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FIRST FORBIDDEN NON-UNIQUE BETA TRANSITIONS IN Re^{186} :
TECHNIQUES IN COINCIDENCE SPECTROSCOPY

by

F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Jr.

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ABSTRACT

The beta spectrum of the main inner group of 89.0-hr Re^{186} ($E_0 = 934.3 \pm 1.3$ kev; 24% of total betas; $\log ft = 8.0$) has been magnetically analyzed in coincidence with the 137.2-kev gamma. The observed spectrum does not have the allowed shape nor the unique ($\Delta I = 2$, yes) shape; its spectrum shape correction factor increases ca 18% from 150 kev to 900 kev. Subtraction of this spectrum, weighted with its measured relative abundance, from the composite spectrum, produces the spectrum of the ground state transition ($E_0 = 1071.5 \pm 1.3$ kev; 76% of total betas; $\log ft = 7.68$). Its similarly non-allowed, non-unique shape factor increases by ca 22% from 150 kev to 1050 kev. Both transitions are interpreted as first forbidden ($\Delta I = 1$, yes); Re^{186} is assigned a ground state spin and parity $1(-)$. A theoretical parametric fitting of these spectra with the combination of scalar and tensor beta interactions is demonstrated; the limited ranges of the fitting parameters, which are functions of the nuclear matrix elements, are obtained.

A comparison is presented between the theoretical fittings of the beta spectrum of the inner group and of the beta-gamma angular correlation.

Pursuant to these studies on Re^{186} we obtained values for the maximum beta energies (see above), gamma energies, conversion coefficients, half-life, and branching ratios, particularly of the ca 0.1% abundant 307-kev beta group (see Table IV).

We discuss in detail the criteria for evaluating the performance of a coincidence spectrometer, particularly the coincidence efficiency as a function of beta energy, and present an analysis of some corrections to coincidence data usually not considered. The performance of the apparatus is exemplified by the spectrum of the main beta transition in Au^{198} ($E_0 = 960$ kev) in coincidence with the 411-kev gammas.

In appendices we present 1) a generalized treatment of the correction for finite spectrometer resolution and 2) the correction to coincidence spectrometer measurements arising from the β - γ directional correlation.

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I. INTRODUCTION

Most of the first forbidden β transitions involving nuclear spin changes of zero or one have experimentally the allowed ("statistical") shape.¹ The notable exception, Bi^{210} (RaE), has recently been discussed extensively;²⁻⁴ only with the presumption that $\Delta I = 1$ rather than 0 has the experimental spectral shape been fitted in a satisfactory way (with a combination of scalar and tensor interaction). The other exception is the 2.317-Mev β transition in Sb^{124} ,⁵ which also exhibits a β - γ directional anisotropy.⁶⁻⁸ The nuclear parameters which account for the shape of the β spectrum are not inconsistent⁹ with those fitting the β - γ correlation for the combination ST and $\Delta I = 1$, in the theoretical approximation $\alpha Z \ll 1$.

It has been pointed out^{1,2} that only in the case of accidental cancellation of the usually dominant energy independent terms (Coulomb terms $\propto \alpha Z/2R$) in the spectrum shape correction factor will the energy dependence of the smaller terms be exhibited. If one fairly expects that present day β spectroscopy is capable of detecting 3 to 5% deviations from the statistical shape, then theory¹⁰ leads one to expect that deviations from statistical shapes should not be as infrequent as seems to be true thus far.

Moreover, as has been pointed out,¹¹ in decays in which β - γ directional anisotropy exists, the associated β spectrum must deviate from the

¹H. M. Mahmoud and E. J. Konopinski, Phys. Rev. 88, 1266 (1952).

²M. Yamada, Prog. Theor. Phys. (Japan), 10, 252 (1953).

³E. A. Plassmann and L. M. Langer, Phys. Rev. 96, 1593 (1954).

⁴G. E. Lee-Whiting, Phys. Rev. 97, 463 (1955).

⁵Langer, Lazar, and Moffet, Phys. Rev. 91, 338 (1953); J. Moreau, Compt. rend., 239, 800 (1954). This transition is uncomplicated by lower energy betas only above 1.60 Mev.

⁶E. K. Darby and W. Opechowski, Phys. Rev. 83, 676 (1951).

⁷D. T. Stevenson and M. Deutsch, Phys. Rev. 83, 1202 (1951).

⁸Kloepper, Lennox, and Wiedenbeck, Phys. Rev. 88, 695 (1952).

⁹M. Morita and M. Yamada, Prog. Theor. Phys. (Japan), 8, 449 (1952); *ibid.*, 10, 641 (1953).

¹⁰Using Eq. (8) of ref. 1 with $W_0 = 2$ and $\alpha Z/2R \approx 10$, then one expects deviations of the order of 3% for (estimated) $x \approx y \approx 1$, $\Delta I = 1$.

¹¹D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. 79, 334 (1950); L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. 25, 729 (1953); H. Frauenfelder, Beta and Gamma Ray Spectroscopy, Ed., Kai. Siegbahn, (Interscience Publishers, Inc., New York, 1955), p. 570

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statistical shape, although Fuchs¹² has shown that this deviation may be small. Of the several known cases of β - γ anisotropy, ¹³⁻¹⁸, K^{42} , As^{76} , Rb^{86} , Sb^{122} , Sb^{124} , I^{126} , Tm^{170} , the associated spectra of all but Sb^{124} have been reported to have the statistical distribution. All the others except Tm^{170} are believed to beta decay from a 2(-) parent state to a 2(+) excited state of the even-even daughter and thus are $\Delta I = 0$ first-forbidden transitions followed by E2 transitions. Tm^{170} differs only in having a 1(-) ground state.¹⁸ Hence $\Delta I = 1$ for the inner group beta transition, which has an allowed shape associated with a β - γ anisotropy. Fujita, Morita, and Yamada¹⁹ have shown that for Tm^{170} the nuclear parameters which produce a fit for the differential β - γ anisotropy permit, but do not require, a spectral shape which deviates by only a few per cent from the allowed distribution. We judge that this slight "shape" might be within the uncertainties of the experiments. Sb^{124} is the only other case also believed to be $\Delta I = 1$; as noted above, it has a non-allowed shape.

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- ¹²M. Fuchs, PhD. Dissertation, University of Michigan (1951).
- ¹³ K^{42} ; $\beta\gamma(\theta)$ - D. T. Stevenson and M. Deutsch, Phys. Rev. 84, 1071 (1951); shape - Koertz, Schwarzschild, Gold, and Wu, Phys. Rev. 95, 612A (1954).
- ¹⁴ As^{76} ; $\beta\gamma(\theta)$ - H. Rose, Phil. Mag. 44, 739 (1953); S. L. Ridgeway and F. M. Pipkin, Phys. Rev. 87, 202 (1952); shape - E. P. Tomlinson and S. L. Ridgeway, Phys. Rev. 88, 170 (1952); E. N. Jensen, Ames Laboratory Report ISC-533 (1954).
- ¹⁵ Rb^{86} ; $\beta\gamma(\theta)$ - D. T. Stevenson and M. Deutsch, Phys. Rev. 83, 1202 (1951); shape - Pohm, Lewis, Talboy, Jr., and Jensen, Phys. Rev. 95, 1523 (1954).
- ¹⁶ Sb^{122} ; $\beta\gamma(\theta)$ - I. Shakhov, Phys. Rev. 83, 333A (1951); shape - M. J. Glaubman, Phys. Rev. 98, 645 (1955).
- ¹⁷ I^{126} ; $\beta\gamma(\theta)$ - D. T. Stevenson and M. Deutsch, Phys. Rev. 84, 1021 (1951); shape - Mitchell, Mei, Maienschein, and Peacock, Phys. Rev. 76, 1450 (1949); Koerts, Macklin, Farrelly, Van Lieshout, and Wu, Phys. Rev. 98, 1230 (1955).
- ¹⁸ Tm^{170} ; $\beta\gamma(\theta)$ - T. B. Novey, Phys. Rev. 78, 66 (1950); H. Rose, Phil. Mag. 43, 1146 (1952); shape - Graham, Wolfson, and Bell, Can. J. Phys. 30, 459 (1952); Pohm, Lewis, Talboy, Jr. and Jensen, op. cit.; R. Richmond and H. Rose, Phil. Mag. 43, 367 (1952).
- ¹⁹Fujita, Morita, and Yamada, Prog. Theor. Phys. (Japan) 11, 219 (1954).

In the present paper we consider the related case of Re^{186} . Previous investigators²⁰ had established the basic features of the decay scheme as given in Fig. 1,²¹ with general agreement that both abundant beta branches have the allowed shape. This led to the assignment of $1(-)$ for the ground state of Re^{186} . R. W. King²² has pointed out that this assignment would violate Nordheim's rule which would predict spin 2 and hence a unique shape for the ground state transition.

The first indication that at least one of the β spectral shapes is not allowed came from Guss, Killion, and Porter²³ who reported that if the ground-state β transition was assumed to have the allowed shape, then the 934-keV group obtained by subtraction exhibited a shape which could be fitted with the unique first forbidden correction factor ($\Delta I = 2$, yes), supporting a $0(-)$ assignment. At the same time, Hurley and Jastram²⁴ reported that the β - γ directional correlation was isotropic ($A < 0.07$). These results are incompatible in the light of β - γ correlation theory which predicts an integral anisotropy of ca 0.4 for the β - γ cascade ($0-, 2+, 0+$). It appeared that a resolution of the problem hinged on a determination of the shape of the first inner group which would be independent of assumptions as to the nature of the ground state transition, i. e., by coincidence spectroscopy.

In the interests of continuity we briefly summarize the results of these experiments here.²⁵ We have observed non-unique, non-allowed, and very similar shapes for both the first inner group and the ground state beta transitions in Re^{186} . These are interpreted as first forbidden transitions with $\Delta I = 1$, requiring a spin assignment, as originally proposed by Metzger and Hill²⁰ and by Steffen²⁰ of $1(-)$ for the Re^{186} ground state. The integral and differential β - γ directional anisotropy has been measured.^{25a} The integral ($E_\beta = 257-934$ keV) anisotropy of 0.105 ± 0.003 and the maximum differential anisotropy of 0.158 (for $E_\beta = 934$ keV) are both too small to be associated with a $\Delta I = 2$, yes beta transition. Both the beta spectra and the differential directional correlation can be reconciled with theory in an internally consistent fashion for the theoretical approximation used.

²⁰F. R. Metzger and R. D. Hill, Phys. Rev. 82, 646 (1951);
R. M. Steffen, Phys. Rev. 82, 827 (1951).

²¹The energies and abundances in Fig. 1 are, for the most part, those obtained in the present work. (See Section V, Table IV.)

²²R. W. King, Dissertation, Washington University (1952).

²³Guss, Killion, and Porter, Phys. Rev. 95, 627A (1954).

²⁴J. P. Hurley and P. S. Jastram, Phys. Rev. 95, 627A (1954).

²⁵Preliminary reports of this work have appeared: Porter, Freedman, Novey, and Wagner, Jr., Phys. Rev. 98, 214 (1955); Phys. Rev. 99, 671(A), (1955).

^{25a}See Novey, T. B., M. S. Freedman, F. T. Porter, and F. Wagner, Jr., "Beta-Gamma Directional Correlation in Re^{186} ," ANL-5523 (Feb., 1956).

Fig. 1. Decay scheme of Re^{186} including results of the present work.

We begin (Section II) with a discussion of the problems of beta spectroscopy and the treatment of coincidence data (Section III) from the special point of view of obtaining accurate spectral distributions from coincidence spectroscopy. We then present (Section IV) an example, Au^{198} , to illustrate the performance of the apparatus, and in Section V we give the detailed results for Re^{186} . Section VI presents the shape-fitting calculations. In Section VII we discuss the relationship of the theoretical fittings of the beta spectrum and the $\beta - \gamma$ correlation and summarize the results.

II. APPARATUS

Spectrometer

The beta spectroscopy described here involves both coincidence operation and the usual (singles) manner of operation.

The Argonne double lens spectrometer, built in 1947,²⁶ has been used in a number of previous investigations involving spectral shape determinations. The distance between source and detector is 102 centimeters and between centers of the 6-in. wide coils is 90 centimeters (which places both source and detector in high magnetic field regions); the diameter of the vacuum chamber is 19 centimeters. The vertical component of the local earth magnetic field (approximately 300 milligauss; field distribution severely distorted by the steel laboratory building) is reduced by compensating coils to less than 3 milligauss over the volume occupied by the electron trajectories. One criterion for the adjustment of the compensating current is the central axial focusing of 1-kev electrons in a 1 meter long x 2.5 cm diameter cathode ray tube placed in the spectrometer chamber in the volume traversed by the beta trajectories. Another criterion is a null response derived from a 6-in. diameter, 3500-turn flip coil which explores the trajectory volume. No ferromagnetic materials were used in the construction of the spectrometer and hence the magnetic field was taken as proportional to the coil current. The all-electronic current servo used to control the generator field current has been described previously.²⁷ Recent modification along the lines suggested by Sommers, Weiss, and Halpern²⁸ include a wide band dynamic filter which further decreases ripple and noise in the spectrometer current to 3×10^{-4} amp peak-to-peak, while the current necessary to focus electrons from 150 kev to 1100 kev is 6 to 25 amp; i.e., noise and ripple are $< 10^{-4}$. The long term stability (over a period of days) is less than one part in 10^4 over the range 1-80 amp. The linearity of the current control (i.e., the ratio of actual spectrometer coil current to control potentiometer setting) is better than one part in 10^4 at all currents greater than 1 amp.

The spectrometer was operated in two different modes in the present experiments. One mode, used in coincidence studies, is characterized by a measured transmission of 5% (of 4π steradians) and a momentum resolution of 6.7% (width at half height) for a source diameter of 6 mm; the other mode has a transmission of 0.8% and resolution of 1.3% with a source

²⁶Freedman, Ramler, and Smaller, Rev. Sci. Instr. 23, 571 (1952).
Describes the spectrometer coils only.

²⁷Jay Wolff and M. S. Freedman, Rev. Sci. Instr. 22, 736 (1951).

²⁸Sommers, Weiss, and Halpern, Rev. Sci. Instr. 20, 244 (1949);
Rev. Sci. Instr. 22, 612 (1951).

diameter of 3 mm. The mode of operation may be changed easily by external controls which can vary (continuously) the inner and outer radii and the axial positions of both the source (transmission regulating) baffle and of the ring focus baffle. In addition, the diameter of the baffle immediately in front of the detector aperture, the "detector baffle," may be selected externally. There are also numerous other anti-scattering baffles disposed inside and outside the electron trajectories. Studies with intense sources of Cs^{137} have demonstrated that no "ghost" lines appear above the normal conversion lines with relative intensity as great as 0.025%. A search for ghosts below the Cs^{137} K-line was made by eliminating the continuum by counting the line in coincidence with K-X rays; it set a maximum relative intensity at any point down to 50 kev of 0.03%. The spectrometer vacuum was maintained at less than 0.1μ .

β - Detectors

One of the two types of beta particle detectors used was an end window, loop-type, low pressure proportional counter, Fig. 2. The designation "proportional" means that the pulse output of the counter is proportional to the total ionization produced by a particle in the counter sensitive volume and not to the particle energy. In fact, since the counter gas presents only 0.5 mg/cm^2 , the range of all but the very lowest energy electrons is much greater than the counter sensitive depth, and the pulse output then depends directly on the specific ionization of the particle, i.e., the pulse amplitude decreases with increasing energy out to a minimum at about 1 Mev.

The window for these experiments, 0.92 mg/cm^2 Mylar, with measured cutoff of 19 kev, was unsupported over the 1-cm diameter aperture. In these experiments we were interested in beta energies greater than 150 kev. The window was made conducting by volatilizing about $20 \mu\text{g/cm}^2$ of gold onto the side facing the counter gas, to avoid hysteresis effects which attend the presence of insulating surfaces in counters. A continuous flow pressure regulating system kept the counter gas, research grade propane, at 18 cm Hg pressure. Pressure regulation was accomplished by a glass Cartesian manostat.

With a non-overloading stabilized amplifier of gain approximately 4×10^4 and input sensitivity of $100 \mu\text{v}$, 200-volt long plateaus were obtained with slope of approximately 1% per 100 volts, with high energy (low pulse amplitude) beta particles. Dead times were approximately 5μ sec (limited by the electronics).²⁹ Backgrounds were approximately 6-9 c/min.

²⁹Except at the upper end of the plateau, where a rapid increase in the counter dead time occurs.

Fig. 2. Cross section of low pressure flow-type gas proportional counter.

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The other beta particle detector was a scintillation counter. Although both detectors were used in the coincidence experiments, emphasis is placed on the results obtained with the scintillation detector. The light output of the 1 mm thick x 11 mm diameter anthracene disc³⁰ (covered by 200 $\mu\text{g}/\text{cm}^2$ aluminum foil reflector, Fig. 3) was led out of the strong magnetic field region by a Lucite light pipe, 24 in. long by 1 in. o.d., to a Dumont 6292 photomultiplier tube. The phototube was cooled to -80°C by enclosing it in a Styrofoam box filled with chipped dry ice. Double shell magnetic shielding (Mu metal) around the phototube was sufficient to insure that the phototube performance was not affected by the magnetic field of the spectrometer. This was checked by exciting the β -detector with an external gamma source and noting that the counting rate was independent (less than 1%) of the magnetic field, from zero to a field necessary to focus 1.2-Mev electrons.

The question of the relative efficiency of the scintillation detector over the beta particle energy range of interest (>150 kev) has been approached in three ways. The first involved pulse height analysis of the output of the scintillation detector; the second was a direct comparison of the anthracene counter to the proportional counter; the third was the performance in the spectrometer with spectra of known shape. We present the first of these in some detail because the knowledge of the pulse height distribution from the scintillation beta detector is necessary for some of the later comments on the over-all coincidence efficiency.

Figures 4 and 5 show the pulse distribution from the anthracene beta detector as electrons of various indicated energies are focused from a beta continuum onto the crystal by the spectrometer. We include one example at 104 kev which is below the limit of our interest, 150 kev. The electrons, of course, are not strictly monoenergetic, the momentum resolution of the spectrometer being 6.7%. The pulses were amplified and fed to a differential pulse height analyzer. The abscissa of Figs. 4 and 5 is the bias setting of the analyzer plus $1/2$ the small "window" width. All curves are normalized to the same area. Note that at energies over 370 kev the pulse height distribution is "double peaked." The peaks near 300 bias units are interpreted as those from electrons which penetrate the 1-mm crystal without large angle scattering except for 370-kev peak, which has not quite penetrated. This "penetration peak" or "minimum path length peak" shifts slightly, as expected, to lower pulse amplitudes as the electron energy approaches minimum ionization. The higher broad "peak" is due to electrons with path lengths in the crystal which, because of scattering, exceed the minimum length, ca 1 mm, by various amounts. The maximum

³⁰ A 5-mm thick clear anthracene crystal gave lower pulse amplitudes for a given beta energy than did the 1-mm crystal, presumably due to lower light collection efficiency.

Fig. 3. Scintillation beta detector assembly. All interfaces in light path joined with 10^6 centi-stoke silicone oil or silicone vacuum grease.

Fig. 4. Pulse height distributions for beta particles of indicated energy focused on 1 mm thick anthracene disc in spectrometer. Fig. 5 shows expansion of low pulse height region.

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Fig. 5. Pulse height distributions for beta particles of indicated energy focused on 1 mm thick anthracene disc in spectrometer. Photomultiplier at -80°C . Upturn below 8 bias units due mainly to afterpulses. Flat tail A due to beta particles backscattered from crystal. B indicates beta pulse analyzer discrimination level (10 bias units). Shaded rectangle designates assumed loss of single counts for 150 keV beta particles due to discrimination at B.

