Astronuclear
Laboratory

WANL-TMI-1501

July, 1965

EFFECTIVE THERMAL CONDUCTIVITY
OF MULTILAYERED HEAT SHIELDS

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INFORMATION CATEGORY

UNCLASSIFIED

Authorized Classifier

Date

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SUMMARY

The "effective" thermal conductivity of multilayers of insulation materials has been derived for the general case where heat transfer occurs by conduction across the solid layers and conduction and radiation across stagnant gas layers. Equations were also given for special cases and specific geometries, i. e., operation in a vacuum, parallel planes, etc.

On the basis of heat conduction across successive layers of pyrolytic graphite foil and stagnant gas, a simple equation was derived for the effective thermal conductivity of pyrofoil laminates:

$$k_{\text{eff}} = \frac{(1 + \alpha) k_s k_g}{k_g + \alpha k_s}$$

where k_s and k_g are the thermal conductivities of pyrolytic graphite (as deposited) and the gas, respectively, and α is a proportionality constant which is a function of the packing density or the "bulk" material density given by the equation

$$\alpha = \frac{\rho_s}{\rho_b} - 1$$

where ρ_s is the density of pyrographite. The theoretical effective thermal conductivities calculated for hydrogen is presented in graphical form for a range of bulk densities.

The results have shown that the effective thermal conductivity varies appreciably with the bulk density. At temperatures below 3500°R, the effective thermal conductivities are lower than those for pyrographite, but above 3500°R, the values are higher than those for pyrographite. The calculated results have been compared favorably with limited experimental data.

INTRODUCTION

In recent years, the use of multilayers of thermal shields for aerospace applications has become commonplace. In cryogenics, the use of "super" insulators consisting of multilayers of aluminized foil wraps has proved to be extremely effective. Currently, at the Astronuclear Laboratory, the application of such insulations in two instances are being considered:

- 1) pyrofoil laminates on the periphery of the NERVA reactor core, and
- 2) multilayers of metal sheets in a vacuum enclosure to provide insulation for a potassium vapor calorimeter now under development.

In order to analyze the thermal characteristics in these designs, the "effective" thermal conductivity of the multilayered materials must be known. In the case of pyrofoil laminates, there is virtually no controlled experimental data available. Some data obtained from Quality Control tests indicated significant scatter; consequently, for design purposes, the thermal conductivity of pyrographite had been used as an approximation.

In this report, the effective thermal conductivity of multilayers of insulating materials is analyzed for the general case, and that of pyrofoil laminates in particular, in an attempt to

- 1) better understand the thermal behavior of multilayered thermal shielding,
- 2) provide general design equations,
- 3) provide effective thermal conductivity data for reactor and component test model design and analysis,
- 4) provide basis for specification of reactor materials, and
- 5) provide a sound basis for evaluation of pyrofoil materials.

GENERAL EQUATION

For the most general case of multilayers of parallel thermal shields with a curvature, the effective thermal conductivity is given by the equation

$$\frac{1}{K_{\text{eff}}} = \frac{D_o + N\delta + (N-1)\lambda}{N\delta + (N-1)\lambda} \left\{ \sum_{n=0}^N \frac{\delta}{k_n [D_o + (2n+1)\delta + 2n\lambda]} \right. \\ \left. + \sum_{i=1}^{N-1} \frac{1}{[D_o + 2i\delta + (2i-1)\lambda] \left[\frac{k_{i,i+1}}{\lambda} + \nabla F_{i,i+1} T_i^3 \right]} \right\} \quad (1)$$

The derivation of Equation (1) is given in the Appendix, and a nomenclature is found on page 14.

Equation (1) can be readily evaluated with the aid of a digital computer. However, for many applications, the equation can be simplified considerably. Some of these cases are discussed in the Appendix.

PYROFOIL LAMINATED LAYERS

For the intended application of pyrofoil layers, i. e., in a hydrogen atmosphere, it can be shown that the conduction term, $\frac{k_{i,i+1}}{\lambda}$, is significantly greater than the radiation term $\nabla F_{i,i+1} T_i^3$. Moreover, the overall thickness of the pyrofoil layers is expected to be small compared to the surface curvature; hence, Equation (1) can be simplified to

$$\frac{d}{k_{\text{eff}}} = \delta \sum_{n=0}^N \frac{1}{k_n} + \lambda \sum_{i=1}^N \frac{1}{k_{i,i+1}} \quad (2)$$

where λ is the average thickness of gas films. By defining an average reciprocal conductivity, i. e.

$$\overline{\frac{1}{k}} = \frac{\sum_{n=0}^N \frac{1}{k_n}}{N}$$

Equation (2) can then be written as

$$d \left(\overline{\frac{1}{k_n}} \right) = N \delta \left(\overline{\frac{1}{k_n}} \right) + N \lambda \left(\overline{\frac{1}{k_i}} \right)$$

From similarity, it shall be assumed that

$$\frac{d}{k_{\text{eff}}} = \frac{N \delta}{\overline{k_n}} + \frac{N \lambda}{\overline{k_i}} \quad (3)^*$$

where the thermal conductivities are evaluated at the average temperature.

