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By
Lawrence Cranberg

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ABSTRACT

A method is discussed for determining the momentum of a beam of charged particles bent by a magnetic field, from the ratio of tension to current in a current-carrying wire which follows the same path in the field as the particles. Details are given of the calculations and apparatus necessary for realization of a precision of a few hundredths of one percent in the measurement of momentum.

PHYSICS

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MAGNET CALIBRATION BY THE FLOATING-WIRE METHOD*

Lawrence Cranberg

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

I. Introduction.

It seems to be generally known that a close analogy exists between the trajectory of a beam of electrons or ions in a magnetic field, and the path followed by a flexible, current-carrying wire in the same field; but very little has appeared in the literature regarding the extent to which the analogy is valid, and the uses to which this analogy has been put. Loeb[(1)] has shown that wire and charged particles are subject to identical variational conditions, namely

$$(1) \quad \delta \int_A^B \vec{A} \cdot \vec{ds} = 0$$

where A and B are two fixed points of the path, $\vec{A}$ is the vector potential and $\vec{ds}$ is the vector element of length, and the length of path between A and B is fixed. Loeb has also discussed application of the analogy to ray-tracing through magnetic lenses, and to calibration of magnetic fields for particle momenta, and describes the details of apparatus for ray-tracing. The work to be described here is an extension of Loeb's, the purpose being to explore the limit to which the wire analogy may be pushed for the precise, accurate measurement of charged particle momenta by bending in magnetic fields.

It is claimed by Loeb that, in general, wire and particle paths may be superimposed because both satisfy Equation 1. It is not clear that this is a sufficient condition, and, in fact, it has been shown by Perry[(2)] that there

(1) J. Loeb, L'Onde Electrique 27, 27 (1947).

(2) J. E. Perry, Los Alamos Scientific Laboratory, private communication

* Work done under the auspices of the AEC.

are special circumstances in which a wire cannot be superimposed on a particle trajectory. Nor is it difficult to understand the reason.

Consider the magnet of Figure 1, on which the work to be described here was performed. It consists of a uniform-field portion which bends the rays 90° on a 1-meter radius of curvature, simultaneously focusing them in the r direction. There is also a fringing field which produces focusing in the z direction[(3)]. Now to support a wire stably in the magnetic field it is necessary that the Amperian force on the wire be directed outward from the center of curvature, whereas to bend the beam along a similar path the Lorentz force must be inward. Therefore the wire and ion-beam currents must be oppositely directed and the z as well as the r forces are opposite for the two cases. This means that the part of the magnetic field which produces z focus of the ion beam will tend to deflect the wire against the pole faces.

Nevertheless there are conditions for which the wire will hang stably in a magnetic field without being deflected against mechanical stops, and for which the wire and particle paths can be superimposed. It has been found experimentally that there is a wide range of support positions for the ends of the wire in the median plane of the gap for which the wire hangs stably, and there is also a wide range of stable positions outside the median plane. For these stable positions one has for the wire,

$$(2) \quad R_w = \frac{T}{H_n i}$$

and for the particles,

$$(3) \quad R_p = \frac{mv}{H_n e}$$

where m, v, and e are the mass, velocity, and charge of the particle, T and i are the tension and current in the wire, H_n is the component of the magnetic field normal to the path, R_p and R_w are the radii of curvature of particle and

(3) M. Camac, R. S. I., 22, 197 (1951).

