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SUBJECT: Redstrom Plot for the Calculation of Laminar Flow
Pressure Drop for Bingham Plastic Materials with
Redstrom Numbers from 0 to 10^{15}

TO: Distribution

FROM: D. G. Thomas

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SUMMARY

The results of a machine calculation of a modified Fanning-friction-factor Hedstrom plot for Hedstrom numbers from 0 to 10^{15} are presented in graphical and tabular form.

INTRODUCTION

The usefulness of the concept proposed by B. O. A. Hedstrom¹ for the calculation of laminar flow pressure drop of Bingham plastic materials in circular conduits using a modified Fanning friction factor plot has been pointed out previously.^{2,3,4} Turbulent flow data for thorium oxide slurries were shown to be between the curves for commercial pipe and smooth tubes provided the density of the slurry and the viscosity of water were used in the calculation of the Reynolds number.⁶ Therefore the Hedstrom plot for plastic slurries was modified² for use in turbulent flow calculations by introducing a set of curves displaced from the smooth tube line by the factor η/μ and retaining the definition of the Reynolds number, $DV\rho/\eta$, proposed by Hedstrom. Curves for Hedstrom numbers up to 10^7 were presented.²

Design calculations for systems having large pipe diameter and high yield stress showed that it was desirable to have curves for Hedstrom numbers at least up to 10^{12} . Therefore calculations were made by J. L. Gabbard, Jr., of the K-25 Numerical Analysis Laboratory for Hedstrom numbers up to 10^{15} .

RESULTS OF CALCULATION

A. B. Metzner has presented a rearranged form of the Buckingham equation in terms of the Hedstrom number, the Reynolds number and the Fanning friction factor, Equation 1

$$\frac{1}{N_{Re}} = \frac{f}{16} - \frac{1}{6} \frac{N_{He}}{(N_{Re})^2} + \frac{1}{3} \frac{N_{He}^4}{f^3 (N_{Re})^3} \quad (1)$$

where

$$f = \frac{2g_c D \Delta p}{4L \rho V^2} \quad (2)$$

$$N_{He} = \frac{g_c \rho \tau_y D^2}{\eta^2} \quad (3)$$

$$N_{Re} = \frac{DV\rho}{\eta} \quad (4)$$

In the course of programming Equation 1 for machine computation it was observed that the expression has two real positive roots in the range of $0 < N_{He} < 10^{15}$.

If the left term of Equation 1 is transposed to the other side of the equality sign and the resultant expression set equal to $f(G)$ then a curve similar to the one below

can be prepared for constant Hedstrom number. The two positive roots are very close together for values of N_{He} greater than 10^9 .

The computation was made by solving for the Reynolds number for powers of 10 of the Hedstrom number from 0 to 10^{15} for Fanning friction factors of 1.0, 0.3, 0.1, 0.03, 0.01, 0.003, 0.001. It was observed the larger of the two positive roots converged toward 4 distinct values as N_{Re} was increased, i.e.

$$\sqrt{2} \times 10^n$$

$$\sqrt{20} \times 10^n$$

$$f = 1 \times 10^8$$

$$\sqrt{2/3} \times 10^n$$

$$\sqrt{20/3} \times 10^n$$

$$f = 3 \times 10^8$$

The values of N_{Re} , calculated to 0.1% accuracy, are presented in Tables 1 and 2 for the smaller and larger positive roots respectively. These data are plotted on the modified Hedstrom plot in Figure 1.

Calculations of the velocity corresponding to the intersection of different Hedstrom lines with the appropriate turbulent friction factor curve (i.e., the critical velocity for the onset of turbulence) were made for slurries having a density of 100 lb/cu ft, yield stresses from 0 to 1.5 lb/sq ft and coefficients of rigidity from 1/2 to 10 centipoise. The viscosity of water was taken to be 1/4 centipoise and the pipe diameters were from 1 to 24 inches. The results are shown in Figure 2, which shows that the major variable is the slurry yield stress. The coefficient of rigidity was of secondary importance and the critical velocity was independent of pipe diameter to within $\pm 10\%$ for the range of pipe diameters covered.

