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TILAK AGERWALA

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THE JOHNS HOPKINS UNIVERSITY

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An Analysis of Controlling Agents for Asynchronous Processes*

Tilak Agerwala
Electrical Engineering Dept.
The Johns Hopkins University
Baltimore, Maryland 21218

MASTER

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AN ANALYSIS OF CONTROLLING AGENTS FOR ASYNCHRONOUS PROCESSES

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1. Introduction

The purpose of this report is to study different models of parallel computation and to analyze the capabilities and limitations of the controlling agents in these models. We are interested in determining, for each such controlling agent, whether it can represent all possible types of interactions between a group of processes. If the answer is negative we will try and determine exactly how powerful the model is. Our main interest is the study of control flow and its representation. The report will thus attempt to classify, according to capabilities, a large number of existing models. The approach will be systematic rather than ad hoc. We will define nine different classes of nets. Each class of nets represents a particular kind of control and is basically an ordinary Petri net with slight modifications. Many of these classes have been introduced and studied earlier, [2, 3, 13, 20], but we present those results here for the sake of completeness. We will establish a hierarchy of these nets based on their capability of representing interaction between processes; at one end are nets that are equivalent to finite state machines and at the other end, we have nets that can simulate arbitrary Turing machines. Establishing this hierarchy of nets will systematize our study of different models since very often, the control part of a particular model will be in one to one correspondence with one of the nine classes. Numerous models of parallel computation have been proposed and studied. Usually such a study has been carried out independent of other models. This paper is a first attempt at comparing these models formally and classifying them according to their modelling power.* It is thus a step in the direction of establishing a uniform, underlying theory for the study of asynchronous systems.

*See important note page 84.

In a previous paper [2] we introduced the extended Petri net model and showed that it was complete [3], i.e., it could represent all possible kinds of interaction between a group of processes and thus provided a correct, formal counterpart to the vague notion of "coordination". The current report establishes the significance of the extended Petri net model by demonstrating that most existing models of parallel computation are incomplete, and the extended Petri net model is one of the very few models that is of practical significance and not introduced for theoretical definitions alone.

Basically, any model for studying parallel systems consists of two parts:

- (1) A control mechanism, C , which decides whether a particular process is to be enabled or not.
- (2) A set of asynchronous processes $PR = \{P_1, P_2, \dots, P_n\}$ that are to be "coordinated".

If C and PR are the only two entities specified, we have an uninterpreted model. In general, each process performs some computation. While studying the representation of coordinations, we are not concerned with the computation performed by the system as a whole, or that performed by each individual process. If the computation is of interest, additional specification must be provided. Given an uninterpreted model, if

- (1) A set of storage variables $M = \{m_i \mid 1 \leq i \leq k\}$ is defined.
- (2) For each process P_i , a set of input and output data sets is specified, where each data set is a subset of M .

Then the model is said to be semi-interpreted. Given a semi-interpreted model, if one defines

- (1) The initial values of variables in M .
- (2) A set of functions, one for each process P_i , which takes the values of the variables in the input data set, performs some transformations on these values,

and modifies the values of the output data set of P_i .

Then an interpreted model results. We are interested only in uninterpreted models since such a model is an abstraction of a real system that allows one to focus attention on the flow of control. Each of the nine classes of nets studied in the next section is an uninterpreted model. When we refer to an existing model (say X), we refer to the model that results when X is stripped off all interpretation.

2. The Nine Classes of Nets

In what follows, we will formally define the nine classes. We assume that the reader is thoroughly familiar with ordinary Petri nets. The definitions serve only to formalize intuitive notions, so that there is no ambiguity.

2.1 Extended Petri Net Model

Definition 2.1.1: The extended Petri net model is a directed graph defined as a six-tuple (T, P, A, M^0, F, f_e) where:

$T = \{t_i \mid 1 \leq i \leq n\}$ is a finite set of "transitions".

$P = \{p_i \mid 1 \leq i \leq m\}$ is a finite set of "places".

$A \subseteq \{(p_i, t_j) \mid p_i \in P, t_j \in T\} \cup \{(t_i, p_j) \mid t_i \in T, p_j \in P\} \ni (p_i, t_j) \in A \Rightarrow (t_j, p_i) \notin A$ and $(t_i, p_j) \in A \Rightarrow (p_j, t_i) \notin A$, is a finite set of directed arcs.

Each arc connects a transition to a place or a place to a transition.

$M^0: P \rightarrow N$ (N is the set of non-negative integers) is a function that gives the number of "markers" in each place initially. M^0 can be represented as

$\{(p_1, n_1) \dots (p_m, n_m)\}$ or where there is no ambiguity, as $(n_1, \dots, n_m)$.

$F = \{F_i \mid 1 \leq i \leq p\}$ is a set of final markings where $F_i: P \rightarrow N \cup \{d\}$. (d denotes "don't care")

$f_e: A \rightarrow \{\text{True}, \text{False}\}$ is a predicate on the arcs.

T, P form the nodes of the graph. The transitions represent processes and the places, conditions.

Definition 2.1.2: $I_1 = \{j \mid (p_j, t_1) \in A\}$ is the set of indices of the input places of t_1 .

Definition 2.1.3: $O_1 = \{j \mid (t_1, p_j) \in A\}$ is the set of indices of the output places of t_1 .

Definition 2.1.4: $I_1' = \{j \mid (p_j, t_1) \in A \text{ and } f_e((p_j, t_1)) = \text{True}\}$ denotes a subset of the input places of t_1 for which f_e is true.

Definition 2.1.5: $I_1'' = \{j \mid (p_j, t_1) \in A \text{ and } f_e((p_j, t_1)) = \text{False}\}$ denotes a subset of the input places of t_1 for which f_e is false.

Definition 2.1.6: P_1 = number of markers in p_1 at any instant.

Simulation Rules:

Transitions represent events or processes. The firing of a transition represents the occurrence of an event. Whenever a transition t_1 desires to fire, it checks its input places and is enabled if and only if $P_j > 0$, for all $j \in I_1'$ and $P_j = 0$, for all $j \in I_1''$. At this stage, a marker is removed from each $p_j, j \in I_1'$ and the transition t_1 starts firing. At some later, finite, though a priori unknown and unbounded time, it completes firing and places a marker in each output place. Firing of a transition thus causes a change of state of the net and other transitions may be enabled or disabled as a result of this firing. In the actual graph, the number of markers in any place is shown by a corresponding number of "dots". The movement of markers clearly shows which conditions cause a transition to fire and also which conditions come into being at the completion of firing.

The notion of conflict is important. During a simulation, two transitions are said to be in conflict if enabling of one prevents the enabling of the other and visa versa. In such a case the decision as to which one will be enabled is completely arbitrary.

Describing the Coordination Represented by a Net in this Model:

There are two ways of doing this. For each of these methods we require the notion of "trans-labelling".

Definition 2.1.7: A trans-labelling I of a net $N = (T, P, A, M^0, F, f)$ is a mapping from T' onto E where

$T' = \{\ell_1, \dots, \ell_a\} \subseteq T$ is a set of transitions.

$E = \{E_1, E_2, \dots, E_k\}$, $k \leq a$ is a set of event or process names.

Thus, the same event or process name may be attached to different transitions and transitions in $T - T'$ have no associated names. This is indicated by $I(t) = \emptyset$ for $t \in T - T'$. It should be clear that the same net may represent different coordinations depending on its trans-labelling. The two ways of specifying the coordination represented by a net are:

- (1) By defining T, P, A, M^0, F, f and giving a translabelling.
- (2) By considering the "process sequences" produced by the net:

Definition 2.1.8: A marking M is covered by some F_i , $1 \leq i \leq p$ if $M(p) = F_i(p)$, for all $p \ni F_i(p) \in N$.

Definition 2.1.9: $\bar{t} = \ell_1, \ell_2, \dots, \ell_n \in T^*$ is said to be produced by a net $N = (T, P, A, M^0, F, f)$ during a simulation if $\exists$ a sequence of markings $M^0, M^1, \dots, M^n \ni \ell_i$ can be enabled in M^{i-1} and firing of ℓ_i in M^{i-1} results in M^i , $1 \leq i \leq n$. In addition, if M^n is covered by some $F_i \in F$, $\bar{t}$ is said to be a simulation sequence of N .

Definition 2.1.10: The set of all simulation sequences of M is called the simulation set of N or SIMSET_N.

Definition 2.1.11: Given a trans-labelling $I(T', E)$ of a net $N = (T, P, A, M^0, F, f_e)$, for each simulation sequence $\bar{t} = \ell_1, \ell_2, \dots, \ell_n$ we define a reduced simulation sequence $\bar{t}'$ where $\bar{t}'$ is obtained from $\bar{t}$ by deleting transitions in $T - T'$.

Definition 2.1.12: Given a trans-labelling $I (T', E)$ of a net $N = (T, P, A, M^0, F, f_e)$, for each reduced simulation sequence $\bar{t}' = \ell_1, \dots, \ell_a$ we define a process sequence $I(\bar{t}') = E_{c_1}, E_{c_2}, \dots, E_{c_m}$ where $I(\ell_i) = E_{c_i}, 1 \leq i \leq a$.

Definition 2.1.13: The set of all process sequences of N given $I (T', E)$ is called $I (SIMSET_N)$.

The coordination represented by a given net N and trans-labelling I is completely specified by $I (SIMSET_N)^*$.

Let us call the class of extended Petri nets TN_{EXT} . In the following four classes, the simulation rules are identical with those of extended Petri nets and definitions 2.1.2 through 2.1.14 carry over without change.

2.2 Ordinary Petri Net Model

Definition 2.2.1: The ordinary Petri net model is directed graph defined as a six-tuple (T, P, A, M^0, F, f_e) where $(T, P, A, M^0, F, f_e) \in TN_{EXT}$ and in addition, $f_e(a) = T$, for all $a \in A$.

Let the class of ordinary Petri nets be TN .

2.3 Bounded Petri Net Model

Definition 2.3.1: The bounded Petri net model is a directed graph defined as a six-tuple (T, P, A, M^0, F, f_e) where $(T, P, A, M^0, F, f_e) \in TN$ and in addition for each net N in this class $\exists$ an a priori known bound $R \geq$

$$(1) \quad M^0: P \rightarrow \{0, 1, \dots, R\}$$

$$(2) \quad F_i \in F \Rightarrow F_i: P \rightarrow \{0, 1, 2, \dots, R\} \cup \{d\}$$

(d denotes "don't care")

(3) There is no sequence $\bar{t} = \ell_1, \ell_2, \dots, \ell_n$ produced by $N \ni M^{n+1}(p) > R$ for any place p in N .

Let the class of bounded Petri nets be TN_{BDD} .

2.4 Safe Petri Net Model

Definition 2.4.1: The safe Petri net model is a directed graph defined as a six-tuple (T, P, A, M^0, F, f_e) where $(T, P, A, M^0, F, f_e) \in TN_{BDD}$ and $R = 1$.

*See note page 83.

Let the class of safe Petri nets be TN_{safe} .

2.5 The Marked Graph Model

Definition 2.5.1: The marked graph model is a directed graph defined as a six-tuple (T, P, A, M^0, F, f_e) where $(T, P, A, M^0, F, f_e) \in TN$ and in addition:

- (1) for all $p_i \in P$, $(t_j, p_i) \in A$ and $(t_k, p_i) \in A \Rightarrow j = k$.
- (2) for all $p_i \in P$, $(p_i, t_j) \in A$ and $(p_i, t_k) \in A \Rightarrow j = k$.

Thus, each place has at most one input transition and one output transition.

Such a net can have no conflicts. Let the class of marked graphs be TN_{MARK} .

2.6 The Complex Petri Net Model

Definition 2.6.1: The complex Petri net model is a directed graph defined as a six-tuple (T, P, A, M^0, F, f_x) where T, P, A, M^0, F are the same as in definition 2.1.1 and $f_x: A \rightarrow N$.

Let I_1 , O_1 and P_1 be as in definitions 2.1.2, 2.1.3, and 2.1.6 respectively.

Simulation Rules:

Whenever a transition t_1 desires to fire, it checks its input places.

If $P_j \geq f_x((p_j, t_1))$ for all $j \in I_1$, then the transition is enabled. At this stage it removes $f_x((p_j, t_1))$ markers from each input place p_j . On completion of firing, t_1 places $f_x((t_1, p_j))$ markers in each output place p_j .

Definitions 2.1.7 through 2.1.14 carry over unchanged. Let the class of complex Petri nets be TN_{PLEX} .

2.7 The OR Input Petri Net Model

Definition 2.7.1: The OR input Petri net model is a directed graph defined as a six-tuple $(T, P, A, M^0, F, f_{XORI})$ where T, P, A, M^0, F are the same as in definition 2.1.1, and $f_{XORI}: A_1 \rightarrow (+, *)$, where $A_1 \subset A$ and consists of all arcs of the form (p_j, t_1) .

Let I_1 , O_1 and P_1 be the same as in definitions 2.1.2, 2.1.3, and 2.1.6 respectively.

Definition 2.7.2: $I_1^* = \{j | j \in I_1 \text{ and } f_{XORI}((p_j, t_1)) = *\}$.

Definition 2.7.3: I_1^+ is the set difference of the sets I_1 and I_1^* .

Simulation Rules:

Whenever a transition t_1 desires to fire it checks its input places. If $P_j > 0$ for all $j \in I_1^*$ and $P_k > 0$ for at least one $k \in I_1^+$, the transition is enabled. At this stage it removes one marker from each $P_j \ni j \in I_1^*$ and one marker from the place p_k , $k \in I_1^+$ which satisfied the check. When it completes firing, it places a marker in each output place.

Definitions 2.1.7 through 2.1.13 carry over unchanged. Let the class of nets be TN_{XORI} .

2.8 The Exclusive OR Output Petri Net Model:

Definition 2.8.1: The exclusive OR output Petri net model is a directed graph defined as a six-tuple $(T, P, A, M^0, F, f_{XORO})$ where T, P, A, M^0, F are the same as in definition 2.1.1 and $f_{XORO}: A_2 \rightarrow (*, +)$ where $A_2 \subset A$ and contains all arc of the form (t_1, p_j) .

Let I_1, O_1 and P_1 be the same as in definitions 2.1.2, 2.1.3 and 2.1.6 respectively.

Definition 2.8.2: $O_1^* = \{j | j \in O_1 \text{ and } f_{XORO}(t_1, p_j) = *\}$.

