Corrections to the Mass and Width of a Resonance*

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Abstract

A resonant amplitude is often parametrized with a Breit-Wigner having an energy-dependent width. This energy dependence usually incorporates the fact that the amplitude behaves at threshold like the momentum raised to the $2l+1$ power, where l is the orbital angular momentum of the resonance. In this case, the mass and width of the resonance, defined in terms of the position of the pole of the amplitude, are smaller than the "mass" and "width" parameters entering the phenomenological Breit-Wigner expression. The lowest-order correction to the mass goes as the width squared, while the lowest-order correction to the width goes as the width cubed, and the corrections act to reduce the mass and width of the resonance. Applications to the $\Lambda(1232)$, $\Sigma^*(1383)$, and $\Xi^*(1530)$ are given.

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I. Introduction

A prominent feature in the scattering of one hadron by another is the appearance of resonant peaks in the cross section. The amplitude describing such a scattering is often parametrized by a Breit-Wigner form A_{BW} in the resonant partial wave, given by

$$A_{BW}(E) = \frac{1}{2} x_0 \Gamma_0 / (M_0 - E - i\Gamma_0/2) \quad (1)$$

where M_0 is the mass of the resonance, Γ_0 is the width and x_0 is the elasticity, and we take $\hbar = c = 1$. The partial wave amplitude A_{BW} has a pole at the complex energy E_0 given by

$$E_0 = M_0 - i\Gamma_0/2 \quad (2)$$

Thus, the mass is the real part of E_0 and the width is minus twice the imaginary part of E_0 .

In the neighborhood of the pole, the Breit-Wigner form is a good approximation to the amplitude $A(E)$. However, unless the width of the resonance is very small, the approximation may be poor on the real energy axis where experiments are done. In particular, at energies just above threshold, A is expected to behave as (in the case of elastic scattering)

$$A(E) \underset{k \rightarrow 0}{\sim} k^{2\ell+1} \quad (3)$$

where k is the momentum of either particle in the c.m. system. On the contrary, the Breit-Wigner expression of Eq. (1) goes to a constant at threshold. This is an important consideration for several observed resonances, such as the well-known $\Delta(1232)$, as an amplitude without the correct threshold dependence does not fit the precise data of Carter et al.¹

To overcome this difficulty, it is customary to let the "width" of the resonance depend on energy. We thus write for the resonant amplitude $A(E)$

$$A(E) = \frac{1}{2} x \Gamma G(E) / [M - E - i\Gamma G(E)/2] \quad (4)$$

where x , Γ , and M are real parameters and $G(E)$ is a real function of E normalized so that

$$G(M) = 1. \quad (5)$$

The parameters M and Γ should not now be regarded as the mass and width of the resonance, because the mass and width are properly defined in terms of the position of the pole of A in the complex energy plane.²

We can obtain the pole position by solving the equation

$$M - E_0 - i\Gamma G(E_0)/2 = 0 \quad (6)$$

for E_0 . We define the "true" mass and width of the resonance in terms of this solution.

II. Approximate Formula for the Mass and Width

We can obtain an approximate solution to Eq. (6) by assuming that E_0 is not very far from M . (This is equivalent to the assumption that the width of the resonance is not very large.) We then expand $G(E_0)$ about M , obtaining

$$G(E_0) = G(M) + G_1(E_0 - M) + G_2(E_0 - M)^2 + \dots \quad (7)$$

where

$$G_n = \frac{1}{n!} (d^n G(E)/dM^n)_{E=M}, \quad n = 1, 2, \dots \quad (8)$$

We substitute Eq. (7) in Eq. (6) and use the definitions in Eqs. (2) and (5). Then, keeping terms only of order Γ^3 or lower,

$$M_0 = M - G_1 \Gamma^2 / 4 \quad (9)$$

$$\Gamma_0 = \Gamma - (G_1^2 + G_2) \Gamma^3 / 4 \quad (10)$$

We see that the leading correction to the mass goes as Γ^2 while the leading correction to the width goes as Γ^3 . Because of the rapid variation with Γ , the corrections can be very important for hadron resonances of width of the order of 100 MeV, while the corrections become negligible for resonances of width below about 10 MeV.

