STRESS ANALYSIS OF A RADIAL NOZZLE ATTACHED TO A CYLINDRICAL SHELL UNDER INTERNAL PRESSURE*

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ABSTRACT

An analytical method for the determination of stresses in the region of intersection of a thin-walled composite shell made from two right circular cylinders intersecting with axes perpendicular and subjected to internal pressure is presented. The governing equations are based on shallow, thin-shell theory and are limited to a ratio of cylinder radii of one-third or less. The formulations are written in terms of dimensionless groups to facilitate use of the analysis method in conducting parameter studies.

Comparisons are made between the results obtained using the method and experimental data. The results of this investigation indicate that the analysis should prove to be a useful tool for conducting parametric studies within the specified range of validity.

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INTRODUCTION

The development of a method for calculating stresses in the region of intersection of two right circular cylinders whose axes intersect at right angles and are subjected to various loads including internal pressure has been a formidable problem for some time. Attempts, by various researchers, along lines similar to those presented here have taken place in recent years, and some of these have found their way into the open literature. The solution of the intersection problem was initially made possible as a result of the solution by Lur² [1] in 1947 of the cylindrical shell with a circular cutout. The primary deficiency of this work was due to the perturbation solution involving a power series in the shell curvature parameter, $\beta\rho_o$, where

$$(\beta\rho_o)^2 = \frac{[3(1 - \nu^2)]^{1/2}}{4} \left(\frac{\rho_o}{R}\right)^2 \left(\frac{R}{T}\right), \quad (1)$$

which limits the solution to small values of $\beta\rho_o$. Withum [2] extended the results in 1958 to larger values of the curvature parameter, and Eringen et al. [3] used this technique for the solution of a pressurized shell with a circular cutout in 1961. Naghdi and Eringen [4] in 1963 produced a solution containing the products of Krylov functions and Hankel functions of the first kind. A collocation method was used whereby the boundary conditions were satisfied in a least-squares sense at a selected number

²Numbers in brackets designate References at end of paper.

of points along the boundary Van Dyke [5] in 1965 worked along similar lines, while Lekkerkerker [6] used Fourier series involving Bessel functions which represented the Krylov functions; hence, each term in the Fourier series satisfied the boundary conditions along the cutout. These latter works are all similarly limited by (1) thin-shell theory, and (2) shallow-shell theory, which implied that the transverse shear along the generating curve is small in comparison with changes of the in-the-surface forces. The latter item mentioned does not seem to be an unreasonable assumption judging from previous experience reported in the literature. These two limitations will be discussed in more detail later. Reidelbach [7] in 1961 presented a serious attempt at treating cylinder-to-cylinder intersections under internal pressure loading. The most significant contribution of this work lies in the boundary conditions used at the intersection of the two cylinders. The boundary conditions presented in Reidelbach's paper represent a fundamental part of all the subsequent work concerning the normal intersection of two cylinders. However, even in the form used by Reidelbach, there is some simplification. The approximation of the governing differential equation of the intersected cylinder limits the usefulness of his development, although it is difficult to accurately assess the error involved. Reidelbach presented a numerical example but did not compare it with experimental work. In 1965, Eringen and Suhubi [8] presented equations for the same configuration by

combining the Krylov-Hankel function type solution of the intersected cylinder with a solution of the intersecting cylinder based on the work of Green and Zerna [9], which is equivalent to that of Hoff [10], and using the boundary conditions of Reidelbach. Although no numerical work or comparisons were presented, the solution using a numerical collocation technique was implied. Eringen et al. [11], in 1969, dealing with the same problem, presented numerical results from the previous development, which was modified by representing the Krylov functions as Fourier series. This eliminated the need of the collocation technique. In addition to the inherent limits imposed by the theory, the analysis method was further limited to $\beta p_0 < 0.5$ by an arbitrary truncation of the Fourier series to a few terms. During the period in which the work presented in this report was taking place, Maya and Eringen [12], in 1970, presented another theoretical analysis method for the intersecting shells in a concise form. Unfortunately that is all it is since, again, there are no numerical results or comparisons. Although the work presented in this paper was performed by the author [13] during the same period, considerable time has been spent in computer program writing and debugging so that the value of the analysis method can be evaluated by comparison with experimental work. The best results of this effort are included here and are compared with experimental results.

