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A Many Body Approach to the Low Energy
Pion-Nucleus Interaction*

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MASTER

1. INTRODUCTION AND MOTIVATION

In this talk, I will confine my attention to the interaction of low energy pions with nuclei. The low energy domain will be somewhat loosely defined as the region between 0-100 MeV pion lab kinetic energy, i.e., energies well below the dominant $J = 3/2$ $T = 3/2$ resonance in the elementary pion-nucleon system. The main experimental information on the low energy pion-nucleus interaction comes from measurements in pi-mesic atoms. The measured energies and level widths in pionic Bohr transitions provide us with information on the strong pion-nucleus interaction at threshold, i.e., essentially at zero kinetic energy. The usual theoretical description of these strong interaction effects involves the solution of a one-particle Klein-Gordon equation of the form

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$$[\nabla^2 - \mu^2 + (\omega - V_c(r))^2] \psi_\pi(r) = 2\omega V_{opt}(r) \psi_\pi(r) \quad (1.1)$$

where $\psi_\pi(r)$ is the pion wave function, $\omega = (p^2 + \mu^2)^{\frac{1}{2}}$ is the total pion energy and $V_{opt}(r)$ and $V_c(r)$ are the strong interaction potential and Coulomb potential, respectively. The energy and width of a given level in a Bohr atom are identified with the complex eigenvalue of Eq. (1.1). The pion optical potential $V_{opt}(r)$ has been parametrized^{2,3} in the form

$$-2\mu V_{opt}(r) = 4\pi[b_s^{(1)}\rho(r) + b_s\rho^2(r) - \tilde{\nabla}[b_p^{(1)}\rho(r) + b_p\rho^2(r)]\tilde{\nabla}] \quad (1.2)$$

where the constants $b_s^{(1)}$, b_s , $b_p^{(1)}$, b_p are allowed to vary in order to provide a best least squares fit to the available pi-mesic data. In the work of Krell and Ericson,² the coefficients $b_s^{(1)}$ and $b_p^{(1)}$ of the terms linear in the total nuclear density $\rho(r)$ are taken to be real; the empirical values are found to be closely related to the elementary s and p wave πN scattering lengths. The coefficients b_s and b_p are chosen to be purely imaginary, representing the real pion absorption present even at threshold. Dispersive effects of order $\rho^2(r)$ are ignored in ref. 2, but are included in ref. 3. The following set of parameters provide a good fit to the data^{2,4}

$$\begin{aligned} b_s^{(1)} &= -0.03 \text{ } 1/\mu \\ b_p^{(1)} &= 0.22 \text{ } 1/\mu^3 \\ b_s &= 0.041 \text{ } 1/\mu^4 \\ b_p &= 0.081 \text{ } 1/\mu^6 \end{aligned} \quad (1.3)$$

Note that the set of strength constants given in Eq. (1.3) are universal parameters which are independent of the particular nucleus under consid-

eration. The ansatz (1.2) thus depends only on rather gross features of nuclear structure, i.e., on single particle densities. This fact enables us to make contact with the results of calculations performed in nuclear matter, using the local density approximation, since the nuclear matter results are also expressible only in terms of the total density ρ . We exhibit this connection in detail in later sections.

Traditionally, the $\rho^2(r)$ terms in the potential (1.2) have been associated with the two nucleon absorption mechanism for pions, i.e., the (π, NN) reaction. Estimates⁵ based on the old quasi-deuteron model relate b_s and b_p to the total cross section for the reaction $\pi^- + d \rightarrow n + n$ near threshold. This gives essential agreement for b_p , but yields a result for b_s which is too small by a factor of two, indicating that some important physical processes are being left out. A many body calculation employing a density expansion gave good results for both the s and p wave parts of the absorptive ρ^2 term.^{6,7} However, as we shall see later, this type of expansion overestimates the absorptive part near threshold, although it is still useful in the low energy region above threshold. Thus, attempts to relate pion absorption on complex nuclei solely to absorption on two-body subsystems within the nucleus seem to provide only part of the physical picture.

Recently, new experimental information has become available which demonstrates that modes of pion absorption other than (π, NN) are important and even dominant in a number of cases.^{8,9} The ρ^2 terms of the phenomenological potential (1.2) must thus bear the burden of accounting for these multiparticle effects as well as the true two body absorption processes; these terms must mock up alpha cluster absorption,¹⁰ for instance, which would not correspond to a ρ^2 contribution in a microscopic approach. Thus we need expect no agreement between the ρ^2 terms

in a fundamental many body approach and the ρ^2 term required by fitting the pi-mesic data within the too restrictive ansatz of Eq. (1.2).

