Contract No. W-7405-eng-92

CRITICAL-ASSEMBLY EXPERIMENTS ON A REFLECTOR CONTROL
SYSTEM FOR A BOILING REACTOR

by

David A. Dingee
William S. Hogan
Robert G. Wilson
William C. Ballowe
Francis J. Jankowski
Joel W. Chastain

December 20, 1957

BATTELLE MEMORIAL INSTITUTE
505 King Avenue
Columbus 1, Ohio

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
PROTOTYPE REACTOR WITH REFLECTOR CONTROL	2
CRITICAL-ASSEMBLY DESCRIPTION	4
EXPERIMENTAL PROGRAM	10
Cold Case	11
Semihot Case	11
Hot Case	11
Nonhomogeneous Boiling Case	14
Homogeneous Boiling Case.	14
EXPERIMENTAL PROCEDURES.	15
Critical-Mass Determinations	15
Incremental Reflector-Worth Determination	15
Flux-Distribution Determination and Counting Technique	15
Boiling Void Distribution	16
EXPERIMENTAL RESULTS	16
Reflector Worth and Critical Mass	16
Boiling Void-Distribution Results	19
Miscellaneous Reflector Experiments	27
THEORY.	29
Calculational Procedures	29
Reflector Worth	29
Disadvantage Factor.	29
Two-Group Constants	30
Results	31
DISCUSSION	31
CONCLUSIONS	35
REFERENCES.	36

TABLE OF CONTENTS
(Continued)

	<u>Page</u>
APPENDIX A	
CALCULATION PROCEDURE FOR VOID DISTRIBUTIONS	A-1
Nomenclature	A-1
APPENDIX B	
SOLUTION OF TWO GROUP-TWO REGION EQUATIONS WITH ANNULAR GAP	B-1

CRITICAL-ASSEMBLY EXPERIMENTS ON A REFLECTOR CONTROL SYSTEM FOR A BOILING REACTOR

David A. Dingee, William S. Hogan, Robert G. Wilson,
William C. Ballowe, Francis J. Jankowski, and Joel W. Chastain

Research directed toward an evaluation of a reflector control scheme for a small boiling heterogeneous reactor is reported. The control method uses the ambient reactor steam pressure to adjust the height of a radial water reflector.

Critical-assembly experiments were conducted to measure the reflector worth and the effect of nonhomogeneous voids.

The critical-assembly core approximated a right circular cylinder 24 in. high and 21 in. in diameter. The core was made up of polyethylene strips to simulate water channels, aluminum strips, and uranium foil. The reflector consisted of polyethylene blocks and two different densities were used to simulate two different water temperatures.

The cold-water, hot non-power-producing, and boiling power-producing reactor conditions were mocked-up. These conditions were obtained by voiding the moderator plastic strips.

The results indicate that sufficient reactivity is available in the reflector to compensate for large power variations and equilibrium poisons. However, the reactivity worth is not great enough in the system considered to include startup from the cold condition or long-time fuel burnup. These experiments also show that in calculations of critical mass and reflector worth the assumption of uniform void distributions seems valid for cores of the size studied here.

INTRODUCTION

The type of reactor system considered here is a relatively small water-moderated and -cooled boiling heterogeneous reactor with a water reflector. All, or a major part, of the control requirements of the system would be achieved by varying the height of the radial reflector. The reflector height is varied on power demand through changes in the core pressure.

Reflector control coupled with a burnable poison (if it is required) could make possible a reactor power system capable of unmanned operation for use in remote areas. Elimination of all or most of the control rods normally required would simplify pressure-vessel design and reduce fuel-handling problems.

Before undertaking the development of such a system, it is desirable to have preliminary information concerning some of the more obvious of the problems.

- (1) Does the system provide adequate control potential?
- (2) To what extent can one ignore the inhomogeneous distribution of voids in predicting reflector worth, void coefficients, fuel loadings, etc.?
- (3) Can the system respond fast enough, and is it stable?

The purpose of the critical experiment described in this report is to provide answers relating to Items (1) and (2) on the preceding page.

A separate study was done with an operational electronic analog coupled to a simulated hydraulic system to provide answers to Item (3). This work is reported in BMI-1236.

PROTOTYPE REACTOR WITH REFLECTOR CONTROL

The reactor assumed for this system is small, developing approximately 2 megawatts heat output. It was assumed to operate at a saturation temperature of 420 F, corresponding to an ambient pressure of 300 psi.

The principle of operation of the reflector control scheme is shown in Figure 1. The reactor core is surrounded by an annular tank which forms the inner reflector region. This reflector tank is connected to an external pressurizing tank and is open to the steam region of the reactor pressure vessel. External to this annular tank is the downcomer region which also serves as a reflector and shield.

The method of operation can be most easily seen by considering how this system behaves in operation. First consider the reactor producing power at a constant rate with all of the parameters in equilibrium. Now, if the throttle is opened the immediate effect will be a reduction in steam pressure within the reactor volume. This will cause the water in the core volume to boil more violently thus reducing reactivity, but it will also cause the water height in the annular reflector tank to rise, which will increase reactivity. The rising reflector level will control the system, thus increasing the overall reactivity and producing more power and more steam until the operating pressure of the system is re-established. After the transients have died out the system will be in equilibrium again with a new power output and new inner reflector level.

Now consider what happens when the throttle is closed. The pressure in the reactor system immediately increases, the boiling is reduced, and the steam fraction in the core will be reduced, thus increasing the reactivity; the reflector level in the annular tank is also forced down, decreasing the reactivity. The total effect is a decrease in reactivity, which decreases the power output until the system pressure is again re-established with a new power level and a new reflector level.

If the throttle is closed completely so that there is no demand on the system there will be enough heat produced to overcome losses in the system; the pressure will be maintained automatically so that the reflector will be at its proper level.

Initial analytical studies were carried out on the core physics and the system dynamics. These showed that the system had considerable promise.

An experimental program was then undertaken. This program has two phases. The first is a critical-assembly study to determine the magnitude of reactivity which can be controlled by the reflector, how the reactivity varies with reflector height, and what the critical mass of the reactor must be, and to indicate how some factors affect the reactivity worth of the reflector.

FIGURE 1. SCHEMATIC DIAGRAM OF REFLECTOR CONTROL SYSTEM

Heterogeneous boiling core.
Natural circulation.

A-26473

Parallel to these critical-assembly studies hydraulic analog studies were undertaken to determine the stability and ability to control the power of the entire system. These studies use a full-sized hydraulic mock-up of the system with variable coupling (for experimental purposes) between the annular reflector and the pressurizing tank. An electronic simulator replaces the reactor in the system. The pressure, reflector height, and steam demand are fed into the simulator. The simulator computes the power and steam production and feeds back into the system the instantaneous steam pressure.

This report describes the critical-assembly studies on this program.

CRITICAL-ASSEMBLY DESCRIPTION

A critical-assembly experiment was designed to simulate the core and reflector of this small boiling heterogeneous reactor. To investigate the reflector control scheme a number of reactor water conditions, i. e., cold, hot, and boiling, had to be investigated. Thus, the special requirements on the critical assembly are that the steam voids which would be present in the boiling reactor be simulatable and that the inner reflector region in the annular tank be variable in height. It is quite important to simulate the steam voids because this affects the neutron leakage which in turn affects the reactivity worth of the reflector.

To meet these special requirements it was decided to use polyethylene to simulate water. The density of the polyethylene was adjusted so that its hydrogen content would match the hydrogen content of water under the operating conditions of the reactor. The steam voids in the system were determined to be the predominant factor affecting the reactivity. The nuclear differences between polyethylene and water were found to have a small effect on the experiment and, consequently, were not corrected for.

The use of polyethylene makes a safe system, in that any voids inserted into the system cannot collapse to increase the reactivity, and the reflector height is fixed by stacking polyethylene blocks, thereby limiting the reactivity available to the operator.

