Study of $d + {}^3\text{H}$ and $d + {}^3\text{He}$ Systems

With the Resonating-Group Method*

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ABSTRACT

The one-channel resonating-group method, together with a phenomenological imaginary potential, is used to study the $d + {}^3\text{H}$ and $d + {}^3\text{He}$ systems. Our calculations show that, because of the Pauli exclusion principle, these systems possess the following rather interesting feature: in the $s = 3/2$ state they behave similar to the $d + \alpha$ system, whereas in the $s = 1/2$ state they behave more like the ${}^3\text{He} + {}^4\text{He}$ system. Phase shifts are computed up to $\ell = 7$, and an R-matrix analysis of these phases indicates that there are a large number of levels with resonance energies below 12 MeV. Differential scattering and total-reaction cross sections are compared with experimental data at several c.m. energies from about 2 to 14 MeV. This comparison shows that the introduction of an odd-even orbital-angular-momentum dependence into the imaginary potential can improve the agreement of the calculation with experiment.

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I. INTRODUCTION

The one-channel resonating-group method,¹ together with a phenomenological imaginary potential containing an odd-even orbital-angular-momentum dependence,² has been applied quite successfully in several few-nucleon systems. In this study we continue such a semi-microscopic investigation by considering the five-nucleon $d + {}^3\text{H}$ and $d + {}^3\text{He}$ problems. We choose these particular systems for detailed examination because they contain interesting features which are not clearly exhibited in the other systems which we have examined so far. These interesting features arise because the Pauli exclusion principle has, as expected, different effects in the channel-spin states $s = 3/2$ and $s = 1/2$. Thus, while it is possible for an additional nucleon to come relatively close to the ${}^3\text{H}$ or ${}^3\text{He}$ cluster in $s = 1/2$ states, a similar occurrence will be much less likely in $s = 3/2$ states. As a consequence, it is anticipated that the phase-shift behavior in the two spin states will be distinctly different, and this is indeed found to be so in the present calculation. We should point out, however, that the occurrence of different phase-shift behavior in different channel-spin states is not a completely general feature of light nuclear systems. For instance, in the case of ${}^3\text{H} + {}^3\text{He}$ scattering,³ phase shifts in both the $s = 0$ and $s = 1$ states have been found to have rather similar behavior.

Resonating-group calculations for the $d + {}^3\text{He}$ system have been carried out by Laskar et al.,⁴ and by Heiss and Hackenbroich.⁵ Although the investigation of Heiss and Hackenbroich⁵ was a rather careful one, it dealt with partial waves only up to $l = 2$; hence for the $d + {}^3\text{He}$ system, which involves diffuse clusters, a comparison with experimental differential cross-section data is not possible even at energies of only a few MeV. In addition, even though these authors⁵ have included two-body reaction channels in their calculation, this is not sufficient to completely account for the influence of reactions on the elastic channel. As has been found experimentally,⁶ reactions proceed mainly through three- or more-particle channels if these channels are open. This means that in the $d + {}^3\text{He}$ system, where the thresholds for these channels are quite low, one must find some means to take them into account. As it is very difficult to include a proper, microscopic description of these channels, we shall include reaction effects by the simple introduction of a phenomenological imaginary potential into the theoretical formulation. This is a rather crude procedure, but previous calculations did show that reasonable results can be obtained in this way.

In the next section, a brief formulation of the problem is given, together with a discussion of the approximations employed. In Sec. III, we present the calculated phase shifts. The level structure of ${}^5\text{He}$ and ${}^5\text{Li}$ is discussed in

Sec. IV, and in Sec. V we compare the calculated differential cross sections with experiment. Finally, in Sec. VI concluding remarks are made.

II. FORMULATION

In the one-channel approximation, the wave function for the $d + {}^3\text{H}$ or $d + {}^3\text{He}$ system is written as

$$\Psi_\lambda = \mathcal{A} \{ \phi_d \phi_3 F_\lambda(\vec{R}_d - \vec{R}_3) \xi_\lambda(\sigma, \tau) \}, \quad (1)$$

where $\mathcal{A}$ is an antisymmetrization operator and ξ_λ is an appropriate spin-isospin function. The subscript λ denotes the channel-spin multiplicity $(2s + 1)$, with s denoting the channel-spin angular-momentum quantum number of the system, which can be either $3/2$ or $1/2$. It is permissible to consider the two spin states as uncoupled here, since we shall employ a central nucleon-nucleon potential in our calculation. In Eq. (1), the functions ϕ_d and ϕ_3 describe the spatial behavior of the deuteron cluster and the three-nucleon cluster (either ${}^3\text{H}$ or ${}^3\text{He}$), respectively, and the function $F_\lambda(\vec{R}_d - \vec{R}_3)$ describes the relative motion of the two clusters.

The deuteron function ϕ_d is taken to have the form

$$\phi_d = \sum_{i=1}^N A_i \exp \left[-\frac{1}{2} \alpha_i \sum_{j=1}^2 (\vec{r}_j - \vec{R}_d)^2 \right], \quad (2)$$

with $\vec{R}_d$ being the position vector of the center of mass of the deuteron cluster. The number of terms in the sum N which is needed to give a good representation of the deuteron eigenfunction has been examined previously.⁷ It was found that for $N = 3$, the values of the parameters A_i and α_i , determined by minimizing the expectation value of the deuteron Hamiltonian, are

$$\begin{aligned} A_1 &= 1.000, & \alpha_1 &= 0.07284 \text{ F}^{-2}, \\ A_2 &= 3.631, & \alpha_2 &= 0.3657 \text{ F}^{-2}, \\ A_3 &= 5.746, & \alpha_3 &= 1.4696 \text{ F}^{-2}. \end{aligned} \quad (3)$$

With these parameters, one obtains a binding energy of 2.20 MeV and an rms radius of 1.90 F for the deuteron. These values compare favorably with the corresponding experimental values of 2.22 MeV and about 1.95 F.

The three-nucleon function ϕ_3 is chosen as

$$\phi_3 = \exp \left[-\frac{1}{2} \gamma \sum_{j=1}^3 (\vec{r}_j - \vec{R}_3)^2 \right], \quad (4)$$

with $\vec{R}_3$ being the position vector of the center of mass of the three-nucleon cluster. As has been shown previously,⁸ such a simple choice of a one-Gaussian function is adequate for describing a scattering process involving the ^3H or the ^3He cluster. The single parameter γ is chosen to yield a correct value for the rms radius of the nucleon distribution;

it is

$$\gamma = 0.378 F^{-2} \quad \text{for } {}^3\text{H}, \quad (5)$$

or

$$\gamma = 0.367 F^{-2} \quad \text{for } {}^3\text{He}. \quad (6)$$

The slight difference in the value of γ for the two clusters comes from the existence of a Coulomb repulsion in ${}^3\text{He}$.⁹

The relative-motion function $F_\lambda(\vec{R}_d - \vec{R}_3)$ is determined from the variational principle

$$\langle \delta \Psi_\lambda | H - E' | \Psi_\lambda \rangle = 0, \quad (7)$$

where E' is the total energy, composed of the internal energies of the clusters and the relative energy E in the c.m. system, and H is the Hamiltonian operator, given by

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^5 \nabla_i^2 + \sum_{i>j=1}^5 V_{ij}. \quad (8)$$

The nucleon-nucleon potential is taken to be

$$V_{ij} = \left[\frac{1}{2}(1 + P_{ij}^\sigma) V_{ij}^t + \frac{1}{2}(1 - P_{ij}^\sigma) V_{ij}^s \right] \\ \times \left[\frac{1}{2}u + \frac{1}{2}(2-u) P_{ij}^\tau \right] + \frac{e^2}{r_{ij}} \frac{1+\tau_{iz}}{2} \frac{1+\tau_{jz}}{2}, \quad (9)$$

where P_{ij}^{σ} and P_{ij}^{τ} are the spin- and space-exchange operators, respectively, and τ_{iz} and τ_{jz} are the z-components of the isospin operators for the i^{th} and j^{th} particles. The quantities V_{ij}^t and V_{ij}^s denote the $\ell = 0$ spin triplet and spin singlet nucleon-nucleon potentials given by

$$\begin{aligned} V_{ij}^t &= -V_0^t \exp(-\kappa_t r_{ij}^2), \\ V_{ij}^s &= -V_0^s \exp(-\kappa_s r_{ij}^2). \end{aligned} \quad (10)$$

