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TECHNICAL REPORT

**ANALYSIS OF PCRV LINERS
BY INITIAL STRESS METHOD**

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and K. L. Johnson

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Technical Report

ANALYSIS OF PCRV LINERS BY INITIAL STRESS METHOD

by

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Iowa State University

Ames, Iowa

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1. INTRODUCTION

The main function of steel liners in prestressed concrete reactor vessels (PCRVS) is to provide leak-tightness for the reactor coolant. To prevent instability problems, the liner is anchored to the concrete by means of shear studs, bars, or angles. In addition, cooling tubes are usually welded to the liner between anchors for control of temperature.

In the design of an anchored liner, analyses are conducted to determine forces on and displacements in the liner components under different loading conditions. Since the deformation of the liner must be compatible with the deformation of the vessel, the liner and the anchors must be designed to accommodate the strain distribution over the lined surface of the concrete vessel. Thus, strains obtained from the analysis of the overall vessel, with or without the liner, are used as "design strains" for the liners (1)*.

The analysis of the liner assembly subjected to the "design strains" is usually done by considering an idealized one-dimensional segment of the liner with appropriate boundary conditions (see Fig. 1). Among the few published methods of one-dimensional liner stress analysis (1, 2, 3, 4). Parker's method (1) appears to be general and most versatile. His method uses a flexibility approach in which equilibrium relations and anchor and liner panel stiffnesses are substituted into the compatibility equations for node displacements. The resulting set of simultaneous nonlinear equations (due to nonlinear stiffness characteristics) stated in terms of unknown liner panel forces are solved by a step-by-step approach. Details of the

* Numbers in parentheses refer to references in Appendix A.

actual numerical solution technique are not given, however.

A new method of one-dimensional liner stress analysis is presented in this paper. The method uses a stiffness approach in which the final set of simultaneous equations are equations of equilibrium written in terms of unknown nodal displacements. The solution of these equations (nonlinear due to stiffness characteristics) is accomplished by the so-called "Initial Stress Method" which has been used in some finite element analysis of nonlinear continua (5).

The development of the "Initial Stress Method of Liner Analysis" and the solution technique used in this method are described in detail in this paper. The method is then used in a parametric study of the effects of liner design variables.

2. INITIAL STRESS METHOD OF ANALYSIS

In the design of PCRV liners, nonlinear and inelastic behavior of liner components must be considered. This prevents the straightforward use of elastic relationships. To consider yielding of liner members and nonlinearity of anchors, the Initial Stress Method of analysis was formulated for application to an "idealized model" section of the liner. The following subsections describe the formulation of the theory.

2.1. Assumptions

The model consists of a one-dimensional segment of the liner (Fig. 1a) where the effects of shear anchors are assumed to act over the model width. Since the model is highly indeterminate, the following assumptions are made:

- a. A sufficient number of panels are analyzed so that the displacement at the endpoints of the strip is negligible (Fig. 1a).

- b. The stress-strain characteristics of a liner panel are bilinear, as shown in Fig. 2.
- c. The load-deformation characteristics of the anchor-in-shear are nonlinear and are represented by four linear segments as shown in Fig. 3.
- d. The liner is subjected to a concrete-imposed design strain which is determined by an analysis of the overall vessel.

2.2. Formulation of the Theory and the Solution Technique

Equilibrium at the nth node (Fig. 2b) requires

$$F_{i+1} - F_i - S_n = 0 \quad (1)$$

where F_{i+1} and F_i are the panel forces in the $i + 1$ th and i th panels, respectively. Compressive panel forces are considered to be positive. S_n represents the shear force in the anchor at the nth node and is considered positive when the corresponding node displacement is positive.

When a completely elastic analysis is considered, the stiffness of the i th panel per unit width, C_i , is given by

$$C_i = \frac{E_i h_i}{1 - \mu^2} \quad (2)$$

where E is the modulus of elasticity, h is the liner thickness, and μ is Poisson's ratio.

By letting K_n represent the anchor stiffness per unit width, and U_n the relative* node displacement at the nth node (positive if in the

*"Relative" implies that the displacement is additional displacement with respect to the deformed concrete and is caused by anchor deformation.

negative x-direction), Eq. (1) can be rewritten as follows:

$$C_{i+1} \epsilon_{i+1} - C_i \epsilon_i - K_n U_n = 0 \quad (3)$$

where ϵ represents the panel strain (compressive positive).

Solving for ϵ_{i+1} gives

$$\epsilon_{i+1} = \frac{C_i}{C_{i+1}} \epsilon_i + \frac{K_n}{C_{i+1}} U_n \quad (4)$$

Compatibility requires that for panel i

$$U_n - U_{n-1} = (\epsilon_i - \epsilon_{ci}) a_i \quad (5)$$

where a_i is the length of the ith panel and ϵ_{ci} is the concrete-imposed design strain in the ith liner panel.

Similarly, for panel i + 1

$$U_{n+1} - U_n = (\epsilon_{i+1} - \epsilon_{c,i+1}) a_{i+1} \quad (6)$$

Eliminating ϵ from (4) through (6),

$$\frac{C_i}{a_i} U_{n-1} - \left(\frac{C_i}{d_i} + K_n + \frac{C_{i+1}}{a_{i+1}} \right) U_n + \frac{C_{i+1}}{a_{i+1}} U_{n+1} = C_i \epsilon_{ci} - C_{i+1} \epsilon_{c,i+1} \quad (7)$$

or in a more simplified form,

$$b_1 U_{n-1} + b_2 U_n + b_3 U_{n+1} = R_n^o \quad (8)$$

where b_1 , b_2 , and b_3 are elastic stiffness coefficients and R_n^o is defined as the residual node force at the nth node caused by the difference in panel forces acting at that node and is positive if directed in the positive x-direction.