In order to go further, we need to say something about the behavior of the function $G(E)$. The simplest such function which incorporates the threshold behavior of Eq. (3) and the condition of Eq. (5) is

$$G(E) = (k/k_M)^{2\ell+1} \quad (11)$$

where k_M is the value of the momentum when $E = M$. With this form, we can verify that

$$G_1 > 0, G_1^2 + G_2 > 0. \quad (12)$$

It then follows from Eqs. (9) and (10) that the true mass and width are smaller than the parameters M and Γ respectively, provided Γ is sufficiently small that terms of order Γ^4 and higher can be neglected.

The function in Eq. (11), although simple, has the defect that if it is substituted into Eq. (4), the high-energy behavior of the resonant amplitude is

$$A(E) \underset{E \rightarrow \infty}{\sim} \text{constant}. \quad (13)$$

Thus, there is a finite contribution to the amplitude at high energy from the low-energy resonance. On the contrary, a Breit-Wigner amplitude behaves like

$$A(E) \underset{E \rightarrow \infty}{\sim} E^{-1}. \quad (14)$$

and becomes negligible at high energy.

On physical grounds, it is unreasonable for the high-energy behavior of the amplitude to be affected by a term arising from a low-energy resonance. We can avoid this if we require that the resonant amplitude increase like E^{-1} at high E just like a Breit-Wigner amplitude. To insure this, we require that $G(E)$ go to a constant at high energy. An example of a function which has the desired threshold and asymptotic properties is ³

$$G(E) = (k_M/k_M E)^{2\ell+1} \quad (15)$$

Another possibility is to include one or more adjustable parameters in $G(E)$. An example is ⁴

$$G(E) = \frac{M}{E} \left(\frac{k}{k_M} \right)^{2\ell+1} \left(\frac{k_M^2 + q^2}{k^2 + q^2} \right)^\ell \quad (16)$$

where q is a parameter. Still other forms for $G(E)$ can readily be written down.

We cannot of course prove that the inequalities (12) remain true independent of the form of $G(E)$. However, we have considered a number of functions leading to resonant amplitudes with the desired threshold and asymptotic properties. We have found that the inequalities (12) hold in applications to the observed hadron resonances.

Often a Breit-Wigner amplitude is not parametrized in the form given in Eq. (1), but by a quadratic form A_{QBW} given by ⁵

$$A_{QBW}(E) = xM\Gamma/(M^2 - E^2 - iM\Gamma) \quad (17)$$

(This form is a Breit-Wigner in $s = E^2$.) In this case the pole position is given by the solution to the equation

$$M^2 - E_0^2 - iM\Gamma = 0 \quad (18)$$

Using Eq. (2), we can obtain the true mass and width. Keeping corrections to second order in Γ , we get

$$M_0 = M \left(1 + \frac{1}{8} \frac{\Gamma^2}{M^2} \right) \quad (19)$$

$$\Gamma_0 = \Gamma \left(1 - \frac{1}{8} \frac{\Gamma^2}{M^2} \right) \quad (20)$$

For typical broad resonances listed in the tables of the Particle Data Group⁶, the corrections to the mass and width arising from Eqs. (19) and (20) are well within the present experimental errors. It follows that, for the purpose of obtaining the masses and widths of the meson and baryon resonances from present experiments, it is immaterial whether a linear or quadratic Breit-Wigner amplitude is used.

If a resonance is observed in a production experiment rather than a formation experiment, the approximate amplitude is no longer given by Eq. (4). Similarly, background effects lead to additional modifications of the amplitude. However, none of these effects should change the position of the pole of the amplitude in the complex energy plane.

We conclude that if a resonance is parametrized with an energy-dependent width incorporating the correct threshold behavior, its true mass and width are smaller than the "mass" and "width" parameters M and Γ in the phenomenological amplitude.

We have examined higher-order corrections to Eq. (9) for the mass of a resonance and Eq. (10) for the width. The next-order mass correction goes as Γ^4 , while the next-order width correction goes as Γ^5 . We have found that, in cases of interest, these higher-order corrections to the mass and width are of opposite sign to the lowest-order corrections. This means that Eqs. (9) and (10) underestimate the true mass and width respectively.

We thus have an estimate of both an upper and a lower bound on the mass and width of a resonance. The parameters M and Γ are an estimate of the upper bound, while the values of M_0 and Γ_0 , calculated from Eqs. (9) and (10), constitute an estimate of the lower bound. These bounds are not completely general, because the actual bounds depend on the form of $G(E)$. Our statement on bounds is a practical one, which should hold in applications to meson and baryon resonances with reasonable functions $G(E)$.

