$, and (4) the dilution, θ_p/θ_o , as a function of downstream distance.

We approached the problem empirically with the goal of collapsing families of curves describing the dependence of a particular characteristic on the various parameters into a single one-dimensional curve by proper scaling. In this we believe we have been reasonably successful. The final result, a series of non-dimensional curves describing the

²

In Carter et al. (1973) only the part of the jet offshore from the centerline, i.e., positive η axis, was used in defining $\eta_{1/2}$ because of the asymmetry of the jet about the centerline as $h \rightarrow D_w$. For the three dimensional cases described herein, however, the asymmetries were not extreme and the full width of the jet was used to define $2\eta_{1/2}$.

characteristics enumerated above, should have transfer or predictive value. A major deficiency in our predictor curves, however, is the omission of bottom slope as a parameter. Adjustment will undoubtedly be necessary as new data become available, particularly from the field.

The data taken during Series X is considered to be of higher quality than that from either Series I, V, or VI because the cross sectional distribution of flow in the flume was more uniform during Series X and because a greater number of crossings for each beam position and thermistor depth were made during this series. Therefore, where at variance, the results from Series X have been chosen in preference to those from either Series I, V, or VI.

a. The vertical growth of the three-dimensional heated jet in a cross flow.

The depth, $z_{1/2}$, has been defined previously as the value of z for which $\theta = \frac{1}{2} \theta_p$ measured on the vertical surface passing through the centerline trajectory, i.e., $\eta = 0$. To determine its value, we proceeded as follows. At a given beam position or downstream distance, ξ , the peak value of θ at each depth was plotted as a function of depth, z , and a smooth curve faired through the points. $z_{1/2}$ was then scaled from the resulting curve. This was done for all beam positions for each test. Since under certain conditions, it was possible for reflections from the bottom of the flume to increase the value of $z_{1/2}$ so obtained over what it would have been if there were no bottom boundary, it was necessary to remove the data so biased. To do this it was noted that the vertical distribution of excess temperature, θ , for $\eta = 0$ could be approximated by a cosine function, or that the vertical dis-

tribution of excess temperature, θ , could be expressed as

$$\theta = \frac{\theta_p}{2} \left(1 + \cos \left(\frac{\pi}{2} \cdot \frac{z}{z_{1/2}} \right) \right) \quad (1)$$

See Figure 11. If there is reflection from a solid boundary located at $z = D_w$ then the increase in θ due to reflection, θ_{ref} is given by

$$\theta_{ref} = \frac{\theta_p}{2} \left(1 + \cos \frac{\pi}{2} \left(\frac{2D_w - z}{z_{1/2}} \right) \right) \text{ for } z \leq D_w \quad (2)$$

From (2) it may be seen that at $z = z_{1/2}$, if $D_w \geq 1.5z_{1/2}$, then $\theta_{ref} = 0$ and the *determination* of $z_{1/2}$ from the vertically measured excess temperature distribution would be unaffected by reflection from the bottom. For those cases where no bottom interference was present, it was clear that, in general, as one proceeded down the jet the half depth increased at first, reached a maximum value, and then decreased to the limits of our measurements. Table 2 lists the maximum value of the half depth, $z_{1/2, \max}$, for the various tests that met the bottom reflection criteria above.

Analysis of the data in Table 2 indicated a linear dependence on the densimetric Froude number and a somewhat smaller dependence on A_r , the aspect ratio. The following relationship was selected as providing the best estimate of our measurements of $z_{1/2, \max}$

$$\frac{z_{1/2, \max}}{\sqrt{A}} = 0.342 A_r^{1/4} F_r \quad (3)$$

Equation (3) and the data are shown in Figure 12.

The data were next examined to establish the dependence of $z_{1/2}$ on ξ , the downstream distance, for a given A_r and F_r . By trial and error it was

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$z_{1/2, \max}/D_w$ for Tests B-V and C-VI are slightly in excess of 0.67; the actual measurements, however, appeared to be unreflected.

Figure 11. The vertical (z) profile of excess temperature, θ/θ_p , at various downstream distances, ξ , for all Series X tests.

TABLE 2. Maximum half depth, $z_{1/2,max}$ as a function of R , F_r , and A_r .

Test	F_r	R	A_r	Measured $z_{1/2,max}/\sqrt{A}$	$D_w/\sqrt{A}$	Measured $z_{1/2,max}/D_w$
B-V	11.34	4.54	2.53	4.77	6.49	0.735
I-V	3.97	1.77	1.04	1.33	3.01	0.441
J-V	4.16	1.79	1.02	1.45	5.05	0.287
A-VI(1)	5.02	1.61	0.50	2.21	7.05	0.313
B-VI(1)	14.74	4.86	1.57	6.20	12.25	0.506
B-VI(2)	14.63	5.02	1.55	6.73	12.14	0.554
C-VI	25.24	7.81	2.62	12.13	15.49	0.783
A-X	17.28	5.22	1.60	5.99	12.44	0.482
C-X	18.06	6.02	1.60	6.14	9.48	0.647
D-X	8.00	2.92	1.00	3.25	6.59	0.493
E-X	15.51	4.98	1.01	5.29	8.60	0.615
F-X	9.94	4.78	4.95	5.30	8.46	0.627
G-X	9.66	3.53	1.03	3.07	7.25	0.423
H-X	9.90	3.09	0.51	2.85	6.77	0.421
J-X	10.31	7.35	1.03	3.58	6.68	0.536
K-X	10.34	1.88	1.02	3.25	7.28	0.446

established that by scaling $z_{1/2}$ by $A_r^{1/4}$, the dependence on aspect ratio was eliminated. The data were now sorted by F_r and plotted versus ξ (see Figures 13, 14, & 15 for example) from which it was determined that