Definition 2.8.3: O_1^+ is the set difference of the sets O_1 and O_1^* .

Simulation rules:

Whenever a transition desires to fire, it checks its input places and is enabled if $P_i > 0$, for all $i \in I_1$. At this stage it removes a marker from each input place and starts firing. On completion of firing it places a marker in each $p_j \ni j \in O_1^*$ and places one marker arbitrarily in one of the places $p_k \in O_1^+$.

Definitions 2.1.7-2.1.14 carry over unchanged. Let the class of nets be TN_{XORO} .

2.9 The Place Restricted Petri Net Model

Definition 2.9.1: The place restricted Petri net model is a directed graph defined as a six tuple $(T, P, A, M^0, F, f_{OUT})$ where T, P, A, M^0, F are the

same as in definitions 2.1.1 and $f_{OUT}: P \rightarrow \{True, False\}$.

Let I_1 , O_1 and P_1 be the same as in definitions 2.1.2, 2.1.3 and 2.1.6.

Simulation rules:

When a transition desires to fire, it checks its input places and is enabled if $P_i > 0$, for all $i \in I_1$. At this stage it removes one marker from each input place. On completion of firing, it places a marker in each output place $p \ni f_{OUT}(p) = True$. If t_1 has an output place $p \ni f_{OUT}(p) = False$, then a marker is placed in p if and only if $P = 0$.

Definitions 2.1.7 through 2.1.13 carry over unchanged. Let the class of nets be TN_{OUT} .

2.10 Summary

TN_{EXT}

$$f(p_1, t_1) = f(t_1, p_3) = T$$

$f(p_2, t_1) = F$. This is indicated by a slash across (p_2, t_1) . t_1 can be enabled if $P_1 > 0$ and $P_2 = 0$

TN

$f(p_1, t_1) = f(p_2, t_1) = f(t_1, p_3) = T$. t_1 can be enabled if $P_1 > 0$ and $P_2 > 0$.

TN_{bdd}

same as TN except there is a bound, R, on the number of markers in any place

TN_{safe}

same as TN_{bdd} except the bound is 1.

TN_{MARK}

same as TN except each place has at most one input transition and one output transition

TN_{PLEX}

$$\begin{aligned} f_x(p_1, t_1) &= n_1 \\ f_x(p_2, t_1) &= n_2 \\ f_x(t_1, p_3) &= n_3 \\ f_x(t_1, p_4) &= n_4 \end{aligned}$$

This is indicated by writing the values next to the arcs.

t_1 can be enabled if $P_1 \geq n_1$, $P_2 \geq n_2$. At this stage it removes n_1 markers from p_1 , n_2 from p_2 . On completion, it places n_3 markers in p_3 and n_4 in p_4 .

TN_{XORI}

$$\begin{aligned} f_{XORI}(p_1, t_1) &= * \\ f_{XORI}(p_2, t_1) &= + \\ f_{XORI}(p_3, t_1) &= + \end{aligned}$$

t_1 can be enabled if $P_1 > 0$ and $(P_2 > 0 \text{ or } P_3 > 0)$. t_1 removes one marker from p_1 and one from either p_2 or p_3

TN_{XORO}

$f_{XORO}(t_1, p_3) = *$, $f_{XORO}(t_1, p_3) = f_{XORO}(t_1, p_4) = +$. When t_1 completes, it places a marker in p_2 and one in p_3 or p_4 .

TN_{OUT}

$$f_{OUT}(p_1) = F_1$$

A marker is placed in p_1 if and only if $P_1 = 0$.

2.11 The Hierarchy of Nets

Definition 2.11.1: A net $N = (T, P, A, M^0, F, f)$ is well behaved (WB) if for any sequence $\ell_1, \ell_2, \dots, \ell_n$ produced by the net either M^n is covered by some $f_1 \in F$ or there exists a way of reaching a marking which is covered by some $f_1 \in F$.

Definition 2.11.2: $TN_A \subseteq TN_B$ if and only if for every $N_A \in TN_A$ and trans-labelling I_A , $\exists$ a net $N_B \in TN_B$ and trans-labelling I_B $\Rightarrow$

- (1) $I_A(\text{SIMSET}_{N_A}) = I_B(\text{SIMSET}_{N_B})$ and
- (2) N_A is WB $\Rightarrow N_B$ is WB.

Definition 2.11.3: $TN_A = TN_B$ if and only if $TN_A \subseteq TN_B$ and $TN_B \subseteq TN_A$.

Definition 2.11.4: $TN_A \rightarrow TN_B$ if and only if $TN_B \subseteq TN_A$ and $TN_A \not\subseteq TN_B$.

Intuitively, $TN_A = TN_B$ if and only if the two classes of nets can represent exactly the same class of coordinations. $TN_A \rightarrow TN_B$ if and only if TN_A is more powerful than TN_B with respect to its capability for representing coordinations. In what follows, we will establish the following hierarchy:

We will not go into detailed proofs in this paper, especially the proofs demonstrating equivalence. The reason is that these proofs are extremely tedious. One such detailed proof has been worked out [4] as an example. Some parts of this hierarchy have already been established earlier and will not be discussed in this report.

Theorem 2.11.1: $TN_{EXT} \rightarrow TN$.

Established in [3].

Theorem 2.11.2: $TN = TN_{OUT}$.

Established in [2, 4].

Theorem 2.11.3: $TN_{SAFE} = TN_{BDD}$

Proof:

Since $N \in TN_{SAFE} \Rightarrow N \in TN_{BDD}$, $TN_{SAFE} \subseteq TN_{BDD}$. Thus we are only required to show that $TN_{BDD} \subseteq TN_{SAFE}$.

Let $N_1 = (T_1, P_1, A_1, M_1^0, F_1, f_e^1) \in TN_{BDD}$ and let I_1 be any trans-labelling of N_1 . Let the bound be R . Then, the maximum number of possible markings of N_1 is $(R+1)^m$ where m is the number of places. Each marking of N_1 can be considered to be one system state of N_1 and can be written as an m -tuple. Let these markings be $Q_1, Q_2, \dots, Q_{(R+1)^m}$. (N_1 may never reach most of these markings but that is not of concern).

A net $N_2 = (T_2, P_2, A_2, M_2^0, F_2, f_e^2)$ is constructed as follows:

$P_2 = \{q_i \mid 1 \leq i \leq (R+1)^m\}$ where each q_i corresponds to a unique marking Q_i of N_1 .

$T_2 = \{t_s(i, j) \mid t_s \in T_1 \text{ can be enabled in } Q_i \text{ and firing of } t_s \text{ in } Q_i \text{ leads to } Q_j\}$

$A_2 = \{(q_i, t_s(i, j)) \mid t_s(i, j) \in T_2\} \cup \{(t_s(i, j), q_j) \mid t_s(i, j) \in T_2\}$

$M_2^0(q) = \begin{cases} 1 & \text{if } q \text{ corresponds to } M_1^0 \\ 0 & \text{otherwise} \end{cases}$

Let F' be the set of k markings of N_1 $\ni Q_r \in F'$ if and only if Q_r is covered by some $F_j \in F_1$, then

$F_2 = \{M_1, M_2, \dots, M_k\}$ where, for $1 \leq j \leq k$

$M_j(q_r) = \begin{cases} 1 & \text{if } Q_r \in F' \\ 0 & \text{otherwise} \end{cases}$

$f_e^2(a) = \text{True}$, for all $a \in A_2$

Thus, each final marking of N_2 assigns one marker to q_i if Q_i is covered by a final marking of N_2 and assigns zero markers to all other places of N_2 .

At any stage in the simulation of N_1 and N_2 , let N_1 have marking Q_i and in

N_2 let q_1 be the only place containing a marker. If t_s can be enabled in N_1 at this stage and firing of t_s results in Q_j , then $t_s(i,j)$ can be enabled in N_2 (and if any $t_s(k,\ell)$ can be enabled then $k = i$ and $\ell = j$) and firing of $t_s(i,j)$ results in a marking that assigns one marker to q_j and zero markers to all other places. It is easy to see (by the definitions of M_2^0 and F_2) that $\ell_1, \ell_2, \dots, \ell_k \in \text{SIMSET}_{N_1}$, with the corresponding sequence of markings $M_1^i = Q_{a_i}$, $0 \leq i \leq k$ if and only if $\ell_1(a_0, a_1), \ell_2(a_1, a_2), \dots, \ell_k(a_{k-1}, a_k) \in \text{SIMSET}_{N_2}$. If I_2 is now defined so that $I_2(t_i(x, y)) = I_1(t_i)$ for all $t_i(x, y) \in T_2$ then $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$.

It is not difficult to see that N_2 is well behaved if N_1 is. Consider any sequence $\bar{t} = \ell_1(a_0, a_1), \ell_2(a_1, a_2), \dots, \ell_k(a_{k-1}, a_k)$ produced by N_2 . Then (see above) the sequence $\ell_1, \ell_2, \dots, \ell_k$ can be produced by N_1 , with Q_{a_i} , $0 \leq i \leq k$ being the corresponding markings. Since N_1 is well behaved, either (1) Q_{a_k} is covered by some final marking of N_1 or (2) $\exists$ a sequence $\ell_{k+1}, \dots, \ell_p \Rightarrow \ell_1, \ell_2, \dots, \ell_k, \ell_{k+1}, \dots, \ell_p$ can be produced by N (with a corresponding sequence of markings Q_{a_i} , $0 \leq i \leq p$) such that Q_{a_p} is covered by a final marking of N_1 .

In case (1), the marking after $\bar{t}$ assign one marker to q_{a_k} and zero markers to all other places. Clearly, this marking is covered by a final marking of N_2 if Q_{a_k} is covered by a final marking of N_1 . In case (2), after $\bar{t}$, the sequence of firings $\ell_{k+1}(a_k, a_{k+1}), \dots, \ell_p(a_{p-1}, a_p)$ can take place. The resulting marking assigns zero markers to all places except q_{a_p} . This marking is also covered by a final marking if Q_{a_p} is.

Example 2.11.1: We will illustrate the above construction by means of an example. Let $N_1 = (T_1, P_1, A_1, M_1^0, F_1, f_e^1) \in \text{TN}_{\text{BDD}}$. The net is given in figure 2.11.1 and $R = 2$.

$$M_1^0 = \{(p_1, 2), (p_2, 0), (p_3, 0)\} \text{ or more briefly, } (2, 0, 0)$$

$$F_1 = \{M_1^0\}$$

Of the 27 markings, the only possible markings that N_1 can attain are:

$Q_1 = (2,0,0)$, $Q_2 = (1,1,0)$, $Q_3 = (0,2,0)$, $Q_4 = (1,0,1)$, $Q_5 = (0,0,2)$,
 $Q_6 = (0,1,1)$. Thus, the net N_2 to be constructed has 6 places $q_1, q_2, q_3,$
 q_4, q_5 and q_6 . F' for this case is the set $\{(2,0,0)\}$. Thus, $M_2^0 = \{(q_1,1),$
 $(q_2,0), (q_3,0), (q_4,0), (q_5,0), (q_6,0)\}$, or more briefly, $(1,0,0,0,0,0)$.

$F_2 = \{(1,0,0,0,0,0)\}..$ The net $N_2 = (T_2, P_2, A_2, M_2^O, F_2, f_e^2)$ is given in figure 2.11.2.

Figure 2.11.2

Let I_1 be a trans-labelling of $N_1 \ni I_1(t_1) = P_1, I_1(t_2) = P_2, I_1(t_3) = P_3$. Then define $I_2 \ni I_2(t_1(i, j)) = P_1, I_2(t_2(i, j)) = P_2, I_2(t_3(i, j)) = P_3$. Then, $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$ and both N_1 and N_2 are well behaved.

Theorem 2.11.4: $\text{TN}_{\text{MARK}} \leftarrow \text{TN}$

Proof:

Since $N \in \text{TN}_{\text{MARK}} \Rightarrow N \in \text{TN}$, $\text{TN}_{\text{MARK}} \subseteq \text{TN}$. Thus, we only have to prove that $\text{TN} \not\subseteq \text{TN}_{\text{MARK}}$. Let $N = (T, P, A, M^0, F, f) \in \text{TN}_{\text{MARK}}$. In N_1 , each place has at most one input transition and one output transition. We can thus talk of any transition being either in the enabled (E) state or disabled (D) state at any stage in a simulation. If a transition is in an E state, no other transition can affect this state. The only way the transition can possibly go into the D state is by firing (itself). Let $\ell^n = \ell_1, \ell_2, \dots, \ell_n$ and $d^n = d_1, d_2, \dots, d_n$ be any strings produced by N . Let $C_1 \sqcup C_2$ indicate the concatenation of two strings C_1 and C_2 .

Algorithm:

Stage (i); Examine ℓ_i and d_i

1. If ℓ_i and d_i already included in C then start stage $i + 1$.
2. If $\ell_i = d_i$ and ℓ_i not included in C , then $C \leftarrow C \sqcup \ell_i$; start stage $i + 1$.
3. If $\ell_i \neq d_i$ and ℓ_i included in C , then $C \leftarrow C \sqcup d_i$; start stage $i + 1$.
4. If $\ell_i \neq d_i$ and d_i included in C then $C \leftarrow C \sqcup \ell_i$; start stage $i + 1$.
5. If $\ell_i \neq d_i$ and neither ℓ_i nor d_i included in C then $C \leftarrow C \sqcup \ell_i \sqcup d_i$; start stage $i + 1$.

Start at stage 1 with $C = \text{null string}$. Continue until stage n . Let C^n be the resultant string C .

Lemma 2.11.1: C^n can be produced by N .

Proof: (by induction)

$n = 1$; $\ell^1 = \ell_1^1 = \ell_1, d^1 = d_1^1 = d_1$. Then ℓ_1 and d_1 are both in the E state in the initial marking. If $\ell_1 = d_1$, the $C^1 (= \ell_1)$ can be produced by N . If $\ell_1 \neq d_1$, then $C^1 = \ell_1, d_1$. In this case also C^1 can be produced by N since ℓ_1 cannot cause d_1 to go from an E state to a D state.

Let $\ell^n = \ell_1, \dots, \ell_n$; $d^n = d_1, \dots, d_n$ and assume C^n can be produced by N. Let $\ell^{n+1} = \ell_1, \dots, \ell_n, \ell_{n+1}$; $d^{n+1} = d_1, \dots, d_n, d_{n+1}$. Applying the algorithm until stage n, we get C^n as an intermediate value of C. By the inductive hypothesis, C^n can be produced by N.

