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THE LONGITUDINAL DISTRIBUTION OF THERMAL NEUTRON FLUX
IN CYLINDRICAL FUEL SPECIMENS DURING IRRADIATION

by

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TABLE OF CONTENTS

	<u>Page</u>
LIST OF SYMBOLS.	5
ABSTRACT	7
INTRODUCTION.	7
EXPERIMENTAL METHOD	8
EXPLANATION OF EXPERIMENTAL DATA.	9
ANALYSIS OF RESULTS	14
SUMMARY	20
ACKNOWLEDGMENTS	21
REFERENCES.	21
APPENDIX	23

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	Schematic Drawing of Assembly for Neutron-flux-distribution Measurements	9
2.	Neutron Flux Perturbations at the Ends of Cylindrical Fuel Specimens during Irradiation	17
3.	Longitudinal Distributions of Neutron Flux at the Ends of Cylindrical, Natural Uranium Specimens of Various Diameters.	18
4.	Longitudinal Distributions of Neutron Flux at the Ends of Fuel Specimens (0.635-cm Dia) Having Various Inverse Diffusion Lengths.	18
5.	Longitudinal Distributions of Neutron Flux at the Ends of Two Natural Uranium Specimens (1.27-cm Dia) Separated by Various Distances	19

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I.	Composition, Macroscopic Absorption Cross Sections and Effective Inverse-Diffusion Lengths of Fuel Test Specimens .	10
II.	Constants Used in Calculating the Macroscopic Absorption and Scattering Cross Sections.	10
III.	Description of the Specimens	10
IV.	The Wilkins Effect and Average Volume Neutron Flux in Absorbing Cylinders for Various Values of τ	12
V.	Comparison Between Experimental and Computed (Least Squares) Values of the Wilkins Effect.	12
VI.	Comparison Between Experimental and Computed (Least Squares) Values of the Average Volume Neutron Fluxes	13
VII.	Comparison Between Experimental and Computed (Least Squares) Values of λ	13
VIII.	Solution of the Equation $x(\tau) = 1/[1-m(\tau)]$ for Various Values of τ	16
IX. to XXII.	Tabulations of Experimental Data	23-25

LIST OF SYMBOLS

a	Radius of Specimen (cm)
Σ_{abs}	Macroscopic Absorption Cross Section (cm^{-1})
Σ_{s}	Macroscopic Scattering Cross Section (cm^{-1})
$\Sigma_{\text{t}} (= \Sigma_{\text{abs}} + \Sigma_{\text{s}})$	Total Macroscopic Cross Section (cm^{-1})
$K_{\text{eff}} (= 0.91 \Sigma_{\text{t}})$	Effective Inverse Diffusion Length (cm^{-1})
$\tau (= a K_{\text{eff}})$	Dimensionless Parameter
z	Position Coordinate in Direction Parallel to Axis of Specimen
λ	Position in Specimen at which Flux Perturbation Equals 3% of Maximum Perturbation (cm)
L	Position in Specimen at which Effects of End Perturbations Become Negligible (cm)
$\bar{n}$	Average Neutron Flux on Cross Section of Specimen
$\bar{n}_{\text{v}}$	Volume-averaged Neutron Flux
ϕ, ψ	Neutron Flux
σ_{abs}	Microscopic Absorption Cross Section (barn)
σ_{s}	Microscopic Scattering Cross Section (barn)

THE LONGITUDINAL DISTRIBUTION OF THERMAL NEUTRON FLUX IN CYLINDRICAL FUEL SPECIMENS DURING IRRADIATION

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ABSTRACT

It has been found that the longitudinal distribution of the average thermal-neutron flux on the cross section of a cylindrical fuel specimen may be expressed by an empirical equation of the type

$$\phi(z, \tau) = A(\tau) \exp[-B(\tau)z] + C(\tau), \quad (0 \leq z \leq \lambda),$$

where $A(\tau)$, $B(\tau)$ and $C(\tau)$ are polynomials of the second degree, $\tau = 0.91a(\Sigma_{\text{abs}} + \Sigma_s)$ and $\lambda = 0.5935 a^2 + 6.429a$. The symbol a is used to denote the radius of the specimen, and z is the axial distance from the end surface.

It has also been found that the distribution of the average neutron flux near the ends of a pair of cylindrical fuel specimens arranged in a column and separated by a nonabsorbing material, such as graphite or aluminum, in which $\Sigma_{\text{abs}} \ll \Sigma_s$, may be expressed by an empirical equation of the type

$$\psi(z, \tau) = (p-1) \exp \left[- \frac{\phi(0, \tau) - 1}{p - 1} B(\tau) z \right] + 1 \quad (0 \leq z \leq \lambda'),$$

where

$$\lambda' = \left(\frac{p - 1}{\phi(0, \tau) - 1} \right).$$

The symbol p is defined as the Wilkins Effect and depends upon the neutron properties and separation of the fuel specimens. The Wilkins Effect must be determined experimentally before the equation may be applied.