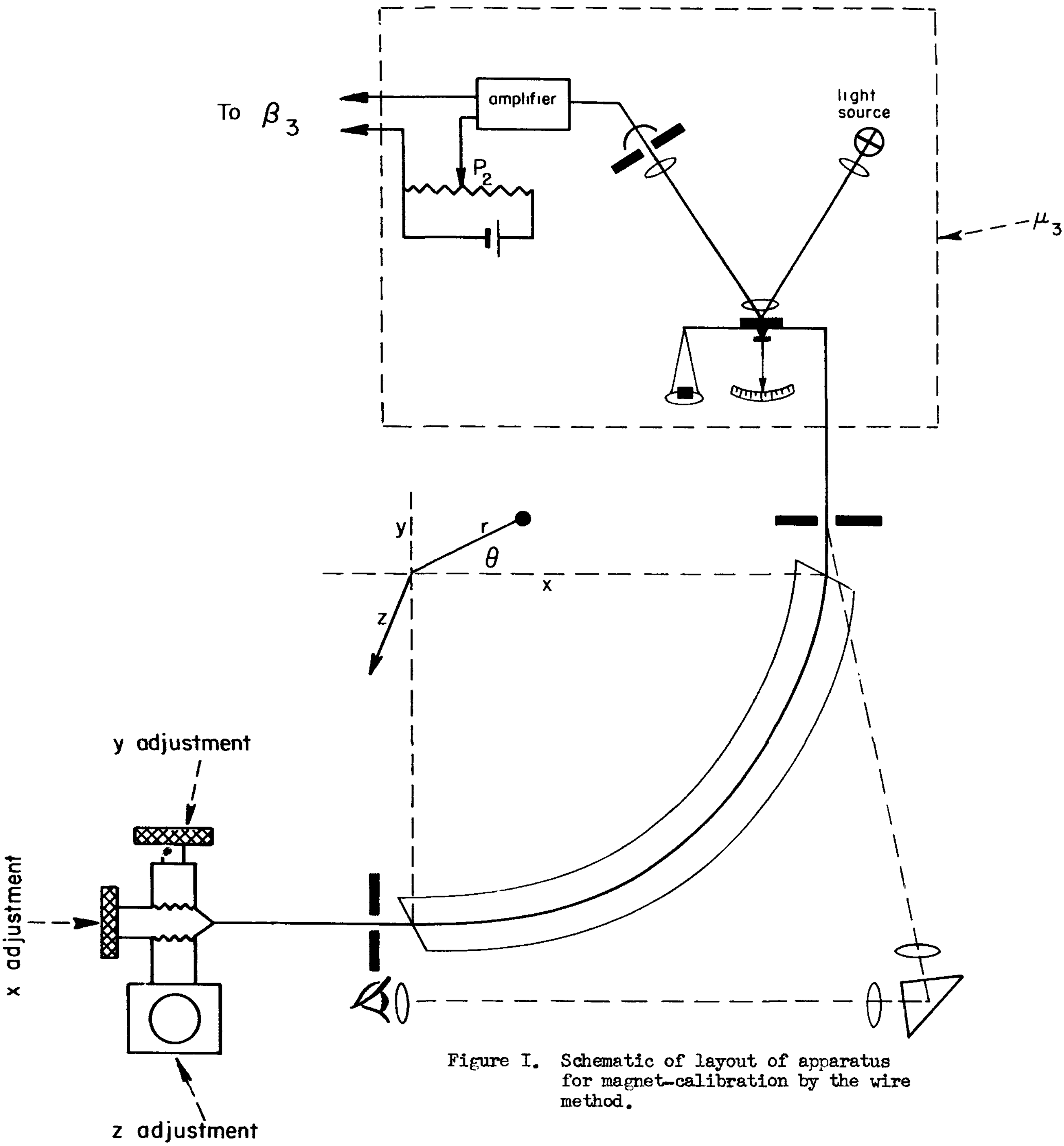

Figure I. Schematic of layout of apparatus for magnet-calibration by the wire method.

wire paths respectively. For the special case of interest here, namely trajectories close to the median plane, $z = 0$, $H_n = H$, and Equations 2 and 3 are differential equations of second order for the trajectory in two dimensions. Particular solutions of 2 and 3 which correspond to the same boundary conditions will be identical if

$$(4) \quad \frac{mv}{e} = \frac{T}{i}$$

a result given by Loeb.

