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for Slug-Type Reactor Fuels
TO: F. L. Culler
FROM: S. H. Jury

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1.0 Introduction

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In any discussion of the theory of dissolvers, mention must be made and account taken of the peculiarities of these operations. In this regard, the simplest dissolver is one charged with something like salt granules and operated under countercurrent conditions with water as solvent. The important point here is that the chemical complexities are avoided and attention may be focused on the progress of granule refinement which precedes in such a tower. Initially, the tower is charged with a certain number of grains or slugs of uniform size and packing density as well as surface area. As the run progresses more slugs are added to the tower to increase the total number contained therein since no slugs are ever withdrawn from the tower. Since the bulk volume of slug bed never changes, the slugs must become smaller during the run in order to accommodate the greater number. At the slug feed point, the slugs are of fixed size so that the refinement in slug size that does occur must do so at the lower levels in the tower. At the top of the dissolver bed the slug velocity must be high, gradually tapering to zero at the bottom. Thus the finest particles concentrate at the bottom. The unsteady state of a dissolver then is one of readjustment of the slug size distribution over the tower.

At the instant of startup, the fluid front passes over a bed which is at rest and of uniform size slugs. Since the area of slugs is at a minimum, the concentration of the first drop of effluent must also be a minimum. During the run, the bed velocities increase and the effluent concentration does likewise, reaching maxima at steady state. The implications are that if one wants to avoid recycle during startup the liquid velocity at startup must be reduced to the point where the initial drop comes out at design concentration and then gradually increases the liquid rate until at steady state one attains the design liquid feed rate.

In batch dissolvers, the steady state is not useful because at steady state, dissolving ceases. The unsteady state involves uniform refinement of slug size with steady increase of concentration of solution.

The packing density of slugs in a dissolver, i.e., the mols or pounds of solid per ft^3 of bed, is a quantity that arises in any theoretical analysis. The real question that arises is as to how this quantity varies with slug size. Theoretically close packed spheres of any size can be packed to about 25% void space. Dumped adsorbent packings normally contain 40-50% voids indicating that theoretical packing density is not attainable under these conditions. Laboratory data show that the packing density for a given slug feed is relatively independent of refinement and this assumption is made in all the analyses which follow.

In regard to rate mechanisms in a dissolver, one might envision two, the one involving solution at the solid surface and the other involving diffusion of dissolved material away from the interface.

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The former is determined by how tenaciously the metal atoms are bound together, this being less for Al than for Zr for example. The latter is a concentration phenomenon. Studies of the evaporation and condensation rates for pure metals and their vapors indicate that except for low reaction rates, the rate is directly proportional to the degree of supersaturation in the vapor phase or unsaturation, in our case. This assumption has also been made in the analyses which follow because one seldom has occasion to become involved in extremely acid-deficient solutions where the rates might become non-linear.

Speaking of solubility of Al, in something like HCl or HNO₃ solution, one is apt to become confused with the complicated chemistry involved and argue that because of hydrolysis, the acid is always regenerated and the solubility of Al in such a solution is indefinitely high. Actually, this is not so under operating conditions because the hot off-gases from the dissolver contain definite amounts of acid vapor or H₂ and nitrogen oxides. From the moles of these to the moles of Al dissolved, one can compute the solubility of the Al in the acid solvent fed initially, i.e., the amount of Al that would be dissolved by the time that the complete solvent power of the solvent had been destroyed by conversion to vapor or oxides and H₂. This is a useful concept which was first recognized by D. L. Foster in connection with his tests of experimental data. It eliminates the necessity for complicated mathematical expressions to cope with the chemistry involved and substitutes instead a principle which permits viewing a metal dissolver as though it was operating on salt and water with a normal solubility.

The gas problem, particularly in batch dissolvers, should be comparable to that of vapor generation in boiling heat transfer. This is to say that at low gas evolution rates the stirring effect of these gases should at first increase the transfer coefficient or reaction rate depending upon how one chooses to view the solution. At the higher evolution rates, where the interface-bulk concentration differences are greater, the beneficial effects of stirring should be overcome by the insulating effect of the gases to reaction. As the concentration difference increases then one might expect a maximum in reaction rate or mass transfer coefficient.

