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Status of ARPA-LLL 4-D Propagation Code Development

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May 22, 1974

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Status of ARPA-LLL 4-D Propagation Code Development

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A four-dimensional description of atmospheric laser propagation is required in three regimes of interest, each of which requires a fundamentally different approach. These are

1. Quasi-CW pulse propagation in the presence of stagnation or dead zones but without supersonic slewing.
2. Long pulse propagation with supersonic slewing.
3. Multipulse propagation.

A code applicable to all three of these categories is now complete and is ready for general use. Additions to the code will still be needed for specific scenario applications.

Whenever changes in the pulse structure are slow on the time scale of the transit of an acoustic wave across the beam, one can use the infinite sound speed approximation for density changes which are then governed by the equation

$$-c_s^2 \left(\frac{\partial \delta \rho}{\partial t} + v_x \frac{\partial \delta \rho}{\partial x} \right) = (\gamma - 1)(\rho_0 \dot{Q}) \quad (1)$$

In this equation c_s is the sound speed, v_x is the local wind velocity assumed without loss of generality to be in the x-direction, γ is the adiabatic constant, ρ_0 the density of unperturbed air, and $\dot{Q}$ is the heating rate per unit mass. When kinetic cooling is unimportant, the quantity $\rho_0 \dot{Q}$ may also be represented as

$$\rho_0 \dot{Q} = \alpha I \quad , \quad (2)$$

where α is the absorption coefficient, and I is the laser intensity.

One wishes to solve Eq. (1) in conjunction with the parabolic propagation equation

$$2ik \frac{\partial E}{\partial z}(t, x, y, z) = \nabla_{\perp}^2 E + k^2 \left(\frac{\partial \epsilon}{\partial \rho} \right) \delta \rho E \quad (3)$$

In Eq. (3) $E(t, x, y, z)$ represents a slowly varying complex electric field amplitude, $\nabla_{\perp}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian operator, and $(\partial \epsilon/\partial \rho)$ is the derivative of permittivity with respect to density. In both Eqs. (1) and (3) the variable t represents a retarded time, i.e. a time measured from the time the front of the pulse arrives at the axial position z . Equation (3) is solved by marching the entire pulse history one axial space step at a time down the z -axis.

The axial advancement of the solution to (3) may be represented formally in the form

$$E^{n+1} = e^{-\frac{i\Delta z}{4K} \nabla_{\perp}^2} e^{-\frac{i\Delta z}{2K} \chi} e^{-\frac{i\Delta z}{4K} \nabla_{\perp}^2} E^n \quad (4)$$

Where E^n and E^{n+1} are vectors representing the entire solution at the z -positions z_n and $z_n + \Delta z$ respectively, and the susceptibility χ depends linearly on the density. Expression (4) is second order accurate with respect to commutation errors between $\nabla_{\perp}^2$ and χ . The free propagation steps in (4) are based on a Fourier transform solution and the use of the fast Fourier transform algorithm.

The phase incrementation step in the solution of (4) requires a numerical solution of Eq. (1) which possesses the properties of accuracy, smoothness, and stability. Finite difference solutions to Eq. (1) are at best only second order accurate and are normally restricted by the condition that

$v_x \Delta t \leq \Delta x$, which is unacceptable for the applications which we have in mind, since along a given propagation trajectory a range of wind conditions may be encountered varying from stagnation to one where the wind may cross the beam in a fraction of an integration time step. A solution of Eq. (1) based on numerical Fourier transforms, on the other hand, appears to possess all of the necessary requirements of accuracy, stability, and flexibility.