INTRODUCTION

In the computation of the temperature distribution within a cylindrical fuel specimen during irradiation⁽¹⁾ an accurate knowledge of the neutron distribution is required. This information is also related to the distribution of

fission events, fission damage and new atoms or isotopes. It, too, may have other engineering uses in the consideration of internal stress, of disadvantage factors and of neutron economy.

Neutron distributions may be computed from diffusion⁽²⁾ or transport theory if the pertinent variables are accurately known. However, the problem usually must be idealized to permit a solution. This is particularly true of distributions in cylindrical rods of finite length. Thus far, the experimental support for such results has not been extensive and there are often large uncertainties in the values of parameters which must be employed.

In view of an increasing need for more accurate definition of the conditions under which test specimens are irradiated, a study of the distribution of neutrons in absorbing cylinders has been in progress at Argonne National Laboratory. A number of radial and longitudinal neutron distributions have been experimentally determined for bars of natural uranium and of magnesium-cadmium alloys. The latter were used to simulate the absorption properties of fuel materials of various enrichments in fissionable atoms. Specimens were activated in the almost pure thermal flux of the CP-5 reactor reflector.

The present report deals only with the longitudinal distributions found in the specimens. A companion report deals with the radial distributions which were observed.⁽³⁾

EXPERIMENTAL METHOD

It was desired to study neutron distributions in cylinders of small, as well as of large, diameter. As the smaller specimens were also to have the larger macro-absorption cross sections, it was necessary to devise an experimental method which would provide a homogeneous bar and yet allow rapid and accurate sampling. This was made possible by assembling the specimens from carefully prepared disks or wafers between which thin monitor foils of the same diameter and material could be sandwiched. The assemblies could then be activated under tight longitudinal compression, and the radioactivity induced in each monitor foil could be determined by gamma counting. The distribution of activity along the lengths of the specimens could then be related to the distribution of the thermal-neutron flux. Other investigators have monitored the neutron fluxes in relatively large absorbing bars with embedded monitors of dissimilar metals or metal oxides,⁽⁴⁾ a method which could not be used satisfactorily in small-diameter rods without causing serious perturbation of the flux.

A detailed description of the experimental procedure is given in a companion report dealing with the radial distribution of neutrons observed in the fuel specimens.⁽³⁾ For the longitudinal work the entire foil was gamma counted, whereas for the radial work "dot" samples were punched from the foils at various radial positions.

Figure 1 is a schematic drawing showing the graphite capsule and the arrangement of fuel specimens within the capsule. The positions of the specimens were fixed through use of graphite spacers and end plugs. The distance between specimens was varied during the course of the investigation by using graphite spacers of various lengths and the resulting flux distributions were determined. The relative increase in the neutron flux observed at the ends of the specimens adjacent to the spacer is defined as the "Wilkins Effect." It was observed that the relative increase of the neutron flux at the ends of the specimens next to the spacer was always less than that observed at the opposite (free) ends. However, as the distance between the specimens became greater, the relative flux adjacent to the spacer also increased and approached a maximum. The maximum value represents the limiting value of the Wilkins Effect.

Figure 1. Schematic Drawing of Assembly for Neutron Flux Distribution Measurements.

A complete tabulation of the experimental data is given in the Appendix.

Diameters of the test bars ranged from 0.449 to 2.540 cm (0.177 to 1.00 in.). Five sets of two each were of natural uranium and covered almost the entire range. Eight sets of two each were of the magnesium-cadmium alloys listed in Table I. Compositions were checked by duplicate chemical analyses from the tops and bottoms of the castings from which the specimens were fabricated. Densities were obtained in duplicate by the immersion method, and macroscopic absorption cross sections were computed from the constants shown in Table II. Small amounts of manganese and cobalt (less than 1/2% each) were added to the alloys to serve as radioactive tracers after exposure to neutrons.

EXPLANATION OF EXPERIMENTAL DATA

A description of the specimens used in the investigation of the distribution of neutrons in finite absorbing cylinders is given in Table III. The diameters of the specimens ranged from 0.449 to 2.540 cm. The ratio

TABLE I

Composition, Macroscopic Absorption Cross Sections and
Effective Inverse Diffusion Lengths of Fuel Test Specimens

Material	Composition, w/o				Density, gm/cm ³	Σ_{abs} , cm ⁻¹	Σ_s , cm ⁻¹	K_{eff} , cm ⁻¹
	Magnesium ^a	Cadmium	Manganese	Cobalt				
Alloy A	92.564 ± 0.057	6.57 ± 0.05	0.475 ± 0.005	0.391 ± 0.002	1.868	1.921	0.160	1.83
Alloy B	87.933 ± 0.074	11.32 ± 0.08	0.410 ± 0.005	0.337 ± 0.004	1.962	3.472	0.163	3.15
Alloy C	81.210 ± 0.074	18.15 ± 0.12	0.345 ± 0.005	0.295 ± 0.002	2.012	5.704	0.160	5.25
Uranium	Natural				19.0	0.3135	0.3991	0.672

^aBy difference

TABLE II

Constants Used in Calculating the Macroscopic Absorption and
Scattering Cross Sections