In continuous dissolvers, the liquid flows are of forced convection rather than free convection and the stirring effects of gases are much less noticeable. The effects of gas phase and liquid phase holdup are taken into account, however.

Another point in connection with dissolvers, particularly the trickle-type, is the question regarding "washed packing area" versus "wetted packing area". At any given instant, the liquid covers or washes a certain fraction of the total packing area. At start-up, the wetted area or portion of surface is the same as the washed area. Although the washed area for a given packing remains constant, with time the wetted area increases toward the maximum available area

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because the fluid streams shift position with time, eventually wetting all the packing. The action is comparable to wetting a floor with a hose, the floor being well drained. As one moves the hose around, more of the floor will be wetted but only a constant portion will be washed or flushed with water at a given instant. In a dissolver, it is logical to base reaction rates on washed area and that is the view adopted in this report.

Sometimes it is advantageous to spike the feed solvent with a soluble catalyst which accelerates the solubilizing action of the solvent but does not change the solubility. The view that has been adopted here is that catalysts increase the rate constant for solution at the interface, whereas only solvent-solid relative velocity can alter the diffusion rate constant.

3.0 List of Symbols for Batch Dissolvers

$a(t)$ = ft^2 slug area/ ft^3 of solution, i.e., total slug area/ V

a_0 = $a(0)$

$C(t)$ = moles solute/ ft^3 solvent, bulk concentration

$C^*(t)$ = moles solute/ ft^3 solvent, interface concentration

C_e^* = moles solute/ ft^3 solvent, equilibrium solubility

K = $\frac{1}{\frac{1}{k_R} + \frac{1}{k_s}}$ where $Ka(t)$ is in reciprocal seconds

k_R = reaction rate constant for solution at interface, $\frac{\text{ft}^3 \text{ solution}}{\text{sec-ft}^2 \text{ slug area}}$

k_s = diffusion rate constant, $\frac{\text{ft}^3 \text{ solution}}{\text{sec-ft}^2 \text{ slug area}}$

$L(t)$ = $l + 2r(t)$ = slug length, ft

l = $L(0) - 2r(0)$

M = molecular weight of slugs, $\frac{\text{lb mass}}{\text{lb mole}}$

N = number of slugs involved in one batch dissolving

$Q(t)$ = lb moles solid metal at time, t

$r(t)$ = radius of cylindrical slug at time, t , ft

t = time, sec

x = variable of integration

V = total volume of solution, ft^3

ρ = density of slug, lb/ft^3

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4.0 Theory of Batch Dissolving

In accord with previous discussions, the rate equation for solution at the interface should be:

$$\frac{dC}{dt} = k_R a(t) (C_e^* - C^*) \quad (1)$$

and that for diffusion away from interface should be:

$$\frac{dC}{dt} = k_g a(t) (C^* - C) \quad (2)$$

Since rates (1) and (2) must be the same, one can eliminate C^* and write

$$\frac{dC}{dt} = K a(t) (C_e^* - C) \quad (3)$$

If K varied with the rate of gas evolution, one might use the parabolic approximation for K , i.e.:

$$\frac{K - K_0}{K_{\max} - K_0} = 1 - \left[1 - \frac{C_e^* - C}{C_e^* - C_{\min}} \right]^2 \quad (4)$$

where pictorially

Now

$$- \int_{Q_0}^{Q(t)} \frac{dQ}{dt} = V \int_0^{C(t)} \frac{dC}{dt}$$

or

$$Q_0 - Q(t) = V C(t) \quad (5)$$

where

$$Q_0 = Q(0)$$

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Next one has that

$$N = \frac{Q(t) M}{\rho [1 + 2r(t)] \pi [r(t)]^2} \quad (6)$$

and

$$a(t) = \frac{2\pi N}{V} \left[r(t) [1 + 2r(t)] + [r(t)]^2 \right] \quad (7)$$

If the slugs are made longer than they are broad, which is usually the case, then

$$N \approx \frac{Q(t) M}{\rho l \pi [r(t)]^2} \quad (8)$$

and

$$a(t) \approx \frac{2\pi N l}{V} r(t) \quad (9)$$

or

$$a(t) \approx \frac{2\pi N l}{V} \sqrt{\frac{Q(t) M}{\rho \pi l N}} \quad (10)$$

or with equation 5

$$a(t) \approx \frac{2\pi N l}{V} \sqrt{\frac{M(Q_0 - VC(t))}{\rho \pi l N}} \quad (11)$$

