WANL-TNR-083

JANUARY, 1963

SUBMITTED BY:

**Westinghouse Electric Corporation
Astronuclear Laboratory
Pittsburgh 36, Pennsylvania**

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**TANK HEATING CODES
TIC-TOC AND TOE FOR
THE IBM 7090 COMPUTER**

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ABSTRACT

Two codes, TIC-TOC and TOE, have been formulated for a 32 K IBM-7090 Computer in order to calculate integrated heating, heating distributions and temperature rise in the hydrogen propellant contained in the propellant tank of a NERVA vehicle produced by incident reactor radiation. TIC-TOC calculates the temperature rise vs. operating time for two fluid flow models, the heating distributions, and the integrated heating in the liquid hydrogen propellant for 3 group neutron and gamma reactor equivalent point sources. TOE calculates the heating density in the hydrogen from n, γ reactions. The approximate running time for TIC-TOC is 1 minute per case while that of TOE is about 1 minute per receiver point or about 20 minutes for a complete map of the secondary gamma heating distribution.

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SUMMARY

TIC-TOC (For Temperature In Coolant Tank and Other Calculations), is a program coded in Fortran II language for an IBM-7090 computer. This program obtains the heating distributions, the integrated heating, and the temperature rise as a function of time for two fluid flow models (complete mix and no mix) of the liquid hydrogen propellant. The basic method of calculating heating densities is to scale the results of the Monte Carlo calculated heating rate functions generated at the George C. Marshall Space Flight Center⁽¹⁾ for inverse square law spreading and account for the actual slant thickness which the gammas or neutrons have penetrated in the hydrogen tank. Thus, it is assumed that the heating densities are a function of only these two variables, the distance to the heating point and the slant thickness of hydrogen penetrated.

Any tank geometry which can be described as a combination of cylinders and truncated cones (up to 10 regions) can be used.

TOE (For Thermal Neutron Originating Energy) is a program, coded in Fortran II for the IBM-7090 computer, which calculates the heating density in the hydrogen propellant due to absorption of secondary gamma rays originating from the n, γ reaction in the hydrogen. It also makes use of the Monte Carlo calculations mentioned above (1) for information of the location and intensity of secondary gammas sources. These secondary sources are again scaled for inverse square spreading and slant thickness penetration. The heating at a point in the tank is calculated using an exponential point kernel with a

buildup factor to allow for the scattered component and an integration is performed over the whole source volume.

The tank geometry is as flexible as in TIC-TOC and input cases can be stacked so that heating densities throughout the tank can be calculated in one run.

Descriptions of the detailed formulation of TIC-TOC and TOE are given in the following section. A description of the mechanics of the codes, the type of input needed, and the output options and format is given in the section entitled, "Program Descriptions". For ease in code operation, the input and output formats are summarized in the sections bearing these same titles.

DISCUSSION

A. TIC-TOC Formulation

The basic equation for calculating heating density in the hydrogen propellant is:

$$H = \sum_{i=1}^3 \int_{\text{vol.}} S(\theta, E_i) H(E_i, R, Z') \frac{r_2^2}{r_1^2} dv \quad (1)$$

where H is the total integrated heating in the hydrogen from gammas (or from neutrons).

$S(\theta, E_i)$ is the source distribution as a function of the polar angle θ (See Figure 1) for the i^{th} energy group.

E_i 's are the energies of the monoenergetic point sources (assumed to be 1, 3, 6 Mev for gammas and 1, 3, 7 Mev for neutrons).

$H(R, Z', E_i)$ is the heating distribution calculated for a point mono-energetic source of gammas (or neutrons) at 11.25 feet from a 30 foot diameter tank of liquid hydrogen by the George C. Marshall Space Flight Center.⁽¹⁾

$r_2^2 = (d_2 + Z')^2 + R^2$, distance squared to volume element in original problem.

$r_1^2 = (d_1 + Z)^2 + R^2$, distance squared to volume element for present study.

$dv = \text{differential volume element} = 2\pi r_1^2 \sin \theta d\theta$ in spherical coordinates.

Z' is the axial distance penetrated in reaching the heating point

(See Figure 1)

Z is the axial distance measured from the bottom of the tank to the heating point (See Figure 1)

d_2 is the axial distance measured from the "equivalent reactor point source to the bottom of the tank, (See Figure 1) for the original study (Ref. 1) = 11.25 ft.

d_1 is the axial distance to the bottom of the tank in the present study

$$\text{Since } r_2 = \frac{d_2 + Z}{\cos \theta}, \quad r_1 = \frac{d_1 + Z}{\cos \theta}$$

$$dr_1 = \frac{dz}{\cos \theta}$$

Equation 1 can be rewritten as:

$$H = \sum_{i=1}^3 2\pi \int_0^{\theta_{\max}} S(\theta, E_i) \frac{\sin \theta d\theta}{\cos^3 \theta} \int_{Z''}^{Z_t} \left[\frac{(d_2 + Z')^2 + R^2}{(d_1 + Z)^2 + R^2} \right] H(E_i, R, Z') \times (d_1 + Z)^2 dZ \quad (2)$$

For the limits on the integrals, See Figure 1

$\theta_{\max}$, and Z_t , the upper limits on the θ and Z integrals, are specified in the input to the problem.

Now Z' and Z'' must be obtained in terms of the geometry of the tank. Expressions for these quantities are derived for each of the tank regions in terms of input parameters $a, b_1, b_2, b_3, \dots, b_{10}, Z_1, Z_2, \dots, Z_{10}$ in Appendix A. These expressions

are given in Section A under Program Description.

Integrated values of heating are given in $\frac{\text{BTU}}{\text{sec}}$ since the original Monte Carlo data heating density functions $H(R, Z', \theta)$ are given in $\frac{\text{BTU}}{(\text{in})^3\text{-sec}}$

B. TOE Formulation

The basic equation for calculating the heating density in the hydrogen due to secondary gammas generated in the hydrogen from n, γ reactions is:

$$H_{s\gamma} = C \mu_a E_\gamma \int_{\text{vol.}} \sum_{i=1}^3 S_{Vi}(E_i, R, Z') A(R, Z, \emptyset) \frac{r_2^2}{r_1^2} dv$$

where $\sum_{i=1}^3 S_{Vi}$ is the number of neutrons absorbed per cm^3 of hydrogen per sec. summed over each of the neutron energy groups E_i as calculated by the Marshall Space Flight Center Monte Carlo Code.⁽¹⁾

(See Figures 9, 10, 11)

E_i, r_2, r_1, Z' are as defined previously in the TIC-FOC code.

E_γ is the gamma energy for hydrogen capture gammas = 2.23 Mev

$A(R, Z, \emptyset)$ is an attenuation factor for the secondary gammas $= \frac{e^{-\mu x}}{4\pi x^2} B_E(\mu x)$

where x is the total distance from source to receiver

μ is the total gamma attenuation coefficient for 2.2 Mev gammas in liquid hydrogen = $.0058 \text{ cm}^{-1}$

μ_a is the gamma energy absorption coefficient for 2.23 Mev gammas
in liquid hydrogen = .00315 cm^{-1}

C is a constant = $1.6 \times 10^{-13} \frac{\text{watt-sec}}{\text{Mev}}$ to convert heating densities
from $\frac{\text{Mev}}{3 \text{ cm}^3 \text{-sec}}$ to watts/cm³.