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amplitude of this broad peak would be expected to increase with the electron energy as is seen in the figure. All the curves in the figure have the background (source out) subtracted. The flat "tail" is caused by electrons which backscatter³¹ and leave only a small portion of their energy in the crystal, and the sharp rise at very low bias settings is caused by afterpulsing in the phototube. The rise is shown only for the 150-keV electrons, but it exists for all incident energies. The "tail" and afterpulse rise of the beta pulse height distribution are obscured by the large room temperature background of the phototube, but at -80°C they are distinct. The integral background of 12,000 c/min above 10 bias units is reduced to ca 500 c/min with cooling. It should be noted here that the beta analyzer has a dead time of 8 microseconds, hence those afterpulses recorded occur with at least an 8 μsec delay. That these are indeed afterpulses has been verified by exhibiting the output of the phototube on a fast oscilloscope, each sweep being triggered by a large pulse. A distribution of small pulses was seen to extend out to approximately 20 μsec , although a majority of these secondary pulses occur closely grouped at approximately 2 μsec . The shape of the afterpulse distribution is a sensitive function of the phototube high voltage. With increasing photomultiplier voltage the afterpulse amplitude increases and the relatively sharp intersection (see Fig. 5) of this distribution with the tail of the real beta pulse distribution disappears. In the interest of maximum gain we operated with the highest phototube voltage (1015 volts) consistent with a clean separation from after pulses at low pulse amplitude. At this voltage the sharp rise in afterpulse distribution occurs below 8 bias units for all energies and so we rejected them by discriminating at 10 bias units. In so doing we must inevitably lose some small fraction of the real pulses, the fraction thus lost depending on the beta energy. In order to make this small correction we extrapolated the flat low tail on the pulse height distributions to zero amplitude. The area cut off from this tail between 0 and 10 bias units compared with the area under the whole curve (not including the afterpulse rise) gives the relative loss for that energy. Figure 6 gives the relative efficiency (η) for counting singles obtained in this way, and it is seen that above 150 keV the relative efficiency is greater than 99% and is known with an accuracy of 0.1%. This operating point of 10 bias units should correspond to the upper end of a gain and bias plateaus. Such plateaus, flat to within $<1\%$ for fivefold bias increases, were observed.

For the second check on the scintillation counter efficiency, the counting rate of the anthracene detector was compared directly with that of the proportional counter at various energies from 100 keV to 700 keV by interchanging the two via the detector vacuum lock with the same sample

³¹The number of counts in the backscattering tail is about one-half the fraction expected from an infinite thickness of low atomic number backscatter (L. Glendenin; private communication). A comparison of pulse height distributions from the crystal edge and center showed no differences, indicating that edge effects were not responsible for the low amplitude tail.

Fig. 6. Efficiency for counting beta particles with the anthracene detector, as a function of beta energy (singles efficiency). This curve applies for discrimination level of 10 bias units (See Figs. 4 and 5). Note the expanded ordinate scale.

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in the spectrometer. The detector baffle limited the image diameter to 7 mm, less than the respective detector apertures. This experiment showed that within the experimental error of the comparison (0.8%), the relative efficiency of the scintillation counter was the same as was shown in Fig. 6.

Finally, independent evidence that the scintillation detector efficiency is known accurately is the performance with known spectra. This performance is not independent of other characteristics of the magnetic spectrometer. However, the fact that straight Kurie plots were obtained for both P^{32} , 300-1712 keV (Fig. 7) and Au^{198} , 400-960 keV (Fig. 8) employing the proportional counter detector and also for Au^{198} (Fig. 19, p. 42) using the scintillation counter supports both the performance of the magnetic lens spectrometer as an instrument for determining spectral distributions and the essentially constant efficiency of the scintillation counter over this energy range. A discussion of the slight upturn in the Au^{198} Kurie plots is given in Section IV.

Gamma Detector and Source Mounts

The gamma detector was a NaI-Tl crystal, 3/4-in. diameter by 3/4-in. long (packaged by Harshaw Chemical Company), located one inch behind the source on the axis of the spectrometer (see Fig. 9). A Lucite light pipe, 24 in. long by 1 in. diameter, made possible the location of the phototube outside the strong field region. This together with double shell Mu-metal shielding rendered the gamma counting rate independent (<1% variation) of the magnetic field strength up to a field necessary to focus 1.2-MeV electrons. The light pipe introduced a factor of 3 loss in optical transmission, but somewhat less than $\sqrt{3}$ loss in resolution, resulting in a half-width of 12% for the Cs^{137} γ ray.

The absence of significant scattering from the whole coincidence source holder is illustrated by the spectrum of P^{32} , which is presumed to have the statistical shape.³² Figure 7 gives the Kurie plot and $\underline{m}$ function³³ of P^{32} taken with the coincidence source holder and proportional counter beta detector. The source was vacuum-volatilized H_3PO_4 ($\sim 40 \mu g/cm^2$) on $200-\mu g/cm^2$ Al, 1/4-in. diameter; the spectrometer was set to 5% transmission. The resolution correction was made by applying constants derived from the line shape of the Re^{186} 63-keV K-conversion line (from another

³²Sheline, Holtzman, and Fan, Phys. Rev. 83, 919 (1951).

³³In the notation of J. P. Davidson and D. C. Peaslee, Phys. Rev. 91, 1232 (1953), $m = N/[f(E_0 - E)^2]$, the experimental spectrum shape factor; f , the Fermi coulomb function as given in the Bureau of Standards tables, includes the usual factor p^2 . With an accurately known end point value, E_0 , the $\underline{m}$ function provides a much more sensitive way of displaying small deviations from the allowed shape than does the conventional Kurie plot.

Fig. 7. Kurie and $\underline{m}$ plots for P^{32} with proportional counter detector; spectrometer transmission = 5%. The straight line in the $\underline{m}$ plot is drawn at the average ordinate of the points.

Fig. 8. Kurie and $\underline{m}$ plot of Au^{198} with proportional counter detector; spectrometer transmission = 5%. Statistical error smaller than circle unless indicated.

Fig. 9. Coincidence source holder and gamma detecting crystal mount. Crystal mount-light guide interface joined with 10^6 centistoke silicone fluid.

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experiment) by the method outlined in Appendix I. We do not present data below 300 kev because of the presence of P^{33} in the sample. Inspection of the figure, particularly of the $\underline{m}$ function, indicates a maximum possible scattering component of 2% at about 300 kev, $< E_0/5$.

Coincidence Circuit

Figure 10 shows a block diagram of the coincidence arrangement used for the coincidence beta spectrum.³⁴ The wide-band distributed type amplifiers are Hewlett Packard 460A (nominal gain 10; saturate at ca 8 volts output), four on the beta side and three on the gamma side, with a G7A diode limiter preceding the last beta amplifier. These provide enough gain to saturate nearly all the smallest pulses, and to oversaturate at least fivefold the average pulse. These saturated pulses are shaped with 30-m μ sec (double travel times) delay lines and are fed to the fast coincidence circuit of the Garwin type.³⁵ With this shaping the resolution time, $2\tau_f$, is of the order of 0.12 μ sec. The resolving time is thus of the order of twice the sum of the shaping times; this is due to the extremely high gain used to insure high coincidence efficiency. This high gain, in combination with a sensitive coincidence mixer,³⁵ causes the great majority of the pulses to be very far above the critical amplitude necessary for registration of a coincidence (if, of course, a pulse of some minimum amplitude is present in the other channel simultaneously). In such circumstances the effective pulse width at the critical amplitude is sensitive to those minor features of circuit operation as tend to stretch or distort the shaped pulse near the base line (imperfectly terminated shaping lines, improperly matched impedances, or base line shift in the amplifiers at high rates). At lower amplifier gains the resolving time more closely equalled the sum of the shaping times.

³⁴ When the proportional counter is the beta detector, a small low noise preamplifier with gain of 35, one-millisecond time constant, and cathode follower output is located on the counter assembly. The preamplifier output is carried by 20 ft of RG-62-U cable to a differentiator-limiter with a time constant of $1/2 \mu$ sec. At this point the signal splits into the fast and slow β channels and the rest of the circuits are the same as in Fig. 10, with the exception of the shaping times. Primary ionization at various distances from the central wire loop of the counter produces pulses which vary in time relationship with the γ pulses originating in the same nuclear decay. This so called "jitter time" of the proportional counter necessitates the use of shaping times for the β and γ pulses about twice as long as for the scintillation detector in order to insure 100% coincidence efficiency.

³⁵ R. L. Garwin, Rev. Sci. Instr. 24, 618 (1953).

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Fig. 10. Block diagram of coincidence circuit and spectrometer.

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Pulse amplitude selection is accomplished by demanding triple coincidences between the output of the fast coincidence circuit and the outputs of the relatively slow, well stabilized pulse height analyzers³⁶ (operated either differentially or integrally) in the beta and gamma channels. In such an arrangement, the effective resolving time, even for analyzed pulses, is that of the fast double coincidence circuit, if that of the triples coincidence circuit is greater. The triple coincidence circuit resolving time, $2\tau_s$, is of the order of $2.5 \mu\text{sec}$. Figure 10 shows that the triples circuit is driven only by trigger-generated pulses; compensation for the relative delays of these pulses is easily accomplished with fixed delay lines. Six scales of 10^3 and mechanical registers record triple coincidences (T), double coincidences (D), analyzed betas (B), analyzed gammas (G), unanalyzed betas (β), and unanalyzed gammas (γ). Careful attention was given to eliminating pulse reflection and circuit interaction. Stabilized ($<0.01\%$ long term drift) high voltage sources supplied the photomultipliers.

Coincidence Efficiency

A question of prime importance for beta coincidence spectroscopy is that of the relative coincidence efficiency as a function of beta energy. The crux of the problem is that the detectors, particularly the beta detector, put out a broad distribution of pulse amplitudes rather than a single pulse amplitude, and that, moreover (Figs. 4 and 5), this distribution changes with beta energy and extends to a small extent down into the noise background and afterpulse region. A criterion for 100% coincidence efficiency that is often used is the existence of a flat top delay curve.³⁷ This criterion is a necessary but certainly not a sufficient condition³⁸ for any pulse distributions. Moreover, with coincidence rates of the order of a few hundred per minute it is not practical to determine the "flatness" of the delay curves to the required accuracy. Figure 11 is a delay curve for 150-keV electrons detected with anthracene in coincidence with 137-keV gammas. Lengths of RG-114U cable inserted before the wide band amplifiers effected the delay. Although this curve is "flat" within the statistics of the counting, we have found that the coincidence efficiency here is 98.8% compared to electrons of energy greater than 600 keV which have equally "flat" top delay curves. This problem has been investigated with much better statistical accuracy by careful exploration of delay curves obtained with a similar, but faster, coincidence arrangement. This has shown that the "reals" counting rate on the flat top may continue slowly to increase by as much

³⁶R. K. Swank, ANL-4303 (1949), describes both the linear amplifier and the pulse height analyzer.

³⁷Bell, Graham, and Petch, Can. J. Phys. 30, 35 (1952).

³⁸This may be most easily seen by considering pulse distributions which consist of discrete groups.

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Fig. 11. Delay curve for 150-keV beta particles in coincidence with the 137-keV gamma of Re^{186} . Apparent "flat" top indicates nearly 100% coincidence efficiency. It is shown (see text) that the coincidence efficiency at this energy is 98.8% compared to beta particles of energy >600 keV.

as 2.5% with increasing resolving time beyond that at which the top is flat to within 1.5%, the maximum slope consistent with the statistics of counting.

Another approach we have used is to demand that the real coincidence rate be independent of the fast amplifier gains above a certain minimum. Gains above these minima insure that the fast coincidence circuit will respond to every pulse which is recorded by B and G. We did obtain gain plateaus sensibly flat, within the statistics of 3%, over a gain factor of approximately 6. Again, to determine within a fraction of a per cent the flatness of these gain plateaus is difficult because of the counting time involved. Also, because of the long tail on the beta pulse distributions (see Figs. 4 and 5) it will be seen that a truly flat gain plateau is unattainable except as all the pulses in the tail down to the B discrimination level (10 bias units) are detected in coincidence. Thus the gain required is also that which forces the noise and afterpulses through the fast coincidence discriminator, with an attendant high rate of double coincidences. This rate, which increases rapidly with further gain, shortly interferes with the reliable operation of the last fast amplifier and burdens the fast coincidence circuit and the slow coincidence discriminator with high chance rates. Practically, therefore, one cannot realize a flat plateau if one must detect even moderately low energy pulses, and one must substitute for the goal of 100% coincidence efficiency the attainment of a high, measurable, and hence correctible, efficiency.

Although the requirements of flat top delay curves and gain plateaus were useful in the early exploration of the apparatus and in establishing optimal values for the delays and gains to be used, the quantitative answer to the question of relative coincidence efficiency was obtained by examining the number of real coincidences per B as a function of pulse height (not energy) selected differentially with the pulse height analyzer. These experiments showed that the number of real coincidences per B was independent of beta pulse height for pulse heights above 60 bias divisions (refer to Figs. 4 and 5 for bias unit scale). Figure 12 shows this differential efficiency curve in the region below 60 bias divisions. In this figure we call 100% coincidence efficiency the constant value of reals/B^{39} obtained for pulse heights over 60 bias units. Since the B pulse height discriminator was operated as an integral discriminator during a spectrum run, accepting only pulse heights over 10 bias units, the efficiency must be taken as zero below 10 bias units. This differential efficiency curve combined with the pulse height distribution of Figs. 4 and 5 was used to obtain the over-all coincidence efficiency as a function of beta energy (Fig. 13). This was done by simple numerical integration with the assumption that the flat low intensity

³⁹The B scaler was driven by the same trigger-generated pulse that was impressed on the triple mixer grid, and similarly for G. Thus any count recorded as B has been effective in questioning the triple coincidence circuit. By "reals" we mean real triple coincidences.

Fig. 12. Differential coincidence efficiency applicable for any beta energy, focused by the spectrometer. This curve is used in determining the over-all coincidence efficiency. The ordinate is the ratio of the triple coincidences ($\underline{T}$) to the beta particles recorded ($\underline{B}$) with those pulse heights indicated on the abscissa, expressed as per cent of the maximum observed. Above 60 bias units the ratio T/B was observed to be independent of the beta pulse height. Bias units are the same as in Figs. 4 and 5. The differential coincidence efficiency value used in computing the over-all coincidence efficiency is taken to be zero below 10 bias units, since the $\underline{B}$ discriminator is adjusted to count integrally above 10 bias units during an actual spectrum run.

Fig. 13. Over-all coincidence efficiency as a function of beta particle energy obtaining during the Re^{186} experiments. This curve is calculated from the observed differential efficiency curve of Fig. 12 in conjunction with the observed pulse height distributions of Figs. 4 and 5.

tail on each curve extends to zero bias setting with constant intensity. Note that because the system has an efficiency so near 100% the over-all coincidence efficiency can be given to 0.1%, even with the uncertainties of the order of 5% in the low energy part of the differential efficiency curve.

In practice we have found that a sensitive and rapid check on the over-all stability of the system is obtained by the differential determination of the ratio reals/B at bias settings below 50; this was done periodically throughout the long runs during which the beta spectra were taken. For this purpose we focused 80-kev beta particles, whose pulse distribution lay mainly in the region 10-50 bias units and thus maximized the counting rate. This check was supplemented by periodic checks of the other factors influencing the coincidence efficiency as a function of beta energy, viz., the pulse height distributions from the beta detector and the resolving time of the coincidence circuit. The former was monitored during the experiments by periodically running a low energy beta pulse height distribution.

The resolving time ($2\tau_f$) of the coincidence circuit can be determined in three ways: (1) chance rate measurement with independent sources; (2) chance rate measurement with single source in which the real coincidences are eliminated by use of a sufficiently long relative delay; (3) measurement of the effective width⁴⁰ of the delay curve for real coincidences. The first has the disadvantage that it is not always convenient; the second has the disadvantage of possible error in 2τ due to attenuation and distortion of the pulses by introduction of the delay; the third is neither convenient nor capable of the required precision if the delay curve is not nearly rectangular. Of these, we have used the more fundamental independent source technique to investigate the dependence of $2\tau_f$ on the beta energy. Figure 14 shows the result of these measurements for the anthracene detector.⁴¹ For the independent source measurements the gamma detector, including light pipe and coincidence source holder, was assembled outside the spectrometer while another source was inserted in the spectrometer. It is very inconvenient to perform this operation during the spectrum runs; therefore we used the delay method during the run to monitor the stability of $2\tau_f$. We have not been able to demonstrate

⁴⁰Bell, Graham, and Petch, op. cit., define effective width as the area of the delay curve divided by the height. The resolving time so defined will be inconsistent with the results of measurement with independent sources unless the coincidence efficiency is 100% at the peak of the delay curve. Bay, Henri, and Kanner, Phys. Rev. 100 1197 (1955).

⁴¹For the proportional counter the dependence of $2\tau_f$ on beta energy shows slightly decreasing resolving time for low energy where it gives maximum pulse amplitude. Similarly the anthracene detector has shorter resolving time for larger pulse amplitudes at high energy.

Fig. 14. Fast resolving time ($2\tau_f$) as a function of beta energy. Note the small increase in $2\tau_f$ at low beta energies. This behavior is observed with the anthracene scintillation counter as beta detector. No variation in $2\tau_f$ with beta energy is observed above beta energies of 300 kev.

conclusively any difference in $2\tau_f$ as measured by the two methods. The relatively weak sources which we used for the spectrum runs made it time-consuming to measure $2\tau_f$ by the delay method for every beta energy taken on the spectrum. We monitored $2\tau_f$ at one energy which is near the peak of the beta spectrum and which is on the flat part of the $2\tau_f$ vs beta-energy curve.⁴² The $2\tau_f$ vs beta-energy curve was then normalized to the value obtained during the actual run. We have not found any evidence for drift in $2\tau_f$ during three day runs, but changes of the order of 5% have been noted over periods of several weeks. In summary, our knowledge of $2\tau_f$ at a particular time for a particular beta energy has at most a 5% uncertainty. Since we used source strengths small enough so that the ratio reals/chance was greater than 10 over most of the spectrum, the error due to the uncertainty in 2τ was reduced to less than 0.5%.

⁴² This spectrometer has a device which enables one to bring a source of beta particles from a shielded source position in the vacuum chamber to a position in front of the beta counter. If at the same time the magnetic field is reduced to zero one has independent sources exciting the beta and gamma counters. Unfortunately, the slight dependence of $2\tau_f$ on energy makes the use of a continuum for the beta source undesirable (we have used Pm^{147} , for example). The same objections also caused us to reject the use of a separate external gamma source to excite the beta counter, even for monitoring purposes.

III. TREATMENT OF DATA

Calculation of Real Coincidences

The treatment of the raw coincidence data to produce a value for the real coincidence rate, R , and for the coincidence momentum spectrum, $M(p)$, is given in Eqs. (1); these represent a degree of approximation which is more than sufficient for most present day coincidence beta spectroscopy. The special effects given consideration are the coincidence efficiency, ϵ , a chance rate, C_s , peculiar to a fast-slow coincidence combination such as ours, and also some usually disregarded dead time effects. Many of these effects are very small for our circuits, but might not be for others. However, in view of the small effects sought in the present experiments we have included these small corrections.⁴³

$$R = T_c - 2\tau_f^*(B^* - R')(G^* - R') \quad (1a)$$

$$T_c = (T/\epsilon - C_s)(1 + \delta_B^C B)(1 + \delta_G^C G) \quad (1b)$$

$$C_s = (D - T) G (2\tau_s) \quad (1c)$$

$$R' = (T/\epsilon) - 2\tau_f (GB) \quad (1d)$$

$$B^* = B(1 + \delta_B B) \quad (1e)$$

$$G^* = G(1 + \delta_G G) \quad (1f)$$

$$2\tau_f^* = 2\tau_f \frac{(1 + \delta_B^C B_T)(1 + \delta_G^C G_T)}{(1 + \delta_B B_T)(1 + \delta_G G_T)} ; \quad 2\tau_f = \frac{T}{B_T G_T} \quad (1g)$$

$$M(p) = \frac{R}{(G - bg) p} \quad (1h)$$

The quantities are defined as follows:

R = real coincidence rate; R' = first approximation of R .