III. Applications

As an example of the shift in pole position, consider the $\Delta(1232) \pi^+ p$ resonance. The 1974 tables of the Particle Data Group⁶ list the mass and width parameters as

$$1230 \leq M(\Delta^{++}) \leq 1236 \text{ MeV}, \quad 110 \leq \Gamma(\Delta^{++}) \leq 122 \text{ MeV}. \quad (21)$$

The range of values correspond to different choices of $A(E)$ and $G(E)$.

On the other hand, the values of the mass and width defined in terms of the pole position are^{6,7}

$$M_0(\Delta^{++}) = 1212 \pm 1 \text{ MeV}, \quad \Gamma_0(\Delta^{++}) = 99 \pm 3 \text{ MeV} \quad (22)$$

in agreement with our result that these values should be smaller than the values of the parameters in the modified Breit-Wigner expression for the amplitude. We advocate calling this resonance the $\Delta(1212)$.

This shift of the mass and width can also account for much of the discrepancy between the mass and width values of resonances reported by different groups. For example, consider the $\Sigma(1383)$ resonance. Borenstein et al.⁸ fit their data to a Breit-Wigner with a constant width and find

$$M_0(\Sigma^{*+}) = 1381 \pm 1 \text{ MeV}, \quad \Gamma_0(\Sigma^{*+}) = 34 \pm 2 \text{ MeV} \quad (23)$$

$$M_0(\Sigma^{*-}) = 1383 \pm 2 \text{ MeV}, \quad \Gamma_0(\Sigma^{*-}) = 35 \pm 3 \text{ MeV}. \quad (24)$$

On the other hand, Baltay et al. and Habibi⁹, using an energy-dependent width, obtain

$$M(\Sigma^{*+}) = 1383 \pm 1 \text{ MeV}, \Gamma(\Sigma^{*+}) = 38 \pm 3 \text{ MeV} \quad (25)$$

$$M(\Sigma^{*-}) = 1391 \pm 1 \text{ MeV}, \Gamma(\Sigma^{*-}) = 52 \pm 3 \text{ MeV}. \quad (26)$$

A good part of the difference between these results lies in the analysis. From the information given by Baltay et al. and Habibi⁹, we can numerically compute the pole positions of the $\Sigma^{*\pm}$ amplitudes. We find

$$M_0(\Sigma^{*+}) = 1379 \pm 1 \text{ MeV}, \Gamma_0(\Sigma^{*+}) = 35 \pm 3 \text{ MeV} \quad (27)$$

$$M_0(\Sigma^{*-}) = 1383 \pm 1 \text{ MeV}, \Gamma_0(\Sigma^{*-}) = 45 \pm 3 \text{ MeV}. \quad (28)$$

As expected, the masses and widths are lower than those given by Baltay et al. and Habibi. The overall agreement with the results of Borenstein et al. [Eqs. (23) and (24)] is considerably improved, although a discrepancy still remains in the width of the Σ^{*-} .

We can apply the same arguments to the $\Xi^{*0}(1532)$ and $\Xi^{*-}(1535)$ which have widths of about 10 MeV. We find that the masses are reduced about $\frac{1}{2}$ MeV and the widths only about .1 MeV, in agreement with our conclusion that the corrections are negligible for narrow resonances.

IV. Conclusions

The results of our examples lead us to believe that several resonances have smaller uncertainties in their masses and widths than those given by the Particle Data Group⁶. The uncertainties listed in the tables of the Particle Data Group arise in part from experimental errors and in part from the fact that different authors analyze their data in different ways. The mass and width of a resonance has a unique definition in terms of the position of the pole in the amplitude.² This pole position will usually be fairly stable, that is, relatively independent of the particular way the amplitude is parametrized.⁷ For parametrizations in which threshold effects are important, the true mass and width will be smaller than the corresponding "mass" and "width" parameters used in phenomenological analyses. The correction to the mass is of order Γ^2 , while the correction to the width is of order Γ^3 .

We suggest the following procedures for analyzing data: If the data can be fit equally well with a constant width and an energy-dependent width, then the constant width is preferred. If the data are sufficiently precise that an energy-dependent width is necessary, then Eq. (6) should be solved to obtain the true mass and width in terms of the pole position. Alternatively, the data may be fit with an amplitude which incorporates the correct threshold dependence and yet explicitly singles out the pole term¹⁰.

We believe that a systematic analysis of the present data according to these ideas will yield improved values of the true masses and widths of resonances, independent of the parametrization. After such analyses, we should be better able to tell how much of the present disagreements

between different experiments arises from experimental error and how much from different interpretations of the data.

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