Thus, according to Equation 4, to obtain the momentum of a beam of ions, it is necessary only to define their path, string a wire to follow the same path, then measure the ratio of wire-tension to wire-current, while the magnetic field is held to the same value as that for which the ions followed the prescribed path. Since tension and current may easily be measured to high precision, and magnetic fields may be held constant and paths may be defined with good precision, one is lead to inquire into the limits to which this method might be pushed, from the point of view of precise accurate determination of ion energies. From this point of view the competition to be met is that of the electrostatic analyzer[(4)], for which the overall accuracy is of the order of 0.1%. The method appears to be attractive also for precise calibration of the mass-analyzing and momentum-measuring magnets usually associated with accelerators, for energies between and beyond the points of the energy scale which are already fixed by carefully measured reaction thresholds.

II. Sources of Error.

The main source of error in applying Equation 4 arises from the assumption implicit in it that the only force acting on the wire is the Amperian force due to interaction of the fields of the magnet and wire. There are

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(4) Herb, Snowdon, and Sala, Phys. Rev., 75, 246 (1949).

actually three other forces to be considered: (a) the extra Amperian forces on the wire due to the self-inductance of the loop and the fact that in practice the wire current will be many orders of magnitude greater than the ion or electron beam current, (b) the elastic forces due to the stiffness of the wire, and (c) the effect of gravity on the wire.

The error due to (a) may be estimated by considering a circular loop and its first images in each of the pole faces, using the results of Stratton[5]. For a current of 1 ampere, radius of wire of 0.1 mm., pole separation of 1-1/2 inches, and trajectory radius of 1 m. the error in the tension should not exceed 1 mg. Interaction of the wire with its images in the pole faces also produces z forces which tend to make the wire unstable in the gap. For the conditions of this work the z force is also about 1 mg., and produced no instability. An independent check on the role of these forces may be obtained experimentally since they depend on the square of the wire current, whereas the main Amperian force depends only on the first power of the current.

To minimize effects of stiffness or springiness of the wire, the wires were always annealed by passing heavy current while they were hanging in the prescribed path. This effectively removed all visible irregularities. An independent check on residual effects is obtained by comparing results obtained with wires of different diameter.

The most troublesome effect is that due to gravity. Even with the finest wires used in this work the effect of gravity was always apparent, and manifested itself in the following way. According to the simple theory of Equation 4, a plot of wire-tension versus wire current should yield a straight line passing through the origin, the slope of which is proportional to the momentum of the ion beam following the same trajectory. The

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(5) J. A. Stratton, Electromagnetic Theory, 1st Ed., McGraw Hill, 1941, pp. 263, 264.

tension-current plots obtained experimentally invariably gave a positive intercept on the tension axis, which was greater the heavier the wire used. This fact is disturbing because it raises the question as to whether the momentum is still given by the slope of the experimental curve. To settle this question of interpretation, and determine the effect of gravity on the wire method, the detailed calculation given in the appendix was carried out. The most important result of the calculation is that with gravity the slope of the tension-current curve still gives the particle momentum. The calculation also gives the value of the intercept on the tension axis, and the true wire trajectory. The results of a comparison of the predicted and measured intercepts given later are reassuring regarding the adequacy with which one may take account of the effect of gravity.

III. Apparatus and Procedure.

a. Choice of Trajectory. If the trajectory is to be defined by three points, these points should be as far apart from one another as possible, since for a given tolerance in position the fractional error in the definition of the radius of curvature is least. To minimize gravitational errors, however, the exit point was taken close to the point of emergence of the wire from the gap, where its height was read against a fixed silvered scale and was kept constant to 0.1 mm.

An aperture 0.030 inch in diameter whose center was 30 inches plumb beneath the point of support of the wire on the balance and 10 inches above the top of the magnet now defines the trajectory. These dimensions imply the tension error due to angular departure from plumb of the wire cannot exceed 1 part per million, and if the wire is maintained in a fixed position in relation to the aperture to 10% of the aperture diameter, the radius of curvature of the trajectory is defined to 1 part in 10,000.

b. Wire Adjustment. All the adjustments necessary to maintain a particular trajectory were made at the exit end of the magnet by three mutually perpendicular adjustment screws supporting the wire at a point close to the scale at which the emergent height is checked. The adjustment mechanism is lined up so that one screw is closely parallel to the emergent wire and one is closely parallel to the magnetic field. This makes negligible any interlock of the adjustments, and usually only the wire-parallel screw need be adjusted to compensate for wire length changes. An improvised 200 power telescope allows the operator making the adjustment also to observe the clearance of the wire in the 0.030 inch aperture. Whenever the tension or wire current is changed this clearance must be checked because of changes in the length of the wire due to thermal and elastic effects.