If m represents the number of interior nodes used in the model, the assumed boundary conditions result in $U_0 = 0$ and $U_{m+1} = 0$ (Fig. 1a).

Using Eq. (8), m simultaneous equations can be written which, in matrix form, will be

$$[B] \{U_n\} = \{R_n^0\} \quad (9)$$

where $[B]$ is a square matrix of elastic stiffness coefficients.

The solution for nodal displacements is given by

$$\{U_n\} = [B]^{-1} \{R_n^0\} \quad (10)$$

providing all panels and anchors remain in the elastic range. After the equation has been solved for nodal displacements, final panel and anchor forces may be obtained using elastic relationships.

In the analysis of PCRV liners, however, the design strains may be beyond yield, so a solution is necessary where one or more panels or anchors have been stressed into the inelastic range. Because of non-linearity involved, Eq. (9) is no longer valid. There are three different methods of non-linear analysis (5) which have been developed mainly for the finite element analysis of inelastic continua. These are:

- a. Variable Stiffness Method,
- b. Initial Strain Method, and
- c. Initial Stress Method.

The first of these methods requires an incremental analysis in which, after each increment, the stiffness matrix is adjusted. This results in a costly computer program. The Initial Strain Method is not suited for analyses where perfect plasticity is involved. For these reasons, the Initial Stress Method of analysis has been adopted in this study.

Application of the Initial Stress Method to the one-dimensional problem is best described in terms of a step-by-step procedure.

Step 1. Assuming everything remains elastic, establish the initial elastic stiffness matrix $[B]$.

Step 2. Assuming all panels remain elastic, calculate the "apparent" (initial) residual node forces $\{R_n^o\}$.

Step 3. a) Calculate the "apparent" (elastic) node displacements $\{U_n^o\}$ by Eq. (10).

b) Calculate the "apparent" (elastic) panel strains by [using Eq. (5) and matrix formulation]

$$\{\epsilon_i^o\} = \{\epsilon_{ci}\} + [A] \{U_n^o - U_{n-1}^o\} \quad (11)$$

where $[A]$ is a diagonal matrix whose nonzero elements are $1/a_i$.

c) The "apparent" (elastic) panel force in the i th panel is given by $\epsilon_i^o C_i$. The actual panel force, F_i , is determined using the actual stress-strain relationships. Since bilinear panel behavior is assumed, the actual panel forces are given by

$$\begin{aligned} F_i &= \epsilon_i^o C_i & |\epsilon_i| &\leq \epsilon_y \\ F_i &= \epsilon_y C_i + (\epsilon_i^o - \epsilon_y) C_i' & |\epsilon_i| &> \epsilon_y \end{aligned} \quad (12)$$

where ϵ_y is the yield strain and C_i' is the stiffness in the inelastic range as shown in Fig. 2.

d) Similarly, the "apparent" (elastic) anchor force at the n th node is given by $U_n^o K_{1n}$, where K_{1n} is the initial anchor stiffness. The actual anchor force, S_n , is given

by one of the following relationships:

$$\begin{aligned}
 S_n &= U_n^0 K_{1n} & |U_n^0| &\leq V_1 \\
 S_n &= A_{1n} + (U_n^0 - V_1) K_{2n} & V_1 < |U_n^0| &\leq V_2 \\
 S_n &= A_{2n} + (U_n^0 - V_2) K_{3n} & V_2 < |U_n^0| &\leq V_3 \\
 S_n &= A_{3n} + (U_n^0 - V_3) K_{4n} & |U_n^0| &> V_3
 \end{aligned} \tag{13}$$

where A_1, A_2, A_3, V_1, V_2 , and V_3 define the line segments in Fig. 3 for load-deformation characteristics, and K_1, K_2, K_3 , and K_4 represent the slope of these line segments.

Step 4. If one or more of the panels or anchors are beyond yield, the difference between actual and apparent forces will result in some fictitious forces at the nodes which must be assumed to exist for equilibrium. Since these forces do not actually exist, they must be removed by application at the relevant nodes of residual forces which are the opposites of the fictitious forces. The force to be applied at the n th node is given by

$$R'_n = (\epsilon_{i+1}^0 C_{i+1} - F_{i+1}) - (\epsilon_i^0 C_i - F_i) - (U_n^0 K_{1n} - S_n) \tag{14}$$

If all $\epsilon_i \leq \epsilon_y$, and all $U_n^0 \leq V_1$, then $\{R_n^1\}$ will be zero, which indicates that the problem is elastic and no further computation is necessary.

Step 5. a) If R_n^1 is not zero, the additional incremental displacements are given by

$$\{\Delta U_n^1\} = [B]^{-1} \{R_n^1\} \tag{15}$$

where, again, the initial elastic stiffness matrix $[B]$ is used.