Element	Density, gm/cm ³	Atomic Weight	σ_{abs} , barns ⁽⁵⁾	$\bar{\sigma}$, barns ^a	σ_s , barns ⁽⁵⁾	$g(T)$ Factors
Magnesium	1.741	24.32	0.063 ± 0.004	0.054	3.6 ± 0.4	1.0
Manganese	7.42	54.94	13.2 ± 0.4	11.3	2.3 ± 0.3	1.0
Cobalt	8.71	58.94	37.0 ± 1.5	31.7	7 ± 1	1.0
Cadmium	8.65	112.41	2450 ± 50	2912	7 ± 1	1.388
Uranium	19.0	238.07	7.68 ± 0.07	6.52	8.3 ± 0.2	0.99

^aMaxwellian-averaged Cross Section,⁽⁶⁾ i.e., $\bar{\sigma}(T) = g(T)\sigma_{abs}(\pi T_0/4T)^{1/2}$
where $T_0 = 293.6^\circ K$

TABLE III

Description of the Specimens

No.	Specimen Material	Specimen Diameter, 2a, cm	Specimen Length ^a , cm	K_{eff} , cm ⁻¹	$\tau (= aK_{eff})$
1	Natural Uranium	2.540	12.7	0.672	0.8534
2	Natural Uranium	1.796	10.8	0.672	0.6035
3	Natural Uranium	1.270	7.6	0.672	0.4267
4	Natural Uranium	0.898	5.4	0.672	0.3017
5	Natural Uranium	0.635	3.8	0.672	0.2134
6	Alloy A	1.270	7.6	1.83	1.162
7	Alloy A	0.898	5.4	1.83	0.8217
8	Alloy A	0.635	3.8	1.83	0.5810
9	Alloy B	0.898	5.4	3.15	1.414
10	Alloy B	0.635	3.8	3.15	1.000
11	Alloy B	0.449	2.7	3.15	0.7072
12	Alloy C	0.635	3.8	5.25	1.667
13	Alloy C	0.449	2.7	5.25	1.179

^aThe ratio of specimen length to diameter is six for specimens No. 2 to No. 13. For specimen No. 1, the ratio is five.

of specimen length to diameter was a constant equal to six, except for specimen No. 1, in which the ratio is equal to five. Quantities designated by the symbols K_{eff} and τ are given in the last two columns of the table. K_{eff} is defined as the effective inverse diffusion length for thermal neutrons and is dependent upon the composition of the specimens. The reciprocal of the inverse diffusion length is a measure of the distance traveled by a thermal neutron during its lifetime. The quantity τ is a dimensionless parameter and is defined as the product of the radius, a , and the effective inverse diffusion length of the specimen, i.e., $\tau = aK_{\text{eff}}$. The variable τ , ranging from about 0.2 to 1.7 for the specimens listed, was proved to be very useful in the analysis of the experimental data.

If a finite absorbing cylinder is placed in a uniform field of thermal neutrons the average neutron flux on the cross section of the cylinder will be greatest at the ends. The relative increase of neutron flux at the ends of the cylinder is referred to as a flux perturbation. Table IV lists the neutron flux perturbations observed at the free ends of a number of different fuel specimens. The magnitude of the perturbation is given by the ratio $\bar{n}(O)/\bar{n}(L)$ which was suggested by the theoretical work of Wilkins.⁽²⁾ The values appearing in the third column of the table represent limiting or maximum values of this ratio. The values are maximum in the sense that they were determined from the neutron flux distributions at the ends of the specimens farthest from the graphite spacer (free ends). The notation $\bar{n}(O)$ is used to designate the average neutron flux on the cross section of the cylindrical specimen at $z = 0$, i.e., at the end or plane face of the cylinder. The term $\bar{n}(L)$ denotes the average flux on the cross section of the specimen at $z = L$, i.e., at a distance L from the plane face of the specimen. The quantity L is defined as the distance within the specimen at which the effect of the largest end perturbation becomes negligible. The ratio appearing in the fourth column of the table is defined as the average neutron flux, $\bar{n}_V$, within the length, L , of the specimen, divided by the average unperturbed flux within the specimen, $\bar{n}(L)$. The quantity $\bar{n}_V$ was computed from the experimentally determined longitudinal distributions by numerical integration. The last column of the table lists λ , the position in the specimen where the flux perturbation is only 3% of the maximum perturbation, and defined implicitly by the equation

$$\frac{\bar{n}(O)}{\bar{n}(L)} - \frac{\bar{n}(\lambda)}{\bar{n}(L)} = 0.97 \left[\frac{\bar{n}(O)}{\bar{n}(L)} - 1 \right] \quad (1)$$

where $\bar{n}(\lambda)$ denotes the average flux on the cross section of the specimen at $z = \lambda$. The other quantities appearing in the equation are defined above.

Tables V and VI show a comparison between the experimentally derived ratios $\bar{n}(O)/\bar{n}(L)$ and $\bar{n}_V/\bar{n}(L)$ and the least-squares-adjusted values of these ratios. Table VII shows a comparison between the experimentally determined and the least-squares-adjusted values of λ .