In view of the new information^{8,9} on pion absorption, and its possible interpretation in terms of cluster structure,¹⁰ it seems relevant to re-examine the many body calculations of the two nucleon absorption process,^{6,7} in order to evaluate as carefully as possible the true contributions of order ρ^2 . We study various forms of low density expansion, such as the strict density expansion, the hole line expansion, and various refinements of these methods. The language in which our discussion is couched is that of many body field theory with propagators. We discuss the physical interpretation of the various graphs which arise in the theory, i.e., their relation to more familiar notions of pion absorption. We emphasize some difficulties of nuclear matter estimates of pion absorption at low energies.

There are a number of advantages of the many-body approach: In a density expansion, the terms of order ρ and ρ^2 in the optical potential are related unambiguously to the free pion-nucleon and nucleon-nucleon scattering amplitude, in general off the energy shell. These are quantities over which we can exercise some theoretical control, i.e., variations in the calculated pion potential due to variations in off-shell behavior are not large. In a hole line expansion, the ρ and ρ^2 terms are expressible in terms of effective pion-nucleon and nucleon-nucleon interactions, which are related to the free space interactions corrected in a calculable way for binding and Pauli principle effects. Also, the many body approach yields the real and imaginary parts of the potential on the same footing, as well as the predicted energy dependence of the ρ and ρ^2 terms at low energy. This latter fact enables us to assess the relative importance of the (π, N) and (π, NN) mechanisms

as a function of energy. Macroscopic approaches based on the threshold cross sections for $\pi^- + d$ or $\pi^- + \text{He}$ ^{4,5,10} provide no information about dispersive contributions to the potential (which turn out to be of the same order as the absorptive part) or the energy dependence.

The general strategy of the many-body approach is to try to calculate as many contributions to the pion potential as we can from first principles; in practice this includes all terms of order ρ and ρ^2 , plus selective summations of certain classes of graphs to all orders in ρ .^{6,11} Complicated multiparticle contributions ("clusters") could in principle also be calculated microscopically, but in practice would be parametrized by a few constants, or estimated as in ref. 10. The net result is to considerably reduce the number of free parameters in the optical potential. One must also try to insert the phenomenological aspects of the potential in a way which does not disturb the physical interpretation of the other parameters. For instance, lumping all absorptive effects into a single term of order ρ^2 , as in Eq. (1.2), disturbs the relation between the phenomenological strength of this term and the underlying two nucleon processes. It would seem more reasonable to parametrize the alpha cluster absorption by a separate term of order $\rho^4(r)$, for instance, and determine its strength empirically, while keeping the theoretical estimate (or close to it) for the ρ^2 term.

There are several sound theoretical reasons for formulating the pion-nucleus problem in the framework of a many body field theory rather than in terms of conventional Watson multiple scattering theory,¹² using static potentials to represent the pion-nucleon interaction. The underlying philosophy has been discussed in some detail in various

6,7,13-16 articles, so only the salient features will be summarized here: A field theory provides a natural description of retardation effects in pionic exchange processes in nuclei, since the use of a pion propagator in the nuclear medium ensures a correct account of the time delay effects between absorption and emission vertices. For the problem of the binding energy of nuclear matter or of nucleon-nucleus scattering, retardation effects are not expected to be important,^{13,16} since virtual pions which are exchanged in the absence of an external "beam" pion generally propagate far off the energy shell. The effect of such strongly virtual pion exchanges can be replaced by a static Yukawa potential, i.e., it is sensible to eliminate mesonic degrees of freedom in favor of static potentials for nuclear problems not involving external pions. The pion-nucleus problem, in the other hand, demands an approach which treats meson degrees of freedom explicitly. After the absorption of an external pion by a nuclear system, intermediate state virtual pions can propagate close to the energy shell (long range in space), since the beam pion has dumped the requisite amount of energy into the system. In this sense, pion-nucleus scattering is rather distinct from the more familiar domain of the nucleon-nucleus problem; it seems rather dangerous to blindly apply the techniques of multiple scattering theory with static potentials to the pion-nucleus problem, even though this treatment of the corresponding nucleon-nucleus situation may be very successful.