B and G = observed analyzed beta rate and analyzed gamma rate, respectively.

bg = appropriate background.

T = observed triple coincidence rate.

T_c = triples rate corrected for coincidence efficiency, slow chance events, and dead time losses.

⁴³See Westcott, Greenberg, and Kirkaldy, Can. J. Phys. 31, 859, (1953) for a discussion of some of these and higher order corrections.

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D = observed double coincidence rate.

ϵ = intrinsic coincidence efficiency, ≤ 1 ; in general a function of beta energy (see Fig. 13).

$2\tau_s$ = resolving time of the slow (triple) coincidence circuit for G pulses and D-B pairs (see text).

$2\tau_f$ = observed resolving time of fast (double) coincidence circuit measured with rates B_T and G_T . In general, a function of beta energy (see Fig. 14).

δ_B and δ_G = effective dead time of B channel and of the G channel.

δ_B^C and δ_G^C = effective dead time for triple coincidences of the B and G channels (see text).

$M(p)$ = observed coincidence momentum distribution.

p = electron momentum.

Our notation (*) implies dead-time-corrected quantities. Dead-time corrections [e.g., Eqs. (1e) and (1f)] in our case are given well by the linear approximation.⁴⁴ The dead time correction factors of the form $(1 + \delta_B B)$ are usually between 1 and 1.01. The measurement of the fast resolving time $2\tau_f$ has been discussed previously.

The real coincidence rate R is obtained in Eq. (1a), in which the second term represents the fast chance coincidences, and T_C is the triple coincidence rate corrected [Eq. (1b)] for coincidence efficiency (ϵ), resolving time losses and slow chance rate C_s [Eq. (1c)]. Equation (1d) represents the usual approximation in treating coincidence data with the exception of the factor ϵ , the over-all coincidence efficiency.

C_s was always less than 0.2% T for our experiments. The origin of C_s counts may be seen by reference to Fig. 15, which depicts typical pulse height distributions in the beta and gamma channels. Real coincidences (R) between pulses in regions II and IV were the object of the calculation, and chance coincidences II-IV were accounted for by the second term of Eq. (1a). Of the other possible combinations I-III, I-IV, and II-III only I-IV gives rise (significantly) to real double (D) coincidences, i.e., between betas and X rays or Compton gammas, at a rate $D - T$.⁴⁵ These doubles, normally rejected

⁴⁴See for example L. G. Rainwater and C. S. Wu, *Nucleonics*, 1, 60 (1947).

⁴⁵Under the conditions of our experiments, real/chance > 10 . Thus in computing this small correction, we have ignored as contributions to the doubles rate D, chance coincidences of types I-III, I-IV, and II-III. A very small contribution to D from reals of type II-III and I-III is also ignored. Recall that T consists mainly of type II-IV coincidences, real plus chance.

Fig. 15. Schematic pulse height distributions for betas (lower curve) and gammas (upper curve) with scintillation detectors. Analyzed beta (B) and analyzed gamma (G) discrimination levels separate regions I (X rays and Comptons) from II (photopeak) and III (noise and afterpulses) from IV (main beta peak and backscattering tail).

in the triple circuit since gamma pulses in region I are below the $\underline{G}$ discrimination level, may occur, by chance, with a $\underline{G}$ pulse from another nuclear disintegration within the triple circuit resolving time $2\tau_s$, and give rise to a C_s count.

The slow resolving time $2\tau_s$ was measured with a double pulser (Berkeley Instrument Model 903) by connecting one pulse to the $\underline{G}$ input, and the other to $\underline{B}$ and to the fast amplifier beta and gamma inputs in parallel (see Fig. 10). The pulses from the double pulser can be moved timewise with respect to each other; either can precede. One notes the range of separation during which coincidences were registered. The result was $2.6 \mu\text{sec}$.

The last two factors in Eq. (1b) account for the loss of triple coincidences which occurs when a II-IV (real or chance) coincidence event follows a single $\underline{B}$ or $\underline{G}$ pulse within a time $\delta_B^C(W \rightarrow \delta_B)$ or $\delta_G^C(W \rightarrow \delta_G)$, and for the restoration of the recorded coincidence if it follows within a time $t < W$ (see Fig. 16). Thus the loss-governing rates are the singles $\underline{B}$ and $\underline{G}$, rather than the coincidence rates.⁴⁶ The dead times δ_B , δ_G and the product $(1 + \delta_B^C \underline{B})(1 + \delta_G^C \underline{G})$ can be measured simultaneously by a "paired source" experiment. For this purpose the paired sources were mounted on halves of a disc each of which could be held independently in the coincidence source holder of the spectrometer. Accurate repositioning of the source holder is a requirement of this technique. We have found this method suitable for δ_B and δ_G , but it was difficult to accumulate enough coincidence counts to determine the quantity $(1 + \delta_B^C \underline{B})(1 + \delta_G^C \underline{G})$ accurately. Another method gives δ_B^C and δ_G^C directly and precisely, and in addition demonstrates clearly that δ_B^C and δ_G^C are not the same as, but rather are smaller than, δ_B and δ_G , respectively. For this method a double pulse generator (Berkeley Instrument Model 903) with two outputs was used; output No. 1 supplied the first pulse and output No. 2 the second pulse at a later time which was variable from 0 to $10 \mu\text{sec}$. In order to measure δ_B and δ_B^C , for example, a suitable resistive pulse divider network allowed both pulse No. 1 and pulse No. 2 to be substituted for the beta detector (see Fig. 10) while only pulse No. 2 was introduced in place of the gamma detector; this simulates a single $\underline{B}$ pulse followed by a coincidence. If the interval between pulses is larger than δ_B (see Fig. 16, region I) and if the double pulse repetition rate is $\underline{N}$, then $B = 2N$ and $G = T = N$. When the spacing is reduced to just below δ_B (region II) then $B = G = N$, because the second $\underline{B}$ pulse is suppressed in the analyzer by the first one, and thus $T = 0$. If now the interval between pulse No. 1 and No. 2 is further reduced to some value less than the pulse width, $\underline{W}$ (region III), then $B = G = N$, as in region II. However, in this case $T = N$ as in region I, i.e., the coincidence registration is restored, now resulting from the $\underline{G}$ pulse and the first $\underline{B}$ pulse, the second $\underline{B}$ pulse still being suppressed.

⁴⁶The loss of coincidences in either the double or triple coincidence circuits due to the dead times of the associated coincidence scalars, i.e., coincidences suppressing coincidences, is completely negligible for our coincidence rates.

Fig. 16. Schematic representation showing the relationship between the analyzer dead time (δ_B) and the effective coincidence dead time (δ_B^C) for the B channel. An initial singles pulse in the B channel is followed after time t by a coincidence event. If the time separation, t , is greater than δ_B (region I) then both the second B count and coincidence are recorded. If $W < t < \delta_B$ (region II), then both the second B count and the coincidence are not recorded. If $t < W$ (region III) then the coincidence registration is restored, the second B pulse still being suppressed.

The time duration of region II is δ_B^C . To measure δ_G and δ_G^C one interchanges B and G in the above procedure. The double pulser was calibrated against a signal generator via oscilloscope display. The double pulser results for δ_B and δ_G agreed with the paired source values. For our circuits we found $\delta_B = 8.0 \mu\text{sec}$; $\delta_G = 5.9 \mu\text{sec}$; $\delta_B^C = 6.3 \mu\text{sec}$; and $\delta_G^C = 3.6 \mu\text{sec}$. These results were essentially independent of the pulse durations and amplitudes over a range encompassing the durations and amplitudes of the detector pulses.

The second term in Eq. (1a) accounts for the chance coincidences of Type II-IV. Subtraction of the real coincidence rate (R' , in first approximation, instead of R , to avoid the quadratic form) from the singles rates B^* and G^* accounts for the impossibility of simultaneous registration of both a real and a chance coincidence when the resolving time of the triples coincident circuit is less than the limiting dead times in the system,⁴³ (either analyzer or triple coincidence scaler).⁴⁷ In our case where reals/chance is large and where the ratios R/B and R/G are small, this correction is very small, approximately 0.1% of R .

$2\tau_f^*$ is computed from observed $2\tau_f$ [Eq. (1g)] by correcting for the dead time losses of the observed singles rates G_T and B_T (denominator) and for the associated loss in coincidences (numerator) where G_T and B_T are the singles rates obtaining in the $2\tau_f$ measurements.

The ratio R/R' , {corrected real coincidence rates [Eq. (1a)]/the usual approximation to R [Eq. (1d)]}, for our experiment lay in the range 0.99-1.02, for B and G rates of $<100,000$ c/min, and the resolving times and dead times given above.

The coincidence rate R is divided by $(G - bg)$ to correct for small drifts in G discriminator level. This eliminates the explicit correction for radioactive decay. Since the G discriminator level is chosen in the trough between the photopeak and the Compton and/or X-ray distribution, small drifts in the G discriminator level are expected to affect the G count only slightly. It is indeed observed that $(G - bg)$ follows the decay curve characteristic of the sample with fluctuations of $\pm 2\%$.

The observed momentum distribution $M(p)$ is obtained from this normalized R by dividing by p , the electron momentum. For each individual spectrum the calibration factor is derived from an accurately known or measured⁴⁸ conversion line in the source, if one exists.

⁴⁷Note that this correction applies also to the cases in which the chance rate is determined by measurement with sufficient delay introduced to eliminate all real coincidences.

⁴⁸The momentum of the conversion line is compared to that of a known accurate line standard, the K-line of Au^{198} , in a mixed source at good resolution. We used the value 2222.4 gauss-cm for the momentum of this line [D. A. Lind and A. Hedgran, Ark. Fys. 5, 29 (1952)].

Correction for Finite Resolution

$M(p)$ must be corrected for the finite momentum resolution of the spectrometer. The details of this correction are given in Appendix I. This correction expresses the true momentum distribution $N(p)$ in a series expansion in derivatives of the measured spectrum $M(p)$; the coefficients are derived from the empirical line shape ("transmission curve") of the spectrometer. The analysis presented requires that the line shape parameters be independent of the momentum as would be the case for a magnetic spectrometer with no scattering and for a thin source. This has been investigated experimentally by comparing in detail conversion line shapes at various energies, with sources of the same diameter, in the high transmission mode of operation of the spectrometer. The sources were all very thin with reference to the energies of the conversion lines of interest (ca $12 \mu\text{g}/\text{cm}^2$ - Re^{186} ; $57 \mu\text{g}/\text{cm}^2$ - Au^{198} ; $20 \mu\text{g}/\text{cm}^2$ - Cs^{137}). They were all prepared by volatilization of the radioactive material in vacuum onto thin backings ($200 \mu\text{g}/\text{cm}^2$ Al foil). The comparison showed, for example, that the K-conversion line at 63 keV in the decay of Re^{186} has very nearly the same shape as the 328-keV conversion line in the decay of Au^{198} and as the 624-keV line in the decay of Cs^{137} (Fig. 17). In the first two examples, however, some uncertainties in the low intensity wings of the lines is introduced by the necessity of subtracting the underlying beta continua. In order to investigate the extent of any low intensity scattered tail in the low energy side of the line, the K-line at 624 keV following the decay of Cs^{137} was examined in coincidence with K-X rays accompanying the conversion. Since the lifetime of the 661-keV level in the daughter Ba^{137} is 2.5 minutes, much longer than the coincidence resolving time, the discrimination against the beta continuum is very effective. An upper limit for the counting rate in the tail of the line is 0.3% of the line peak.

For the large transmission mode of operation of the spectrometer, the details of the line shapes, particularly near the peaks, will be sensitive to the detailed source distribution, even for volatilized sources of the same diameter.⁴⁹ We attribute the small differences in shape near the peaks (Fig. 17) to such small and smooth variations. In spite of the subtraction difficulty mentioned above, the line wings are seen to agree very well, and the half widths are very reproducible. In view of the aforementioned effects, comparison of the line shape parameters as a function of energy, which is the only point of concern for the spectrum resolution correction, cannot be made unequivocally using different sources. However, since the differences in the line shapes are quite small (those shown in

⁴⁹ The high geometry conditions (yields >30%) obtaining in our volatilization apparatus sometimes produced visible structure in the source patterns (interference fringes). The variations from quoted average source thicknesses were indeed small and the maximum thickness was negligible with respect to source scattering effects.

Fig. 17. Line shapes for 63-(Re^{186}), 328-(Au^{198}), and 624-(Ba^{137}) kev electrons in the 5% transmission mode of operation of the spectrometer. All sources were $1/4$ in. in diameter, volatilized onto $200\text{-}\mu\text{g}/\text{cm}^2$ Al; source thicknesses were 12, 57, and $20\text{ }\mu\text{g}/\text{cm}^2$, respectively. This comparison is taken as evidence that the line shape parameters used in the resolution correction can be considered independent of the electron energy for the samples used in the Re and Au experiments. The 624-kev line (K-line of the 2.5-minute 661-kev transition in Ba^{137}) was obtained free of L-line and the continuum of the parent Cs^{137} by counting in coincidence with K-X rays. Representative statistical errors near the peak are indicated. Note that the resolution correction parameters are derived from a "counting rate" plot rather than a "counting rate per unit momentum" plot.

Fig. 17, plus a number of others run with different Re^{186} and Au^{198} sources) and exhibit no trends with energy (down to 63 kev) such as might be expected, e.g., scattering tails at low energy, we assume that the parameters are independent of energy for such sources. We therefore apply the parameters as constants, but use those derived from a conversion line (if one exists) in the actual source to correct the spectrum of that source. Table I lists the constants (defined in Appendix I) for the curves of Fig. 17.

TABLE I
Parameters for Spectrometer Resolution Correction
Derived from Indicated Line Shapes Plotted in
Fig. 17 (See Appendix I)

	Line energy (kev)	α_1/p	α_2/p^2
Re^{186}	63	0.00171	-0.000344
Au^{198}	328	0.00165	-0.000325
Cs^{137}	624	0.00128	-0.000356

In order to assess the quality of the resolution correction, it was applied to the spectra of Au^{198} and of P^{32} . All previous investigations have indicated that both these spectra have the statistical shape above 500 kev. Figure 18 displays Kurie plots for Au^{198} , with and without the resolution correction, for the 5% transmission mode of operation of the spectrometer. This shows how effectively the resolution correction handles the large curvature near the end point of the uncorrected spectrum. The resolution correction, expressed as $N(p)/M(p)$, varied from 1.001 at 150 kev to 0.51 at 930 kev; the latter point is within 2% of the straight Kurie plot. Figures 8 and 19 show $\underline{m}$ function plots for Au^{198} with the proportional counter and scintillation counter detectors, respectively, which exhibit in a more sensitive manner the corrected distributions. These figures show that the spectrum shape factor, as corrected, is constant ($\pm 2\%$) from 400 to 900 kev. For P^{32} , which has no conversion lines, the resolution correction was made using the constants from the Re^{186} 63-kev K-line; the $\underline{m}$ plot of Fig. 7 shows the effectiveness of the resolution correction. Our E_0 value used in computing the $\underline{m}$ function, 1712 kev, agrees exactly with the recent Iowa State measurement.¹⁴

To assess further the errors involved in the resolution correction, the spectrum of Re^{186} inner group in coincidence was calculated using the

Fig. 18. Effect of resolution correction on the Kurie plot of the Au^{198} spectrum with scintillation detector; spectrometer resolution = 6.7%. Curve (a), no correction for finite spectrometer resolution. Curve (b), resolution correction applied, via method of Appendix I, using parameters of Table I.

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constants from its own line and from the Au^{198} and Cs^{137} lines (Table I). The comparison, in the least favorable case, Re^{186} constants vs Cs^{137} constants, showed the variation in the ratio of the corrected spectra to be $< 0.2\%$ from 150 kev to 600 kev, $< 0.5\%$ up to 785 kev and to attain a maximum of 1.5% at 900 kev ($E_0 = 934$ kev). When the Au^{198} constants were applied to the Re^{186} spectrum the differences were smaller.

While these differences (Cs vs Re constants) are invisible in the Kurie representation, they are significant in the m plot (spectrum correction factor) near the end point where the correction is large. The most conservative view, which we adopt, is that these differences represent the uncertainty in the resolution correction. In view of the remarks about the effects of source distribution on line shape parameters, this estimate of uncertainty may be excessive for a spectrum corrected with an internal line shape.

Correction for $\beta - \gamma$ Angular Correlation

Because of the beta-energy dependence of a beta-gamma angular correlation (if one exists) examination of the beta spectrum in coincidence with a gamma at a particular angle (or in a range of angles) will result in a spectrum shape which is a function of the angles employed. To obtain the shape associated with gammas at all angles (i.e., the "singles" shape, if the spectral group could be seen in singles counting not in combination with interfering beta transitions), one must apply a further correction to the observed momentum spectrum. The mathematical details of the correction and evaluation for our spectrometer are given in Appendix II. For the Re^{186} spectrum in our spectrometer the correction amounted to ca -2% at 150 kev and increased to -6% at 900 kev, as the beta-gamma anisotropy increases with increasing beta energy.^{25a}

Inner Bremsstrahlung

Consideration was also given to an effect which pertains to beta spectroscopy in general and to coincidence beta spectroscopy in particular. The continuous low intensity photon spectrum, which accompanies all beta decay processes (inner bremsstrahlung) is usually ignored in computing theoretical beta spectral distributions. Theoretical evaluations⁵⁰ of the relative number of inner bremsstrahlung (I.B.) photons per beta decay, well substantiated by experiment,⁵¹ give ≈ 0.002 for the fraction of beta decays associated with quanta of energy $\geq W_0/10$; this fraction decreases rapidly with increasing I.B. energy. On the assumption that no radical

⁵⁰C. S. Wang Chang and D. L. Falkoff, Phys. Rev. 76, 365 (1949).

⁵¹T. B. Novey, Phys. Rev. 89, 672 (1953); Bolgiano, Madansky, and Rasetti, Phys. Rev. 89, 679 (1953); and K. Liden and N. Starfelt, Phys. Rev. 97 419 (1955).

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change in the shape of the beta energy distribution associated with I.B. occurs, beyond the necessary lowering of the maximum beta energy to $E_0 - E_{I.B.}$, it is evident that detection of the distortion of a spectrum distribution seen in singles counting is considerably below present spectroscopic standards. If, however, a beta group seen in coincidence with a gamma is associated with a more abundant beta group, the β - I.B. coincidences from the latter will augment this distortion in proportion to the relative branching ratios for the two beta groups; e.g., in Re^{186} the approximately threefold more abundant ground transition will, in coincidence with I.B. of energy ≥ 137 kev, give rise to ≈ 0.008 relative contribution to the coincidence spectrum over all angles. However, a strong β - I.B. angular correlation is known,⁵¹ such that no I.B. photons emerge at angle π to the betas, and in a lens coincidence spectrometer arrangement like ours (see Appendix II) a severe discrimination (approximately fivefold) against β - I.B. coincidences exists. This reduces the relative β - I.B. contribution to the coincidence spectrum to a negligible ca 0.1%. In the case of Au^{198} , the contribution is ca 0.01%.

