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REPRESSURIZATION CHARACTERISTICS OF A ROCKET ENGINE TEST COMPARTMENT
OPERATING AT SUBATMOSPHERIC PRESSURE

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TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION	1
BASIC CONSIDERATIONS	3
Lumped Parameter Acoustic Circuit Elements	3
Differential Equations Describing Duct Response to Flow Upset	5
Extension of Method to Include Ramping Down Ejector Flow	8
More Advanced Considerations with Ramp Upset	9
COMPARISON WITH EXPERIMENT	11
Comparison with Subscale Test Results	11
Comparison with Full Scale Test Results	12
CONCLUSIONS AND SUMMARY	14
NOMENCLATURE	15
REFERENCES	16

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LIST OF FIGURES

<u>Figure No.</u>	<u>Title</u>
1	Sketch of a Short Open-Ended Tube
2	Sketch of a Short Closed-Ended Tube
3	Sketch of a Long Open Ended Tube
4	Sketch of the ETS-1 Duct
5	Sketch of the ETS-1 Duct Model
6	Schematic of the ETS-1 Duct
7	Sketch of Actual Driving and Approximate Functions
8	Ramp Upset and Pressure Response
9	Comparison of the Size of the ETS-1 and the Subscale Model
10	A Comparison of Predicted and Experimental Results for ETS-1

<u>Table No.</u>	
1	Engine Test Compartment Repressurization Following Loss of Primary or Secondary Pumping Fluids

REPRESSURIZATION CHARACTERISTICS OF A ROCKET ENGINE TEST COMPARTMENT

OPERATING AT SUBATMOSPHERIC PRESSURE

INTRODUCTION

Ground testing of a nuclear rocket engine in a space-simulating environment requires an Engine Test Compartment (ETC) to enclose and shield the engine and a Nuclear Exhaust System (NES) to convey the heated hydrogen propellant away from the engine and test stand and to reduce the ETC pressure. Typically, the ETC is inerted by purging with nitrogen and the pressure reduced to about 7 psia by a secondary ejector in the exhaust duct; the secondary ejector fluid is high-temperature steam produced by a Steam Generator System (SGS). Propellant flowing from the engine nozzle through the primary diffuser of the duct further reduces the pressure in the ETC to about 1 psia at full engine chamber pressure. The sudden loss of either of these pumping fluids at certain times in the engine operation cycle can result in backflow of air and duct gases through the duct into the ETC and can lead to overpressurization of the ETC, with possible structural failure, or to a test stand fire or explosion.

In order to properly test a nuclear-powered space engine, it is necessary that the nature of the resulting pressure oscillations in the ETC due to such backflow be understood. This knowledge (1) will permit prediction of structural loads and air intrusion prior to committing a design to construction, and (2) will suggest pretest modifications in the system or in test procedures to reduce or eliminate any overpressure effects in existing test stands. The analysis requires the development of a suitable mathematical model by formulation of the mass, momentum, and state equations and the solution of these for the appropriate boundary conditions.

This technical memorandum presents an analytical technique originally developed for application to the Engine Test Compartment and Nuclear Exhaust System of Engine Test Stand No. 1 (ETS-1) at the Nuclear Rocket Development Station, Jackass Flats, Nevada. The technique under report is thus largely in terms of this specific system. However, the methods of analysis are

generally applicable to all test stands wherein a rocket engine is fired into a compartment at subatmospheric pressure. The technique also has been used to predict repressurization characteristics for the larger rocket engine test stands currently in design.

Network analysis, which is the basis of the study presented herein, is not new; only the solution of familiar differential equations with some new boundary conditions and the current application are a late development. The basic methods of solving network problems were laid down by Heaviside¹ and Alford². Heaviside calculus has extensive applications to electrical problems but has had less application to pneumatic problems. Gardner and Barnes³ introduce a formal method of solving the differential equations which is simple and valid for a wide range of driving functions. Blackburn⁴ considered the basic components of any network and derived the performance equation of each. Meisenholder⁵ formalized the method of determining the system resistance, inductance, and capacitance, and included the matrix formulation of the system equations to machine solution in the frequency domain. These frequency domain methods allow the frequency response and stability characteristics of a complex interacting fluid flow system to be determined.