$B_E (\mu x)$ is an energy absorption buildup factor

$B_E (\mu x)$ can be approximated by a quadratic of the form

$$B_E (\mu x) = d + \beta (\mu x) + \gamma (\mu x)^2$$

d, β, γ are empirical constants

$$d = 1.1$$

$$\beta = 0.77$$

$$\gamma = .0267$$

but x (in cylindrical coordinates)

$$= \sqrt{(Z_0 - Z)^2 + \rho_0^2 + \rho^2 - 2\rho\rho_0 \cos \phi} \quad (\text{See Figure 2})$$

With these substitutions, Equation 3 becomes:

$$H_s \phi = 2\mu_a E \gamma C \int_{Z''}^{Z_t} dZ \int_{\rho=0}^{R_{\max}} \rho d\rho \int_{\phi=0}^{\pi/3} \sum_{i=1}^3 \frac{S_{Vi}(\rho, Z', E_i) e^{-\mu x}}{4\pi x^2(\rho, \phi, Z)} d\phi \quad (4)$$

$$\text{where } x = \sqrt{(Z - Z_0)^2 + \rho_0^2 + \rho^2 - 2\rho\rho_0 \cos \phi} \quad (5)$$

Z_o and ρ_o specify the "detector" location or the point at which the heating density is to be calculated - Z_o being the axial distance measured from the bottom of the tank and ρ_o being the radial distance measured from the cylindrical axis. (See Figure 2)

Limits on R are given in Section A under Program Description.

The integration indicated in Eq. 4 is carried out over all source volume except for the volume within one Δr , Δz , and $\Delta \phi$ of the detector which is carried out analytically and added to the source integration results. Since the heating in a sphere of radius r is given by:

$$H = \mu a \int_0^r \frac{S_V e^{-\mu R} dv}{4 \pi R^2} = \frac{S_V \mu a}{\mu} (1 - e^{-\mu r})$$

where the volume has been assumed small enough for the source density to be constant throughout the volume.

Now the source volume for a sphere one Δr around the detector is

$$\frac{4}{3}\pi(\Delta r)^3 = 4.19 (\Delta r)^3$$

The source volume for a cylinder for $\Delta Z = 2 \Delta r = 2\pi(\Delta r)^3$

Hence, a correction of $\frac{2\pi}{4/3\pi} = 1.5$ has been applied to include heating from a cylindrical rather than spherical volume around the source. This added contribution to the heating is usually small (on the order of 10% or less) unless the Δr 's, Δz 's and $\Delta \phi$ are unreasonably large.

C. SPECIAL OPTIONS FOR TIC-TOC

a. Heating Densities

Besides providing the total integrated heating in the tank for each neutron and gamma energy group and the total gamma and neutron heating as given by Eq. 1, the code provides an option to print out the heating densities throughout the tank for each gamma and neutron energy group and for the total heating densities summed over all groups.

Using the same notation as before, these heating densities can be represented as:

$$H_V = S(\theta, E_i) H(E_i, R, Z') \frac{r_2^2}{r_1^2} \quad \text{and are given in units of } \frac{\text{BTU}}{\text{in.}^3 \text{-sec}}$$

b. Temperature Rise Vs. Time

Another option provided by the code, is a calculation of the temperature rise in the hydrogen coolant for two fluid mix conditions: a no-mix model in which case the heat deposited is assumed to remain in the same area until that incremental volume of coolant is pumped out of the tank and a complete mix model in which case the total heat deposited is assumed to be instantaneously mixed throughout the tank. The actual conditions probably lie somewhere between these two extremes but since little is known at present concerning the exact mechanics of the mixing process, these two models serve to illustrate two extreme cases.

The equations used to calculate the temperature rise ΔT for each of these models are as follow:

$$\Delta T = \frac{1}{C_p W} \int_0^{Z_t} \sum_{j+n} \sum_{i=1}^3 S_{j(n)}(\theta) H_{j(n)}(E, R, Z') \left(\frac{r_2}{r_1} \right)^2 dv \quad (8)$$

$$\text{and} \quad t = \frac{\pi \rho}{W} \int_0^{Z_t} R_{\max}^2(Z) dZ \quad (9)$$

where ΔT is the temperature rise in $^{\circ}\text{F}$

t is the time at full flow rate conditions, in sec.

C_p is the heat capacity at constant pressure of hydrogen, BTU/lb- $^{\circ}\text{R}$

W is the flow rate of hydrogen, lbs/sec

ρ is the density of liquid hydrogen, lbs/ft³

$$\int_0^{Z_t} \sum_{j+n} \sum_{i=1}^3 (S(\theta) H_{j(n)}(E, R, Z') \frac{r_1^2}{r_2^2} dv \text{ is the}$$

total integrated heating (gammas + neutrons) up to the level in the tank $Z = Z_t$.

Thus, the time is determined through the auxiliary equation -

$$t = \frac{\pi \rho}{W} \int_0^{Z_t} R_{\max}^2(Z) dZ \text{ by the ratio of integrated volume to flow rate.}$$

For a series of Z_t values, a series of times are printed out corresponding to these Z_t 's and the flow rate. The equation for $R_{\max}$ are the same as those given in Equation 6.

For the Complete Mix Model -

$$\Delta T = \frac{1}{C_p W} \int_{Z_0}^L \frac{\int_0^{Z_0} \sum_{j=1}^n \sum_{i=1}^3 S_j(n) (\theta, E_i) H_j(n) (E_i, R, Z') \left(\frac{r_2}{r_1} \right)^2 dv}{\int_0^{Z_0} R_{\max}^2 (Z) dZ} R_{\max}^2 dZ$$

and $t = \frac{\pi \rho}{W} \int_{Z_0}^L R_{\max}^2 (Z) dZ$

where L is the initial height of hydrogen above the bottom of the tank

t is the time (in sec.) for the hydrogen level to drop from L to Z_0

Other terms are defined previously.

Hence, the temperature rise ΔT is obtained for each Z_0 specified, with each Z_0 related to the time of operation at full flow conditions through the auxiliary equation -

$$t = \frac{\pi \rho}{W} \int_{Z_0}^L R_{\max}^2 (Z) dZ$$

D. GENERAL DISCUSSION OF PROGRAM'S ACCURACY

The accuracy of the programs TIC-TOC and TOE is limited by five factors:

- 1.) The accuracy of the Monte Carlo data heating rate functions which are expressed by empirical fits in the programs
- 2.) the closeness of these fits to the Monte Carlo data
- 3.) the accuracy of the numerical

integrations 4.) the accuracy of representing the incident flux spectra with only three energy groups for gammas and neutrons originating from a point source and 5.) the inaccuracy occurring because of the basic assumption that the heating rates are only a function of the distance of the heated point from the source and the slant thickness penetrated rather than of the tank boundary shape.

There is an additional inaccuracy imposed by the inaccuracy of knowing the gamma and neutron leakage spectra from the reactor but this is considered to be a separate problem. Let us consider each of the above factors in detail.

First, the accuracy of the Monte Carlo data is, in general, considered to be within a few percent of the true values except where statistics are poor. The data was, in fact, smoothed by plotting and cross plotting before being fitted by empirical expressions. The 6 Mev gamma data appears to be the only data which has such poor statistics as to make this smoothing process questionable. This data after being smoothed has a different shape than the other data (See Figure 5) which also lends suspicion to the original data. Fortunately, for most reactors, the gamma leakage is predominantly lower in energy than 6 Mev. This group could become important and hence a better knowledge of the basic heating densities for this group is desirable. A better estimate could be obtained by running the Burrell Monte Carlo code for more histories than were run previously for the 6 Mev gamma group in order to provide better statistics.