Miscellaneous

Kurie plots were calculated using the Bureau of Standards tables⁵² for the Fermi function and for the screening correction. Energy values were obtained from the tables of Wolfson and Gellman.⁵³

⁵¹T. B. Novey, Phys. Rev. 89, 672 (1953); Bolgiano, Madansky, and Rasetti, Phys. Rev. 89, 679 (1953); and K. Liden and N. Starfelt, Phys. Rev. 97, 419 (1955).

⁵²National Bureau of Standards Applied Mathematics Series 13 (1952). For the screening correction we chose the V_0 constant from the "K binding energy data" curve of Fig. 13 therein.

⁵³J. L. Wolfson and H. S. Gellman, Chalk River Report PD-255, "Five Figure Tables for the Conversion of Electron Momentum to Electron Kinetic Energy."

IV. Au¹⁹⁸

An over-all test of the equipment was obtained from the coincidence spectrum of Au¹⁹⁸, under conditions as nearly as possible like those used in the Re¹⁸⁶ experiment. For Au¹⁹⁸, $E_0 = 960$ kev; for Re¹⁸⁶, $E_0 = 934$ kev for the main inner group.

The abundant 960-kev beta group (see Fig. 19 for the decay scheme), which exhibits the statistical shape (see Table II for references), carries 99.0%⁵⁴ of the disintegrations to the well-studied 412-kev level in Hg¹⁹⁸. The beta-412-kev gamma coincidence data, when corrected for the coincidence efficiency (section II), should exhibit a constant number of coincidences per beta recorded except for the (practically) negligible contribution to the singles count from the 0.02% ground state transition and for the enhancement of the low intensity 282-kev beta group in the coincidence spectrum. This latter is due to the added possibility of its detection⁵⁵ in coincidence with the 677-kev and/or 1089-kev gammas; we computed the enhancement to be ~40-50%, leading to at most a 1% increase in the coincidence count per single beta recorded at 150 kev.

The coincidence/singles ratio would also vary if other beta emitters not recorded in coincidence were present. The production of appreciable amounts of Au¹⁹⁹ ($E_0 = 460$ kev, 5%; $E_0 = 297$ kev, 73%)⁵⁶ by neutron capture

⁵⁴Elliott, Preston, and Wolfson, Can. J. Phys. 32, 153 (1954)

⁵⁵From reference 54 we obtain the relative intensities of the 677- and 1089-kev gammas as 0.82% and 0.16%, respectively. Since the gamma analyzer is operated integrally, accepting the 412-kev photopeak and all pulse heights above it, the 282-kev beta group is recorded in coincidence not only with the 412-kev gamma photopeak, but also with portions of the Compton distributions and with the photopeaks of the higher energy gammas. For our size of crystal, we computed an efficiency relative to that for the 412-kev photopeak of ca 2/3 for the 677-kev gamma and of ca 2/5 for the 1089-kev gamma. We use the data of McLaughlin and O'Kelley, MTA-40, California Research and Development Company Report on "Counting Efficiencies for Sodium Iodide Crystal." Thus the enhancement factor, relative to the coincidence efficiency for the 960-kev beta group is

$$[(1 + 2/3)(0.0082) + 2/5 (0.0016)]/0.0098 \approx 1.4,$$

a 40% increase. Using the data of M. Kalkstein and J. Hollander, UCRL-2764, we calculated an enhancement factor of 1.55.

⁵⁶P. M. Sherk and R. D. Hill, Phys. Rev. 83, 1097 (1951); de Shalit, Huber, and Schneider, Helv. Phys. Acta 25, 279 (1952).

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Fig. 19. Spectrum of Au^{198} . 1) Kurie plot of Au^{198} in coincidence with the 412-keV gamma. 2) Kurie plot of Au^{198} in "singles." 3) Ratio of coincidences to singles $R/(B-bg)$ vs beta energy. 4) m function plot of Au^{198} in coincidence. 5) Decay scheme of Au^{198} .

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in Au^{198} is well known.⁵⁷ The fraction of Au^{199} in our samples was reduced by short, low flux irradiation. Assuming a cross section for Au^{198} of 35,000 barns,⁵⁷ we calculated that our samples had ca 0.05% of the total disintegrations as Au^{199} . The change in the ratio, coincidence/singles, due to the presence of Au^{199} (whose gamma rays are below the G threshold) is -0.1% at 150 keV.

The 6-mm diameter sources were prepared by vacuum volatilization of the neutron-irradiated Au foil onto 200- $\mu\text{g}/\text{cm}^2$ Al backings. The source thickness, ca 57 $\mu\text{g}/\text{cm}^2$, was obtained from the known weight of gold placed on the filament and the measured fraction of the activity collected; this was checked by an activation comparison with a weighed gold standard.

In order that the gamma pulse amplitude, the fast gamma amplifier gain, and the gamma photomultiplier voltage would be the same as for the Re^{186} experiment, the pulse amplitude adjustment was made by artificially reducing the light collection efficiency of the gamma counter by inserting a perforated metal screen of ca 20% light transmission between the light pipe and photomultiplier while holding the other gain variables constant. Reduction of the light collection efficiency rather than reduction of photomultiplier voltage was chosen in order to match more closely the statistics of gamma pulse rise to that of the 137-keV gamma of Re^{186} . The fluctuations in numbers of photocathode electrons influence the effective average pulse width and hence the coincidence resolving time. This procedure made the 412-keV photopeak pulse amplitude ca 60% of the 137-keV photopeak of Re^{186} instead of threefold higher. We believe this would emphasize any tendency towards lowering the coincidence efficiency and thus test the overall performance more severely. The differential coincidence efficiency curve (Section II) applicable to the gold experiment was determined with this lower gamma pulse height and was essentially the same as for the Re^{186} experiment.

The data are the combination of three successive spectrum "runs," the total covering a period of 24 hours. Maximum beta (B) counting rates were 65,000 per minute and the initial G rate was 135,000 per minute. Real/chance coincidence rates exceed 10/1 for every point. No β - γ angular correlation correction was necessary.⁵⁸ The data were treated as in Section III.

Figure 19 shows the Kurie plots of the Au^{198} spectrum in coincidence and in singles and the m function plot of the coincidence spectrum. The inset gives the ratio coincidence/singles [$R/(B - bg)$] vs beta energy.

⁵⁷R. D. Hill and J. W. Mihelich, Phys. Rev. 79, 275 (1950); AECU-2040 Supp. 2 (June, 1953).

⁵⁸S. L. Ridgeway, Phys. Rev. 78, 821 (1950); R. Stump and S. Frankel, Phys. Rev. 79, 243 (1950) report no β - γ correlation for Au^{198} .

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This ratio is seen to be constant from 150 keV to 900 keV within the statistical fluctuations of 1.5% (ca 2.5% above 800 keV). The Kurie and m plots for the coincidence spectrum indicate an allowed shape to within 2% for $E_0 = 960 \pm 2$ keV over the range 300 keV to 900 keV. Note in the coincidence Kurie plot the complete suppression of the conversion lines because of the discrimination in G against the X rays.

An upturn in the singles spectrum appears below ca 400 keV, amounting to ca 5.5% (in the counting rate) at 180 keV, roughly $E_0/5$. The singles spectrum taken with proportional counter detector (Fig. 8) shows the same deviations. The coincidence Kurie plot also deviates at 180 keV by the same amount, but appears not to deviate above 300 keV. This difference (if real) is submerged in the statistical fluctuations of R in $R/(B-b_g)$, and the comparison is unavailable in the range 300-400 keV due to the presence of the conversion lines in the singles spectrum; the deviations may well be identical. Such upturns may be ascribed to five effects: 1) the beta particles of Au^{199} (or other activities); 2) the 282-keV group in Au^{198} ; 3) source thickness; 4) curvature of the main group in Au^{198} ; or 5) an effect introduced by the apparatus.

Since Au^{199} is not counted in coincidence, it can only contribute to the singles upturn and there only to the extent of 0.05%. We have searched for other activities in this neutron-irradiated gold and found none. The relative intensity of an impurity of half-life >10 days could not have exceeded 1%. The 282-keV group in Au^{198} accounts for about 40%⁵⁹ of the upturn in the coincidence Kurie plot at 180 keV on the basis of the branching ratio value of 0.98% of Elliott *et al.*⁵⁴ and the enhancement factor of 1.4.⁵⁵ P. F. Cavanaugh,⁶⁰ also on the basis of gamma counting, has reported a 1.8% branching ratio for the 282-keV group; this would give an apparent⁵⁵ deviation on the Kurie plot at 180 keV of 70% of our Kurie deviation. Our source thickness was $57 \mu\text{g}/\text{cm}^2$, uniformly deposited; this would not be expected to give an upward deviation due to source scattering of as much as 2% at 180 keV on the Kurie plot,⁶¹ or $<40\%$ of the upturn.

⁵⁹In the preliminary report of this work, Ref. 25, it was stated only that all the upturn could be ascribed to the 282-keV group in Au^{198} , on the basis of Cavanaugh's⁶⁰ value for the branching and an overestimated enhancement factor of 1.75.

⁶⁰P. F. Cavanaugh, Phys. Rev. 82, 791 (1951).

⁶¹Compare for example the results of Langer, Motz and Price, Phys. Rev. 77, 798 (1950). A $50\text{-}\mu\text{g}/\text{cm}^2$ source of Pm^{147} ($E_0 = 225$ keV) formed by deposition from solution exhibited a scattering deviation on the Kurie plot of ca 2% at $1/5 E_0$. Assuming that our $57\text{-}\mu\text{g}/\text{cm}^2$ volatilized sources are at least as good as these $50\text{-}\mu\text{g}/\text{cm}^2$ sources and that the comparison is valid at the same fraction of E_0 , the 2% deviation would occur at 180 keV for Au^{198} . Actually, a smaller

Thus, ruling out Au^{199} and other impurities we see that the deviation from the statistical shape exhibited by the coincidence and singles data for Au^{198} might be accounted for by the 282-kev group and by source thickness, taking the maximum abundance value of $1.8\%^{60}$ and the upper limit for source scattering. However, we regard this abundance of the 282-kev group as probably too high and this amount of source scattering is improbable, and believe that the deviation may not be completely accounted for thus. The possibility that the residual upturn below 400 kev (at least 40% is due to the 282-kev group and perhaps ca 20% due to source scattering) originates in an instrumental effect cannot be eliminated by the available information. On the other hand, the possibility that real curvature (of the order of 2 or 3% variation in the spectrum correction factor) may exist in the 960-kev group in Au^{198} ($\Delta I = 1$, yes) cannot in our opinion be ruled out by these or any previous experiments (Table II). The answer to this problem, which is of fundamental interest per se, must await experiments with much thinner samples.

In summary, the fundamental result of the Au^{198} experiments, relative to the Re^{186} experiments, is seen in the constant ratio, real coincidences/singles, as a function of beta energy above 150 kev (inset of Fig. 19) and in the closely statistical shape obtained for Au^{198} down to at least 400 kev. The data from the P^{32} experiment gives independent evidence, for thinner sources, relating to the lack of scattering in the spectrometer and the straightness of the Kurie plots obtained therein down to at least 400 kev.

scattering component is anticipated for the higher energy spectrum. Further, C. S. Wu and R. P. Albert, Phys. Rev. 75, 1107 (1949), found that $100 \mu\text{g}/\text{cm}^2 \text{Cu}^{64}$, chemically deposited, deviated 1.5% upward in counting rates at ca 115 kev, $E_0/5$, whereas Langer, Moffat and Price, Phys. Rev. 76, 1725 (1949), with $75 \mu\text{g}/\text{cm}^2 \text{Cu}^{64}$, volatilized, observed no deviation at $E_0/5$.

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TABLE II

Summary of Au¹⁹⁸ Beta Spectroscopy

All work on Au¹⁹⁸, with reasonably thin sources, reported in the literature is listed with relevant data here. Inspection of the column "Kurie Plot" with reference to the source thickness and mode of deposition used indicates the uncertain character of the shape of the 960-kev transition below 400 kev. We have listed selected values for E_0 (Column 6) where resolutions, line shapes, and calibrations could be ascertained. Note that the 960-kev average is lower than the unselected average of 963 kev of Hollander, Perlman, and Seaborg.⁶²

Reference	Source thickness (mg/cm ²) and mode of deposition.	Spectrometer and estimated resolution	Kurie plot straight (?)	E_0 reported	E_0 corrected by us where resolution correction and calibration known.
63	0.4 liquid drop	180° 1% (?)	> 200 kev	960 ± 5	960 ± 5
64	0.26 foil	180° 1.5%	> 600 ^a	966 ± 10	
65	≈ 0.1 liquid drop	180° 0.6%	> 120 kev ^b	956 ± 5	
66	0.6	lens 4%	> 250	975 ± 15	
67	? liquid drop	double lens 2%	> 400 ^c	970 ± 10	
68	?	thick lens 5.5%	> 400 ^c	967 ± 5	959 ± 2.5
54	0.2 foil	double lens 0.5%	> 400	959 ± 2.5	
69	? liquid drop	180° 1.6%	> 200 ^c	952	
70	0.020 volatilized	lens 3%	> 400	960	
71	0.057 volatilized	double lens 6.7%	> 400		
					average $E_0 = 960 ± 1$

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Table II. References

- ⁶²Hollander, Perlman, and Seaborg, Rev. Modern Phys. 25, 469 (1953)
- ⁶³D. Saxon, Phys. Rev. 73, 811 (1948)
- ⁶⁴P. W. Levy and E. Greuling, Phys. Rev. 75, 819 (1949)
- ⁶⁵Langer, Motz, and Price, Phys. Rev. 76, 641 (1949)
- ⁶⁶Steffen, Huber, and Humbel, Helv. Phys. Acta 22, 167 (1949).
- ⁶⁷C. Y. Fan, Phys. Rev. 87, 252 (1952).
- ⁶⁸Pohm, Lewis, Talboy, Jr., and Jensen, Phys. Rev. 95, 1523 (1954).
- ⁶⁹F. T. Porter, private communication (Washington University).
- ⁷⁰Broyles, Thomas, and Haynes, Phys. Rev. 89, 715 (1953).
- ⁷¹Present work
- ^aAbundant group of $E_0 = 600$ kev reported
- ^bNo 282 group evident.
- ^cAu¹⁹⁹ present.

Inner Group Beta Transition

The decay scheme and beta spectral shapes of Re¹⁸⁶ have been the subject of considerable experimental investigation. Some of the early beta spectroscopy^{65,72} indicated a single beta group of the allowed shape, and other early work⁷⁵ pointed to a complex beta spectrum. The basic features of the decay scheme (Fig. 1) were proposed by Metzger and Hill²⁰ and by Steffen,²⁰ who simultaneously discovered that the 122-keV gamma is associated with orbital capture decay to W¹⁸⁶. Both Metzger and Hill, and Steffen, analyzed the beta spectrum into two groups with allowed shapes having end points differing by 137 keV, the energy of the gamma ray in Os¹⁸⁶. In addition, Steffen²⁰ obtained the beta spectrum in coincidence with the 137-keV gamma ray, but no definite conclusions about the shape of the spectrum could be drawn from the coincidence data. As described in the Introduction, the work of Guss, Killion, and Porter²³ indicated that at least one of the shapes is not allowed; subtraction of an allowed shape ground transition, on the basis of careful study in the small energy range beyond the inner group ($E_0 = 934$ keV), seemed to yield an "alpha" shape ($\Delta I = 2$, yes) for the latter, and thus favored a Re¹⁸⁶ spin assignment of 0(-). The conflict of the results of Hurley and Jastram²⁴ on the β - γ directional correlation (see Introduction) prompted this investigation.

Re¹⁸⁶ is conveniently produced in high specific activity [$\sigma(\text{Re}^{185}) = 100$ barns; isotopic abundance = 37%] by slow neutron irradiation of natural rhenium. We irradiated KReO₄ as the dry salt for five days at a flux of 10^{13} neutrons/cm²sec in the Argonne heavy water reactor, CP-5. Spectrochemical analyses on the salt set an upper limit of 1 in 10^4 for any impurity. After "cooling" for ten days, the KReO₄ was taken up in water. Assay by beta counting gave on the average over a number of such irradiations 10μ curies/ μ g after the ten-day cooling period; ten days of decay reduces the Re¹⁸⁸ [half-life 16.9 hrs, $E_0 = 2.1$ MeV; $\sigma(\text{Re}^{187}) = 63$ barns, isotopic abundance = 63%] to <0.1% of the Re¹⁸⁶ activity. No beta spectroscopy on Re¹⁸⁶ was done until the counting rate beyond the end point of the Re¹⁸⁶ spectrum was indistinguishable from background. This also guaranteed the absence of K⁴² activity [half-life 12.4 hrs, $E_0 = 3.58$ MeV; $\sigma(\text{K}^{41}) = 1.2$ barns, isotopic abundance = 7%]. Consideration of the products of other reactions such as (n,p), (n, α), and (n,2n) indicated that only 40-day Re¹⁸⁴, 73-day W¹⁸⁵, and 117-day Ta¹⁸² could possibly be of concern during the beta spectroscopy (10-15 days after irradiation). The activity of a 10-millicurie sample of irradiated salt was followed for 300 days, and no trace of long lived contamination was discernible. The decay of the sample obtained during the spectroscopy, and from separate samples followed for seven half-lives, showed only an 89-hr period.

⁷²Beach, Peacock, and Wilkinson, Phys. Rev. 76, 1585 (1949).

⁷³P. J. Grant and R. Richmond, Proc. Roy. Soc. (London) 62A, 573 (1949).

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Sources for the coincidence spectroscopy were made by placing 10 μg of KReO_4 in 3 λ of water in a "dimple" impressed in a platinum ribbon from which the volatilization took place. The source backing (200- $\mu\text{g}/\text{cm}^2$ Al) and mask (Pt or Al disc with appropriate size hole punched out) were put into position over the Pt filament inside a vacuum chamber. The source backing-to-mask spacing was 1/2 mm and the mask-to-filament spacing was 2 mm. During the volatilization the filament temperature was raised to ca 600°C for 5 sec. We believe the volatilized form was Re_2O_7 . For a 6-mm diameter source this arrangement yields 50% (measured) of the activity on the source backing. Average source thicknesses of about 12 $\mu\text{g}/\text{cm}^2$ were calculated for the Re^{186} sources. The deposits were barely visible with strong illumination.

Figure 20 shows the pulse height distribution from the gamma detector as it was assembled in the coincidence source holder in the spectrometer with a 0.021 in. cadmium disc absorber in addition to the normal 2-mm glass vacuum seal between the source and the NaI crystal container. With this Cd absorber the over-all efficiency for the 137-keV gamma was ca 1.8%. The crosses (X) show the spectrum with no Cd absorber (normalized at the 137-keV photopeak). The arrow (G) shows the discrimination level, chosen to reject the X-ray pulses. These X rays are both Os (conversion) and W (conversion and electron capture) K-radiation. While the coincidence rate could be increased ca 35% by including the Os K-X rays, the applicable β -photon directional correlation would then be a combination of the β - γ and the isotropic β -X ray correlation. This introduces the problem that the β - γ correlation must then be measured with just the same admixture of 137-keV gamma and X rays if the spectrum is to be corrected properly. We avoided this by accepting only the 137-keV gamma. This choice also, as pointed out above, renders the G count insensitive to small drifts in the G discrimination level. The cadmium absorber was introduced so that the double coincidences due to X- β events (type I-IV coincidences, Fig. 15) were reduced with respect to γ - β events and hence "slow" chance events were minimized (see Section III).