The critical-assembly core approximated a right circular cylinder 24 in. high and 21 in. in diameter. The core materials were strips of aluminum simulating fuel-element cladding, polyethylene representing the water channels, and fully enriched uranium foil as the fuel. These strips were contained in aluminum boxes 1-1/2 by 1-1/2 by 43 in. long. A cross section of the core configuration is shown in Figure 2.

In addition to the simulated fuel elements, which were 24 in. long, the boxes contained 6 in. of polyethylene on the bottom and 12 in. on the top to simulate the axial reflectors. The structure of a typical element is shown in Figure 3.

The bottom section was made up of ten strips of polyethylene 1/8 by 1-5/16 by 6 in. long. The next 24 in. consisted of a sandwich of four strips of aluminum 13/64 by 1-5/16 in., ten strips of polyethylene 1/10 by 1-5/16 in., and several uranium foils 0.004 by 1/2 in. The actual number of uranium foils was determined by the loading. The final section contained ten strips of polyethylene 1/8 by 1-5/16 by 12 in. long.

5

A-26475

FIGURE 2. A CROSS-SECTIONAL VIEW OF THE REACTOR CORE

N43834

FIGURE 3. STRUCTURE OF A TYPICAL ELEMENT

The core was surrounded by a radial polyethylene reflector 12 in. thick. The inner 6 in. of the reflector were removable so that the height of polyethylene in this annular gap could be varied by hand stacking. The outer 6 in. simulated the down-comer. A 1/8-in. gap between the inner and outer radial reflectors was filled in most experiments with a neutron absorber consisting of a 1/8-in. sheet of polyethylene impregnated with 0.119 g per cm³ of boron (based on B₄C content). The radial reflector extended 6 in. below the active portion of the core. The fixed outer radial reflector extended 12 in. above the core. A small gap was generally present between the core and inner reflector because of the difficulty of fitting the reflector to the irregular plan view of the core.

The density of the polyethylene in the core was varied by making transverse or longitudinal grooves in the plastic strips. These density changes were used to simulate temperature changes and voids formed in boiling. High-temperature water in the reflector was simulated by using foamed (low density) plastic blocks.

For the experiments the core and reflector were divided into two halves along a vertical plane and each half mounted on a table. One of these tables was stationary while the other could be driven horizontally on rails. See Figure 4.

The support for the radial reflector was separate from the core support structure and a means was provided to allow the reflector to be moved remotely about 0.3 in. upward. This upward motion enabled the differential reactivity worth of the reflector to be measured at different heights of the radial-reflector annulus. The position could be determined to ± 0.002 inch.

Critical-assembly control was provided by four vertically operating control blades, two in each core half. Each of the rods had its own drive unit but only one power supply was provided. Consequently, the power was available to only one unit at a time. The rods could be driven upward or downward at 10 in. per min. When the power was cut to the magnetic clutch which held the rods, the control rods fell under the action of gravity with an acceleration of approximately 0.7 g to scram the reactor.

A remotely controlled neutron source in each core half was used for loading measurements, startup, and experimental purposes. Rod and source positions were indicated by two sets of selsyns to obtain the required accuracy.

The nuclear instrumentation is shown by the block diagram in Figure 5. Three ion chambers were used as neutron detectors and monitors. A gamma channel consisting of a scintillation detector connected to a selenium rectifier and meter gave an indication of the gamma flux.

In addition to the safety and control circuits, two research circuits were provided. These channels provided accurate low-level information for startup and accurate neutron indications at operating levels. Both of these channels used pulse-counting instruments and were independent of the safety instrumentation.

A system of electrical controls and interlocks in the instrumentation indicated to the operator the condition of the reactor at any instant and prevented unsafe operation of the assembly.

N37278

FIGURE 4. TABLE MECHANISM AFTER CORE INSTALLATION

A 19384

FIGURE 5. BLOCK DIAGRAM OF INSTRUMENTATION

EXPERIMENTAL PROGRAM

The primary objective in the experimental program was to determine the reactivity worth of an annular radial reflector and to evaluate this as a means of controlling a power reactor. The reactivity worth of the reflector depends on core neutron leakage and therefore on the moderator density. Since this control scheme is used with a boiling core the water density in the core and, consequently, the leakage will vary widely. Thus, the reflector worth might be expected to vary by a significant amount and a thorough evaluation of the system requires a knowledge of the annular reflector worth over a wide range of moderator and reflector densities. Experiments were designed to cover this wide range, i. e., the cold, the hot, and the boiling core conditions.

The experimental cases are labeled according to the condition being simulated. Therefore, even though the critical assembly was never operated above room temperature and never developed more than 10 watts of power, the experimental cases are labeled "cold", "hot", "boiling", etc.

In an analysis of boiling reactor systems it is desirable to assume the steam voids to be uniformly distributed throughout the core, even though this is known not to be the case. In making the measurement of reactivity worth of the annular reflector, the core was first set up with the voids distributed as they would be in an actual boiling system. Then, to determine the validity of assuming these voids to be uniformly distributed, the core was set up with uniform distribution of voids and the reflector reactivity worth again determined. This created two boiling cases, a homogeneous and a nonhomogeneous case. The homogeneous boiling case was further broken down into a symmetrical and an asymmetrical case. The former has the top axial reflector with a hydrogen density similar to the lower axial reflector while the asymmetrical case has the top reflector with the density that would be expected in a power-producing reactor (water plus steam), but homogeneously distributed in the radial direction.

The core flux distribution from a hot case was needed for computing the void distribution for the boiling case. Two experimental variations of the hot case were measured. One experiment was conducted with the core and axial reflectors simulating the hot condition while the radial reflector was of full-density polyethylene simulating cold water. This has been referred to as the "semihot" case. Later low-density polyethylene was used for the radial reflector and a true hot case was run to compare the results with the semihot case. These experiments provide an observation on the effect of reflector density on reflector worth.

The flux patterns in the core were measured as part of the program. As previously stated, these were needed to permit the calculation of the void distribution for the boiling cases. The flux distributions were also of interest in seeing how the variation in reflector height affected the power distribution, which would also have a bearing on the general operation of the core, e. g., peak-to-average power ratios.

The critical mass was obtained for each of the cases at several reflector heights. Most of the experimental work was done with all the safety rods fully withdrawn.

A number of miscellaneous experiments were carried out as an aid to understanding the operation of the reactor and its control elements. These included measurements without the boron absorber between the two reflector regions and measurements without the outer reflector.

Table 1 summarizes the characteristics of the important cases which were used in this program. Further comments on each case appear below.

Cold Case

In the cold case no attempt was made to duplicate a given water temperature. The extruded plastic tapes for the core and the cut plastic pieces for the axial reflector were packed into the aluminum cans as tightly as could easily be done. The core tapes were not as wide as the inside dimension of the can and there was a fair amount of void present. Therefore, when compared on the basis of hydrogen density, the core plastic in position was equivalent to water at 165 F. However, the strips cut for the axial reflector were wider and packed more tightly. Their hydrogen density was 6 per cent greater than water at 0 C.

The radial reflector pieces were cut to the required shape and packed tightly in place. This was solid polyethylene having a hydrogen density 18 per cent greater than cold water.

Semihot Case

In the semihot case the radial reflector was left in its solid polyethylene form having a hydrogen density 18 per cent greater than cold water. The plastic tapes going into the core and axial reflectors were voided by making cuts perpendicular to the length of the plastic pieces. These cuts were made at random and the strips were weighed after cutting to determine the amount of polyethylene removed. This procedure resulted in the core and the axial reflectors having hydrogen densities corresponding to water at 420 F.

Strips of plastic having these cross cuts can be seen in the reflector strips shown in Figure 3.