c. Choice of Wire. Loeb used silver wire of 0.0008 inches diameter capable of carrying 0.5 amperes. For such wire δ, the ratio of gravitational to Amperian force per unit length, at 600 gauss, is 10^{-3}. For the work done here commercial aluminum wire down to 0.0055 inch diameter was conveniently available, and was used although for it δ is about 10 times greater than for Loeb's wire. Table I gives the results of measurements made of the currents which will promptly fuse Al wire of different diameters under 50 grams tension in open air.

TABLE I

Wire Diam. Inch	Prompt Fusing Current, Amp.
0.0055	2.2
0.0082	3.5
0.012	6.0

From the log-log plot of these data we have roughly

$$(5) \quad I = 2.2 \left(\frac{D}{.055}\right)^{1.3}$$

where D is in inches and I in amperes. Thus $\frac{I}{D^2} \propto D^{-0.7}$ indicates the way in which the quality criterion varies with wire diameter, and fine wire is favored. For good wire life the current should not exceed about 1.5 ampere maximum for .0055 wire. These values could be increased (almost doubled) with forced air cooling provided by a fan, but this was not resorted to.

Small wire diameter is essential also from the point of view of meeting the flexibility requirement, since the stiffness of a cylinder goes as the cube of the diameter.

The annealing current for 0.0055 inch wire was 1.8 amperes for 1 minute, and 2.6 amperes for one minute for 0.0082 wire. The annealing current has to be shunted around the point of support of the wire on the hook of the balance to prevent breakage at that point.

d. Current Stabilization The current stabilizer for the magnet was of the conventional 100% negative feedback type, the feedback voltage being developed across a precision 0.1 ohm resistance in series with the magnet current.

The wire current-stabilizer and tension measurement are tied together[6], and some of the details of the arrangement may be of interest. An ordinary analytical balance of 200 gram capacity and 0.1 mg accuracy was stripped down by removal of the 1.0 g. slider and chain mechanism, and fitted with reflecting mirror close to the axis of rotation. A light beam and photocell arrangement give an electrical signal indicating the beam position, and this signal is used to help control the wire current. In addition to this feedback loop, a conventional current stabilizer is used which is similar to that used for the magnet. The block diagram is shown in Figure 2.

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(6) Following the suggestion of Mr. George W. Downs of William Miller, Inc.