1. ℓ_{n+1} and d_{n+1} already included in C^n . Then $C^{n+1} = C^n$ and C^{n+1} can be produced by N.

2. $\ell_{n+1} = d_{n+1}$ and ℓ_{n+1} not included in C^n . Then $C^{n+1} = C^n \sqcup \ell_{n+1}$. ℓ_{n+1} does not occur in ℓ^n either. ℓ_{n+1} is in an E state after ℓ^n . Let $p_1, p_2, \dots, p_r$ be the input places of ℓ_{n+1} . Without loss of generality out of these places, let $p_1, p_2, \dots, p_u$ be the places that were empty in the initial marking. Let $t_1, \dots, t_s$ be the input places of these transitions. Then, $t_1, \dots, t_s$ must occur in some order in ℓ^n , since it is only by the firing of these transitions that ℓ_{n+1} can reach the E state. Once enabled, ℓ_{n+1} cannot go to the D state, since it does not occur in ℓ^n . However, the algorithm guarantees that $t_1, \dots, t_s$ occur in C^n (in some order). Firing of these transitions must enable ℓ_{n+1} . Therefore ℓ_{n+1} will be in the E state after C^n . Therefore $C^n \ell_{n+1} = C^{n+1}$ can be produced by N.

The arguments for cases 3 and 4 of the algorithm are similar to those for case 2. In case 5, the argument can be applied twice to prove that after C^n , both ℓ_{n+1} and d_{n+1} are in the E state. Since $\ell_{n+1} \neq d_{n+1}$, d_{n+1} will be in the E state after $C^n \ell_{n+1}$ and hence $C^{n+1} = C^n \ell_{n+1} d_{n+1}$ can be produced by N. This completes the proof of lemma 2.11.1.

Let $\ell^n = \ell_1, \ell_2, \dots, \ell_n$ and $d^m = d_1, \dots, d_m$ be two strings produced by N, $n < m$. It can be shown that if $\ell^m = \ell_1, \dots, \ell_n \ell_{n+1}, \dots, \ell_m$, where $\ell_{n+1}, \dots, \ell_m$ are "null" and the algorithm applied to ℓ^m and d^m to yield C^m , then C^m can be produced by N.

Consider $N_1 = (T_1, P_1, A_1, M_1^0, F_1, f_e^1) \in \text{TN}_{\text{SAFE}}$ given in figure 2.11.3.

Figure 2.11.3

$$M_1^0 = \{(p_1, 1), (p_2, 0), (p_3, 0)\}$$

$$F_1 = \{M_1^0\}$$

Let I_1 be a trans-labelling of $N_1 \ni I_1(t_1) = P_1, I_1(t_2) = P_2, I_1(t_3) = P_3$.

Any sequence belonging to I_1 (SIMSET_{N_1}) has the following characteristics. The sequence starts with P_2 or P_3 and between any two occurrences of P_2 and P_3

there must be at least one occurrence of P_1 . Thus, for example, $P_2P_1P_3P_1 \in$

$I_1(\text{SIMSET}_{N_1})$ and $P_3P_1P_2P_1 \in I_1(\text{SIMSET}_{N_1})$. Also, N_1 is well behaved. Let

$N_2 = (T_2, P_2, A_2, M_2^0, F_2, f_e^2) \in \text{TN}_{\text{MARK}}$ and let I_2 be a trans-labelling of $N_2 \ni$

$I_2(\text{SIMSET}_{N_2}) = I_1(\text{SIMSET}_{N_1})$ and N_2 is well behaved. Then, $P_2P_1P_3P_1 \in I_2$

(SIMSET_{N_2}) and $P_3P_1P_2P_1 \in I_2(\text{SIMSET}_{N_2})$. Let $\ell_1, \ell_2, \dots, \ell_p$ be the simulation

sequence from which $P_2P_1P_3P_1$ was obtained and let $d_1, d_2, \dots, d_q$ be the sequence

from which $P_3P_1P_2P_1$ was obtained. Let $I_2(\ell_n) = P_2$ for some $n < p$ and let I_2

$(\ell_i) = \emptyset, 1 \leq i < n$. Let $I_2(d_m) = P_3$ for some $m < q$ and let $I_2(d_i) =$

$\emptyset, 1 \leq i < m$. Both these sequences are produced by N_2 . Without loss of

generality, let $n \leq m$. Applying the algorithm to ℓ^n and d^m yields C^m , where all

distinct transitions in ℓ^n and d^m appear once and only once, and C^m can be

produced by N_2 . Since N_2 is well behaved by assumption, either the marking

after C^m is covered by a final marking or $\exists$ a sequence $C_{m+1}, \dots, C_n \ni$ the

marking after C^m $C_{m+1}, \dots, C_n$ is covered by a final marking of N_2 . Thus either

one of the two sequences P_2P_3 and P_3P_2 belongs to $I_2(\text{SIMSET}_{N_2})$ or one of the

two sequence P_2P_3 and P_3P_2 is a prefix of a process sequence belonging to I_2

(SIMSET_{N_2}) . In each case, $I_1(\text{SIMSET}_{N_1}) \neq I_2(\text{SIMSET}_{N_2})$. $N_1 \in \text{TN}_{\text{SAFE}}$, therefore $N_1 \in \text{TN}$. Thus $\text{TN} \not\subseteq \text{TN}_{\text{MARK}}$ and $\text{TN} \rightarrow \text{TN}_{\text{MARK}}$. This completes the proof of theorem 2.11.4.

Theorem 2.11.5: $\text{TN}_{\text{SAFE}} \leftarrow \text{TN}$.

Proof:

A trans-labelled net $N = (T, P, A, M^0, F, f)$ can be considered to be a language generator, where $I(\text{SIMSET}_N)$ is the language generated. We will show first that the class of languages generated by trans-labelled, safe Petri nets is the class of regular languages.

1. Let $M = (K, \Sigma, \delta, q_0, F)$ be any finite state machine where

$K = \{q_i \mid 0 \leq i \leq n\}$ is a finite set of states

$\Sigma = \{a_i \mid 1 \leq i \leq m\}$ is a finite alphabet

$\delta : K \times \Sigma \rightarrow K$

q_0 : initial state

$F = \{b_1, b_2, \dots, b_r\} \subseteq K$, is the set of final states.

If $\delta(q_i, a_j) = q_k$, the machine M in state q_i scanning a_j goes to state q_k . $\bar{a} \in \Sigma^*$ is a string accepted by M if $\bar{a}$ causes M to go into a state q starting from $q_0 \Rightarrow q \in F$. The set of all strings accepted by M defines the language L_M accepted by M . We will construct a trans-labelled, safe Petri net $N = (T, P, A, M^0, F, f_e)$ which generates the same language that M accepts. N is defined as follows:

$T = \{\ell_i \mid 1 \leq i \leq m\} \cup \{t_{ij} \mid 0 \leq i \leq n, 1 \leq j \leq m\}$

$P = \{q_i \mid 0 \leq i \leq n\} \cup \{p_{ij} \mid 0 \leq i \leq n, 1 \leq j \leq m\}$

$A = \{(q_i, \ell_j) \mid 0 \leq i \leq n, 1 \leq j \leq m\} \cup$

$\{(p_{ij}, t_{ij}) \mid 0 \leq i \leq n, 1 \leq j \leq m\} \cup$

$\{(\ell_j, p_{ij}) \mid 0 \leq i \leq n, 1 \leq j \leq m\} \cup$

$\{(t_{ij}, q_k) \mid \delta(q_i, a_j) = q_k\}$

$M^0 = \{(q_0, 1), (q_1, 0), \dots, (q_n, 0)\} \cup \{(p_{ij}, 0) \mid 0 \leq i \leq n, 1 \leq j \leq m\}$

$F = \{M_i \mid 1 \leq i \leq r\}$ where

$$F_1(x) = \begin{cases} 1 & \text{if } x \text{ is } b_1 \\ 0 & \text{otherwise} \end{cases}$$

$f_e(a) = \text{True}$, for all $a \in A$.

The trans-labelling I is defined as follows:

$$I(\ell_{1j}) = a_j, \quad 1 \leq j \leq m, \quad 0 \leq i \leq n$$

$$I(t_{1j}) = \emptyset, \quad 1 \leq j \leq m, \quad 0 \leq i \leq n.$$

N is safe and in fact at any stage of a simulation, N has exactly one marker. Clearly, $I(\text{SIMSET}_N) = L_M$ and an example will make this clear.

Example 2.11.2: Figure 2.11.4 gives the state diagram of a finite state automaton M . The nodes represent states. An arc labelled a_k exists between q_i and q_j if and only if $\delta(q_i, a_k) = q_j$. The final states are circled. q_0 is the initial state.

Figure 2.11.4

A trans-labelled safe Petri net is given in figure 2.11.5.

Figure 2.11.5

$$M^0 = \{(q_0, 1), (q_1, 0)\}$$

$$F = \{M^0\}$$

$$I(\ell_{01}) = I(\ell_{11}) = a_1$$

$$I(\ell_{02}) = I(\ell_{12}) = a_2$$

$$I(t_{ij}) = \emptyset, i = 0, 1, j = 1, 2.$$

The sequence $\ell_{1j} \xrightarrow{p_{1j}} t_{1j}$ is required between q_i and q_k because the same place cannot be an input and output place of the same transition. The reader can easily verify that the process sequences of N are exactly the sentences recognized by M .

2. Let $N = (T, P, A, M^0, F, f_e)$ be any safe Petri net with trans-labelling I .

In the proof 2.11.3, given a trans-labelled net $N_1 \in TN_{BDD}$, a net $N_2 \in TN_{SAFE}$ was constructed $\Rightarrow$

- a) N_2 contained exactly one marker at any stage in a simulation.
- b) $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$
- c) N_1 is well behaved $\Rightarrow N_2$ is well behaved
- d) Each transition has exactly one input and one output place.

Since $N \in TN_{BDD}$, the construction yields a similar net N_2 . I_2 can be redefined as follows: $I_2(t_S(i, j)) = I_1(t_S)$ if $I_1(t_S) \neq \emptyset$ and $I_2(t_S(i, j)) = PR$ otherwise where $PR \neq I_1(t_S)$ for all S . Clearly, $I(\text{SIMSET}_N) = I_2(\text{SIMSET}_{N_2})$ if all occurrences of PR are deleted from each process sequence belonging to $I_2(\text{SIMSET}_{N_2})$. A state diagram can be obtained from N_2 by removing the transitions and labelling each arc (q_i, q_j) by t where t was the transition between q_i and q_j . The q_i 's form the states of the state diagram. The initial state of this state diagram is the place that initially contained a marker in N_2 . The final states are those states assigned a marker by the final markings of N_2 . The finite state machine so obtained recognizes a language L_M . Clearly, $L_M = I(\text{SIMSET}_N)$ if all occurrences of PR in each sentence of L_M are

deleted. Since regular languages are closed under homomorphisms, $\exists$ a regular language L_M' (obtained from L_M by deleting all occurrences of PR) $\ni L_M = I(\text{SIMSET}_N)$.

1 and 2 above prove that the class of languages generated by trans-labelled, safe Petri nets is the class of regular languages.

Consider the net $N_1 \in \text{TN}$ given in Figure 2.11.6.

Figure 2.11.6

$$M^0 = \{(p_1, 1), (p_2, 0), (p_3, 1), (p_4, 0), (p_5, 0)\}$$

$$F = \{M^0\}$$

Let I_1 be a translabelling of $N_1 \ni I_1(t_1) = "(", I_1(t_3) = ")", I_1(t_2) = I_1(t_4) = \emptyset$. Then, $I_1(\text{SIMSET}_{N_1})$ is the set of well formed parenthesis expressions. This language is not regular. Therefore, no trans-labelled, safe Petri net can generate this language. This concludes the proof of Theorem 2.11.5

Theorem 2.11.6: $\text{TN} = \text{TN}_{\text{XORO}}$

Proof:

$N \in \text{TN} \Rightarrow N \in \text{TN}_{\text{XORO}}$, therefore $\text{TN} \subseteq \text{TN}_{\text{XORO}}$. We are only required to show

that $TN_{XORO} \subseteq TN$. Consider any $N_1 = (T_1, P_1, A_1, M_1^O, F_1, f_{XORO}^1) \in TN_{XORO}$. Let t_1 be any transition in this net, $\exists I_1 = \{1, 2, \dots, s\}$, $O_1^* = \{s+1, \dots, t\}$, $O_1^+ = \{t+1, \dots, t+r\}$. Thus, local to t_1 , the net is as shown in Figure 2.11.7.

Figure 2.11.7

Let the net $N_2 = (T_2, P_2, A_2, M_2^O, F_2, f_{XORO}^2)$ be obtained by replacing t_1 by the net in figure 2.11.8.

Figure 2.11.8

If M_2 is a marking of N_2 and M_1 of N_1 then we say $M_2(P_1 \cap P_2) = M_1(P_1)$ if $M_2(p_i) = M_1(p_i)$ for all $p_i \in P_1$. Similarly, if I_2 is a trans-labelling of N_2 and I_1 of N_1 then $I_2(T_1 \cap T_2) = I_1(T_1)$ if $I_2(t_i) = I_1(t_i)$ for all $t_i \in T_1$.

The construction of N_2 defines T_2 , P_2 , A_2 , and f_{xoro} .

$M_2^0(P_1 \cap P_2) = M_1^0(P_1)$, $M_2^0(p_d) = 1$, $M_2^0(p_d^i) = 0$, $1 \leq i \leq r$. Let each final marking M of N_2 be the same as that of N_1 except that M assigns 1 marker to p_d and zero markers to all the p_d^i 's.