During the 70-hr experiments from which the data were obtained the following items were monitored every four hours: the photomultiplier voltages; the spectrometer current control potentiometer; the fast resolving time; differential coincidence efficiency; monoenergetic beta pulse height distribution; B background; and 137-keV photopeak amplitude. Those drifts in the γ photopeak which never exceeded 3% were compensated by adjustment of the G discrimination level; no other drifts were observed. For details refer to Section II. The 30 momentum values were run as a complete spectrum eight times during an experiment. Each of the eight runs was programmed so that every other point on the spectrum was taken with increasing spectrometer magnetic field and the remaining points with decreasing field. This procedure was adapted so that the final data would reflect no short or long term instrumental drifts or

Fig. 20 Low energy gamma pulse height distribution from Re¹⁸⁶ with the 1-inch diameter by 24-inch long light guide. The curves with and without 460 mg/cm² Cd absorber have been normalized at the 137-kev γ peak. The arrow labeled G indicates the discrimination level used in the coincidence beta spectral measurements. The Cd absorber was used to reduce slow chance rate (see text) as well as to reduce the contribution of X rays to the counting rate in the "trough."

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hysteresis effects of any kind. The maximum $\underline{B}$ rate was 78,000 counts per minute and the initial $\underline{G}$ rate 60,000 counts per minute. Between 4000 and 6000 coincidences were accumulated at each point below 840 kev, decreasing to 750 coincidences at 895 kev, the highest energy recorded. Real to chance ratios exceeded 10 below 760 kev, and dropped to 2/1 at 895 kev. Values of R/R' ranged between 1.010 and 0.994 (see Section III). As a check on the statistical reliability of the data the eight values of $R/(G - bg)$ at each momentum value were examined for trends and abnormal fluctuations. Applying standard criteria we found that the distribution was perfectly normal (Gaussian) and exhibited no trends; only 2% of the values deviated from the average by more than twice the standard error and none by more than 2.5 times.

The beta spectrum in coincidence with the 137-kev gamma is given in Table III, columns 5 and 6. The data have been treated as noted in Section III with the spectrometer resolution correction applied and with correction for β - γ directional correlation made (see Appendix II). The β - γ correlation as a function of beta energy is shown in Fig. 5e of Ref. 25a.^{73a} Actual experimental values (not a smooth curve) are tabulated. The conventional uncorrected Kurie plot is shown in Fig. 21(a). The plot does not have the allowed shape, nor does it exhibit the unique " α " shape (associated with $\Delta I = 2$, yes).

The character and quantitative measure of the deviation from the allowed shape is best discussed in terms of the plot (Fig. 22) of the experimental values $N/[f(E_0 - E)^2] = \underline{m}$, the experimental (shape) correction factor. The maximum beta energy E_0 as read from the "allowed" Kurie plot is 933.5 ± 3.0 kev. The experimental correction factor $\underline{m}$ is sensitive to the value of E_0 for $\underline{E}$ near E_0 . In another experiment, to be described below, we have narrowed the limits of experimental error, obtaining $E_0 = 934.3 \pm 1.3$ kev. The experimental points have been computed with this value for E_0 and are given in column 7, Table III. The solid curve of Fig. 22 gives the estimated best fit to these points.

^{73a}Reference to Fig. 5e, Ref. 25a, will disclose that the β - γ angular correlation curve used to correct the spectrometer coincidence data differs from the final best evaluation of this function. The curve used was based on earlier measurements. The lengthy computations of the spectrum shape and the theoretical fittings thereto were not repeated when the better data on the β - γ correlation became available. The difference in the $\underline{m}$ function of the inner group of Re^{186} , as corrected with the final β - γ anisotropy, would be very small, about 0.5% reduction in the maximum span of $\underline{m}$ (Fig. 22). This recorrection would actually slightly improve the fit of the experimental $\underline{m}$ function with the theoretical curves (Fig. 28). The effect on the ground state $\underline{m}$ function is of still lesser importance, because of the relative branching ratios.

Fig. 21 Kurie plots of Re^{186} . Data in Table III.

Curve (a), main inner group in coincidence with the 137-kev gamma.

Curve (b), ground state transition by subtraction of (a) from (c) assuming ground state transition abundance of 76% of total betas.

Curve (c), total singles spectrum. Solid circle points obtained from another experiment (E_0 measurement) and normalized.

Fig. 22. Experimental spectrum correction factor ($\underline{m}$) for Re^{186} inner group in coincidence. Solid curve represents best fit to data. Data in Table III. Curves labeled 933.0 kev and 935.6 kev give estimated upper and lower limits between which the true $\underline{m}$ function is judged to lie. The curve labeled 937 kev indicates that the $\underline{m}$ function cannot be made constant even for a higher E_0 value.

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Table III

BETA SPECTRUM OF Re^{186}

1	2	Total spectrum		Main inner group (coinc.) $E_0 = 934.3 \text{ kev}$			Ground state transition $E_0 = 1071.5 \text{ kev}$ 76% of total betas		
		3	4	5	6	7	8	9	10
E (kev)	p (mc)	$N_T(p)$	$\sqrt{\frac{N_T}{f}}$	$N_C(p)$	$\sqrt{\frac{N_C}{f}}$	$\frac{N_C}{f(E_0-E)^2}$ ($\times 10^5$)	$N_G(p)$	$\sqrt{\frac{N_G}{f}}$	$\frac{N_G}{f(E_0-E)^2}$ ($\times 10^5$)
148.6	0.8159	- ^a	-	915	15.29	37.87	- ^a	- ^a	-
167.9	0.8742	3461	28.47	962	15.01	38.35	2499	24.19	71.68
187.6	0.9325	3621	27.94	957	14.36	36.99	2664	23.97	73.53
208.4	0.9908	3744	27.30	1006	14.15	38.01	2738	23.34	73.15
229.6	1.0491	3848	26.65	1012	13.67	37.62	2836	22.88	73.86
251.3	1.1074	3939	26.02	1000	13.11	36.84	2939	22.48	75.12
273.8	1.1656	4007	25.33	1042	12.92	38.25	2965	21.79	74.64
296.6	1.2239	4057	24.66	1066	12.64	39.29	2991	21.17	74.64
319.8	1.2822	4076	23.90	1019	11.95	37.82	3057	20.70	75.83
343.6	1.3405	4085	23.19	1000	11.47	37.73	3085	20.15	76.67
367.6	1.3988	4061	22.44	983	11.04	37.95	3078	19.54	77.03
391.9	1.4570	4019	21.68	990	10.76	39.34	3029	18.82	76.66
416.8	1.5153	3964	20.91	932	10.14	38.40	3032	18.29	78.07
441.7	1.5736	3870	20.09	897	9.68	38.59	2973	17.61	78.21
466.9	1.6319	3772	19.32	870	9.28	39.37	2902	16.94	78.55
492.6	1.6902	3645	18.50	839	8.87	40.37	2806	16.23	78.63
518.2	1.7485	3492	17.66	778	8.34	40.16	2714	15.57	79.25
544.3	1.8067	3334	16.83	698	7.70	39.00	2636	14.97	80.60
570.4	1.8650	3154	16.05	662	7.35	40.86	2492	14.27	81.04
596.6	1.9233	2955	15.12	598	6.80	40.59	2357	13.50	80.80
623.3	1.9816	2750	14.30	541	6.34	41.61	2209	12.82	81.83
649.9	2.0399	2529	13.37	471	5.77	41.20	2058	12.06	81.88
676.6	2.0981	2306	12.49	413	5.29	42.09	1893	11.32	82.17
703.8	2.1564	2077	11.60	347	4.74	42.31	1730	10.59	82.91
730.8	2.2147	1839	10.70	286	4.22	42.98	1553	9.83	83.20
757.8	2.2730	1603	9.79	224	3.66	43.02	1379	9.08	83.73
785.3	2.3313	1371	8.88	176.7	3.19	45.79	1194	8.29	83.87
812.7	2.3896	1151	7.97	114.3	2.51	42.70	1037	7.57	85.55
840.1	2.4478	933	7.04	69.9	1.93	41.9	863	6.77	85.71
867.9	2.5061	732	6.12	37.3	1.38	43.4	695	5.97	85.9
895.5	2.5644	550	5.22	15.5	0.88	51.0	534	5.14	85.2
915.9	2.6039	444	4.62				444	4.62	87.2
934.5	2.6429	347	4.04				347	4.04	85.7
945.8	2.6664	303	3.75				303	3.75	87.8
957.0	2.6899	254	3.41				254	3.41	87.6
968.3	2.7134	211	3.09				211	3.09	88.4
979.6	2.7368	165	2.71				165	2.71	86.0
990.8	2.7601	133	2.42				133	2.42	88.4
1001.9	2.7833	97.8	2.06				97.8	2.06	86.5
1013.1	2.8065	71.8	1.75				71.8	1.75	88.9
1024.2	2.8296	46.2	1.40				46.2	1.40	86.1
1035.3	2.8526	27.6	1.07				27.6	1.07	86.6
1046.4	2.8757	13.5	0.75				13.5	0.75	87.2
1057.7	2.8988	5.2	0.46				5.2	0.46	110.3

^a"M" line of 137.2-kev gamma present.

Columns 3 and 4 give the total (singles) momentum distribution and Kurie representation.

Columns 5, 6, and 7 give the momentum distribution, Kurie representation and $\underline{m}$ function for the main inner group in coincidence with the 137-kev gamma.

Columns 8, 9, and 10 give the analogous data for ground state transition computed using a relative abundance of 76% of total betas.

The data are normalized such that $N_T = N_C + N_G$. Actual experimental data (not smooth curve values) are given. The Fermi function $\underline{f}$ is corrected for orbital electron screening.

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The error flags in Fig. 22 show the statistical counting error ($T^{1/2}/R$) only. The two dashed curves labeled $E_0 = 933.0$ kev and $E_0 = 935.6$ kev give our estimates of the safe limits of error. The $\underline{m}$ function is, we believe, established within the accuracy represented by the error band between these two curves.⁷⁴ The total error was computed taking account of the errors in: 1) the E_0 value, 934.3 ± 1.3 kev; 2) the resolution correction; 3) β - γ directional correlation corrections; 4) the ca 5% error in 2τ ; and 5) reasonable statistical limits. The individual contributions (% of $\underline{m}$) to the total error in $\underline{m}$ are shown in Fig. 23, together with the total error, as a function of energy. For the resolution correction we have conservatively assigned one-third of the total correction as error arising from erroneous determinations of the derivatives of the observed spectrum and to evaluations of the line shape parameters (see Appendix I). The error found in this correction applied to the Au^{198} and P^{32} spectra is nowhere more than one-fifth of the correction. The β - γ directional correlation errors are discussed in Ref. 25a. The contribution of the ca 5% error in 2τ to the error in $\underline{m}$ depends on the observed fraction of total coincidences registered which are accidentals. The statistical error at each point is the standard (68% confidence) error associated with the observed total coincidences. The assigned statistical error in Fig. 23 was obtained as the difference between the "best fit" (solid) curve of Fig. 22 and similar smooth curves drawn at reasonable upper (and lower) statistical limits; it was clearly less than the standard error at each point because account was taken of the influence of neighboring points in locating the curves. The total error curve was computed as the error in the $\underline{m}$ function at each point, combining the individual errors of the several terms and factors by the usual methods for propagation of errors. It was assumed that errors in interpolation of the Fermi function from the Bureau of Standards tables⁵² are negligible. The major error in the screening correction is associated with the choice of the potential value V_0 ;⁵² we estimated this contributed at most 0.2% uncertainty in $\underline{m}$, and so it was omitted from the estimated total error.

The variation in the experimental correction factor $\underline{m}$ (solid curve) was ca 18% from 150-934 kev.

Also shown (Fig. 22) is a dashed curve labeled $E_0 = 937$ kev. We present this to illustrate, in a more graphic manner than is possible on a Kurie plot, the impossibility of eliminating the deviation from the allowed spectral shape (constant $\underline{m}$ plot) by any choice of end point value, i.e., even

⁷⁴This is to be understood in the restricted sense that experimentally acceptable curves all lie within the bounded region, and not in the sense that any conceivable curve within the region is admissible. For example, a curve inside the band which requires variation in E_0 is inadmissible even though its other parameters lie within the estimated limits of error.

Fig. 23. Experimental errors in Re^{186} inner group $\bar{m}$ plots as a function of beta energy, individual and total. Total error curve used in computing error band limiting curves in Fig. 22.

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with this choice ($E_0 = 937$ kev), such that the high energy end of the $\underline{m}$ plot is below the low energy end, the mid range maintains the same upward trend. This curve represents the solid curve transformed only by change of E_0 value.

The experimental points, even taking into account the net uncertainty in the data, seem to demand the indicated curvature (solid curve) in the region from 150 to 300 kev. Consideration of relative abundances show that this trend cannot be due to the 307-kev group in Re^{186} . Such a rise towards low energy is not readily attributable either to source thickness effects in the approximately $12\text{-}\mu\text{g}/\text{cm}^2$ sources used or to contaminant activities detected in coincidence. As will be shown in the $\underline{m}$ plot (Fig. 25) for the ground state transition its experimental shape correction factor is, in contrast, linear in this region. We believe this to be a real difference between the two spectra. However, we note that the evidence presented from the spectra of Au^{198} and P^{32} (see Sections II and IV) does not rule out the possibility that purely instrumental effects, e.g., scattering from the coincidence source holder, as distinguished from source thickness effects, may contribute upturns of 1-2% around 300 kev.

As has been emphasized the results are critically dependent on the relative coincidence efficiency as a function of β energy for the particular combination of detectors and circuits employed. Our measurements showed that inclusion of the correction for the small variation of the relative coincidence efficiency (Fig. 13) actually did not materially affect the shape of the coincidence spectrum for the scintillation counter. In view of the decreasing coincidence efficiency with decreasing β energy (i.e., decreasing pulse amplitudes) shown by the β scintillation counter and of the opposite trend of pulse amplitude with β energy for the proportional counter (see Section II), a comparison of the results with the two detectors might be decisive in establishing the validity of the above results. As noted in Section II, use of the proportional counter presents difficulties associated with the jitter time³⁴ (longer resolving times, poorer coincidence plateaus, and delay curves). Consequently the coincidence efficiency for the proportional counter as a function of β energy was not established beyond the certainty afforded by a relatively poor gain plateau for coincidences. Even though no correction for coincidence efficiency was applied, every experiment using the proportional counter showed curvature of the Kurie plot concave toward the energy axis. The character and magnitude of the curvature was not reproducible; hence the proportional counter was replaced with a scintillation counter. In the theoretical considerations (Section VI) we used the data of Table III, based on the scintillation counter results only.

The criticisms leveled at the coincidence efficiency for the proportional counter are of course not applicable to the "singles" counts. Indeed, the total singles spectrum of Re^{186} on both scintillation and proportional counter are identical within 1%. (See discussion under Singles Spectrum.)

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During the course of the present work Koerts⁷⁵ examined the beta spectrum in coincidence with the 137-keV gamma and found it to have the allowed shape, not in agreement with the present work. The reasons for the disparity in the experimental results are not evident.

Composite Beta Spectrum

The singles or total beta spectrum was recorded at the same time as the coincidence data; it is just $[(B^*/\eta) - bg]$; η is the singles efficiency (Section II, Fig. 6). For the measured total beta momentum distribution, $M(p) = e^{\lambda t} [(B^*/\eta) - bg]/p$, one needs accurate values for the decay constant λ , contrary to the case of the coincidence spectrum where we correct for decay by dividing by $(G - bg)$.⁷⁶ Since the number of B counts recorded during the counting at a given point is the order of 10^5 - 10^6 , and since each of the 30 momentum values is counted eight times over the seventy-hour experiment, we have narrow limits, statistically, in the B data on the value

⁷⁵L. Koerts, Phys. Rev. 95, 1358 (1954). We do not presume to present an explanation of the disparity. We note here some items which may be relevant. The coincidence spectroscopy reported employed a GM beta detector. The dead time losses for the singles rates required to obtain practical coincidence rates are, for a Geiger counter, rather severe and rather difficult to evaluate accurately. Also, for a large counter tube, pulse jitter may be so large as to make very difficult the attainment of high coincidence efficiency. (See our comments on our small 1 centimeter radius proportional counter.³⁴) Koerts shows a spectrum (curve (c), Fig. 2 of the reference) in which the 307-keV group has been emphasized by partial discrimination against the 137-keV gamma. The very low intensity of this group, 0.10% established by the present work [0.08% from the work of Robinson, Whittle, and Jastram, Phys. Rev. 91, 498A (1953)] makes it unlikely that this group could have been seen in coincidence except with very intense sources and by (practically) complete suppression of the 934 keV (24%) group by gamma discrimination. We estimate that to obtain one coincidence per minute on the 307-keV group, a singles rate of approximately 10^5 - 10^6 c/min of betas would be necessary, an impractical rate for GM tubes. The "307-keV group" is also seen in the singles spectrum of Koerts (curve (b), Fig. 2), suggesting the presence of an impurity or of an apparatus effect generating an upturn and thus possibly masking a downward deviation such as we find.

⁷⁶This normalization with respect to G is preferable, for the coincidence spectrum, to correction for decay explicitly, as it corrects also for drifts in the G discrimination level (and thus in the G rate, to which the coincidence rate is proportional). For the singles spectrum, these same small drifts in G (ca 2%) introduce too large an error in the decay correction, and one must use the $e^{\lambda t}$ factor.

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of the half-life which can correct the data. From this data we found the half-life of Re^{186} to be 88.9 ± 0.2 hours.⁷⁷ From a least squares analysis of data from 5 samples followed for 300 hours on an automatic recording flow type proportional beta counter,⁷⁸ the value 88.5 ± 0.2 hours was obtained. We used the former value in the computations. B^* represents $\underline{B}$ corrected for dead time losses. Recall that the background, bg , is ca 500 c/min, <3% of the lowest $\underline{B}$ rate. The data from this experiment extends only up to 895 keV.

The conventional Kurie plot (Fig. 21, curve c) clearly demonstrates a definite continuous curvature concave towards the energy axis. The identical (within 1%) curvature was obtained with the proportional counter. This curvature furnished what is probably the strongest evidence that one or both components are curved concave downwards, in the opposite sense to the curvatures usually ascribed to effects such as scattering and source thickness. The Au^{198} and P^{32} spectra demonstrate the absence of instrumentally introduced curvature above 400 keV, and certainly of any downward curvature even below 400 keV. The points from the experiment described above are represented by circles (Fig. 21, curve c). The statistical error in these points is everywhere small compared to the size of the circle. The resolution correction to the singles data is smaller than that applied to the coincidence data in the neighborhood of 900 keV, since the singles spectrum extends to higher energies. However, the resolution correction is too large in the region of the end point to obtain the requisite accuracy in the end point measurement using the high transmission mode of operation of the spectrometer.

Determination of Transition Energies

For a determination of E_0 for the ground state transition a 3-mm diameter, $30\text{-}\mu\text{g}/\text{cm}^2$ thick volatilized Re source on $200\text{-}\mu\text{g}/\text{cm}^2$ Al foil was prepared. The Re^{188} content was carefully checked and the sample was allowed to decay for 14 days until the calculated $\text{Re}^{188}/\text{Re}^{186}$ was 0.005% and the count 160 keV beyond the Re^{186} end point was indistinguishable from background (9 c/min). The spectrometer was adjusted to give 0.8% transmission and 2.0% resolution and the proportional counter beta detector was used. In order to establish the resolution correction as well as possible, the 63-keV K-line profile was run before and after the experiment. Counts were taken above 730 keV (5000 c/min), overlapping the

⁷⁷The decay of the $\underline{B}$ counts demonstrates considerably smaller fluctuations than that of the $\underline{G}$ count, as expected from the nature of the pulse height distributions (see Fig. 15). The long very low tail on the $\underline{B}$ distribution makes the $\underline{B}$ count very insensitive to relatively large changes in the $\underline{B}$ discrimination level.