* Equation (3) can also be derived in a different manner; consequently, the assumption used is valid.

Whence

$$\bar{k}_{\text{eff}} = \frac{d}{N \left[\frac{\delta}{\bar{k}_n} + \frac{\lambda}{\bar{k}_{i, i+1}} \right]} \quad (4)$$

Let $\alpha = \frac{\lambda}{\delta}$ denote the void fraction, then Equation (4) can be written in terms of α to give

$$k_{\text{eff}} = \frac{d}{N \delta \left[\frac{1}{\bar{k}_s} + \frac{\alpha}{\bar{k}_g} \right]} \quad (5)$$

where k_s and k_g are the thermal conductivities of the solid and the gas, respectively.

But due to the fact that the number of layers is large compared to 1 ($N \gg 1$), one can write

$$d = N \delta + (N - 1) \lambda \approx N (\delta + \lambda) \quad (6)$$

Equation (5) then becomes

$$\bar{k}_{\text{eff}} = \frac{(1 + \alpha) \bar{k}_n \bar{k}_i}{\bar{k}_i + \alpha \bar{k}_n} \quad (7)$$

The Constant α

In order to use Equation (7), the proportionality constant α must be known.

This constant is merely the volume fraction gas; consequently, it is related to the bulk density of pyrofoil layers by the equation:

$$\alpha = \frac{\lambda}{\delta} = \frac{\rho_p}{\rho_b} - 1 \quad (8)$$

where ρ_p is the density of pyrofoil. Thus if ρ_b is known, α can be calculated from Equation (8). Or conversely, if α is fixed by thermal design, then the required bulk density can be determined from Equation (8).

Thermal Conductivity of Pyrographite

Controlled experimental data on the thermal conductivity of pyrofoil as such is non-existent at the present time. As for pyrographite, only one set of well-documented experimental data has been published.¹ The results are reproduced in Figure 1. The pyrographite studied consisted of material "as deposited" and heat treated at 2900°C for three hours. It is evident from Figure 1 that there is considerable differences in the thermal conductivity in the two cases; the thermal conductivities of the heat-treated material were several times lower than those values for the material "as deposited." Hence it is expected that the thermal conductivity of untreated material would be unstable. That this occurs is evident from other properties of pyrographite, i. e. electrical resistivity. However, the data for pyrographite "as deposited" has been generally recommended^{3,4} for analysis and design; consequently, the theoretical effective thermal conductivities shall be calculated on the basis of this data.

EFFECTIVE THERMAL CONDUCTIVITY OF PYROFOIL LAYERS

The thermal conductivity of gases is a function of pressure as well as the temperature. Assuming that the bulk density of the pyrofoil layers does not change significantly with temperature, then for a given pressure, the effective thermal conductivity can be calculated from the bulk density of the material, determined at room temperature. Theoretical, ideal values of k_{eff} at constant pressure for several values of bulk densities were calculated

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* AR with
Fig 1.1
+ MDW Reliability
2000

Figure 1

Thermal Conductivity of Pyrographite Materials
Perpendicular to Basal Planes, "C" Direction

from Equation (7) and the results are presented in Figure 2 as a function of temperature. In the range of pressures of interest to NERVA reactor design, the thermal conductivity of hydrogen does not vary significantly with pressure. Hence the values at 600 psia are representative of values in the range of pressures of interest.

It can be seen from Figure 2 that at temperatures below 3500°R , the "effective" thermal conductivity increases with the bulk density whereas at temperatures greater than 3500°R , the effective thermal conductivity decreases with increasing bulk densities.

Two experimental data points obtained at 2460°R by High Temperature Materials⁵ are plotted on Figure 2 for a comparison. Note that there is consistent deviation between the data and the calculated values. However, since the type of gas in which the samples were tested and the pressure are unknown, a direct comparison cannot be made. The discrepancy may be due to the use of a gas other than hydrogen, which is more than likely the case. It is evident that the general trend is in agreement, i. e., at temperatures below 3500°R , the effective thermal conductivity decreases with decreasing bulk densities.