Let us write Eq. (1) in the form

$$\frac{\partial}{\partial t} \delta \rho + v_x \frac{\partial \delta \rho}{\partial x} = f \quad (5)$$

If we take

$$\delta \rho = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} \phi_n(t) e^{\frac{2\pi i n x}{L}} \quad (6)$$

$$f = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} f_n(t) e^{\frac{2\pi i n x}{L}}$$

then $\phi_n(t)$ must satisfy the ordinary differential equation

$$\frac{\partial \phi_n}{\partial t} + \frac{2\pi i}{L} n v_x \phi_n = f_n(t) \quad (7)$$

If Eq. (7) is integrated from t^n to $t^{n+1} = t^n + \Delta t$, the following result is obtained

$$\phi_n(t^{m+1}) = \phi_n(t^m) e^{-2\pi i n v_x \Delta t} + \int_{t^m}^{t^{m+1}} f_m(t') e^{-\frac{2\pi i n v_x (t-t')}{L}} dt' \quad (8)$$

If $f_n(t')$ is assumed to vary linearly with t over the interval, the solution

(8) can be expressed in the form

$$\phi_n(t^{m+1}) = \phi_n e^{-\frac{2\pi i v_x \Delta t}{L}} + A_n f_n(t^m) + B_n f_n(t^{m+1}) \quad (9)$$

where

$$A_n = \frac{L}{2\pi i v_x} \left[\left(\frac{L}{2\pi i v_x \Delta t} \right) \left(1 - e^{-\frac{2\pi i v_x \Delta t}{L}} \right) - e^{-\frac{2\pi i v_x \Delta t}{L}} \right]$$

$$B_n = \frac{L}{2\pi i v_x} \left[1 - \left(\frac{L}{2\pi i v_x \Delta t} \right) \left(1 - e^{-\frac{2\pi i v_x \Delta t}{L}} \right) \right] \quad (10)$$

The numerical evaluation of the Fourier transform (9) will be discussed in a later report. The accuracy of the solution is independent of time step Δt as long as $v_x \Delta t = n\Delta x$; the only significant approximation in the solution obtained from (9) is the assumption of linearity in $f(t)$. In other words the solution is essentially exact for a ramp source in time. A quasi CW laser pulse, i.e. one of ~ 1 second in duration can be represented as a piecewise linear function in time and the solution (9) used to define the density.

The evaluation of Eq. (4) is now obtained by successively advancing the density and then the field for each time in the pulse.

It is also of interest to apply this method to the case of a multiple pulse laser. In that case the function $f(\underline{r}, t)$ can be expressed to good approximation as

$$f(\underline{r}, t) = \sum_{m=1}^M \delta(t - t_m) g^m(\underline{r}) \quad (11)$$

Writing ϕ_n as

$$\phi_n = \int_0^t f_n(t') e^{-\frac{2\pi i \nu_x}{L} (t-t')} dt' \quad (12)$$

and utilizing (11), we obtain

$$\phi_n(t) = \sum_{m=1}^{m'} g_n^m e^{-\frac{2\pi i \nu_x}{L} (t-t_m)} \quad (13)$$

for $t^{m'} < t < t^{m'+1}$.

For $t = t^{m+1}$ we may write Eq. (13) as

$$\phi_n(t^{m+1}) = \phi_n(t^n) e^{-\frac{2\pi i \nu_x \Delta t}{L}} + g_n^{m+1} \quad (14)$$

It will be noted that Eq. (14) can be represented by the same computational algorithm for the Fourier component $\phi_n(t^{m+1})$ as is used in Eq. (9) provided $A_n = 0$ and $B_n = 1$. However, in applying Eq. (14) one must be careful not to count the contribution g_n^{m+1} when blooming the "current" pulse in the train, since that will lead to an overestimate of the blooming effect of a given pulse on itself. It is better to ignore g_n^{m+1} when the pulse at t^{m+1} is bloomed and include the density contribution afterwards.