As with a tuned pipe organ, the ETS-1 ETC/NES will resonate within a certain frequency band, which is centered near 1.0 Hz, if the flow in the ejector supply line is suddenly stopped. The general expression for the mass flow of a frictionless gas in a resonator shows this effect more clearly than a verbal description.

$$\dot{m} = \dot{m}_0 \sin W_n t \quad (1)$$

where: $W_n = \frac{1}{\sqrt{L C}} \quad (2)$

The subscript is attached to the expression to emphasize that this is the natural undamped frequency. Therefore the resonator will resonate at a frequency given by Equation (2). Actually friction will cause the oscillation eventually to become undetectable.

This general technique was applied to the problem of eliminating system-coupled instabilities involving the propellant system, combustion chamber, pump turbine, and structure in the Titan II Program. The successful results of this program and the subsequent flights of the Gemini Launch Vehicle demonstrate the validity of the concepts of linear, lumped parameter, network analysis. A description of the methodology resulting from this effort is presented in this report.

BASIC CONSIDERATIONS

Lumped Parameter Acoustic Circuit Elements

The "lumped parameter" system is one whose performance may be described with one or more ordinary differential equations. It is common to characterize acoustic elements as the "lumped" type if their effective length is short relative to one wave length at the frequency of interest.

A brief derivation of the expression for the impedance of pneumatic elements will clarify the later sections which deal with applications of the analysis. A sketch of a fluid-filled short open-ended tube is shown in Figure 1. Gas in a short open tube of length L and cross-sectional area A behaves as an incompressible (rigid) body, so that continuity is achieved ($U_1 = U_2$). Therefore, Newton's second law may be written:

$$\sum F = (P_1 - P_2) A = \frac{m \dot{U}}{g_c} = \frac{L}{g_c} \frac{d \dot{m}}{d t}$$

Then, defining fluid inductance/inertance, $\bar{L}$, as the constant of proportionality relating pressure drop to the rate of mass acceleration:

$$\Delta P = \bar{L} \frac{d \dot{m}}{d t}$$

and,

$$\Delta P A = F$$

then,

$$\bar{L} = \frac{\Delta P}{d \dot{m} / d t} = \frac{L}{A g_c} \quad (4)$$

Capacitance

The derivation of the expression for the compliance of the gas in a short closed-ended tube can be understood best by referring to Figure 2. The gas contained in a short, closed-ended tube of length L and flow area A, behaves as a spring, with the fluid strain being proportional to the applied pressure.

Equation (5) relates the difference in flow rates to the change in pressure.

$$\dot{m} = \frac{\rho A L}{K} \frac{d P}{d t} \quad (5)$$

where: K = effective bulk modulus, lbf/ft²

We may then define compliance, C as:

$$C \equiv \frac{1}{\Delta P} \int \dot{m} \, d t$$

then, from Equation (5) and this definition

$$C = \frac{\rho A L}{K} \quad (5a)$$

The capacitance can be calculated from Equation (5a) or determined by measuring the mass of fluid that must be introduced into a tube, slowly, in order to produce a unit pressure rise.

Resistance

The derivation of the resistance equation is presented below (please refer to Figure 3). The pressure loss for laminar flow from Poiseuille's Law is stated:

$$\Delta P = \frac{8 \pi \mu L \dot{m}}{A^2 \rho} \equiv \dot{m} R \quad (6)$$

Therefore

$$R = \frac{8 \pi \mu L}{A^2 \rho} \quad (7)$$

For turbulent flow

$$\Delta P = \left(\frac{\dot{m}}{A} \right)^2 \frac{1}{2g_c \rho} \left[\frac{f L}{D} + K_e + K_c \right] \quad (8)$$

Therefore

$$R = \frac{\dot{m}}{A^2 2g_c \rho} \left[f \frac{L}{D} + K_e + K_c \right] \quad (9)$$

Impedance

The general impedance law in complex form is stated

$$\Delta P = Z \dot{m}$$

where:

$$Z \equiv R + j W \bar{L} - \frac{j}{W C}$$

Differential Equations Describing Duct Response to Flow Upset

This section is intended to show how the factors governing back flow in a duct and ejector may be represented by a differential equation which in turn can be solved to give flow characteristics as a function of time. As a first approximation in analyzing the transient response of the ETS-1 engine compartment and exhaust system in order to predict its repressurization characteristics, the fluid capacitance, inertance, and resistance can be mathematically lumped. This means that each element is assumed to be concentrated at a single location and interactions are assumed to be negligible. The effect of these assumptions can then be considered separately.

The lumping approximations are strictly valid only when the significant wave lengths of all variables are large compared with the actual dimensions of the system. The undamped, natural wave length of the fundamental response mode of the ETS-1 compartment-exhaust system is approximately 1100 feet. The most important dimension is the 18 ft height of the Engine Test Compartment (ETC).