- (1) there was no dependence of $z_{1/2}$ on R anywhere,
- (2) initially, $(z_{1/2} / \sqrt{A})(1/A_r^{1/4})$ was proportional to $(\xi/\sqrt{A})^{1/2}$ and did not depend on F_r ,
- (3) $(z_{1/2} / \sqrt{A})(1/A_r^{1/4})$ reached a maximum in accordance with equation (3), and

MEASURED MAXIMUM DEPTH, $Z_{1/2,max} / \sqrt{A} \cdot Ar^{1/4}$
 VERSUS INITIAL DENSIMETRIC FROUDE NUMBER, Fr

Figure 12.

	<u>TEST</u>	<u>A_r</u>	<u>F_r</u>
○	I-V	1.04	3.97
□	J-V	1.02	4.16

Figure 13. The half depth, as given by $z_{1/2}/\sqrt{A} A_r^{1/4}$, versus downstream distance, $\xi/\sqrt{A}$, for a F_r of 4.

Figure 14. The half depth, as given by $z_{1/2} / \sqrt{A} A_r^{1/4}$, versus downstream distance, $\xi / \sqrt{A}$, for a F_r of 10.

Figure 15. The half depth, as given by $z_{1/2} / \sqrt{A} A_r^{1/4}$, versus downstream distance, $\xi / \sqrt{A}$, for a F_r of 18.

(4) ultimately, $(z_{1/2} / \sqrt{A}) (1/A_r)^{1/4}$ decreased in proportion to

$$F_r^{3/2} (\xi / \sqrt{A})^{-1/2}.$$

The dependence of $z_{1/2}$ on ξ , F_r , and A_r is tabulated more formally below.

Initial zone:
$$\frac{z_{1/2}}{\sqrt{A}} \frac{1}{A_r^{1/4}} = 0.87 \left(\frac{\xi}{\sqrt{A}} \right)^{1/2} \quad (4)$$

Zone of maximum penetration:
$$\frac{z_{1/2, \max}}{\sqrt{A}} \frac{1}{A_r^{1/4}} = 0.342 F_r$$

Final zone:
$$\frac{z_{1/2}}{\sqrt{A}} \frac{1}{A_r^{1/4}} = 0.80 F_r^{3/2} \left(\frac{\xi}{\sqrt{A}} \right)^{-1/2} \quad (5)$$

Equations (3), (4), and (5) together with the data are shown on Figure 16.

There are few data from other investigations suitable for direct comparison with our measurements of $z_{1/2}$. Stölzenbach et al., 1972 proposed the following theoretical relationship for $z_{\max}$ or $2z_{1/2, \max}$ in our symbols

$$\frac{2z_{1/2, \max}}{\sqrt{A/2}} = 0.42 F_r (2A_r)^{1/4} \quad \text{for } F_r (2A_r)^{1/4} > 3$$

or

$$\frac{z_{1/2, \max}}{\sqrt{A}} = 0.177 F_r A_r^{1/4} \quad \text{for } F_r A_r^{1/4} > 2.52$$

which provides values of $z_{1/2, \max}$ one half as large as equation (3).

Figure 16. The half depth, as given by $z_{1/2} / \sqrt{A} A_r^{1/4}$, versus downstream distance, $\xi / \sqrt{A}$, for various F_r between 4 and 25.

D.W. Pritchard (personal communication) finds that

$$\frac{z_{1/2, \max}}{\sqrt{A}} = 0.3 A_r^{1/4} (F_r^{3/4} - 1.5 F_r^{-3/4})$$

based on Stolzenbach and Harleman 1971 data and unpublished data from a field study at Oyster Creek by Pritchard-Carpenter, Consultants. This relationship gives values of $z_{1/2, \max} / \sqrt{A}$ which are low by a factor of $0.88/F_r^{1/4}$ which is approximately 0.5 at a F_r of 10.

b. The lateral spread of the three-dimensional heated jet in a cross flow.

The width, $2\eta_{1/2}$, has been defined previously. See Page 20 of this report. To determine its value as a function of ξ , R , F_r , and A_r we proceeded as follows. At a given beam position, i.e., ξ , the value of $2\eta_{1/2}$ was scaled directly from the beam thermistor temperature records between the $\theta = \frac{1}{2} \theta_p$ values converting the horizontal time axis of these records to distance through the known thermistor speed of travel. See Figure 10. In the event the plume crossing was not normal to the centerline, i.e., small ξ , the value of $2\eta_{1/2}$ measured was reduced to a more correct value graphically. For the tests run, there was no observable bias in the measurements of $2\eta_{1/2}$ because of near- or farshore reflections.