Another way of emphasizing the same point is as follows: we start from an interaction Hamiltonian coupling pion and nucleon fields rather than a static pion-nucleon potential. If we choose a particular form of this interaction, as for example the linear coupling of the meson field to the nucleon density suggested by the Chew-Low theory, we

can construct a unique off-energy-shell continuation of the elementary pion-nucleon amplitude. These off-shell amplitudes are needed in the pion-nucleus problem, since the pion propagates off-shell in the nuclear medium. The important point is now that the off-shell pion-nucleon amplitudes generated by simple field theories ^{14,15} cannot be reproduced by means of energy independent potentials, although the same on-shell scattering can be obtained. Thus there is a fundamental difference in principle between field theories and static potential models; it remains to be seen whether or not this difference of philosophy has any notable consequences for the actual calculated pion-nucleus cross sections.

We close this introduction with some comments on the advantages of probing nuclei with low energy pions. Since the pion-nucleus interaction is relatively weaker at low energies than in the (3,3) resonance region, greater sensitivity to details of nuclear structure is to be expected. Near the (3,3) resonance region, the nucleus is largely opaque, and one is primarily sensitive to global nuclear properties such as the total density. The main challenge of the (3,3) resonance region is to develop a method of calculating the optical potential in the presence of a strongly absorptive elementary amplitude, since density expansions and hole line expansions may not converge in this region. At low energies, on the other hand, a systematic density expansion seems to converge and the terms of this expansion can be calculated from a many body theory or estimated empirically. The low energy potential is quite sensitive to nucleon-nucleon correlation terms ($\propto \rho^2(r)$); these contributions are relatively less important in the (3,3) resonance region, where the main contribution to the absorption comes from the elementary amplitude. Since the low energy potential can be fairly reliably calculated, one can use this potential

with some confidence to generate pion distorted waves, which in turn are used as input in calculations of various reaction processes, such as (p, π^+) . If the potential is known, one can use reaction processes to test more detailed aspects of nuclear wave functions.

2. THEORETICAL FRAMEWORK

The interaction Hamiltonian H_{int} which underlies all of our considerations has the general form

$$H_{\text{int}} = \sum_{k\alpha} \sum_{p\nu\nu'} a_{p+k, \nu'}^+ g_{\nu, \nu'}^\alpha(k) a_{p\nu} Q_{k\alpha} \quad (2.1)$$

of a non-relativistic linear coupling of a set of meson field operators $Q_{k\alpha}$ to the nucleon density $a_{p+k}^+ a_p$. In Eq. (2.1), $Q_{k\alpha} = (2\omega_{k\alpha})^{-\frac{1}{2}} (B_{k\alpha} + B_{-k\alpha}^+)$, where $B_{k\alpha}^+$ and $B_{k\alpha}$ are creation and destruction operators for mesons of momentum k and type α ; $a_{p\nu}^+$ and $a_{p\nu}$ are nucleon creation and destruction operators for a nucleon of momentum p and spin-isospin ν ; the vertex function $g_{\nu, \nu'}^\alpha(k)$ is of the form $\lambda \delta_{\nu, \nu'}$ for neutral scalar coupling and $i f / \mu (g \cdot k)_{\sigma' \sigma} (\tau_\alpha')_{\tau' \tau}$ for charged pseudoscalar coupling. The Hamiltonian (2.1) can also accommodate couplings to heavy mesons such as the ρ , ω , ϕ , η (treated as elementary particles) as well as the pion.

Most of our discussion is phrased in terms of the one nucleon propagator G defined by

$$G(p\nu t, p'\nu' t') = i \langle T \{ a_{p\nu}(t) a_{p'\nu'}^+(t') \} \rangle \quad (2.2)$$

and the pion propagator D given by

$$D(q\alpha t, q'\alpha' t') = i \langle T \{ Q_{q\alpha}(t) Q_{q'\alpha'}^+(t') \} \rangle \quad (2.3)$$