THEORY

The geometry of the middle surfaces of the two intersecting

cylinders is shown in Fig. 1. The thickness of the nozzle (intersecting cylinder) is t and the thickness of the shell (intersected cylinder) is T . The ratio of the radii, $m = \rho_0/R$, together with the parameters ρ_0/t and R/T characterize the intersecting shell configuration. The midsurface radius, ρ_0 , of the nozzle will be used to construct a nondimensional coordinate system such that y/ρ_0 , x/ρ_0 , ϕ , and, eventually, ρ/ρ_0 (after transformation of the shell coordinates to polar form) become the independent variables. Thus

$$\begin{aligned}\bar{y} &= y/\rho_0, \\ \bar{x} &= x/\rho_0, \\ \bar{\rho} &= \rho/\rho_0.\end{aligned}\tag{2}$$

The coordinate systems and the associated convention for the displacements and stress resultants are shown in Fig. 2 for both the nozzle and the shell.

The differential equations describing the separate behavior of the nozzle and of the shell may be written, within the limits of shallow, thin-shell theory, as single complex differential equations. For the nozzle, the single equation is

$$\nabla^4 \bar{\Omega} + 18\alpha^2 \bar{\Omega}_{,yy} = \frac{-12(1-\nu^2)}{Et^3\rho_0} \bar{p},\tag{3}$$

where

$$\bar{\Omega} = \bar{w} + ikf,$$

$$\alpha^2 = \frac{[3(1 - \nu^2)]^{1/2}}{4\rho_0 t}$$

$$k = \frac{[12(1 - \nu^2)]^{1/2}}{E\rho_0 t^2}$$

$$\bar{w} = \frac{w}{\rho_0}$$

The function $f(y, \phi)$ is such that

$$f_{,yy} = n_\phi$$

$$\frac{1}{\rho_0^2} f_{,\phi\phi} = n_y \quad (4)$$

$$-\frac{1}{\rho_0} f_{,y\phi} = n_{y\phi}$$

which is necessary to satisfy the following simplified equilibrium equations of Donnell:

$$n_{y,y} + \frac{1}{\rho_0} n_{y\phi,\phi} = 0$$

$$\frac{1}{\rho_0} n_{\phi,\phi} + n_{y\phi,y} = 0 \quad (5)$$

For the shell, the following single equation applies:

$$\nabla^4 \psi + 18\beta^2 \psi_{,xx} = -\frac{12(1 - \nu^2)}{ET^3R} P \quad (6)$$

where

$$\psi = \bar{w} + 1KF$$

$$\beta^2 = \frac{[3(1 - \nu^2)]^{1/2}}{4RT}$$

$$K = \frac{[12(1 - \nu^2)]^{1/2}}{E\rho_0 T^2}$$

$$\bar{W} = \frac{W}{\rho_0} ,$$

the function $F(\rho, \phi)$ is such that

$$N_{\rho} = \frac{1}{\rho^2} F_{,\phi\phi} + \frac{1}{\rho} F_{,\rho} ,$$

$$N_{\phi} = F_{,\rho\rho} , \quad (7)$$

$$N_{\rho\phi} = - \frac{1}{\rho} F_{,\phi} ,_{\rho} ,$$

which is necessary to satisfy the simplified Donnell equations in polar coordinates.

The governing differential equations are solved by first obtaining the particular solutions, representing the membrane state of stress or the stress in regions of the shells which are unaffected by the behavior at the intersection. The particular solutions are then added to the homogeneous solution representing the bending behavior of the nozzle and shell. The particular solution for the nozzle is

$$\begin{aligned} \frac{w'}{\rho_0} &= \frac{P\rho_0}{2Et} (\nu-2) , \\ u' &= \frac{P\rho_0}{2Et} (1-2\nu) y , \\ v' &= 0 , \end{aligned} \quad (8)$$

$$n'_y = \frac{P\rho_o}{2} ,$$

$$n'_\phi = P\rho_o , \quad (9)$$

$$n'_{y\phi} = m'_y = m'_\phi = m'_{y\phi} = 0 .$$

For the shell, the particular solution is

$$\frac{W'}{\rho_o} = \frac{P}{2E} \frac{R}{\rho_o} \frac{R}{T} (\nu-2) ,$$

$$U'_\rho = \frac{PR}{4ET} (1-2\nu) (1 + \cos 2\phi) \rho , \quad (10)$$

$$U'_\phi = - \frac{PR}{4ET} (1-2\nu) \rho \sin 2\phi ,$$

$$N'_\rho = \frac{PR}{4} (3 - \cos 2\phi) ,$$

$$N'_\phi = \frac{PR}{4} (3 + \cos 2\phi) ,$$

$$N'_{\rho\phi} = \frac{PR}{4} \sin 2\phi , \quad (11)$$

$$M'_\rho = M'_\phi = M'_{\rho\phi} = 0 .$$

These results are written here in terms of coordinates transformed from the cylindrical (x,θ) system into a polar coordinate (ρ,ϕ) system. Both the nozzle and shell are assumed to be capped at the ends.