Second, the accuracy of the empirical fits to the Monte Carlo data are believed to be within the accuracy of the Monte Carlo data itself and are in general within $\pm 5\%$ of the smoothed Monte Carlo values. A comparison of the empirical fits to the smoothed Monte Carlo data are shown in Figures 3 to 11, the smoothed data being represented by solid curves and the empirical fits by x's. These fits in general are on the high side in order to be slightly conservative. Thus, integrations performed over a 30 ft. diameter-20 ft. long tank using these empirical fits are slightly higher than those obtained directly from the Monte Carlo calculations. (See Table I)

Third, the numerical integration accuracy is good if small enough increments are used. Since there is a compromise between accuracy and machine time involved in any numerical integration, it is advisable to set forth certain guide lines in order to achieve reasonable accuracy in a near minimum of machine time.

For TIC-TOC, it has been found that ΔZ for gammas should be about 1 ft. ; for neutrons about 0.25 ft. A $\Delta \theta$ of 5° is usually sufficiently small. Smaller values will not give significantly better values and will require more running time in the ratio to the total number of points to be integrated.

For TOE, it has been found that for off-axis points, a $\Delta \theta = 15^\circ$, $\Delta Z = 0.25$ ft. $\Delta \rho = 0.5$ ft. are satisfactorily small values. ΔZ appears to be more sensitive than $\Delta \rho$. On center line axis, any value of $\Delta \theta$ can be used since the source attenuation is symmetric about this axis. A $\Delta \theta$ of 60° is suggested. Again, smaller values of $\Delta \theta$, and ΔZ will give slightly better results but the machine time will increase according to the total number of source points.

Fourth, the accuracy of representing the flux spectra incident on the tank by only 3 energy groups for neutrons and for gammas is considered no worse than the assumption that the uncollided gamma flux times a buildup factor represents a true gamma spectrum emerging from the reactor. When better values of the reactor leakage become available, then it would perhaps be wise to include heating density functions for more gamma and neutron energy groups in order to better treat these spectral effects. This could be done by running the Burrell Monte Carlo code for other energy groups and incorporating this data into the code. The inaccuracy caused by representing the reactor as a point source has been investigated to some extent and is believed to be one of the more important assumptions. With the reactor located some 10-12 feet away from the near part of the tank, the fall-off of the flux in the tank with distance follows the inverse square law fairly well but the question of where to locate the equivalent point source in the reactor is a bit of a problem. The total integrated heating with the equivalent point source located at the center of the reactor has been found to be some 25% higher than with the point source located at the top edge of the core. * Hence, the total integrated heating is not strongly dependent on the location of the point source and by locating the source in the center of the core, the results should be conservative.

Fifth, the inaccuracy occurring because of the basic assumption that heating rates are only a function of distance of the heating points from the source and the slant

*The total source strength in either case is obtained by multiplying fluxes calculated at 10 ft. from the center of the reactor core by $4\pi d^2$ where d is the assumed tank-source point separation distance. Thus the sources are considerably higher when scaled to the center of the core.

thickness penetrated is difficult to evaluate without very extensive, elaborate calculations and/or measurements. Larger tank diameters than that originally calculated would be expected to decrease the radial leakage while conical shaped tanks would be expected to increase the axial leakage due to the greater surface area involved. However, shaping the tank in the form of a cone rather than a cylinder would be expected to decrease the leakage of gamma rays through the corners of the tank. Hence, it is difficult to say what effect the shape of the tank would have on the integrated heating and on the heating distributions. Some attempt to include some of this axial leakage has been made in this code by calculating Z' , the effective axial distance to the heating point where Z' is in general less than Z thus increasing the axial leakage. It is expected that center line heating densities would be little disturbed by changes in the tank geometry. How far this region of "non disturbances" extends, it is difficult to say at the present time.

PROGRAM DESCRIPTION

A. PROGRAM DESCRIPTION OF TIC-TOC

TIC-TOC is a program written in Fortran II for the IBM-7090 which calculates both neutron and primary gamma heating and also temperature rise as a function of time for a tank consisting of a series of truncated conical regions.

The following basic equations are those used in the calculation of gamma and neutron heating.

$$H_{\gamma} = \sum_{i=1}^3 \int_V S(\theta, E_i) H(E_i, R, Z') \left(\frac{r_2}{r_1} \right)^2 dv \quad (1)$$

where $dv = 2\pi r_1^2 dr \sin \theta d\theta$

$$r_1 = \left(\frac{d_1 + Z}{\cos \theta} \right)$$

$$r_2 = \left(\frac{d_2 + Z}{\cos \theta} \right)$$

$$dr = \frac{dZ}{\cos \theta}$$

$$H_i = 2\pi \int_0^{\theta_{\max}} S(\theta, E) \sin \theta d\theta \int_{Z''}^{Z'} \left[\frac{(d_2 + Z')^2 + R^2}{(d_1 + Z)^2 + R^2} \right] H(E_i, R, Z') \frac{(d_1 + Z)^2}{\cos^3 \theta} dZ$$

$$= 2\pi \int_0^{\theta_{\max}} S(\theta, E) \frac{\sin \theta d\theta}{\cos^3 \theta} \int_{Z''}^{Z'} \left[\frac{(d_2 + Z')^2 + R^2}{(d_1 + Z)^2 + R^2} \right] H(E_i, R, Z') (d_1 + Z)^2 dZ$$

$$H_{\gamma} = \sum_{i=1}^3 H_i(E_i)$$

where $E_i = 1, 3, \text{ and } 6 \text{ Mev for gammas}$
and $1, 3, \text{ and } 7 \text{ Mev for neutrons}$

Z' , Z'' , and Z_1 are determined in the following manner. Letting a denote the lower radius of the first (zero) region, $b_1, b_2, \dots, b_n$, the radii of each of the n regions, Z_1 , the axial distance from a to b_1 , Z_2 , the axial distance from a to b_2 , $\dots$ and Z_n , the axial distance from a to b_n , and defining

$$\theta_1 = \tan^{-1} \left(\frac{a}{d_1} \right)$$

$$\theta_2 = \tan^{-1} \left(\frac{b_1}{d_1 + Z_1} \right)$$

$$\theta_n = \tan^{-1} \left(\frac{b_n}{d_1 + Z_{n-1}} \right)$$

$$\theta_{n+1} = \tan^{-1} \left(\frac{b_n}{d_1 + Z_n} \right)$$

We then have:

$$\text{For } 0 \leq \theta \leq \theta_1 \quad (\text{Region 0}) \quad Z'' = 0$$

$$Z' = Z$$

$$\text{For } \theta_1 \leq \theta \leq \theta_2 \quad (\text{Region 1}) \quad Z'' = \frac{\left(d_1 - \frac{aZ_1}{b_1 - a} \right)}{(b_1 - a) \cot \theta - Z_1} - \frac{aZ_1}{b_1 - a}$$

$$Z' = Z - Z''$$

$$\text{For } \theta_2 < \theta \leq \theta_3 \quad (\text{Region 2})$$

$$Z'' = (Z_2 - Z_1) \left[\frac{(d_1 + Z_1) - \frac{b_1(Z_2 - Z_1)}{b_2 - b_1}}{(b_2 - b_1) \cot \theta - (Z_2 - Z_1)} \right] - \frac{b_1}{b_2 - b_1} (Z_2 - Z_1)$$

$$Z' = Z - Z_1 - Z''$$

Retaining the numbering system on regions, then, in general, for region n ($n \geq 2$), we have:

For $\theta_n < \theta \leq \theta_{n+1}$

$$Z'' = (Z_n - Z_{n-1}) \left[\frac{(d_1 + Z_{n-1}) - b_{n-1} \frac{(Z_n - Z_{n-1})}{(b_n - b_{n-1})}}{(b_n - b_{n-1}) \cot \theta - (Z_n - Z_{n-1})} \right] - b_{n-1} \frac{(Z_n - Z_{n-1})}{(b_n - b_{n-1})}$$

$$Z' = Z - Z_{n-1} - Z''$$

The upper limit on the integral over Z is obtained from the relation:

$$Z_t = \frac{R \max(Z)}{\tan \theta} - d_1$$

where For $0 \leq Z \leq Z_1$ $R \max = a + \frac{Z}{Z_1} (b_1 - a)$

For $Z_1 < Z \leq Z_2$ $R \max = b_1 + \left(\frac{Z - Z_1}{Z_2 - Z_1} \right) (b_2 - b_1)$