Hot Case

In the hot case the core and axial reflectors had the same polyethylene density as the semihot case. The only difference was in the radial reflector where low-density plastic was used to simulate the hot water. The density of the plastic in the radial reflector was selected to give a hydrogen density corresponding to water at 420 F. Thus, the entire reactor (core and reflectors) in the hot case had a hydrogen density simulating 420 F water.

TABLE 1. CORE AND

Case	Core Averages						Metal- Water Ratio	Equivalent Temperature ^(b) , F
	Metal Volume			Moderator Volume				
	Aluminum	Uranium-235	Total Metal	Polyethylene	Void	Total Water ^(a)		
Cold	0.276	~0.002	0.278	0.595	0.127	0.722	0.385	165
Semihot	0.276	~0.002	0.278	0.516	0.206	0.722	0.385	420
Hot	0.276	~0.003	0.279	0.516	0.205	0.721	0.387	420
Boiling nonhomogeneous	0.276	~0.003	0.279	0.428	0.293	0.721	0.387	420
Boiling homogeneous	0.276	~0.003	0.279	0.429	0.292	0.721	0.387	420

(a) This is the volume which would be occupied by water (or water and steam) in a prototype power reactor.

(b) This is the temperature at which water would have a hydrogen density equal to the hydrogen density used in the experiment.

(c) This hydrogen density is 6 per cent greater than that of water at 0 C.

(d) This hydrogen density is 18 per cent greater than that of water at 0 C.

REFLECTOR COMPOSITION

Steam Volume, per cent	Metal Volume, Aluminum	Axial-Reflector Averages				Steam Volume, per cent	Radial Reflector	
		Water Volume			Equivalent Temperature ^(b) , F		Polyethylene Density	Equivalent Temperature, F
		Polyethylene	Void	Total Water ^(a)				
0	0.174	0.740	0.086	0.826	-- (c)	0	0.920	-- (d)
0	0.174	0.590	0.236	0.826	420	0	0.920	-- (d)
0	0.174	0.590	0.236	0.826	420	0	0.657	420
17.0	0.174	0.377	0.449	0.826	420	36.1 top	0.657	420
	0.174	0.590	0.236	0.826	420	0 bottom		
16.9	0.174	0.377	0.449	0.826	420	36.1 top	0.657	420
	0.174	0.590	0.236	0.826	420	0 bottom		

Nonhomogeneous Boiling Case

In the boiling nonhomogeneous case the bottom axial reflector and the radial reflectors were the same as in the hot case. The core and the top axial reflector was voided to approximate the void distribution which was calculated (see Appendix A) for a boiling reactor producing 1.8 megawatts of power. This power was considered to be a reasonable upper limit for this core, giving a void fraction of about 18 per cent. It was questionable at the outset of the program whether a higher power and void could be used since the unreflected core critical mass might have exceeded the fuel available.

In a power-producing reactor the void distribution would be a continuous function of position; in this experiment the core was divided into five axial regions and seven radial regions for voiding purposes. The top axial reflector was divided into one axial region and seven radial regions.

The core plastic was voided by making cuts in the surface of the plastic parallel to the length of the plastic. The axial regions were obtained by varying the depth of the cut in each of the five regions. The strips were checked by weighing to see if the amount of plastic removal was the correct amount for the desired radial position.

Core plastic strips voided for the boiling case can be seen in Figure 3.

The top axial reflector was voided by making cuts across the surface of the plastic strip perpendicular to the length of the plastic strip. These were made with uniform spacings rather than the random spacing used for the hot case.

Homogeneous Boiling Case

In this case the radial reflector and bottom axial reflector were again identical to the hot case. The core was voided in the same manner as the boiling nonhomogeneous case except there was a single region in the core instead of the multiple regions used for the nonhomogeneous case. All plastic strips were cut in an identical fashion (several cuts parallel to the long dimension) to produce the desired void fraction.

The top axial reflector was voided to produce the same average void as the top axial reflector in the boiling nonhomogeneous case. This was done by taking the same reflector plastic and rearranging it so that there were no radial variations in the void fraction.

A variation of the boiling homogeneous case was also tested. In this case the top axial reflector was voided to simulate hot water rather than water containing steam voids. Only the fully reflected condition was tested. It was felt that this case could be more easily calculated in an analysis of the reflector control system because of its symmetry. The results of this experiment are discussed on page 34.

EXPERIMENTAL PROCEDURES

Critical-Mass Determinations

The uranium-235 mass was calculated* for each of 754 fuel strips and catalogued by strip number. Fuel loadings were made by distributing fuel strips as uniformly in the case as was convenient and changes were made by a uniform addition or removal of fuel strips throughout the core. Fuel masses were determined by totaling the uranium-235 masses for all strips in the core for each criticality attempt. Critical masses were taken to be the fuel masses when less than 2 in. of one control blade remained in the core. This amount of control was found to be worth less than 15 g of fuel.

Incremental Reflector-Worth Determination

The inner radial reflector was constructed of 2-in. -thick horizontal slabs which were stacked and fastened to the reflector base plate by vertical Ready-bolts. The reflector height was changed in 2-in. increments by the addition or removal of these slabs. The zero point for the reflector height was taken to be the bottom of the active core region, which left approximately 6 in. of radial reflector below the zero point.

For determination of the reflector worth at any given nominal height the radial reflector was raised a distance of about 0.3 in. This distance was measured either by a linear variable differential-transformer circuit or by a rack and pinion-selsyn system. The calibration of both systems was checked frequently with a micrometer. The worth of this reflector bump was measured in terms of reactivity by the positive-period method.

Checks were made frequently on these measurements by a second method using calibrated control blades. This consisted of determining the position of the control blade which produced a very nearly infinite period for a given height. Then the height of the radial reflector was raised by several 2-in. slabs and the control-blade position which produced a very nearly infinite period determined again. The difference in the integral control-blade worth for these two control-blade positions gave the reactivity worth for the change in reflector height.

Flux-Distribution Determination and Counting Technique

Iron-manganese-molybdenum alloy wires 0.033 in. in diameter and containing about 15 w/o manganese and 2 w/o molybdenum were activated in the reactor to determine flux distributions. In some cases, the wires were cut into 1-in. lengths and stacked vertically in 0.038-in. -ID aluminum tubes to any desired height. The aluminum tubes were labeled and placed vertically in slots made in the aluminum strips in the core-element boxes. If a two-dimensional distribution was desired, a number of tubes were placed at measured distances along a radius of the core. After irradiation, the aluminum tubes were removed from the core and taken into the counting room where the 1-in. lengths of wire were removed in order and placed in numbered trays in a sample changer.

* The strips were weighed individually and corrected for the weight of the plastic coating and uranium-238 content. The average atomic per cent of uranium-235 in the ingots used for making the strips was 93.16.

In a second method, the wires were cut into 48-in. -lengths, and taped to a core face in the small opening formed by the rounded corners of the boxes.

After irradiation, the wires were removed from the core face and taken into the counting room where they were cut into 1-in. -lengths and placed in numbered trays in a sample changer.

The wires were activated by operating the reactor at a power of about 5 watts for between 20 and 60 min. All data were normalized to give the same volume integral of flux for each determination of flux distribution. All data are presented in this normalized form.

Whenever wires were activated in the core, an extra wire was included to serve as a source for the decay-correcting timing channel. This timing channel activated a sample changer and two counting channels at the beginning and end of a preset count.

Boiling Void Distribution

The calculational method used to determine the void distribution for the boiling system is described in Appendix A. This section discusses the experimental iterations necessary to develop the distribution.

The normalized flux for the semihot case with the reflector at 10 in. was used to calculate a void distribution. The reactor was then loaded with this void distribution and another two-dimensional flux plot was made. From this plot a second void distribution was calculated and approximated in the core. The resulting flux plot was used to calculate a new void distribution. The experimental iterative process was to be continued until acceptable convergence occurred between consecutive flux plots or void distributions.