⁷⁸The counting and analysis of the data were kindly performed for us by Jerome Lerner of this laboratory on equipment of his own design.

points of the high transmission experiment; the highest energy point counted was 1058 kev, at which point the rate was 21.7 c/min above the 9-c/min background of the proportional counter. Below 980 kev from 10,000-30,000 counts were taken at each point, and, at the 1058-kev point, for example, ca 800 counts. The duration of the experiment was five hours, so the decay correction was small. A careful check of the zero setting, linearity, and calibration of the spectrometer current control immediately preceded the experiment and established maximum errors of 0.01% on the current values.

The transformation to momentum units requires an absolute calibration of the reference line in the Re^{186} source, which was accomplished in a separate experiment with a mixed Re^{186} - Au^{198} source. The standard K-line momentum in Au^{198} was taken as 2222.4 ± 0.4 gauss cm.^{48,79}

The source for the calibration experiment (1.5 mm diameter, ca 150 $\mu\text{g}/\text{cm}^2$ total thickness) was prepared by volatilizing Au^{198} (ca 130 $\mu\text{g}/\text{cm}^2$) onto 200- $\mu\text{g}/\text{cm}^2$ Al foil, and without moving the foil or mask in the volatilization apparatus, volatilizing Re^{186} ($< 20 \mu\text{g}/\text{cm}^2$) over the gold. The observed resolutions were $1.27 \pm 0.02\%$ for the Au^{198} K-line (328 kev) and $1.30 \pm 0.03\%$ for the 63-kev Re^{186} K-line, and no evidence for tailing out or scattering was visible in the line profiles. The fiducial point on each line was taken at the half height median. The $H\rho$ value obtained for the Re^{186} K-line from the comparison was $874.40 \pm 0.41 H\rho$. Using the Cauchois and Hulubei value for the K binding energy of Os, 73.87 kev, as computed by Slack,⁸⁰ we obtained for the energy of the first excited state in Os^{186} , 137.19 ± 0.07 kev. Paul Day of this laboratory has measured the energy of the gamma ray on a Cauchois bent crystal spectrometer,⁸¹ obtaining 137.19 ± 0.12 kev.

Using the value obtained for the $H\rho$ of the line, applying resolution and decay correction and a carefully determined value for the counter background, we calculated the conventional Kurie plot of Fig. 24 for the E_0 experiment. No deviation from linearity can be seen above 935 kev on a very expanded plot. Note that the counts extend to within 1.5% of the end point; for the last point the resolution correction is ca 15%, and at 4% below the end point, about 4%. The data for this experiment, normalized to that of the high transmission singles experiment over the range 730-935 kev, is shown as solid circles on Fig. 21, curves b and c. The

⁷⁹The gamma-ray energy is quoted to somewhat better precision; Muller, Hoyt, Klein, and DuMond, Phys. Rev. 88, 775 (1952). However, the uncertainty in the K binding energy of Hg limits the accuracy of the calculated conversion line to about the same limits as those quoted for the direct measurement of the $H\rho$ of the conversion line.⁴⁸

⁸⁰L. Slack, privately circulated tables.

⁸¹P. Day, Phys. Rev. 97, 689 (1955).

Fig. 24 Maximum beta particle energy determination via Kurie plot of ground state transition in Re¹⁸⁶. Expanded view of data near end point from experiment at 2.0% resolution. Data in Table III.

normalizing factor obtained by comparison of the data from the low and high resolution modes of operation of the spectrometer showed a mean deviation of 0.2% over this range, and no evidence of a trend.

Linear, graphical extrapolation on Fig. 24 indicated $E_0 = 1071.5 \pm 1.0$ kev, while an unweighted least squares fitting of the data above 935 kev to a straight line gave $E_0 = 1072.2 \pm 0.2$ kev. We indicate a final weighted value of $E_0 = 1071.5 \pm 1.3$ kev, including estimated contributions to the error from the uncertainties of the Au^{198} standard and from the calibration procedure. From the gamma energy of 137.19 ± 0.07 kev the energy of the main inner group is computed to be $E_0 = 934.3 \pm 1.3$ kev, in comparison to the observed 933.5 ± 3.0 kev.

Ground State Beta Transition

By subtracting the inner group spectrum from the singles data, with an appropriate normalization to take account of the beta branching ratio, the shape of the ground-to-ground beta distribution can be obtained. Analysis of the experimental possibility of a direct counting determination of the branching ratio disclosed that uncertainties in Auger yields, electron capture branching ratio, conversion coefficients, and counting efficiencies for X rays and gammas combined to prevent attainment of the requisite accuracy of ca 1% (in the ca 20% inner group, i.e., $20 \pm 1\%$). Metzger and Hill²⁰ and Steffen,²⁰ from analysis of the spectrum in terms of allowed shapes for both main components, proposed branching ratios of 19% and of 27% for the 934-kev group, respectively.

In lieu of the direct measurement of the branching ratio, a criterion was sought for obtaining a normalization factor without reference to independent data. Such a criterion is that the ground state m plot (Fig. 25) or Kurie plot (Fig. 21) behave smoothly in the neighborhood of 934 kev (E_0 of inner group). Of values around 25% indicated by earlier work, the value of 24% for the relative abundance of the inner group was judged to affect the smoothest transition between the regions above and below 934 kev. In order to demonstrate that the method is not very sensitive we have shown in Fig. 25 the curve labeled 79% (experimental points not plotted), which demonstrates the effect of a 3% change in the relative abundance of the ground state transition.

The slope of the "76%" curve near the end point is not in conflict, within experimental error, with the observed straightness of the Kurie plot near E_0 .

A more sensitive determination of the branching ratio can be obtained from the beta spectrometer coincidence data along with the known

Fig. 25 Experimental spectrum correction factor ($\underline{m}$) for the Re¹⁸⁶ ground state transition. The solid curve represents the best fit to the data. Curves labeled 1070 kev and 1073 kev give estimated upper and lower limits between which the true $\underline{m}$ function is judged to lie, for a branching ratio of 76% of total betas. The curve labeled "79%" represents the best fit to the data (experimental points not shown) for a branching ratio of 79% of total betas. Data in Table III.

efficiencies for counting betas and gammas. The following equation is employed:

$$\frac{\beta^-(934)}{\beta^-(934) + \beta^-(1071)} = \frac{(G/\epsilon_\gamma) + (e^-/\epsilon_\beta)}{1/\epsilon_\beta \int_0^{p_{\max}} (B/\Delta p) dp}, \quad (1)$$

where B and G are the beta and analysed 137-keV gamma photopeak counts recorded in the spectrometer (corrected for dead time losses and decay) at efficiencies ϵ_β and ϵ_γ , respectively, and e^- is the total K + L + M conversion line area. Δp is the "resolution width" of the spectrometer, i.e., the area of the $p = \text{constant}$ section of the surface $L(p, p')$ divided by the peak counting rate (see Appendix I). ϵ_β was obtained by counting an absolutely standardized Ba^{137m} source in the spectrometer, and ϵ_γ was computed from the source-crystal geometry, taking account of absorption and edge effects⁵⁵ in the crystal. G is corrected for the presence of ca 10% of 122-keV gamma counts in the photopeak. The calculation of the integral of the total singles spectrum required an extrapolation to zero momentum. We obtain for the inner group a branching ratio of 23.4 (± 1.5)%, and for the ground state 76.8 (± 1.5)% of total betas, and use the value 76% in further discussion.

Curve (b), Fig. 21, shows the Kurie plot of the ground state beta transition obtained by subtraction of the inner group from the composite spectrum using a multiplying factor which yields the 76% branching ratio. This factor is obtained by trial and error comparison of the areas of the momentum spectra. This Kurie plot is very similar in shape to that of the inner group, being concave downward toward the energy axis. In Fig. 25, the $\underline{m}$ plot for the ground state transition, we indicate by dashed curves labeled $E_0 = 1072.8$ keV, and $E_0 = 1070.2$ keV, as in the $\underline{m}$ plot of the inner group (Fig. 22), the extremes of the "safe" error band for the ground transition. These take into account the uncertainty in E_0 , the uncertainties in the resolution correction of the total spectrum, the assigned uncertainty in the inner group, and reasonable upper and lower limits consistent with the statistical uncertainties of the counts. The errors are combined as discussed with reference to the inner group. Note that the uncertainties associated specifically with the inner group (its statistical errors, and the errors in its β - γ angular correlation, 2τ and resolution corrections) are reflected much reduced in the ground state curve because of the relatively small branching ratio of the inner group. The contribution to the total error due to the statistical uncertainty of the singles counts is negligible below 900 keV. In computing the error limits, we have arbitrarily ignored the contribution to the total error due to the uncertainty in the branching ratio. In Table III, columns 8-10, we present the relevant data on the ground transition. The values listed are the actual experimental numbers (not smooth curve). The variation in the $\underline{m}$ function for the ground state transition is ca 23% from 150 keV to E_0 .

Conventional Subtraction Analysis

The disparity between the results of Guss, Killion and Porter²³ (in which an "α" shape was found for the inner group on subtraction of an allowed shape for ground state group) and the allowed shape found by other investigators^{20,75} was checked by application of the same procedure to our total singles data. The results of subtracting the least square allowed shape fit to the high energy end (>934 keV) of the singles spectrum (Fig. 24) is shown in Fig. 26, curve (a). How nearly the difference spectrum approximates the "α" shape may be seen in curve (b), Fig. 26, which is obtained from curve (a) via the correction factor⁸² $C_{1T}^{(2)} = p^2 + \lambda K^2$. Close examination of the curve discloses only a very slight curvature over the mid-energy range in addition to the drop off below 400 keV, and an E_0 value of 915 keV, 19 keV too low. This remarkable accidental fit lends a note of caution to accepting the concordance with theory as proof of the validity of an extended extrapolation of data from a limited spectral region.

Additional Data on Re¹⁸⁶

As the relative intensity of the 307-keV beta group is of concern in connection with the aforementioned curvature in the coincidence spectrum near 200 keV, a measurement of the intensity of gammas following the 307-keV beta was made. Standard scintillation spectrometry techniques employing a 2-9/16 in. long x 2-15/32 in. diameter NaI·Tl crystal with DuMont #6363 3 in. diameter cathode photomultiplier and single channel analyser were used to compare the areas of the 764- and 627-keV photopeaks to that of the 137-keV gamma. We calculate a value of 0.34 for the relative photopeak (only) efficiencies of ca 700-keV gammas compared to the 137-keV photopeak for this crystal. The 627-keV γ /764-keV γ ratio obtained was 1.95 ± 0.17 . The ratio of 627-keV γ /137-keV γ was 7.0×10^{-3} , leading to a 307-keV β per total beta intensity of $0.105 \pm (0.03)\%$ (conversion of the 627- and 764-keV gammas neglected).

We present here (Table IV) a summary of data on the decay of Re¹⁸⁶ including a number of values obtained incidental to the spectrum shape studies. We list first in Eqs. (1)-(16) the primary experimental (from present experiment unless otherwise indicated) and theoretical quantities needed for the calculations.

It will be noted from Table IV that the values agree well with those of Metzger and Hill²⁰ and Steffen²⁰ except for the quantities 21) and 22) where the agreement is better with Steffen and for the quantities 25), 26), and 27) where they favor the results of Robinson, Whittle, and Jastram.⁸³

⁸² λ factor obtained from J. P. Davidson, Phys. Rev. 82, 48 (1951).

⁸³Robinson, Whittle, and Jastram, Phys. Rev. 91, 498A (1953).

Fig. 26 Conventional subtraction analysis for Re^{186} . Kurie plots for main inner group. Curve (a) is obtained by subtraction from the total singles spectrum assuming the ground state transition has an allowed shape. Curve (b) is curve (a) corrected with the unique first forbidden spectrum shape factor $C_{1T}^{(2)} = p^2 + \lambda K^2$.

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Table IV

1) $e_K^-(137)/[\beta^-(934) + \beta^-(1071)] = 0.042_8$	
2) $e_L^-(137)/[\beta^-(934) + \beta^-(1071)] = 0.065_7$	
3) $e_L^-(137)/e_M^-(137)$ [Ref. 20] = 5	
4) $e_K^-(122)/[\beta^-(934) + \beta^-(1071)] = 0.0042_9$	
5) $e_L^-(122)/[\beta^-(934) + \beta^-(1071)] = 0.0059_5$	
6) $e_L^-(122)/e_M^-(122)$ [Ref. 20] = 5	
7) $\beta^-(934)/[\beta^-(934) + \beta^-(1071)] = 0.24 \pm 0.015$	
8) K - X rays (W)/K - X rays (Os) [Ref. 84] = 1.9	
9) $\gamma_{627}/\gamma_{137} = 0.0070$	
10) $\gamma_{764}/\gamma_{137} = 0.0034$	
11) E_γ [Metzger and Hill, Ref. 20] = 764 keV	
12) Electron capture ⁸⁵ [total L/K] = 0.18	
13) $e_K^-(137)/e_K^-(122) = 10.0$	
14) $\gamma(137)/\gamma(122)$ [Ref. 20] = 9	
15) $[e_{K+L+M}^-(122)/\gamma(122)]/[e_{K+L+M}^-(137)/\gamma(137)] \approx 1$ [Estimated from (13) and (14)]	
16) K - X rays/K vacancies (Os and W) ⁸⁶ = 0.93	
17) From 1), 2), 3)	$K/L/M(137) = 0.65/1/0.2$
18) From 4), 5), 6)	$K/L/M(122) = 0.72/1/0.2$
19) From 1), 2), 3), 7), 9)	$\alpha_K(137) = 0.36_4$
20) From 1), 2), 3), 7), 9)	$\alpha_L(137) = 0.53_7$
21) From 7), 8), 12), 15), 16)	$E.C. \text{ to ground/decay} = 0.063$
22) From 7), 8), 12), 15), 16)	$E.C. \text{ to 122 level/decay} = 0.021$
23) From 7), 8), 12), 15), 16)	$\beta^-(934 \text{ keV})/\text{decay} = 0.22 \pm 0.02$
24) From 7), 8), 12), 15), 16)	$\beta^-(1071 \text{ keV})/\text{decay} = 0.70 \pm 0.02$
25) From 7), 8), 12), 15), 16)	$\beta^-(307 \text{ keV})/\text{decay} = 0.0010$
26) From 7), 8), 12), 15), 16), 10)	$\gamma_{764}/\text{decay} = 0.00034$
27) From 7), 8), 9), 12), 15), 16)	$\gamma_{627}/\text{decay} = 0.00070$
28)	$E_0\beta_1 = 1071.5 \pm 1.3 \text{ keV}$
29)	$E_0\beta_2 = 934.3 \pm 1.3 \text{ keV}$
30) From 28) and 11)	$E_0\beta_3 = 307 \text{ keV}$
31)	$E(\gamma_1) = 137.19 \pm 0.07 \text{ keV}$
32)	$E(\gamma_2) = 122.67 \pm 0.21 \text{ keV}$
33)	Half-life = $88.9 \pm 0.2 \text{ hrs}$
34) Curves and nomographs of S. Moszkowski ⁸⁷	$\log f_0t(\beta 934) = 8.00$
35) Curves and nomographs of S. Moszkowski ⁸⁷	$\log f_0t(\beta 1071) = 7.68$
36) Curves and nomographs of S. Moszkowski ⁸⁷	$\log f_0t(\beta 307) = 8.95$
37)	upper limit $\beta^+/\text{decay} = 2 \times 10^{-6}$

⁸⁴Paul Day, Cauchois spectrometer; private communication.

⁸⁵H. Brysk and M. E. Rose, ORNL-1830, "Theoretical Results on Electron Capture." (Jan., 1955)

⁸⁶From curve in Ref. 70.

⁸⁷S. Moszkowski, Phys. Rev. 82, 35 (1951).

VI. THEORETICAL INTERPRETATIONS OF BETA SPECTRAL SHAPES OF Re^{186}

From the ft value of the two main beta transitions the spectra are classified as first forbidden; the shell model prediction of the Re^{186} ground state parity (odd) together with the even parities of the ground and first excited states in Os^{186} support the first forbidden character of the transitions. The non-unique shapes found for both transitions justify the assignment of a $1(-)$ character to the ground state of Re^{186} . (Either $0-$ or $2-$ assignments would require an " α " transition for one or the other beta groups.) From the parametric fitting of these beta distributions to theoretical spectral shapes, values of ratios of nuclear matrix elements are obtained. From the similar fitting to theory of the directional correlation of inner group beta and 137-kev gamma independent values for the same nuclear matrix element ratios governing the beta transition were derived;^{25a} these results are compared in Section VII.

The experimental correction factors used in the fittings are the solid curves of Figs. 22 and 25.

We calculated theoretical correction factors⁸⁸ for both ground state and main inner group transitions involving the combination of scalar and tensor interactions only, $G_S S + G_T T$. This choice was based on the weight of current evidence⁸⁹ that these two interactions contribute to the beta process rather than the VA combination⁹⁰ and that the pseudoscalar invariant has not been found necessary^{91,92,93} in the fitting of the spectrum of RaE , which is judged also to be a $\Delta I = 1$, yes transition.

From Konopinski and Uhlenbeck⁸⁸ we obtained the direct terms and from Pursey⁹⁴ the cross terms and write for the spectrum correction factor for a transition with spin change one with a parity change:

⁸⁸See E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 60, 308 (1941) for definition and notation.

⁸⁹E. J. Konopinski and L. M. Langer, Annual Reviews of Nuclear Science 2, 261 (1953).

⁹⁰B. M. Rustad and S. L. Ruby, Phys. Rev. 97, 991 (1955) show that the tensor predominates over the axial vector.

⁹¹M. Yamada, Prog. Theo. Phys. Japan 10, 252 (1953).

⁹²E. A. Plassmann and L. M. Langer, Phys. Rev. 96, 1593 (1954).

⁹³G. E. Lee-Whiting, Phys. Rev. 97, 463 (1954).

⁹⁴D. L. Pursey, Phil. Mag. 42, 1193 (1951).

$$\begin{aligned}
C^1_{ST} = & G_S^2 \left| \int \beta_{\underline{r}} \right|^2 \left\{ \frac{1}{3} K^2 L_0 + 2L_1 + M_0 + \frac{2}{3} KN_0 \right\} \\
& + G_T^2 \left[\left| \int \beta_{\underline{\sigma}} \times \underline{r} \right|^2 \left\{ \frac{1}{6} K^2 L_0 + \frac{1}{2} L_1 + M_0 - \frac{2}{3} KN_0 \right\} \right. \\
& - \left\{ \left(\int \beta_{\underline{\sigma}} \times \underline{r} \right) \left(\int \beta_{\underline{\alpha}} \right)^* + \text{C.C.} \right\} \left\{ \frac{1}{3} KL_0 - N_0 \right\} + \left| \int \beta_{\underline{\alpha}} \right|^2 L_0 \Big] \\
& + G_S G_T i \left[\left\{ \left(\int \beta_{\underline{r}} \right)^* \left(\int \beta_{\underline{\sigma}} \times \underline{r} \right) - \text{C.C.} \right\} (L_1 - M_0) \right. \\
& - \left. \left\{ \left(\int \beta_{\underline{r}} \right)^* \left(\int \beta_{\underline{\alpha}} \right) - \text{C.C.} \right\} \times \left(\frac{1}{3} KL_0 + N_0 \right) \right] \quad (2)
\end{aligned}$$

where $K = W_0 - W$ and L_0, N_0, M_0 , and L_1 are combinations of the electronic radial wave functions.