The total corrected node displacement is then given by

$$\{U_n^1\} = \{U_n^0\} + \{\Delta U_n^1\} \quad (16)$$

- b) Due to $\{\Delta U_n^1\}$, there will be changes in panel strains given by

$$\{\Delta \epsilon_i^1\} = [A] \{\Delta U_n^1 - \Delta U_{n-1}^1\} \quad (17)$$

and the total panel strains are

$$\{\epsilon_i^1\} = \{\epsilon_i^0\} + \{\Delta \epsilon_i^1\} \quad (18)$$

- c) From these new values of panel strains and node displacements, the new actual and apparent panel and anchor forces are computed as before.

- d) A new set of residual nodal forces, $\{R_n^2\}$, can now be obtained. If $\{R_n^2\}$ is negligibly small, no further computation is necessary.

Step 6. If $\{R_n^2\}$ is not negligible, repeat step 5, as the next iteration.

In general, Eq. (15) can be written as

$$\{\Delta U_n^j\} = [B]^{-1} \{R_n^j\} \quad (19)$$

where j represents the j th iteration. The node displacements are continuously summed by

$$\{U_n^j\} = \{U_n^{j-1}\} + \{\Delta U_n^j\} \quad (20)$$

and the panel strains by

$$\{\epsilon_i^j\} = \{\epsilon_i^{j-1}\} + \{\Delta\epsilon_i^j\} \quad (21)$$

This process is repeated until

$$\{R_n^j\} = \{\epsilon_{i+1}^{j-1} C_{i+1} - F_{i+1}^{j-1}\} - \{\epsilon_i^{j-1} C_i - F_i^{j-1}\} - \{U_n^{j-1} K_{ln} - S_n^{j-1}\} \quad (22)$$

becomes negligible. The results are sets of displacements, strains, panel forces, and anchor forces which satisfy all equilibrium and compatibility conditions, as well as the nonlinear panel and anchor characteristics.

2.3. Computer Program

A computer program was prepared in order to use the Initial Stress Method formulated above for the analysis of one-dimensional liner systems. The complete listing of the computer program and instructions for input/output are given in Appendix D.

The computer program, as prepared, uses individual input for each anchor and each panel. This permits the analysis of the liner systems for any local variations such as

- a. Variation in panel characteristics
- b. Variation in anchor characteristics
- c. Variation in anchor spacing
- d. Variation in liner thickness
- e. Variation in design strain
- f. Any combination of the above.

2.4. Criteria for Solution Accuracy

The magnitudes of residual nodal forces, R_n^j , served as the natural criterion for convergence since iteration is to be stopped when all values of R_n^j are negligible. It was found in this study that, within the range of variables studied, the computed maximum anchor force is within 1% of the estimated correct anchor force when all values of residual nodal forces are less than 0.05 kips. The number of iterations required to obtain this value varies considerably depending on the magnitudes of various design parameters. In most cases, however, the convergence was reached in less than 200 iterations.

In Subsection 2.1.a, it was assumed that a sufficient number of panels are analyzed so that the displacements at the endpoints of the strip are negligible. The criterion used in this study is that the end anchor displacement should be within 2% of the displacement of the maximum stressed anchor. In most cases analyzed it was found that the 2% limit was satisfied with a model consisting of 30 nodes. It should be pointed out that Doyle and Chu have also used 2% criterion in their containment liner analysis (2).

2.5. Comparison to Another Method

The following case was analyzed by Doyle and Chu (2).

Liner thickness = 0.375 in.

Anchor spacing in the direction of model = 21.0 in.

Anchor spacing in the width direction of model = 10.5

Size of anchors = 3/4-in. diameter studs

Liner yield stress = 43.0 ksi

Design strain = uniaxial yield strain

Weak panel has zero load carrying capacity

Number of anchors in model = 20.

The results obtained using the initial stress method compare to Doyle and Chu's results as follows:

<u>Method</u>	<u>Maximum anchor force (kips)</u>	<u>Maximum anchor displacement (in.)</u>
Doyle and Chu	25.433	0.1103
Initial stress method	25.904	0.118

As can be seen from this table, the analysis results from the two methods agree very well. The differences are probably due to differences in idealizing the anchor load-displacement characteristics.

3. PARAMETRIC STUDY

3.1. Parameters Studied

A study has been conducted to assess the effect of various design parameters on the magnitude of anchor forces and displacements in PCR/V liners. The range of parameters was chosen to be compatible with most PCR/V liners designed to date. Specifically, the parameters used are as follows:

- a. Liner plate yield stress (σ_y): 40 and 60 ksi
- b. Liner plate thickness (h): 1/2 and 3/4 in.
- c. Anchor spacing (a): 5, 7.5, and 10 in.
- d. Anchor size (ϕ): 3/4- and 1-in. diameter studs.
- e. Design strains (ϵ_c): 1000, 1200, 1400, 1600, and 1800 $\mu\text{in./in.}$

3.2. Weak Panels

In design analysis, several local variations must be estimated (such as thickness or yield point variations and initial lateral deflection). When local variations adversely affect a panel, its in-plane load carrying capacity will be less than that of other panels. Such a panel is usually referred to as a "weak panel" with the remainder being "strong panels."

The study of these local effects is beyond the scope of this paper. For calculation purposes, the weak panel strength is assumed to be a fraction of the strong panel strength. Specifically, the following weak panel apparent yield points are studied:

$$\sigma_{yw} = (0, 0.25, 0.5, 0.75)\sigma_y .$$

It is to be noted that anchor forces and displacements will be zero under uniform design strains and identical panel and anchor characteristics.