TABLE V

Comparison between Experimental and Computed
(Least Squares) Values of the Wilkins Effect

τ	$\bar{n}(O)/\bar{n}(L) \equiv \phi(O, \tau)$		Relative Deviation, % ^b
	Experimental Ratios	Least-squares- adjusted Ratios ^a	
0.2134	1.054	1.073	-1.8
0.3017	1.082	1.111	-2.6
0.4267	1.186	1.173	+1.1
0.5810	1.300	1.261	+3.1
0.6035	1.233	1.275	-3.3
0.7072	1.395	1.343	+3.9
0.8217	1.459	1.426	+2.3
0.8534	1.402	1.450	-3.3
1.000	1.499	1.570	-4.5
1.162	1.612	1.716	-6.1
1.179	1.805	1.733	+4.2
1.414	2.131	1.975	+2.8
1.667	2.195	2.271	-3.4

$$^a \phi(O, \tau) = 1 + 0.28155\tau + 0.28838\tau^2$$

^bRMS Deviation, 3.5%.

TABLE IV

The Wilkins Effect and Average Volume Neutron Flux
in Absorbing Cylinders for Various Values of τ

Specimen No.	τ	$\bar{n}(O)/\bar{n}(L) \equiv \phi(O, \tau)$	$\bar{n}_V/\bar{n}(L) \equiv \phi_V(\tau)$	λ , cm
1	0.8534	1.402	1.095	9.1
2	0.6035	1.233	1.028	6.3
3	0.4267	1.186	1.033	4.1
4	0.3017	1.082	1.019	3.2
5	0.2134	1.054	1.013	2.1
6	1.162	1.612	1.098	4.5
7	0.8217	1.459	1.038	3.1
8	0.5810	1.300	1.023	2.1
9	1.414	2.131	1.173	2.9
10	1.000	1.499	1.076	2.2
11	0.7072	1.395	1.066	1.4
12	1.667	2.195	1.165	2.1
13	1.179	1.805	1.126	1.4

TABLE VI

Comparison between Experimental and Computed
(Least Squares) Values of the Average Volume
Neutron Fluxes

τ	$\bar{n}_v/\bar{n}(L) \equiv \phi_v(\tau)$		Relative Deviation, % ^b
	Experimental Ratios	Least-squares- adjusted Ratios ^a	
0.2134	1.013	1.012	+0.1
0.3017	1.019	1.018	+0.1
0.4267	1.033	1.027	+0.6
0.5810	1.023	1.041	-1.7
0.6035	1.028	1.043	-1.4
0.7072	1.066	1.053	+1.2
0.8217	1.038	1.065	-2.5
0.8534	1.095	1.068	+2.5
1.000	1.076	1.086	-0.9
1.162	1.098	1.107	-0.8
1.179	1.126	1.109	+1.5
1.414	1.173	1.144	+2.5
1.667	1.165	1.186	-1.8

$$^a\phi_v(\tau) = 1 + 0.047763\tau + 0.038126\tau^2$$

^bRMS Deviation, 1.2%

TABLE VII

Comparison between Experimental and Computed
(Least Squares) Values of λ

Specimen No.	a, cm	λ , cm		Relative Deviation, % ^b
		Experimental	Least-squares- adjusted ^a	
1	1.270	9.1	9.12	-0.2
2	0.898	6.3	6.25	+0.8
3	0.635	4.1	4.32	-5.1
4	0.449	3.2	3.01	+6.4
5	0.3175	2.1	2.10	+0.0
6	0.635	4.5	4.32	+4.1
7	0.449	3.1	3.01	+3.1
8	0.3175	2.1	2.10	+0.0
9	0.449	2.9	3.01	-3.5
10	0.3175	2.2	2.10	+4.7
11	0.2245	1.4	1.47	-4.8
12	0.3175	2.1	2.10	+0.0
13	0.2245	1.4	1.47	-4.8

$$^a\lambda = 0.5935 a^2 + 6.429a$$

^bRMS Deviation, 3.6%

It should be pointed out that the experimental results of this investigation are strictly applicable to graphite only, that is, to graphite moderator and spacers. However, other materials having similar neutron-scattering properties may be substituted for graphite without invalidating the above results.

ANALYSIS OF RESULTS

A. Derivation of an Empirical Formula for Computing Longitudinal Flux Distributions in Cylindrical Specimens

The longitudinal distribution of the average neutron flux on the cross section of a cylindrical fuel specimen during irradiation may be expressed by an empirical equation of the form

$$\phi(z, \tau) = A(\tau) \exp[-B(\tau) z] + C(\tau) \quad , \quad (2)$$

for $0 \leq z \leq \lambda$, where λ is defined by the equation $\lambda = 0.5935a^2 + 6.429a$. The function $\phi(z, \tau)$ expresses the (relative) average flux on the cross section of the specimen in terms of the position coordinate, z , and the dimensionless parameter $\tau (= aK_{\text{eff}})$. The position coordinate, z , is measured in the direction parallel to the axis of the specimen. The plane face of the specimen is assumed to be located at $z = 0$.

The effective inverse diffusion length is given by the empirical equation

$$K_{\text{eff}} = 0.91 (\Sigma_{\text{abs}} + \Sigma_{\text{s}}) = 0.91 \Sigma_{\text{t}} \quad (3)$$

and is valid for $0.7 < K_{\text{eff}} < 5.2$.