DISCUSSIONS AND CONCLUSIONS

Comparison of Thermal Conductivities

The theoretical ideal thermal conductivities of pyrofoil laminates are compared with those of hydrogen and "as deposited" pyrographite in Figure 3. It is apparent from this figure that the thermal conductivities of the first material is lower than that of pyrographite below 3500°R , but it is higher at temperatures above 3500°R . At all temperatures, the values are between those of pyrographite and hydrogen gas. It becomes evident that pyrofoil laminates are inferior to pyrographite in insulation qualities at high temperatures.

In view of the unstable nature of pyrographite "as deposited", it can be concluded that the ideal effective thermal conductivities calculated are the

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Figure 2

Effect of Bulk Density on Effective Thermal Conductivity of Pyrofoil Laminates

Figure 3

Comparison of Thermal Conductivities

maximum values. In high temperature service of relatively long durations, the thermal conductivity of pyrofoil laminates is expected to decrease with time. For this reason, heat-treated pyrographite would appear to be the preferred material, as it has a stable thermal conductivity at high temperatures and their values are considerably lower than those of hydrogen gas (see Figure 1). Further evidence can be seen from data published recently.⁶

Actual Effective Thermal Conductivities

The derivation of Equation (7) implies that the pyrofoils are parallel, evenly spaced and separated by stagnant gas. In reality, commercial pyrofoil laminates contain a large number of contact areas as a result of the use of bonding materials. Because of this fact, the thermal conductivity of pyrofoil laminates is expected to vary randomly over a wide range; the actual range being dependent on the thickness of the pyrofoil used, the overall thickness of the laminates, the material used for bonding, the degree of surface contact, etc.

Since the number and extent of pyrofoil contact areas are unknown and not controlled during the manufacture of pyrofoil laminates, the effect of contact areas on the thermal conductivity may be evaluated from the general equation

$$\frac{d}{k_{\text{eff}}} = \delta \sum_{n=0}^N \frac{1}{k_n} + \sum_{i=1}^{N-1} \frac{\lambda_{i,i+1}}{k_{i,i+1}} \quad (9)$$

where $\lambda_{i,i+1}$, the thickness of the gas space, can be given a random value, subjected to the following constraint:

$$\sum_{i=0}^{N-1} \lambda_{i,i+1} = d - N\delta \quad (10)$$

From such an approach, a "most probable" effective thermal conductivity can be calculated. However, the computations required would be excessive. In view of the fact that the production of pyrofoil laminated insulators is still in the development stage, such refined calculations are not justified at the present time.

Experimental data² from a typical commercial pyrofoil laminate is compared with the ideal calculated values in Figure 4. The data were obtained from pyrofoil laminates that were cemented together with furfural. It is evident that there is significant scatter in the data. Note that the experimental apparent thermal conductivities fall on both sides of the predicted values. The lower observed values are due to the fact that the samples were tested in an argon atmosphere and argon is known to have significantly lower thermal conductivities than hydrogen.

The higher values can be attributed to the increased conductivity resulting from the presence of the bonding material. Post-test observations showed that the furfural used to cement the pyrofoils tended to decompose at higher temperatures. In view of these facts, the observed scatter in the data is not surprising.

Various types of pyrofoil laminates are currently being developed.^{7,8} Among these is a hot-pressed material in which no cement is used. The pyrofoil laminates are merely pressed together to the desired bulk density. It is believed that the thermal conductivities of this type of material should closely approach the ideal theoretical values.

From these considerations, it appears that in a hydrogen atmosphere, the thermal conductivity of pyrofoil laminates when used at the reactor periphery can be expected to be relatively more stable than that indicated by data currently obtained during Quality Control testing of pyrofoil sleeves.

The analysis presented here on the effective thermal conductivity of pyrofoil laminates may serve as a basis for preliminary design or for the specification of bulk densities required to meet reactor design conditions.

Figure 4

Comparison of Observed and Calculated
Effective Thermal Conductivities for Pyrofoil Laminates
 $\rho_b \approx 80 \text{ lb/ft}^3$

The use of Equation (7) to calculate the effective thermal conductivity of pyrofoil layers is restricted to constant heat transfer areas. This is a good approximation for the intended pyrofoil applications at the periphery of the core where the thickness to diameter ratio is exceedingly small. However, in the design of small scale models of reactor components or for use in the center hex, care must be taken to evaluate the effect of the diameter changes on the effective thermal conductivity.