The constants V_0^t , κ_t , V_0^s , and κ_s are adjusted to yield the correct values for the two-nucleon effective-range parameters; they are found to be

$$\begin{aligned} V_0^t &= 66.92 \text{ MeV}, & \kappa_t &= 0.415 \text{ F}^{-2}, \\ V_0^s &= 29.05 \text{ MeV}, & \kappa_s &= 0.292 \text{ F}^{-2}. \end{aligned} \quad (11)$$

The quantity u , which fixes the exchange mixture in the nucleon-nucleon potential, will be treated as an adjustable parameter in our calculation. Its value will be determined by the requirement that the experimental resonance energies of the $\ell = 0$, $s = 3/2$ states in ${}^5\text{He}$ and ${}^5\text{Li}$ be nearly reproduced. As has been mentioned previously,⁸ this procedure of varying u is certainly a rather undesirable feature in a

microscopic calculation. It is, however, a necessary one in order to make crude compensation for the defects in our calculation caused by the lack of consideration of specific distortion effects.¹⁰ We should emphasize that the value of u so determined should be close to 1, which corresponds to a Serber exchange mixture. If this should turn out not to be the case, then varying u is too crude to compensate for these defects and a more elaborate calculation should be performed.

The nucleon-nucleon potential of Eq. (9) does not have a noncentral component. This should not be a serious defect here, because it has been shown by Heiss and Hackenbroich⁵ that the diffuse nature of the $d + {}^3\text{H}$ and $d + {}^3\text{He}$ systems causes the amount of spin-orbit splitting of the phase shifts to be rather small. Consequently, one expects that the lack of spin-orbit splitting in our calculation will only cause a notable discrepancy between calculated and experimental differential cross-sections in energy regions where sharp resonance levels exist. Indeed, the conclusions reached from a recent calculation¹¹ for ${}^3\text{He} + {}^4\text{He}$ scattering, in which a nucleon-nucleon spin-orbit potential was included, have fully supported this assertion. Thus, the fact that our calculation yields no sharp resonance levels in ${}^5\text{He}$ and

^5Li with non-zero orbital angular momentum indicates that no serious deficiency is introduced by the use of a purely central nucleon-nucleon potential.

From Eq. (7) an integrodifferential equation for the function $F_\lambda(\vec{r})$ can be derived. Upon performing a partial-wave expansion and introducing a phenomenological imaginary potential $iW_{\ell\lambda}(r)$ to take approximate account of reaction effects, we obtain

$$\left\{ \frac{\hbar^2}{2\mu} \left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + E - V_{N\lambda}(r) - V_C(r) - iW_{\ell\lambda}(r) \right\} f_{\ell\lambda}(r) = \int_0^\infty k_{\ell\lambda}(r, r') f_{\ell\lambda}(r') dr', \quad (12)$$

where $f_{\ell\lambda}(r)$ is related to $F_\lambda(\vec{r})$ by the equation

$$F_\lambda(\vec{r}) = \sum_\ell \frac{1}{r} f_{\ell\lambda}(r) P_\ell(\cos\theta). \quad (13)$$

In Eq. (12), the functions $V_{N\lambda}$ and V_C are the direct nuclear potential and the direct Coulomb potential, respectively, while the function $k_{\ell\lambda}$ represents the kernel function for the nonlocal interaction arising from the exchange character of the nucleon-nucleon potential and the anti-symmetrization procedure. The expressions for these functions are quite lengthy and will be given in the Appendix.

The phenomenological imaginary potential is chosen as

$$W_{\ell\lambda}(r) = [1 + C_{\lambda}(-1)^{\ell}]U_{\lambda}(r), \quad (14)$$

which contains an odd-even orbital angular-momentum dependence. This odd-even feature has been found to be desirable in a number of previous calculations^{2,7} in order to achieve a detailed fit to the experimental data. The function $U_{\lambda}(r)$ is taken to be

$$U_{\lambda}(r) = -U_{0\lambda} \left\{ \frac{1}{1 + \exp[(r-R)/a]} + \frac{4 \exp[(r-R)/a]}{\{1 + \exp[(r-R)/a]\}^2} \right\}, \quad (15)$$

with

$$\begin{aligned} R &= 3.5 F, \\ a &= 0.5 F. \end{aligned} \quad (16)$$

The rationale for choosing this particular form of U_{λ} is given in Ref. 7 and will not be gone into here. The parameters contained in the imaginary potential are the potential depth $U_{0\lambda}$ and the odd-even parameter C_{λ} ; these will be adjusted to yield the best fit to the experimental differential cross-section data at each energy under consideration.

By solving Eq. (12) subject to the scattering boundary condition, we can compute phase shifts δ_{ℓ} and the scattering amplitude $f_{\lambda}(\theta)$ in each spin state. The differential cross section for elastic scattering is then given by

$$\sigma(\theta) = \frac{2}{3} \sigma_4(\theta) + \frac{1}{3} \sigma_2(\theta), \quad (17)$$

with

$$\sigma_\lambda(\theta) = |f_\lambda(\theta)|^2. \quad (18)$$

III. PHASE SHIFTS

It is known from experiment¹² that there are $\ell = 0$, $s = 3/2$ resonance levels at 0.068 and 0.262 MeV¹³ in ^5He and ^5Li , respectively, which have predominantly a deuteron plus ^3H or ^3He cluster structure.¹⁴ As was mentioned in Sec. II, we shall use this information to determine the appropriate value of the exchange-mixture parameter u to be used in this calculation. We do this by computing the phase shift $^4\delta_0$ for several values of u with U_{04} in Eq. (15) set to zero and by then determining the corresponding resonance energies through a one-level R-matrix analysis. The result of this procedure is that with u equal to 1.1, the resonance energies in ^5He and ^5Li are equal to 0.054 and 0.301 MeV, respectively, which compare favorably with the experimentally determined values.

Using $u = 1.1$ and no imaginary potential,¹⁵ we have computed the $d + ^3\text{H}$ and $d + ^3\text{He}$ phase shifts $^{\lambda}\delta_{\ell}$ for a wide range of energies. The result for the $d + ^3\text{He}$ phase shifts from 0 to 15 MeV is illustrated in Fig. 1, where we

have adopted the convention¹⁶

$$\lim_{E \rightarrow 0} \lambda \delta_l = n \pi, \quad (19)$$

with n being the number of real bound states plus the number of Pauli-forbidden bound states. Here it is seen that the phase-shift behavior in the two spin states is indeed quite different. For example, the value of ${}^4\delta_l$ is larger than the value of ${}^2\delta_l$ in even- l states, whereas the reverse is true in odd- l states.

As in the case of $d + d$ scattering,⁷ we here also find anomalous phase-shift behavior in those states where the effect of the Pauli-exclusion principle is particularly important. In the $d + {}^3\text{He}$ case, for example, these anomalies occur at about 26 MeV in the ${}^4\text{S}$ state, 33 MeV in the ${}^4\text{P}$ state, and 22 MeV in the ${}^2\text{S}$ state. They have a fairly wide spread in energy, but their effects at energies outside of the range of about 16 to 40 MeV are quite minor. At this moment we do not have a clear understanding of the origin of these anomalies, but it is tempting to associate their presence with the adoption of a one-channel approximation in our calculation.

The difference in the phase-shift behavior of the two spin states is further demonstrated in Fig. 2. In this figure we plot $\lambda \delta_l$ as a function of l at an energy of 50 MeV.