Let us now determine the functions $A(\tau)$, $B(\tau)$ and $C(\tau)$ appearing in equation (2). If the values $z = 0$ and $z = \lambda$ are substituted in equation (2), we get

$$\phi(0, \tau) = A(\tau) + C(\tau) \quad , \quad (4)$$

and

$$\phi(\lambda, \tau) = A(\tau) \exp[-B(\tau) \lambda] + C(\tau) \quad . \quad (5)$$

The left member of equation (4) has been determined experimentally for various values of τ and its values are given in the third column of Table IV. The left member of equation (5) may be computed from the relation

$$\phi(0, \tau) - \phi(\lambda, \tau) = 0.97 [\phi(0, \tau) - 1] \quad , \quad (6)$$

which is equivalent to equation (1).

Next, consider the mathematical expression for the average flux in the specimen, i.e., between $z = 0$ and $z = \lambda$:

$$\phi_v(\lambda, \tau) = \frac{1}{\lambda} \int_0^{\lambda} \phi(z, \tau) dz \quad (7)$$

If the indicated integration is performed, we obtain

$$\phi_v(\lambda, \tau) = \frac{A(\tau)}{\lambda B(\tau)} \left[1 - \exp \left\{ -B(\tau) \lambda \right\} \right] + C(\tau) \quad (8)$$

The quantity appearing on the left side of the equation has been determined for the specimens listed in Table III by numerical integration of the experimentally determined flux distributions. Values are given in the fourth column of Table IV. Actually $\phi_v(\lambda, \tau)$ is a function of τ only, since λ is functionally related to τ . The functions $A(\tau)$, $B(\tau)$ and $C(\tau)$ may now be determined from equations (4), (5) and (8). If $C(\tau)$ is eliminated from equations (5) and (8), using the relation given in equation (4), we get

$$\phi(0, \tau) - \phi(\lambda, \tau) = A(\tau) \left\{ 1 - \exp [-B(\tau) \lambda] \right\} \quad (9)$$

and

$$\phi(0, \tau) - \phi_v(\lambda, \tau) = A(\tau) \left\{ 1 - \frac{1}{\lambda B(\tau)} \left[1 - \exp \left\{ -B(\tau) \lambda \right\} \right] \right\} \quad (10)$$

We may now eliminate $A(\tau)$ by dividing each side of equation (10) by the corresponding members of equation (9). Thus,

$$m(\tau) \left[1 - e^{-x(\tau)} \right] = 1 - \frac{1}{x(\tau)} \left[1 - e^{-x(\tau)} \right] \quad (11)$$

where $m(\tau)$ is defined as the ratio of the left members of equations (10) and (9), and $x(\tau) = \lambda B(\tau)$. Equation (11) must now be solved for $x(\tau)$. If the equation is solved for particular values of $m(\tau)$, where $0.2 < \tau < 1.7$, it is seen that $x(\tau)$ varies continuously between 7.3 and 8.4. The equation cannot be solved explicitly for x in terms of m . Hence, since $x(\tau)$ is relatively large, $e^{-x(\tau)}$ will be very small (of the order 10^{-4}) and may be assumed equal to zero. Thus, equation (11) simplifies to

$$m(\tau) = 1 - 1/[x(\tau)] \quad (12)$$

Therefore,

$$x(\tau) \equiv \lambda B(\tau) = 1/[1 - m(\tau)] \quad (13)$$

Table VIII lists the values of $x(\tau)$ and $m(\tau)$ for various values of $0.2 < \tau < 1.7$. The functions $A(\tau)$ and $C(\tau)$ may now be found quite easily from equations (9) and (4). Thus,

$$A(\tau) = \phi(O, \tau) - \phi(\lambda, \tau) \quad (14)$$

and

$$C(\tau) = \phi(O, \tau) - A(\tau) = \phi(\lambda, \tau) \quad (15)$$

TABLE VIII

Solution of the Equation $x(\tau) = 1/[1 - m(\tau)]$
for Various Values of τ

τ	$m(\tau)$	$x(\tau)$	τ	$m(\tau)$	$x(\tau)$
0.2134	0.8630	7.299	0.8534	0.8740	7.936
0.3017	0.8652	7.418	1.000	0.8756	8.039
0.4267	0.8678	7.564	1.162	0.8770	8.130
0.5810	0.8704	7.716	1.179	0.8772	8.143
0.6035	0.8708	7.740	1.414	0.8789	8.258
0.7072	0.8723	7.831	1.667	0.8804	8.361
0.8217	0.8737	7.918			