The homogeneous solution to Eq. (3) can be obtained for the

nozzle by first writing the equation in factored form such as

$$\left[\nabla^2 - 2(1 - i) \alpha \frac{\partial}{\partial y} \right] \left[\nabla^2 + 2(1 - i) \alpha \frac{\partial}{\partial y} \right] \Omega = 0 . \quad (12)$$

Each parenthesized term operating on Ω can be set equal to zero which results in

$$\left[\nabla^2 - 2(1 - i) \alpha \frac{\partial}{\partial y} \right] \Omega^1 = \left[\nabla^2 + 2(1 - i) \alpha \frac{\partial}{\partial y} \right] \Omega^2 = 0 , \quad (13)$$

where $\Omega = \Omega^1 + \Omega^2$. These equations can be solved by assuming their solution to be in the form of the product of two functions, one a function of y and the other a function of ϕ , as follows:

$$\Omega^1(y, \phi) = g^{(1)}(y) h^{(1)}(\phi) \quad (14)$$

and

$$\Omega^2(y, \phi) = g^{(2)}(y) h^{(2)}(\phi) . \quad (15)$$

Substitution of these assumed product solutions into the previous two equations yields

$$\begin{aligned} \Omega = \sum_{n=0}^{\infty} \sum_{j=1}^4 \left[(A_{n,j} + iB_{n,j}) \cos \omega_{n,j} \bar{y} \right. \\ \left. + i (A_{n,j} + iB_{n,j}) \sin \omega_{n,j} \bar{y} \right] e^{\pi_{n,j} \bar{y}} \cos n\phi , \end{aligned} \quad (16)$$

where $\omega_{n,j}$ and $\pi_{n,j}$ are found in the appendix.

In order to assure that the bending effects of the intersection are absent in the nozzle at a sufficient distance from the region of influence of the intersection, the coefficients $A_{n,3}$, $A_{n,4}$, $B_{n,3}$, $B_{n,4}$ will be set equal to zero. This insures that Ω

diminishes as $\bar{y}$ increases since $\pi_{n,3}$ and $\pi_{n,4}$ being positive would result in an exponentially increasing behavior. Since the pressure loading represents an even function with regards to both the longitudinal and transverse planes passing through the intersection, only the even values of n will give an acceptable solution to the preceding equation. Thus

$$\bar{w} = \sum_{n=0}^{\infty} \sum_{j=1}^2 (A_{2n,j} \cos \omega_{2n,j} \bar{y} - B_{2n,j} \sin \omega_{2n,j} \bar{y}) e^{\pi_{2n,j} \bar{y}} \cos 2n\phi, \quad (17)$$

and

$$f = \frac{1}{k} \sum_{n=0}^{\infty} \sum_{j=1}^2 (B_{2n,j} \cos \omega_{2n,j} \bar{y} + A_{2n,j} \sin \omega_{2n,j} \bar{y}) e^{\pi_{2n,j} \bar{y}} \cos 2n\phi. \quad (18)$$

The stress resultants can then be evaluated with the use of Eqs. (17) and (18). The displacements of the nozzle in its longitudinal and circumferential directions may be obtained by integrating the stress-strain relationships

$$\begin{aligned} \frac{1}{\rho_0} v_{,\phi} &= \frac{1}{Et} (n_{\phi} - \nu n_y) + \bar{w}, \\ v_{,y} + \frac{1}{\rho_0} u_{,\phi} &= \frac{2}{Et} (1 + \nu) n_{y\phi}, \\ u_{,y} &= \frac{1}{Et} (n_y - \nu n_{\phi}). \end{aligned} \quad (19)$$

and selecting the boundary conditions as $v = 0$, $u = E_0$ at $y = 0$.