If this result is used to eliminate $a(t)$ from equation 3, one can integrate for the case of constant K thus

$$\frac{1}{\sqrt{C_e^* V - Q_0}} \tan^{-1} \frac{-\sqrt{Q_0 - VC} + \sqrt{Q_0}}{\sqrt{C_e^* V - Q_0} \left[1 + \frac{\sqrt{Q_0 (Q_0 - VC)}}{C_e^* V - Q_0} \right]} = \frac{K}{V} \sqrt{\frac{M \pi l N}{\rho}} t \quad (12)$$

where

$$a(0) \approx \frac{2\pi N l}{V} \sqrt{\frac{M Q_0}{\rho \pi l N}}$$

$$C(0) = 0$$

An alternative form of 12 is

$$\begin{aligned} & \frac{1}{\sqrt{Q_0 - C_e^* V}} \ln \left[\frac{\sqrt{Q_0} - \sqrt{Q_0 - C_e^* V}}{\sqrt{Q_0} + \sqrt{Q_0 - C_e^* V}} \right] \left[\frac{\sqrt{Q_0 - CV} + \sqrt{Q_0 - C_e^* V}}{\sqrt{Q_0 - CV} - \sqrt{Q_0 - C_e^* V}} \right] \\ & = \frac{2Kt}{V} \sqrt{\frac{MN\pi l}{\rho}} t \end{aligned} \quad (12a)$$

Whether one uses 12 or 12a is determined by whether $\sqrt{C_e^* V - Q_0}$ is real or imaginary.

A plot of the arctangent function versus t should be linear and K may be determined from the slope.

If

$$Q_0 = C_e^* V$$

then

$$\frac{1}{\sqrt{1 - \frac{VC}{Q_0}}} = 1 + \frac{K}{V} \sqrt{\frac{M \pi l N Q_0}{\rho}} t \quad (13)$$

and a plot of the reciprocal square root function versus t should be linear and the evaluation of K is obvious.

If equation 4 governs, i.e., K varies with $(C_e^* - C)$, then this fact along with the other substitutions must be made in equation 3 to obtain

$$\int_{\sqrt{Q_0}}^{\sqrt{Q_0 - VC}} \frac{dx}{(a + x^2) [a' - (b - x^2)^2]} = - \frac{K_{max} - K_0}{(a + b)^2} \frac{N \pi l}{V} \sqrt{\frac{M}{\rho \pi l N}} t \quad (14)$$

where

$$x = \sqrt{Q_0 - VC}$$

$$a = C_e^* V - Q_0$$

$$a' = \frac{K_{max}}{K_{max} - K_0} (a + b)^2$$

$$b = Q_0 - C_{min} V$$

Expanding 14 into its partial fractions and integrating, one finds that

$$\frac{1}{[a' - (b + a)^2] \sqrt{a}} \left[\tan^{-1} \sqrt{\frac{Q_0 - VC}{a}} - \tan^{-1} \sqrt{\frac{Q_0}{a}} \right] - \frac{1}{2 \sqrt{a'} [\sqrt{a'} - (b + a)] \sqrt{a' - b}} \left[\tan^{-1} \sqrt{\frac{Q_0 - VC}{a' - b}} - \tan^{-1} \sqrt{\frac{Q_0}{a' - b}} \right]$$

$$+ \frac{1}{2\sqrt{a'} \left[\sqrt{a'} + (b+a) \right] \sqrt{a'+b}} \left[\tanh^{-1} \sqrt{\frac{Q_0 - VC}{a'+b}} \right. \\ \left. - \tanh^{-1} \sqrt{\frac{Q_0}{a'+b}} \right] = - \frac{K_{\max} - K_0}{(a+b)^2} \frac{N\pi l}{V} \sqrt{\frac{M}{\rho \pi l N}} t \quad (15)$$

Since

$$\tan^{-1} \theta - \tan^{-1} \phi = \tan^{-1} \frac{\theta - \phi}{1 + \theta\phi} \quad (16)$$

one can write 15 in the compact form characterizing equation 12 for the case of constant K.