Consider any sequence $\ell^n = \ell_1, \ell_2, \dots, \ell_n \in \text{SIMSET}_{N_1}$. Let ℓ_{i+1} be the first occurrence of t_1 . Clearly, the sequence ℓ^1 can be produced by N_2 , $\exists$ for $1 \leq k \leq i$. $M_2^k(P_1 \cap P_2) = M_1^k(P_1)$, $M_2^k(p_d) = 1$, $M_2^k(p_d^j) = 0$, $1 \leq j \leq r$. When ℓ_{i+1} is enabled it removes a marker from each of $p_1, p_2, \dots, p_s$. On completion of firing, let ℓ_{i+1} place one marker in p_{t+k} and one in each of $p_{s+1}, \dots, p_t$. Then, in N_2 , t_d^k can be enabled after ℓ^1 . Firing of t_d^k removes one marker from each of $p_1, \dots, p_s$, one from p_d and places a marker in p_d^k enabling t_1^k . Firing of t_1^k replaces the marker in p_d and, places a marker in each of $p_{s+1}, \dots, p_t$ and one marker in p_{t+k} . Let M_2 be the marking of N_2 after $\ell^{i-1} t_d^k t_1^k$. Then $M_2(P_1 \cap P_2) = M_1^{i+1}(P_1)$, $M_2(p_d) = 1$, $M_2(p_d^j) = 0$, $1 \leq j \leq r$. Continuing in this manner, if $\ell_{i_1}, \ell_{i_2}, \dots, \ell_{i_m}$ are occurrences of t_1 in ℓ^n then the sequence $d^{n+m} = \ell^{i-1} t_d^{j_1} t_1^{j_1} \ell_{i_1+1}, \dots, t_d^{j_m} t_1^{j_m} \ell_{i_m+1}, \dots, \ell_n$ can be produced by N_2 . By appropriately selecting $j_1, \dots, j_m$, $M_2^{n+m}(P_1 \cap P_2) = M_1^n(P_1)$, $M_2^{n+m}(p_d) = 1$, $M_2^{n+m}(p_d^i) = 0$, $1 \leq i \leq r$. By definition of F_2 , $d^{n+m} \in \text{SIMSET}_{N_2}$.

Let $d^n = d_1, d_2, \dots, d_n$ be any sequence $\in \text{SIMSET}_{N_2}$. Let d_a be the first occurrence of some t_d^1 . Then the sequence d^{a-1} can be produced by $N_1 \ni M_2^{a-1}(P_1 \cap P_2) = M_1^{a-1}(P_1)$, $M_2^{a-1}(p_d^j) = 0$, $1 \leq j \leq r$. Firing of t_d^1 removes the marker from p_d and one marker from each of $p_1, \dots, p_s$. Thus $M_2^a(p_j) = M_1^{a-1}(p_j) - 1$, $1 \leq i \leq s$. $M_2^a(p_j) = M_1^{a-1}(p_j)$ for all other p_j 's in $P_1 \cap P_2$. Clearly, no t_d^k can fire until t_1^1 fires replacing the marker in p_d . Let d_b be the first occurrence of t_1^1 . The sequence $d_1, d_2, \dots, d_{a-1}, d_{a+1}, \dots, d_{b-1}$ can be produced by

$N_1 \ni$ if M is the marking of N_1 after this sequence then $M_2^{b-1}(p_i) = M(p_i) - 1$, $1 \leq i \leq s$ and M assigns the same markers to all other places in $P_1 \cap P_2$. Firing of t_1^i places a marker in p_d , one marker in each of $p_{s+1}, \dots, p_t$ and a marker in p_{t+i} . Since $M_2^{b-1}(p_i) \geq 0$, $1 \leq i \leq s$, $M(p_i) \geq 1$, $1 \leq i \leq s$. Therefore t_1 can fire in N_1 at this stage. It removes a marker from each of $p_1, \dots, p_s$. On completion of firing let the resulting marking be M_1^b where p_{t+i} is selected to receive a marker. $M_1^b(p_1) = M_2^b(p_1 \cap P_2)$, $M_2^b(p_d) = 1$, $M_2^b(p_d^i) = 0$, $1 \leq i \leq r$. Continuing in this manner, it can be seen that the sequence $\ell^m = \ell_1, \ell_2, \dots, \ell_m$ obtained by deleting the t_d^j 's in d^n and replacing the t_1^j 's by t_1 can be produced by N_1 . Since $d^n \in \text{SIMSET}_{N_2}$, the last occurrence of t_d^j in d^n must be followed by a t_1^j else $P_d = 0$ and each final marking of N_2 assigns a marker to p_d . By appropriately selecting the place to receive a marker after each firing of t_1 in N_1 the marking M_1^m is obtained $\ni M_1^m(p_1) = M_2^n(p_1 \cap P_2)$, $M_2^n(p_d) = 1$, $M_2^n(p_d^j) = 0$, $1 \leq j \leq r$. By definition of F_2 , since $d^n \in \text{SIMSET}_{N_1}$, ℓ^m must belong to SIMSET .

Let I_1 be any trans-labelling of N_1 . Let I_2 be a trans-labelling of N_2 defined as follows: $I_2(t_1^j) = I_1(t_1)$, $1 \leq j \leq r$. $I_2(t) = I_1(t)$ for all other t 's belonging to $T_1 \cap T_2$, $I_2(t_d^j) = \emptyset$, $1 \leq j \leq r$. The construction $\ell^m \in \text{SIMSET}_{N_1}$ given $d^n \in \text{SIMSET}_{N_2}$ and that of $d^{n+m} \in \text{SIMSET}_{N_2}$ given $\ell^n \in \text{SIMSET}_{N_1}$ clearly shows that $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$.

The reader can easily verify that N_2 is well behaved if N_1 is. Let N_1 have k transitions. If the above replacement procedure is carried out k times, once for each transition in N_1 , we end up with a net $N_{k+1} \in \text{TN}$ and interpretation $I_{k+1} \ni I_{k+1}(\text{SIMSET}_{N_{k+1}}) = I_1(\text{SIMSET}_{N_1})$ and N_{k+1} is well behaved if N_1 is. This completes the proof of theorem 2.11.6.

Theorem 2.11.7: $TN = TN_{XORI}$

Proof:

$N \in TN \Rightarrow N \in TN_{XORI}$, therefore $TN \subseteq TN_{XORI}$ and we are only required to show that $TN_{XORI} \subseteq TN$. Consider any $N_1 = (T_1, P_1, A_1, M_1^0, F_1, f_{XORI}^1) \in TN_{XORI}$. Let t_1 be any transition of $N_1 \ni I_1^* = \{p_1, \dots, p_s\}$, $I_1^+ = \{p_{s+1}, \dots, p_{s+t}\}$, $O_1 = \{p_{s+t+1}, \dots, p_{s+r}\}$. Local to t_1 the net is as shown in Figure 2.11.9.

Figure 2.11.9

Let $N_2 = (T_2, P_2, A_2, M_2^0, F_2, f_{XORI}^2)$ be the net obtained when t_1 is replaced by the net in Figure 2.11.10.

Figure 2.11.10

$t_1^1, \dots, t_1^r$ correspond to t_1 in Figure 2.11.10. $M_2^0(P_1 \cap P_2) = M_1^0(P_1)$, $M_2^0(p_d) = 1$, $M_2^0(p_d^i) = 0$, $1 \leq i \leq r$. The final markings of N_2 are the same as those of N_1 except that each marking in M_F^2 assigns zero markers to p_d^i , $1 \leq i \leq r$ and one marker to p_d . Let I_1 be any trans-labelling of N_1 . Then $I_1(t_1^i) = I_1(t_1)$, $1 \leq i \leq r$, $I_2(T_1 \cap T_2) = I_1(t_1)$ and $I_2(t_d^i) = \emptyset$, $1 \leq i \leq r$. Then an argument, similar to that in theorem 2.11.6 can be given to prove that $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$ and N_2 is well behaved if N_1 is. Let k be the number of transitions in N_1 . Then if the above replacement procedure is repeated k times, once for each transition in N_1 , a net $N_{k+1} \in \text{TN}$ with trans-labelling I_{k+1} is obtained $\ni I_1(\text{SIMSET}_{N_1}) = I_{k+1}(\text{SIMSET}_{N_{k+1}})$ and N_{k+1} is well behaved if N_1 is. This completes the proof of theorem 2.11.7.

Theorem 2.11.8: $\text{TN}_{\text{PLEX}} = \text{TN}$

Proof:

$N \in \text{TN} \Rightarrow N \in \text{TN}_{\text{PLEX}}$. Therefore $\text{TN} \subseteq \text{TN}_{\text{PLEX}}$ and we need only show that $\text{TN}_{\text{PLEX}} \subseteq \text{TN}$. We will illustrate the construction and proof by means of an example. The proof for the general case is similar. Let $N_1 = (T_1, P_1, A_1, M_1^0, F_1, f_x^1) \in \text{TN}_{\text{PLEX}}$ and let I_1 be any trans-labelling. Let a particular transition t_1 have p_1 and p_2 as input places and p_3 and p_4 as output places. Let $f_x((p_1, t_1)) = f_x((t_1, p_3)) = 1$, $f_x((p_2, t_1)) = f_x((t_1, p_4)) = 2$. Thus, local to t_1 , the net N_1 is as shown in Figure 2.11.11.

Figure 2.11.11

The goal now is to replace t_1 by a net where the condition for any transition to fire is that no input place be empty. By defining a suitable interpretation I_2 for the net N_2 obtained by the replacement, we have to ensure that N_2 produces the same process sequences as N_1 and if N_1 is well behaved then so is N_2 . Each transition can then be replaced in a similar manner and the final net will belong to the class TN. Let us try to replace t_1 by the net in Figure 2.11.12.

Figure 2.11.12

Let the resulting net be N_2 . $M_2^0(P_1 \cap P_2) = M_1^0(P_1)$, $M_2^0(p_d^i) = 0$, $1 \leq i \leq 4$.

F_2 is the same as F_1 except that each marking in F_2 assigns zero markers to p_d^i , $1 \leq i \leq 4$. $I_2(T_1 \cap T_2) = I_1(T_1 \cap T_2)$, $I_2(t_d^i) = \emptyset$, $1 \leq i \leq 4$, $I_2(t_1') = I_1(t_1)$. Thus, t_1' corresponds to t_1 . In N_1 , $P_1 \geq 1$ and $P_2 \geq 2 \Leftrightarrow t_1$ can fire. In N_2 , the check for $P_2 \geq 2$ is made by transitions t_d^1 and t_d^2 . A marker can be placed in p_d^2 if and only if $P_2 \geq 2$. Thus t_1' can be enabled if and only if $P_1 \geq 1$ and $P_2 \geq 2$. When t_1 fires, it places a marker in p_3 and 2 markers in p_4 . When t_1' fires, it places a marker in p_3 and enables t_d^3 and t_d^4 , firing of which places 2 markers in p_4 .

Let $\ell^n = \ell_1, \ell_2, \dots, \ell_n \in \text{SIMSET}_{N_1}$. Let ℓ_1 be the first occurrence of t_1 . The sequence ℓ^{i-1} can be produced by N_2 and $M_2^{i-1}(P_1 \cap P_2) = M_1^{i-1}(P_1)$. Since $\ell_1 = t_1$, $M_1^{i-1}(p_2) \geq 2$ and t_d^1 can fire in M_2^{i-1} removing a marker from p_2 . Since $M_2^{i-1}(p_2) \geq 2$, t_d^2 can subsequently fire placing a marker in p_d^2 , and enabling t_1' . t_d^3 and t_d^4 are enabled by t_1' and on completion of firing, a marking M of N_2 results $\ni M(P_1 \cap P_2) = M(P_1)$ and $M(p_d^i) = 0$, $1 \leq i \leq 4$.

Continuing in this manner, it can be seen that if $\ell_1, \dots, \ell_n \in \text{SIMSET}_{N_1}$ where $\ell_{a_1} = \ell_{a_2} = \dots = \ell_{a_m} = t_1$, $1 \leq a_1 < a_2, \dots, < a_m \leq n$ then the sequence $d^{n+4m} = \ell_1, \dots, \ell_{a_1-1} S \ell_{a_1+1}, \dots, \ell_{a_m-1} S \ell_{a_m+1}, \dots, \ell_n$ where $S = t_d^1, t_d^2, t_d^3, t_d^4$ can be produced by $N_2 \ni d_{n+4m} \in \text{SIMSET}_{N_2}$. By definition of I_2 , $\bar{P} \in I_1(\text{SIMSET}_{N_1}) \Rightarrow \bar{P} \in I_2(\text{SIMSET}_{N_2})$.

Consider any sequence $d^n = d_1, d_2, \dots, d_n \in \text{SIMSET}_{N_2}$. If t_1 does not occur anywhere in d^n and one or more of t_d^i occur, then $M_2^n(p_d^j) > 0$ for some j and M_2^n will not be covered by a final marking of N_2 . If t_1 does not occur anywhere in d^n (then neither do any t_d^i 's) and the sequence d^n can be produced by N_2 , $M_2^n(P_1 \cap P_2) = M_1^n(P_1)$, $M_2^n(p_d^i) = 0$, $1 \leq i \leq n$. Let d_1 be the first occurrence of t_1 in d^n . Then d^n must be of the form $d^a t_d^1 d_{a+2}, \dots, d_b t_d^2 d_{b+2} \dots d_c t_1' d_{c+2}, \dots, d_e (t_d^3 \text{ or } t_d^4) d_{e+2}, \dots, d_f (t_d^4 \text{ or } t_d^3) d_{f+2}, \dots, d_n$ else once again M_2^n will not be covered by any final marking. Clearly, $\ell^{c-2} =$

$d_1, \dots, d_a d_{a+2}, \dots, d_b d_{b+2}, \dots, d_c$ can be produced by N_1 since firing t_d^1 or t_d^2 cannot enable any transition of N_2 in $T_1 \cap T_2$. $M_1^{c-2}(p_2) = M_2^c(p_2) + 2$ and $M_1^{c-2}(p) = M_2^c(p)$ for all other $p \in P_1 \cap P_2$. If t_1 can fire in M_2^c then since $M_2^c(p_2) \geq 0$, t_1 can fire in M_1^{c-2} . Firing of t_1 yields the following situation: $M_2^{c+1}(p_4) = M_1^{c-1}(p_4) - 2$, $M_2^{c+1}(p) = M_1^{c-1}(p)$ for all $p \in P_1 \cap P_2$. Any transition in $T_1 \cap T_2$ that can be enabled in N_1 with marking M_1^{c-1} can be enabled in N_2 with marking M_2^{c+1} . Continuing this argument, it can be shown that the sequence $\ell^{f-3} = d_1, \dots, d_a d_{a+2}, \dots, d_b d_{b+2}, \dots, d_c, t_1, d_{c+2}, \dots, d_e, d_{e+2}, \dots, d_f$ can be produced by N_1 and $M_1^{f-3}(P_1 \cap P_2) = M_2^{f+1}(P_1 \cap P_2)$ and $M_2^{f+1}(p_d^1) = 0$, $1 \leq i \leq d$. Thus, if d^n has m occurrences of t_1^1 then the sequence ℓ^{n-4m} obtained by deleting all occurrences of t_d^1 's and replacing t_1^1 by t_1 can be produced by N_1 and $M_1^n(P_1) = M_2^{n-4m}(P_1 \cap P_2)$, $M_2^{n-4m}(p_d^1) = 0$, $1 \leq i \leq 4$. If $d^n \in \text{SIMSET}_{N_2}$ then by definition of F_2 , $\ell^{n-4m} \in \text{SIMSET}_{N_1}$. Thus, by definition of I_2 , if $\bar{P} \in I_2(\text{SIMSET}_{N_2})$ then $\bar{P} \in I_1(\text{SIMSET}_{N_1})$.