Therefore, since 18 ft is a small fraction of 1100 ft, the requirement that the wave length be large compared to the important dimension is satisfied, at least for the fundamental wave length and most important dimension.

Let all the frictional losses be represented by an equivalent resistance and consider the inertia of the gas flow in the long duct, shown in Figure 4, leading to the much larger compartment volume. Figure 5 shows an idealized model of the ETS-1 duct which is more practical to analyze, while Figure 6 shows a schematic of the duct which helps explain the formulation of Equation (10) below.

The sum of the pressure losses is equated to the line terminal pressure difference, the left hand side of the expression representing the sum of the inertial, resistance and capacitive pressure losses respectively.

$$\bar{L} \frac{d \dot{m}}{dt} + R \dot{m} + \frac{1}{C} \int_0^t \dot{m} dt = P - \gamma \quad (10)$$

Clement and Johnson (6), p. 265, show for the cell pressure the expression*

$$P = \gamma + (P_{\infty} - \gamma) \left[1 - e^{-\alpha t} \left(\cos Wt + \frac{\alpha}{W} \sin Wt \right) \right] \quad (11)$$

where:

$$\alpha \equiv R/2 \bar{L}$$

$$W \equiv \sqrt{(1/\bar{L}C) - (R^2/4\bar{L}^2)}$$

In applying this method to the ETS-1 system, the equivalent resistance of the duct is taken from the calculations of mass flow and pressure loss when the pressure in the cell is 85% of atmosphere. The entrance and exit loss coefficients, K_e and K_c respectively, are taken from London and Kays⁷. Detailed calculations show that:

*If the atmospheric pressure is γ for all negative time and P_{∞} for all positive time.

$$f = 0.015$$

$$K_e + K_c = 0.411$$

$$\Delta P = 1.98 \text{ psi}$$

$$m = 360 \text{ pps}$$

Therefore:

$$R = \frac{\Delta P}{m} = 0.00550 \text{ psi/pps}$$

The inertance of the duct is found by summing the $L/(A g_c)$ terms of all the sections.

<u>Section</u>	<u>Name</u>	<u>Length (ft)</u>	<u>Area ft²</u>	<u>Inductance (lbf-sec²/lb-ft²)</u>
1	Diffuser	38.10	8.18	.1146
2	90° Elbow	6.28	25.30	.0077
3	Accelerator	8.38	17.19	.0151
4	Mixer	46.28	14.75	.0974
5	45° Elbow	5.65	14.75	.0119
Total				.2767

The compliance of the duct is determined from the average change in pressure which results from the addition of one pound of gas, or by use of Equation (5a)

ETC Volume	5925 ft ³
Temperature	520°R
Gas Constant	53 ft-lbf/lb°R
Compliance (ETC + Duct)	0.2065 lb-ft ² /lbf

The damping constant was calculated from the resistance and inductance,

$$R/2 \bar{L} = (0.0055)(144)/(2)(0.2767)$$

$$= 1.43 \text{ per sec}$$

The natural circular frequency, W

$$W = \sqrt{\frac{1}{LC} - \alpha^2} = \sqrt{17.5 - 2.04} = 3.93 \text{ per sec}$$

The response of the ETS-1 duct shown later in this report was calculated utilizing these values of the damping constant and natural circular frequency.

Extension of Method to Include Ramping Down Ejector Flow

If the steam ejector flow is "slowly" decreased to zero (by closing a supply valve, for instance) the pressure response will not be as given by Equation (11) but will show a lower amplitude. Time did not permit an accurate determination of the response for this case, so an approximate technique was used based on the superposition principle given by Gardner and Barnes³.

The superposition principle states that if the system is linear, the total response is equal to the sum of the responses of the two or more functions which when summed equal the function. Mathematically this can be expressed as:

$$P(t) = f(t)_0 P_u(t)_0 + \int_0^t (df/dt)_\tau P_u(t-\tau) d\tau \quad (12)$$

$P(t)$ is the response to the series of functions, $P_u(t)$ is the response of the network to a unit step function, τ is the elapsed time prior to the start of each driving function, and $f(t)$ is the driving function. Note that $P_u(t)$ at $t = 0$ is unity for a passive network such as is exemplified by the ETS-1 duct system.