It has been found by running nonlinear hydrodynamics calculations with an existing LLL Eulerian code using a typical laser heat source and a sonic wind that the density distribution comes to a steady state in the order of a (sonic) wind transit time, and that, furthermore, no shocks are generated. Indeed density changes are generally confined to about 5 orders of magnitude below normal density air. This suggests that one can describe the flow in

connection with near sonic winds or slewing by means of the steady state linearized hydro equations

$$v_x \frac{\partial \rho_1}{\partial x} = -\rho_0 \nabla \cdot v_1 \quad (15)$$

$$\rho_0 v_x \frac{\partial v_1}{\partial x} = - \frac{\partial p_1}{\partial x} \quad (16)$$

$$v_x \frac{\partial}{\partial x} (p_1 - c_s^2 \rho_1) = -(\gamma - 1) \rho_0 \dot{Q} \quad (17)$$

These equations are easily solved and a routine for solving them is now incorporated in the computer code. Near sonic slewing can be treated as a succession of steady states each described by Eqs. (15)-(17).

Numerical Examples

The figures which follow illustrate the performance of the code and are for the most part self-explanatory. Shown are plots of intensity and (negative) density along the wind axis, intensity, density, and fluence contours at different times. Figures labeled 1 and 2 refer, respectively, to time dependent calculations for the equilibration to a steady state of a CW pulse and a train of short pulses. Figures labeled 3 refer to steady state calculations in the transonic regime. For both Figs. 1 and 2 the initial average beam power is 100 KW concentrated in a Gaussian beam having a 1/e radius of 14.4 cm and focused at a distance of .6 Km. The focusing of the beam is contained directly within the calculation, i.e. no transformation has been used to remove the converging wave front imposed by the lens. The absorption coefficient is $.2 \times 10^{-5} \text{ cm}^{-1}$. The assumed wind speed is 4.69 m/sec. For the CW beam time increments are taken to be .02 sec, and the calculation is carried on until time $t = .14 \text{ sec}$. In the case of the

multi-pulse beam, pulses are assumed to be separated by .02 sec and to be 32 μ sec in duration and to have a peak intensity of 100 Kw/cm², yielding the same average power as the CW case.

In the transonic calculations the sound speed is taken to be 3.45×10^4 cm/sec and the normal viscosity of air is used. One calculation refers to the slightly supersonic case of 3.46×10^4 cm/sec, the other to the slightly subsonic case of 3.40×10^4 cm/sec. The exact transonic case is of course unstable in the linearized approximation, and will not support a steady state. The slightly supersonic case leads to a slight compression and a deflection of the beam in the direction of the wind. The slightly subsonic case leads to a rarefaction and a deflection in a direction opposite to the wind. For the case of large Mach numbers, e.g. 2-3, the downstream air is again rarified and deflection returns to the upwind side. In both the calculations of Fig. 3 a lens transformation is used leading to a dilating grid. Focusing is assumed at 2 Km.

We are also developing a capability for solving the exact hydrodynamic equations in the acoustic approximation using a fast Fourier transform. The numerical problem of inverting the transform in one dimension for a ramp heat source has been solved. This solution is now being extended to 2-dimensions.

1.1

1.8

1.10

6.13 1.13

1.14

1.15

1.18

1.22


```

LAMBDA=1.E-3
IMP=1
ETA=1.
LMAX=7
ICASE=1
ZF=6.E5
SIGX=14.14
SIGY=14.14
LX=100.
LY=100.
IMAX=.1
BTIMP=32.E-6
DT=.02
ISYMM=1
WI(1)=1.
WI(2)=1.
WI(3)=1.
WI(4)=1.
WI(5)=1.
WI(6)=1.
WI(7)=1.
NELEM=1
DZ=.073E5
NEDIT=20
NCONT=20
NFL=500.
NF=41
SIGNA=.2E-5
NFFT=64
END ARMY MULTIPULSE PROBLEM

```

04

1.9

2.10

17

2.12

44

9th

2.16

2.18

Y-COORDINATE IN CM.

Y-COORDINATE IN CM.

ISOINTENSITY CONTOURS

Z = $1.5000\text{E}+05$ T (SEC) = $2.5000\text{E}-01$

(NEGATIVE) DENSITY

Y-COORDINATE IN CM.

ISOINTENSITY CONTOURS

Z = $1.5000E+05$ T (SEC) = $2.5000E-01$

3.4

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