The width data were first analyzed for a dependence on R at a densimetric Froude number of approximately 10 and an aspect ratio of 1. For this purpose, tests D-X, G-X, J-X, and K-X were utilized. The data from these tests are plotted in Figure 17 from which it may be seen that for small ξ , the spread is linear with ξ and for large ξ , the spread is parabolic. The following relationships were selected as providing the best

Figures 17 & 18. The lateral spread, $2n_{1/2}/\sqrt{A}$ or $2n_{1/2}/R^2\sqrt{A}$, as a function of downstream distance, $\xi/\sqrt{A}$ or $\xi/R^2\sqrt{A}$, for various R and for $F_r \approx 10$ and $A_r \approx 1$.

estimate of the dependence of $2\eta_{1/2}$ on ξ and R for a constant F_r and A_r .

$$\text{Small } \xi: \quad \frac{2\eta_{1/2}}{\sqrt{A}} = 0.58 \frac{\xi}{\sqrt{A}} \quad (6a)$$

$$\text{Large } \xi: \quad \frac{2\eta_{1/2}}{\sqrt{A}} = 0.90 R \sqrt{\frac{\xi}{A}} \quad (6b)$$

for constant F_r and A_r .

The family of curves represented by equation (6b) may be collapsed to a single curve if we scale $2\eta_{1/2}$ and ξ by $R^2\sqrt{A}$ or for

$$\text{Small } \xi: \quad \frac{2\eta_{1/2}}{R^2\sqrt{A}} = 0.58 \frac{\xi}{R^2\sqrt{A}}, \quad (7a)$$

and for

$$\text{Large } \xi: \quad \frac{2\eta_{1/2}}{R^2\sqrt{A}} = 0.90 \sqrt{\frac{\xi}{R^2\sqrt{A}}} \quad (7b)$$

for constant F_r and A_r . Equations (7a) and (7b) are shown in Figure 18 together with the data.

The data were next sorted to reveal any dependence on aspect ratio, A_r . For this purpose tests C-V, F-X, G-X, and H-X (nominal values of R and F_r of 4 and 10, respectively) were used. There was no dependence of $2\eta_{1/2}$ on A_r for small ξ but at large ξ , $2\eta_{1/2}$ was inversely proportional to $A_r^{1/4}$ for a given value of ξ , R and F_r . This is shown in Figure 19.

Equation (7b) was therefore modified in light of the results from Figure 19 to read for

$$\text{Large } \xi: \quad \frac{2\eta_{1/2}}{R^2\sqrt{A}} = \frac{0.90}{A_r^{1/4}} \sqrt{\frac{\xi}{R^2\sqrt{A}}} \quad (8)$$

for constant F_r . Since $\sqrt{A} / \sqrt{A_r} = b_o$, equations (7a) and (8) may be re-

Figures 19 & 20. The lateral spread, $2\eta_{1/2}/R^2\sqrt{A}$ or $2\eta_{1/2}/R^2b_0$, as a function of downstream distance, $\xi/R^2\sqrt{A}$ or ξ/R^2b_0 , for various Ar and a Fr of 10.

arranged to read for

$$\text{Small } \xi: \quad \frac{2\eta_{1/2}}{R^2 b_o} = 0.58 \frac{\xi}{R^2 b_o}, \quad (9a)$$

and for

$$\text{Large } \xi: \quad \frac{2\eta_{1/2}}{R^2 b_o} = 0.90 \sqrt{\frac{\xi}{R^2 b_o}} \quad (9b)$$

for constant F_r . Equations (9a) and (9b) are shown in Figure 20 together with the data.

It remains to consider the effect of densimetric Froude number, F_r , on $2\eta_{1/2}$. If our previous analysis is correct, plots of $2\eta_{1/2} / R^2 b_o$ versus $\xi/R^2 b_o$ will be independent of R and A_r . To assess the effect of F_r on $2\eta_{1/2}$, we selected 3 tests where $F_r \geq 10$ (tests A-X, E-X, and F-X) and 3 tests where $F_r \leq 10$ (tests J-V, A-VI(2), and K-X).

$2\eta_{1/2} / R^2 b_o$ versus $\xi/R^2 b_o$ for tests J-V, A-VI(2), and K-X are shown in Figure 21; similar data for tests A-X, E-X, and F-X are shown in Figure 22. $2\eta_{1/2}$ appears to be inversely related to $F_r^{1/2}$ at large ξ ; the situation with respect to small ξ is not clear cut, the data suggesting that $2\eta_{1/2}$ may not be independent of F_r at low F_r . Notwithstanding, the data in Figures 21 and 22 are reasonably well represented by the expressions for

$$\text{Small } \xi: \quad \frac{2\eta_{1/2}}{R^2 b_o} = 0.58 \frac{\xi}{R^2 b_o}, \quad (9a)$$

and for

$$\text{Large } \xi: \quad \frac{2\eta_{1/2}}{R^2 b_o} = \frac{2.85}{\sqrt{F_r}} \sqrt{\frac{\xi}{R^2 b_o}} \quad (10)$$

Figures 21 & 22. The lateral spread, $2n_{1/2}/R^2 b_0$, as a function of downstream distance, $\xi/R^2 b_0$, for various F_r .

for all R , A_r , and F_r . It may be seen from equations (9a) and (10) that our data may be rendered independent of R , A_r , and F_r if we scale $2\eta_{1/2}$ and ξ by $\frac{F_r}{R^2 b_o}$ or for

$$\text{Small } \xi: \quad \frac{2\eta_{1/2} F_r}{R^2 b_o} = 0.58 \frac{\xi F_r}{R^2 b_o}, \quad (11)$$

and for

$$\text{Large } \xi: \quad \frac{2\eta_{1/2} F_r}{R^2 b_o} = 2.85 \sqrt{\frac{\xi F_r}{R^2 b_o}} \quad (12)$$

$\frac{2\eta_{1/2} F_r}{R^2 b_o}$ versus $\frac{\xi F_r}{R^2 b_o}$ has been plotted for test D-X and for 5 tests not previously used in the development of equations (11) and (12) in Figure 23 together with equations (11) and (12).