To facilitate comparison, dashed lines have been drawn through the phase-shift points. Here one sees that the odd-even feature in the quartet spin state ($\lambda = 4$) is entirely different from that in the doublet spin state ($\lambda = 2$). In fact, by comparing with the odd-even phase-shift features existing in other light nuclear systems, we find that the odd-even behavior in the quartet state is similar to that of the $d + \alpha$ system,¹⁷ whereas the odd-even behavior in the doublet state is similar to that of the ${}^3\text{He} + {}^4\text{He}$ and $p + \alpha$ systems.^{18,19} This therefore supports our qualitative picture (see Sec. I) in which, in the quartet spin state, the three-nucleon cluster (${}^3\text{H}$ or ${}^3\text{He}$) acts as a spin and isospin saturated substructure, but, in the doublet spin state, an extra nucleon can have an appreciable probability to form with the three-nucleon cluster an entity which has the correlation nature of a relatively diffuse α cluster.

Another striking way to demonstrate the different behavior in the two spin states is shown in Fig. 3. Here we depict the partial differential scattering cross sections $\sigma_4(\theta)$ and $\sigma_2(\theta)$ at 15 MeV. As a comparison, we show also the differential cross sections for $d + \alpha$ and ${}^3\text{He} + {}^4\text{He}$ scattering^{20,9} at the same energy. From this figure one sees clearly that the shape of the σ_4 curve is similar to that of the $d + \alpha$ curve, and the shape of the σ_2 curve is similar to that of the ${}^3\text{He} + {}^4\text{He}$ curve, a result which is in full accord with the discussion given in the preceding paragraphs.

IV. LEVEL STRUCTURE OF ^5He AND ^5Li

In order to gain some understanding of the level structure of ^5He and ^5Li , we have fit the $d + ^3\text{H}$ and $d + ^3\text{He}$ phase shifts, calculated using $u = 1.1$ and no imaginary potential, with a two-level formula,²¹ given by

$$\delta_\ell = -\phi_\ell + \tan^{-1} \left[\frac{P_\ell R_{\ell\lambda}}{1 - (S_\ell + \ell) R_{\ell\lambda}} \right], \quad (20)$$

where ϕ_ℓ is the hard-sphere phase shift, characterized by an interaction radius a , P_ℓ and S_ℓ are the penetration and shift functions, respectively, and

$$R_{\ell\lambda} = \sum_{i=1}^2 \frac{\gamma_{i\ell\lambda}^2}{E_{i\ell\lambda} - E}, \quad (21)$$

with $E_{i\ell\lambda}$ and $\gamma_{i\ell\lambda}^2$ being the characteristic energy and the reduced width of the level i , respectively. The fitting is performed by varying the parameters a , $E_{1\ell\lambda}$, $\gamma_{1\ell\lambda}^2$, $E_{2\ell\lambda}$, and $\gamma_{2\ell\lambda}^2$, until near perfect agreement with the phase-shift values in an appropriately chosen energy range is achieved. The results for various (ℓ, s) states are given in Table I, where we have also listed the values of the resonance energy E_{res} , defined as the energy at which the resonant part of the phase shift goes through 90° .

We now discuss the experimental evidence for the existence of levels in ^5He and ^5Li above the $d + ^3\text{H}$ and $d + ^3\text{He}$ thresholds, respectively. In this paragraph and in Fig. 4, excitation

energies are referred to the $\ell = 0$, $s = 3/2$ state. Resonance levels have been reported in ${}^5\text{He}$ at about 2.3 MeV²² with an orbital angular momentum of $\ell = 0$ or 2, at 3.5 and 7.1 MeV²³ with rather large level widths, and at about 3.8 MeV²⁴ with negative parity. In ${}^5\text{Li}$ there is evidence for the existence of resonance levels at about 1.4 MeV^{25,26} with $J^\pi = 1/2^+$, at about 3.4 MeV²⁷ with $J^\pi = 3/2^+$ or $5/2^+$, and at about 5.3 and 8.3 MeV.²⁸ In Fig. 4 we summarize the experimental information, together with our theoretical predictions.²⁹ Dashed lines are drawn in this figure to show possible correspondences of the experimentally determined and theoretically calculated levels. As is seen, the agreement between theory and experiment is fairly satisfactory. In particular, the prediction of the presence of negative-parity levels with rather low excitation energies seems to be verified by experiment.²⁴

V. DIFFERENTIAL AND TOTAL-REACTION CROSS SECTIONS

In this section we compare calculated and experimental differential cross sections at several c.m. energies from 2 to 14 MeV where accurate experimental data^{22,30,31} exist. At these energies, reaction effects are quite important and, as has been shown before,^{7,18} an account, even an approximate one, of these effects is necessary in order to achieve reasonable agreement between theoretical and experimental

results. In this study we take reaction channels approximately into account by the introduction of a phenomenological imaginary potential $iW_{\ell\lambda}$. As is seen from Eqs. (14)-(16) in Sec. II, this potential has a number of parameters to be adjusted at each energy to fit the experimental data. These adjustable parameters are the depth parameters U_{04} and U_{02} , and the odd-even parameters C_4 and C_2 .

As in any study involving a phenomenological potential, it is desirable to try to reduce the number of adjustable parameters. This will be done here by using our knowledge (see Sec. III) that the $d + {}^3\text{He}$ system behaves similar to the $d + \alpha$ system in the $s = 3/2$ state and to the ${}^3\text{He} + {}^4\text{He}$ system in the $s = 1/2$ state. For the $d + \alpha$ system, a calculation³² similar to the present one revealed that the value of the odd-even parameter in the absorptive potential is positive, whereas for the ${}^3\text{He} + {}^4\text{He}$ system,⁹ a negative value for the odd-even parameter has been found to yield improved agreement with experiment. In addition, both of these calculations have shown that, with differential scattering cross-section data alone, there is a rather wide range of acceptable values for the odd-even parameters. Thus, in the present calculation, we shall assume

$$C_4 = -C_2 = C_1. \quad (22)$$

With this additional restriction, the number of adjustable parameters is then reduced to three, namely, U_{04} , U_{02} , and C_I .

Differential scattering and total-reaction cross sections are calculated at 2.02, 4.19, and 6.6 MeV for the $d + {}^3\text{H}$ system, and 8.64, 10.0, and 13.84 MeV for the $d + {}^3\text{He}$ system. At the three lower energies, our calculation shows that C_I can be taken to be zero, without very much compromising the fit to the data. On the other hand, we find that at the three higher energies, it is more appropriate to use a non-zero but common value of 0.7 for C_I . In Fig. 5 we illustrate the effect of C_I at a scattering energy of 10 MeV. In this figure, the solid and dashed lines represent the results obtained with C_I equal to 0.7 and 0, respectively. In both of these calculations the parameters U_{04} and U_{02} have been adjusted to yield optimum fits to the experimental data. Here it is seen that even though the use of a non-zero C_I value does not have a drastic effect, it does yield an improvement in the fit.

Optimum values for U_{04} and U_{02} are given in Tables II and III, together with the resultant values for the total-reaction cross section σ_R and the complex phase shifts λ_{δ_ℓ} . At 2.02 MeV, the optimum values are obtained by using both the experimental differential scattering²² and total-reaction cross-section³⁰ data. At the other energies, three- and more-particle

breakup channels are open and total-reaction cross-section data are not available, and hence, only differential scattering cross-section data have been used to determine the optimum U_{04} and U_{02} values.