EXPERIMENTAL RESULTS

Reflector Worth and Critical Mass

Generally, the reflector worth was determined for each condition of the reactor by the reflector-bump method. The reactivity change associated with each positive period was then found with the in-hour equation. Constants for the in-hour equation were obtained from the literature.⁽¹⁾ In the hot case the data by the reflector bump method were not considered reliable because of a failure in the reflector mechanism. Data for this case were obtained using the calibrated control blades. This method was used as a check on the reflector method for the homogeneous boiling case and gave results in good agreement.

Figure 6 is a graph of the differential reflector worth for five cases. Figure 7 is the integral reflector worth for these cases and is obtained by straight-forward integration of the curves on Figure 6.

(1) References at end of report.

FIGURE 6. DIFFERENTIAL REFLECTOR WORTH FOR FIVE CASES

FIGURE 7. INTEGRAL REFLECTOR WORTH FOR FIVE CASES

It should be noted that the peak in the curves of the differential reflector worth of Figure 6 occurs, in general, below the center of the core. This is believed to result from the fact that the data were obtained with the safety rods held at the core-axial reflector interface. This tends to shift the flux distribution toward the bottom of the core. Consequently, the reflector movement has a greater effect near the bottom. For a similar reason, caused by the moderator density gradient, the nonhomogeneous boiling case has a more asymmetric curve for differential reflector worth than the homogeneous case.

The critical mass for all five cases is given in Figure 8, as a function of reflector height. An associated error is assigned to the critical mass based on the inhomogeneity of the fuel distribution. Experience showed that this variation in the critical mass was approximately 1 per cent.

The results of the reflector-worth and critical-mass measurements for the five cases, i. e., the cold, semihot, hot, nonhomogeneous boiling second iteration, and the homogeneous boiling cases are summarized in Table 2. The B₄C curtain between inner and outer radial reflectors was present in all cases.

Boiling Void-Distribution Results

The iterative procedure to arrive at the boiling void distribution has been described above. Three critical-assembly iterations were performed. The steps in the approach to the final distribution are shown on Figure 9. The normalized flux for the semihot case, Figure 10, was used to calculate a void distribution by the method of Appendix A. This is denoted "first iteration" on Figure 9. With this void distribution loaded in the core another flux plot was made (see Figure 11). From this plot a second void distribution was calculated and appears on Figure 9 as a short dashed line. This distribution differed significantly from the first iteration in only the outer radial region so that a second experimental iteration in this region was inserted in the core. This is shown by the solid curve approximating the short dashed curve. The two-dimensional flux plot resulting from this second iteration is shown in Figure 12. The boiling void distribution calculated from this second iteration is shown in Figure 9 as alternate long and short dashes. The iteration was stopped at this point. Stopping the boiling void determination at this point was not a result of a convergence of the experimental results but was based on the equivalence of critical-mass and reflector-worth values for these various iterations.

The estimated "true-void" distribution curve is also drawn in on Figure 9. This curve results from a consideration of the iterative behavior of the three cases. The radial-density variation of the moderator in the first iteration was apparently too great and caused a severe peaking of the flux at the core-reflector interface. The second iteration apparently overcompensates for this peaking since the calculated void distribution for this case does not follow the experimental voids. The true-void distribution is, therefore, estimated to be between these cases as is shown.

The variation in void in a vertical direction may be seen in the typical void-contour curves of Figure 13. Since all the iterations used the same moderator strips the average axial-void variation was constant.

FIGURE 8. CRITICAL MASS FOR FIVE CASES

TABLE 2. REFLECTOR WORTH AND CRITICAL MASS

Case	Reflector Position From Bottom of Core, in.	Reflector Worth, per cent	Critical Mass, g
Cold	0		5964 $\pm$ 60
	24	6.20	5060 $\pm$ 51
Semihot	0		6607 $\pm$ 66
	24	6.77	5270 $\pm$ 53
Hot	0		6550 $\pm$ 65
	24	7.31	5460 $\pm$ 55
Nonhomogeneous boiling - second iteration	0		7889 $\pm$ 79
	24	8.44	5620 $\pm$ 56
Homogeneous boiling	0		7863 $\pm$ 79
	24	8.50	5658 $\pm$ 57

FIGURE 9. AXIALLY AVERAGED VOIDS IN THE CORE VERSUS RADIUS

FIGURE 10. THERMAL-FLUX DISTRIBUTION FOR THE SEMIHOT CONDITION

10-in. reflector height.

FIGURE 11. THERMAL-FLUX DISTRIBUTION FOR THE NONHOMOGENEOUS BOILING CONDITION

First iteration, reflector at 10 in.

FIGURE 12. THERMAL-FLUX DISTRIBUTION FOR THE NONUNIFORM BOILING CONDITION

Second iteration, reflector at 10 in.

FIGURE 13. VOID CONTOURS FOR FIRST ITERATION

Miscellaneous Reflector Experiments

During the reflector-worth experiments the lateral reflector was always stacked from 0 in. (corresponding to the bottom of the active core) to 24 in. (the top of the active core), hence the reflector worth was measured only over this range. Measurements of the critical mass were made in the homogeneous "boiling" condition with the B₄C poison sheet removed, the B₄C poison sheet and the 6 in. of inner reflector below the active core level removed, and the reactor laterally unreflected. Table 3 gives the results of these measurements. $\frac{\Delta k}{k}$ in Table 3 was calculated from the assumed relationship

$$\frac{\Delta k}{k} = C \frac{\Delta m}{m}, \text{ where } m = \text{linear average of the initial and final values and}$$

$C = 0.260$ was obtained from the measurements of the homogeneous boiling case.

Flux measurements obtained for the various conditions of the radial reflector are shown in Figure 14. It is interesting to note the influence of the gap between the core and radial reflector. A considerable flux flattening within the core is noted even with the radial reflector as much as 6 in. away from the core. This is further evidence of the 4 per cent worth of this outer reflector region.

TABLE 3. RESULTS OF MISCELLANEOUS EXPERIMENTS

Item Measured	Conditions	Δm	$\bar{m}$, g	$\frac{\Delta k}{k}$, per cent
Worth of B ₄ C curtain	Initial -- inner radial reflector at 0 in., B ₄ C poison sheet in	-923	7401	-3.24
	Final -- inner radial reflector at 0 in., B ₄ C poison sheet out			
Worth of lower 6 in. of inner reflector	Initial -- inner radial reflector at 0 in., B ₄ C poison sheet out	+124	7002	+0.46
	Final -- inner radial reflector at -6 in., B ₄ C poison sheet out			
Worth of outer reflector	Initial -- inner radial reflector at 0 in., B ₄ C poison sheet out	+1210	7669	+4.09
	Final -- radial reflector (inner and outer) completely removed			

Note: The combined worth of the poison sheet and the outer reflector is estimated to be +0.85 per cent by taking the sum of these two cases as given in Table 3.

FIGURE 14. RADIAL-FLUX DISTRIBUTIONS AT HEIGHT OF 12 IN. WITH CORE IN HOMOGENEOUS BOILING CONDITION

THEORY

An analysis was made of the cold, semihot, hot, and homogeneous boiling cases. The nonhomogeneous boiling case was not attempted analytically.

Much of the analysis was carried out to give estimates of critical mass and reflector worth prior to running the experiment. However, some of the calculations were performed at the conclusion of the experiments to check the agreement between theoretical and experimental results. Generally, techniques which prove adequate in checking experimental data may be extended to predict results where data are not available. It was felt that such would be the case here.