There are few published data of the lateral spread of a *heated* jet in a cross flow with which to compare our results. There are data, however, from tests of nonbuoyant jets in cross flows (see Pratte et al., 1967) and from tests of heated jets in stagnant fluid (see Dornhelm et al., 1972 and Tamai et al., 1969). Dornhelm et al., 1972 concluded that the effect of the densimetric Froude number on the mixing of a surface jet did not significantly alter the mixing for F_r as low as 12. However, for $F_r < 12$, the mixing appeared to increase markedly for low values of downstream distance, possibly due to the dominance of surface spreading at low F_r ($6 < F_r < 12$) and small ξ . Our data suggest that this holds true also for the heated jet in a cross flow. See for example, the close in data points for tests J-V and A-VI(2) on Figure 21 and for tests A-VI(1) and D-X on Figure 23. Equation (11) should be used with caution, therefore, for $F_r < 10$.

Figure 23. The lateral spread, $2\eta_{1/2} F_r / R^2 b_0$, as a function of downstream distance, $\xi F_r / R^2 b_0$, for various R, A_r , and F_r .

Although not shown, a plot of $2\eta/2\eta_{1/2}$ versus θ/θ_p for all ξ of all Series X tests cluster about the relationship

$$\frac{\theta}{\theta_p} = \frac{1}{2} \left(1 + \cos \left(\frac{\pi}{2} \cdot \frac{\eta}{\eta_{1/2}} \right) \right) \quad (13)$$

c. *The centerline trajectory of the three-dimensional heated jet in a cross flow.*

It was argued in Carter et al., 1973 that a hypothesis of drag without entrainment led to tangential momentum being conserved along a centerline trajectory that was parabolic in the limit for any reasonable assumption regarding the characteristic velocity in the drag force. Whether the jet was 2- or 3-dimensional was not considered pertinent; the absence of entrainment was the essential factor. It was also argued that a hypothesis of *lateral* entrainment without drag led to offshore or y-momentum being conserved along a centerline trajectory that was also parabolic. It was felt intuitively that any vertical entrainment would modify this result in that the trajectory would depart from a parabolic trajectory in the near-shore or -y-direction.

An examination of the literature for trajectory measurements of jets in cross flows indicates that most of the data could be characterized by a relationship of the form

$$\frac{y}{\sqrt{A}} = k \left(\frac{x}{\sqrt{A}} \right)^m \quad (14)$$

with m varying between $1/2$ and $1/3$. Accordingly, we have confined our attention to quantitating the constant of proportionality, k , and the slope, m , in equation (14) and their dependence, if any, on R , A_r and F_r . For this purpose, we have considered only Series X results in view of the

excessive nonuniformity of the ambient flow in the lateral or y-direction during Series V & VI. Unfortunately, this denies us a reasonably good set of data not used in quantifying k and m for testing our results. This verification must therefore await additional testing.

Centerline trajectory data were processed as follows. For each run or plume crossing of Series X, the x and y coordinates were computed and tabulated. It was assumed that the trajectory was of the form of equation (14) and the results scaled by $\sqrt{A}$, logarithms to the base 10 computed, and a linear regression of y on x calculated for each test separately. The slope of this regression line is, of course, m in equation (14) and the antilog of the intercept of the regression line is k . The results of these calculations are listed in Table 3, below.

TABLE 3. m and k (equation (14)) as functions of R , A_r , & F_r .

Test	m	k	F_r	A_r	R	k/R	$(k/R)A_r^{\frac{m}{2}}$
A-X	0.380	5.420	17.28	1.60	5.22	1.038	1.135
C-X	0.353	6.095	18.06	1.60	6.02	1.012	1.100
D-X	0.498	2.886	8.00	1.00	2.92	0.988	0.988
E-X	0.419	5.121	15.51	1.01	4.98	1.028	1.030
F-X	0.471	3.140	9.94	4.95	4.78	0.657	0.957
G-X	0.466	3.523	9.66	1.03	3.53	0.998	1.005
H-X	0.479	3.670	9.90	0.51	3.09	1.188	1.011
J-X	0.383	7.812	10.31	1.03	7.35	1.063	1.069
K-X	0.489	1.854	10.34	1.02	1.88	0.986	<u>0.991</u>