3.3. Details of Analysis

All cases included in this study were analyzed using the Initial Stress Method. The specific conditions common to all cases analyzed are as follows:

- a. The model consists of 30 anchors (nodes) and 31 panels with the end panels being fixed.
- b. The center (or 16th) panel is assumed to be weaker than the other panels.
- c. All panels are assumed to be elastic-perfectly plastic, with the weak panel yield point being a fraction of the strong panel yield point (Fig. 4). The modulus of elasticity for strong panels is taken to be 29,000 ksi and for weak panels, 28,000 ksi.

- d. Anchors are placed on the liner in a square pattern.
- e. Anchor force-deformation characteristics are represented by four line segments, based on test results given in Refs. 6 and 7. The actual and approximate anchor characteristics are shown in Fig. 5.
- f. Stiffness of the cooling tubes is ignored.

3.4. Results of Parametric Study

The anchor forces and displacements obtained from the parametric study are graphically presented in Figs. 6 through 11. Some discussion of these figures follows.

(a) The dependence of the magnitude of maximum anchor force on the magnitudes of design strain, liner yield point, and weak panel load carrying capacity is clearly illustrated in Fig. 6. It is observed from this figure that:

1) When the weak panel carries no load, the maximum anchor force is practically independent of the liner yield point. In contrast, when the weak panel capacity is nearly that of the strong panel, the 60 ksi steel results in much lower anchor forces than the 40 ksi steel. However, this trend may be correct only for the particular range of design strains studied. The most pronounced effect is that anchor forces increase considerably with decreasing weak panel load capacity.

2) The slope of the curves is much greater for the higher strength steel, reflecting the fact that for a given σ_{yw}/σ_y ratio the actual change in weak panel capacity (in ksi) is more severe for higher strength steel. Increasing the design strain also causes greater increases

in anchor force when high strength steel is used. This is an important fatigue consideration since the values of design strains are expected to vary considerably during normal and accident conditions. Generally, once an anchor is stressed, it is not desirable to have the forces fluctuating over a wide range of values. This consideration indicates that it would be better to use the lower strength steel where the change in anchor force, for a given change in design strain or weak panel characteristics, is not as great.

3) For a given weak panel capacity, an increase in design strain results in higher anchor forces. The shape of the curves for the high strength steel are different from those for the lower strength steel because the values of design strain are less than the yield strain for the high strength steel.

(b) The effect of different anchor spacing is shown in Fig. 7. The larger anchor spacing naturally results in higher anchor forces. Again, it is shown that the lower weak panel strength results in higher anchor forces.

(c) Figure 8 shows the effect of liner thickness on the magnitude of anchor force. It is seen that a thicker liner actually results in higher anchor forces. Furthermore, for most cases analyzed, the change of thickness from 1/2 in. to 3/4 in. resulted in roughly a 10% increase in anchor force. The increase did not appear to be influenced by design strain or weak panel capacity to any great extent.

(d) All other parameters being identical, the anchor forces would be larger for larger anchors. However, the anchor force as a fraction of its ultimate strength is actually smaller for a larger anchor. From Fig. 9 it

can be seen that the 3/4-in. diameter studs are stressed considerably higher with respect to their ultimate strength than the 1-in. diameter studs. On this basis alone, it would be better to use the larger anchors. However, the anchor diameter to liner thickness ratio should be limited to about 2 in order to prevent tearing of the liner (2).

(e) The curves in Fig. 10 indicate the interaction of main design parameters. In addition to the trends noted in (a), Fig. 10 shows that:

1) If a is increased by 50%, from 5 in. to 7.5 in., while holding h constant at 1/2 in., the increase in anchor force is approximately 4 kips ($\epsilon_c = 1000 \mu\text{in./in.}$, $\sigma_{yw} = 0$). On the other hand, if h is increased 50%, from 1/2 in. to 3/4 in., while holding a constant at 5 in., the anchor force is only increased by 2 kips. This indicates that the selection of anchor spacing in design will be more critical with respect to anchor force than the selection of liner thickness.

2) Design of anchors cannot be done on the basis of a/h since two sets of a 's and h 's may have the same a/h value yet the corresponding anchor forces are quite different.

3) If the design strain is increased from 1000 $\mu\text{in./in.}$ to 1800 $\mu\text{in./in.}$, the increase in anchor force is more or less uniform.

(f) Figure 11 is similar to Fig. 10 except that it is for anchor displacements instead of anchor forces. It can be seen that anchor displacements are quite sensitive to the value of the design strain chosen. This may be explained by considering the shape of the load-slip curve for anchors (Fig. 5) where, in the high range of anchor forces, an increase in anchor force corresponds to a large increase in displacement. The trends noted for Fig. 10 generally apply to Fig. 11, also.

(g) For a given set of magnitudes of design strains (reflecting different loading conditions), liner yield point and thickness, anchor spacing, and weak panel capacity, the ranges of maximum anchor forces expected may be quickly obtained from curves such as those given in Fig. 10. Figure 11 gives similar interaction curves for anchor displacements. A complete set of similar interaction curves covering all ranges of variables could be a very useful design aid.