In Figs. 5 and 6, calculated and experimental differential cross sections for elastic scattering are compared. Data points in these figures are those of Ivanovich et al.²² at 2.02, 4.19, and 6.6 MeV, those of Brolley et al.³⁰ at 8.64 MeV, and those of King and Smythe³¹ at 10.0 and 13.84 MeV. It is seen that the agreement is quite good at the four intermediate energies, but is only fair at the lowest and highest energies. At 2.02 MeV, the lack of close agreement is likely a consequence of the fact that the specific distortion of the deuteron cluster has not been properly taken into account. Indeed, recent calculations¹⁰ on $d + \alpha$ scattering have shown that for a loosely bound cluster, specific distortion can become quite important at low energies. As to the relatively poor fit at 13.84 MeV, we feel that the probable cause is that there may be resonance levels near this energy which do not have predominantly a $d + {}^3\text{He}$ cluster structure. In fact, we have encountered a similar experience in the case of $d + \alpha$ scattering. In that investigation¹⁷ we have found that the fit to experiment was satisfactory at energies of 5.98 and 8.0 MeV, but became rather poor at the higher energies of 14 and 16.58 MeV.

Because of the similarity in behavior of the $d + {}^3\text{He}$ system in the $s = 3/2$ state to the $d + \alpha$ system, we are reasonably sure that the discrepancy at higher energies between theory and experiment has a common origin for these two systems.

VI. CONCLUSION

One of the most interesting findings of this investigation is that, because of the Pauli exclusion principle, the $d + {}^3\text{He}$ (or $d + {}^3\text{H}$) system behaves quite differently in the $s = 3/2$ and $s = 1/2$ states. Thus, though it has a behavior in the $s = 3/2$ state similar to that of the $d + \alpha$ system, it has a distinctly different behavior in the $s = 1/2$ state similar to that of the ${}^3\text{He} + {}^4\text{He}$ system. It is quite conceivable that the spin effects found here will be important in many other scattering problems, such as, for example, the $d + {}^{15}\text{N}$ scattering problem. This means that if one wishes to conduct a simple, macroscopic examination of these problems, then according to the conclusions reached in our present calculation, one will have to use effective interactions between the nuclei which are different in different channel-spin states.

The calculated phase shifts have been fit by a two-level R-matrix formula. From this analysis we find a number of levels with resonance energies below 12 MeV. In particular, the theory predicts the existence of some negative-parity levels in this energy region, one of which seems to be verified by recent experimental findings.

As in previous instances, our present investigation shows that the one-channel resonating-group method, together with a phenomenological imaginary potential, can be successfully used to explain the features of the experimental results. Here also, we find that the introduction of an odd-even orbital angular-momentum dependence into the imaginary potential is desirable and does lead to improved agreement with experiment.

This calculation is the latest in a series of investigations using the one-channel resonating-group method to examine the properties of the scattering of two light nuclei A and B, with A and B being the nuclei n , ${}^1\text{H}$, ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$. The success of these investigations indicates to us that it might be quite profitable to proceed to examine more complicated systems involving larger clusters, such as $d + {}^{16}\text{O}$ scattering, etc. For this type of calculation, the procedure of antisymmetrization is more difficult than for lighter systems, but a recent discussion by Sünkel and Wildermuth³³ did show that it can be performed with reasonable effort. In addition, the one-channel calculations can be improved by considering specific distortion effects and by explicitly including inelastic channels. It is our feeling that the latter type of improvement should be particularly interesting, since it will yield information not only on effects of inelastic channels on the elastic channel, but also on the nature of reaction mechanisms.

APPENDIX

I. DIRECT POTENTIALS

A. Direct nuclear potential $V_{N\lambda}(r)$

$$V_{N\lambda}(r) = \sum_{j=1}^2 \left(-\frac{V_{0j}}{G_0} \lambda w_j \right) \sum_{i=1}^3 \sum_{k=1}^3 \left\{ A_i A_k \left(\frac{\pi^2}{3\gamma^2} \right)^{\frac{3}{2}} \left(\frac{\pi}{\alpha_i + \alpha_k} \right)^{\frac{3}{2}} \right. \\ \times \left[\frac{3\delta(\alpha_i + \alpha_k)}{3\delta(\alpha_i + \alpha_k) + \kappa_j(3\delta + 2\alpha_i + 2\alpha_k)} \right]^{\frac{3}{2}} \\ \times \exp \left[-\frac{3\kappa_j\delta(\alpha_i + \alpha_k)}{3\delta(\alpha_i + \alpha_k) + \kappa_j(3\delta + 2\alpha_i + 2\alpha_k)} r^2 \right] \Bigg\}$$

where

$$G_0 = \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \left(\frac{\pi^2}{3\gamma^2} \right)^{\frac{3}{2}} \left(\frac{\pi}{\alpha_i + \alpha_k} \right)^{\frac{3}{2}},$$

$$V_{01} = V_0^{\tau}, \quad V_{02} = V_0^{\rho},$$

$$\kappa_1 = \kappa_t, \quad \kappa_2 = \kappa_e,$$

and

$${}^4w_j = 6w_j - 2m_j + 4b_j - 3h_j,$$

$${}^2w_j = 6w_j - \frac{1}{2}m_j + b_j - 3h_j,$$

with

$$w_1 = b_1 = w_2 = -b_2 = \frac{1}{4}u,$$

$$m_1 = h_1 = m_2 = -h_2 = \frac{1}{4}(2-u).$$

B. Direct Coulomb potential $V_C(r)$.

$$V_C(r) = \frac{1}{G_0} \frac{zz'e^2}{r} \sum_{i=1}^3 \sum_{k=1}^3 \left\{ A_i A_k \left(\frac{\pi^2}{3\gamma^2} \right)^{\frac{3}{2}} \left(\frac{\pi}{\alpha_i + \alpha_k} \right)^{\frac{3}{2}} \right. \\ \left. \times \Phi \left[\left(\frac{3\gamma(\alpha_i + \alpha_k)}{3\gamma + 2\alpha_i + 2\alpha_k} \right)^{\frac{1}{2}} r \right] \right\}$$

with z and z' being the atomic numbers of the three-nucleon cluster and the deuteron cluster, respectively, and

$$\Phi(\nu) = \frac{2}{\sqrt{\pi}} \int_0^\nu e^{-t^2} dt$$

II. KERNEL FUNCTIONS

The kernel function $k_{\ell\lambda}(r, r')$ is composed of two terms $k_{\ell\lambda}^N$ and $k_{\ell\lambda}^C$, i.e.,

$$k_{\ell\lambda}(r, r') = k_{\ell\lambda}^N(r, r') + k_{\ell\lambda}^C(r, r')$$

A. Partial-wave kernel function $k_{\ell\lambda}^N(r, r')$

$$k_{\ell\lambda}^N(r, r') = \frac{1}{G_0} \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \left(-\frac{\hbar^2}{2M} {}^\lambda J_{ik} - \sum_{j=1}^2 {}^\lambda V_{j,ik} V_{0j} + E' {}^\lambda C_{ik} \right)$$

where

$$E' = E + E_3 + E_d,$$

with

$$E_3 = \frac{\hbar^2}{2M} 3\gamma - \sum_{j=1}^2 (3w_j + 3m_j) \left(\frac{\gamma}{\gamma + 2\kappa_j} \right)^{\frac{3}{2}} V_{0j} + \frac{z(z-1)e^2}{2} \left(\frac{2\gamma}{\pi} \right)^{\frac{1}{2}}$$

and

$$E_d = \frac{1}{G_0} \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \left(\frac{\pi^2}{3\gamma^2} \right)^{\frac{3}{2}} \left(\frac{\pi}{\alpha_i + \alpha_k} \right)^{\frac{3}{2}} \left[\frac{\hbar^2}{2M} \frac{3\alpha_i \alpha_k}{\alpha_i + \alpha_k} \right. \\ \left. - \sum_{j=1}^2 (w_j + m_j + b_j + h_j) \left(\frac{\alpha_i + \alpha_k}{\alpha_i + \alpha_k + 4\kappa_j} \right)^{\frac{3}{2}} V_{0j} \right].$$