The constant E_0 represents the displacement of the coordinate system of the nozzle resulting from expansion of the shell under the influence of the pressure loading.

The governing differential equation for the shell, Eq. (6), can be written in its factored form for the purpose of obtaining the homogeneous solution.

$$\left[\nabla^2 - 2(1 - i)\beta \frac{\partial}{\partial x} \right] \left[\nabla^2 + 2(1 - i)\beta \frac{\partial}{\partial x} \right] \psi = 0 \quad (20)$$

Assuming that $\psi = \psi^1 + \psi^2$ where

$$\begin{aligned} \psi^1 &= e^{(1 - i)\beta x} \pi^1(x, \theta) \\ \psi^2 &= e^{-(1 + i)\beta x} \pi^2(x, \theta) \end{aligned} \quad (21)$$

and substituting these expressions into the governing equation, the relationship

$$[\nabla^2 \pi^1 + 12\beta \pi^1] = [\nabla^2 \pi^2 + 12\beta \pi^2] = [\nabla^2 \pi + 12\beta \pi] = 0 \quad (22)$$

is obtained. The solution for ψ can then be written as a linear combination of all the π 's. If the approximation is made that

$$\nabla^2 = \frac{\partial^2}{R^2 \partial \theta^2} + \frac{\partial^2}{\partial X^2} \approx \frac{\partial^2}{\partial Z^2} + \frac{\partial^2}{\partial X^2} \quad (23)$$

which can then be expressed in polar coordinate form as

$$\nabla^2 = \frac{1}{\rho^2} \frac{\partial}{\partial \phi^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \rho^2}, \quad (24)$$

the resulting differential equation becomes

$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 12\beta \right] \pi(\rho, \phi) = 0 \quad (25)$$

This equation can be solved by a product solution of the form

$$\pi(\rho, \phi) = H(\rho) G(\phi) \quad (26)$$

Substitution of the polar Laplacian operator into the equation limits the range of parameters over which this solution is applicable. This limitation will be discussed in a later section. Finally,

$$\Psi = \sum_{n=0}^{\infty} \left\{ C_n' \cosh [(1 + i)\beta \rho_0 \bar{\rho} \cos \phi] \right. \quad (27)$$

$$\left. + D_n' (1 + i) \sinh [(1 + i)\beta \rho_0 \bar{\rho} \cos \phi] \right\} H_n^{(1)}(\sqrt{2i} \beta \rho_0 \bar{\rho}) \frac{\sin n\phi}{\cos n\phi} ,$$

where $H_n^{(1)}(\sqrt{2i} \beta \rho_0 \bar{\rho})$ is the Hankel function of the first kind.

The coefficients on the Hankel functions of the second kind have been set equal to zero so that the solution vanishes as $\bar{\rho}$ becomes large. Following the suggestion of Lekkerkerker [6], the hyperbolic relationships are replaced by the following infinite series:

$$\cosh [(1 - i)\beta \rho_0 \bar{\rho} \cos \phi] = \sum_{m=0}^{\infty} \gamma_m \cos m\phi \quad (28)$$

$$(1 + i) \sinh [(1 - i)\beta \rho_0 \bar{\rho} \cos \phi] = \sum_{m=0}^{\infty} \delta_m \cos m\phi ,$$

so that Ψ may be written as a function of ρ times a function of ϕ . Noting the symmetry of the pressure loading, the solution

can finally be written as

$$\psi = \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} [(\alpha_{n,\ell} C_{\ell} + \beta_{n,\ell} D_{\ell}) + i(-\beta_{n,\ell} C_{\ell} + \alpha_{n,\ell} D_{\ell})] \cos 2n\phi, \quad (29)$$

where $\alpha_{n,\ell}$ and $\beta_{n,\ell}$ are combinations of Bessel and Hankel functions whose complex arguments depend upon the shell parameters $\beta\rho_0$ and $\bar{\rho}$ only. The values of $\alpha_{n,\ell}$ and $\beta_{n,\ell}$ are defined in the appendix.