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Table 1

Reynolds Numbers for Hedstrom Numbers from 0 to 10^9 and Fanning Friction Factors from 0.001 to 1.0 (Smaller Positive Root of Equation 1)

f, Fanning Friction Factor

NHe	1.0	0.3	0.1	0.03	0.01	0.003	0.001
10^0	8.382×10^{-1}	1.414×10^0	2.274×10^0	3.822×10^0	6.131×10^0	1.028×10^1	1.648×10^1
10^1	3.051×10^0	5.186×10^0	8.382×10^0	1.414×10^1	2.274×10^1	3.822×10^1	6.131×10^1
10^2	1.087×10^1	1.871×10^1	3.051×10^1	5.186×10^1	8.382×10^1	1.414×10^2	2.274×10^2
10^3	3.774×10^1	6.586×10^1	1.087×10^2	1.871×10^2	3.051×10^2	5.186×10^2	8.382×10^2
10^4	1.274×10^2	2.255×10^2	3.774×10^2	6.587×10^2	1.087×10^3	1.871×10^3	3.051×10^3
10^5	4.203×10^2	7.525×10^2	1.274×10^3	2.255×10^3	3.774×10^3	6.587×10^3	1.087×10^4
10^6	1.364×10^3	2.461×10^3	4.203×10^3	7.524×10^3	1.274×10^4	2.255×10^4	3.774×10^4
10^7	4.381×10^3	7.942×10^3	1.364×10^4	2.461×10^4	4.203×10^4	7.524×10^4	1.274×10^5
10^8	1.398×10^4	2.542×10^4	4.381×10^4	7.942×10^4	1.364×10^5	2.461×10^5	4.203×10^5
10^9	4.443×10^4	8.092×10^4	1.398×10^5	2.542×10^5	4.381×10^5	7.943×10^5	1.364×10^6

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Table 2

Reynolds Numbers for Hedstrom Numbers from 0 to 10^{15} (Larger Positive Root of Equation 1.)
 Fanning Friction Factor, f

NRe	1.0	0.3	0.1	0.03	0.01	0.003	0.001
10^0	1.617×10^1	5.350×10^1	1.602×10^2	5.335×10^2	1.600×10^3	5.333×10^3	1.600×10^4
10^1	1.752×10^1	5.495×10^1	1.617×10^2	5.350×10^2	1.602×10^3	5.335×10^3	1.600×10^4
10^2	2.614×10^1	6.667×10^1	1.752×10^2	5.495×10^2	1.616×10^3	5.350×10^3	1.602×10^4
10^3	5.904×10^1	1.237×10^2	2.614×10^2	6.667×10^2	1.752×10^3	5.495×10^3	1.616×10^4
10^4	1.624×10^2	3.144×10^2	5.902×10^2	1.236×10^3	2.614×10^3	6.666×10^3	1.752×10^4
10^5	4.810×10^2	9.036×10^2	1.624×10^3	3.146×10^3	5.901×10^3	1.237×10^4	2.615×10^4
10^6	1.471×10^3	2.724×10^3	4.807×10^3	9.033×10^3	1.623×10^4	3.144×10^4	5.901×10^4
10^7	4.567×10^3	8.413×10^3	1.471×10^4	2.727×10^4	4.807×10^4	9.030×10^4	1.623×10^5
10^8	1.432×10^4	2.627×10^4	4.573×10^4	8.412×10^4	1.471×10^5	2.727×10^5	4.806×10^5
10^9	4.505×10^4	8.242×10^4	1.431×10^5	2.623×10^5	4.571×10^5	8.409×10^5	1.471×10^6
10^{10}	1.418×10^5	2.596×10^5	4.500×10^5	8.234×10^5	1.432×10^6	2.626×10^6	4.568×10^6
10^{11}	4.484×10^5	8.192×10^5	1.420×10^6	2.594×10^6	4.505×10^6	8.244×10^6	1.432×10^7
10^{12}	1.417×10^6	2.587×10^6	4.485×10^6	8.189×10^6	1.420×10^7	2.597×10^7	4.503×10^7
10^{13}	4.476×10^6	8.180×10^6	1.417×10^7	2.587×10^7	4.482×10^7	8.189×10^7	1.420×10^8
10^{14}	1.415×10^7	2.584×10^7	4.476×10^7	8.180×10^7	1.417×10^8	2.587×10^8	4.485×10^8
10^{15}	4.477×10^7	8.172×10^7	1.415×10^8	2.584×10^8	4.476×10^8	8.180×10^8	1.416×10^9

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Fig. 1. Friction Factor - Reynolds Number Diagram for Bingham Plastic Slurries in Smooth Pipes

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Fig. 2. Effect of Slurry Physical Properties on Velocity for Onset of Turbulence

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