It is evident that experimental verification of the results obtained here would be required, and that experimental tests have to be carried out in a hydrogen atmosphere. When evaluating the relative merits of various types of pyrofoil laminates, care must be taken to ensure that the bases for comparison are the same, i. e. , the pressure and the gas used during testing must be identical.

NOMENCLATURE

D_o	twice the radius of curvature
d	overall thickness of multilayered thermal shields
F	view factor
k	thermal conductivity
L	axial length of thermal shield
N	total number of thermal shields
q	rate of heat flow
T	temperature

Greek Letters

α	a proportionality constant, ratio of gas space to thermal shield thickness
δ	thickness of thermal shield
λ	thickness of gas or vacuum space
ρ	specific gravity
σ	Stefan-Boltzman constant

Subscripts

b	bulk
eff	effective
g	gas
i	the i th gas space
$n, \text{ or } N$	the n th thermal shield
o	outside or high temperature side
P	pyrographite
S	solid or shield

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APPENDIX

DERIVATION OF GENERAL EQUATIONS

Consider thermal insulations made up of N layers of thin shields of thickness δ , as shown in the following sketch:

For the most general case, it shall be assumed that the shields have a curvature. If the shields operate in a vacuum, then the layers may be supported by ribs in the shields as shown below:

In this case, heat transfer is by conduction across the shields and radiation across the vacuum. Conduction through the ribs are generally negligible. However, if the shields were supported by a gas, then there is additional heat transfer due to conduction across the gas. It is evident that the latter situation represents the most general case, giving the greatest number of modes of heat transfer.

The heat transfer per unit length through the first shield is merely by conduction,
or

$$\frac{q}{L} = \frac{k_s}{\delta} (D_o + \delta) (T_o - T_1) \quad (1)$$

where δ is the thickness of the shield, D_o is twice the radius of curvature, and k_s is the thermal conductivity of the shield material. Now the heat transfer across the gas space is by conduction and radiation, whence

$$\frac{q}{L} = (D_o + 2\delta + \lambda) \left[\frac{k_G}{\lambda} (T_1 - T_2) + \nabla F_{12} (T_1^4 - T_2^4) \right] \quad (2)$$

where λ is the thickness of the gas space, ∇ is the Stefan-Boltzman constant, and F_{12} is the standard view factor for parallel planes or concentric cylinders. Similarly, for heat transfer through the second shield, we have

$$\frac{q}{L} = \frac{k_s}{\delta} (T_2 - T_3) (D_o + 2\delta + 2\lambda + \delta) \quad (3)$$

etc.

Since the shields are spaced closely together, the difference between the temperatures of adjacent shields is small and the term $T_i^4 - T_{i+1}^4$ can be expanded in a Binomial series and truncated after the second term without introducing significant errors:

$$T_i^4 - T_{i+1}^4 \approx T_i^4 - T_i^3 T_{i+1} \quad (4)$$

Using this form, Equation (2) can be written

$$\begin{aligned} \frac{q}{L} &= \left[\frac{k_G}{\lambda} (T_1 - T_2) + \nabla F_{12} T_1^3 (T_1 - T_2) \right] (D_o + 2\delta + \lambda) \\ &= \left(\frac{k_G}{\lambda} + \nabla F_{12} T_1^3 \right) (T_1 - T_2) (D_o + 2\delta + \lambda) \end{aligned} \quad (5)$$

Equations (1) and (5) and similar ones can be solved for the temperature difference as follows:

$$T_o - T_1 = \frac{\frac{q}{L}}{\frac{k_s}{\delta} (D_o + \delta)} \quad (6)$$

$$T_1 - T_2 = \frac{\frac{q}{L}}{\left(\frac{k_{1,2}}{\lambda} + \nabla F_{1,2} T_1^3 \right) (D_o + 2\delta + \lambda)} \quad (7)$$

$$T_2 - T_3 = \frac{\frac{q}{L}}{\frac{k_s}{\lambda} (D_o + 3\delta + 2\lambda)} \quad (8)$$

$$T_3 - T_4 = \frac{\frac{q}{L}}{\left(\frac{k_{3,4}}{\lambda} + \nabla F_{3,4} T_3^3 \right) (D_o + 4\delta + 3\lambda)} \quad (9)$$

where $k_{i, i+1}$ denotes the thermal conductivity of the gas between the i th and the $i+1$ shields.

$$T_4 - T_5 = \frac{\frac{q}{L}}{\frac{k_s}{\delta} (D_o + 5\delta + 4\lambda)} \quad (10)$$

etc.