$Z_{n-1} < Z \leq Z_n$ $R \max = b_{n-1} + \left(\frac{Z - Z_{n-1}}{Z_n - Z_{n-1}} \right) (b_n - b_{n-1})$

The program uses the following curve fits in approximating the function $H(E, R, Z')$. (R and Z' assumed to be in feet).

$$\begin{aligned} H(1, R, Z') &= (7.2 - 0.25 R - 0.005 R^2) \frac{\text{Gammas}}{e^{\frac{-Z'}{4.0}}} \times 10^{-24} \\ H(3, R, Z') &= (1.2 - .02 R - .0017 R^2) \frac{\text{Gammas}}{e^{\frac{-Z'}{5.7}}} \times 10^{-23} \\ H(6, R, Z') &= \left[(1.7 - .0844 R) \frac{\text{Gammas}}{e^{\frac{-Z'}{5.47}}} + (0.18 + 0.004375 R + 0.000391 R^2) \frac{\text{Gammas}}{e^{\frac{-Z'}{14.3125 R}}} \right] \times 10^{-23} \end{aligned}$$

$$\begin{aligned}
 H(1, R, Z') &= \left[(6.6 - 0.01R - 0.0275 R^2) \overset{\text{Neutrons}}{e^{-Z' (5.18 + 0.125 R + 0.003 R^2)}} \right. \\
 &\quad \left. - (5.2 + 0.03 R - 0.026 R^2) e^{-Z' (7.23 + 0.148 R + 0.0054 R^2)} \right] \times 10^{-22} \\
 H(3, R, Z') &= \left[(6.6 - 0.015 R - 0.025 R^2) e^{-Z' (2.55 + 0.022 R + 0.0044 R^2)} \right. \\
 &\quad \left. - (4.2 + 0.025 R - 0.021 R^2) e^{-Z' (3.91 - 0.0285 R + 0.0085 R^2)} \right] \times 10^{-22} \\
 H(7, R, Z') &= \left[(8.8 - 0.57 R) e^{-Z' (1.547 - 0.0411 R + 0.00434 R^2)} \right. \\
 &\quad \left. - (5.8 - 0.395 R - 0.005 R^2) e^{-Z' (2.725 - 0.02 R + 0.0089 R^2)} \right] \times 10^{-22}
 \end{aligned}$$

TIC-TOC prints out the contribution of each of the three energy groups for gammas and neutrons as well as their sum in BTU/sec.

Temperature rise as a function of time is calculated for both the complete mix and the no-mix models.

The following equations are used for the no-mix model:

$$\Delta T_i = \frac{1}{C_p W} \int_0^{Z_{Ti}} S(\theta, E) H(E, R, Z') \left(\frac{r_2}{r_1} \right)^2 dv$$

and

$$t_i = \frac{\pi e}{W} \int_0^{Z_{Ti}} R_{\max}^2(Z) dZ$$

where ΔT is temperature change in $^{\circ}\text{F}$ and t is time in seconds. ΔT vs. t is printed out for each Z_t .

Equations used in the complete mix model are:

$$\Delta T = \frac{1}{C_p W} \int_{Z_o}^L \left[\frac{\int_0^{Z_o} S(\theta, E) H(E, R, Z') \left(\frac{r_2}{r_1} \right)^2 dv}{\int_0^{Z_o} R_{\max}^2(Z) dZ} \right] R_{\max}^2(Z) dZ$$

and

$$t = \frac{\pi \rho}{W} \int_{Z_o}^L R_{\max}^2(Z) dZ$$

where ΔT is again temperature change in $^{\circ}\text{F}$ and t is time in seconds. ΔT vs. t is printed out for each Z_o .

All integrations in TIC-TOC are performed with Simpson's rule with two possible exceptions. In the event that an unequal increment is required ($Z'' \neq 0$, $Z_T \neq n \Delta Z$, $\theta_{\max} \neq m \Delta \theta$ where n, m are integers) or an even number of points occur, one of the following approximations was used to handle the integration between these end points:

Integrations Over Z

$$\int_{Z_i}^{Z_{i+1}} f(Z) dZ = \left[\frac{f(Z_{i+1}) - f(Z_i)}{\ln \frac{f(Z_{i+1})}{f(Z_i)}} \right] [Z_{i+1} - Z_i]$$

Integrations Over θ

$$\int_{\theta_m}^{\theta_{m+1}} f(\theta) d\theta = \left[\frac{f(\theta_{m+1}) + f(\theta_m)}{2} \right] (\theta_{m+1} - \theta_m)$$

While the program provides for different mesh spacing in the Z direction for neutrons and gammas, the finer of the two is chosen in evaluating the integral which yields time in the temperature rise vs. time option.

A list of input parameters required by the code is given below: The input format itself is described in the next section.

n (No. of regions)

d_1

d_2

ΔZ neutrons

ΔZ gammas

$\Delta \theta$

a

$b_1, \dots, b_n$

$Z_1, \dots, Z_n$

$S(E)$ or $S(\theta, E)$

$\theta_{\max}$

Z_t

L, C_p, W (required only for ΔT vs. t option)

All linear measurements should be given in feet and angular measurements in degrees. The program may be used to approximate a cylindrical tank by specifying a zero number of regions. While a θ max is provided as input, the program tests the input θ max against the $\tan^{-1} \left(\frac{bn}{d_1 + Z_n} \right)$, where n refers to the last region. In the event that the input θ max is larger than this calculated value, the calculated value will be used. This adjustment will be indicated on the output returned to the requestor, immediately following the print-out of input and preceding any program results.

B. PROGRAM LIMITATIONS OF TIC-TOC

$$0 \leq \text{number of regions} \leq 10$$

$$\frac{Z_T}{\min \{ \Delta Z_i, \Delta Z_n \}} \leq 499$$

$$\frac{\theta \text{ max}}{\Delta \theta} \leq 49$$

In the event that a pointwise print-out is requested to obtain $S(\theta, E)$
 $H(E, R, Z) \times \left(\frac{r_2}{r_1} \right)^2$ for each point, an additional restriction is imposed:

$$\left(\frac{Z_T}{\min \{ \Delta Z_i, \Delta Z_n \}} + 2 \right) \left(\frac{\theta \text{ max}}{\Delta \theta} + 2 \right) \leq 2500$$

If the ΔT vs. t option is used, the number of Z_{Ti} 's used for the no-mix model must not exceed 50, and the number of Z_{oi} 's used for the complete mix model

also should not exceed 50. In this option, each Z_0 (complete mix) must correspond to some Z_t used for the no-mix model. In addition to this, the increment between any two consecutive Z_t 's must always be the same; i. e., $Z_{t_i+1} \equiv Z_{t_i} + \Delta Z_t \equiv Z_{t_i-1} + \Delta Z_t \dots$

NOTE: $\left(\frac{\theta_{\max}}{\Delta \theta} \right)$ must not necessarily be an integer

$\left(\frac{Z_t}{\Delta Z} \right)$ must not necessarily be an integer

C. INPUT FORMAT FOR TIC TOC

Card 1: Identification Card. Column 1 must contain a 1. Any combination of alpha-numeric data may be used in Columns 2-72.