The function $B(\tau)$ is defined by equation (13). Now let

$$\phi(O, \tau) = 1 + a_1 \tau + a_2 \tau^2 \quad (16)$$

and

$$\phi_v(\lambda, \tau) = 1 + b_1 \tau + b_2 \tau^2. \quad (17)$$

Substitution in equations (14), (13) and (15) gives the following expressions for the functions $A(\tau)$, $B(\tau)$ and $C(\tau)$ in terms of the variable τ :

$$A(\tau) = 0.97 \tau (a_2 \tau + a_1) \quad (18)$$

$$B(\tau) = \frac{1}{\lambda} \left[\frac{0.97(a_2 \tau + a_1)}{(b_1 - 0.03a_1) + \tau(b_2 - 0.03a_2)} \right] \quad (19)$$

and

$$C(\tau) = 0.03 \tau (a_2 \tau + a_1) + 1 \quad (20)$$

The coefficients a_1 , a_2 , b_1 , and b_2 appearing in the equations were determined from the experimental data given in Table IV by the method of least squares. The numerical values are $a_1 = 0.28155$, $a_2 = 0.28838$, $b_1 = 0.047763$ and $b_2 = 0.038126$. If these values are substituted in the above equations we get

$$A(\tau) = \tau(0.27973 \tau + 0.27310) \quad , \quad (21)$$

$$B(\tau) = \frac{1}{\tau} \left(\frac{0.27973 \tau + 0.27310}{0.029475 \tau + 0.039317} \right) \quad , \quad (22)$$

and

$$C(\tau) = \tau(0.0086514 \tau + 0.0084465) + 1 \quad (23)$$

The empirical formulas derived in the above manner permit the calculation of the neutron flux distributions in cylindrical fuel specimens which are in good agreement with the experimental data.

Figures 2, 3 and 4 show several neutron flux distributions computed from the various empirical relationships derived in this section. Figure 2 is based on equations (16) and (17). The curves shown in the other two figures were derived from equations (2), (21), (22) and (23).

Figure 2

Neutron Flux Perturbations at the Ends of Cylindrical Fuel Specimens during Irradiation

Figure 3

Longitudinal Distributions
of Neutron Flux at the Ends
of Cylindrical, Natural Ura-
nium Specimens of Various
Diameters.

Figure 4

Longitudinal Distributions
of Neutron Flux at the Ends
of Fuel Specimens (0.635-cm
dia) Having Various Inverse
Diffusion Lengths.

B. An Empirical Formula for Computing Longitudinal Flux Distributions at the Ends of Cylindrical Fuel Specimens Separated by a Nonabsorbing Material

The formulas developed in the preceding section may be slightly modified to give the neutron flux distribution at the ends of specimens arranged in a column and separated by a nonabsorbing material, i.e., in cases for which $\Sigma_{\text{abs}} \ll \Sigma_s$. The average flux on the cross section of the specimens may be computed with the aid of the following formula:

$$\psi(z, \tau) = (p - 1) \exp \left[- \frac{\phi(0, \tau) - 1}{p - 1} B(\tau) z \right] + 1 \quad (24)$$

for $0 \leq z \leq \lambda'$. The function $B(\tau)$ is given by equation (22) and λ' may be computed from the following empirical relationship between λ and λ' :

$$\lambda' = \left[\frac{p - 1}{\phi(0, \tau) - 1} \right] \lambda \quad (25)$$

Figure 5 Longitudinal Distributions of Neutron Flux at the Ends of Two Natural Uranium Specimens (1.27-cm Dia) Separated by Various Distances

The symbol p appearing in equations (24) and (25) is defined as the Wilkins Effect (i.e., flux perturbation adjacent to the spacer) and depends upon the neutron properties and the separation of the fuel specimens. The quantity p must be found experimentally before the above formulae may be applied. It should be pointed out again that the Wilkins Effect continues to increase as the distance between the ends of the two adjacent specimens increases, and eventually reaches a maximum value given by $\phi(0, \tau)$. Figure 5 shows a series of flux distributions for a pair of natural uranium cylinders separated (using graphite spacers) by various distances. The curves in the figure were derived from equations (24) and (25).

A tabulation of the experimental data related to this section appears in the Appendix.

SUMMARY

I. The longitudinal distribution of the average thermal neutron flux on the cross section of a cylindrical fuel specimen was found to depend only upon the material comprising the specimen and may be computed from the empirically derived formula

$$\phi(z, \tau) = A(\tau)e^{-B(\tau)z} + C(\tau)$$

for $0 \leq z \leq \lambda$, where τ is a dimensionless parameter defined as the product $0.91a(\Sigma_{\text{abs}} + \Sigma_s)$ and $\lambda = 0.5935 a^2 + 6.429a$, the quantity a denoting the radius of the cylindrical specimen. The coefficients appearing in the formula are given by

$$A(\tau) = \tau(0.27973\tau + 0.27310) \quad ,$$

$$B(\tau) = \frac{1}{\lambda} \left(\frac{0.27973\tau + 0.27310}{0.029475\tau + 0.039317} \right) \quad ,$$

and

$$C(\tau) = \tau(0.0086514\tau + 0.0084465) + 1 \quad .$$

The above formulas are valid for $0.2 < \tau < 1.7$ and $1 < \lambda(\text{cm}) < 9$.