A summation of the equations yields

$$\begin{aligned} T_o - T_N &= \delta \frac{q}{L} \sum_{n=0}^N \frac{1}{k_n [D_o + (2n+1)\delta + 2n\lambda]} \\ &+ \frac{q}{L} \sum_{i=0}^{N-1} \frac{1}{[D_o + 2_i\delta + (2_i - 1)\lambda] \left[\frac{k_{i, i+1}}{\lambda} + \nabla F_{i, i+1} T_i^3 \right]} \\ &= \frac{q}{L} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n [D_o + (2n+1)\delta + 2N\lambda]} \right. \\ &\quad \left. + \sum_{i=1}^{N-1} \frac{1}{[D_o + 2_i\delta + (2_i - 1)\lambda] \left[\frac{k_{i, i+1}}{\lambda} + \nabla F_{i, i+1} T_i^3 \right]} \right\} \quad (11) \end{aligned}$$

Now the heat transfer across the N layers of thermal shields can be written in terms of an "effective" thermal conductivity, K_{eff} :

$$\frac{q}{L} = \frac{q}{A} (D_o + d) = \frac{K_{\text{eff}}}{d} (D_o + d) (T_o - T_N) \quad (12)$$

or

$$T_o - T_N = \frac{q}{L} \left(\frac{d}{D_o + d} \right) \left(\frac{1}{K_{\text{eff}}} \right) \quad (13)$$

Comparing with Equation (11) yields

$$\begin{aligned} \frac{1}{K_{\text{eff}}} = \frac{D_o + d}{d} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n [D_o + (2n+1)\delta + 2n\lambda]} \right. \\ \left. + \sum_{i=1}^{N-1} \frac{1}{[D_o + 2_i\delta + (2_i - 1)\lambda] \left[\frac{k_{i,i+1}}{\lambda} + \nabla F_{i,i+1} T_i^3 \right]} \right\} \quad (14) \end{aligned}$$

But the overall thickness of the laminates, d , is a function of the number of shields and the thickness of the shields and gas spaces

$$d = N\delta + (N-1)\lambda \quad (15)$$

whence

$$\frac{1}{K_{\text{eff}}} = \frac{D_o + N\delta + (N-1)\lambda}{N\delta + (N-1)\lambda} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n [D_o + (2n+1)\delta + 2n\lambda]} \right. \\ \left. + \sum_{i=1}^{N-1} \frac{1}{[D_o + 2_i\delta + (2_i-1)\lambda] \left[\frac{k_{i,i+1}}{\lambda} + \nabla F_{i,i+1} T_i^3 \right]} \right\} \quad (16)$$

Equation (16) gives the effective thermal conductivity of multilayered heat shields as a function of the number of shields, the thickness of the shields and gas spaces, and the thermal conductivity of the shield and the gas. The application of this equation is quite involved; however, it can be readily programmed for computer analysis. This general equation must be used when D_o is small and N relatively large, i. e., small tubular shields. However, in many applications, Equation (16) can be simplified. Some of these special cases shall be considered below:

Operation in a Vacuum

For this case, conduction across gas films is eliminated, and Equation (16) reduces to

$$\frac{1}{K_{\text{eff}}} = \frac{D_o + N\delta + (N-1)\lambda}{N\delta + (N-1)\lambda} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n [D_o + (2n+1)\delta + 2n\lambda]} \right. \\ \left. + \sum_{i=1}^{N-1} \frac{1}{[D_o + 2_i\delta + (2_i-1)\lambda] [\nabla F_{i,i+1} T_i^3]} \right\} \quad (17)$$

Parallel Planes

If the multilayers of shielding consists of parallel planes or if D_o is large and N is relatively small, Equation (16) can be simplified considerably. In these cases, it can be readily shown that Equation (16) reduces to

$$\frac{1}{K_{\text{eff}}} = \frac{1}{d} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n} + \sum_{i=1}^N \frac{1}{\frac{k_{i,i+1}}{\lambda_{i,i+1}} + \nabla F_{i,i+1} T_i^3} \right\} \quad (18)$$

For the case of operation in a vacuum, Equation (18) can be further simplified to give

$$\frac{1}{K_{\text{eff}}} = \frac{1}{d} \left\{ \delta \sum_{n=0}^N \frac{1}{k_n} + \sum_{i=1}^N \frac{1}{\nabla F_{i,i+1} T_i^3} \right\} \quad (19)$$