NOTE 1: All input cards except Card 3 and Card 4 of the ΔT vs. t option should appear in floating point form, with a maximum of five words per card, 14 bits per word; i. e., word 1 in columns 1-14, word 2 in columns 15-28, word 3 in columns 29-42, word 4 in columns 43-56, and word 5 in columns 57-70. Where a card calls for less than 5 words, it should not be completed with zeroes. The last card of each data set actually referred to as a card may contain any number of words, less than or equal to five.

NOTE 2: All linear measurements should be in feet, and all angular measurements in degrees.

Card 2: $d_1, d_2, \theta_{\max}, R_{\max}, a$

where d_1 = point source - tank separation distance for calculation in feet

d_2 = point source - tank separation distance for reference sources = 11.25'

$\theta_{\max}$ = maximum angle (may be adjusted by code)

$R_{\max}$ = maximum radius

a = lower radius of "0" region

Card 3: Option Card. Two columns are allocated for each input parameter appearing on this card; where only one digit is used, it should appear in the right-most of the two columns; i. e., the even numbered column.

Columns 1-2: NR = number of regions. $0 \leq NR \leq 10$. This number should correspond to the number of b_n 's and Z_n 's supplied as input.

NOTE: A "0" may be used here to indicate cylindrical geometry.

Columns 3-4: NA = number of angles. $\left(\frac{\theta_{\max}}{\Delta\theta}\right) + 1 = NA$

$$1 \leq NA \leq 50$$

Column 6: Contains a 1 or 2 punch.

1 = Print integrals only

2 = Print pointwise values of $H(E, R, Z') \left(\frac{r_2}{r_1}\right)^2$

Column 8: Contains a 1, 2, or 3 punch

1 = Gamma heating only

2 = Neutron Heating only

3 = Both neutron and gamma heating

Column 10: Contains a 1 or 2 punch

1 = Calculate pointwise values only

2 = Calculate integrals

Column 12: Contains a 1 or 2 punch

1 = input source will be a function of θ

2 = input source will be constant over θ

Column 14: Contains a 1 or 2 punch

1 = Do not print pointwise values for expression described below:

2 = Print pointwise values of $S(\phi, E) H(E, R, Z') \left(\frac{r_2}{r_1} \right)^2$

Column 16: Contains a 1 or 2 punch

1 = Don't want ΔT vs. t option

2 = Do want ΔT vs. t option

NOTE: The ΔT vs. t option (2 in col. 16) may be chosen only if both gamma and neutron heating are desired (Col. 8 contains a 3).

The remainder of the input format is dependent on the option chosen as specified in Card 3.

If the ΔT vs. t option is chosen (Col. 16 of Card 3 contains a 2), the remainder of the input is as follows:

Card 4:

Columns 1-2: NZT = number of Z_T 's to be used as upper limit of the integral for ΔT , no mix model. $1 \leq \text{NZT} \leq 50$

Card 4: (Cont'd.)

Columns 3-4: NZ_0 = number of Z_0 's to be used as lower limit of the integral for ΔT , complete mix model. $1 \leq NZ_0 \leq 50$

Card 5: $Z_{T1}, Z_{T2}, \dots, Z_{TNZT}$ = Upper limits on the ΔT integral for the no-mix model, listed in ascending order of magnitude. NOTE: A constant increment must be used between all Z_T 's.

Card 6: $Z_{01}, Z_{02}, \dots, Z_{0NZ0}$ = Lower limits on the ΔT integral for the complete mix model, listed in ascending order of magnitude. NOTE: Each Z_0 must correspond to the product of some integer and some Z_T .

Card 7: $L, C_p, W, \rho, \Delta Z_n$

where L = upper limit on ΔT integral for complete mix model

NOTE: $L \equiv Z_{TNZT}$

C_p = heat capacity in BTU/lbs/ $^{\circ}$ R

W = liquid hydrogen flow rate in lbs/sec

ρ = liquid hydrogen density in lbs/ft³

ΔZ_n = ΔZ to be used for neutrons

Card 8: $Z_{in}, \theta_{in}, \Delta \theta, Z_T, \Delta Z_\gamma$

where Z_{in} = lower limit of integral on Z for determination of $H(E)$.

Should be 0.

θ_{in} = lower limit of integral on θ for determination of $H(E)$.

Should be 0.

$\Delta\theta$ = increment on θ

Z_T = upper limit of integral on Z for determination of $H(E)$.

NOTE: $Z_t \equiv L = Z_T \text{ NZT}$

ΔZ_f = increment on Z for gamma heating

Card 9: $b_1, b_2, \dots b_n$ = Lower radius of each region. The number of words appearing in this set should correspond to the number of regions specified.

Omit this card if a "0" number of regions was specified.

Card 10: $Z_1, Z_2, \dots Z_n$ = Height of each region as measured from bottom of tank. The number of words in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 11: Sources. If source is constant over θ , (col. 12 on Card 3 contains a 2), the first card of this set should contain the 1, 3, and 6 Mev gamma source listed in that order, and the second card the 1, 3, and 7 Mev neutron source also listed in that order. If source is a function of θ , the 1, 3, and 6 Mev gamma source should appear for $\theta = 0^\circ$, followed by 1, 3 and 6 Mev gamma source for $\theta = \Delta\theta, \dots \theta = \theta \text{ max}$, listed 5 words per card. Starting on a new card, the 1, 3, and 7 Mev neutron source should then be listed for $\theta = 0^\circ$, followed by the 1, 3, and 7 Mev neutron source for $\theta = \Delta\theta \dots \theta = \theta \text{ max}$, listed 5 words per card. Card 11 is the

last card required for the ΔT vs. t option. If the ΔT vs. t option is not requested, but both neutron and gamma heating are desired (Col. 8 of Card 3 contains a 3), the remainder of the input is as follows:

Card 5: $Z_{in}, \theta_{in}, \Delta\theta, Z_T, \Delta Z\gamma$

where Z_{in} = lower limit of integral on Z for determination of $H(E)$.
Should be 0.

Card 6: $b_1, b_2, \dots, b_n$ = Lower radius of each region. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 7: Z_1, Z_2, Z_n = Height of each region as measured from bottom of tank. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 8: Gamma sources, neutron sources. Follow the instructions given for Card 11 of the ΔT vs. t option.

Card 8 is the last card required for this option.

If the ΔT vs. t option was not chosen, and only one of the two types of heating is desired (Col. 8 of Card 3 $\neq 3$), the remainder of the input is as follows:

Card 4 : $Z_{in}, \theta_{in}, \Delta\theta, Z_T, \Delta Z$

where Z_{in} = lower limit of integral on Z for determination of H (E).

Should be 0.

θ_{in} = lower limit of integral on θ for determination of H (E).

Should be 0.

$\Delta\theta$ = increment on θ

Z_T = upper limit of integral on Z for determination of H (E).

ΔZ = increment on Z

Card 5: $b_1, b_2, \dots b_n$ = lower radius of each region. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 6: $Z_1, Z_2, \dots Z_n$ = Height of each region as measured from bottom of tank. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 7: Sources. If source is constant over θ (Col. 12 of Card 3 contains a 2), list the 1, 3, and 6 Mev gamma sources if the gamma option is being used, or the 1, 3, and 7 Mev neutron sources if the neutron option is being used. If source varies with θ , list the 1, 3, and (6, 7) Mev (gamma, neutron) source for $\theta = 0^\circ$, the 1, 3, and (6, 7) Mev (gamma, neutron) source for $\theta = \Delta\theta, \dots \theta = \theta_{max}$.

Card 7 is the last input card required for this option.

Input problems may be stacked, thus enabling the requestor to run any number of problems at a given time.

Note that if input sources are to be a function of angle, they must be provided for each angle generated by the program. For Example: if $\theta_{\max} = 65^\circ$, and $\Delta\theta = 10^\circ$, sources must be supplied for 8 angles, i. e., for $\theta = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ$, and 65° .