4. ACKNOWLEDGMENT

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APPENDIX A. REFERENCES CITED

1. Parker, J. V., "Stress Analysis of Liners for Prestressed Concrete Pressure Vessels," Proc. of First Int. Conf. on Struc. Mech. in Reactor Tech., Paper H 6/1 (September 1971).
2. Doyle, J. M., and S. L. Chu, "Some Structural Considerations in the Design of Nuclear Containment Liners," Nuclear Engr. and Design, 16, 294-300 (1971).
3. Chapman, J. C., and A. Carter, "Interaction between a Pressure Vessel and Its Liners," Conf. on Prestressed Concrete Pressure Vessels, Paper 58, London (March 1967).
4. Tan, C. P., "A Study of the Design and Construction Practices of Prestressed Concrete and Reinforced Containment Vessels," Clearinghouse for Federal Scientific and Technical Information, Report No. TID 25176 (1969).
5. Zienkiewicz, O. C., The Finite Element Method in Engineering Science, London (1971).
6. Viest, I. M., "Investigation of Stud Shear Connectors for Composite Concrete and Steel T-Beams," ACI Journal, Proc., V. 52, pp. 875-891 (April 1956).
7. Design Data, Nelson Concrete Anchor Studs, Nelson Study Welding, Division of Gregory Industries, Manual No. 21 (1961).

APPENDIX B. FIGURES

Fig. 1. One-dimensional stress analysis model.

Fig. 2. Panel characteristics used in the computer program.

Fig. 3. Anchor characteristics used in the computer program.

Fig. 4: Panel characteristics used in parametric study.

Fig. 5. Actual and idealized anchor characteristics used in parametric study (Refs. 6 and 7).

Fig. 6. Anchor force - weak panel strength as influenced by design strain and liner yield point.

Fig. 7. Anchor force - design strain relationships as influenced by anchor spacing.

Fig. 8. Anchor force - design strain relationship as influenced by liner thickness.

Fig. 9. Maximum anchor force/ultimate ratio - design strain relationship as influenced by anchor size.

Fig. 10. Anchor force - a/h ratio as influenced by values of a , h , σ_{yw} , and design strain.

Fig. 11. Anchor displacement - a/h ratio as influenced by various values of a and h .

APPENDIX C.

COMPUTER PROGRAM FOR INITIAL STRESS METHOD

C.1. Input and Output

1. Input:

A. Control Card (2I5, 2F10.5)

Col. 1-5 Number of panels (maximum = 31)
 Col. 6-10 Number of cases to be run with the same anchor stiffness properties
 Col. 11-20 Poisson's ratio
 Col. 21-30 Maximum residual node force to end iteration, kips

B. Anchor Cards (8F10.6): One card per anchor is needed

Col. 1-10 Maximum anchor force in first linear segment, kips
 Col. 11-20 Corresponding displacement, in.
 Col. 21-30 Maximum anchor force in second linear segment, kips
 Col. 31-40 Corresponding displacement, in.
 Col. 41-50 Maximum anchor force in third linear segment, kips
 Col. 51-60 Corresponding displacement, in.
 Col. 61-70 Maximum anchor force in fourth linear segment, kips
 Col. 71-80 Corresponding displacement, in.

C*. Anchor Spacing Card (F10.3)

Col. 1-10 Spacing of anchors perpendicular to the model, in.

D*. Panel Cards (3F10.3, 4F10.6): One card per panel is needed

Col. 1-10 Modulus of elasticity, ksi
 Col. 11-20 Yield stress, ksi
 Col. 21-30 Ultimate strength, ksi
 Col. 31-40 Ultimate strain, in./in.
 Col. 41-50 Panel length, in.
 Col. 51-60 Panel thickness, in.
 Col. 61-70 Design strain, in./in.

Notes: 1) The data input is for a general case. If many anchor and panel cards are the same, the program may easily be altered to avoid repetitious input.

2) For each set of different anchor stiffness properties, a new computer run should be conducted.

*Other cases: Repeat cards C and D for each case.

2. Output: There are three pages of output.

Page 1: Number of panels (input)
Anchor forces and displacements (input)

Page 2: Lists data for each panel (input)

Page 3: Number of Iterations (computed)
Panel width (input)
Panel design strains (input)
Panel final strains and forces (output)
Node displacements, anchor forces, and residual node forces
(computed)

```

C   ONE DIMENSIONAL LINER STRESS ANALYSIS USING THE
C   ITERATIVE INITIAL STRESS METHOD
C   ASSUMPTIONS
C   MODEL CONSISTS OF NP PANELS AND NP-1 ANCHORS
C   END POINTS HAVE NO RELATIVE DISPLACEMENTS
C   PANEL CHARACTERISTICS ARE BILINEAR
C   ANCHOR CHARACTERISTICS ARE REPRESENTED BY FOUR LINE SEGMENTS
C   NOMENCLATURE
C   NP   -NUMBER OF PANELS
C   E    -MODULUS OF ELASTICITY, KSI
C   FY   -YIELD POINT, KSI
C   FU   -ULTIMATE STRENGTH, KSI
C   AA   -PANEL LENGTH, IN
C   T    -PANEL THICKNESS, IN
C   W    -ANCHOR SPACING PERPENDICULAR TO THE STRIP, IN
C   DS   -DESIGN STRAIN, IN/IN
C   POI  -POISSON'S RATIO
C   SA   -ANCHOR FORCE FOR DISPLACEMENT UA, KIPS
C   UA   -ANCHOR DISPLACEMENT CORRESPONDING TO SA, IN
C   ERR  -ACCEPTIBLE RESIDUAL NODE FORCE, KIPS
C
C   DIMENSION SA1(32), SA2(32), SA3(32), SA4(32)
C   DIMENSION UA1(32), UA2(32), UA3(32), UA4(32)
C   DIMENSION UEL(32), EPSA(32), DELU(32), DELE(32)
C   DIMENSION C1(32), C2(32), AA(32), DS(32)
C   DIMENSION EY(32), FACT(32), BF(32), SBF(32)
C   DIMENSION LW(31), MW(31)
C   DIMENSION ANCFO(32), STIFF(31,31)
C *****
C *DATA INPUT*
C *****
C   READ (5,10) NP, NCASE, POI, ERR
C   NA=NP-1
C   B=1.0
C   WRITE(6,22)
C   WRITE(6,23) NP
C   WRITE(6,24)
C   DO 40 I=1, NA