This method has many valuable applications. For example, experimental data can be considered to be a series of step functions, each data point representing a function. These functions can be positive or negative and effects can reinforce or cancel previous oscillations. An example is shown in Figure 7.

For a single-loop passive network, such as is considered here, an approximation by the use of such a series of step responses can be used to represent a driving function. This driving function can be a Steam Generator System (SGS) shut down or other event which causes a non-equilibrium condition. If the driving function is represented by P and the mode of step response is selected, then the amplitude of the driving function will be P_i and Equation (12) becomes:

$$P = \gamma + \sum \Delta P_i \left[(1 - e^{-\alpha(t-\tau_i)}) \left(\cos W(t - \tau_i) + \frac{\alpha}{W} \sin(t - \tau_i) \right) \right] \quad (12a)$$

where:

$$\Delta P_i = \frac{P_\infty - \gamma}{N}$$

$$\tau = \left[\frac{2 i-1}{2 n} \right]$$

N = number of approximations

i = i^{th} step forcing function

a = elapsed time between step responses, sec

The predictions utilizing this equation as shown elsewhere in this report agree quite well with the test results.

More Advanced Considerations with Ramp Upset

If instead of an instantaneous change in the driving function, i.e., the steam line pressure, the duct system experiences a change linear with time (a ramp upset), the expression for the pressure in the cell will not be as calculated using Equation (11) but rather will be that determined from:

$$P = (\bar{A} + \bar{B} t) U(t) - [\bar{A} + \bar{B}(t-b)] U(t-b) + \left[\frac{\bar{D} - \bar{C}\alpha}{W} \left\{ 1 - U(t-b) e^{\alpha b} \cos(Wb) \right\} \right] \sin(Wt) e^{-\alpha t} + [\bar{C} U(t-b) e^{\alpha b} \sin(Wb)] \sin(Wt) e^{-\alpha t} \quad (13)$$

continued

$$\begin{aligned}
 & + \left[\bar{C} \{1 - U(t-b) e^{\alpha b} \cos(Wb)\} \right] \cos(Wt) e^{-\alpha t} \\
 & + \left[\frac{\bar{D} - \bar{C}\alpha}{W} \{e^{\alpha b} \sin(Wb)\} \right] \cos(Wt) e^{-\alpha t}
 \end{aligned} \tag{13}$$

continued

where:

$$\begin{aligned}
 \bar{A} & \equiv - \frac{2}{(\alpha^2 + W^2)} \\
 b & \equiv \text{ramp time, sec} \\
 \bar{B} & \equiv \frac{1}{\alpha^2 + W^2} \\
 \bar{C} & \equiv 2\alpha \bar{B}^2 \\
 \bar{D} & \equiv (4\alpha^2 \bar{B} - 1) \bar{B} \\
 U(t-b) & \equiv 1 \quad \text{when } t > b \\
 & \equiv 0 \quad \text{when } t \leq b
 \end{aligned}$$

In order to simplify the derivation of the case of ramp upset only the response function has been considered. Equation (13) can be derived by equating the sum of the pressure losses to the line terminal pressure difference and by solving the resulting equation in a conventional manner, e.g., Gardner and Barnes³, p. 196. The mathematical details are omitted here as not pertinent to this presentation.

Equation (13) may appear formidable but modern high speed computers can determine the complete response for a typical ramp upset (30 points for a curve) in 0.3 to 3 seconds machine time, depending on the accuracy required and the speed of the computer.

As an independent check on the derivation of Equation (13), the length of the pulse was allowed to approach zero. It is intuitively obvious that the ramp solution must approach the step solution for this case and in the limit Equation (13) should equal Equation (11). This limiting procedure has been carried out and the conditions are satisfied. The details of the proof are again omitted here to save space.

The applications of this equation are many. The majority of steam generator shutdowns can be satisfactorily approximated as one ramp, and the more complicated can be analyzed as a series of ramps of different durations and slopes.

COMPARISON WITH EXPERIMENT

The realistic assessment of these analytical techniques required the application of the method to specific flow upsets in test hardware for which the flow factors contributing to repressurization characteristics were known. Two sources of experimental data were available: (1) a subscale program conducted at Von Karman Center in which a large number of tests were performed over a wide range of operating parameters, and (2) a limited number of tests of the ETS-1 system itself. The majority of the subscale tests did not produce detectable oscillations; only those which did were selected for analytical evaluation. Five of the test conditions as run with the fullscale system produced oscillations; the results of all of these runs were compared with the results obtained in applying the theoretical model. A comparison of the predictions with the results of the test programs is presented in this section.