For the inner group the B_{ij} matrix element also contributes to the beta process (but not for the ground state⁹⁵ decay). Inclusion of B_{ij} would add a third nuclear matrix element ratio parameter to the fitting procedure. Justification for avoiding this complication is derived from consideration of the ca tenfold higher ft values for unique ($\Delta I = 2$, yes) transitions which depend only on B_{ij} . While we recognize that the B_{ij} term contribution may be significant, we are able to match the theoretical and experimental shape factors for the inner group without it. By extension of the same argument we omit the third forbidden matrix elements.⁹¹

Following Lee-Whiting⁹³ we define ratios:

$$\begin{aligned}
x &= i \frac{G_S}{G_T} \frac{\int \beta_{\underline{r}}}{\int \beta_{\underline{\sigma}} \times \underline{r}} \\
\Lambda &= \frac{2\rho}{\alpha Z} \frac{\int \beta_{\underline{\alpha}}}{\int \beta_{\underline{\sigma}} \times \underline{r}} \quad ; \quad (3)
\end{aligned}$$

⁹⁵ $B_{ij} = 0$ for $I = 1 \longleftrightarrow I = 0$ transitions.

ρ = nuclear radius. We equate⁹⁶ $\rho = \alpha/2 (\hbar/mc) A^{1/3}$ cm, as do Rose et al.⁹⁷
 $\rho = 1.41 \times 10^{-13} A^{1/3}$ cm for $\alpha = 1/137.00$.

The large leading terms in the expansions of L_0 , M_0 , and N_0 give rise to the dominant energy-independent terms in C_{ST}^1 , Eq. (2). The energy dependence of the smaller terms can confer a significant energy dependence on C_{ST}^1 only if the coefficients of the large terms ($\alpha\rho^{-2}$, $\rho \ll 1$) lie within the relatively restricted range for which extensive cancellation of these large terms occurs. Following Yamada⁹¹ and Lee-Whiting⁹³ we effect a transformation of the parameters such that if complete cancellation of the largest ($\alpha\rho^{-2}$) energy-independent terms occurs then the new parameter vanishes. In terms of the new parameter, y , the total ρ^{-2} term in Eq. (2) becomes $(y^2/\rho^2) [\alpha^2 Z^2/2(1+S)]$, where

$$y = [(1+S)/2] \Lambda - 1 - x \quad (4)$$

and

$$S = \sqrt{1 - \alpha^2 Z^2}$$

As Yamada⁹¹ has shown, this transformation also causes the smaller terms proportional to ρ^{-1} to vanish, as y vanishes, i.e., in the resulting correction factor terms in ρ^{-2} and in ρ^{-1} are eliminated explicitly.

Taking the coupling constants⁹⁸ and the quantities x and Λ ⁹⁹ as real, we can write,

$$C_{ST}^1 = \left[G^2 S/x^2 \right] \left| \int \beta_{\underline{x}} \right|^2 \left[2/(1+S) \right] (\alpha Z/2\rho)^2 U \quad , \quad (5)$$

where the energy dependent (shape determining) terms are confined to

⁹⁶We have not investigated the sensitivity of the fitting to the choice of ρ value. Lee-Whiting (Ref. 93) has stated that the fitting is fairly insensitive for RaE over a factor of two variation in ρ . However, in the fitting of the second forbidden ground state transition in Cs¹³⁷ the possibility of obtaining a fit at all is dependent on the selection of a value for $\rho \approx 1.2 \times 10^{-13} A^{1/3}$ cm instead of $\rho \approx 1.4 \times 10^{-13} A^{1/3}$ cm. (M. Freedman - unpublished work.)

⁹⁷Rose, Perry, and Dismuke, ORNL-1459 (1953).

⁹⁸L. C. Biedenharn and M. E. Rose, Phys. Rev. 83, 459 (1951).

⁹⁹C. L. Longmire and A. M. L. Messiah, Phys. Rev. 83, 464 (1951).

$$\begin{aligned}
U = & y^2 \left[2L_0/(1+S) \right] + xy \left\{ \left[4L_0/(1+S) \right] + 2\zeta \left[KL_0/3 + N_0 \right] \right\} \\
& + x^2 \left\{ \left[\zeta^2(1+S)/2 \right] \left[(K^2L_0/3) + 2L_1 + M_0 + 2KN_0/3 \right] \right. \\
& \quad \left. + \left[2L_0/(1+S) \right] + 2\zeta \left[(KL_0/3) + N_0 \right] \right\} \\
& + y \left\{ 4L_0/(1+S) - 2\zeta \left[(KL_0/3) - N_0 \right] \right\} \\
& + x \left\{ 4L_0/(1+S) + 4\zeta N_0 - \zeta^2(1+S)(L_1 - M_0) \right\} \\
& + \left\{ \left[\zeta^2(1+S)/2 \right] \left[(K^2L_0/6) + (L_1/2) + M_0 - 2KN_0/3 \right] \right. \\
& \quad \left. - 2\zeta \left[(KL_0/3) - N_0 \right] + 2L_0/(1+S) \right\} \tag{6}
\end{aligned}$$

and $\zeta = 2\rho/\alpha Z = 0.07511$ for Os¹⁸⁶.

For values of the parameters x and y for which the energy dependence of U is significant (y small) the six terms of Eq. (6) in powers of x and y do not suffer severe mutual cancellation in computing U . However, in evaluating the six coefficients of these terms, in all except the y^2 term one finds that the coefficients represent very small differences between the quantities involved. To retain, say, 2-figure accuracy in the coefficients requires that at least 5 significant figures be employed in evaluating the L_0 , M_0 , N_0 functions. The advantage of transformation (4) is then that fewer figures need be carried through the rest of the computation. To obtain 5 significant figures in the L_0 , M_0 , and N_0 one must retain higher order terms in the expansion of the L_0 , M_0 , and N_0 functions in powers of ρ . This has been called "taking account of the finite de-Broglie wavelength of the electron in comparison to the nuclear radius."^{91,100} L_0 and N_0 are given in Yamada,⁹¹ Eqs. (1) and (2), and M_0 in Konopinski and Uhlenbeck.⁸⁸ We use the first three terms in each expansion, so that Eq. (6) is complete in terms to ρ^2 and contains some terms in ρ^3 and ρ^4 . L_0 and M_0 so computed are valid to 5 figures and N_0 to 4 or 5. We obtain the relatively small quantity L_1 by graphical interpolation from the table of Rose, Perry, and Dismuke.⁹⁷ Unfortunately, these 4-figure tables do not quite carry enough significant figures for L_0 , M_0 , and N_0 .

Expression (5) is identical to Lee-Whiting's Eq. (1) if in evaluating U we keep only terms through ρ^2 .

¹⁰⁰M. E. Rose and C. L. Perry, Phys. Rev. 90, 479 (1953).

The expansions of the electronic radial wave function combinations L_0 , M_0 , and N_0 within the nuclear volume are given for the Coulomb field case only, i.e., the nuclear charge is considered to be concentrated at the center of the nucleus. Rose and Holmes¹⁰¹ have shown how sensitive the L_0 , M_0 , and N_0 functions are to changes in the charge distributions. For example, the assumption of a uniform charge distribution leads to ca 50% alteration in M_0 . In view of the lack of a well-established model for the nuclear charge distribution it appears that one cannot take account of this effect of finite nuclear size with the necessary accuracy. Two arguments may be adduced to justify the utility of precise calculations based on the unrealistic point charge model which surely yields grossly erroneous wave function combinations, in comparison to the stated requirements of precision. Yamada⁹¹ argued that in the case of RaE the parameter values for which a fit was obtained were not sensitive to the nature of the nuclear charge distribution, as anticipated theoretically for cases, such as RaE, where extensive deviation from the allowed shape indicates near cancellation of the large energy independent terms, the ones most sensitive to the detailed nature of the charge distribution. We cannot with certainty apply the argument to the case of Re¹⁸⁶ where the deviation from the allowed shape is slight. However, as we also compute^{25a} the nuclear parameters $\underline{x}$ and $\underline{y}$ by fitting the experimental differential β - γ directional correlation assuming point nuclear charge, a comparison of the fittings may be useful even if the absolute magnitudes of the parameters derived from either are not accurate.

The ratio of the values of the experimental correction factor, $\underline{m}$, at two energies, E_i and E_j , can be reproduced for a set of (x,y) values by the theoretical expression (6); i.e.,

$$m_i/m_j = \left[U_i(x,y) \right] / \left[U_j(x,y) \right] \quad . \quad (7)$$

This set generates a conic section in the x,y plane. Several pairs of points on the experimental correction factor are chosen and the conic section generated by each pair is plotted. Since actually the experimental correction factor has uncertainty, each pair of energy values determines a range for the correction factor ratio; thus a band rather than a single conic is obtained. The range used was restricted to within the "safe" error limits of Figs. 22 and 25. It is in the common area of the bands that values of $\underline{x}$ and $\underline{y}$ lie which may produce fits to the experimental correction factor over the entire energy range. Figures 27a and 27b show these conic section plots for the inner group of Re¹⁸⁶ and the ground state transition, respectively. Two hatched conic section bands are shown in each figure; for band A, $\underline{i}$ and $\underline{j}$ of Eq. (7) correspond to $p_i = 1.2$ and $p_j = 1.8$ mc; for band B, $p_i = 1.8$ mc and $p_j = 2.4$ mc.

¹⁰¹ M. E. Rose and D. K. Holmes, Phys. Rev. 83, 190 (1951).

Fig. 27. Quadratics in x and y for which theory reproduces ratio of experimental shape correction factor at two energies (Eq. 7). Because of experimental uncertainty, a band (hatched) is generated rather than single conic section. Figure 27a is for the inner group and 27b is for the ground state transition in Re^{186} . Experimental ratios are taken at $(p = 1.2, p = 1.8 \text{ mc})$ for bands A and at $(p = 1.8, p = 2.4 \text{ mc})$ for bands B. All conics are hyperbolae. Points (x, y) in crosshatched region may produce good fits to the experimental shape correction factor. Theoretical correction factors are displayed in Figs. 28 and 29 for the indicated points.

These bounding curves are all hyperbolae. In the computation of the theoretical correction factors $U(x,y)$ we have used only the "best" E_0 values, 934.3 kev and 1071.5 kev, respectively.

Not all the points (x,y) in overlap regions (crosshatched) of Fig. 27 are expected to be equally good in fitting the entire spectrum, since all the details of the experimental correction factor are not contained in the points chosen for the conic section plots. With selected values of $\underline{x}$, $\underline{y}$ from Fig. 27 (indicated points) the theoretical correction factor was calculated from (6) and compared in Fig. 28 to the experimental data for the inner group and in Fig. 29 for the ground state transition. Experimental and theoretical curves are normalized at 541.2 kev ($p = 1.8$ mc). Each heavy solid curve in Fig. 28 or 29 is a reproduction of the solid curve in Fig. 22 or 25, respectively. In Fig. 28 one experimental curve is shifted vertically by 3.6 ordinate units and one by 7.2 units for clarity; the ordinate scale applies to the lowest set of curves. We have not included the error bands of Fig. 22 or 25 in Figs. 28 and 29. In judging the quality of the fit one must keep the error bands in mind. Each curve is labeled with the appropriate value of $\underline{x}$ and $\underline{y}$.

In each of Figs. 27a and 27b the overlap area (crosshatched) is seen to extend (in fact, infinitely) to both positive and negative values of $\underline{x}$, along straight lines. Theoretical shape factor curves calculated for points far out on these lines give satisfactory fits to the data, differing only slightly from the curves calculated for the indicated outermost points in Figs. 27a and 27b. In Fig. 29 we show one curve calculated for (x,y) points on the line $x/y = -1.25$ which for $x > 2$ or $x < -5$ lies well within the extrapolated overlap bands of Fig. 27b. The implication of the good fits obtained for these high values of $\underline{x}$ and $\underline{y}$ for both transitions is that the matrix element $\int \beta \sigma_{\mu} x r_{\mu}$ need not contribute to these beta transitions.

One curve in Fig. 29 represents the almost indistinguishable fits of three distinct points in Fig. 27b: $(0, -0.0975)$, $(0.25, -0.26)$, and $(1, -0.83)$.

In Fig. 28 one curve has been computed with the more approximate Eq. (8) of Mahmoud and Konopinski.¹ It is seen that although the point (x,y) ¹⁰² is outside the bands of Fig. 27a [and hence would give a poor fit using Eq. (6)], a fit is obtained. The curve marked "scalar only" clearly shows the need for including substantial contribution from tensor interaction. The curves for $x = 0$ (no scalar) in Figs. 28 and 29 show that a fair fit can be obtained with tensor interaction only.

¹⁰² Note that $\underline{x}$ and $\underline{y}$ are given. They are related to the parameters x_1 and y_1 of Ref. 1 by the equations $x_1 = x$, $y_1 = (2/1 + S)(y + 1 + x)$.

Fig. 28. Main inner group. Comparison of experimental and theoretical shape factors. Solid curves are tracings of "best fit" curve of Fig. 22 (experimental shape correction factor). Lighter curves are theoretical shapes calculated with indicated values of nuclear matrix element ratio parameters x and y . Middle group of curves is shifted vertically by 3.6 ordinate units and top group by 7.2 ordinate units; scale applies to lower group.

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Fig. 29. Re¹⁸⁶ ground state transition. Comparison of experimental and theoretical shape factors. Solid curve is tracing of "best fit" curve of Fig. 25 experimental shape correction factor. Lighter curves are theoretical shapes calculated with indicated values of nuclear matrix element ratio parameters $\underline{x}$ and $\underline{y}$.

With the sole exception of the "scalar only" curve, the fits to the innergroup must be regarded as more than satisfactory at energy >300 kev; the calculated curves lie well within the error bands of Fig. 22. In section V we noted that the flattening of the experimental correction factor below 300 kev could not be due to the low abundance 307-kev beta group, and thus the theoretical fitting actually is poorer below 300 kev than above. Two examples, $(x = -0.205, y = 0.173)$ and $(x = -0.5, y = 0.37)$, are shown of calculated C^1_{ST} which turn upwards above the experimental curve below 300 kev. All x, y pairs from the overlap area in the upper branch of the hyperbolae give such upward deviating theoretical curves at low energies, while the downward turning theoretical curves arise from the overlap area on the lower branch of the hyperbolae. No overlap region exists which will give a theoretical curve lying between these extremes at low energy better fitting the experimental data. Oscillations like those exhibited in the $x = -0.5, y = 0.37$ curve can occur since the U function employed is fifth degree in W owing to the inclusion of all the terms in the three term expansion of L_0, M_0 , and N_0 . Inclusion of higher order terms in the series may reduce these oscillations. Some support of this may be found in the fact that recomputing this curve with tables of L_0, M_0 , and N_0 extended to five-figure accuracy¹⁰³ eliminated the oscillations, while preserving the same general trend of the shape.

The relatively poorer fit obtained below 300 kev for the inner group compared to the ground state may reflect the omission of a possibly significant B_{ij} contribution for the former.

In Fig. 30 we show in an alternative representation of the fitting, the corrected Kurie plot for the inner group with $x = -0.1$ and $y = -0.046$. Any other x, y pair from the overlap regions of Fig. 27 would give as straight Kurie plots.

¹⁰³ These five-figure values were obtained from raw data of the ORNL-1459 tables⁹⁷ kindly supplied to us by Dr. M. E. Rose. The existence of this more accurate data was discovered only after the computations of this section were complete.

Fig. 30. Corrected Kurie plot of main inner group of Re¹⁸⁶. The correction factor C_{ST}^1 and the definition of the parameters $\underline{x}$ and $\underline{y}$ are given in Eqs. (3 - 6).

VII. DISCUSSION AND SUMMARY

We have demonstrated that it is possible, within the stated theoretical approximations, to fit the experimental energy dependence of both abundant beta spectra in Re^{186} with a precision comparable to that obtained with allowed shapes with current beta spectroscopic techniques. We now show that for the inner group transition the simultaneous theoretical fitting of the spectrum shape and of the beta-gamma angular correlation^{25a} serves to reject most of the range of the nuclear parameters for which a fitting of either separately is possible.

In a manner analogous to the solution of Eq. (7), the fitting of the angular correlation energy dependence proceeds graphically, resulting in conic section bands (as in Fig. 27) in x and y for each momentum p . These hyperbolae are shown in Fig. 9, Ref. 25a. Only within the overlap areas of the bands of the latter figure [including (0.695, 0.035) and (-1.0, 0.58)] can fits be obtained for the angular correlation. Comparison¹⁰⁴ of these conics to those for the spectrum shape (Fig. 27a) shows no common overlap areas, but the overlap areas of both conic sets do approximate each other in the second quadrant. We have calculated the spectrum shape (Fig. 31a) and the correlation (Fig. 31b) for the x, y set (-0.6, 0.44) selected at the point of closest approach of the overlap areas. The quality of the fit for the spectrum shape and for the angular correlation is not as good as for points selected within the overlap areas, particularly for the angular correlation, but is perhaps acceptable in each case.

In view of the recognized approximations in the theoretical development (omission of finite nuclear size effects, B_{ij} matrix element, other interactions, etc.) it is probably correct to regard this common "near" fitting as indicating that a theory including such refinements will probably fit both experiments closely. However, the range in x, y for which there might be a common fitting would almost certainly differ from that found here. A relatively small shift in the x, y value is anticipated, by generalizing from the results of Yamada⁹¹ on the small influence of finite nuclear size corrections in RaE and from Lee-Whiting's⁹³ calculations on the small effect associated with a factor of two change in nuclear radius value. This expectation

¹⁰⁴The validity of the comparison is limited by the fact that slightly different electronic wave function evaluations were used in computing the conic sets. Those for the angular correlation were the five-figure tables of Rose, et al.,¹⁰³ computed for mass number $A = 189$; those for the spectrum shape were the Yamada⁹¹ 3-term expansions using $A = 186$. The mass number difference corresponds to a small (0.6%) difference in nuclear radius used. Some evidence that the influence of these differences is small is derived from the comparison of the theoretical shape factors for the point (-0.5, 0.37) described above.

Fig. 31a. Spectrum shape correction factor for inner group of Re^{186} vs beta energy. Solid curves are the experimental error bands of Fig. 22. Dashed curve (I) is theoretical shape for a point in the overlap region of Fig. 27a ($x = -0.1$, $y = 0.046$). Dashed curve (II) is the theoretical shape for the point of closest approach of the shape factor and angular correlation conics, $(-0.6, 0.44)$, computed with the five figure tables of Rose et al. (103)

Fig. 31b. β - γ angular correlation for Re^{186} vs beta energy. Solid curves are experimental error bands of Fig. 5, Ref. 25a. Dashed curve (III) is theoretical correlation for a point in the overlap region of the "correlation conics," $(0.695, 0.035)$. Dashed curve (IV) is the correlation for the point $(-0.6, 0.44)$.

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is probably most optimistic with regard to the change in the angular correlation fitting associated with introducing the βB_{ij} matrix element, inasmuch as it enters there linearly (in cross terms) as well as to second power, while only the squared term enters the spectrum shape factor. Since the βB_{ij} influence is small compared to $\int \beta \alpha$, the linear terms may be significant to the angular correlation where the squared terms are negligible in the shape factor. Indeed, some preliminary attempts^{25a} to include very small contributions of βB_{ij} interaction to the angular correlation fitting [$i\beta B_{ij}/\int \beta \alpha \times r = \pm 0.1$, too small to affect the spectrum shape fitting significantly] resulted in marked changes in the overlap areas of the conics. A more refined attempt at simultaneous fitting must take the βB_{ij} interaction into account.