Equation (29) can then be conveniently written as

$$\psi = \sum_{n=0}^{\infty} (X_n + iY_n) \cos 2n\phi, \quad (30)$$

where

$$X_n = \sum_{\ell=0}^{\infty} \alpha_{n,\ell} C_{\ell} + \beta_{n,\ell} D_{\ell},$$

$$Y_n = \sum_{\ell=0}^{\infty} -\beta_{n,\ell} C_{\ell} + \alpha_{n,\ell} D_{\ell}.$$

The displacements of the shell due to the local bending can be obtained from the following equations which represent the stress-strain relationships in polar coordinates.

$$\frac{1}{\rho_0} u_{\rho, \bar{\rho}} = \frac{1}{ET} [N_{\rho} - \nu N_{\phi}] + \frac{W}{2R} (1 - \cos 2\phi) \quad (31)$$

$$\frac{1}{\rho_0} \left[\frac{u_{\phi, \phi}}{\bar{\rho}} + \frac{u_{\rho}}{\rho} \right] = \frac{1}{ET} [N_{\phi} - \nu N_{\rho}] + \frac{W}{2R} (1 + \cos 2\phi)$$

$$\frac{1}{\rho_0} \left[\frac{u_{\rho, \phi}}{\rho} + u_{\phi, \rho} - \frac{u_{\phi}}{\rho} \right] = \frac{2(1+\nu)}{ET} N_{\rho\phi} + \frac{W}{R} \sin 2\phi$$

(31 cont'd)

Using Eq. (30), the stress resultants for the shell may be obtained, completing the solution of the homogeneous equation. Having then the complete solutions for the nozzle and shell differential equations, all that remains is the proper substitutions into the boundary conditions at the intersection so that the constants of integration may be evaluated.

BOUNDARY CONDITIONS

The geometry of the intersection is clearly described in Reidelbach's [7] work of 1961. The eight equations describing the compatibility at the juncture are

$$U_{\phi} - \nu + u \frac{m}{2} C \sin 2\phi = 0, \quad (32)$$

$$U_{\rho} + u \frac{m}{2} (1 - \cos 2\phi) + w = 0, \quad (33)$$

$$W + \nu \frac{m}{2} C \sin 2\phi + u - w \frac{m}{2} (1 - \cos 2\phi) = 0, \quad (34)$$

$$N_{\rho} - n_y \frac{m}{2} (1 - \cos 2\phi) - q_{\phi} \frac{m}{2} C \sin 2\phi - q_y = 0, \quad (35)$$

$$N_{\rho\phi} + n_{y\phi} + (n_{\phi} - n_y) \frac{m}{2} C \sin 2\phi = 0, \quad (36)$$

$$Q_{\rho} + \frac{m}{2} N_{\rho} (1 - \cos 2\phi) - (n_y + n_{y\phi} m C \sin 2\phi) = 0, \quad (37)$$

$$M_\rho - m_y - m_y \phi \frac{m}{2} C \sin 2\phi = 0 , \quad (38)$$

$$\rho_0 w_{,\rho} + \rho_0 w_{,y} + w_{,\phi} \frac{m}{2} C \sin 2\phi = 0 , \quad (39)$$

where

$$C = \frac{1}{\sqrt{1 - m^2 \sin^4 \phi}} . \quad (40)$$

The first three equations, (32), (33), and (34), result from displacement considerations, while the next three, (35), (36), and (37), result from force equilibrium. Equation (38) results from moment equilibrium about an axis tangent to the intersection curve. The last, Eq. (39), represents continuity of slopes. An additional moment equilibrium equation about an axis normal to the intersection yields information equivalent to Eq. (36), and hence is not needed. In reality, the equations given here are approximations because the terms containing powers of m greater than unity have been neglected. Equations (32) through (39) were further simplified in this work by setting the value of C equal to unity. These approximations obviously restrict the applicability of the results, as discussed later.

It should be recalled that the forms of the equations for the nozzle and for the shell were chosen to satisfy the membrane conditions at regions removed from the intersection. Hence, the intersection conditions are all that require further satisfaction.

FINAL EQUATIONS

The substitution of the displacements, stress resultants, and slopes into the boundary conditions results in a number of equations that when solved yield the coefficients $A_{n,j}$, $B_{n,j}$, C_ℓ , and D_ℓ of the series expressions. The final set of equations resulting from the substitution can be written in a rather compact form. Thus,

$$\sum_{j=1}^2 \sum_{m=n}^{n+2} \mu_{m,j}^{n,p} A_{2(m-1),j} + \lambda_{m,j}^{n,p} B_{2(m-1),j} + \sum_{\ell=0}^{\infty} \mu_{\ell}^{n,p} C_{\ell} + \lambda_{\ell}^{n,p} D_{\ell} + \sigma^{n,p} E_0 = \tau^{n,p} \quad (41)$$