D. OUTPUT FORMAT FOR TIC-TOC

The problem identification card is printed out, followed by all input data with self-explanatory labels.

Numerous input checks are provided in the program, and in the event that an error is detected, an instruction to check the erroneous word on the appropriate card will be printed, and the problem will be discontinued.

If $\theta_{\max}$ has been adjusted, this will be noted on the output following the printout of input and preceding the appearance of any calculated output. The value of the maximum angle used by the program will be printed out, and the problem will be run using this adjusted value.

Calculated output is a function of the options chosen.

If the print option specified by a 2 in column 6 of Card 3 is used, the value of $H(E, R, Z') \left(\frac{(d_2 + Z')^2 + R^2}{(d_1 + Z)^2 + R^2} \right)$ will be printed for each point, for each of the 3 energy levels. Each pointwise value will be preceded by the information Energy

(1, 3, 6 or 7), Radius R, Theta (angle in radians), Z, and finally Gamma (or neutron)

Heating Rate $\frac{\text{BTU}}{\text{In}^3 \cdot \text{sec}}$ which will give the pointwise value at the point described
 $\frac{1}{\theta \text{ or } n/\text{sec}}$

by the preceding parameter. These will appear in order of increasing Z for each θ until θ max is attained. The last angle will be followed by a printout of the words Energy (stating the energy level) and Gamma (or Neutron) Heating which gives the integrated value of H (E) for that particular energy level. This will be repeated for each of the three energy levels. After pointwise values for the 3 levels have been printed, the sum of the integrals of all 3 levels will be printed following the label "Total Gamma (or Neutron) Heating. In the event that both neutron and gamma heating are requested, this output will appear first for the gamma heating, then for the neutron. The total of the three energy levels for both neutron and gamma heating will appear following the label "total neutron plus gamma heating." In the event that this option is used in conjunction with the ΔT vs. t option, the pointwise values will appear only once, for the final iteration on Z_T , i. e., $Z_T = L$.

If the print option specified by a 2 in Column 14 of Card 3 is requested, the value of $S(\theta, E) H(E, R, Z) \left(\frac{r_2}{r_1} \right)^2 \frac{\text{BTU}}{\text{In}^3 \cdot \text{sec}}$ will be printed out for each point.

(Due to the bulk of output produced, it is suggested that this option not be used concurrently with the previously described pointwise print option.) The value of $S(\theta, E)$

$H(E, R, Z) \left(\frac{r_2}{r_1} \right)^2$ will appear for each value of Z, for each angle, for each energy

level. If both gamma and neutron options are being used, these values will appear first for the gammas (all energies), then for the neutrons. For the first two energy

levels of both, the output will appear as E (energy level), R (radius), A (angle in radians), Z (position along Z axis), Point Source Gamma (or Neutron) Heating Rates. The last angle of each energy level will be followed by a line consisting of Energy (1, 3) and (Gamma, Neutron) Heating which gives H (E) for that energy level. For the last energy level of each type of heating, the printout gives the pointwise heating at that energy level plus the 3 group total for each point. The format is R (radius), A (angle in radians) Z (position along Z axis), 6 (or 7) Mev, (pointwise value for the specified energy group), Total Point Source (Gamma, Neutron) Heating Rate (sum of the three energy groups at that point). That will again be followed by Energy E (6 or 7), and (Gamma, Neutron) Heating which gives the integrated value for H (E) for the last energy group. The 3 group integrated total will be printed after the identification "Total (Gamma, Neutron) Heating". In the event that this option is used in conjunction with the ΔT vs. t option, the pointwise values will appear only once, for the final iteration on Z_T . If pointwise values go to zero for some Z, the print option is terminated for that particular angle, thus minimizing the bulk of the output, which would only consist of zeroes for all subsequent Z's for that particular angle.

Printout for the ΔT vs. t option is as follows:

Energy 1	Gamma Heating	Integrated H (E) for that energy
Energy 3	Gamma Heating	Integrated H (E) for that energy
Energy 6	Gamma Heating	Integrated H (E) for that energy
Total Gamma Heating	(Sum of three energy groups)	
Energy 1	Neutron Heating	Integrated H (E) for that level
Energy 3	Neutron Heating	Integrated H (E) for that level
Energy 7	Neutron Heating	Integrated H (E) for that level
Total Neutron Heating	(Sum of H (E)'s for the 3 groups)	
Height	(Z _T for the first iteration) Total Neutron and Gamma Heating	
	(The Sum of the 2 types of heating for that height)	

This is printed out for each Z_T , starting with the lowest. If a pointwise printout is requested, it will precede the printout for the last Z_T . After the above format is repeated for each Z_T , and following a pointwise print if it has been requested, the words "no mix model" will be printed followed by a table consisting of Upper limit on Z (Z_T), time t, Delta T (ΔT) which prints ΔT (temperature rise in $^{\circ}F$) versus time in seconds for each Z_T . "Complete mix model" will then be printed out, followed by a table consisting of Lower limit on Z (Z_0), time t, Delta T (ΔT) which prints ΔT (temperature rise in $^{\circ}F$) vs. time in seconds for each Z_0 .

E. PROGRAM DESCRIPTION OF TOE

TOE (Thermal Neutron Originating Energy) is a separate Fortran Program written for the IBM 7090 which calculates heating density due to secondary gammas by evaluating the expression:

$$H_{\gamma} = 2 \mu_a E_{\gamma} C \int_0^{Z_{\max}} dZ \int_0^{R_{\max}} \rho d\rho \int_0^{\pi} \frac{S_i(E) S_V(\rho, Z') e^{-\mu x} B_E(\mu x)}{4 \pi x^2} \frac{r_2^2}{r_1^2} d\theta$$

where
$$x = \sqrt{(Z_0 - Z)^2 + \rho_0^2 + \rho^2 - 2 \rho \rho_0 \cos \theta}$$

ρ_0, Z_0 - coordinates of detector point

$$B_E(\mu x) = \alpha + \beta(\mu x) + \gamma(\mu x)^2$$

$$E_{\gamma} = 2.23 \text{ Mev}$$

$$\mu_a = .00315 \text{ cm}^{-1}$$

$$\alpha = 1.1$$

$$\beta = 0.77$$

$$\gamma = .026$$

$$C = 1.6 \times 10^{-13} \frac{\text{watt-sec}}{\text{Mev}}$$

The geometry of TOE is identical to that of TIC-TOC consisting of a series of truncated conical regions. The method of determining Z' and $R_{\max}$ is discussed in the program description section of TIC-TOC.

The following curve fits are used to approximate the function $S_V(E, \rho, Z')$ (ρ, Z' assumed to be in feet).

$$\begin{array}{l} E = 1 \text{ Mev} \\ 0 \leq Z' \leq 0.833' \\ S(1, \rho, Z') = \left[(2.27 - 0.115 \rho) e^{-(3.76 + 0.114 \rho) Z'} \right. \\ \quad \left. - (2.22 - 0.113 \rho) \times \right. \\ \quad \left. e^{-(5.24 + 0.166 \rho) Z'} \right] \times 10^{-7} \end{array}$$

$$Z > 0.833$$

$$S(1, \rho, Z') = 0$$

$$\begin{array}{l} E = 3 \text{ Mev} \\ 0 \leq Z' \leq 2.92' \\ S(3, \rho, Z') = \left[(9.0 - 0.41 \rho) e^{-(2.107 + 0.0382 \rho + 0.001368 \rho^2) Z'} \right. \\ \quad \left. - (8.5 - 0.39 \rho) e^{-(3.547 + 0.0683 \rho - 0.000925 \rho^2) Z'} \right] \times 10^{-8} \end{array}$$

$$Z > 2.92'$$

$$S(3, \rho, Z') = 0$$

$$\begin{array}{l} E = 7 \text{ Mev} \\ S(7, \rho, Z') = \left[(1.14 - 0.058 \rho) e^{-(1.42 + 0.036 \rho) Z'} \right. \\ \quad \left. - (1.11 - 0.057 \rho) \times \right. \\ \quad \left. e^{-(1.82 + 0.046 \rho) Z'} \right] \times 10^{-7} \end{array}$$

For each point, TOE evaluates the expression $\sum_E S(E, \rho, Z')$ and performs the integration first on ρ , secondly on ρ , and last on Z , as indicated by the equation. No point within a $\Delta\rho$, ΔZ of the detector point is included in the integration. This point is evaluated separately by the expression:

$$H_{pt} = \frac{1.5 \mu_a C E_f}{\mu} \left(\sum_{i=1}^3 S_V(\rho, Z') S_i(E) (1 - e^{-\mu \Delta\rho}) \right)$$

and is added to H_f , the result of the integration.