It will be noted then that the final arctangent term of 15 is identical with that of 12. Hence, if a given set of experimental data is plotted as

$$\tan^{-1} \frac{\sqrt{Q_0} - \sqrt{Q_0 - VC}}{\sqrt{C_0^* V - Q_0} \left[1 + \frac{\sqrt{Q_0(Q_0 - VC)}}{C_0^* V - Q_0} \right]} \text{ versus } t$$

then the plot will be linear if 12 applies and will show curvature if 15 applies. In the latter case, the data would have to be re-evaluated using 15 in its entirety in order to determine the numerical value of the parameters involved.

In those cases where jacketed slugs are involved, the foregoing equations apply. The only difference in this application is the run data must be divided into two parts, that for the first part of the run applying to the jacket only and the latter portion applying to the core only. Either 12 or 15 is then applied to each of the two portions to obtain two sets of parameters, the one characterizing the jacket and the other the core. If a bonding layer is also included then a third portion of data must be analyzed separately for the parameters characterizing the bond.

If a catalyst is added to the solvent, then in the laboratory one should make several runs at various catalyst concentrations, then analyze each set of data and plot k_R or K as the case may be versus catalyst concentration. By so doing, one could see at a glance the optimum catalyst concentration for the design of large-scale batch dissolvers. Various catalysts could also be properly evaluated by this method.

One obvious point that might be overlooked in regard to equation 15 is that dissolvings of this type should be arranged so the $K_{\max}$ is attained and even exceeded so that K average for the run is as great as is economically feasible, all other matters taken into consideration.

A final point to consider is that D. L. Foster in some of his runs found that

$$C = \frac{C_e^* t}{b + t}$$

or

$$\frac{dC}{dt} = \frac{1}{C_e^* b} (C_e^* - C)^2$$

where b is inversely proportional to a (t).

5.0 List of Symbols for Cocurrent Dissolvers

a, b, a', b', c', a'', b''	= parameters defined in text where they are employed
a (s, t)	= ft ² slug surface area/ft ³ of tower
a _w (s, t)	= washed surface area of slugs/ft ³ of tower
A	= cross area of tower, ft ²
C (s, t)	= moles of metal/ft ³ solution (lb mole basis)
C* (s, t)	= moles solute/ft ³ solvent, interface concentration
C _e [*]	= moles solute/ft ³ solvent, equilibrium solubility
f	= friction factor, dimensionless
g	= acceleration due to gravity, ft/sec ²
g _c	= 32.1740 $\frac{\text{lb mass}}{\text{lb force}} \frac{\text{ft}}{\text{sec}^2}$
K, k _s , k _R , 1 L(s, t), r(s, t), t=	same definitions as for batch dissolvers
k	= proportionality constant
M _g	= pound mole wt of gas
M _s	= pound mole wt of slug
N (s, t)	= number of slugs/ft ³ of tower at s and t
O (s, t)	= slug linear velocity at s, t; ft/sec
P	= pressure at right angle to flow, consistent units
Q = $\frac{\alpha p s}{M_s}$	= moles of slugs/ft ³ of tower
t' = t - $\frac{A \rho Z}{V_L}$	

V_L	= liquid flow rate, ft^3/sec
$V_g (z, t)$	= gas flow rate, ft^3/sec
$U_L (z, t)$	= liquid linear velocity, ft/sec
$U_g (z, t)$	= gas linear velocity, ft/sec
X, Y	= variables of integration or as defined in text
z	= linear measure down from top of tower, ft
μ_{V-L}	= liquid viscosity, $\frac{\text{pound mass}}{\text{ft sec}}$
μ_{V-g}	= gas viscosity, $\frac{\text{pound mass}}{\text{ft sec}}$
α	= ft^3 of solid metal/ ft^3 of tower
β	= ft^3 of liquid/ ft^3 of tower, liquid phase hold-up
δ	= proportionality rate constant as defined in text
ϕ	= $\sin \theta$ where θ is angle between horizontal and direction of flow
ρ_g	= gas density which may vary with z and t , lb/ft^3
ρ_L	= liquid density, lb/ft^3
ρ_s	= solid density, lb/ft^3