AVERAGE ≈ 1

It is readily apparent from Table 3 that the penetration of the jet as measured by k is strongly correlated with R , uncorrelated with F_r (see Tests G-X, J-X, and K-X) and somewhat dependent upon A_r (see Tests F-X & H-X). We

have chosen for the relationship between k , R , and A_r the following

$$k A_r^{\frac{m}{2}} = R \quad (15)$$

Equation (13) now becomes in light of (15)

$$\frac{Y}{\sqrt{A}} = \frac{R}{A_r^{m/2}} \left(\frac{x}{\sqrt{A}} \right)^m$$

Rearranging (15) and remembering that $\sqrt{A} \sqrt{A_r} = h_o$, equation (15) becomes

$$\frac{Y}{h_o} = \frac{R}{\sqrt{A_r}} \left(\frac{x}{h_o} \right)^m, \quad (16a)$$

or

$$\frac{Y}{b_o} = R A_r^{\left(\frac{1}{2} - m\right)} \left(\frac{x}{b_o} \right)^m, \quad (16b)$$

or

$$\frac{Y}{\sqrt{A}} = R \left(\frac{x}{h_o} \right)^m. \quad (16c)$$

A plot of m versus A_r and R reveals no significant dependence on these parameters. See results in Table 3 for Tests F-X, G-X, and H-X for A_r dependence and with the exception of Test J-X which is considered anomalous, there is no obvious dependence on R . The regression line of m on F_r is

$$m = 0.609 - 0.013 F_r. \quad (17)$$

Equation (17) is shown on Figure 24.

Figure 24.

If confidence limits at the 5% level are computed for the slopes listed in Table 3, they are of the order of ± 0.02 to ± 0.03 with a high value of ± 0.102 for Test J-X and a low of ± 0.009 for Test K-X. It is of significance, we believe, that the ambient velocity, u_a , for Test J-X was extremely low, 0.093 fs^{-1} , and for Test K-X was the highest of any test in the program, 0.366 fs^{-1} . Quite possibly, there is large scale eddying motion in the flume at very low ambient velocities which significantly affects the centerline trajectory. At any rate, in view of the foregoing, and as was pointed out previously, we have chosen to regard the centerline trajectory results from Test J-X as anomalous.

To illustrate the variability in the measurements of the position of the centerline trajectory, we have plotted the data from Tests F-X, G-X, H-X, and K-X ($F_r \approx 10$) and the data from Tests A-X and C-X ($F_r \approx 18$) together with equations (16c) and (17) on Figures 25 and 26.

d. Dilution, θ_p/θ_o , as a function of downstream distance, ξ , for the three-dimensional heated jet in a cross flow.

For purposes of analyzing the dilution, we have chosen to divide the jet into three regions which we have labelled (after Stolzenbach and Harleman, 1971) (1) the *core* region where the centerline temperatures are essentially constant, (2) the *entrainment* region where the temperatures drop sharply with downstream distance, and (3) a *stable* region where the temperature drops more slowly than in the entrainment region. For purposes of illustration, data from Tests B-V, G-X, and H-X are shown in Figure 27. Data from these tests were used since their initial densimetric Froude numbers and velocity ratios were approximately the same.

The solid curves were fitted by eye; determination of the slopes and

Figure 25. The centerline trajectory for various R and A_r and $F_r \approx 10$.

Figure 26. The centerline trajectory for various R and A_r and $F_r \approx 18$.

TYPICAL DILUTION CURVE FOR A 3-DIMENSIONAL HEATED JET IN A CROSS FLOW

Figure 27.

intercepts of these solid curves is, of course, the goal of this subsection. The regions described above are also delineated on the figure.

1. The entrainment region.

Dilution in the entrainment region will be treated first since its intercept with the line $\theta_p/\theta_o = 1$, i.e., no dilution, is used to define the length of the core region. For analysis of the entrainment region, the data were separated into two parts, Series X and pre-Series X tests. The basic analysis was carried out on the pre-Series X data with information from Series X tests utilized for verification.

The θ_p/θ_o versus $\xi/\sqrt{A}$ data from the pre-Series X tests were first sorted with respect to region and plotted. It was clear from the plot that a relationship of the form

$$\frac{\theta_p}{\theta_o} \sim \left(\frac{\xi}{\sqrt{A}}\right)^{-2/3}$$

was a reasonable representation of the data in the entrainment region with deviations principally occurring near the boundary between the core region and the entrainment region. No dependence of the constant of proportionality upon either R or A_r could be established but there was a strong inverse relationship with the cube root of densimetric Froude number. That is

$$\frac{\theta_p}{\theta_o} = \frac{4.744}{F_r^{1/3}} \left(\frac{\xi}{\sqrt{A}}\right)^{-2/3} \quad (18)$$

Figure 28 is a plot of all the data versus predicted values from equation (18). It may be seen from Figure 28 that, as was stated earlier, the errors of estimation are largest for values of θ_p/θ_o near 1. A statistical analysis

Figure 28. Comparison of computed values of θ_p/θ_0 (equation (18)) and measured values of θ_p/θ_0 in the entrainment region.

of the variance, however, indicated that in excess of 93% of the variation could be accounted for by equation (18).