If, nevertheless, one assumes that the region in the x, y plane for which the inner group beta transition would be best fit is approximately located as found here near $(-0.6, 0.44)$, then some significance attaches to the following remarks.

For $y = 0.44$, $\Lambda = (2\rho/\alpha Z) (\int \beta \alpha / \int \beta \alpha \times r) = 0.90$. Rose and Osborn,¹⁰⁵ Ahrens and Feenberg¹⁰⁶ and Pursey⁹⁴ predict Λ lies in the range 1 to 3.

The quantity G_S/G_T can be obtained from the value for x (-0.6) if the quantity $\epsilon = i(\int \beta r / \int \beta \alpha \times r)$ is available. This latter can be estimated on the basis of the single particle nuclear model if only one nuclear orbital is involved in the beta transition,^{105,106} which may not be at all realistic for an odd-odd nucleus like Re^{186} , so far removed from shell edges. Assigning the proton and neutron orbitals as $h_{11/2}$ and $i_{13/2}$, respectively,¹⁰⁷ one obtains $\epsilon = -1$. Thus $G_S/G_T = +0.6$, in agreement with Lee-Whiting's⁹³ evaluation from the RaE analysis as regards sign and well inside his indicated range for the magnitude.

The fitting of the ground state transition (Fig. 27b) gives equally good results with either sign of x and hence furnishes no information on the sign of G_S/G_T . The theoretically predicted^{94,105,106} range of Λ , 1 - 3, would be consistent with values of x in the range 0 to +10, using Eq. (4) and the relation $x/y = -1.25$ for which the ground state can be fit. Thus all high positive and all negative values of x would be rejected. For positive x , G_S/G_T is negative (for $\epsilon = -1$), in contrast to the result for the inner group. For $\Lambda < 1$, however, G_S/G_T is positive, for $x/y = -1.25$.

¹⁰⁵M. E. Rose and R. K. Osborn, Phys. Rev. 93, 1326 (1954).

¹⁰⁶T. Ahrens and E. Feenberg, Phys. Rev. 86, 64 (1952).

¹⁰⁷P. F. A. Klinkenberg, Rev. Mod. Phys. 24, 63 (1952).

Transitions to members of a rotational band have been discussed by Alaga et al.;¹⁰⁸ they predict, for transitions with the spin and parity sequence of the ground state and main inner group betas of Re^{186} , a ratio of 2.0 for the reduced transition probabilities. This probability varies inversely as f_1 for a first forbidden transition. The ratio of the f_0 values for these beta transitions in Re^{186} is 2.1 ± 0.2 assuming the beta multipolarity $L = 1$. The error in the ratio is mainly that in the beta branching ratio. The ratio f_1/f_0 should be close to unity because of the small deviation from the allowed shape. Thus the transitions are consistent with the intensity rule. The similarity in spectrum shapes is perhaps correlatable by the collective model; the intensity relation indicates that the daughter states are pure rotational states, and thus $\Delta K = \Delta I = 1$ for both transitions.

Quite in contrast is the f_0 value of the 307-kev beta transition to the $2+$ level at 764 kev in Os^{186} , in relation to the ground state transition. Here the f_0 ratio is ca 20, where the daughter states are not members of the same rotational band, even though the spins of this level and of the 137-kev level are the same.

It is noteworthy that preliminary work on the beta spectrum of Re^{188} gives indication²³ of similar concave downward deviations from the allowed shape for the transitions to ground and first excited states of Os^{188} . We are pursuing investigation of this isotope, and of Tm^{170} , searching for smaller deviations that have thus far been sought in the latter. We feel also that a more careful investigation of the shape of the main transition in Au^{198} is merited.

Acknowledgement

We wish to thank Gordon Goodman for his assistance with some of the computations.

¹⁰⁸Alaga, Alder, Bohr, and Mottelson, Dan. Mat. Fys. Medd. 29, #9, (1955).

APPENDIX I

Correction for Finite Spectrometer Resolution

It is generally recognized that the finite resolution of a spectrometer introduces distortions in a continuous spectrum for which correction must be made. Owen and Primakoff,¹⁰⁹ who have pointed the way to the general treatment of the problem, have shown that for an asymmetrical transmission curve, the correction is not small even for fair resolution, although it is usually so small as to be negligible, except close to E_0 , for an approximately symmetrical line such as is obtained in lens type instruments. All coincidence spectrometers of which we are aware are lens instruments, but necessarily operated at such high transmission as to give poor resolution. The transmission curves published for such instruments indicate generally nearly symmetrical (Gaussian) lines, as for our instrument (see Fig. 17). The correction for the best Gaussian fit to this shape, as given by Eq. (11) in Ref. 109, when applied to the spectra of P^{32} and Au^{198} , gave E_0 values about 1.5% too high and noticeable curvature to the Kurie plots within 20% of E_0 .

A more elaborate treatment of the resolution correction valid for any reasonable line shape has been given by Owen and Primakoff.¹¹⁰ In this treatment, a spectrum to be compared with the observed distribution is derived from an assumed form for the true spectrum; the method is not practical in cases where the true spectrum cannot be accurately presumed, as in the case of Re^{186} . An extension of the method to the inverse calculation, in which the form of the true spectrum is deduced from the observed spectrum, has been given by Hinman.¹¹¹ As this has not been published in the general literature, we present it here, and indicate the detailed procedure for the case of interest, a line shape or window curve whose width Δp is proportional to the momentum, p' , of the electrons being focused.

$M(p)$ = observed momentum spectrum as a function of p , the spectrometer setting.

$n(p')$ = true momentum spectrum as a function of the momentum p' .

$L(p, p')$ = observed transmission function of spectrometer, the surface generated by the line shapes at every p' (not restricted to the case of constant fractional line widths, $\Delta p/p = \text{constant}$).

¹⁰⁹G. E. Owen and H. Primakoff, *Phys. Rev.* 74, 1406 (1948).

¹¹⁰G. E. Owen and H. Primakoff, *Rev. Sci. Instr.* 21, 447 (1950).

¹¹¹George W. Hinman, Carnegie Institute of Technology Report No. NYO-91, Appendix VII, (1951).

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It should be noted specifically that for $p' = p_i'$ (the momentum of the conversion line being focused) the section of the $L(p, p')$ surface, $L(p, p_i')$, is the experimental counting rate spectrum of the line (line shape) and not the counting rate per unit momentum interval spectrum ("momentum plot"). The assignment of the momentum being focused (p_i') to a particular spectrometer setting, p_i , is arbitrary.

Following Owen and Primakoff we write

$$M(p) = \int_0^\infty N(p') L(p, p') dp' / \int_0^\infty L(p, p') dp' \quad , \quad (1)$$

i.e., with the spectrometer set at $\underline{p}$, the observed counting rate per unit momentum interval is obtained by convoluting the true momentum spectrum with the transmission function, $\underline{L}$, over the variable p' . Thus the section of the $L(p, p')$ surface used in the integration is the cut at constant $p = p_i$; experimentally, $L(p_i, p')$ would be the counting rate spectrum obtained with fixed spectrometer setting p_i with a constant intensity source of variable momentum p' (available practically only at low energies in the form of an electron gun). The normalization factor in the denominator, which differs from that of Owen and Primakoff, has been chosen to effect a simplification in Eq. (5). Expanding $N(p')$ in a Taylor series around $\underline{p}$, we write

$$M(p) = \int_0^\infty L(p, p') \left[\sum_{k=0}^\infty \frac{(p' - p)^k}{k!} N^{(k)}(p) \right] dp' / \int_0^\infty L(p, p') dp' \quad (2)$$

where

$$N^{(k)}(p) = \left[\frac{d^k N(p')}{dp'^k} \right]_{p' = p}$$

Let

$$\lambda_k(p) = p^k \int_0^\infty \left(\frac{p' - p}{p} \right)^k \frac{1}{k!} L(p, p') dp' / \int_0^\infty L(p, p') dp' \quad (3)$$

which for purposes of numerical integration can be written

$$\lambda_k(p) = \frac{p^k}{k!} \sum_i \left(\frac{p_i' - p}{p} \right)^k L_i(p, p') \Delta p_i' / \sum_i L(p, p') \Delta p_i' \quad . \quad (4)$$

Since p' is the variable in this summation, the quantities $L(p, p')$ are obtained each from an individual line shape measured with $p'_i = p'_i$. Thus

$$M(p) = \sum_k \lambda_k N^{(k)}(p), \quad (5)$$

which is equivalent to Eq. (2) of Owen and Primakoff.¹¹⁰ In the normalization employed, $\lambda_0 = 1$.

Now, if as is generally true, the data from the experimental determination of the $L(p, p')$ function from measurements of a number of conversion line shapes is available in graphical or tabular rather than analytic form, the evaluation of the $\lambda_k(p)$ from (4) is simplified by plotting the constant p sections of $L(p, p')$ vs p' as derived from the experimental (constant p') sections, $L(p, p')$ vs p . From one such plot for each p the $\lambda_k(p)$ are obtainable. The transformation is in general difficult, unless the properties of source and spectrometer give some simple relationship between the line shapes (constant p' sections) measured at various p' over the range of interest.

Such a relation has been shown in Section III, to hold for our spectrometer, viz., the experimental line shapes are independent of the electron momentum p' (above 60 kev) when viewed as a function of p/p' (Fig. 17), i.e., $L(p, p') = L(p/p')$. In this case, one plotting of a constant p section, $L(p/p')$ vs p'/p , would suffice to determine the quantities λ_k/p^k .¹¹² Thus the constant p section is just $L(p/p')$ plotted at the reciprocal of its argument. The function $L(p/p')$, the constant p , and the constant p' sections are illustrated in Fig. 32. Note in Eq. (4) that $k! \lambda_k$ is the k^{th} moment of the constant p section of $L(p, p')$. The virtue of this transformation to the constant p section is that its moments can be directly visualized, and that the numerical integration of Eq. (4) is simplified in that the $\Delta p'$ can be taken as constant abscissal increments. Note, for example, that the transformation of a symmetrical line shape (constant p' section) about its axis of symmetry leads to an asymmetrical constant p section, and hence a non-zero first moment. Thus a Gaussian line shape calibrated at its peak leads to a first derivative correction term in addition to the (usually) larger second derivative term derived by Owen and Primakoff.¹⁰⁹

In case the resolution correction is small, Eq. (5) is sometimes solved for $N(p)$ using the approximation $N^{(k)}(p) \cong M^{(k)}(p)$, as in Ref. 109. To avoid this approximation we follow Hinman¹¹¹ and write

$$N(p) = \sum_{\ell} \alpha_{\ell} M^{(\ell)}(p). \quad (7)$$

¹¹² That λ_k/p^k is independent of p can be seen in Eq. (3) with $L(p, p') = L(p/p')$ and by making the transformation $p/p' = x$.

Fig. 32. Transmission function L of magnetic spectrometer as function of spectrometer current p and electron momentum p' , for case $L(p, p') = L(p/p')$. Sections of L surface at constant p (I) and at constant p' (II-experimental "line shape"), intersecting at calibration point (p_i, p'_i) , also shown in inset.

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From (7) and (5),

$$M(p) = \sum_{k, \ell=0}^{\infty} \alpha_{\ell} \lambda_k M^{(k+\ell)}(p) \quad (8)$$

Equating coefficients of like derivatives on both sides of (7) one obtains

$$\begin{aligned} \alpha_0 \lambda_0 &= 1 \\ \sum_{k+\ell=n} \alpha_{\ell} \lambda_k &= 0, \text{ for } n \neq 0. \end{aligned}$$

Solve for α_{ℓ} , and note from (4) that $\lambda_0 = 1$;

$$\begin{aligned} \alpha_0 &= 1 \\ \alpha_1 &= -\lambda_1 \\ \alpha_2 &= \lambda_1^2 - \lambda_2 \\ \alpha_3 &= -\lambda_3 + 2\lambda_1 \lambda_2 - \lambda_1^3 \end{aligned} \quad (9)$$

Thus by evaluating the λ_k via (4), and employing (9), we have in Eq. (7) the desired result. Questions of uniqueness of the solution and of convergence of (7) are discussed by Hinman.¹¹¹

We have found it sufficiently accurate to divide the line profile into about thirty intervals for numerical integration; this determines the λ_k to $\pm 0.1\%$, less than the uncertainty in determining the line profile, $L(p/p')$, superposed on the continuum.

It is useful to note that, within limits set by considerations of convergence of Eq. (7), the choice of the fiducial or calibration point, to which is assigned the momentum of the line, is arbitrary. Thus the choice may be made either (a) to achieve a simplification in the computations of the resolution correction, such as by choosing the centroid of the constant p section as fiducial point, for which $\lambda_1 = -\alpha_1 = 0$, or (b) to achieve the most sensitive determination of the fiducial point. We have made the latter choice, and calibrated our momentum scale by assigning the momentum of the line to the median at one-half peak amplitude of the line profile.

We used the first four terms in Eq. (7), evaluating the first three derivatives of $M(p)$ graphically. In all cases the sum has converged sufficiently rapidly so that the omitted terms are probably small compared to the errors in the included terms. Employment of this method of resolution correction instead of the Gaussian line shape approximation has resulted in material improvement in the linearity of the Kurie plots of Au^{198} and P^{32} , and has given the proper end points, for data taken in the lens spectrometer at 6.7% resolution.⁶⁹ In Section III we discuss the character of Kurie plots obtained with this correction (see Fig. 18).

APPENDIX II

BETA-GAMMA DIRECTIONAL CORRELATION CORRECTION

Presented here are the details of the correction to the beta spectrum observed in coincidence with a gamma ray if a beta-energy-dependent directional correlation exists between the beta and gamma of interest. The problem involves the integration of the correlation function over the beta and gamma detector solid angles. We are indebted to Roland F. Herbst, formerly of this Laboratory, for the present formulation.

The correlation function, $W(\Theta, E)$ can always be written

$$W(\Theta, E) = \sum_{\ell=0}^k a_{2\ell}(E) P_{2\ell}(\cos \Theta), \quad (1)$$

where $P_{2\ell}$ are the Legendre polynomials of order 2ℓ . The coefficients $a_{2\ell}(E)$ are determined in a differential correlation experiment. In an instrument in which the γ 's are detected if they are emitted into the solid angle $\Omega_1(\theta_1, \phi_1)$ and the β 's detected if emitted into the solid angle $\Omega_2(\theta_2, \phi)$, then from a point source the observed energy spectrum of beta particles is

$$N'(E) dE = \frac{1}{(4\pi)^2} N(E) dE \int_{\Omega_1} \int_{\Omega_2} W(\Theta, E) d\Omega_1 d\Omega_2; \quad (2)$$

i.e., $N(E)$, the true distribution, is weighted with the relative probability that a beta particle of energy E is associated with a gamma ray at angle within the acceptance angle of the detecting apparatus. Here spherical coordinates are adopted, and $d\Omega = \sin \theta d\theta d\phi$. The integration is carried out for the β geometry which obtains in most lens type spectrometers and for a circular aperture gamma detector. (It is assumed that the efficiency for gamma detection is a constant over the aperture.) Figure 33 defines the coordinates and shows schematically that β 's are accepted between two cones of half angle $c = \pi - b_1$, and $d = \pi - b_2$. The center of the gamma aperture, the source, and the axis of the β acceptance cones are taken as colinear. The source is the origin of coordinates and the polar axis extends toward the center of the γ aperture. Equations (1) and (2) yield

$$N'(E) dE = \frac{N(E) dE}{(4\pi)^2} \sum_{\ell=0}^k a_{2\ell}(E) \int_{\Omega_1} \int_{\Omega_2} P_{2\ell}(\cos \Theta) d\Omega_1 d\Omega_2. \quad (3)$$

Making use of the addition theorem for Legendre polynomials and performing the ϕ_1 and ϕ_2 integrations, (3) becomes,

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Fig. 33. Geometrical arrangements of source, gamma detector and beta acceptance baffles in coincidence spectrometer.

$$N'(E) dE = \frac{N(E) dE}{4} \sum_{\ell=0}^k a_{2\ell}(E) \int_0^\alpha \int_{b_1}^{b_2} P_{2\ell}(\cos \theta_1) P_{2\ell}(\cos \theta_2) \times \\ \sin \theta_1 \sin \theta_2 d\theta_1 d\theta_2 \quad . \quad (4)$$

It is convenient to let

$$\cos \theta_1 \equiv y_1 \text{ and } \cos \alpha \equiv s, \cos b_1 \equiv t, \cos b_2 \equiv u; \quad (5a)$$

then

$$N'(E) dE = \frac{N(E) dE}{4} \sum_{\ell=0}^k a_{2\ell}(E) \int_s^1 P_{2\ell}(y_1) dy_1 \int_u^t P_{2\ell}(y_2) dy_2. \quad (5b)$$

Consider now the case which is of interest in the Re^{186} experiment, namely,

$$W(\Theta, E) = 1 + a_2(E) P_2(\cos \Theta); \quad (6)$$

[see Fig. 4 of Ref. 25a for evidence that the P_4 term is absent in Re^{186} $\beta - \gamma$ correlation]. Performing the integration indicated in (5b), we obtain

$$N'(E) dE = \frac{N(E) dE}{4} (1-s)(t-u) \left\{ 1 + \frac{a_2(E)}{4} s(1+s)(t^2 + tu + u^2 - 1) \right\}. \quad (7)$$

The energy independent factors are not important in the spectral shape correction. The quantities s , t , and u are known for the particular instrument (Eq. 5a and Fig. 33) and $a_2(E)$ is obtained from independent experiments which give the $\beta - \gamma$ direction correlation as a function of β energy. Since $P_2(\cos \theta) = 1/2 (3 \cos^2 \theta - 1)$ the correlation functions can be written

$$W(\Theta, E) = 1 + A(E) \cos^2 \Theta, \quad (8)$$

where $a_2 = 2A/(3 + A)$. Then $A(E)$ has a form convenient for experimental use:

$$A(E) = \frac{W(\pi, E) - W(\pi/2, E)}{W(\pi/2, E)} \quad .$$

Finally in terms of the experimental quantity $A(E)$,

$$N'(E) dE \propto N(E) dE \left\{ 1 + \frac{A(E)}{6 + 2A(E)} s(1+s)(t^2 + tu + u^2 - 1) \right\} \quad . \quad (9)$$

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For the Argonne double lens with $3/4$ " diameter γ aperture 1" behind the source and β acceptance between cones of half angle 13° and 30° , Eq. (9) becomes

$$N(E) dE \propto N'(E) dE / [1 + 0.43 A(E) + \text{smaller terms}]. \quad (10)$$

The uncertainties in the coefficient (0.43) associated with errors in angle determinations and with the variation in gamma detection efficiency over the gamma aperture are small compared to the uncertainty in $A(E)$. For small $A(E)$ the series in Eq. (10) converges rapidly; we use only the linear term.

In the case of Re^{186} the anisotropy $A(E) \geq 0$ and increases with increasing beta energy, so that according to Eq. (10) the observed spectrum is reduced at high energies more than at low energies. The net result is that the $\beta - \gamma$ correlation correction reduced, by about 20%, the total variation in the observed experimental correction factor for Re^{186} .

The extent to which the correction for the correlation is reduced for larger beta and gamma solid angles in the same geometrical arrangement is illustrated by the following example: for a beta solid angle of 12% of 4π centered about $\Theta \approx 135^\circ$ and a gamma solid angle = 10%, the correction becomes

$$N'(E) = N(E) [1 + 0.123 A(E) + \dots] \quad .$$