The quantities ${}^\lambda \mathcal{J}_{ik}$, ${}^\lambda \mathcal{V}_{j,ik}$, and ${}^\lambda \mathcal{C}_{ik}$ are defined as follows:

$$\begin{aligned} {}^\lambda \mathcal{J}_{ik} = & {}^\lambda t_1 \epsilon_1 \left\{ \left[\frac{9}{2} \gamma + 3\alpha_k - \frac{9\gamma^2 + 12\alpha_k^2}{6\gamma + 4\alpha_i + 4\alpha_k} + 3a_1 \right. \right. \\ & - \left(\frac{3}{5} T_a + \frac{6}{5} a_1^2 \right) r^2 - \left(\frac{3}{5} T_b + \frac{3}{10} c_1^2 \right) r'^2 \left. \right] S_\lambda \left(-\frac{3}{5} c_1 \right) \\ & + \left(\frac{3}{5} T_c + \frac{6}{5} a_1 c_1 \right) r r' T_\lambda \left(-\frac{3}{5} c_1 \right) \left. \right\} e^{-\frac{3}{5} (a_1 r^2 + b_1 r'^2)} \\ & + {}^\lambda t_2 \epsilon_2 \left\{ \left[6\gamma + 3\alpha_k - \frac{3\gamma^2}{\gamma + \alpha_i} - \frac{3\alpha_k^2}{\gamma + \alpha_k} + 3a_2 \right. \right. \\ & - \left(\frac{24}{25} \gamma^2 + \frac{6}{5} a_2^2 \right) r^2 - \left(\frac{54}{25} \gamma^2 + \frac{3}{10} c_2^2 \right) r'^2 \left. \right] S_\lambda \left(-\frac{3}{5} c_2 \right) \\ & - \left(\frac{72}{25} \gamma^2 - \frac{6}{5} a_2 c_2 \right) r r' T_\lambda \left(-\frac{3}{5} c_2 \right) \left. \right\} e^{-\frac{3}{5} (a_2 r^2 + b_2 r'^2)} \end{aligned}$$

where

$$\begin{aligned} {}^4 t_1 &= 2, & {}^4 t_2 &= -1, \\ {}^2 t_1 &= {}^2 t_2 = \frac{1}{2}, \end{aligned}$$

$$\begin{aligned} {}^\lambda \mathcal{V}_{j,ik} = & \epsilon_1 \left\{ \left[{}^\lambda w'_{12} \epsilon'_{12} \exp \left[-\frac{3}{5} (a'_{12} r^2 + b'_{12} r'^2) \right] S_\lambda \left(-\frac{3}{5} c'_{12} \right) \right. \right. \\ & + {}^\lambda w'_{25} \epsilon'_{25} \exp \left[-\frac{3}{5} (a'_{25} r^2 + b'_{25} r'^2) \right] S_\lambda \left(-\frac{3}{5} c'_{25} \right) \left. \right] \\ & + {}^\lambda w'_{15} \epsilon'_{15} \exp \left[-\frac{3}{5} (a'_{15} r^2 + b'_{15} r'^2) \right] S_\lambda \left(-\frac{3}{5} c'_{15} \right) \\ & + \left[{}^\lambda w'_{16} \epsilon'_{16} \exp \left[-\frac{3}{5} (a'_{16} r^2 + b'_{16} r'^2) \right] S_\lambda \left(-\frac{3}{5} c'_{16} \right) \right. \end{aligned}$$

$$\begin{aligned}
 & + \lambda w'_{56} \epsilon'_{56} \exp \left[-\frac{3}{5} (a'_{56} r^2 + b'_{56} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c'_{56} \right) \Big] \\
 & + \lambda w'_{23} \epsilon'_{23} \exp \left[-\frac{3}{5} (a'_{23} r^2 + b'_{23} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c'_{23} \right) \\
 & + \lambda w'_{26} \epsilon'_{26} \exp \left[-\frac{3}{5} (a'_{26} r^2 + b'_{26} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c'_{26} \right) \Big\} \\
 & + \epsilon_2 \left\{ \left[\lambda w^2_{12} \epsilon^2_{12} \exp \left[-\frac{3}{5} (a^2_{12} r^2 + b^2_{12} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c^2_{12} \right) \right. \right. \\
 & + \lambda w^2_{56} \epsilon^2_{56} \exp \left[-\frac{3}{5} (a^2_{56} r^2 + b^2_{56} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c^2_{56} \right) \\
 & + \left[\lambda w^2_{13} \epsilon^2_{13} \exp \left[-\frac{3}{5} (a^2_{13} r^2 + b^2_{13} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c^2_{13} \right) \right. \\
 & + \lambda w^2_{35} \epsilon^2_{35} \exp \left[-\frac{3}{5} (a^2_{35} r^2 + b^2_{35} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c^2_{35} \right) \\
 & \left. + \lambda w^2_{15} \epsilon^2_{15} \exp \left[-\frac{3}{5} (a^2_{15} r^2 + b^2_{15} r'^2) \right] S_{\lambda} \left(-\frac{3}{5} c^2_{15} \right) \right\}
 \end{aligned}$$

$$\begin{aligned}
 {}^{\lambda} \mathcal{E}_{ik} &= \lambda t_i \epsilon_i e^{-\frac{3}{5} (a_i r^2 + b_i r'^2)} S_{\lambda} \left(-\frac{3}{5} c_i \right) \\
 &+ \lambda t_2 \epsilon_2 e^{-\frac{3}{5} (a_2 r^2 + b_2 r'^2)} S_{\lambda} \left(-\frac{3}{5} c_2 \right)
 \end{aligned}$$

with

$$S_{\lambda}(\omega) = \frac{4\pi}{\omega} \int_{\lambda+\frac{1}{2}} (\omega r r')$$

and

$$T_l(\omega) = \frac{4\pi}{\omega} \left[\int_{l+\frac{3}{2}} (\omega r r') - \frac{l}{\omega r r'} \int_{l+\frac{1}{2}} (\omega r r') \right]$$

where $j(x)$ is a hyperbolic spherical Bessel function. Also, in the above equations, the following definitions have been made:

$$e_1 = \left(\frac{6}{5}\right)^3 \left(\frac{\pi}{2\gamma}\right)^{\frac{3}{2}} \left(\frac{2\pi}{3\gamma + 2\alpha_i + 2\alpha_k}\right)^{\frac{3}{2}},$$

$$e_2 = \left(\frac{6}{5}\right)^3 \left(\frac{\pi}{\gamma + \alpha_i}\right)^{\frac{3}{2}} \left(\frac{\pi}{\gamma + \alpha_k}\right)^{\frac{3}{2}},$$

$$a_1 = f_1 / e_1,$$

$$b_1 = g_1 / e_1,$$

$$c_1 = \gamma_1 / e_1,$$

$$a_2 = b_2 = \frac{13}{5} \gamma,$$

$$c_2 = -\frac{24}{5} \gamma,$$

$$T_a = \frac{\gamma^2(12\gamma + 4\alpha_i + \alpha_k) + \alpha_k^2(10\gamma + 12\alpha_i)}{e_1} - \frac{5f_1(3\gamma^2 + 4\alpha_k^2)}{e_1^2}$$

$$T_b = \frac{\gamma^2(12\gamma + 9\alpha_i + 36\alpha_k) + \alpha_k^2(40\gamma + 12\alpha_i)}{e_1} - \frac{5g_1(3\gamma^2 + 4\alpha_k^2)}{e_1^2}$$

$$T_c = \frac{\gamma^2(24\gamma - 12\alpha_i + 12\alpha_k) + \alpha_k^2(24\alpha_i)}{e_1} - \frac{5\gamma_1(3\gamma^2 + 4\alpha_k^2)}{e_1^2}$$