The above two results indicate that $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$. The problem with this replacement procedure, however, is that N_2 need not be well behaved (even though N_1 is). Let t_2 be a transition of $N_1 \ni I_2 = \{1, 2\}$ and $f_x((p_1, t_2)) = f_x((p_2, t_2)) = 1$. At some stage in the simulation of N_1 , after sequence $\ell^n = \ell_1, \dots, \ell_n$, let $P_1 = 1$, $P_2 = 2$. Assume $\ell_i \neq t_1$, $1 \leq i \leq n$ and p_2 is not the output place of any transition. $d^n = \ell_1, \dots, \ell_n$ can be produced by N_2 (as we have shown) $\Rightarrow$ after this sequence $P_1 = 1$ and $P_2 = 2$. After d^n , suppose t_2 fires in N_1 , followed by the firing of t_d^1 . Thus, the sequence $d^{n+1} = d^n t_2 t_d^1$ can be produced by N_2 . After this sequence, $P_2 = 0$. Since no transition has p_2 as an output place, p_2 will remain empty. Thus, after d^{n+1} , p_d^1 will always contain a marker irrespective of simulation. Since each final marking assigns zero markers to p_d^1 , there is no way of reaching a marking covered by a final marking of N_2 . Thus N_2 is not

Let the resulting net be $N_2 = (T_2, P_2, A_2, M_2^0, F_2, f_x^2), M_2^0(P_1 \cap P_2) = M_1^0(P_1), M_2^0(p_d^1) = 0, 1 \leq i \leq 6$. F_2 is the same as F_1 except that each marking in F_2 assigns zero markers to $p_d^1, 1 \leq i \leq 6$. $I_2(T_1 \cap T_2) = I_1(T_1 \cap T_2), I_2(t_1') = I_1(t_1)$ and $I_1(t_d^1) = \emptyset, 1 \leq i \leq 8$. As before, it can be shown that $I_1(\text{SIMSET}_{N_1}) = I_2(\text{SIMSET}_{N_2})$. However, in this case, if N_1 is well behaved, then so is N_2 . In the previous construction, N_2 was not well behaved because checking if P_2 was greater than 1 was achieved by the sequential firing of two transitions t_d^1 and t_d^2 . Thus, if p_1 and p_2 were also input places of other transitions then "dead-lock" could occur if the same replacement technique is used for the other transitions. The replacement in figure 2.11.13, however, avoids this. At each step of the checking procedure, it is possible to revert back to the start of the check. Thus, if $P_2 > 1$, then the check can be completed and t_1' can be enabled. If $P_2 = 1$, firing of t_d^1 will not lead to dead lock because t_d^5 can fire causing the marker in p_2 to be replaced. At each stage in the simulation of N_2 , either a transition in $(T_1 \cap T_2)$ is fired or t_d^1 followed by t_d^2 can be fired enabling t_1' or a sequence of t_d^1 's can be fired returning the net to the marking at the start of the stage. It can easily be seen now that if N_1 is well behaved. N_2 must also be well behaved.

For the general case, a transition t_1 having input places $p_1, \dots, p_s$ and output places $p_{s+1}, \dots, p_{s+t} \ni f_x((p_i, t_1)) = n_i, 1 \leq i \leq s$ and $f_x((t_1, p_i)) = n_i, s+1 \leq i \leq t$, can be replaced by the net in figure 2.11.14 to yield a net N_2 .

Figure 2.11.14

$M_2^0(p_1 \cap p_2) = M_1^0(p_1)$, $M_2^0(p_d) = 0$. $I_2(T_1 \cap T_2) = I_1(T_1 \cap T_2)$, $I_2(t_1') = I_1(t_1)$, $I_2(t_d) = \emptyset$. F_2 is the same as F_1 except each final marking in F_2 assigns zero markers to the p_d 's

As we have shown, $I_2(\text{SIMSET}_{N_2}) = I_1(\text{SIMSET}_{N_1})$ and N_2 is well behaved if N_1 is.

If N_1 contains k transitions, the replacement procedure can be carried out

k times once for each transition of N_1 to yield a net N_{k+1} and interpretation

$I_{k+1} \ni I_1(\text{SIMSET}_{N_1}) = I_{k+1}(\text{SIMSET}_{N_{k+1}})$ and N_{k+1} is well behaved if N_1 is.

3. Summary

We have defined nine classes of Petri nets and have established the following hierarchy:

As will be demonstrated in subsequent chapters, this hierarchy will be useful in establishing the capabilities of different controlling agents. For example, we will show that the complex Biologic Directed Graph model of UCLA, Petersons P-nets and ordinary Petri nets are equivalent with respect to their power for representing coordinations. We feel this is a significant result and is derived directly from the above hierarchy.

4. Controlling agents in different models

4.1 U.C.L.A. studies

In the past decade, a vast amount of literature (on graph models of computations) has emerged from U.C.L.A. as an outgrowth of the variable structure computer project. The intent of the first model by Estrin and Turn was to give a representation of programs to be run on a variable structure computer. Over the years, the model has been extended and refined and different aspects have been studied by different investigators. For a good survey, the reader is referred to [6]. We will proceed directly to the latest and most general model, the Graph Model of Computation from U.C.L.A., studied by Cerf in his doctoral dissertation, [10]. This Graph Model of Computation is based on the works of Martin [17], Baer [5], Bovet [9], Russel [22], Volansky [24], Costelow [12], Slutz [23], and Holt [13]. The main objective of Cerf's dissertation was to seek methods for analyzing program control flow based on its representation in the model, to study the practical implications of the model, and to discover potentially correctable control anomalies such as deadlock, etc.

The basic controlling agent in the Graph Model of Computation is the Complex, Bilogic Directed Graph. We will proceed with the definitions as presented in [10].

Definition 4.1.1: A complex directed graph (digraph) is a pair (V, A)

where

- (1) V is a finite set of vertices of cardinality $|V|$
- (2) A is a finite set of arcs and if $a \in A$ then a is an ordered pair of sets

$$a = (I, J)$$

$$I \subseteq V, J \subseteq V$$

(3) If $I = \{i\}$, $J = \{j\}$ we write $a = (i, j)$

(4) We permit $I = \phi$ or $J = \phi$, but not both

(5) When $|J| > 1$ we have a multi-headed arc

(6) When $|I| > 1$ we have a multi-tailed arc.

Definition 4.1.2: A complex, multiple-entry, multiple-exit digraph is a four-tuple, $G = (V, I, T, A)$ where

1. (V, A) is a digraph
2. I is a set of entry arcs
3. T is a set of exit arcs.

Definition 4.1.3: A complex, single-entry, single exit digraph (SESX) is a MEMX, $G = (V, I, T, A)$ such that

1. $I = \{S\}$ and $S = (\phi, v_1)$
2. $T = \{X\}$ and $X = (v_j, \phi)$

Definition 4.1.4: A complex, bilogic, directed graph (bigraph) is a triple (G, L, Q) where:

1. $G = (V, A)$ is a SESX digraph
2. $L = (L^-, L^+)$ is a total, single-valued mapping from the set of vertices, V , into the set $\{+, *\}$.
3. $Q = (Q^-, Q^+)$ is a partial single valued mapping from $V \times A$ into the integers.

The simulation rules are given by Cerf in terms of a Token Machine after defining the graph model of computation (GMC) which is semiinterpreted and then defining an interpretation. Without going into these details, we will present the simulation rules informally here.

Definition 4.1.5: The inbranching set of vertex v is $A^-(v) = \{a \mid a \in A \text{ and } a = (I, J) \text{ and } v \in J\}$.

Definition 4.1.6: The outbranching set of vertex v is $A^+(v) = \{a \mid a \in A \text{ and } a = (I, J) \text{ and } v \in I\}$.

A simulation starts by placing one "token" on the entry arc. All other arcs have zero tokens. At any stage in a simulation let $N(a)$ be the number of tokens on arc a . A vertex v , with $L^-(v) = +$ can initiate if $\exists a \in A^-(v) \ni N(a) \geq Q^-(v, a)$. When it initiates, it removes $Q^-(v, a)$ tokens from a . A vertex v with $L^-(v) = *$ can initiate if for all $a \in A^-(v)$, $N(a) \geq Q^-(v, a)$. When it initiates, it removes $Q^-(v, a)$ tokens from each $a \in A$. When v terminates, if $L^+(v) = +$ then arbitrarily, a particular $a \in A^+(v)$ is selected and $Q^+(v, a)$ tokens are placed on a . When v terminates, if $L^+(v) = *$ then $Q^+(v, a)$ tokens are placed on each arc $a \in A^+(v)$. Initiations and terminations cannot occur simultaneously. The graph terminates when no vertex can initiate or terminate.

A computation sequence is a sequence of initiations and terminations at the end of which no vertex can initiate or terminate. An initiation of v corresponds to the symbol $\bar{v}$ and a termination of v to the symbol $\underline{v}$. Cerf also defines a properly terminating graph where one of the conditions is that whenever the graph terminates, it terminates such that there is a token on the exit arc and no tokens on any other arc. Thus, it seems reasonable to define a simulation sequence of such a graph as a sequence of initiations and terminations that, starting with a single token on the entry arc, causes the graph to end up with one token on the exit arc and no tokens elsewhere. The set of all sequences of a bigraph B is

called S_B . According to our definitions then, a graph is well behaved if it satisfies the condition for proper termination stated above.

Figure 4.1.1 is an example of a bigraph.

$Q^+(1, b) = 2 = Q^+(2, c) = Q^-(6, e)$
 $Q^-(1, s) = 1 = Q^+(1, a) = Q^-(2, a) = Q^-(3, b) = Q^+(3, b) = Q^+(3, d)$
 $= Q^-(4, c) = Q^-(4, d) = Q^+(4, e) = Q^+(5, e) = Q^+(6, x)$. (The values,
 other than 1, are indicated by small numerals).

Figure 4.1.1

Given a bigraph B, it is easy to construct trans-labelled ordinary Petri net N which produces the same sequences as B. Let us proceed in stages.

Stage A. Each arc of B corresponds to a place of N, each node of B corresponds to a transition of N. An arc is drawn from a place p to a transition t if and only if the arc corresponding to p is in the inbranching set of the node corresponding to transition t. An arc is drawn from a transition t to a place p if and only if the arc corresponding to p is in the outbranching set of the node corresponding to t. M^0 , the initial marking of N is defined as follows:

$$M^0(x) = \begin{cases} 1 & \text{if } x \text{ corresponds to the entry arc} \\ 0 & \text{otherwise.} \end{cases}$$

The set of final markings $F = \{F_1\}$ where

$$F_1(x) = \begin{cases} 1 & \text{if } x \text{ corresponds to the exit arc} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, corresponding to the graph B in Figure 4.1.1, we get a net N shown in Figure 4.1.2.

Figure 4.1.2

$$M^0(p_s) = 1$$

$$M^0(p) = 0, \quad p \neq p_s$$

$$F = \{F_1\}$$

$$F_1(p_x) = 1$$

$$F_1(p) = 0, \quad p \neq p_x$$

The net obtained does not really fall into any of the 9 classes discussed earlier but is really a combination of TN_{PLEX} , TN_{XORO} , TN_{XORI} . The input and output logics are indicated. A numeral next to arc (t, p) indicates the number of markers placed in p when t terminates. A numeral near (p, t) indicates the number of markers that must be in p before t can initiate. This is also the number of markers removed from p when t is enabled.

Stage B. In order to be able to talk of initiations and terminations separately, each transition in the net (obtained in stage A) is split into two as indicated in figure 4.1.3.

Figure 4.1.3 (contd. on next page)

Figure 4.1.3 (contd.)

Let the new net obtained be $N_1 = (T_1, P_1, A_1, M_1^0, F^1, f_e^1)$. Define a trans-labelling $I_1 \ni I_1(\bar{t}) = \bar{v}$ and $I_1(t) = v$ where t corresponds to vertex v of B . Let X be the set of new places introduced in N_1 . Then M_1^0 is the same as M^0 except that all places in X are assigned zero markers. F^1 is the same as F except that the final marking M in F^1 assigns zero markers to each place in X . It should be obvious that $I_1(\text{SIMSET}_{N_1}) = S_B$, and N_1 is well behaved if B is. For the net N in Figure 4.1.2, we get the net N_1 shown in Figure 4.1.4. The numerals near the arcs have the same meanings as before.

Theorems 2.11.6 and 2.11.7 and the respective stage by stage replacements indicate that a net $N_2 = (T_2, P_2, A_2, M_2^0, F^2, f_e^2) \in \text{TN}_{\text{PLEX}}$ can be constructed with trans-labelling $I_2 \ni I_2(\text{SIMSET}_{N_2}) = I_1(\text{SIMSET}_{N_1})$ and N_2 is well behaved if N_1 is. In particular, the replacements to be used to obtain N_2 are shown in Figure 4.1.5.

Figure 4.1.5

M_2^0, F^2 and I_2 can be suitably defined so that $I_2(\text{SIMSET}_{N_2}) = I_1(\text{SIMSET}_{N_1})$ and N_2 is well behaved if N_1 is. We can now directly apply Theorem 2.11.8 to conclude that there exists a net $N_3 \in \text{TN}_{\text{PLEX}}$ and

trans-labelling $I_3 \ni I_3(\text{SIMSET}_{N_3}) = I_2(\text{SIMSET}_{N_2})$ and N_3 is well behaved if N_2 is. Thus, there exists a net $N_3 \in \text{TN}_{\text{PLEX}}$ and trans-labelling $I_3 \ni I_3(\text{SIMSET}_{N_3}) = S_B$ and N_3 is well behaved if B is.