2. The core region.

The core region is by definition, the region where $\theta_p/\theta_o = 1$. At $\theta_p/\theta_o = 1$, equation (18) becomes

$$\frac{\xi_o}{\sqrt{A}} = \frac{10.334}{\sqrt{F_r}} \quad (19)$$

where $\xi_o/\sqrt{A}$ is defined as the value of $\xi/\sqrt{A}$ obtained from equation (18) when $\theta_p/\theta_o = 1$. As we have defined it, $\xi_o/\sqrt{A}$ is *not* a quantity measured in the core region but rather is inferred from measurements made in the entrainment region and fitted to equation (18). That is, $\xi_o/\sqrt{A}$ was determined for each test by computing the geometric means of θ_p/θ_o and $\xi/\sqrt{A}$ in the entrainment region and then passing a curve of slope - 2/3 through this point. The value of $\xi/\sqrt{A}$ at $\theta_p/\theta_o = 1$ was by definition $\xi_o/\sqrt{A}$ and was tabulated as a function of F_r . The values of $\xi_o/\sqrt{A}$ versus F_r so obtained from pre-Series X tests are plotted in the upper portion of Figure 29; data from Series X tests are plotted on the lower portion of Figure 29. The solid lines are equation (19). *To repeat, data points on this figure should be viewed as estimates of the core length if the transition were as sharp as we have assumed.*

Shirazi and Davis, 1974 measured ξ_o in a still water tank, i.e., $u_a = 0$, and obtained (in our symbols)

$$\frac{\xi_o}{\sqrt{A}} = \frac{5.4}{F_r^{1/3} A^{1/6}} \quad (20)$$

Figure 29. Comparison of predicted core lengths (equation (19)) and measured values for pre-series X tests (upper) and series X tests (lower).

This relationship produces values of $\xi_o/\sqrt{A}$ that are approximately 30% shorter than equation (19) at an A_r of 1 and F_r of 10. Prych, according to Shirazi and Davis, 1974, uses for $\xi_o/\sqrt{A}$ (again in our symbols)

$$\frac{\xi_o}{\sqrt{A}} = 6.38 \quad . \quad (21)$$

Verification of the dependence of $\xi_o/\sqrt{A}$ in equation (20) on A_r , would require experimental detection of a difference of 0.06 feet in ξ_o over a range in A_r of 0.5 to 5 at a F_r of 9. It is not felt that this precision is obtainable in a setup comparable to ours, at least with temperature sensors, since the transition is apparently gradual and θ_o tends to fluctuate somewhat. However, as was shown earlier, significant error was not introduced by inferring the length of the core from measurements in the entrainment zone as we have done and predicting values of θ_p/θ_o from $\theta_p/\theta_o = 1$ for $0 < \xi/\sqrt{A} < 10.334/\sqrt{F_r}$ and from equation (18) in the entrainment zone.

3. The stable region.

According to Stolzenbach et al., 1971 and 1973, the stable region is characterized by *relatively* constant centerline temperatures, θ_p , and dilutions, θ_p/θ_o . Our measurements in this region (see Figure 27) show that the centerline temperatures continue to decrease but that the rate of decrease is greatly reduced. Foregoing for the moment an explanation which accounts for this reduced dilution rate, we shall attempt a qualitative explanation based on our measurements. For this purpose we have plotted data and our deduced relationships for θ_p/θ_o , $z_{1/2}/\sqrt{A}$, and $2\eta_{1/2}/\sqrt{A}$ against a common $\xi/\sqrt{A}$ axis on Figure 30 for a densimetric Froude number of 10, a velocity ratio R of 3.5 and an aspect ratio A_r of 1.

It appears from the curve labelled $z_{1/2}/\sqrt{A}$ on Figure 30 that although

Figure 30. θ_p/θ_o , $2\eta_{1/2}/\sqrt{A}$, and $z_{1/2}/\sqrt{A}$ versus $\xi/\sqrt{A}$ for $Fr \approx 10$, $A_r = 1$, and $R \approx 3.5$.

the density difference between the jetted fluid and the ambient fluid is reduced by dilution, its importance relative to inertia increases until at about $\xi / \sqrt{A} \approx 15$ (for the example shown) vertical growth of the jet ceases. This should occur at a local densimetric Froude number, $F_{r,\ell}$, of order 1 and a simple analysis of the ratio, $F_{r,\ell}/F_r$, in the entrainment region based on our measurements of dilution, lateral spread, and vertical growth with downstream distance, $\xi/\sqrt{A}$, confirm this.

The cessation of vertical mixing appears to be followed almost immediately by a reduction in lateral growth as indicated in Figure 30 by the curve labelled $2\eta_{1/2}/\sqrt{A}$. That is, the vertical surfaces across which lateral entrainment takes place are no longer increasing in vertical extent and consequently the rate of lateral mixing is reduced. In addition, the jetted fluid continues to spread due to gravitational convection and, as a result, continuity requires the depth of the jet to decrease.

The fact that the centerline temperatures continue to decrease in the stable region indicates that fluid is still being entrained even though the product of $2\eta_{1/2}/\sqrt{A}$ and $z_{1/2}/\sqrt{A}$ is now constant. In all probability, the shape functions f_2 and g_2 as given by

$$\theta(\xi, \eta, z) = \theta_p(\xi) f_2\left(\frac{\eta}{\eta_{1/2}}\right) g_2\left(\frac{z}{z_{1/2}}\right) \quad (22)$$

are not independent of ξ (see equations (1) and (13) and Figure 11) and our measurements were not sensitive enough to detect such a dependence. If the shape functions were self preserving, as is invariably assumed in theoretical analyses of jets, the measured decrease in centerline temperatures in the stable region would have to be the result of surface cooling. () First order calculations, however, show that cooling could not account for the observed decrease in temperatures in this region.

Figure 31 is presented for comparison with Figure 30 and shows the relationship between θ_p/θ_o , $2\eta_{1/2}/\sqrt{A}$, and $z_{1/2}/\sqrt{A}$ for a densimetric Froude number of approximately 6. As noted earlier, equation (11) does not predict the spreading rate particularly well at low densimetric Froude numbers.