$$\begin{aligned}
 {}^4w'_{12} &= -4w_j - 4m_j - b_j - h_j, & {}^2w'_{12} &= -w_j - m_j + 2b_j + 2h_j, \\
 {}^4w'_{25} &= {}^4w'_{12}, & {}^2w'_{25} &= {}^2w'_{12}, \\
 {}^4w'_{15} &= -2w_j + 6m_j - 3b_j + 4h_j, & {}^2w'_{15} &= -\frac{1}{2}w_j + 6m_j - 3b_j + h_j, \\
 {}^4w'_{16} &= -2w_j - 2m_j - 2b_j - 2h_j, & {}^2w'_{16} &= -\frac{1}{2}w_j - \frac{1}{2}m_j - \frac{1}{2}b_j - \frac{1}{2}h_j, \\
 {}^4w'_{56} &= {}^4w'_{16}, & {}^2w'_{56} &= {}^2w'_{16}, \\
 {}^4w'_{23} &= -2w_j - 2m_j + b_j + h_j, & {}^2w'_{23} &= -\frac{1}{2}w_j - \frac{1}{2}m_j - 2b_j - 2h_j, \\
 {}^4w'_{26} &= -4w_j + 4m_j - 2b_j + 5h_j, & {}^2w'_{26} &= -w_j - 2m_j + b_j - h_j, \\
 {}^4w'^2_{12} &= w_j + m_j + b_j + h_j, & {}^2w'^2_{12} &= -\frac{1}{2}w_j - \frac{1}{2}m_j - \frac{1}{2}b_j - \frac{1}{2}h_j, \\
 {}^4w'^2_{56} &= {}^4w'^2_{12}, & {}^2w'^2_{56} &= {}^2w'^2_{12}, \\
 {}^4w'^2_{13} &= 2w_j + 2m_j - b_j - h_j, & {}^2w'^2_{13} &= -w_j - m_j + \frac{1}{2}b_j + \frac{1}{2}h_j, \\
 {}^4w'^2_{35} &= {}^4w'^2_{13}, & {}^2w'^2_{35} &= {}^2w'^2_{13}, \\
 {}^4w'^2_{15} &= 4w_j - 4m_j + 5b_j - 2h_j, & {}^2w'^2_{15} &= -2w_j - m_j - b_j + h_j.
 \end{aligned}$$

$$\epsilon_{12}^i = \left[\frac{re_i}{re_i + \kappa_j (30\gamma + 5\alpha_i + 5\alpha_R)} \right]^{\frac{3}{2}}$$

$$a_{12}^i = \frac{\gamma f_i + \kappa_j (21\gamma^2 + 16\gamma\alpha_i + 4\gamma\alpha_R + 3\alpha_i\alpha_R)}{re_i + \kappa_j (30\gamma + 5\alpha_i + 5\alpha_R)}$$

$$b_{12}^i = \frac{\gamma g_i + \kappa_j (21\gamma^2 + 16\gamma\alpha_i + 64\gamma\alpha_R + 3\alpha_i\alpha_R)}{re_i + \kappa_j (30\gamma + 5\alpha_i + 5\alpha_R)}$$

$$c_{12}^i = \frac{\gamma y_i + \kappa_j (42\gamma^2 - 18\gamma\alpha_i + 18\gamma\alpha_R + 6\alpha_i\alpha_R)}{re_i + \kappa_j (30\gamma + 5\alpha_i + 5\alpha_R)}$$

$$\epsilon'_{25} = \epsilon'_{12} \quad \text{with} \quad \alpha_i \leftrightarrow \alpha_k,$$

$$a'_{25} = b'_{12} \quad \text{with} \quad \alpha_i \leftrightarrow \alpha_k,$$

$$b'_{25} = a'_{12} \quad \text{with} \quad \alpha_i \leftrightarrow \alpha_k,$$

$$c'_{25} = c'_{12} \quad \text{with} \quad \alpha_i \leftrightarrow \alpha_k,$$

$$\epsilon'_{15} = 1,$$

$$a'_{15} = \frac{f_i + \kappa_j (36\gamma + 24\alpha_i + 24\alpha_k)}{e_i},$$

$$b'_{15} = \frac{g_i + \kappa_j (36\gamma + 24\alpha_i + 24\alpha_k)}{e_i},$$

$$c'_{15} = \frac{y_i + \kappa_j (72\gamma + 48\alpha_i + 48\alpha_k)}{e_i},$$

$$\epsilon'_{16} = \left(\frac{e_i}{e_i + 40\kappa_j} \right)^{\frac{3}{2}},$$

$$a'_{16} = \frac{f_i + \kappa_j (80\gamma + 24\alpha_k)}{e_i + 40\kappa_j},$$

$$b'_{16} = \frac{g_i + \kappa_j (20\gamma + 24\alpha_k)}{e_i + 40\kappa_j},$$

$$c'_{16} = \frac{y_i + \kappa_j (48\alpha_k)}{e_i + 40\kappa_j},$$

$$\epsilon_{56}^i = \epsilon_{16}^i \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$a_{56}^i = b_{16}^i \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$b_{56}^i = a_{16}^i \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$c_{56}^i = c_{16}^i \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$\epsilon_{23}^i = \left(\frac{\gamma}{\gamma + 2\kappa_j} \right)^{3/2},$$

$$a_{23}^i = a_i,$$

$$b_{23}^i = b_i,$$

$$c_{23}^i = c_i,$$

$$\epsilon_{26}^i = \left[\frac{\gamma e_i}{\gamma e_i + \kappa_j (10\gamma + 5\alpha_i + 5\alpha_k)} \right]^{\frac{3}{2}},$$

$$a_{26}^i = \frac{\gamma f_i + \kappa_j (29\gamma^2 + 16\gamma\alpha_i + 16\gamma\alpha_k + 3\alpha_i\alpha_k)}{\gamma e_i + \kappa_j (10\gamma + 5\alpha_i + 5\alpha_k)},$$

$$b_{26}^i = \frac{\gamma g_i + \kappa_j (29\gamma^2 + 16\gamma\alpha_i + 16\gamma\alpha_k + 3\alpha_i\alpha_k)}{\gamma e_i + \kappa_j (10\gamma + 5\alpha_i + 5\alpha_k)},$$

$$c_{26}^i = \frac{\gamma y_i + \kappa_j (-42\gamma^2 - 18\gamma\alpha_i - 18\gamma\alpha_k + 6\alpha_i\alpha_k)}{\gamma e_i + \kappa_j (10\gamma + 5\alpha_i + 5\alpha_k)},$$

$$\epsilon_{12}^2 = \left(\frac{\gamma + \alpha_i}{\gamma + \alpha_i + 4\kappa_j} \right)^{\frac{3}{2}},$$

$$a_{12}^2 = b_{12}^2 = \frac{13}{5} \gamma,$$

$$c_{12}^2 = -\frac{24}{5} \gamma,$$

$$E_{56}^2 = E_{12}^2 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$a_{56}^2 = b_{56}^2 = \frac{13}{5} \gamma,$$

$$c_{56}^2 = -\frac{24}{5} \gamma,$$

$$E_{13}^2 = \left(\frac{\gamma + \alpha_i}{\gamma + \alpha_i + \kappa_j} \right)^{\frac{3}{2}},$$

$$a_{13}^2 = \frac{1}{5} \frac{13\gamma(\gamma + \alpha_i) + \kappa_j(25\gamma + 12\alpha_i)}{\gamma + \alpha_i + \kappa_j}$$

$$b_{13}^2 = \frac{1}{5} \frac{13\gamma(\gamma + \alpha_i) + \kappa_j(40\gamma + 27\alpha_i)}{\gamma + \alpha_i + \kappa_j}$$

$$c_{13}^2 = -\frac{1}{5} \frac{24\gamma(\gamma + \alpha_i) + \kappa_j(60\gamma + 36\alpha_i)}{\gamma + \alpha_i + \kappa_j},$$

$$E_{35}^2 = E_{13}^2 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$a_{35}^2 = b_{13}^2 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$b_{35}^2 = a_{13}^2 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$c_{35}^2 = c_{13}^2 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$E_{15}^2 = \left[\frac{(\gamma + \alpha_i)(\gamma + \alpha_k)}{(\gamma + \alpha_i)(\gamma + \alpha_k) + \kappa_j(2\gamma + \alpha_i + \alpha_k)} \right]^{\frac{3}{2}},$$