II. If cylindrical fuel specimens are arranged in a column and are separated by a nonabsorbing material, e.g., graphite or aluminum, the average neutron flux on the cross section of the specimens (adjacent to the spacer) may be determined with the aid of the following formula:

$$\psi(z, \tau) = (p - 1) e^{-\frac{\phi(O, \tau) - 1}{p - 1} B(\tau)z} + 1 \quad , \quad 0 \leq z \leq \lambda' \quad ,$$

where $B(\tau)$, λ and τ are defined above, and

$$\lambda' = \left[\frac{p - 1}{\phi(O, \tau) - 1} \right] \lambda$$

with

$$\phi(O, \tau) = 1 + 0.28155\tau + 0.28838 \tau^2 \quad .$$

The symbol p is defined as the Wilkins Effect and depends upon the neutron properties and separation of the fuel specimens. The quantity p must be determined experimentally before the formula may be applied. The Wilkins Effect is equal to the average flux at the ends of the fuel specimens, adjacent to the spacer, divided by the average flux in the specimens remote from the spacer.

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APPENDIX

Tabulation of Experimental Data - The Longitudinal Distribution of
Specific Activity in Monitor Foils Taken from Various Positions
along Specimens No. 1 through 13.

The tables give the longitudinal distributions of specific activity observed at the ends of fuel specimens arranged in columns and separated by graphite. The experimental data (specific activity or counting rate divided by weight) are given in units of 10^3 counts/min-gm.

Table XXII, appearing at the end of the Appendix, lists the lengths of the graphite spacers used in the investigation. The graphite spacers listed in columns 1-4 of the table are to be associated with the neutron flux distributions appearing in the corresponding columns of Tables IX-XXI, i.e., $\phi_1(z)$, $\phi_2(z)$, etc.

TABLE IX

Specimen No. 1; $a = 1.27$ cm, $K_{\text{eff}} = 0.672$ cm $^{-1}$

z, cm	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$	$\phi_4(z)$
0	3.7116	3.6634	3.6217	3.8644
0.1363	3.5754	3.5247	3.4698	3.7767
0.2727	3.4667	3.4135	3.4047	3.6743
0.4092	3.4147	3.3202	3.3010	3.5964
0.5455	3.3809	3.2658	3.1520	3.5376
0.8087	3.3444	3.2134	3.0715	3.2896
1.0723	3.3407	3.1719	3.0906	3.2352
1.3359	3.3254	3.1507	3.0387	3.0913
1.9804	3.3250	3.1267	2.9800	2.9787
2.6247	3.3227	3.1201	2.9611	2.8892
3.9041	3.3229	3.1252	2.9484	2.8088
5.1834			2.9342	2.7450
6.4626			2.9265	2.7202
7.7416				2.7100
9.0208				2.6853

TABLE X

Specimen No. 2; $a = 0.898$ cm,
 $K_{\text{eff}} = 0.672$ cm $^{-1}$

z, cm	$\phi_1(z)$	$\phi_2(z)$
0	2.7339	3.6432
0.1365	2.6451	3.6049
0.2729	2.5660	3.4870
0.4092	2.5352	3.4254
0.5456	2.4839	3.3213
0.8091	2.4621	3.1937
1.0724	2.4306	3.1823
1.3356	2.4149	3.1390
1.9801	2.4141	3.0134
2.6247	2.4135	2.9448
3.9041	2.4137	2.9104
5.1836		2.8917
6.4629		2.8710

TABLE XI

Specimen No. 3; $a = 0.635$ cm,
 $K_{\text{eff}} = 0.672$ cm $^{-1}$

z, cm	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$	$\phi_4(z)$
0	4.3502	4.2397	4.7467	5.2712
0.1363	4.2400	4.1101	4.5957	5.1511
0.2727	4.1835	4.0461	4.5102	4.9438
0.4092	4.1530	4.0095	4.4379	4.8021
0.5457	4.1405	3.9883	4.4000	4.7884
0.6820	4.1297	3.9779	4.3762	4.7027
0.9453	4.1287	3.9684	4.3412	4.7006
1.2088	4.1277	3.9657	4.3322	4.6276
1.4723	4.1269	3.9640	4.3230	4.5795
2.7516		3.9642	4.3189	4.4614
4.0309			4.3191	4.4204

TABLE XII

Specimen No. 4; $a = 0.449$ cm,
 $K_{\text{eff}} = 0.672$ cm $^{-1}$

z, cm	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$
0	4.5482	4.3727	4.9635
0.1365	4.4310	4.2559	4.9100
0.2730	4.3714	4.1900	4.8408
0.4093	4.3406	4.1530	4.8145
0.5456	4.3265	4.1316	4.7348
0.6821	4.3191	4.1199	4.6852
0.9457	4.3131	4.1105	4.6167
1.2092	4.3112	4.1071	4.5947
1.4725	4.3107	4.1064	4.5553
1.7358	4.3104	4.1057	4.5420
3.0154	4.3103	4.1052	4.5101
4.2947			4.5022

TABLE XIII

Specimen No. 5; $a = 0.3175$ cm,
 $K_{\text{eff}} = 0.672$ cm $^{-1}$

z , cm	$\phi_1(z)$
0	5.0616
0.1365	4.9616
0.2730	4.9395
0.4096	4.8931
0.5460	4.8718
0.6822	4.8505
0.9455	4.8443
1.2087	4.8174
1.7356	4.8007
2.6809	4.7982