The subscripts here are consistent with those used previously. The superscripts are used to identify the particular equation of the intersection conditions, where the value of n is associated with the coefficients, and p , which ranges in value from 1 to 8, is associated with Eqs. (32) through (39), respectively. The coefficients $\mu_{m,j}^{n,p}$, $\lambda_{m,j}^{n,p}$, $\mu_{\ell}^{n,p}$, $\lambda_{\ell}^{n,p}$, $\sigma^{n,p}$, and $\tau^{n,p}$ are tabulated in the previously mentioned work of the author [13]. Also listed are the derivatives of R_{nm} and S_{nm} with respect to $\bar{\rho}$ which are required to determine $\alpha_{n,\ell}$ and $\beta_{n,\ell}$ for the evaluation of the stress resultants in the shell.

In order to complete the equations, it is necessary to choose proper values for the number of terms in the series so

that adequate accuracy is obtained. The choice with reference to Eq. (41) involves $n_{\max}$ and $l_{\max}$. This choice is somewhat arbitrary since the number of terms in the nozzle series does not have to be the same as the number of terms in the shell series; however, it is necessary to choose $n_{\max}$ and $l_{\max}$ such that Eq. (41) yields more equations than unknown A, B, C, and D terms.

REGION OF APPLICABILITY

Several matters discussed in the derivation of the shell equations which relate to the range of parameters over which reasonable results should be expected need to be considered. Taking all of them into consideration should enable one to determine how good the results are.

First of all, the substitution of the Laplacian operator in polar form into the shell equation implies that the intersection curve essentially lies in a plane and has the form of a circle. An examination of the intersection curve after flattening the shell to a plane surface reveals that the curve is nearly circular for $m = \rho_0/R < \frac{1}{2}$; however, a value of $m = 1/3$ appears to represent a much more practical limit.

Another related matter involves the boundary equations at the intersection. Equations (25) through (32) are written in a form which assumes that terms containing m raised to a power greater than unity are small compared with the retained terms.

Therefore, taking the intersection function C to be equal to unity is consistent with this assumption; since C is everywhere multiplied by m , it can be written in series form as

$$C = 1 - \frac{1}{2} m \sin^2 \phi + \frac{3}{8} m^2 \sin^4 \phi + \dots \quad (42)$$

Further consideration of C , however, may give some valuable insight into what one can expect from the limitation of the radius ratio, ρ_0/R . The first two terms in the series representation of C can be easily compared. If m is $1/2$, terms whose magnitude are $1/4$ of those being retained are neglected. Clearly, a limit of $1/3$ on m represents a more practical value.

It is also necessary to consider the shallowness criterion of a shallow shell. Novozhilov [14] suggests that the ratio of rise to base length ($h:L$ as defined in Fig. 3) of 1:5 satisfies the criteria. Van Dyke, following Reissner [15], uses a ratio of 1:8 in his considerations. Ratios ranging between these two apparently represent a gray area. It should also be expected that the bending behavior developed at the intersection should be attenuated within the shallow shell region of the shell. A ratio of 1:10 represents a reasonable criterion for this. All of these matters are considered simultaneously in Fig. 3. The region between the two curves and to the left of $\rho_0/R = 1/3$ represents parameters for which results are questionable but possible. The region to the left of the 1:8

curve and to the left of $\rho_0/R = 1/3$ represents parameters for which reasonable results are expected.

RESULTS AND COMPARISONS

The results of the computer analysis can be evaluated to some extent by comparing them to experimental data. For this to be of significant value, the experimental model must agree geometrically with the mathematical model and it must be carefully tested. Efforts to obtain reliable experimental data were made at ORNL on a steel model designated as the cylinder-to-cylinder model 4 [16]. The geometry is described by the following dimensions and ratios.

Nozzle: Outside diameter = 1.29 in.

Thickness = 0.064 in.

$\rho_0 = 0.613$ in.

$\rho_0/t = 9.58$

Shell: Outside diameter = 10.0 in.

Thickness = 0.2 in.

$R = 4.90$ in.