```



```

40 READ(5,30)SA1(I),UA1(I),SA2(I),UA2(I),SA3(I),UA3(I),SA4(I),UA4(I)
DO 999 KJ=1,NCASE
READ (5,20) W
B=W/B
DO 45 M=1,NA
SA1(M)=SA1(M)/B
SA2(M)=SA2(M)/B
SA3(M)=SA3(M)/B
SA4(M)=SA4(M)/B
45 WRITE(6,25)SA1(M),SA2(M),SA3(M),SA4(M),UA1(M),UA2(M),UA3(M),UA4(M)
WRITE (6,42)KJ
WRITE(6,43)
WRITE(6,44)
DO 60 I=1,NP
READ (5,50) E,FY,FU,EU,AA(I),T,DS(I)
IF (T.GT.0.) GO TO 52
FY=0.
52 EY(I)=(FY/E)*(1.-POI**2)
C1(I)= (E/(1.-POI**2))*T
C2(I)=(FU-FY)*T /(EU-EY(I))
F=DS(I)*C1(I)
60 WRITE (6,55) C1(I),C2(I),AA(I),T,FY,EY(I),DS(I),F
DO 65 J=1,NA
DO 65 I=1,NA
65 STIFF(I,J)=0.
C *****
C *FORM AND INVERT STIFFNESS MATRIX - INSITU*
C *****
K=1
DO 80 J=1,NA
IF (K.EQ.1) GO TO 70
STIFF(J,K-1)=C1(J)/AA(J)
70 STIFF(J,K)=-SA1(J)/UA1(J)-C1(J)/AA(J)-C1(J+1)/AA(J+1)
IF (K.EQ.NA) GO TO 80
STIFF(J,K+1)=C1(J+1)/AA(J+1)
80 K=K+1
NNN=31

```

```

      DO 92 I=1,NA
      DO 92 J=1,NA
92  STIFF(I,J)=STIFF(I,J)/1000.
      CALL UMINV(STIFF,NA,NNN,DET,LW,MW)
      DO 94 I=1,NA
      DO 94 J=1,NA
94  STIFF(I,J)=STIFF(I,J)/1000.
C  ****
C  *INITIAL FORCES AND DISPLACEMENTS*
C  ****
      DO 100 I=1,NA
100 BF(I)=C1(I)*DS(I)-C1(I+1)*DS(I+1)
      DO 104 I=1,NA
      SUM=0.
      DO 102 J=1,NA
102 SUM=SUM+STIFF(I,J)*BF(J)
104 UEL(I)=SUM
C  ****
C  *INITIALIZATION*
C  ****
      EPSA(1)=DS(1)+UEL(1)/AA(1)
      EPSA(NP)=DS(NP)-UEL(NP-1)/AA(NP)
      DO 120 I=2,NA
120 EPSA(I)=DS(I)+(UEL(I)-UEL(I-1))/AA(I)
      DO 130 I=1,NP
      BF(I)=0.
      DELU(I)=0.
      DELE(I)=0.
      FACT(I)=EPSA(I)*C1(I)
      IF (I.EQ.NP) GO TO 130
      ANCF0(I)=UEL(I)*(SA1(I)/UA1(I))
130 CONTINUE
      UEL(NP)=0.
      ANCF0(NP)=0.
140 IMAX=0
C  ****
C  *ITERATION*