To study the capabilities of CBD graphs, we need not be concerned with initiations and terminations of computations but with process sequences. We can also generalize the CBD by considering a triple $((V, A), L, Q)$ where (V, A) is a digraph. Initial and final markings can be appropriately defined, where an initial marking is an initial distribution of tokens on arcs and final markings are final distributions of tokens on arcs where for each final marking the number of tokens on some arcs may be left unspecified. The terms "trans-labelling", "simulation sequence", "process sequence" and $I(\text{SIMSET}_B)$ can be defined as in Section 2.11.

Clearly, given any such graph B and trans-labelling I' a net $N \in \text{TN}$ can be constructed with trans-labelling $I \ni I(\text{SIMSET}_N) = I'(\text{SIMSET}_B)$ and N is well behaved if B is. The construction is similar to that described earlier in this section. Also, given any $N \in \text{TN}$ and trans-labelling I , a graph B can be constructed with trans-labelling $I' \ni I'(\text{SIMSET}_B) = I(\text{SIMSET}_N)$ and B is well behaved if N is. Here also, places of N correspond to multi-head/multi-tail arcs of B and transitions of N to vertices of B .

The above discussion establishes that the controlling agent in bi-graphs is essentially an ordinary Petri net and thus has the same capabilities and limitations.

4.2 The Karp and Miller Program Graph Model

This model was proposed by Karp and Miller [14] in 1966 to describe and analyze parallel computations. The emphasis in the report was to

show that each graph represented a unique computation and to investigate termination properties. Data queues are associated with arcs in the graph. Nodes represent computation steps. Whether a particular step can be initiated or not depends on the lengths of the queues on branches directed into the associated node. The type of control is severely restricted in that each queue has a unique "source" and a unique "sink" node. Data dependent conditional transfers cannot be represented. More formally, a computation graph G is defined as follows:

Definition 4.2.1: A computation graph G is a directed graph consisting of:

- (i) nodes $n_1, n_2, \dots, n_\ell$;
- (ii) branches $d_1, \dots, d_t$, where any branch d_p is directed from a specified node n_i to a specified node n_j ;
- (iii) four nonnegative integers A_p, U_p, W_p, T_p , where $T_p \geq W_p$, associated with each branch d_p .

For $d_p = (n_i, n_j)$, the parameters have the following interpretation:

A_p gives the number of data words initially in the first-in, first-out queue associated with d_p . U_p gives the number of words added to the queue whenever the operation O_i associated with n_i terminates; W_p gives the number of words removed from the queue whenever the operation O_j is initiated; and T_p is a threshold giving the minimum queue length of d_p which permits the initiation of O_j .

In the uninterpreted version of this model the arcs can be considered to carry markers rather than be associated with queues of data. Removal and placing of data then correspond to removal and placing of markers. The nodes can have processes associated, rather than steps of an operation.

If each node is replaced by a "transition" and in each arc, a "place" is inserted to hold the markers, then the resulting net closely resembles the marked graph model.

For any graph G , final markings, simulation sequences, trans-labeling, process sequences, $I(\text{SIMSET}_G)$ etc. can be defined as in Section 2.11.

Let the following restrictions be placed on the parameters A_p, U_p, T_p and W_p : $A_p = 0$ or $1, U_p = W_p = T_p = 1$ for all arcs d_p . Let the resulting class of graphs be called GR1 . Then clearly the control representable by a graph $G \in \text{GR1}$ can be represented by a marked graph and vice-versa. Thus, $\text{GR1} = \text{TN}_{\text{MARK}}$.

Let A_p, U_p, W_p, T_p be nonnegative integers, but let W_p be restricted to be equal to T_p . Let the resulting class of graphs be GR2 . This class of graphs has already been studied in the proof of Theorem 2.11.8. Notice that a graph $G \in \text{GR2}$ can be considered to be a net $N \in \text{TN}_{\text{PLEX}}$ with the restriction that each place has exactly one input and one output transition. It was shown in Theorem 2.11.8 that the control provided by such a net N can also be provided by a net $N' \in \text{TN}$. However, as in the proof of Theorem 2.11.4, it can be shown that GR2 cannot represent all the coordinations representable by TN . Thus GR2 lies between TN_{MARK} and TN in our hierarchy.

Let us now consider the unrestricted model. Figure 4.2.1 gives a node in some particular graph G .

$$\begin{aligned} A_1 &= A_2 = A_3 = 0 \\ U_1 &= U_2 = 1 \\ T_1 &= 3, T_2 = 2 \\ W_1 &= 2, W_2 = 1 \\ U_3 &= 2, W_3 = T_3 = 1 \end{aligned}$$

Figure 4.2.1

Replacing arcs by places and the node by a transition yields the net in Figure 4.2.2.

Figure 4.2.2

If $W_1 = 3$ and $W_2 = 2$ then the replacement shown in Figure 4.2.3 (ignoring dotted lines) is a valid one as demonstrated in the proof of Theorem 2.11.8.

Figure 4.2.3

If $W_1 = 2$ and $W_2 = 1$ then after the test for $P_1 \geq 3$ and $P_2 \geq 2$ is made, 1 marker must be replaced in each of p_1 and p_2 . This is achieved by the dotted lines in Figure 4.2.3. Since the difference $T_p - W_p$ is always finite and a priori known, after the tests for initiation are made and a particular transition enabled, $T_p - W_p$ markers can be replaced in each place representing arc d_p . Thus, the control representable by any graph in the Karp and Miller program graph model can be represented by an ordinary Petri net. The reverse, however, is not true as can be shown in a manner similar to the proof of $TN \rightarrow TN_{MARK}$. Thus the control of the Karp and Miller program graph model is less powerful than ordinary Petri nets and lies between TN_{MARK} and TN in our hierarchy.

4.3 The Karp and Miller parallel program schemata

The parallel program schema introduced and studied by Karp and Miller in 1968 [15] consists essentially of: a set M of memory locations; a finite set A of operations; and T , a control for sequencing the initiations and terminations of operations. Associated with each operation $a \in A$ is a set of domain locations $D(a) \subseteq M$ and a set of range locations $R(a) \subseteq M$. In its unrestricted form, T is a very general control defined as a four-tuple (Q, q_0, Σ, τ) where Q is a set of states, q_0 is the initial state, Σ is an alphabet containing, for each $a \in A$, an initiation symbol $\bar{a}$ and the termination symbols $a_1, a_2, \dots, a_{K(a)}$. τ is a partial transition function from $Q \times \Sigma$ to Q which is total on termination symbols. If the control is in some state q_i then some operation $a \in A$ can initiate if $\tau(q_i, \bar{a})$ is defined. On initiation the control proceeds to state $\tau(q_i, \bar{a})$. Once an operation a initiates, it obtains its operands from $D(a)$ and at some later time places the results in $R(a)$. The model can

represent conditional transfers since each operation a can terminate with any one of the symbols $a_1, \dots, a_{K(a)}$. Also, several initiations of the same operation are possible before termination. More formally,

Definition 4.3.1: A (parallel program) schema $S = (M, A, T)$ is specified by

- (1) A set M of memory locations
- (2) A finite set $A = \{a, b, \dots\}$ of operations and for each operation a in A :

- (i) a positive integer $K(a)$ called the number of outcomes of a ;
- (ii) a set $D(a) \subseteq M$ whose elements are the domain locations for a ;
- (iii) a set $R(a) \subseteq M$ whose elements are the range locations for a .

- (3) A quadruple $T = (Q, q_0, \Sigma, \tau)$ called the control where:

- (i) Q is a set of states;
- (ii) q_0 is a designated state called the initial state;
- (iii) Σ , the alphabet, is the union of $\Sigma_1 = \bigcup_{a \in A} \{\bar{a}\}$ the initiation symbols, and $\Sigma_t = \bigcup_{a \in A} \{a_1, a_2, \dots, a_{K(a)}\}$ the termination symbols.
- (iv) τ , the transition function, is a partial function from $Q \times \Sigma$ into Q which is total on Σ_t .

Clearly, T is an extremely general control since absolutely no restrictions are placed either on the cardinality of Q or on the transition function τ . The specification of T in general does not consist of a "finite description". We will thus move directly to more restricted controls studied by Karp and Miller.

Definition 4.3.2: $S = (M, A, T)$ is a finite state schema if S is a schema that has a finite number of states.

We have already established the equivalence of finite state automata and safe Petri nets earlier in this report, in the sense that every regular language can be generated by a trans-labelled safe Petri net and vice versa. Thus, the finite state schema of Karp and Miller = TN_{SAFE} . However, we will illustrate with an example [18] the construction of a Petri net, given a finite state control T of the Karp and Miller schema. This example also illustrates how sequences of initiations and terminations (rather than process sequences) are modeled using Petri nets.

Example 4.3.1

The graph in Figure 4.3.1. is the control T of a Karp and Miller parallel program schema.

Figure 4.3.1

Each of the operations a , b , c has only one possible outcome, d has two. Note that the control specifies that a and b can proceed concurrently and c can proceed only on completion of a and b . The schema thus represents a simple fork-join pair.

A trans-labelled Petri net representing the same coordination and that has an obvious one to one correspondence with the net in Figure 4.3.1 is given in Figure 4.3.2. A simpler Petri net representing essentially the same coordination without going to the level of initiations and terminations is given in Figure 4.3.3.

Figure 4.3.2

Figure 4.3.3

The nets in Figures 4.3.2 and 4.3.3 essentially provide distributed controls. A central control which is a (safe) Petri net can also be constructed, that controls the operations a, b, c and d shown in Figure 4.3.4. a, b, c and d can be viewed as processes not represented by a single transition as in Figure 4.3.3 but split into a set of transitions to represent initiation, terminations, etc.

Figure 4.3.4

We shall proceed now with another restriction of parallel graph schemas, namely the parallel flow charts.

Definition 4.3.3: A parallel flow chart is a three-tuple $FC = (M, A, T)$ where

(1) M is a finite set of memory locations.

(2) A is a finite set of operations and for each $a \in A$,

(i) $K(a)$ is a positive integer specifying the number of outcomes of a

(ii) $D(a) \subseteq M$ is the domain of a

(iii) $R(a) \subseteq M$ is the range of a .

(3) $T = (Q, q_0, \Sigma, \tau)$, the control is specified in terms of a nonnegative integer p and a function $v: \Sigma \rightarrow \{-1, 0, 1\}^p$ as follows:

(i) $Q = N^p$ (where N is the set of nonnegative integers) is the set of control states

(ii) $q_0 \in Q$ is the initial control state

(iii) Σ is the union of

$$\Sigma_i = \bigcup_{a \in A} \{\bar{a}\}$$

the set of initiation symbols and

$$\Sigma_t = \bigcup_{a \in A} \{a_1, \dots, a_{K(a)}\}$$

the set of termination symbols.

(iv) τ is a partial function, $\tau: Q \times \Sigma \rightarrow Q$ called the transition

function. $\tau(q, \sigma)$ is defined if $q + v(\sigma) \geq 0$ and in this case,

$\tau(q, \sigma) = q + v(\sigma)$. The function v has the following constraints:

for all $a \in A$, $v: \Sigma_i \rightarrow \{0, -1\}^p$; $v: \Sigma_t \rightarrow \{0, 1\}^p$; for any two

distinct initiation symbols σ and σ' , if $v(\sigma)_i = -1$ then $v(\sigma')_i = 0$

[x_i denotes the i th element of vector x].

In this model, the control essentially consists of p counters which always have positive values. The state of the control is then specified at any stage by giving the p -tuple of values of the counters. Each operation σ has a p dimensional vector $v(\sigma)$ associated with it. A particular transition (initiation or termination) is defined if $q + v(\sigma) \geq 0$. Thus terminations are always defined since for $\sigma \in \Sigma_t$, $v(\sigma) \geq 0$. Initiation of a particular operation may or may not be defined in some control state. If it is defined, that initiation may take place leading to a new control state. Once an operation initiates it must terminate in some finite time. In the framework of Petri nets, an operation a can be viewed as in Figure 4.3.5.

Figure 4.3.5

The set $\{i \mid v(\bar{\sigma})_i = -1\}$ can be viewed as the set of input counters of operation σ . The set $\{i \mid v(\sigma_j)_i = +1\}$ can be viewed as output counters for the termination σ_j of σ . Thus, an operation σ can initiate only if all the input counters have values greater than zero. When it initiates, it decrements the values of the input counters by 1. Subsequently, σ terminates with σ_j and increments the values of all output counters of σ_j . The restriction on v , that for any two $\bar{a}$ and $\bar{b}$, if $v(\bar{a})_i = -1$ then $v(\bar{b})_i = 0$ implies that initiations do not share input counters. If the operations are viewed as in Figure 4.3.5, the counters as places, and

of input/output counter relationships are expressed by means of directed arcs, then clearly, a Petri net $\in \text{TN}_{\text{XORO}}$ results. To investigate the capabilities and limitations of the resulting control, we will not talk about initiations and terminations separately but assume that each transition represents some process. From this viewpoint, the definition of the control of parallel flow charts corresponds to the definition of the class TN_{XORO} with the added constraint that no place has more than one output transition.

Figure 4.3.6 represents a well behaved trans-labelled net from this latter class. As in Theorem 2.11.4, it can be shown that no well behaved trans-labelled net $\in \text{TN}_{\text{MARK}}$ can represent the same coordination. Thus we conclude that the control of parallel flow charts is more powerful than TN_{MARK} and lies somewhere between TN_{MARK} and TN .

Initial marking is shown in figure. $F = \{M^0\}$

Figure 4.3.6

4.4 Slutz' Flow Graph Schemata

The flow graph schemata model, presented and studied by Slutz in 1970 [23], is essentially a generalization of the Karp and Miller parallel flow charts analyzed in the previous section. The model was developed for two reasons. First, it was intended to model more closely certain aspects of parallelism in existing and anticipated systems. Second, it was structured in such a way that properties like determinacy and equivalence could be investigated more fully.

There are two significant differences between FGS's and parallel flow charts. In Karp and Miller's parallel flow charts, terminations are always defined. In FGS's terminations are "controlled" just like initiations. The second significant difference is that a counter can be an input counter of more than one initiation or termination.