Calculational Procedures

Reflector Worth

To obtain the total reactivity worth of the inner radial reflector, the critical conditions of the reactor with full reflector and with the inner 6 in. of the radial reflector removed were calculated. In the latter case, the reactor core was surrounded by a 6-in. annular gap bounded by the core on the inner surface and by an absorbing sheet black to thermal neutrons on the outside surface.

The active core was assumed to be a cylinder 21 in. in diameter and 24 in. high. An axial-reflector savings of 7 cm was assumed for all cases. This assumption is justified by the final results which are relatively insensitive to axial savings.

In the fully reflected case the radial reflector was assumed to be in contact with the core, i. e., no radial gap existed between core and reflector. In the critical-assembly experiment a small gap of about 1/8 in. was always present.

For the case with the inner reflector removed, the calculations considered an outer 6 in. radial reflector (behind the absorbing sheet) equal to the core height plus the axial-reflector savings. The inner reflector was considered to be a void everywhere. In the actual experiment, the inner reflector extended 6 in. below the core while the outer reflector extended 6 in. below and 12 in. above the core.

For the calculations on the fully reflected case, the standard two group-two region problem was solved on an IBM-650 using the Paracantor Code.⁽²⁾

The problem with no inner radial reflector was solved similarly, but with different boundary conditions. The details of this problem are given in Appendix B.

Disadvantage Factor

The disadvantage factor, defined as the ratio of the average thermal-neutron flux in the moderator to the average thermal-neutron flux in the fuel was calculated using the so-called P_3 approximation to solve the neutron-transport equation. The critical-assembly geometry was too difficult to attempt, so the problem was solved for a

one-dimensional slab geometry for 0.004-in. -thick fuel foils and varying amounts of polyethylene. For these calculations, the ratio of neutron temperature to moderator temperature was taken⁽³⁾ as:

$$\frac{T_n}{T_m} = \left(1 + 0.75 \frac{\Sigma_a}{\xi \Sigma_s}\right)^2,$$

where $\xi \Sigma_s$ is evaluated for the moderator just above thermal energy where Σ_s is constant, and Σ_a is evaluated at T_m . Cross sections for the P_3 calculation were then evaluated assuming a Maxwell-Boltzmann spectrum around T_n . It appears reasonable using a one-dimensional geometry to assume that the disadvantage factor is a function of only the moderator to fuel ratio.

Two-Group Constants

The validity of using polyethylene to simulate water and the calculation of two-group constants for polyethylene systems were given careful consideration. Investigation of this problem at KAPL⁽⁴⁾ included age calculations by the moments method for different volume ratios of moderator to aluminum or zirconium which indicated that for the same hydrogen density $\frac{\tau_{\text{plastic}}}{\tau_{\text{water}}} = 0.96$ to 0.98 . Measurements of reactivity coefficients per hydrogen atom done at KAPL indicated that polyethylene and water were essentially equivalent nuclearwise.

Measurements of hydrogen-transport cross sections in various hydrocarbons done at BNL⁽⁵⁾ indicate that the thermal diffusion lengths in polyethylene and water are the same, for the same hydrogen density.

To arrive at a method for finding age in the various cases, the ages for polyethylene and water were calculated by the moments method⁽⁶⁾, assuming isotropic scattering by oxygen and carbon, and assuming hydrogen slowing down only. These calculations gave $\tau_{\text{H}_2\text{O}} = 27.5 \text{ cm}^2$, and $\tau_{\text{CH}_2} = 21.7 \text{ cm}^2$ for the fission spectrum to thermal-energy ages. It was assumed, however, that the correct age to thermal in water was 32 cm^2 , and that the ratio of correct value for polyethylene to calculated value would be the same as that ratio for water. This gave a value of $\tau_{\text{CH}_2} = 25.2 \text{ cm}^2$.

Experimental values⁽⁷⁾ of age for aluminum-water mixtures were found to be given quite well over a wide range of volume ratios by the equation,

$$\tau = \tau_{\text{H}_2\text{O}} \left[\frac{1 + R}{\rho_{\text{H}_2\text{O}} + 0.33 R} \right]^2,$$

where R is the aluminum-to-water volume ratio. This is an equivalence-factor type of formula. Applying this to aluminum-polyethylene mixtures, the result is:

$$\tau = \tau_{\text{CH}_2} \left[\frac{0.92 + R}{\rho + 0.29 R} \right]^2.$$

Fast-group diffusion coefficients for the calculations were found by using the results of calculations of fast diffusion coefficients for a fission source in an infinite medium of aluminum and water. These values were corrected to be consistent with experimental age measurements. The calculation involved solution of the space-independent energy-dependent transport equation. Four-hundred energy groups were used and the calculations were done on an IBM-650. The fast diffusion coefficients used for Aluminum-CH₂ mixtures were found from the water coefficients by assuming that, for the same aluminum-to-hydrogen atomic ratio, the transport mean free path is inversely proportional to the atomic density of hydrogen.

The thermal-group constants were found by assuming a Maxwell-Boltzmann distribution at room temperature and using cross sections from BNL-325.

Table 4 lists volume fractions of materials for the cases calculated and the appropriate group constants used.

Results

The results of the calculations are listed in Table 5. For purposes of comparison with measured reflector worths, the calculated reflector worth is taken to be the natural logarithm of the ratio of k_{∞} with the radial gap and k_{∞} fully reflected case. The quantity k_{∞} is defined as:

$$k_{\infty} = \eta f = (2.077) \times \frac{\text{Thermal absorptions in uranium-235}}{\text{Total thermal absorptions}}$$

The critical masses in the first column are the results of the calculations on a homogeneous core and reflector case. That is, no consideration was made for a flux depression or disadvantage factor caused by lumped fuel. The calculated disadvantage factors are listed and have been multiplied by the homogeneous critical mass to obtain a critical mass for the heterogeneous case. The experimentally determined critical masses are given for comparison.

DISCUSSION

The curves of Figures 7 and 8 provide a graphical method of evaluating the control potential of the inner radial reflector for the reactor which was studied. Consider for example a reactor which has 1 per cent negative reactivity in the cold, clean condition with the inner radial reflector at zero. Thus, the reactor will go critical at a position intermediate between 0 and 24 in. so that some reactivity will be available to provide for shutdown safety. Figure 7 indicates the reactor will be critical at a reflector height of 6 in. The fuel loading corresponding to a 6-in. reflector in the cold case is 5.67 kg. With this loading, the reflector will have to rise to about 16.75 in. to remain critical when the hot state is reached. In the power-producing boiling case the reflector position for this fuel loading will be 24 in. so that no control, poison override, or fuel-burnup potential exists. Thus, for this reactor system it is not feasible to consider a complete reactor-startup operation.

TABLE 4. CORE COMPOSITIONS AND NUCLEAR PARAMETERS

Case	V_{CH_2}	V_{A_1}	V_{CH_2}	τ_c, cm^2	τ_r, cm^2	D_{fc}, cm	D_{fr}, cm	L_c^2, cm^2	L_r^2, cm^2	D_{sc}, cm	D_{sr}, cm
	$V_{\text{total core}}$	$V_{\text{total core}}$	$V_{\text{total reflector}}$								
Cold	0.602	0.276	1	55.5	25.2	1.67	1.31	3.5 ^(a)	5.20	0.197	0.120
Semihot	0.516	0.276	1	72.5	25.2	1.91	1.31	3.5	5.20	0.229	0.120
Hot	0.516	0.276	0.714	72.5	49.4	1.91	1.83	3.5	10.20	0.229	0.168
Boiling- homogeneous	0.434	0.276	0.714	97.4	49.4	2.17	1.83	3.5	10.20	0.271	0.168

(a) Approximate values — these varied slightly with the fuel loading.