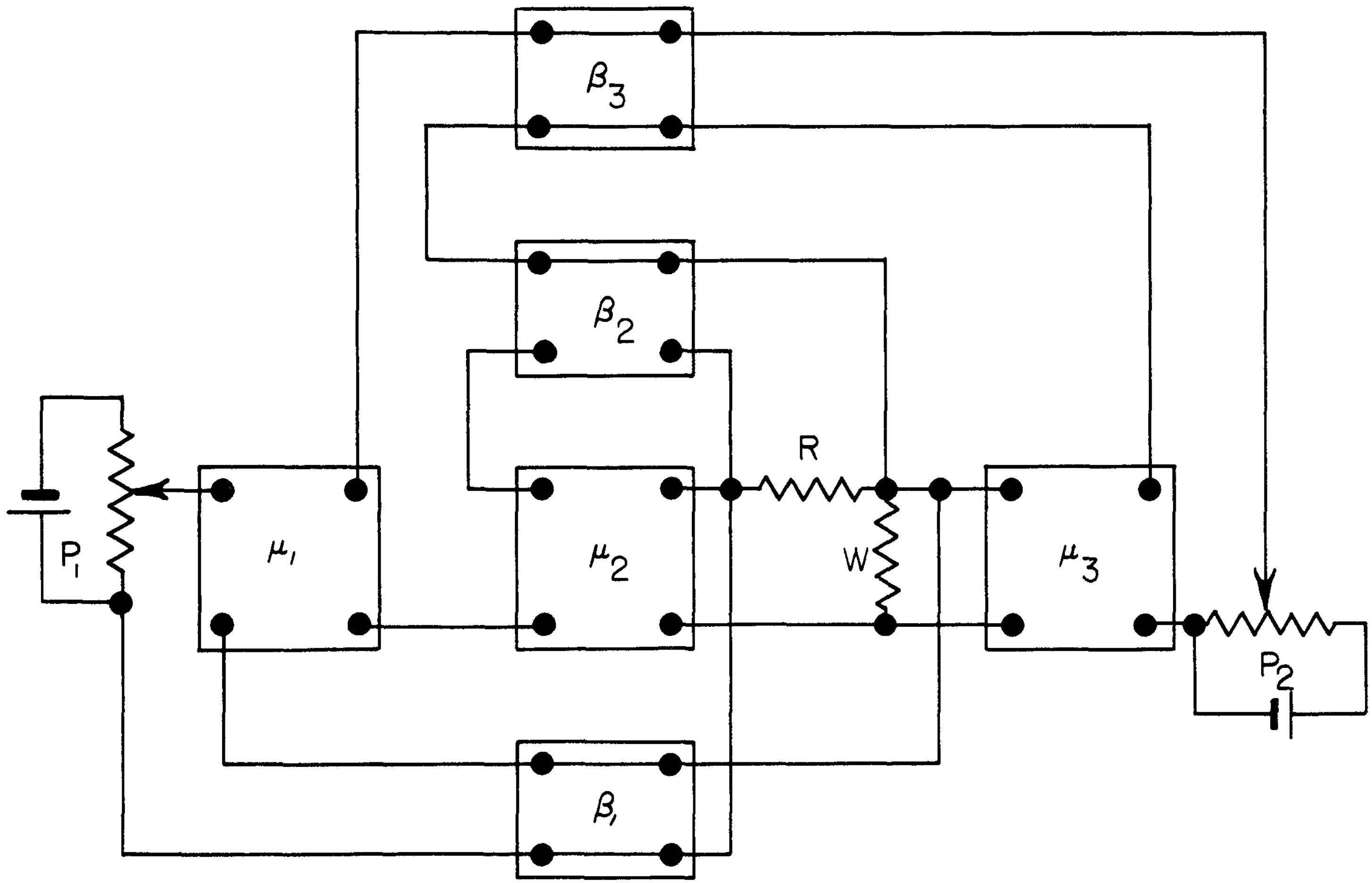

Figure II. Block diagram of components of wire-current stabilizer.

The output current passes through the wire W in series with the precision 1 ohm resistance R, and the voltage developed across the latter is bucked against the voltage developed across the potentiometer P_1, the difference being amplified by μ_1, a Perkin-Elmer breaker-type amplifier, and by μ_2 which consists of a bank of paralleled regulator tubes. β_2 is a differentiating circuit which feeds back out-of-phase voltage proportional to the rate of change of the wire current. Amplifier μ_3 represents the balance-photocell-amplifier combination, the output of which feeds into μ_2 through the loop β_3. The input connection to μ_3 is actually a mechanical linkage through the wire, but since the balance displacement is proportional to the wire current, it may be represented as shown, the electrical input impedance to μ_3 being infinite.

Operation is as follows. Potentiometer P_1 is adjusted to bring the balance, with the desired weight on the counterpan, within the control range of potentiometer P_2 which is then adjusted to zero the balance. The way in which P_2 controls may be understood as follows. A change in voltage ΔV_2 of P_2 is introduced into the $\mu_1\mu_2\beta_1$ loop, and this loop then adjusts itself so that the change in voltage appearing at the input to μ_2 is only $\dfrac{\Delta V_2}{1 - \mu_1\mu_2\beta_1}$. The change in wire current is then proportional to this quantity. Since $\mu_1\mu_2\beta_1$ is of the order of a thousand this provides a very fine control of the current. The loop $\mu_2\mu_3\beta_3$ acts to stabilize the wire current in the same manner as $\mu_1\mu_2\beta_1$.