Definition 4.4.1: A Flow Graph Schema F is a three-tuple $F = (M, A, T)$ where

- (1) M is a finite set of memory locations.
- (2) A is a finite set of operations and for each $a \in A$:
 - (i) $K(a)$ is a positive integer specifying the outcomes of a ;
 - (ii) $D(a) \subseteq M$ is the domain of a ;
 - (iii) $R(a) \subseteq M$ is the range of a .
- (3) $T = (Q, q_0, \Sigma, \tau)$, the control is specified in terms of a nonnegative integer p and a function $v : \Sigma \rightarrow (\{-1\} \cup \mathbb{N})^p$ as follows:
 - (i) $Q = \mathbb{N}^p$ is the set of control states;
 - (ii) $q_0 \in Q$ is the initial control state. For each $a \in A$ there is a coordinate j_a such that $(q_0)_{j_a} = 0$;
 - (iii) $\Sigma = \bigcup_{a \in A} \{\bar{a}, a_1, \dots, a_{K(a)}\}$ is the alphabet;

(iv) τ is a partial function, $\tau: Q \times \Sigma \rightarrow Q$ is called the transition

function. $\tau(q, \sigma)$ is defined if $q + v(\sigma) \geq 0$ in which case

$\tau(q, \sigma) = q + v(\sigma)$. The function v has the following constraints:

(a) $(v(a_1))_k = -1 \Rightarrow (v(a_j))_k = -1, 1 \leq i, j \leq K(a)$

(b) for all $b \in \Sigma$ where $b \neq \bar{a}, a_1, \dots, a_{K(a)}, (v(\bar{a}))_{j_a} = 1,$

$(v(a_1))_{j_a} = -1$ and $(v(b))_{j_a} = 0$.

Restriction (a) on v is the requirement that all terminations of an operation have the same set of input counters. Restriction (b) is the requirement that a counter j_a exist (which is initially empty) that is an output counter only for $\bar{a}$ and an input counter only for terminations of a .

Each operation a , in the Petri net framework, can be viewed as in

Figure 4.4.1.

Figure 4.4.1

The counters can be viewed as places. The input places of $\bar{a}$ consist of the set $\{i \mid (v(\bar{a}))_i = -1\}$ and the output places form the set $\{i \mid (v(\bar{a}))_i > 0\}$. The set of input places of a' is $\{i \mid (v(a_1))_i = -1\}$; a' has $K(a)$ output places and the output logic is EOR. The set of output

places of a_j is $\{1 \mid (v(a_j))_1 \geq 0\}, 1 \leq j \leq K(a)$. Thus, after initiation of a , termination is controlled by a' . Firing of a' arbitrarily enables one of the termination symbols. Each of the σ' transitions can be replaced by a net belonging to TN_{PLEX} in a manner similar to the replacement procedure in the proof of Theorem 2.11.6. The resulting net is a member of TN_{PLEX} and represents the same control as the original FGS. Once again, if the transitions represent arbitrary processes rather than initiations and terminations it is easy to see that the definition of the control in an FGS corresponds exactly to the class TN_{PLEX} .

4.5 Luconi's Computational Schemata

This is another graph model, presented in 1968 [16] with the intent to describe finite, discrete information systems. The model consists of a set of operators standing in fixed relation to one another with respect to paths of communication. The connecting lines may carry data, operator sequencing and control information, timing sequences, and so on. A partial function is associated with each operator defining when and how each operator can process information appearing on the communication lines constituting its inputs. The results of processing are distributed to the operator's output lines (which may include some of the input lines). Because the timing of this processing is arbitrary, operators act asynchronously with respect to each other. More formally:

Definition 4.5.1: A computational schema is an ordered 5-tuple

$S = (A, L, T, \delta, \rho)$ where

- (1) $A = \{a_1, \dots, a_n\}$ is a finite set of operators;
- (2) $L = \{l_1, \dots, l_m\}$ is a finite set of links;

(3) $T \subseteq L$ is a set of links specifying the schema interface or terminal set,

(4) $\delta : A \rightarrow L^d$ is a function specifying the links constituting each operators input set,

(5) $\rho : A \rightarrow L^r$ is a function specifying the links constituting each operators output set.

In addition, a discrete value can be associated with each link. No restrictions are placed on the nature of this value. $V(l_i)$ denotes the set containing every value that could be associated with link l_i . Each operator $a \in A$ has a vector of input links:

$$\delta(a) = (\delta^1(a), \dots, \delta^d(a)), \delta^j(a) \in L$$

and a vector of output links:

$$\rho(a) = (\rho^1(a), \dots, \rho^r(a)), \rho^j(a) \in L.$$

The local configuration $r(a)$ defined for operator a is the vector:

$$r(a) = (\gamma^1(a), \dots, \gamma^d(a))$$

where each $\gamma^1(a)$ is the value associated with link $\delta^1(a)$. A partial function $f(a)$ is defined for each operator $a \in A$. Thus:

$$f_a : D_a \rightarrow R_a$$

where

$$D_a \subseteq \bigtimes_{i=1}^d [V(\delta^i(a))]$$

$$R_a \subseteq \bigtimes_{i=1}^r [V(\rho^i(a)) \cup \{(-)\}]$$

If operator a has local configuration $\gamma(a)$ and if $\gamma(a) \in D_a$ then operator a is applicable. When an operator a applies, it replaces the values associated with the links of its output set according to f_a .

($(-)$ indicates that the value of the associated output link is not to be changed).

A schema defined in the above manner operates as follows. Initially, each link ℓ_i is associated with a value belonging to $V(\ell_i)$. The link-value associations at any stage define a local configuration for each operator $a \in A$. An operator may or may not be applicable in a given local configuration. If it is, and if it applies, it redefines the values associated with some links. The conditions for applicability being local, the operators proceed asynchronously.

With the above definition of a schema we have two alternatives. The schema can be considered to be an interpreted model, where the function associated with each operator a can be thought of as specifying the behavior of a . In this case, the schema can clearly be considered to represent a computation. The second alternative is to consider the function f_a as merely determining when a particular operation a can be initiated and the effect it has on its environment on terminating. Here each operator can be thought of as representing a particular process (that is activated when a initiates and on whose completion, a terminates) that affects entities completely distinct from anything in the schema. Neither the effects nor the entities affected are known. Looking at the schema from this perspective yields an uninterpreted model. It is this uninterpreted model, representing a controlling agent that will be analyzed here. In the second half of his thesis Luconi views a schema as composed of two parts called the Interpretation schema (I-schema) and the Control schema (C-schema). Such a schema is called a structural schema. The I-schema performs the computations and the C-schema determines when operators in the I-schema are enabled. A connection can exist between an I-operator a and a C-operator b through a common link,

control link. (See Figure 4.5.1). In a restricted form each operation in either schema can have at most one input and one output control link.

Figure 4.5.1

Before an operator in the I-schema is eligible for initiation the common link must have a value 0. When the I-schema operator is eligible for initiation the value of the control link is set to one by the C-schema operator. When the I-schema operator terminates its execution, it sets the value of the control variable to 2. It should be noted that the uninterpreted schema we are considering does not correspond exactly to a C-schema.

To proceed with the analysis of the general computational schema viewed as a controlling agent, in the definition of the schema, no restrictions are placed either on the values that may be associated with link or on the function f_a associated with each operator $a \in A$. In this general form, the specification corresponds to a controlling agent as powerful as the Turing machine central controller [3], and hence as powerful as the extended Petri nets. (Whether such a control is useful other than for theoretical purposes and for studying questions regarding decidability is a different matter and will be discussed elsewhere.) One restricted class of computational schemas is the class of finite state computational schemas.

Definition 4.5.2: A finite state computational schema, S , is a schema in which all values that can be associated with a link are required to be members of some finite set V_g , i.e., $V(l_i) \subseteq V_g$ for all i .

As Luconi shows, a finite state transition graph $G(S)$ can be defined for any finite state schema S as follows:

- (i) A node in $G(S)$ corresponds to a state of S where a state is defined as a particular set of link value associations. Since each link can take on only a finite number of values, there are only a finite number of states.
- (ii) A directed arc is drawn from node i to node j and labelled a if operator a is applicable in the state corresponding to node i and if application of a in this state leads to the state corresponding to node j .

Clearly, $G(S)$ represents the same control as S . Similarly, for every finite state machine (representing a control) a finite state schema can be given that represents the same control. We can thus conclude that finite state schemas = TN_{SAFE} .

The other restricted schemas studied by Luconi are well-formed schemas. A schema is well-formed if it is conflict-free and transformational-lossless. A schema is conflict-free if for any pair of operators a and b that can become applicable simultaneously, either (i) a and b have no output links in common, or (ii) a and b do not alter the value of a common output link, or (iii) both a and b indicate identical value changes to all common output links. A schema is transformational-lossless if for any pair of operators a and b that can become applicable simultaneously either (i) a and b do not communicate or (ii) the application of neither can affect the value changes about to be made by the other.

Thus, in well-formed schemas, once an operation becomes applicable, it continues to remain applicable. A very formal analysis has not been conducted but the above fact seems to indicate that there exist problems where the desired control can be represented by ordinary Petri nets but not by well-formed schemas. Whether the reverse is true is not known. Clearly, the coordination represented by the net in Figure 4.3.6 can be represented by a well-formed schema. Thus, well-formed schemas are more powerful than marked graphs.

4.6 Rodriguez' Graph Model of Computation

This model was proposed and studied by Rodriguez in 1969 [22]. The underlying control is basically a finite, well-formed schema. In Section 4.5 we considered a schema to correspond to a controlling agent. Each activation of an operator was considered to correspond to the initiation of some one process. An operation was activated when the vector of values of the input links belonged to a specific set. On termination values were assigned to the output links depending on the values of the input links at the time of activation. Irrespective of the conditions under which a node was activated, the same process was initiated. In the Rodriguez model, the process initiated will depend, in general, on the value of the input links at the time of activation. Thus the underlying control here corresponds more closely to a C-schema.

The model is composed of nodes (which correspond to computational elements) and links (which correspond to storage-transmission elements). There are two types of links: a data link and a control link. Associated with both data and control links is a quantity called link status. At any given time the link status of a link assumes precisely one of the

four possible values -1, 0, 1, 2. These values are called disabled, idle, enabled and blocked respectively. In addition, data links can carry data. There are seven types of nodes. Nodes have specific points of attachment called connectors. Connectors are either input or output connectors. The activation of a node is determined by the link status of the links attached to the node. A node is in an active configuration when the statuses of its connectors is such that the node becomes active. The occurrence of an active configuration initiates a transition of arbitrary time length. On completion of the transition, the status of each connector (and perhaps the data contents of the attached data links as well) are changed in a way specified by the transition table for that type of node.

The program graph nodes and their transition tables are given in Figure 4.6.1. Thus, for the function operator one transition is

$$1\ 1\ 0 \rightarrow 0\ 0\ 1 \quad \$3 \leftarrow f(\$1, \$2)$$

This transition indicates that if the statuses of links 1, 2 and 3 are 1, 1, 0 respectively, then the data contents of connector 3 are replaced by the result of applying f to the data contents of connectors 2 and 1, and the statuses of links 1, 2 and 3 are given the values 0, 0, and 1 respectively. For an in-depth explanation of the different nodes, the reader is referred to Rodriguez' thesis.

A program graph is a finite set of input terminals, output terminals and nodes interconnected by data links and control links according to the following rules:

- (1) The root of a data link must be attached to an input terminal or to a data output connector and its tip must be attached to

Operator

Data operator

Function

1 1 0 → 0 0 1 \$3 ← f(\$1, \$2)

1 -1 0 → 0 0 -1

-1 1 0 → 0 0 -1

-1 -1 0 → 0 0 -1

Identity

1 0 0 → 0 1 1 \$2, \$3 ← \$1

-1 0 0 → 0 -1 -1

Control operator

And

1 1 0 → 0 0 1

1 -1 0 → 0 0 -1

-1 1 0 → 0 0 -1

-1 -1 0 → 0 0 -1

Or

1 1 0 → 0 0 1

1 -1 0 → 0 0 1

-1 1 0 → 0 0 1

-1 -1 0 → 0 0 -1

Selector

1 1 0 0 → { 0 0 -1 1 if β(\$1, \$2) = true
 0 0 1 -1 if β(\$1, \$2) = false

1 -1 0 0 → 0 0 -1 -1

-1 1 0 0 → 0 0 -1 -1

-1 -1 0 0 → 0 0 -1 -1

Junction

1 -1 0 → 0 0 1 \$3 ← \$1

-1 1 0 → 0 0 1 \$3 ← \$2

-1 -1 0 → 0 0 -1

Figure 4.6.1

Loop Junction

$1\ 0\ 0\ 0 \rightarrow 2\ 0\ 2\ 1$ $\$4 \leftarrow \1
 $1\ 1\ 0\ 0 \rightarrow 2\ 1\ 2\ 1$ $\$4 \leftarrow \1
 $1\ -1\ 0\ 0 \rightarrow 2\ -1\ 2\ 1$ $\$4 \leftarrow \1
 $-1\ 0\ 0\ 0 \rightarrow 2\ 0\ 2\ -1$
 $-1\ 1\ 0\ 0 \rightarrow 2\ 1\ 2\ -1$
 $-1\ -1\ 0\ 0 \rightarrow 2\ -1\ 2\ -1$
 $2\ 1\ 0\ 0 \rightarrow 2\ 0\ -1\ 1$ $\$4 \leftarrow \2
 $2\ -1\ 0\ 0 \rightarrow 0\ 0\ 1\ 0$

Loop Output

$1\ 1\ 0 \rightarrow 0\ 0\ 1$ $\$3 \leftarrow \1
 $1\ -1\ 0 \rightarrow 0\ 0\ -1$
 $-1\ 1\ 0 \rightarrow 2\ 0\ 0$
 $-1\ -1\ 0 \rightarrow 2\ 0\ 0$
 $2\ 1\ 0 \rightarrow 0\ 1\ 0$
 $2\ -1\ 0 \rightarrow 0\ -1\ 0$

Input Terminal

Output Terminal

$1 \rightarrow 0$
 $-1 \rightarrow 0$

Figure 4.6.1 (contd.)

an output terminal or to a data input connector.

- (2) The root of a control link must be attached to a control output connector and its tip must be attached to a control input connector.
- (3) Every input connector of a node must be attached to a control input connector.

All links are in the idle (0) state initially. At some time t_0 , all input terminals are placed in an enabled state simultaneously and execution begins. Figure 4.6.2 gives an example of a program graph.

The underlying controlling agent is now obtained as follows: No distinction is made between data links and control links.