TABLE 5. RESULTS OF CALCULATIONS COMPARED WITH EXPERIMENT

Case	Inner Reflector Height, in.	k_{∞}	Critical Mass, No Disadvantage Factor, g	Disadvantage Factor	Critical Mass With Disadvantage Factor, g		Reflector Worth	
					Calculated	Measured	Calculated	ρ , Measured (a)
Cold	24	1.399	3180	1.75	5560	5060	0.0595	0.062
	0	1.485	3860	1.67	6440	5964		
Semihot	24	1.451	3160	1.65	5210	5270	0.0850	0.0677
	0	1.580	4325	1.55	6700	6607		
Hot	24	1.462	3240	1.64	5310	5460	0.0775	0.0731
	0	1.580	4325	1.56	6750	6550		
Boiling-homogeneous	24	1.618	4200	1.55	6500	5658	0.0777	0.085
	0	1.749	6340	1.44	9130	7860	0.0451 ^(b)	0.0085
	Completely bare	1.830	8835	1.43	12610	8290		

(a) From reflector-worth curves.

(b) Calculated worth of the outer reflector and poison sheet in the boiling case. This calculation was done using two-group theory where different extrapolation lengths were used for the thermal and fast fluxes.

As a second example consider a reactor which is taken to the hot condition by other techniques, such as control rods, soluble poisons in the water, or partial water heights in the core. With the reactor critical in the hot condition and with the reflector at 0 in., the total reflector worth is available to take the reactor to a power-producing condition and for shimmed control.

The critical mass for the hot zero-reflector-height case is about 6.5 kg. To maintain a critical state with 6.5 kg in the nonhomogeneous boiling (power-producing) case the reflector height will have to be 11-1/2 in. from Figure 8. Figure 7 shows that about 4 per cent reactivity will still be available in this condition. At least 3 per cent of this reactivity will be used to override poisons and only 1 per cent will be left for uranium burnup. Figures 7 and 8 show that 1 per cent of reactivity corresponds to about 150 g of uranium. At 2 megawatts of heat power this burnup amounts to about 70 days of full-power operation.

This example indicates sufficient control is available for power changes from zero to 100 per cent and for equilibrium poison override but not enough for long-time burnup.

Improvements in the control potential probably can be realized by increasing the core height-to-diameter ratio, by using zone loading and by increasing the void volume in the core. Soluble poisons in the reflector water whose concentration can be varied may be another way of increasing the reflector worth.

Other results from the experimental work are noteworthy. A comparison between the homogeneous and nonhomogeneous boiling cases can be made by referring to Figures 7 and 8. These figures provide experimental evidence that for this core almost identical critical masses and reflector worths are obtained for the homogeneous and nonhomogeneous void distributions. This is significant in that it provides the analyst with some evidence of the validity of the results obtained from the generally assumed uniform distribution of boiling voids. This conclusion is probably not valid for larger boiling cores having large nonuniformities in the void distribution.

One experimental point of critical mass was obtained for the homogeneously voided core with identical upper and lower axial reflectors and a complete radial reflector. The critical mass was essentially identical to the case of nonsymmetric axial-reflector voids (e.g., the lower reflector voided to represent hot water and the upper reflector voided to represent outlet steam voids). Although only this one point was taken because of lack of time, it seems justifiable to conclude that the symmetric-core assumption, which is convenient to make analytically, will not lead to appreciable errors in calculations regarding boiling cores.

From the experimental results, there is apparently an enhancement of the reactivity worth of the movable reflector with decreases in the hydrogen density in both the core and in the reflector (the reflector density was changed between the semihot and hot core conditions). It would appear reasonable that the reactivity worth of the movable reflector should increase as the hydrogen density in the core decreases, but it is difficult to justify the increase when the hydrogen density in the reflector decreases. Further, a comparison of results in Table 4 indicates that the critical-mass changes and the reactivity worths for the semihot and hot cases are apparently inconsistent since the larger mass change accompanies the small measured reactivity worth.

Several factors are probably significant here. As already noted, the reactivity result for the hot case was based upon control-rod measurements since the reflector bump measurements were suspected of being in error for this case. And further, there was a difference in the gap separating the core and the movable reflector which resulted from changes made in the movable reflector after the semihot experiments were performed. At the time the hot radial reflector plastic was put in place considerable effort was taken to positioning it as close to the core as possible. Considering these factors, the inconsistency noted, and the precision of the measurements, no conclusion regarding the comparison of results between the hot and semihot cases seems justifiable.

CONCLUSIONS

On the basis of the experimental and analytical work done the following conclusions may be drawn:

- (1) The use of a movable radial reflector will probably not supply sufficient control to include startup operations.
- (2) Sufficient reactivity is available in this system to compensate for large power variations and equilibrium poisons. However, the system does not provide enough reactivity to compensate for long-time burnup. There appear to be some modifications which may increase reflector worth.
- (3) The nonuniformity of the void distribution in the power-producing case for cores of the size considered here does not have a significant effect upon the gross reactor properties such as critical mass and reflector worth. However, these nonuniformities may have significant effects upon the detailed properties such as the flux and power distribution.
- (4) For calculational purposes for cores of the size studied, the assumptions of uniform void distributions during boiling and of identical axial reflectors above and below the core appear to be justified.
- (5) The use of a thermal-neutron-absorber curtain between the movable and stationary radial reflectors of the reactor considered adds a significant amount to the reactivity worth of the inner, movable reflector.
- (6) The results of the reactor experiments supply useful data which can be used to aid in the design of a power reactor to utilize the reflector control concept. The reactor experiments combined with the results from the simulation studies of the system kinetics (BMI-1236) provides a good foundation for a detailed feasibility study of the reflector-controlled reactor concept. Such a study is necessary to make the most intelligent utilization of the design features peculiar to this reactor concept. On the basis of the studies already completed it is reasonable to expect that an attractive system may result from such a feasibility study.

REFERENCES

- (1) Keepin, G. R. , Progress in Nuclear Energy, Vol I, Physics and Mathematics, p 206, McGraw-Hill Book Company, New York (1956).
- (2) Stone, Stuart P. , "Paracantor, A Two Group, Two Region Reactor Code", UCRL, Livermore, California, 650 Program Library File No. 8.2.002.
- (3) Deutsch, R. W. , Nucleonics, p 48 (January, 1957).
- (4) "S3G/S4G Program Progress Report", March and April, 1956, KAPL-1535, pp 203-204. Secret.
- (5) "Quarterly Progress Report", November, 1954, to February, 1955, BNL-333, p 11.
- (6) Hurwitz, Henry, Jr. , and Zweifel, P. F. , "Slowing Down of Neutrons by Hydrogenous Moderators", KAPL-1269 (February, 1955).
- (7) Roberts, L. D. , and Fitch, T. E. , "The Slowing Down Distribution to Indium Resonance of Fission Neutrons From a Point Fission Source in Two Mixtures of Aluminum and Light Water, 1:2 by Volume", ORNL-294 (February, 1949).

DAD:WSH:RGW:WCB:FJJ:JWC/mln

APPENDIX A

CALCULATION PROCEDURE FOR VOID DISTRIBUTIONS

APPENDIX A

CALCULATION PROCEDURE FOR VOID DISTRIBUTIONS

In order to calculate the boiling-void distribution the reactor core was divided into seven radial regions as shown in Figure A-1. Each of these regions was treated independently and the void distribution in each was calculated on the basis of the following conditions and assumptions:

- (1) The equivalent diameter for assumed fuel channels is 0.4 in.
- (2) Fluid enters the flow channels at the saturation condition.
- (3) The liquid-vapor mixture in the entire upper axial reflector has the same properties as the mixture leaving the active core region.
- (4) The kinetic energy of the liquid phase leaving the flow channels is dissipated into internal energy by friction as the liquid leaves the channel.
- (5) The kinetic energy of the vapor phase leaving the flow channels is lost from the system directly.
- (6) Natural convection cooling is assumed.
- (7) The outlet steam quality from any fuel element is small.