Comparison with Subscale Test Results

The subscale model contained an Engine Test Compartment (ETC), a supersonic diffuser, a 90° elbow, an ejector, a mixing section, and a 45° elbow as does the full scale duct; this test hardware was scaled linearly at 1/8 of the ETS-1 system. For convenience some changes were made in the facilities and procedures; for example, only nitrogen gas was used as a fluid, and a rocket engine nozzle with a non-scale expansion ratio was mounted in the ETC. The volume of gas trapped between the shut-off valves was quite small, permitting rates of depressurization as high as 3200 and 2100 psi per sec for the secondary and primary flow respectively. Purge flow could also be introduced in the ETC as required.

In order to perform the model studies, the duct constants ($\bar{L}$, R and C) were calculated by use of the formulae given earlier. These constants for the basic subscale model with the 18,000 in.³ compartment volume are:

$$\begin{aligned}\alpha &= 14.28 \text{ rad per sec} \\ W &= 49.4 \text{ rad per sec} \\ \bar{L} &= 1.414 \text{ lbf-sec}^2/\text{ft}^2\text{-lb}\end{aligned}$$

Table I summarizes the results of the selected ten tests and corresponding predictions. Results from the VKC tests were provided by the Facilities Engineering Department. The times to depressurize the simulated engine chamber pressure for Test Nos. 16 and 44 were difficult to determine because of high frequency oscillations; for these two cases the time required to stop the primary (chamber) gas flow was determined from the digital data from averaged pressure values. A number of terms used on Table I should be defined. "Primary", as used herein, refers to engine, "Secondary" refers to ejector gas, "Amplitude" to overpressure, and "Response time" to elapsed time from the start of upset.

From the comparison of predicted vs observed overpressures and response times, it may be concluded that the method of analysis adequately predicts overpressures and refill durations for the engine test compartment. Except for one case of the ten, where the prediction is conservative (i.e., predicted amplitude is greater than measured), agreement is quite satisfactory between predicted and measured values.

Comparison with Full Scale Test Results

All five full scale tests were evaluated similarly and again the predictions of theory agree quite well with the experimental results. Only the most representative test is discussed herein as all results were quite similar. This test, as performed at ETS-1, is described in Test 1 of Specification No. 202A, "Nuclear Exhaust System Design Demonstration Tests" Phase II, May 1967. In this test the flow upset resulted from the rapid closing of the primary fluid supply valve. The steam system was not in operation. Purge gas at a high flow rate was flowing into the compartment.

Primary Gas	N ₂
Initial Primary Pressure, psia	122
Initial Purge Flow, pps	35
Initial Cell Pressure, psia	6.0
Purge Gas	N ₂
Chamber Decay Rate, psi/sec	200
Chamber Ramp Time, sec	0.6
Steam Pressure, psig	0

The analysis procedure employed was similar to that previously described for the subscale study. The magnitude of the overpressure and the time at which it would be expected to occur were predicted using the "quick-look" measured initial and upset conditions, i.e., chamber decay rate and ramp time. The analytical model employed was the linear ramp-superposition, lumped parameter, network (discussed in detail previously). Analytic and test results are shown on Figure 10. The analysis shows that ETC repressurizes to a maximum of 14.5 psia (1.7 psi overpressure), reaching this pressure level 1.1 sec after the primary flow starts to decrease. The initial surge is followed by a series of pressure oscillations which are effectively damped in 5.5 sec.

Also shown are the measured values of compartment pressure and primary chamber pressure as replotted from the final published test data. Note that the first cycle of the predicted and measured pressure traces are in excellent agreement, and would essentially coincide if the initial P_v of the calculation ($P_v = 6.0$ psia as initially quoted) were displaced toward the actual initial measured value. The second peak in the measured P_v is higher than predicted and is displaced in time. This is due to the fact that primary flow was continuing at a low rate, changing the frequency response.

It is concluded that the analytical technique predicted compartment repressurization reasonably well, agreement being within 6% or 0.1 psi and the prediction of time being within 0.05 sec.