All links are control links. Let $\{\ell_1, \dots, \ell_n\}$ denote the set of nodes in any program graph. Each node ℓ_i has an associated transition table t_i containing a set of transitions $\{t_{i_1}, \dots, t_{i_{k(i)}}\}$. The t_{i_j} 's are as shown in Figure 4.6.1 except that in each transition t_{i_j} , if there is a data transformation specified, it is replaced by a symbol σ_{i_j} . Thus if ℓ_i is a function node, the transition $t_{i_1} = 1\ 1\ 0 \rightarrow 0\ 0\ 1\ \$3 \leftarrow f(\$1, \$2)$ is replaced by $1\ 1\ 0 \rightarrow 0\ 0\ 1\ \sigma_{i_1}$. Each σ_{i_j} can now refer to an arbitrary process. For each selector node, the transition

$$1\ 1\ 0\ 0 \rightarrow \begin{cases} 0\ 0\ -1\ 1 & \text{if } \beta(\$1, \$2) = \underline{\text{True}} \\ 0\ 0\ 1\ -1 & \text{if } \beta(\$1, \$2) = \underline{\text{False}} \end{cases}$$

is replaced by two transitions

$$1\ 1\ 0\ 0 \rightarrow 0\ 0\ -1\ 1$$

$$1\ 1\ 0\ 0 \rightarrow 0\ 0\ 1\ -1$$

Thus, if the configuration of the selector is 1 1 0 0, either one of the two transitions could take place arbitrarily. If a function A is defined

$$\text{Sqrt} (\text{Sin}(X \uparrow 2) + \text{cos} (1 - \text{tan}(Y) \uparrow 2))$$

Figure 4.6.2

for a graph G such that A maps the set of σ_{ij} 's (say Ω) onto a set $\{P_1, \dots, P_k\}$ where $k \leq \# \Omega$ then G and A represent a coordination of the processes $\{P_1, \dots, P_n\}$ rather than a computation.

Let a program graph G have p links. Since each link can take on at most 4 status values, the p -tuple of status values of the link can be considered to constitute a "system state" of G . Clearly, G has a finite number of system states (say $\{q_1, \dots, q_s\}$). A finite state transition diagram, G' , corresponding to G , can be constructed as follows:

G' has S nodes, $q_1, \dots, q_s$, each corresponding to a unique system state of G . An arc is drawn between q_i and q_j and labelled x_{km} if node l_k of G is in an active configuration in state q_i and the m th transition of table t_k causes the state to change from q_i to q_j . When all the arcs are drawn each label x_{km} is replaced by P if $A(\sigma_{km}) = P$ and is deleted if $A(\sigma_{km})$ is undefined. Clearly, G' represents the same coordination as G . Since TN_{SAFE} can represent any coordination that can be represented by a finite state transition diagram, we conclude that the underlying control in Rodriguez' model $\subseteq TN_{SAFE}$.

4.7 Adam's Model

Adam's model presented in 1970 [1] is an interpreted model, the motivation once again being the study of parallel computations. The model is fairly complex and we shall not attempt to formally define it. The main features will be explained and the underlying controlling agent extracted.

The model consists of a directed graph with the nodes representing computation steps and the arcs, FIFO queues of data. In the Adam model each node can be either a primitive node or a procedure where the latter

is also defined as an interpreted graph. For our purpose we will consider each node to be a primitive node representing as usual, some arbitrary process rather than a particular computation.

The possible data types are defined separately by means of a context free grammar. Each edge e is of a particular data type d and the parameter d identifies the structure of the elements found on the queue associated with e . Primitive nodes are of two types, r or s . r nodes are computational and s nodes, both computational and logical. The inputs of each s node carry status values. The status of a particular input edge can be locked (L) or unlocked (U). With each r node is associated a function f and with each s node, two functions f and g . The f function receives an argument from each input edge that is unlocked. f may produce output on only some or possibly none of the output edges. More than one element may be placed on an edge as output. The g function associated with an s node allows the status settings of that node to be changed as a function of the current settings and the input arguments of the node. For a given ordered set of inputs, both f and g have uniquely defined outputs.

Informally, the program graph executes as follows: The initial data for the program is placed on the appropriate edges. During the execution of the graph program, a node in the graph may be executed whenever there is at least one element of data on each of its unlocked input edges. When the node executes, it removes one element of data from each unlocked input, performs the indicated computation, and places the results on the appropriate edges leading out from the node. For the s -nodes, the status settings may also be changed. When no further nodes

can be executed, the computation is complete, and the results will be left on the nodes designated as the output set of G .

For simplicity, let us assume that no activation of a node places more than one result on any output arc. In the Petri net framework, each edge in a graph G can be replaced by a place. Each data element is a marker. Each node corresponds to a net, as will be explained. For any node, let $I = \{e_1, e_2, \dots, e_k\}$ be the set of input edges and $O = \{e_{k+1}, \dots, e_{k+m}\}$ the set of output edges. Then an r -node can be represented as shown in Figure 4.7.1.

Figure 4.7.1

Each t_1 has as its output places a particular subset of 2^O . The transition r has an exclusive -OR output because, depending on f and the arguments on the input edges of r , any subset of the output edges would receive data. The s -node is represented by a more complicated network, because of the associated status values. The net to replace the s -node is given

in Figure 4.7.2. Each place $q_i, 1 \leq i \leq 2^k$ represents a particular assignment of status values to the edges $e_1, \dots, e_k$, i.e., represents a particular subset K_i of 2^I . Each transition S_i has as inputs, place q_i and the set of places K_i . Each of $S_1, S_2, \dots, S_{2^k}$ represents node s . Firing of one of these activates transition t which selects the

Figure 4.7.2

subset of 2^0 to receive markers. When markers have been placed in the appropriate output places, t' is also activated. t' arbitrarily selects the next set of status values. We will assume that given a graph model, the underlying controlling agent is obtained by the above two replacement procedures. The resulting control is then clearly a subset of TN_{XORO} , where once a process has been enabled, it cannot be disabled by any other process.

4.8 Peterson's P-nets

This model was studied by Peterson in 1973 [19], who viewed it as the control structure of a model of parallel computation. The p-net is essentially an ordinary Petri net where multiple arcs are allowed between places and transitions and between transitions and places.

The p-net is a directed bi-partite graph with two types of nodes, places (represented by circles) and transitions (represented by bars). Arcs are directed from one type of node to the other. Arcs directed into a node define its inputs while arcs directed out of a node define its outputs. Each transition has an operation or label associated with it. A start place and a set of final places are defined for each p-net. Figure 4.8.1 gives an example of a p-net.

Figure 4.8.1

The simulation rules are the same as for ordinary Petri nets except that multiple arcs require multiple tokens. The labels associated with the arcs correspond to a particular trans-labelling. Corresponding to initial and final places, initial and final markings can be defined. Multiple arcs, as between t_1 and p_3 , can be replaced by a single arc with a numeral indicating the number of markers required. If instead of a start place and final places, initial and final markings are defined, then clearly, p-nets = TN_{PLEX} .

4.9 Baer's Switch Petri nets

This modified form of Petri nets was introduced by Baer in 1973 [7]. In addition to ordinary transitions and places Baer allows the following: Disjunctive input transitions (Figure 4.9.1), disjunctive output transitions (Figure 4.9.1), switches (Figure 4.9.2) and token absorbers (Figure 4.9.3).

Input Disjunctive Logic.
Only one of the input
places can be full.
Then a can fire.

Output Disjunctive Logic
After firing of a, only
one of the output places
will receive a token.

Figure 4.9.1

(a) Empty Switch

(b) Full Switch

Figure 4.9.2

token absorber

When t fires, it places one marker in each of p_1 and p_2 and removes all markers from p_3 and p_4 .

Figure 4.9.3

Formally:

Definition 4.9.1: The switch Petri net is a triple $P = (X, A, C)$ where:

$X = P \cup T$ is a finite set of vertices with

$P = \{p_1, \dots, p_n\}$ being a finite set of places;

$T = \{t_1, \dots, t_m\}$ being a finite set of transitions;

S : a possibly empty proper subset of P is a set of switches;

$A = I \cup O \cup N$ is a finite set of arcs with

$I = \{(p_i, t_j) \mid p_i \in P, t_j \in T\}$ being the input arc set and p_i being an input place to t_j ;

$O = \{(t_i, p_j) \mid t_i \in T \text{ and } p_j \in P\}$ being the output arc set and p_j being an output place to T_i .

$N = \{(t_i, [p_1, p_j, \dots, p_k]) \mid t_i \in T \text{ and } p_1, p_j, \dots, p_k \in P\}$ being the token absorber set and $p_1, p_j, \dots, p_k$ being the cancelled places.

C : is the control which associates with each transition a pair of logics, i.e., one of the possible combinations $\{(AND, AND), (AND EOR), (EOR, AND), (EOR, EOR)\}$.

The following topological restrictions are imposed. If $(p_i, t_j) \in I$, then $(t_j, p_i) \notin O$. If $p_i \in S$ and $(p_i, t_j) \in I$, then no other switch is an input place to t_j ; t_j is either of (AND, EOR) or (EOR, EOR) logic and there are only two places to t_j , the two output arcs leading from t_j , being labelled respectively f and e .

In his paper, Baer states the following: "although the EOR and AND logics have sufficient properties to show the flow of control in algorithms, we introduce nevertheless a new type of place that we call switches."

This statement indicates that the switch Petri nets are not really more powerful than Petri nets with mixed input, output logics. This is not true. In a previous report [3], we proved that extended Petri nets (TN_{EXT}) were more powerful than ordinary Petri nets (TN) with respect to their capabilities of representing coordinations and in fact were complete, i.e., they could represent all possible types of interaction between a group of processes. Just as extended Petri nets were used to construct a central controller as powerful as a Turing machine, switch Petri nets can also be used to construct a similar controller. This report is not the place to go into the details of such a construction but the interested reader can easily see how it is accomplished by referring to our earlier

report. Thus we conclude that switch Petri nets (TN_{SWITCH}) are complete and $TN_{\text{SWITCH}} = TN_{\text{EXT}}$.

4.10 Other models

In 1970 Brédet and McCluskey proposed a model [8] where flow table methods from sequential circuit theory are applied to describe a control mechanism for asynchronous processes. The mechanism can be implemented either as a control circuit or a control program but in either case, it represents a finite state control and is thus equivalent to TN_{SAFE} .

Extensions to Dijkstra's P and V primitives [11] for synchronizing have been studied by Cerf [10]. He proposes PA, PX and V primitives. These are equivalent to the CBD model. The extended primitives under study by us are defined as follows:

$$PE(S_1, S_2, \dots, S_n, S_{n+1}, \dots, S_{n+m}) = [\text{if } S_i > 0, 1 \leq i \leq n \text{ and } S_{n+i} = 0, 1 \leq i \leq m, \text{ then } S_i \leftarrow S_i - 1, 1 \leq i \leq n \text{ else Wait}]$$

$$V(S_1, \dots, S_n) = [S_i \leftarrow S_i + 1, 1 \leq i \leq n]$$

As usual, these operations are "indivisible." It is easy to see that these primitives are equivalent to TN_{EXT} and therefore also complete.

5. Summary and Conclusions

The final hierarchy of nets and the relative positions of different controlling agents with respect to the hierarchy is given on page 78 and the abbreviations are explained on page 79. At the top of the hierarchy we have seven models. PE, V primitives, ASSM, ATMC, LGCS, GKMPs, TN_{EXT} and BSPN. Of these PE, V primitives, TN_{EXT} and BSPN are probably the only ones that have significant practical usefulness in modelling and representing parallel systems. This will be the subject of a future technical report.

As mentioned in the introduction, this report is a first step towards establishing a uniform underlying theory for the study of asynchronous systems. The results show the close relationship between various models. In the future, isolated studies on different models should not be the direction taken by researchers. Rather, more effort should be directed towards establishing a theory for modelling, analyzing, synthesizing and understanding systems of asynchronous processes.

CBD

UCLA : Complex Bilogic Directed Graph, UCLA

KMPGM : Karp and Miller Program Graph Model

GR1 : Restricted KMPGM, $A_p = 0$ or 1 , $U_p = W_p = T_p = 1$

GR2 : Restricted KMPGM, $W_p = T_p$

GKMPs : General Karp and Miller Parallel Program Schema

FSKMPs : Finite State Karp and Miller Parallel Program Schema

KMPF : Karp and Miller Parallel Flow Charts

SFGS : Slutz Flow Graph Schema

LGCS : Luconi's General Computational Schema

LFCS : Luconi's Finite State Computational Schema

LWFCs : Luconi's Well formed Computational Schema

AGM : Adam's Graph Model

RGM : Rodriguez' Graph Model

PPN : Peterson's P-Nets

BSPN : Baer's Switch Petri Nets

BMFSC : The Bredt and McCluskey Finite State Control

ATMC : Agerwala's Turing Machine Control

ASSM : Agerwala's SS Model

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NOTE 1

For theoretical questions concerning capabilities, limitations, etc. this assumption is valid. It essentially amounts to saying that one further restriction has to be placed in the simulation rules, i.e., only one transition at a time is allowed to fire. This does not mean that the net cannot represent asynchrony in the theoretical context. A process

can easily be represented as

P^i indicates initiation and P^t , termination. Now, the rule that transitions cannot fire at the same time simply reduces to the rule that initiations and terminations do not occur at the same instant. Processes that have initiated, but not terminated, are running and a number of processes may be running at the same time, thus representing asynchronous operation. Thus the assumption does not affect the modelling power of a net. In a net used to represent some practical problem, the restriction may be removed. Even so, we do assume that removal of markers from input places and placement of markers in output places by different transitions do not occur at the same time. Also, in a practical situation one would normally want well behaved nets.

NOTE 2

Important note introduced at time of publication

It has been brought to my notice that J.L. Peterson and T.H. Bredt have performed a similar comparison of models of computations in their paper "A comparison of models of parallel computations" preprints, IFIP Congress 1974, August 5-10, 1974, Stockholm, Sweden. The portion of their results that overlap with those presented here is given below:

In addition, they also consider Vector Addition Systems, Vector Replacement Systems, T-counter Transition Systems, Coordination Nets, Finishing Schemata, and Message Transmission Systems. The major differences are that their paper is less formal and specific than this one. They do not discuss restricted subclasses within a particular model. The extended Petri nets and Baer's Switch Petri nets are not discussed and we believe both these models are exceedingly important because of their completeness and practical usefulness. Our results also indicate that multiple arcs between a place and a transition do not provide any extra power since $TN = PPN$. Also, the generalized primitives are not mentioned. Overall, the major results are remarkably similar. Both papers establish the

need for a uniform underlying theory of models for parallel systems and provide the first steps in this direction.