The calculation procedure is a trial-and-error technique. A flow velocity in each fuel channel is assumed and the coupled hydrodynamic and thermodynamic equations are solved for the difference between pressure loss of the fluid in flowing through this channel and the available hydrostatic driving pressure. This difference is made to approach zero by proper choice of the inlet velocity. The method given here is a compromise between the desires for a good representation of a boiling system and for a direct computational method.

Nomenclature

a_i = area of i^{th} radial sector in the reactor

A = flow-channel area, ft^2

$A_f(z)$ = flow area for liquid at z , ft^2

$A_g(z)$ = flow area for vapor at z , ft^2

D = equivalent diameter of represented fuel channel, ft

D_f = equivalent diameter for liquid phase, ft

Scale: 1 cm = 1.5 in.

Note: Alternate shaded and unshaded areas denote fuel elements assigned to the various regions.

A-2

A-26474

FIGURE A-1. REACTOR-CORE CONFIGURATION

f = friction factor = $0.046 N_{Re}^{-0.2}$

g_c = gravitational constant, slugs per lb_m

h_{fg} = latent heat of vaporization, Btu per lb_m

L = channel length, including upper axial reflector, ft

L_c = active core length, ft

M_{in} = mass flow rate in the channel, lb per sec

$M_f(z)$ = mass flow rate of liquid at z , lb per sec

$M_g(z)$ = mass flow rate of vapor at z , lb per sec

N = a normalization constant Btu per sec per count per ft^3

N_{Re} = Reynolds number = $\rho \frac{VD}{\mu}$

P = core power, Btu/sec

$P(z)$ = power per unit length at z , Btu/(sec)(ft)

ΔP_a = pressure loss due to acceleration of the fluid in the channel, lb_f per ft^2

ΔP_f = pressure loss due to friction of the fluid in the channel, lb_f per ft^2

$S(z)$ = slip ratio at z , $V_g(z)/V_f(z)$

$V_f(z)$ = velocity of liquid at z , ft per sec

$V_g(z)$ = velocity of vapor at z , ft per sec

V_{in} = inlet liquid velocity = $V_f(0)$, ft per sec

$x(z)$ = weight per cent vapor flowing at z = lb per sec vapor per lb per sec mixture

z = vertical distance in the channel from the inlet, ft

$\alpha(z)$ = volume per cent vapor flowing at z = ft^3 vapor per ft^3 mixture at z

$\bar{\rho}$ = average density of mixture in the channel lb_m per ft^3

ρ_f = liquid density, lb_m per ft^3

ρ_g = vapor density, lb_m per ft^3

μ = liquid viscosity, lb_m per ft-sec

$\phi_1(z)$ = a quantity proportional to the thermal neutron flux, count

Consider a channel of length L . Saturated liquid enters at the bottom ($z = 0$) and a mixture of liquid and vapor leaves the top ($z = L$). For a power distribution $P(z)$ over the active core ($0 < z < L$), mass and heat-balance conditions give

$$M_{in} = M_g(z) + M_f(z), \quad (A-1)$$

$$h_{fg} M_g(z) = \int_0^z P(z') dz' . \quad (A-2)$$

Equation (A-1) can be written in terms of volume flow rates of the liquid and vapor by using the quantities defined above. This relation may be solved for the vapor volume per cent $\alpha(z)$ as

$$\alpha(z) = \frac{1}{1 + \left\{ \frac{1 - x(z)}{x(z)} \right\} \frac{\rho_g}{\rho_f} S(z)} . \quad (A-3)$$

Similarly Equation (A-2) can be rewritten to solve for the weight per cent vapor flow at z , $x(z)$ as

$$x(z) = \frac{M_g(z)}{M_{in}} = \frac{\int_0^z P(z') dz'}{h_{fg} M_{in}} . \quad (A-4)$$

Equations (A-3) and (A-4) suffice to determine $\alpha(z)$ if the slip ratio $S(z)$ and the inlet flow M_{in} are known. Fortunately, it is possible to assume that the slip is independent of z and can be related to only M_{in} . The relationship used for computation is*

$$S(z) = 5.07 \left(\frac{M_{in}}{\rho_f A} \right)^{-1.08} , \quad 0 < z < L . \quad (A-5)$$

As a result of the Equations (A-3), (A-4), and (A-5) the problem of finding $\alpha(z)$ has been reduced to one unknown, namely M_{in} . In order to determine M_{in} it is necessary to consider the fluid dynamics of the system. The application form of the Bernoulli equation for this system may be written as

$$(\rho_f - \bar{\rho}) L = \Delta P_a + \Delta P_f , \quad (A-6)$$

where

$$L\bar{\rho} = \int_0^L \rho(z) dz , \quad (A-7)$$

and

$$\rho(z) = \rho_f \left\{ 1 - \alpha(z) \right\} + \rho_g \alpha(z) . \quad (A-8)$$

*See ANL-5522.

The total kinetic-energy increase of the mixture in the channel can be represented as a pressure loss by*

$$\Delta P_a = \left\{ \frac{[1 - x(L)]^2}{1 - \alpha(L)} + \frac{x^2(L)}{\alpha(L)} \frac{\rho_f}{\rho_g} - 1 \right\} \frac{\rho_f V_{in}^2}{2 g_c} . \quad (A-9)$$

It is assumed that the remaining kinetic energy in the system in the liquid phase at the outlet from the channel is lost by internal friction at the exit. This energy will presumably give rise to cross mixing of the outlet flow from channel to channel across the core at some distance from the outlet.

The two-phase friction pressure loss is regarded to be primarily a result of shear stresses of the liquid phase at the walls of the flow channel. As such this pressure loss is represented by

$$\Delta P_f = \int_0^{L_c} \frac{\rho_f V_f^2(z)}{2 g_c} \frac{4 f(z)}{D_f(z)} dz . \quad (A-10)$$

Equation (A-1) is further reduced by assuming that the Fanning friction fraction $f(z)$ is constant throughout the length (actual changes here will incur to about the 0.2 power), that is

$$D_f(z) = \frac{4 A_f(z)}{\text{wetted perimeter}} = \frac{A_f(z)}{A} D = [1 - \alpha(z)] D, \quad (A-11)$$

and that

$$\frac{V_f(z)}{V_{in}} = \frac{1 - \alpha(z)}{1 - \alpha(z)} \sim \frac{1}{1 - \alpha(z)} \text{ for small } \alpha(z) . \quad (A-12)$$

Combining Equations (A-11) and (A-12) into (A-10) gives

$$\Delta P_f = \frac{4f}{D} \frac{\rho_f V_{in}^2}{2 g_c} \int_0^{L_c} \left[\frac{1}{1 - \alpha(z)} \right]^3 dz . \quad (A-13)$$

The final relationship that must be satisfied by the assumed inlet mass flow rate M_{in} (or equivalently V_{in}) is then obtained from Equation (A-6) utilizing Equations (A-8), (A-9), and (A-13).

In practice the following computational procedure was used:

- (1) The core was divided into seven radial regions. The two dimensional flux plots were normalized by assuming power and flux distributions are macroscopically the same. The normalization constant is defined as

* Trans. ASME, 70, 695-702 (1948).

$$N \left(\frac{\text{Btu/sec}}{\text{count-ft}^3} \right) = \frac{P}{\sum_{i=1}^7 a_i \int_0^{L_c} \phi_i(z) dz} ,$$

so that

$$P_i(z) = N \phi_i(z) a_i \quad (i = 1, \dots, 7)$$

- (2) From Equation (A-2) $M_g(z)$ was determined.
- (3) A value of V_{in} was assumed.
- (4) From Equation (A-4) $x(z)$ was obtained.
- (5) From Equation (A-5) a value for S was calculated.
- (6) Using the results from Steps (4) and (5) and Equation (A-3) the function $\alpha(z)$ was obtained.
- (7) Next $\rho(z)$ was computed from Equation (A-8).
- (8) The average fluid density $\bar{\rho}$ was determined by using Equation (A-7) and assuming that for some distance beyond the outlet from the core (through 12 in. of axial reflector) the density remains at the value $\alpha(L_c)$.
- (9) ΔP_a was calculated from Equation (A-9).
- (10) ΔP_f was calculated from Equation (A-13).
- (11) Equation (A-6) was examined and a new V_{in} was assumed based on the outcome of the pressure-loss balance.
- (12) Steps (1) through (11) were repeated until Equation (A-6) was satisfied to within about 10 per cent.