CONCLUSIONS AND SUMMARY

Development testing of the NERVA nuclear-powered technology engine requires a test stand incorporating a compartment to enclose the engine and a duct to conduct the heated propellant away from the stand. During system operation the compartment and part of the duct are at subatmospheric pressure. Theory predicts and tests confirm that relatively rapid changes in the flow in the duct result in pressure oscillations whose magnitude depends on the nature and rapidity of the change. In many cases which could be expected to occur as a result of a specific component or subsystem malfunction (and in a few which have occurred), the overpressures resulting from the inertia of gas backflow can produce significant stress on the ETC. In addition, certain modes of system shutdown can result in similar repressurization characteristics. Fortunately, linear network theory appears to satisfactorily predict the magnitude of overpressures and the system response time for both planned operations and unplanned component malfunction. This knowledge enables steps to be taken in system modification and/or operating procedures to minimize deleterious repressurization effects.

NOMENCLATURE

<u>English</u>		<u>Greek</u>	
A	cross-sectional area of resonator, ft^2	α	damping constant, $1/\text{sec}$ see Equation (11)
a	acoustic velocity, fps	γ	initial pressure, psfa
B	duct constant, see Equation (13)	ρ	density, lb/ft^3
b	ramp time, sec	τ	variable, sec
C	compliance, $\text{lb-ft}^2/\text{lbf}$	μ	viscosity, $\text{lbf-sec}/\text{ft}^2$
D	diameter, ft		
F	force, lbf	<u>Subscripts</u>	
f	friction factor	∞	steady state after an upset
g_c	Newton's conversion constant, $\text{ft-lb}/\text{lbf-sec}^2$	n	natural
I	fluid inertance, $\text{lbf-sec}^2/\text{lb-ft}^2$	<u>Superscripts</u>	
j	$\sqrt{-1}$		
K	effective bulk modulus, psf	-	duct constant
K_c	contraction coefficient		
K_e	entrance coefficient		
L	length of resonator, ft		
$\bar{L}$	inductance, $\text{lbf-sec}^2/\text{lb-ft}^2$		
m	mass, lb		
$\dot{m}$	mass flow, pps		
$\dot{m}_0$	initial mass flow, pps		
P	pressure, psfa		
R	resistance, $\text{lbf-sec}/\text{lb-ft}^2$		
t	time, sec		
U	velocity, fps, or a unit step		
$\dot{U}$	acceleration, dU/dt , ft/sec^2		
W	circular frequency, rad/sec		
X	distance, ft		
Z	impedance, $\text{lbf-sec}/\text{lb-ft}^2$		

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TABLE I

ENGINE TEST COMPARTMENT REPRESSURIZATION FOLLOWING LOSS OF PRIMARY OR SECONDARY PUMPING FLUIDS
Comparison of Analytical Predictions and Test Data - VKC 1/8 Scale Model Duct Test Program

VKC Run No.	Primary Chamber Pressure P_c (psig)	Secondary Chamber Pressure P_{sc} (psig)	Primary Pressure Decay Rate $d P_c / d \theta$ (psi/sec)	Secondary Pressure Decay Rate $d P_{sc} / d \theta$ (psi/sec)	Compartment Purge Flow w_p (lb/sec)	Repressurization			
						Predicted	Response	Measured	Response
						Amplitude of 1st Cycle (psig)	Time (sec)	Amplitude of 1st Cycle (psig)	Time (sec)
16	600	0	1250	0	0	0.90	.520	1.00	.675
14	595	0	875	0	0	0.01	.600	0	.535
19	0	110	0	3140	0	1.70	.110	1.40	.102
30	0	100	0	3040	.75	1.25	.102	1.70	.098
22	0	100	0	2850	.60	1.90	.104	1.68	.095
28	0	105	0	2770	.45	1.88	.108	1.70	.095
55	0	110	0	262	0	0.01	.450	0	.445
57	0	110	0	205	0	0.01	.530	0	.530
39	0	110	0	204	.75	0.10	.600	0	.535
79*	0	110	0	2820	0	1.00	.120	0.54	.185

*Engine compartment has additional volume.

Note: (1) All gas flows are nitrogen.

(2) Times are measured after start of primary or secondary pressure decay.

Short Open Ended Tube

Figure 1__

Short Closed Ended Tube

Figure 2__

LONG OPEN ENDED TUBE

Figure 3

EPS-1 DUCT

Figure 4

ETS-1 DUCT MODEL

Figure 5

ETS-1 DUCT

Figure 6

ACTUAL DRIVING AND APPROXIMATE FUNCTIONS

Figure 7

RAMP UPSET AND PRECURE RESPONSE

Figure 8

COMPARISON OF THE SIZE OF THE ETS-1 AND THE SUBSCALE MODEL.

A COMPARISON OF PREDICTED AND EXPERIMENTAL RESULTS FOR ETS-1