All integrations are performed using Simpson's rule. In the event that an even number of points occur (or an odd increment was taken), the following approximations are used to handle these end points:

$$\int_{Z_i}^{Z_{i+1}} f(Z) dZ = \left[\frac{f(Z_{i+1}) - f(Z_i)}{\ln \frac{f(Z_{i+1})}{f(Z_i)}} \right] (Z_{i+1} - Z_i)$$

$$\int_{\rho_i}^{\rho_{i+1}} f(\rho) d\rho = \left[\frac{f(\rho_{i+1}) + f(\rho_i)}{2} \right] (\rho_{i+1} - \rho_i)$$

A list of the input parameters appears below:

N_R	}	Parameters Common With TIC-TOC
d_1		
d_2		
a		
$b_1, \dots, b_n$		
$Z_1, \dots, Z_n$		
$\Delta\theta$	}	Parameters Unique to TOE
$\Delta\rho$		
ΔZ		
$Z_{\max}$		
$R_{\max}$		
$S(E_1), S(E_2), S(E_3)$		
ρ_o		
Z_o		

As in TIC TOC, a cylindrical problem may be run by specifying a "0" number of regions. All linear measurements should again be in feet, and all angular measurements in degrees. A pointwise printout is available which yields the expression -

$$\sum_{i=1}^3 \frac{S_i(E, \rho, Z') e^{-\mu x} B_E(\mu x)}{4\pi x^2} \frac{r_2^2}{r_1^2} \quad \text{for each point.}$$

F. PROGRAM LIMITATIONS OF TOE

$$N_R = \text{number of regions} \quad 0 \leq N_R \leq 10$$

$$N_1 = \frac{\pi}{\Delta\theta}; \quad N_1 \text{ not necessarily an integer} \quad 1 \leq N_1 \leq 179$$

$$N_2 = \frac{R_{\max}}{\Delta\rho}; \quad N_2 \text{ not necessarily an integer} \quad 1 \leq N_2 \leq 999$$

$$N_3 = \frac{Z_{\max}}{\Delta Z}; \quad N_3 \text{ not necessarily an integer} \quad 1 \leq N_3 \leq 999$$

G. INPUT FORMAT FOR TOE

Card 1: Identification Card. Column 1 must contain a 1. Any combination of alpha-numeric data may appear in columns 2-72.

NOTE: With the exception of Card 3, all data cards should appear in floating point form, with a maximum of 5 words per card, 14 bits per word; i. e., word 1 should appear in Columns 1-14, word 2 in Columns 15-28, word 3 in Columns 29-42, word 4 in Columns 43-56, and word 5 in Columns 57-70.

Card 2: d_1, d_2, a

where d_1 = point source-tank separation distance for calculation, ft.

d_2 = point source-tank separation distance for reference sources = 11.25'

a = lower radius of "0" region

Card 3:

Columns 1-2 NR, number of regions

$$0 \leq NR \leq 10$$

To indicate a cylinder, Column 2 should contain a "0"

Column 4 Contains a 1 or 2 punch

1 = Print pointwise values of

$$\sum_{i=1}^3 \frac{S(E_i, \rho, Z_i) e^{-\mu x} B_E(\mu x)}{4 \pi x^2} \left(\frac{r_2^2}{r_1^2} \right)$$

2 = no pointwise printout desired

Card 4:

$b_1, b_2, \dots, b_n$ = lower radius of each region. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if "0" number of regions was specified.

Card 5:

$Z_1, Z_2, \dots, Z_n$ = Height of each region as measured from bottom of tank. The number of words appearing in this set should correspond to the number of regions specified. Omit this card if a "0" number of regions was specified.

Card 6:

$$\rho_o, Z_o, \Delta Z, \Delta \rho, \Delta \theta$$

where ρ_o = radial position of detector point

Z_o = axial position of detector point

ΔZ = increment on Z

$\Delta \rho$ = increment on ρ

$\Delta \theta$ = increment on θ

Card 7: $R_{\max}, Z_{\max}$

where $R_{\max}$ = upper limit of integral on ρ

$Z_{\max}$ = upper limit of integral on Z

Card 8: $S_1(E), S_2(E), S_3(E)$

where $S_1(E) = 1 \text{ Mev Source}$

$S_2(E) = 3 \text{ Mev Source}$

$S_3(E) = 7 \text{ Mev Source}$

Card 8 is the last input card required.

Problems may be stacked, provided each one is preceded by a problem identification card, thus permitting the processing of any number of sets of data at a time.

H. OUTPUT FORMAT FOR TOE

The printout for each problem consists of printing of the problem identification card, as well as the input, with self-explanatory labels.

Several input error checks were incorporated into the program. In the event that one of them is encountered, a statement will be printed indicating the erroneous card and word. The problem will then be discontinued.

Printout of calculated output for the normal print option (a 2 in Col. 4 of Card 3) consists of the values of H_{pt} and $H_{pt} + H_{\gamma}$. H_{pt} will appear beside the ρ_0, Z_0 coordinates of the detector point, labeled as "Pointwise Heating Rate", in units of

watts/cm³. $H_{\gamma} + H_{pt}$ appear in units of watts/cm³, labeled as "Total Heating Due to Capture Gammas".

If a pointwise printout has been indicated by the presence of a 1 in Column 4 of Card 3, values of

$$\sum_{i=1}^3 \frac{S_i(E) S_V(\rho, Z') e^{-\mu x} B(\mu x) \left(\frac{r_2}{r_1} \right)^2}{4\pi x^2}$$

will be printed for each point. The identifying format used is RH 0 ____ Z ____

PHI ____ Pointwise Heating Rate _____. For a given Z, values will appear first

for all ρ 's, and then for all ρ 's. After each iteration on ρ , the result of the integra-

tion of ρ will be printed; this is also true for each integration on ρ . The pointwise

printout will be followed by the normal printout as described in the preceding paragraph.