Admittedly the relationship used to calculate the void distributions are not precise. The number of gross assumptions is large. It is believed, however, on the basis of the results, that the over-all reactivity worth of the radial reflector is not really very sensitive to the distribution of voids throughout the moderator region providing the over-all void in the core is constant. Inasmuch as the present program was aimed at determining the reflector-worth properties, adequate representation of the void distribution is believed to have been obtained. In problems dealing with boiling by itself, such as may be encountered in large boiling reactor systems, the problem must be treated with a good deal more care.

APPENDIX B

SOLUTION OF TWO GROUP-TWO REGION
EQUATIONS WITH ANNULAR GAP

APPENDIX B

SOLUTION OF TWO GROUP-TWO REGION
EQUATIONS WITH ANNULAR GAP

The two-group diffusion equations with solutions are:

$$\left. \begin{aligned} D_{fc} \nabla^2 \phi_{fc} - \Sigma_{fc} \phi_{fc} + k_{\infty} \Sigma_{tc} \phi_{tc} &= 0 \\ D_{tc} \nabla^2 \phi_{tc} - \Sigma_{tc} \phi_{tc} + \Sigma_{fc} \phi_{fc} &= 0 \\ D_{fr} \nabla^2 \phi_{fr} - \Sigma_{fr} \phi_{fr} &= 0 \\ D_{tr} \nabla^2 \phi_{tr} - \Sigma_{tr} \phi_{tr} + \Sigma_{fr} \phi_{fr} &= 0 \end{aligned} \right\} \begin{array}{l} \text{core,} \\ \text{reflector,} \end{array}$$

$$\phi_{fc} = S_1 A_1 J_0(\alpha_1 r) + S_2 A_2 I_0(\alpha_2 r),$$

$$\phi_{tc} = A_1 J_0(\alpha_1 r) + A_2 I_0(\alpha_2 r),$$

$$\phi_{fr} = A_3 K_0(\gamma_1 r),$$

$$\phi_{tr} = S_3 A_3 K_0(\gamma_1 r) + A_4 K_0(\gamma_2 r),$$

where

$$\alpha_1^2 = \mu^2 - \left(\frac{\pi}{H} \right)^2 \quad (H = \text{core height} + 2 \times \text{axial reflector savings})$$

$$\alpha_2^2 = v^2 + \left(\frac{\pi}{H} \right)^2$$

$$\gamma_1^2 = \frac{1}{\tau_r} = \left(\frac{\pi}{H} \right)^2, \quad \tau_i = \frac{D_{fi}}{\Sigma_{fi}}, \quad i = r \text{ or } c$$

$$\gamma_2^2 = \frac{1}{L_r^2} + \left(\frac{\pi}{H} \right)^2, \quad L_i^2 = \frac{D_{si}}{\Sigma_{si}}, \quad i = r \text{ or } c$$

$$\mu^2 = \frac{1}{2} \left\{ -\frac{1}{\tau_c} - \frac{1}{L_c^2} + \left[\frac{1}{\tau_c} + \frac{1}{L_c^2} + \frac{4(k-1)}{L_c^2 \tau_c} \right]^{1/2} \right\}$$

$$v^2 = \frac{1}{2} \left\{ \frac{1}{\tau_c} + \frac{1}{L_c^2} + \left[\frac{1}{\tau_c} + \frac{1}{L_c^2} + \frac{4(k-1)}{L_c^2 \tau_c} \right]^{1/2} \right\}$$

$$S_1 = \frac{k \Sigma_{tc}}{D_{fc}} \left(\frac{1}{\mu^2 + \frac{1}{\tau_c}} \right)$$

$$S_2 = \frac{k \Sigma_{tc}}{D_{fc}} \left(\frac{1}{\frac{1}{\tau_c} - v^2} \right)$$

$$S_3 = \frac{\frac{D_{fr}}{D_{tr}} \left(\frac{1}{\tau_r} \right)}{\gamma_2^2 - \gamma_1^2}$$

The boundary conditions are obtained as follows:

Let a = core radius

b = core and gap radius

P_{cr} = probability a neutron leaving the core will reach the reflector

P_{rc} = probability a neutron leaving the reflector will reach the core

P_{rr} = probability a neutron leaving the reflector will re-enter the reflector

P_{co} = probability a neutron leaving the core will be lost

P_{ro} = probability a neutron leaving the reflector will be lost.

Then

$$P_{co} + P_{cr} = 1,$$

$$P_{ro} + P_{rc} + P_{rr} = 1.$$

Further define

J_+ as the outward radial current,

J_- as the inward radial current,

where

$$J_+ = \phi/4 - \frac{D}{2} \frac{d\phi}{dr}$$

$$J_- = \phi/4 + \frac{D}{2} \frac{d\phi}{dr}.$$

Then

$$b J_+(b) = a P_{cr} J_+(a) + b P_{rr} J_-(b),$$

$$a J_-(a) = b P_{rc} J_-(b).$$

Substituting this into the equations relating current to flux yields:

$$\phi(a) = \rho_1 \phi(b) - \rho_2 D_r \left(\frac{d\phi}{dr} \right)_b,$$

$$D_c \left(\frac{d\phi}{dr} \right)_a = -\rho_3 \phi(b) + \rho_4 D_r \left(\frac{d\phi}{dr} \right)_b,$$

where

$$\rho_1 = \frac{b}{2a} \left[\frac{1}{P_{cr}} - \frac{P_{rr}}{P_{cr}} + P_{rc} \right]$$

$$\rho_2 = \frac{b}{a} \left[\frac{1}{P_{cr}} + \frac{P_{rr}}{P_{cr}} - P_{rc} \right]$$

$$\rho_3 = \frac{b}{4a} \left[\frac{1}{P_{cr}} - \frac{P_{rr}}{P_{cr}} - P_{rc} \right]$$

$$\rho_4 = \frac{b}{2a} \left[\frac{1}{P_{cr}} + \frac{P_{rr}}{P_{cr}} + P_{rc} \right]$$

In the case studied, with the annular gap from a to b and with a black thermal absorber at b, the specific boundary conditions are:

$$\phi_{fc}(a) = \rho_1 \phi_{fr}(b) - \rho_2 D_{fr} \left(\frac{d\phi_{fr}}{dr} \right)_b,$$

$$D_{fc} \frac{d\phi_{fc}}{dr}_a = -\rho_3 \phi_{fr}(b) + \rho_4 D_{fr} \left(\frac{d\phi_{tc}}{dr} \right)_b,$$

$$\phi_{tc}(a) = (-0.71)(3D_{tc}) \frac{d\phi_{tc}}{dr}_a = -2.13 D_{tc} \left(\frac{d\phi_{tc}}{dr} \right)_a.$$

The solutions for the fluxes can be substituted into these equations and the resulting third-order determinant is the critical equation.

The probabilities, P_{rc} , P_{rr} , and P_{cr} , were found numerically assuming the angular-dependent neutron fluxes at a and b were isotropic, and assuming the axial flux to be given by a cosine distribution.

The values used are:

$$P_{cr} = 0.8334$$

$$P_{rc} = 0.3970$$

$$P_{rr} = 0.3752.$$