TABLE XIV

Specimen No. 6; $a = 0.635$ cm,
 $K_{\text{eff}} = 1.83$ cm $^{-1}$

z , cm	$\phi_1(z)$
0	13.190
0.1363	12.531
0.2727	11.562
0.4092	10.882
0.5457	10.324
0.6820	9.6501
0.9453	8.9484
1.2088	8.6411
1.4723	8.4605
2.7516	8.2491
3.2788	8.2212
4.3335	8.1204

TABLE XV

Specimen No. 7; $a = 0.449$ cm,
 $K_{\text{eff}} = 1.83$ cm $^{-1}$

z , cm	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$
0	8.7294	9.1495	9.4917
0.07607	8.2401	8.6300	9.2992
0.1521	7.9496	8.2349	8.7573
0.2917	7.7207	7.7485	8.0774
0.4310	7.6355	7.2891	7.6833
0.5704	7.6110	7.1332	7.3176
0.8367	7.5948	7.0748	7.1545
1.1031	7.5932	7.0539	7.0069
1.3697	7.5924	7.0457	6.7965
1.6363		7.0436	6.8130
1.9028		7.0411	6.7074
3.1852			6.4530
4.4674			6.3911

TABLE XVI

Specimen No. 8; $a = 0.3175$ cm,
 $K_{\text{eff}} = 1.83$ cm $^{-1}$

z , cm	$\phi_1(z)$	$\phi_2(z)$
0	8.7850	10.010
0.07607	8.3512	9.5881
0.1520	8.0891	9.1326
0.2916	7.8552	8.5979
0.4313	7.7652	8.3392
0.5709	7.7281	8.0781
0.8373	7.7100	8.0797
1.1036	7.7084	7.9105
1.3699	7.7065	7.8682
1.6365		7.7534
2.1695		7.6507
2.7024		7.6112

TABLE XVII

Specimen No. 9; $a = 0.449$ cm,
 $K_{\text{eff}} = 3.15$ cm $^{-1}$

z , cm	$\phi_1(z)$
0	13.842
0.07607	12.476
0.1521	11.356
0.2917	9.8805
0.4310	9.0591
0.5704	8.4395
0.8367	7.7100
1.1031	7.2810
1.3700	7.0535
1.6363	7.0081
1.9028	6.9532
2.1696	6.9487
2.9702	6.8041
3.7706	6.7286

TABLE XVIII

Specimen No. 10; $a = 0.3175$ cm,
 $K_{\text{eff}} = 3.15$ cm $^{-1}$

z , cm	$\phi_1(z)$
0	11.275
0.07607	10.779
0.1520	10.040
0.2916	9.1271
0.4313	8.5813
0.5709	8.1958
0.8373	7.7811
1.1036	7.7430
1.3700	7.5539
1.6365	7.5128
1.9027	7.4510
2.4349	7.4022

TABLE XIX

Specimen No. 11; $a = 0.2245$ cm,
 $K_{\text{eff}} = 3.15 \text{ cm}^{-1}$

z , cm	$\phi_1(z)$
0	10.441
0.07595	9.7014
0.1518	8.9803
0.2278	8.4549
0.3039	8.4492
0.4436	7.9796
0.5829	7.9041
0.7226	7.7004
0.8621	7.7701
1.1284	7.5932
1.6741	7.5334
2.2198	7.4885

TABLE XX

Specimen No. 12; $a = 0.3175$ cm,
 $K_{\text{eff}} = 5.25 \text{ cm}^{-1}$

z , cm	$\phi_1(z)$	$\phi_2(z)$
0	11.609	17.854
0.07607	9.5510	15.656
0.1520	8.5833	13.761
0.2916	7.9500	11.470
0.4313	7.7897	9.9324
0.5709	7.7537	9.0432
0.8373	7.7482	8.5299
1.1036	7.7414	8.4023
1.3700		8.3104
1.9024		8.1670
2.4345		8.0500

TABLE XXI

Specimen No. 13; $a = 0.2245$ cm,
 $K_{\text{eff}} = 5.25 \text{ cm}^{-1}$

z , cm	$\phi_1(z)$	$\phi_2(z)$	$\phi_3(z)$
0	9.0982	11.207	14.107
0.07595	7.8300	9.5203	11.960
0.1518	7.4413	8.6100	10.478
0.2278	7.3320	8.1309	9.5548
0.3039	7.3007	7.8725	8.9235
0.4434	7.2861	7.6684	8.3188
0.5829	7.2857	7.6081	8.1441
0.7226		7.5880	8.1193
0.8621		7.5839	8.0511
1.0022		7.5792	7.9987
1.4217			7.9660
1.8412			7.8812

TABLE XXII

Tabulation of Graphite Spacers Used in the
Investigation of the Wilkins Effect in
Cylindrical Fuel Specimens

Specimen	Graphite Spacers, cm			
1	0.381	0.762	1.27	Large
2	0.762	Large		
3	0.127	0.254	0.762	Large
4	0.254	0.762	Large	
5	Large			
6	Large			
7	0.127	0.254	Large	
8	0.127	Large		
9	Large			
10	Large			
11	Large			
12	0.127	Large		
13	0.127	0.254	Large	