It was argued earlier that the bottom reflection criteria for bias in the measured value of $z_{1/2}$ was that $z_{1/2}/D_w$ be equal to or less than 0.67 (see page 22). A slightly different criterion applies in the case of bias in measurements of θ_p/θ_o at the surface as may be seen from equation (2). That is, θ_{ref} equals 0 at $z = 0$, when $z_{1/2}/D_w$ is equal to or less than 1. Accordingly, we have sorted our stable region test data for analysis as to whether or not θ_{ref} equals 0 at $z = 0$. That is, when measured values of $z_{1/2, max}/D_w$ were equal to or less than 1, θ_{ref} was considered to be equal to 0; when predicted values of $z_{1/2, max}/D_w$ from equation (3) exceeded 1, surface values of θ_p/θ_o were considered to be biased by bottom reflection.

$$A. \quad \underline{z_{1/2, max}/D_w \leq 1}$$

For this analysis, all of Series VI and X tests met the required criteria in addition to Tests A-V, B-V, C-V, I-V, and J-V. All have been used except B-V & C-V which are redundant and B-X which is considered anomalous.

It was assumed that the functional dependence of θ_p/θ_o on $\xi/\sqrt{A}$ could be expressed by a relationship of the form

$$\frac{\theta_p}{\theta_o} \left(\frac{\xi}{\sqrt{A}} \right)^{1/3} = f(F_r, R, A_r) \quad (23)$$

which passed through the geometric mean of the values of θ_p/θ_o and $\xi/\sqrt{A}$. From these geometric means of θ_p/θ_o and of $\xi/\sqrt{A}$ and equation (23), f was then calculated for each test.

Figure 31. θ_p/θ_o , $2\eta_{1/2}/\sqrt{A}$, and $z_{1/2}/\sqrt{A}$ versus $\xi/\sqrt{A}$ for $Fr \approx 5$, $A_r = 0.50$, and $R \approx 1.6$.

Values of f were next analyzed with respect to R dependence at a densimetric Froude number of 10 and an aspect ratio of 1. For this purpose, Tests D-X, G-X, J-X, and K-X were used. f values from these test data are plotted in Figure 32 (lower portion) from which it may be seen that there is slight dependence of f on R with a reasonable fit to the data points provided by assuming $f \sim R^{-1/6}$.

Values of $fR^{1/6}$ at a constant F_r were next examined for dependence on A_r . For this purpose, Tests A-V, F-X, G-X and H-X were used. Values of $fR^{1/6}$ for these tests are plotted on Figure 32 (upper portion). It is clear from this figure that there is no apparent dependence of the data points on A_r over a range of 0.5 to 5.

Finally, values of $fR^{1/6}$ were examined for dependence on F_r at various R and A_r . Figure 33 shows the results of this analysis with $fR^{1/6}$ plotted versus F_r . Except for I-V and J-V, a good fit to the plotted points is provided by assuming $fR^{1/6} \sim F_r^{-1/3}$. A line of slope $-1/3$ and passing through the geometric means of $fR^{1/6}$ and F_r is given by

$$fR^{1/6} = 1.721 F_r^{-1/3} \quad (24)$$

Combining equations (23) and (24) we obtain

$$\frac{\theta_p}{\theta_o} = \frac{1.721}{R^{1/6} F_r^{1/3}} \left(\frac{\xi}{\sqrt{A}} \right)^{-1/3} \quad (25)$$

which is the predictor relation for unbiased values of θ_p/θ_o versus downstream distance, $\xi/\sqrt{A}$, in the stable region.

Figure 34 is a plot of all the data versus predicted values of θ_p/θ_o from equation (25). Figure 34 suggests that equation (25) provides values of θ_p/θ_o that are lower than the measured values in the area where the stable region and entrainment region join indicating the boundary is not

Figure 32. f , the constant of proportionality between θ_p/θ_o and $\xi/\sqrt{A}$ in the stable region as a function of A_r and R for a F_r of 10 when $z_{1/2, \max} \leq D_w$.

Figure 33. $fR^{1/6}$, the constant of proportionality (corrected for R dependence) between θ/θ_0 and $\xi/\sqrt{A}$ in the stable region as a function of F_r when $P_{z_0} \leq P_{z_{1/2, \max}} \leq D_w$.

Figure 34. Comparison of computed values of θ_p/θ_o (equation (25)) and measured values of θ_p/θ_o in the stable region when $z_{1/2,max} \leq D_w$.

as sharp as that indicated by the intersection of equation (25) and (18).

A statistical analysis of the variance indicated that 84% of the variation could be accounted for by equation (25).

$$B. \quad \underline{z_{1/2, \max} / D_w} > 1$$

In Table 4 are listed the tests for which the surface values of θ_p / θ_o are biased by bottom reflection in the stable region according to our criterion.

TABLE 4. Tests for which predicted $z_{1/2, \max} / \sqrt{A}$ exceeds 1.

Test	A_r	R	F_r	PREDICTED $z_{1/2, \max} / \sqrt{A}$	$D_w / \sqrt{A}$	PREDICTED $z_{1/2, \max} / D_w$
M-I	2.64	4.45	8.54	3.72	3.15	1.18
O-I	0.60	4.53	12.47	3.75	3.16	1.19
Q-I	5.56	4.47	6.85	3.60	3.14	1.15
D-V	2.55	4.61	11.13	4.81	3.26	1.47
H-V	5.15	8.77	20.80	10.72	6.70	1.60

Notes: $z_{1/2, \max} / \sqrt{A}$ computed from equation (3),

Series I tests from Carter et al., 1973.