$$a_{15}^2 = b_{15}^2 = \frac{1}{5} \frac{13\gamma(\gamma + \alpha_i)(\gamma + \alpha_k) + \kappa_j(29\gamma^2 + 16\gamma\alpha_i + 16\gamma\alpha_k + 3\alpha_i\alpha_k)}{(\gamma + \alpha_i)(\gamma + \alpha_k) + \kappa_j(2\gamma + \alpha_i + \alpha_k)}$$

$$c_{15}^2 = -\frac{1}{5} \frac{24\gamma(\gamma + \alpha_i)(\gamma + \alpha_k) + \kappa_j(42\gamma^2 + 17\gamma\alpha_i + 18\gamma\alpha_k - 6\alpha_i\alpha_k)}{(\gamma + \alpha_i)(\gamma + \alpha_k) + \kappa_j(2\gamma + \alpha_i + \alpha_k)}$$

with

$$f_1 = 6\gamma^2 + 20\gamma\alpha_i + 5\gamma\alpha_k + 6\alpha_i\alpha_k,$$

$$g_1 = 6\gamma^2 + 5\gamma\alpha_i + 20\gamma\alpha_k + 6\alpha_i\alpha_k,$$

$$y_1 = 12\gamma^2 + 12\alpha_i\alpha_k,$$

$$e_1 = 5(3\gamma + 2\alpha_i + 2\alpha_k).$$

B. Partial-wave kernel function $k_{\ell\lambda}^C(r, r')$

$$k_{\ell 4}^C(r, r') = k_{\ell}^a(r, r') + k_{\ell}^b(r, r'),$$

$$k_{\ell 2}^C(r, r') = k_{\ell}^a(r, r') - \frac{1}{2} k_{\ell}^b(r, r').$$

B.1 d + ^3He system.

$$k_{\ell}^a(r, r') = - \frac{2\pi e^2}{G_0} \left(\frac{6}{5}\right)^3 \pi^3 r r'$$

$$\times \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \int_{-1}^1 \left\{ p'_{12} \left[\frac{\Phi(q_1)}{q_1} + \frac{\Phi(q_6)}{q_6} \right] + p'_{15} \frac{1}{q_2} \right\} \\ \times e^{-\frac{3}{5}(a_1 r^2 + b_1 r'^2)} P_{\ell}(\mu) d\mu,$$

$$k_{\ell}^b(r, r') = - \frac{2\pi e^2}{G_0} \left(\frac{6}{5}\right)^3 \pi^3$$

$$\times \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \left\{ r r' \int_{-1}^1 p'_{26} \frac{\Phi(q_3)}{q_3} e^{-\frac{3}{5}(a_1 r^2 + b_1 r'^2)} P_{\ell}(\mu) d\mu \right. \\ + p'_{23} e^{-\frac{3}{5}(a_1 r^2 + b_1 r'^2)} S_{\ell}(-\frac{3}{5}c_1) \\ + r r' \int_{-1}^1 \left[p_{35}^2 \frac{\Phi(q_4)}{q_4} + p_{13}^2 \frac{\Phi(q_7)}{q_7} + p_{15}^2 \frac{\Phi(q_5)}{q_5} \right] \\ \left. \times e^{-\frac{3}{5}(a_2 r^2 + b_2 r'^2)} P_{\ell}(\mu) d\mu \right\},$$

where

$$p'_{12} = \frac{1}{\gamma} \frac{1}{3\gamma + 2\alpha_i + 2\alpha_k} \left(\frac{1}{6\gamma + \alpha_i + \alpha_k} \right)^{\frac{1}{2}}$$

$$p'_{15} = \frac{5}{6\gamma} \left(\frac{1}{3\gamma + 2\alpha_i + 2\alpha_k} \right)^{\frac{3}{2}},$$

$$p'_{26} = \frac{2}{\gamma} \frac{1}{3\gamma + 2\alpha_i + 2\alpha_k} \left(\frac{1}{2\gamma + \alpha_i + \alpha_k} \right)^{\frac{1}{2}},$$

$$p'_{23} = \frac{1}{(2\pi^3)^{\frac{1}{2}} \gamma} \left(\frac{1}{3\gamma + 2\alpha_i + 2\alpha_k} \right)^{\frac{3}{2}},$$

$$p^2_{35} = - \frac{1}{\gamma + \alpha_k} \left(\frac{1}{\gamma + \alpha_i} \right)^{\frac{3}{2}}$$

$$p^2_{13} = - \frac{1}{\gamma + \alpha_i} \left(\frac{1}{\gamma + \alpha_k} \right)^{\frac{3}{2}},$$

$$p^2_{15} = - \frac{1}{(\gamma + \alpha_i)(\gamma + \alpha_k)} \left(\frac{1}{2\gamma + \alpha_i + \alpha_k} \right)^{\frac{1}{2}},$$

and

$$q_i = |L_i \vec{r} + M_i \vec{r}'|,$$

With

$$L_1 = \frac{3}{5} (-3\gamma + 2\alpha_i - \alpha_k) \left[\frac{\gamma}{(6\gamma + \alpha_i + \alpha_k)(3\gamma + 2\alpha_i + 2\alpha_k)} \right]^{\frac{1}{2}}$$

$$M_1 = \frac{3}{5} (3\gamma + 3\alpha_i + 6\alpha_k) \left[\frac{\gamma}{(6\gamma + \alpha_i + \alpha_k)(3\gamma + 2\alpha_i + 2\alpha_k)} \right]^{\frac{1}{2}}$$

$$L_2 = \sqrt{\gamma},$$

$$M_2 = -L_2,$$

$$L_3 = \frac{3}{10} \left[\frac{\gamma (10\gamma + 4\alpha_i + 6\alpha_k)^2}{(2\gamma + \alpha_i + \alpha_k)(3\gamma + 2\alpha_i + 2\alpha_k)} \right]^{\frac{1}{2}}$$

$$M_3 = \frac{3}{10} \left[\frac{\gamma (10\gamma + 6\alpha_i + 4\alpha_k)^2}{(2\gamma + \alpha_i + \alpha_k)(3\gamma + 2\alpha_i + 2\alpha_k)} \right]^{\frac{1}{2}},$$

$$L_4 = \frac{9}{5} (\gamma + \alpha_k)^{\frac{1}{2}},$$

$$M_4 = \frac{6}{5} (\gamma + \alpha_k)^{\frac{1}{2}},$$

$$L_5 = \frac{3}{5} \left[\frac{(\gamma + \alpha_i)(\gamma + \alpha_k)}{2\gamma + \alpha_i + \alpha_k} \right]^{\frac{1}{2}},$$

$$M_5 = -L_5,$$

$$L_6 = M_1 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$M_6 = L_1 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$L_7 = M_4 \quad \text{with } \alpha_i \leftrightarrow \alpha_k,$$

$$M_7 = L_4 \quad \text{with } \alpha_i \leftrightarrow \alpha_k.$$

B.2. $d + {}^3\text{H}$ system.

$$k_{\ell}^a(r, r') = - \frac{2\pi e^2}{G_0} \left(\frac{6}{5}\right)^3 \pi^3 r r' \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k p'_{26} \\ \times \int_{-1}^1 \frac{\Phi(q_3)}{q_3} e^{-\frac{3}{5}(a_1 r^2 + b_1 r'^2)} P_{\ell}(\mu) d\mu,$$

$$k_{\ell}^b(r, r') = - \frac{2\pi e^2}{G_0} \left(\frac{6}{5}\right)^3 \pi^3 r r' \\ \times \sum_{i=1}^3 \sum_{k=1}^3 A_i A_k \int_{-1}^1 \left[p'_{15} \frac{1}{q_2} e^{-\frac{3}{5}(a_1 r^2 + b_1 r'^2)} \right. \\ \left. + p_{15}^2 \frac{\Phi(q_5)}{q_5} e^{-\frac{3}{5}(a_2 r^2 + b_2 r'^2)} \right] P_{\ell}(\mu) d\mu$$

FOOTNOTES AND REFERENCES

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TABLE I. Level parameters for the resonance levels in ^5He and ^5Li .