It is important to note that it is not advantageous to include μ_3 in a high-gain feedback loop, since the sensitivity of the balance is then greatly reduced. This may be understood as follows. The sensitivity of the balance, i.e., its angular deflection per unit weight unbalance, is determined in normal use by the dynamics of the balance itself, and depends on how far the axis of rotation is above the center of mass of the beam. In the arrangement used the sensitivity may be calculated as follows.

Let H r be the tension-to-current ratio for the wire and σ be the voltage output of μ_3 per unit angular deflection of the beam. The sensitivity S is then given by

$$(6) \quad S = \frac{1}{S_2} \; \frac{1}{1 - \mu_1 \mu_2 \beta_1} \; \frac{1}{\sigma H r}$$

where S_2 is the transconductance of amplifier μ_2. The electro-mechanical gain σ must therefor be adjusted to provide satisfactory sensitivity over the desired range of H r. This is done in practice by adjusting the illumination. Working values of S were in the range of 5 to 30 milligrams per millimeter deflection of the scale pointer, which could be adjusted and stabilized to 0.1 mm.

To eliminate hunting it was found necessary to dampen the balance and this was done electromagnetically.

III. Results and Conclusions.

The results obtained on a number of runs for a range of tensions from 20 grams to 100 grams are summarized in Table II. The first run was taken with 0.0082 inch wire, and all the others with 0.0055 wire. The quantities a and $\frac{H r}{g}$ are the intercept on the tension axis and the slope, respectively, of the best straight-line fit to the data as determined by the method of least squares. Under Δ are given the r.m.s. deviations of the experimental points from the best straight-line fit.

The spread in momentum values was measured for a very small range of magnet currents in the neighborhood of 11 amperes to determine the magnitude of the hysteresis effects without careful cycling. These amounted to about 1 per cent.

The r.m.s. errors listed in Table II, which are a few hundredths of one per cent, measure the precision with which a linear fit to the tension-wire-current data is possible, and place an upper limit on the magnitude of any quadratic component. Since this upper limit is of the same order of magnitude as the precision of the measurements, the quadratic component must have been negligible for this experimental situation.

TABLE II

Magnet Current Amperes	Number of Observations	Intercept "a" grams	$\frac{Hr}{g}$ gauss sec.^{2}	Δ gauss sec.^{2}
11.3484	6	0.170	974.90	0.13
11.3628	6	0.110	980.20	0.16
11.3627	8	0.095	972.50	0.30
11.3712	6	0.10	980.22	0.10
11.9394	4	0.09	1012.26	0.21
10.2510	3	0.08	892.92	0.05
8.4686	3	0.07	742.88	0.51
6.8079	3	0.08	599.51	0.03
5.1877	3	0.08	459.21	0.11

Equation 13 of the appendix gives for the value of the intercept the weight of the vertically projected length of wire. Weighing of the wire gave 185 mg and 94 mg for the vertically projected lengths of heavy and light wire respectively. This compares with 170 mg and 95 $\pm$ 15 mg from the data of Table II. Agreement between the experimental and calculated values of the intercept for the light wire is in the range to be expected if the agreement is limited by observational precision. It seems reasonable to conclude therefor that the slope of the tension-current curve is also given accurately by equation 13 to the precision of the measurements, so that one may expect accuracy of the order of a few hundredths of a percent for a momentum determination with instruments of the sort described here.

IV. Acknowledgments.

Thanks are due to J. L. McKibben, who initiated this work, for criticism and encouragement, to R. Landshoff for checking the calculations, and to T. Jenkins and J. Kopecek for assistance in constructing the apparatus and in taking data.