Subscripts:

T = top of tower
B = bottom of tower

6.0 Theory of Cocurrent Dissolvers

The unsteady state solid phase material balance is

$$- \frac{\partial (A O (z, t) Q)}{\partial z} = K a_w (z, t) \beta A (C_e^* - C) \quad (17)$$

and for the liquid phase it is

$$A \beta \frac{\partial C}{\partial t} = K a_w (z, t) \beta A (C_e^* - C) - \frac{\partial C V_L}{\partial z} \quad (18)$$

The unsteady state balance for the gas phase is

$$\frac{A}{M_g} \frac{\partial (1 - \alpha - \beta) \rho_g}{\partial t} = - \frac{1}{M_g} \frac{\partial V_{gsg}}{\partial z} + K a_w (z, t) A \beta (C_e^* - C) \quad (19)$$

The continuity equation is

$$\frac{\partial N (z, t)}{\partial t} = - \frac{\partial O (z, t) N (z, t)}{\partial z} \quad (20)$$

Since only the steady state is considered in that which follows, one may write the corresponding steady state version of 17 as

$$- \frac{dAOQ}{dz} = K a_w \beta A (C_e^* - C) \quad (21)$$

and 18 as

$$\frac{d C V_L}{dz} = K a_w \beta A (C_e^* - C) \quad (22)$$

and 19 as

$$\frac{1}{Mg} \frac{d Vg \rho g}{dz} = k K a_w \beta A (C_e^* - C) \quad (23)$$

and for 20

$$\frac{d O N}{dz} = \text{zero} \quad (24)$$

Although as in batch dissolvers one still has for cylindrical slugs

$$a \approx 2 \sqrt{\pi \alpha l} \sqrt{N} \quad (25)$$

it remains to be shown how "a" is related to a_w and to do this, one must consider the hydrodynamics of flow down through the tower.

The flow pattern for solid liquid and gas phases varies from instant to instant and place to place so that at best one can only hope to consider average effects. To this end, let's consider the element dZ of tower and assume that the slugs, instead of being arranged in mosaic fashion, can be cut vertically in half and rearranged as follows:

Let's further idealize by stretching the two rows of ribs to two flat plates of the same surface area thus:

Experimentally, it is known that liquid tends to flow in ribbons as follows:

For simplicity's sake, one might envision all ribbons grouped as follows:

The total volume of the slot is

$$(1 - \alpha)AdZ$$

The interface area in dZ is

$$aAdZ$$

Thus the width of slot is

$$\frac{2(1 - \alpha)A dZ}{aAdZ} = 2 \frac{(1 - \alpha)}{a}$$

The volume of liquid in dZ is

$$BA dZ$$

Thus

$$a_w = \frac{2 \beta A dZ}{2 \frac{1 - \alpha}{a} A dZ}$$

or

$$a_w = \frac{\beta}{1 - \alpha} a \quad (26)$$

and this is the relationship desired as well as the proof for the general belief that washed area is proportional to liquid phase hold-up. The principle dimensions of the flows are:

In general, for each of the two flows, i.e., liquid and gas, one can write an energy balance having the form

$$\frac{1}{\rho} dP + \frac{v dv}{g_c} = - \left[\frac{f v_R^2}{2 g_c R_H} + \phi \right] dz \quad (27)$$

where

$$R_H = \frac{\text{cross area of the flow being considered}}{\text{periphery wetted by that flow}}$$

$$= \frac{2 \frac{1 - \alpha}{a} \frac{\beta}{1 - \alpha} \frac{aA}{2}}{2 \frac{\beta}{1 - \alpha} \frac{aA}{2}} \quad \text{for liquid based on solid}$$

$$= \frac{(1 - \frac{\beta}{1 - \alpha}) \frac{aA}{2} 2 \frac{1 - \alpha}{a}}{2(1 - \frac{\beta}{1 - \alpha}) \frac{aA}{2}} \quad \text{for gas based on solid}$$

$$= \frac{1 - \alpha}{a} \quad \text{in general}$$

v_R = average velocity of fluid under consideration relative to solid velocity, $0. (= v - 0)$