Nucleus	Level Description	a (fm)	$E_{1\ell\lambda}$ (MeV)	$\gamma_{1\ell\lambda}^2$ (MeV)	$E_{2\ell\lambda}$ (MeV)	$\gamma_{2\ell\lambda}^2$ (MeV)	E_{res} (MeV)
^5He	^4S	6.0	-0.42	1.27			0.054
^5He	^4D	5.5	5.7	0.8	60.6	10.7	3.99
^5He	^2S	6.0	2.5	1.0	18.3	2.3	2.52
^5He	^2P	5.5	6.8	1.7	112.1	15.6	5.12
^5He	^2D	5.0	9.1	0.8	35.4	3.7	7.40
^5He	^2F	5.5	14.3	0.9	97.1	11.1	11.04
^5Li	^4S	6.0	-0.15	1.40			0.301
^5Li	^4D	5.5	6.1	0.9	79.7	14.6	4.36
^5Li	^2S	6.0	2.9	1.0	19.0	2.3	2.90
^5Li	^2P	5.5	7.2	1.7	149.4	21.3	5.52
^5Li	^2D	5.0	9.4	0.8	35.7	3.7	7.70
^5Li	^2F	5.5	14.6	0.9	109.2	12.6	11.34

TABLE II. Values of U_{04} , U_{02} , σ_R , and λ_{δ_l} for the $d + {}^3\text{H}$ system at 2.02, 4.19, and 6.6 MeV.

E (MeV)	2.02	4.19	6.6
U_{04} (MeV)	0.12	0.25	0.45
U_{02} (MeV)	0.15	0.30	0.70
σ_R (mb)	87	205	299
${}^4\delta_0$ (deg)	221.37+i1.96	192.49+i3.84	174.08+i6.93
${}^4\delta_1$ (deg)	157.78+i0.11	138.03+i0.61	123.87+i2.02
${}^4\delta_2$ (deg)	14.97+i0.45	61.10+i4.11	80.52+i6.23
${}^4\delta_3$ (deg)	-0.50+i0	-2.96+i0.05	-6.19+i0.33
${}^4\delta_4$ (deg)	0.06+i0	0.84+i0.01	3.21+i0.07
${}^4\delta_5$ (deg)	0	-0.08+i0	-0.45+i0.01
${}^4\delta_6$ (deg)	0	0.01+i0	0.11+i0
${}^4\delta_7$ (deg)	0	0	-0.01+i0
${}^2\delta_0$ (deg)	153.41+i3.03	149.38+i8.58	143.09+i18.71
${}^2\delta_1$ (deg)	186.35+i0.58	179.91+i1.67	168.91+i4.28
${}^2\delta_2$ (deg)	-0.16+i0.13	4.98+i1.88	22.24+i14.22
${}^2\delta_3$ (deg)	0.52+i0.01	4.24+i0.15	12.37+i1.47
${}^2\delta_4$ (deg)	-0.02+i0	-0.19+i0.01	-0.49+i0.08
${}^2\delta_5$ (deg)	0	0.07+i0	0.39+i0.01
${}^2\delta_6$ (deg)	0	0	-0.03+i0
${}^2\delta_7$ (deg)	0	0	0.01+i0

TABLE III. Values of U_{04} , U_{02} , σ_R , and $\lambda \delta_\ell$ for the $d + {}^3\text{He}$ system at 8.64, 10.0, and 13.84 MeV.

E (MeV)	8.64	10.0	13.84
U_{04} (MeV)	0.40	0.55	0.75
U_{02} (MeV)	1.30	1.70	1.80
σ_R (mb)	297	327	296
${}^4\delta_0$ (deg)	166.48+i10.19	158.69+i12.63	142.45+i14.25
${}^4\delta_1$ (deg)	117.25+i0.71	112.89+i1.18	103.90+i2.35
${}^4\delta_2$ (deg)	82.87+i7.86	84.62+i9.55	85.38+i9.49
${}^4\delta_3$ (deg)	-8.13+i0.17	-8.63+i0.35	-6.72+i1.14
${}^4\delta_4$ (deg)	6.04+i0.26	8.10+i0.58	13.51+i2.00
${}^4\delta_5$ (deg)	-1.03+i0	-1.46+i0.01	-2.53+i0.05
${}^4\delta_6$ (deg)	0.35+i0	0.60+i0.01	1.65+i0.06
${}^4\delta_7$ (deg)	-0.06+i0	-0.11+i0	-0.36+i0
${}^2\delta_0$ (deg)	144.21+i10.18	140.40+i12.56	131.16+i12.91
${}^2\delta_1$ (deg)	161.05+i13.61	155.34+i18.12	143.28+i19.20
${}^2\delta_2$ (deg)	43.63+i10.54	55.46+i13.05	70.37+i8.60
${}^2\delta_3$ (deg)	18.20+i8.18	21.39+i14.36	34.00+i25.68
${}^2\delta_4$ (deg)	-0.64+i0.11	-0.49+i0.24	1.17+i0.72
${}^2\delta_5$ (deg)	0.95+i0.09	1.46+i0.21	3.33+i0.83
${}^2\delta_6$ (deg)	-0.10+i0	-0.16+i0	-0.30+i0.02
${}^2\delta_7$ (deg)	0.04+i0	0.08+i0	0.32+i0.02

FIGURE CAPTIONS

- Fig. 1. Calculated phase shifts $\lambda\delta_\ell$ for $d + {}^3\text{He}$ elastic scattering from 0 to 15 MeV (c.m.). The spin multiplicity λ is equal to $2s + 1$, where s is the channel-spin quantum number and ℓ is the quantum number of the relative orbital angular momentum between the d and ${}^3\text{He}$ nuclei. No imaginary potential was used in the calculation.
- Fig. 2. Calculated phase shifts for $d + {}^3\text{He}$ elastic scattering at 50 MeV (c.m.). s is the channel-spin quantum number, and ℓ is the quantum number of the relative orbital angular momentum between the d and ${}^3\text{He}$ nuclei. No imaginary potential was used in the calculation.
- Fig. 3. Comparison of calculated quartet ($s = 3/2$) and doublet ($s = 1/2$) cross sections for $d + {}^3\text{He}$ scattering with calculated cross sections for $d + {}^4\text{He}$ and ${}^3\text{He} + {}^4\text{He}$ scattering, all at 15 MeV (c.m.). No imaginary potential was used in the calculation.
- Fig. 4. Comparison of properties of calculated resonance levels with properties of levels suggested from experimental work.²²⁻²⁸ The level widths; some of which are rather large, are not indicated. The excitation energy is referred to the $\ell = 0$, $s = 3/2$ level. Possible correspondences between experiment and theory are indicated by dashed lines.

Fig. 5. Comparison of calculated cross sections (curves) for $d + {}^3\text{He}$ elastic scattering with experimental cross sections^{30,31} (points) at c.m. energies of 8.64, 10.0, and 13.84 MeV. For the solid curves, C_I of Eq. (22) is taken to be 0.7, and other parameters are given in Table III. The dashed curve for 10.0 MeV represents a calculation with $C_I = 0$ and imaginary-potential depths optimized.

Fig. 6. Comparison of calculated cross sections (curves) for $d + {}^3\text{H}$ elastic scattering with experimental cross sections²² (points) at c.m. energies of 2.02, 4.19, and 6.6 MeV. C_I of Eq. (22) is taken to be 0, and other parameters are given in Table II.

Fig. 1

E: a. 2.

Fig. 3

Fig. 5

Fig. 6