$R/T = 24.5$

Intersection: $\rho_0/R = 1/8$

It can thus be seen that the model falls into the region in Fig. 3 bordered by the two curves defining the shallowness criterion mentioned before. Dimensionally, the nozzle is also

near the limit for which thinshell theory is applicable. The ratio of the radii, however, is well within the limit of the theory. The theoretical analysis was obtained using 10 harmonics in the nozzle series and 8 in the shell series. This requires 44 of the A and B constants and 32 of the C and D constants in the series. Along with the rigid body displacement E_0 of the shell, 77 unknown constants were evaluated in a "least squares" sense with 80 equations.

The experimental model was instrumented with foil strain gages such that the stresses could be determined on the 0, 45, 90, 180, and 270° planes as defined by the angle ϕ in Fig. 1. The average values for both sets of data on the planes of symmetry, that is, the longitudinal and transverse planes, are plotted along with the theoretical stresses calculated by the programmed analysis. These comparisons are shown on Figs. 4 and 5. Here the stresses due to an internal pressure of 300 psi are plotted vs the normalized distance from the intersection; that is y/ρ_0 and ρ/ρ_0 .

The circumferential stress in the shell in a region undisturbed by the intersection is 7350 lb/in.² calculated by the nominal equation, $s = PR/T$. If this value is used as the normalizing factor, the greatest normalized stress is 5.27; it is the transverse stress on the inside surface of the shell located at $\rho/\rho_0 = 1$ along the longitudinal plane. This is illustrated in Fig. 4(c) where it can be seen that the theoretical curve agrees

quite well with the experimental data.

In general, the theoretical curves, Figs. 4 and 5, agree reasonably well with the experimental data. However, there are a few instances where there is poor agreement between the theoretical curves and the experimental data. The comparisons between the predicted stresses and the experimental stresses could be improved, perhaps, if more terms in the series were used or if the number of equations used in the "least squares" determination of the A, B, C, and D constants could be increased in relation to the number of the constants.

Normally, a "least squares" solution is improved if the system is significantly overdetermined. In this analysis, the number of terms in the series solutions can be chosen somewhat arbitrarily; however, because of the method of generating the coefficient array, as described by Eq. (41), the number of equations will be limited to only slightly larger than the number of constants to be determined. Since computer results taking more series terms than the solution illustrated here in the comparisons did not yield significantly different values, it is felt that the "least squares" solution is responsible for some of the poor agreement between the theoretical and experimental results, particularly in regions slightly removed from the intersection.

SUMMARY

An analytical method for the determination of the stress distribution in the region of the intersection of two right circular cylinders has been developed. The analysis is based on shallow, thin-shell theory and the intersection is assumed to be given by polar coordinates in the plane of the intersection; however, certain characteristics associated with the curvature of the intersected cylinder are not eliminated completely. The analysis is thus limited to geometries where the ratio of the radii of the two cylinders is less than $1/3$.

From the comparisons of the analytically and experimentally determined stress distributions, it can be seen that the analysis results in predicting the maximum stresses that occur at the intersection for the steel model used in these comparisons. The results of this investigation would then indicate that the analysis should prove to be useful for conducting parametric studies within the specified range of validity.

APPENDIX

Solution Parameters

A. Nozzle:

$$\omega_{n,1} = -\alpha\rho_0 - \left[-\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2},$$

$$\omega_{n,2} = -\alpha\rho_0 + \left[-\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2},$$

$$\pi_{n,1} = -\alpha\rho_0 - \left[\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2}$$

$$\pi_{n,2} = \alpha\rho_0 - \left[\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2}, \quad n = 0, 1, 2, \dots$$

$$\omega_{n,3} = \alpha\rho_0 + \left[-\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2},$$

$$\omega_{n,4} = \alpha\rho_0 - \left[-\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2},$$

(43)

$$\pi_{n,3} = \alpha\rho_0 + \left[\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2},$$

$$\pi_{n,4} = -\alpha\rho_0 + \left[\frac{n^2}{2} + \left(\frac{n^4}{4} + \alpha^4\rho_0^4 \right)^{1/2} \right]^{1/2}.$$