```

C *****

```
200 DO 270 I=1,NP
    EPSA(I)=EPSA(I)+DELE(I)
    FI=FACT(I)+DELE(I)*C1(I)
    IF (EPSA(I).GT.EY(I)) GO TO 210
    FACT(I)=EPSA(I)*C1(I)
    GO TO 220
210 FACT(I)=(C1(I)*EY(I))+(EPSA(I)-EY(I))*C2(I)
    IF (FACT(I))215,220,220
215 FACT(I)=0.
220 BF(I)=FI-FACT(I)
    IF(I.EQ.NP) GO TO 270
    UEL(I)=UEL(I)+DELU(I)
    ABSU=ABS(UEL(I))
    SI=ANCFO(I)+DELU(I)*(SA1(I)/UA1(I))
    IF (ABSU.GT.UA1(I)) GO TO 230
    ANCFO(I)=ABSU *(SA1(I)/UA1(I))
    GO TO 260
230 IF (ABSU.GT.UA2(I)) GO TO 240
    ANCFO(I)=SA1(I)+( ABSU -UA1(I))*(SA2(I)-SA1(I))/(UA2(I)-UA1(I))
    GO TO 260
240 IF (ABSU.GT.UA3(I)) GO TO 250
    ANCFO(I)=SA2(I)+( ABSU -UA2(I))*(SA3(I)-SA2(I))/(UA3(I)-UA2(I))
    GO TO 260
250 ANCFO(I)=SA3(I)+( ABSU -UA3(I))*(SA4(I)-SA3(I))/(UA4(I)-UA3(I))
260 ANCFO(I)=ANCFO(I)*(UEL(I)/ABSU)
270 SBF(I)=SI-ANCFO(I)
    IK=1
    IMAX=IMAX+1
    DO 300 N=1,NA
        BF(N)=+BF(N+1)-BF(N)-SBF(N)
        IF (BF(N).LT.ERR) GO TO 300
        IK=IK+1
300 CONTINUE
    IF (1-IK)150,400,400
150 CONTINUE
    IF(IMAX.GT.900) GO TO 400
```

```

DO 154 I=1,NA
SUM=0.
DO 152 J=1,NA
152 SUM=SUM+STIFF(I,J)*BF(J)
154 DELU(I)=SUM
DELE(1)=DELU(1)/AA(1)
DELE(NP)=-DELU(NP-1)/AA(NP)
DO 160 M=2,NA
160 DELE(M)=(DELU(M)-DELU(M-1))/AA(M)
GO TO 200

```

```

C *****
C *OUTPUT*
C *****

```

```

400 BF(NP)=0.
WRITE(6,410)
WRITE(6,420)IMAX
WRITE(6,430)W
WRITE(6,440)
WRITE(6,450)
WRITE(6,460)(DS(I),EPSA(I),FACT(I),UEL(I),ANCF0(I),BF(I),I=1,NP)
10 FORMAT (2I5,2F10.5)
30 FORMAT(8F10.6)
22 FORMAT(1H1,40X,'ONE DIMENSIONAL STRESS ANALYSIS',//,53X,'DATA',/)
23 FORMAT(5X,'NUMBER OF PANELS=',I3,/)
24 FORMAT('0',20X,'ANCHOR FORCES,KIPS',35X,'ANCHOR DISPLACEMENTS,INCH
1ES',/)
20 FORMAT (F10.5)
25 FORMAT(8F15.6)
42 FORMAT ('1',50X,'CASE NO. ',I3,/)
43 FORMAT('0',2X,'STIFFNESS C1',3X,'STIFFNESS C2',2X,'PANEL LENGTH ',
11X,'PANEL THICKNESS',4X,'YIELD POINT',3X,'YIELD STRAIN',2X,'DESIGN
2 STRAIN',2X,'INITIAL FORCE',/)
44 FORMAT(5X,'KIPS/IN',8X,'KIPS/IN',10X,'IN',12X,'IN',9X,'KIP/IN WIDT
1H',7X,'IN/IN',9X,'IN/IN',9X,'KIPS',/)
50 FORMAT (3F10.3,4F10.6)
55 FORMAT (2F15.2,6F15.6)
410 FORMAT('1',50X , '***OUTPUT***',/)

```

```
420 FORMAT('0',5X,'NUMBER OF ITERATIONS=',I5,/)
430 FORMAT('0',5X,'PANEL WIDTH=',F10.3,/)
440 FORMAT(1X,'DESIGN STRAINS',2X,'FINAL STRAINS',3X,'PANEL FORCES',5X
1,'NODE DISP',2X,'ANCHOR FORCES',2X,'UNBALANCED FORCES',/)
450 FORMAT(9X,'IN/IN',10X,'IN/IN',11X,'KIPS',9X,'INCHES',11X,'KIPS',11
1X,'KIPS',/)
460 FORMAT(2F15.6,F15.3,F15.6,2F15.3)
999 CONTINUE
STOP
END
```

APPENDIX D. TABULATED RESULTS OF PARAMETRIC STUDY

Table I. Anchor forces using 3/4-in. diameter studs.