Also

$$v_L = \frac{V_L}{A\beta} \quad (28)$$

$$v_g = \frac{V_g}{A[1 - \alpha - \beta]} \quad (29)$$

If we knew the two friction factors, f , for the two fluid flows, then one could proceed with the simultaneous solution of equations 21, 22, 23, 24, 25, 26, the two equations 27, 28 and 29. Because of the non-linear nature of the problem, one would be wise if he would set up the electronic analog, better known as an analog computer, rather than attempting the analytical solution. In this regard, one would also want to take account of the fact that

$$[dP]_{\text{Liquid}} = [dP]_{\text{Gas}} \quad (30)$$

Since equations 27 involve drag law theory, which, in itself, is not completely rigorous, one might adopt the view that additional assumptions may be made so long as they are reasonable and then let the final result stand or fall based on how well it explains actual experimental results. To this end, equations 27 may be written and equated as in 30 to show that

$$\rho_L v_L \frac{dv_L}{dz} - \rho_g v_g \frac{dv_g}{dz} = \frac{1}{2R_H} [f_g \rho_g v_g^2 - f_L \rho_L v_L^2] \quad (31)$$

where 0 has been neglected because it is small relative to fluid velocities. Now if ρ_g and ρ_L are assumed constant and

$$f = f_g = f_L$$

and

$\frac{f}{R_H}$ is independent of variables involved in integration, then one may integrate 31 to show that

$$\frac{\rho_L v_L^2 - \rho_g v_g^2}{2} = \frac{f}{R_H} z$$

For large values of fz/R_H , this result predicts that

$$v_L = \sqrt{\frac{\rho_g}{\rho_L}} v_g \quad (33)$$

Oddly enough 33 is one of the assumptions involved in the drag law theory, i.e., that v_L and v_g are proportional in a two-phase system. Apparently, this is also true in our three-phase system. Result 33 would also be expected from the consideration that as two fluids become more intimately mixed, they tend to act as one fluid with a single velocity. Since at $z = 0$, v_L is not zero and v_g is zero, one might adopt the view that, in general, v_L and v_g are linearly related, i.e., that

$$v_L = a' + b' v_g \quad (34)$$

Then from 28, 29 and 34

$$\frac{v_L}{A\beta} = a' + b' \frac{Vg}{A[1 - \alpha - \beta]} \quad (35)$$

Also from 21 and 22

$$1 - \frac{O}{O_T} = \frac{CV_L}{AO_T Q} \quad (36)$$

and from 22 and 23

$$CV_L = \frac{Vg g}{KMG} \quad (37)$$

Integration of 24 leads to

$$\frac{O}{O_T} = \frac{N_T}{N} \quad (38)$$

At the bottom of the tower

$$C_B v_L = AO_T Q \quad (39)$$

and

$$\beta = \beta_B$$

At $z = 0$

$$Vg = 0$$

and

$$a' = \frac{v_L}{A\beta_T} \quad (40)$$

Substituting for Vg in 35 and subjecting the result to conditions 39 through 40 shows that

$$\frac{\beta_T}{\beta} = 1 + \left[\frac{\beta_T}{\beta_B} - 1 \right] \left[\frac{(1-\alpha) - \beta_B}{(1-\alpha) - \beta} \right] \left(1 - \frac{N_T}{N} \right) \quad (41)$$

This result may be rearranged to read

$$1 - \frac{N_T}{N} = -a' + b' \beta + c' \frac{1}{\beta} \quad (42)$$

$$= \frac{\beta_B}{\beta} \left[\frac{\beta_T - \beta}{\beta_T - \beta_B} \right] \left[\frac{(1 - \alpha) - \beta}{(1 - \alpha) - \beta_B} \right]$$

where now

$$a' = \frac{1 - \alpha + \beta_T}{\left(\frac{\beta_T}{\beta_B} - 1 \right) (1 - \alpha - \beta_B)}$$

$$b' = \frac{1}{\left(\frac{\beta_T}{\beta_B} - 1 \right) (1 - \alpha - \beta_B)}$$

$$c' = \frac{\beta_T (1 - \alpha)}{\left(\frac{\beta_T}{\beta_B} - 1 \right) (1 - \alpha - \beta_B)} \quad (43)$$

Pictorially, 42 predicts

where the equation for the dotted straight line is

$$\beta = \beta_B - \left[\beta_B - \beta_T \right] \frac{N_T}{N} \quad (44)$$

This equation is employed in that which follows because it appears to be a reasonable approximation to 42 and at the same time avoids an otherwise insuperable mathematical situation.