REFERENCES:

1. NASA TND-1115 Nuclear Radiation Transfer and Heat Deposition
Rates in Liquid Hydrogen. M. O. Burrell -
George C. Marshall Space Flight Center,
Huntsville, Alabama - August, 1962

TABLE I

COMPARISON OF INTEGRATED EMPIRICALLY FITTED
HEATING DENSITIES WITH MONTE CARLO RESULTS

<u>Gamma Energy</u>	<u>Monte Carlo</u>	<u>Integration of Empirical Fit</u>
<u>Mev</u>	<u>BTU/Sec</u>	<u>BTU/Sec</u>
	Source Gamma/Sec	Source Gamma/Sec
1	2.00×10^{-17}	2.04×10^{-17}
3	5.46×10^{-17}	5.53×10^{-17}
6	9.28×10^{-17}	8.63×10^{-17}

<u>Neutron Energy</u>	<u>Monte Carlo</u>	<u>Integration of Empirical Fit</u>
	<u>BTU/Sec</u>	<u>BTU/Sec</u>
	Source Neutron/Sec	Source Neutron/Sec
1	2.94×10^{-17}	2.95×10^{-17}
3	8.73×10^{-17}	8.46×10^{-17}
7	2.00×10^{-16}	1.95×10^{-16}

FIGURE 1

GEOMETRY FOR TIC-TOC CODE

$$x = \sqrt{(Z - Z_o)^2 + r_o^2 + r^2 - 2rr_o \cos \phi}$$

FIGURE 2

GEOMETRY FOR TOE CODE

FIGURE 3
 HEATING DISTRIBUTION FUNCTIONS
 FOR 1 MEV GAMMA RAYS

FIGURE 4
 HEATING DISTRIBUTION FUNCTIONS
 FOR 3 MEV GAMMA RAYS

FIGURE 5
HEATING DISTRIBUTION FUNCTIONS
FOR 6 GAMMA RAYS

FIGURE 6
 HEATING DISTRIBUTION FUNCTIONS
 FOR 1 MEV NEUTRONS

FIGURE 7
HEATING DISTRIBUTION FUNCTIONS
FOR 3 MEV NEUTRONS

FIGURE 8
HEATING DISTRIBUTION FUNCTIONS
FOR 7 MEV NEUTRONS

FIGURE 9
SECONDARY GAMMA SOURCE DISTRIBUTION
FUNCTIONS FOR 1 MEV NEUTRONS

FIGURE 10
 SECONDARY GAMMA SOURCE DISTRIBUTION FUNCTIONS
 FOR 3 MEV NEUTRONS

FIGURE 11

SECONDARY GAMMA SOURCE DISTRIBUTION FUNCTIONS
 FOR 7 MFV NEUTRONS

APPENDIX A

Derivation of Z' and Z''

$$\begin{aligned}
 &\text{For } \theta_1 < \theta \leq \theta_2 \quad \text{where } \theta_1 = \tan^{-1} \frac{a}{d_1} \\
 &\quad \text{(region)} \quad \theta_2 = \tan^{-1} \frac{b_1}{d_1 + Z_1} \\
 &Z' = Z - \left(\frac{R_1 - a}{b_1 - a} \right) Z_1 \quad \text{(See Fig. 1)}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 &\text{but } R_1 = (d_1 + Z'') \tan \theta \\
 &Z'' = Z - Z' = (R_1 - a) \tan \alpha_1
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 &\text{also } Z'' = Y \cot \theta \\
 &\quad Y = R_1 - d_1 \tan \theta \\
 &\text{Hence } Z'' = R_1 \cot \theta - d_1 \tan \theta \cot \theta = \frac{R_1 - d_1 \tan \theta}{\tan \theta} \\
 &\quad \quad \quad = \frac{R_1 - d_1}{\tan \theta}
 \end{aligned} \tag{3}$$

Combining Equations (2) and (3), we get -

$$R_1 \tan \alpha_1 - a \tan \alpha_1 = R_1 \cot \theta - d_1$$

$$R_1 (\tan \alpha_1 - \cot \theta) = a \tan \alpha_1 - d_1$$

$$R_1 = \frac{a \tan \alpha_1 - d_1}{\tan \alpha_1 - \cot \theta}$$

$$\text{but } \tan \alpha_1 = \left(\frac{Z_1}{b_1 - a} \right)$$

$$R_1 = \frac{\frac{a Z_1}{b_1 - a} - d_1}{\frac{Z_1}{b_1 - a} - \cot \theta} = \frac{d_1 (b_1 - a) - a Z_1}{(b_1 - a) \cot \theta - Z_1} \quad (4)$$

Substituting this into Equation 1, we get:

$$Z' = Z + \frac{a Z_1}{b_1 - a} - \frac{(d_1 - \frac{a Z_1}{b_1 - a})}{(b_1 - a) \cot \theta - Z_1}; \quad Z'' = Z - Z' = \frac{d_1 - \frac{a Z_1}{b_1 - a}}{(b_1 - a) \cot \theta - Z_1} - \frac{a Z_1}{b_1 - a}$$

For $\theta_2 < \theta \leq \theta_3$
(region 2)

$$\text{where } \theta_2 = \tan^{-1} \left(\frac{b_1}{d_1 + Z_1} \right) \\ \theta_3 = \tan^{-1} \left(\frac{b_2}{d_1 + Z_2} \right)$$

$$Z' = Z - Z_1 - \frac{(R_2 - b_1)}{(b_2 - b_1)} (Z_2 - Z_1) \quad (\text{See Fig. 1}) \quad (1)$$

$$Z'' = (R_2 - b_1) \tan \alpha'_2 \quad (2)$$

$$(d_1 + Z_1 + Z'') \tan \theta = R_2 \quad (3)$$

Substituting Z'' from Equation 2 into Equation 3, we get:

$$d_1 + Z_1 + (R_2 - b_1) \tan \alpha'_2 = R_2 \cot \theta \quad (4)$$

Solving for R_2

$$R_2 = \left[\frac{d_1 + Z_1 - b_1 \tan \alpha'_2}{\cot \theta - \tan \alpha'_2} \right] \quad (5)$$

$$\text{but } \alpha'_2 = \frac{(Z_2 - Z_1)}{(b_2 - b_1)}$$

Hence
$$R_2 = \frac{(d_1 + Z_1) - b_1 \frac{(Z_2 - Z_1)}{b_2 - b_1}}{\cot \theta - \left(\frac{Z_2 - Z_1}{b_2 - b_1} \right)} = \frac{(d_1 + Z_1)(b_2 - b_1) - b_1(Z_2 - Z_1)}{(b_2 - b_1) \cot \theta - (Z_2 - Z_1)}$$

Substituting into Equation 1, we get:

$$Z' = Z - Z_1 - (Z_2 - Z_1) \left[\frac{(d_1 + Z_1) - \frac{b_1}{b_2 - b_1} (Z_2 - Z_1)}{(b_2 - b_1) \cot \theta - (Z_2 - Z_1)} \right] + \frac{b_2}{b_2 - b_1} (Z_2 - Z_1)$$

$$Z'' = Z - Z_1 - Z' = (Z_2 - Z_1) \left[\frac{(d_1 + Z_1) - \frac{b_1}{b_2 - b_1} (Z_2 - Z_1)}{(b_2 - b_1) \cot \theta - (Z_2 - Z_1)} \right] - \frac{b_2}{b_2 - b_1} (Z_2 - Z_1)$$

Hence, in general, for $N \geq 2$

$$\theta_n < \theta \leq \theta_{n+1}$$

$$Z' = Z - Z_{n-1} - (Z_n - Z_{n-1}) \left[\frac{(d_1 + Z_{n-1}) - \frac{b_{n-1}}{b_n - b_{n-1}} (Z_n - Z_{n-1})}{(b_n - b_{n-1}) \cot \theta - (Z_n - Z_{n-1})} \right] + \left(\frac{b_{n-1}}{b_n - b_{n-1}} \right) (Z_n - Z_{n-1})$$

$$Z'' = Z - Z_{n-1} - Z'$$

$$= (Z_n - Z_{n-1}) \left[\frac{(d_1 + Z_{n-1}) - \frac{b_{n-1}}{b_n - b_{n-1}} (Z_n - Z_{n-1})}{(b_n - b_{n-1}) \cot \theta - (Z_n - Z_{n-1})} \right] - \left(\frac{b_{n-1}}{b_n - b_{n-1}} \right) (Z_n - Z_{n-1})$$