σ_y , ksi	σ_{yw}/σ_y	h	a	ϵ_{co}				
				1000	1200	1400	1600	1800
40	0.0	0.5	5	13.96	15.57	17.22	18.64	19.13
			7.5	18.15	19.42	20.48	21.48	22.43
			10	20.13	21.70	23.39	24.81	25.49
		0.75	5	15.92	18.05	19.19	19.92	20.61
			7.5	19.61	20.95	22.43	23.90	24.94
			10	21.92	23.98	25.37	26.43	27.49
	0.25	0.5	5	12.25	13.69	15.20	16.57	17.81
			7.5	15.12	17.68	19.20	20.06	20.87
			10	18.40	19.88	21.35	22.72	24.01
		0.75	5	13.59	15.55	17.62	18.89	19.48
			7.5	17.46	19.38	20.71	21.93	23.08
			10	19.73	21.57	23.61	25.12	25.93
	0.50	0.5	5	9.96	12.02	13.32	14.52	15.60
			7.5	12.45	14.68	17.06	18.79	19.45
			10	14.39	17.77	19.54	20.68	21.74
		0.75	5	11.61	13.27	15.10	16.72	18.17
			7.5	13.85	16.89	19.12	20.12	21.08
			10	16.50	19.42	21.20	22.81	24.30
	0.75	0.5	5	5.51	9.29	11.70	12.61	13.46
			7.5	8.26	12.08	14.08	15.77	17.27
			10	11.01	13.85	16.86	18.88	19.69
		0.75	5	6.92	11.34	12.81	14.12	15.29
			7.5	10.38	13.36	16.11	18.42	19.25
			10	12.04	15.77	19.03	20.28	21.44
60	0.0	0.5	5	13.96	15.57	17.28	18.77	19.42
			7.5	18.15	19.42	20.51	21.70	22.98
			10	20.13	21.70	23.41	24.94	25.83
		0.75	5	15.92	18.05	19.19	20.04	20.96
			7.5	19.61	20.95	22.43	23.99	25.12
			10	21.92	23.98	25.37	26.43	27.52
	0.25	0.5	5	10.26	12.12	13.54	15.12	16.80
			7.5	12.62	14.88	17.43	19.15	20.20
			10	14.65	18.06	19.74	21.25	22.93
		0.75	5	11.73	13.42	15.35	17.46	18.97
			7.5	14.08	17.15	19.26	20.58	21.18
			10	16.85	19.56	21.39	23.41	25.09
	0.50	0.5	5	1.67	5.45	9.23	11.79	13.14
			7.5	2.51	8.18	12.06	14.24	16.71
			10	3.34	10.91	13.80	17.10	19.35
		0.75	5	2.11	6.85	11.32	12.92	14.81
			7.5	3.17	10.28	13.33	16.28	18.93
			10	4.22	11.99	15.71	19.11	20.86
	0.75	0.5	5	0.09	0.11	0.64	4.42	8.20
			7.5	0.14	0.16	0.96	6.63	11.56
			10	0.18	0.22	1.28	8.84	12.99
		0.75	5	0.09	0.11	0.82	5.56	10.30
			7.5	0.14	0.17	1.23	8.34	12.60
			10	0.19	0.22	1.65	11.12	14.61

Table II. Anchor forces using 1-in. diameter studs.

σ_y , ksi	σ_{yw}/σ_y	h	a	ϵ_{co}				
				1000	1200	1400	1600	1800
40	0.0	0.5	5	18.35	20.08	21.86	23.51	25.02
			7.5	22.91	25.98	29.06	31.81	33.32
			10	28.08	32.49	34.30	35.89	37.38
		0.75	5	20.54	22.91	25.33	27.52	29.53
			7.5	26.71	30.75	33.49	34.89	36.24
			10	33.01	35.22	37.59	39.82	41.52
	0.25	0.5	5	16.58	18.08	19.68	21.13	22.47
			7.5	19.60	22.40	25.26	27.82	30.13
			10	23.18	27.32	31.49	33.64	34.90
		0.75	5	18.01	20.14	22.35	24.36	26.19
			7.5	22.25	26.03	29.84	32.89	34.04
			10	27.08	32.53	34.71	36.64	38.48
	0.50	0.5	5	13.83	16.36	17.70	18.90	20.04
			7.5	16.76	19.13	21.68	23.95	26.00
			10	18.80	22.49	26.31	29.63	32.48
		0.75	5	15.98	17.68	19.60	21.35	22.93
			7.5	18.31	21.60	25.09	28.12	30.76
			10	21.19	26.16	31.22	33.78	35.28
	0.75	0.5	5	7.64	12.90	16.02	16.91	17.76
			7.5	11.46	16.43	18.46	20.26	21.90
			10	15.28	18.23	21.45	24.22	26.60
		0.75	5	5.74	15.71	17.20	18.55	19.78
			7.5	14.60	17.78	20.70	23.24	25.49
			10	16.41	20.38	24.82	28.51	31.75
60	0.0	0.5	5	18.35	20.08	21.95	23.91	26.00
			7.5	22.91	25.98	29.19	32.50	33.94
			10	28.08	32.49	34.37	36.29	38.35
		0.75	5	20.54	22.91	25.43	28.06	30.76
			7.5	26.71	30.75	33.54	35.22	37.04
			10	33.01	35.22	37.66	40.29	41.80
	0.25	0.5	5	14.25	16.46	17.93	19.60	21.42
			7.5	16.93	19.34	22.11	25.13	28.30
			10	19.08	22.81	26.92	31.28	33.89
		0.75	5	16.10	17.83	19.92	22.24	24.72
			7.5	18.56	21.89	25.65	29.64	33.11
			10	21.56	26.59	32.00	34.59	36.98
	0.50	0.5	5	2.32	7.57	12.83	16.12	17.51
			7.5	3.48	11.35	16.40	18.62	21.31
			10	4.64	15.13	18.19	21.74	25.77
		0.75	5	2.98	9.64	15.69	17.33	19.33
			7.5	4.47	14.46	17.74	20.95	24.60
			10	5.95	16.36	20.31	25.17	30.49
	0.75	0.5	5	0.18	0.22	0.89	6.14	11.39
			7.5	0.28	0.33	1.33	9.20	15.88
			10	0.37	0.44	1.78	12.27	17.35
		0.75	5	0.19	0.23	1.16	7.83	14.50
			7.5	0.29	0.35	1.74	11.74	16.99
			10	0.38	0.46	2.32	15.53	19.12