For these tests, predicted values of θ_p / θ_o were calculated from equation (25) for comparison with measured values; Figure 35 shows these predicted values of θ_p / θ_o (ordinate) plotted versus the measured values of θ_p / θ_o (abscissa). Geometric means of the ratios of θ_p / θ_o (predicted) to θ_p / θ_o (measured) for each of these tests were calculated and are given in

Table 5.

Figure 35. Comparison of computed values of θ_p / θ_o (equation (25)) and measured values of θ_p / θ_o in the stable region when $z_{1/2, \text{max}} > D_w$.

TABLE 5. β , the mean ratio of predicted to measured values of θ_p/θ_o compared to estimates from one and two reflections.

Test	β	Single Reflection	Double Reflection
		$\theta_p/(\theta_p + \theta_{ref})$	$\theta_p/(\theta_p + 2\theta_{ref})$
M-I	0.896	0.947	0.899
O-I	0.827	0.942	0.890
Q-I	0.785	0.960	0.924
D-V	0.790	0.812	0.683
H-V	0.981	0.764	0.618

The increase in θ_p at $z < D_w$ due to bottom reflection, θ_{ref} , was given by equation (2) for the cosine shape function. At $z = 0$, equation (2) reduces to

$$\theta_{ref} = \frac{\theta_p}{2} \left(1 + \cos \frac{\pi}{2} \left(\frac{2D_w}{z_{1/2}} \right) \right) \quad (26)$$

Therefore, the ratio of unbiased to biased values of θ_p are $\theta_p/(\theta_p + \theta_{ref})$ for a single reflection from the bottom and $\theta_p/(\theta_p + 2\theta_{ref})$ for a first reflection from the bottom and a second reflection at the surface. These ratios are also listed in Table 5; there is no obvious relationship between the measured ratio, β , and the ratios computed from equation (26).

It is clear from Figure 35 that when $z_{1/2}/D_w$ exceeds 1 the surface temperature, θ_p , is consistently higher than that predicted by equation (25). J.E. Edinger (1974) has provided a possible theoretical explanation for this in some unpublished material which follows.

Edinger assumes that the momentum balance within the jet may be written as

$$Q_o q_o = Q(\xi) q(\xi) \quad (27)$$

where

$$Q_0 = q_0 b_0 h_0 \quad \text{and} \quad Q(\xi) = q(\xi) A_\xi .$$

Assuming that the area of the jet in the ηz plane, A_ξ , may be expressed as

$$\frac{A_\xi}{A} = \left(\frac{\xi}{\xi_0}\right)^{2n} \quad (28)$$

where ξ_0 is given by equation (19), equations (27) and (28) give

$$\frac{Q(\xi)}{Q_0} = \left(\frac{\xi}{\xi_0}\right)^n . \quad (29)$$

From equation (29), the rate of entrainment is therefore

$$\frac{dQ(\xi)}{d\xi} = \frac{nQ_0}{\xi} \left(\frac{\xi}{\xi_0}\right)^{n-1} . \quad (30)$$

At the orifice, $A_\xi = A$ and is rectangular; ultimately it is quasi-elliptical. Assuming that A_ξ can be replaced by a semicircular area of equivalent size but with radius r_ξ we have

$$2A_\xi = \pi r_\xi^2 . \quad (31)$$

The rate of entrainment per unit of perimeter of this semicircular area is therefore from (30) and (31)

$$\frac{1}{r_\xi} \frac{dQ(\xi)}{d\xi} = \frac{Q_0 n}{\xi r_\xi} \left(\frac{\xi}{\xi_0}\right)^{n-1} = \frac{Q_0 n}{\xi r_0} \quad (32)$$

as $r_\xi = r_0 \left(\frac{\xi}{\xi_0}\right)^n$ from equations (28) and (31). In terms of L , the perimeter of the jet, equation (32) is

$$\frac{dQ(\xi)}{d\xi} = \frac{L Q_o^n}{\pi \xi r_o} \quad (33)$$

When the semicircular jet intercepts the bottom, its geometry will be as shown in Figure 36, below.

Figure 36. Definition sketch looking into jet.

That is,

$$L = \pi r_\xi \left(1 - \frac{2}{\pi} \cos^{-1} \left(\frac{D_w}{r_\xi} \right) \right) \quad (34)$$

From equations (34) and (33) we have that

$$\frac{dQ(\xi)}{d\xi} = - \frac{D_w}{r_o} \frac{Q_o}{P^2} \left\{ 1 - \frac{2}{\pi} \cos^{-1} P \right\} \quad (35)$$

where

$$P \equiv \frac{D_w}{r_o} \left(\frac{\xi_o}{\xi} \right)^n \quad (36)$$

Equation (35) may be integrated to give

$$\frac{Q(\xi)}{Q_o} = \frac{D_w}{r_o} \left\{ \frac{1}{P} + \frac{2}{\pi} \left(\ln \frac{1 + \sqrt{1 - P^2}}{P} - \frac{\cos^{-1} P}{P} \right) \right\} \quad (37)$$

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