APPENDIX

TRAJECTORY OF A CURRENT-CARRYING WIRE IN CROSSED UNIFORM, MAGNETIC AND GRAVITATIONAL FIELDS

Assume the wire lies in the xy plane, that the magnetic field is in the positive z direction, and that the gravitational field is in the negative y direction (see Figure 1). If the current in the wire flows in the counter-clockwise direction there will be a radial force in the outward direction acting on the wire, of magnitude Hi per unit length. The conditions for static equilibrium of a wire element of length ds at azimuth θ may be written (resolving stresses normal to and tangential to the wire):

$$(1) \quad \frac{dT}{ds} = tg \sin\phi$$

$$(2) \quad T = Hi \frac{ds}{d\phi} + tg \cos\phi \frac{ds}{d\phi}$$

where T is the point value of the tension in the wire, t is the mass per unit length of the wire, and ϕ is the slope angle. (s and ϕ are reckoned positive in the counter-clockwise direction.) Combining (1) and (3), substituting $\frac{ds}{d\phi} = R$, the radius of curvature, dividing through by Hi, and setting $\frac{tg}{Hi} = \delta$, a dimensionless quantity, we obtain

$$(3) \quad \frac{dR}{d\phi}(1 + \delta \cos\phi) - 2\delta \sin\phi\, R = 0$$

This integrates at once to

$$(4) \quad R = C(1 + \delta \cos\phi)^{-2}$$

where c, is the constant of integration. To the first order in δ, assuming $\delta \ll 1$ (in practice δ may be made as small as 10^{-3}):

$$(5) \quad R = C(1 - 2\delta \cos\phi)$$

substituting solution (4) in (2) we have, to first order in δ:

$$(6) \quad T = C\,Hi(1 - \delta \cos\phi)$$

To determine the effect of gravity on the wire tension measured at angle θ it is necessary to know the relation between ϕ and θ and the value of C. To obtain the latter the equation of the trajectory is desired.

To obtain the expression for the trajectory in plane polar coordinates r, θ, we start from Equation 5, which is correct only to first order in δ, and in what follows we retain only terms to first order in δ. The transformation to plane-polar coordinate gives

$$(7) \quad r = C\left(1 + 2\delta \sin\theta - \frac{1}{r}\frac{d^2r}{d\theta^2}\right)$$

This second-order, second-degree equation may be solved as follows: assume the solution to be of the following form:

$$(8) \quad r = C\left\{1 + 2\delta \left[f(\theta)\right]\right\}$$

Substituting this in (7) gives for f (θ),

$$(9) \quad \frac{d^2f}{d\theta^2} + f = \sin\theta$$

the solution to which can be written at once as:

$$(10) \quad f = A\cos\theta + B\sin\theta - \frac{\theta\cos\theta}{2}$$

Hence we have as the general solution of (7):

$$(11) \quad r = C\left[1 + 2\delta\left(A\cos\theta + B\sin\theta - \frac{\theta\cos\theta}{2}\right)\right]$$

This solution for the trajectory has three arbitrary constants, to be determined from the given initial conditions.

The initial conditions assumed for the case of a 90° magnet are as follows:

$$r = r_1 \text{ for } \theta = 0^\circ,\ -90^\circ;\ \text{and at } \theta = 0,\ \phi = 90^\circ$$

This gives $A = -1/2$, $B = 1/2$, $C = r_1(1 + \delta)$

$$(12) \quad r = r_1\left[1 + \delta\left(1 + \sin\theta - \cos\theta - \theta\cos\theta\right)\right]$$

and substituting in (6) we obtain

$$(13) \quad T = Hi\, r_1 + tg\, r_1$$

so that the slope of the tension-current wire gives correctly the quantity proportional to the momentum of the particles, and the intercept on the tension axis is the vertically projected wire weight and independent of the magnetic field strength, in agreement with experiment.

If boundary conditions were imposed entirely in the uniform-field region, Equation 13 would be directly applicable. In practice the first and third boundary conditions were imposed in the relatively field free space above the magnet, so that on entering the uniform field after traversal of the fringing field the slope and coordinate conditions were changed. It is easy to show that if the boundary conditions are changed so that for $\theta = 0$ we substitute $\theta = -a$ where $a \ll 1$, Equation 13 is unchanged except for a higher order correction to the intercept.

From Equation 12 the maximum deviation of the actual wire trajectory from that of the ion beam occurs at - 1 radian, and amounts to $0.16\, \delta\, r_1$. It is clear the deviation can be made extremely small, and this is desirable particularly where there is an appreciable radial component of the magnetic field gradient.

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