The next point to be considered before solution to the problem at hand is possible is that concerning the dependence of k_s on $\mathcal{U}_L$. In lieu of anything better we take

$$k_s = \gamma \mathcal{U}_L = \gamma \frac{V_L}{A\beta} \quad (45)$$

because the literature suggests that for liquid diffusions k_s and $\mathcal{U}_L$ are linearly dependent.

The solution to the problem involves using equations 45, 44, 38, 36, 26 and 25 to convert equation 22 to an integrable form. The solution is

$$\begin{aligned} & - \frac{\sqrt{C} (a + c)}{(b + c)^2} \left[\tan^{-1} \sqrt{\frac{NT/N}{C}} - \tan^{-1} \sqrt{\frac{1}{C}} \right] \\ & + \frac{1}{2\sqrt{b}} \left[\frac{(c - b)(a + c)}{(b + c)^2} + 1 \right] \left[\tanh^{-1} \sqrt{\frac{1}{b}} - \tanh^{-1} \sqrt{\frac{NT/N}{b}} \right] \\ & + \frac{1}{2} \left[\frac{a + c}{b + c} + 1 \right] \left[\frac{\sqrt{NT/N}}{b - NT/N} - \frac{1}{b - 1} \right] = \frac{\sqrt{\pi \alpha 1 NT (\beta_B - \beta_T)} z}{1 - \alpha} \end{aligned} \quad (46)$$

where

$$a = \frac{\frac{V_L}{Ak_R} + \beta_B}{\beta_B - \beta_T}$$

$$b = \frac{\beta_B}{\beta_B - \beta_T}$$

$$c = \frac{Ce^* V_L}{AO_T Q} - 1$$

At the bottom of the dissolver

$$NT/N = 0$$

and 46 can be simplified accordingly. For the special case wherein β is constant down through the tower, the solution becomes

$$\begin{aligned} & 1 - \sqrt{\frac{N_T}{N}} - \sqrt{\frac{Ce^* V_L}{AO_T Q} - 1} \left[\tan^{-1} \sqrt{\frac{1}{\frac{Ce^* V_L}{AO_T Q} - 1}} - \tan^{-1} \sqrt{\frac{NT/N}{\frac{Ce^* V_L}{AO_T Q} - 1}} \right] \\ & = \frac{K\beta^2 \sqrt{\pi \alpha 1 N_T}}{(1 - \alpha) V_L} z \end{aligned} \quad (47)$$

This solution also has additional simplicity at the bottom, i.e., where

$$NT/N = 0$$

Obviously in the analysis of any data one would try equation 47 first and then 46 only if 47 failed to give an adequate fit of the facts. D. L. Foster has already done considerable work with 47 and has shown that in many of his runs 47 gives a good representation of the facts at hand.

When a cocurrent dissolver is first set into operation with a fresh charge of slugs, the first drop of fluid to pass down over the slugs of uniform size should have a concentration

$$\frac{C_{e*} - C}{C_{e*}} = e^{-\frac{K\alpha\beta^2 AZ}{V_L(1-\alpha)}} \quad (48)$$

In other words, equations 21 and 26 can be solved directly for this special case.

In connection with equations involving arctan and arctanh forms, it is wise to remember that

$$\int \frac{dx}{b - x^2} = - \int \frac{dx}{-b + x^2} = - \frac{1}{\sqrt{-b}} \tan^{-1} \frac{x\sqrt{-b}}{-b} = \frac{1}{\sqrt{b}} \tanh^{-1} \frac{x\sqrt{b}}{b}$$

in the event that the argument involved becomes negative.

SHJ:mcj

S. H. Jury

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