B. Shell:

$$\alpha_{0,2\ell} = \frac{(-1)^\ell}{2} R_{0,2\ell},$$

$$\alpha_{0,2\ell+1} = \frac{(-1)^\ell}{2} (R_{0,2\ell+1} + S_{0,2\ell+1}),$$

$$\alpha_{0,2\ell} = (-1)^{n+\ell} R_{2n,2\ell} ,$$

$$\alpha_{n,2\ell+1} = (-1)^{n+\ell} (R_{2n,2\ell+1} + S_{2n,2\ell+1}) ,$$

$$\beta_{0,2\ell} = \frac{(-1)^{\ell+1}}{2} S_{0,2\ell} ,$$

$$\beta_{0,2\ell+1} = \frac{(-1)^{\ell+1}}{2} (-R_{0,2\ell+1} + S_{0,2\ell+1}) , \quad (44)$$

$$\beta_{n,2\ell} = (-1)^{n+\ell+1} S_{2n,2\ell} ,$$

$$\beta_{n,2\ell+1} = (-1)^{n+\ell+1} (-R_{2n,2\ell+1} + S_{2n,2\ell+1}) ,$$

where

$$R_{nm} = \Gamma_m (P_{m-n} + P_{m+n}) - \Delta_m (Q_{m-n} + Q_{m+n}) ,$$

$$S_{nm} = \Gamma_m (Q_{m-n} + Q_{m+n}) + \Delta_m (P_{m-n} + P_{m+n}) ,$$

and the real (R_e) and imaginary (I_m) parts are

$$R_e J_n(\sqrt{2i} \beta \rho_0 \bar{\rho}) = P_n ; \quad I_m J_n(\sqrt{2i} \beta \rho_0 \bar{\rho}) = Q_n ; \quad (45)$$

$$R_e H_n^{(1)}(\sqrt{2i} \beta \rho_0 \bar{\rho}) = \Gamma_n ; \quad I_m H_n^{(1)}(\sqrt{2i} \beta \rho_0 \bar{\rho}) = \Delta_n .$$

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CAPTIONS FOR ILLUSTRATIONS

- Fig. 1 Geometry of middle surfaces of intersecting cylinders
- Fig. 2 Coordinate systems and stress resultants
- Fig. 3 Region of validity for closed-form solution
- Fig. 4 Comparison of experimental and analytical results
 for ORNL Model 4 - longitudinal plane
- Fig. 5 Comparison of experimental and analytical results
 for ORNL Model 4 - transverse plane

NOTE: Fig. 4 and Fig 5 should be double column width.

NOMENCLATURE

$\omega_{n,j}$	}	= prescribed nozzle functions	k	=	$\frac{[12(1 - \nu^2)]^{1/2}}{E\rho_0 t^2}$
$\pi_{n,j}$			K	=	$\frac{[12(1 - \nu^2)]^{1/2}}{E\rho_0 T^2}$
$A_{n,j}$	}	= constants of integration	D_n	=	$\frac{Et^3}{12(1 - \nu^2)}$
$B_{n,j}$			D_s	=	$\frac{ET^3}{12(1 - \nu^2)}$
C_ℓ			ν	=	Poisson's ratio
D_ℓ			E	=	Young's modulus of elasticity
$\alpha_{n,l}$	}	= prescribed shell functions	P	=	internal pressure
$\beta_{n,l}$			u	}	= nozzle displacements
$R_{n,l}$			v		
$S_{n,l}$			w		
t		= nozzle thickness	U_ρ	}	= shell displacements
ρ_0		= nozzle midsurface radius	U_ϕ		
T		= shell thickness	W		
R		= shell midsurface radius	f	}	= stress functions
m		= ρ_0/R	F		
C		= intersection function	n_y	}	= nozzle stress resultants
x	}	= intersection coordinates	n_ϕ		
y			$n_{y\phi}$		
ρ			m_y		
ϕ			m_ϕ		
α^2		= $\frac{[3(1 - \nu^2)]^{1/2}}{4\rho_0 t}$	$M_{y\phi}$		
β^2		= $\frac{[3(1 - \nu^2)]^{1/2}}{4RT}$	q_y		
			q_ϕ		

$$\left. \begin{array}{l} N_{\rho} \\ N_{\phi} \\ N_{\rho\phi} \\ M_{\rho} \\ M_{y\phi} \\ Q_{\rho} \\ Q_{\phi} \end{array} \right\} = \text{shell stress resultants}$$

$$E_0 = \text{rigid body displacement}$$

$$\Omega = w/\rho_0 + ikf$$

$$\Psi = W/\rho_0 + iKF$$

$$H_n^{(1)} = \text{Hankel function of first kind}$$

$$J_n = \text{Bessel function of first kind}$$

$$\nabla^2 = \text{Laplacian operator}$$

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(a) NOZZLE

(b) SHELL, CYLINDRICAL

(c) SHELL, POLAR