In Eqs. (2.2) and (2.3), T is a time-ordering operator and $|>$ represents the interacting ground state of the system. For later reference, it is useful to recall the form of the Fourier transform of the pion propagator in free space

$$D_0(q, \omega) = \frac{1}{(\omega - i\eta)^2 - \omega_q^2} \quad (2.4)$$

where $\omega_q = (\mu^2 + q^2)^{\frac{1}{2}}$ and μ is the pion mass. The corresponding propagator $G(p, \epsilon)$ for a nucleon in a non-interacting Fermi gas is given by

$$G_0(p, \epsilon) = \frac{n_<(p)}{\epsilon_p - \epsilon + i\delta} + \frac{n_>(p)}{\epsilon_p - \epsilon - i\delta} \quad (2.5)$$

where $n_<(p) = n(p)$ and $n_>(p) = 1 - n(p)$, $n(p)$ being the Fermi gas occupation number defined by

$$n(p) = \begin{cases} 1 & |p| < p_F \\ 0 & |p| > p_F \end{cases} \quad (2.6)$$

In Eq. (2.5), the $n_<$ term corresponds to hole propagation in the Fermi sea and $n_>$ refers to particle propagation above the Fermi sea. The single particle energy ϵ_p can be taken to be $p^2/2m$ as for the free Fermi gas or generalized to include a Hartree-type single particle potential $U(p)$. It should be noted that the use of the propagator G_0 ,⁶ in the context of a set of many-body Feynman rules, guarantees that the requirements of the Pauli principle are satisfied exactly.

The self-energy part $\pi(q)$ is related to the full propagator $D(q)$ via the relation $[q = (q, \omega)]$

$$D(q) = D_0(q) + D_0(q)\pi(q)D(q) \quad (2.7)$$

The energy-momentum relation of a pion in the nucleus is determined by the position of the pole in $D(q)$, i.e., for a given energy ω , the complex momentum q is determined by the self consistent solution of the equation

$$q^2 + \mu^2 - \omega^2 = \pi(q, \omega) \quad (2.8)$$

Equations (2.4)-(2.8) refer to an infinitely extended system of nuclear matter. To make contact with the Klein-Gordon equation (1.1) for a finite nucleus, we recall that $\psi_\pi(r) \propto \exp(iq \cdot r)$ for an infinite system, so Eqs. (1.1) and (2.8) are completely equivalent if we identify

$$2\omega \, v_{p \text{ opt}} = -\pi(q, \omega_p) \quad (2.9)$$

Thus, except for a multiplicative constant, the self energy $\pi(q)$ is identical to the usual optical potential.

In the following development, we will perform all calculations for the nuclear matter limit. This has the advantage that most of the program can be carried out analytically; the main features of the results can then be easily seen without recourse to excessive numerical calculation. Contact with finite nuclei is then made via the local density approximation, i.e., the optical potential is calculated as a function of p_F , which is then replaced by the local Fermi momentum $p_F(r) = (3\pi^2 \rho(r)/2)^{1/3}$ of a finite system at the end of the calculation. The local density approximation may be poor, however, for the low density region (10% of central density, say) of the nuclear surface, particularly if clusters are present. Also, the local density approximation may not adequately represent the physics of even the more dense part of the surface region ($\approx 50\%$ of central density), since the true potential, calculated in a basis of states appropriate to a finite

nucleus, is non-local and energy dependent. The local density approximation of course yields a local potential (except for the trivial $\nabla\rho(r)\nabla$ type of non-locality). However, this approximation is probably adequate for the present purposes, since it yields a potential of the form of the empirical ansatz (1.2), with a clear identification of the constants which appear in Eq. (1.2) in terms of more fundamental amplitudes. No empirical or theoretical studies have yet been made which explore the consequences of using a more general non-local ansatz for the pion-nucleus potential. The present formalism can be easily generalized in order to calculate the pion self energy in a shell model or Hartree-Fock basis of states; this more realistic approach always yields a non-local potential. Finite nucleus calculations are now under way in order to assess the consequences of the non-localities which arise in a microscopic shell model calculation of $V_{\text{opt}}(r)$. Even though a local equivalent potential can be found which yields the same elastic scattering phase shifts for pion-nucleus scattering as a non-local potential, the "near zone" behavior of the distorted waves generated by non-local and equivalent local potentials is different. To the extent that processes such as (p, π^+) are sensitive to this "near zone" behavior, quite different reaction cross sections may be obtained. This is a problem which warrants close study.

A variety of models for the self energy $\pi(q)$ have been developed. We examine several models in the next section which are appropriate for low energy pions, i.e., the density and hole line expansions. We will not discuss the problem of the (3,3) resonance region, which has been studied by a generalization of the Chew-Low theory to pion-nucleus scattering.

3. DENSITY AND HOLE LINE EXPANSIONS

In this section, we present a short summary of the theoretical techniques relating to the calculation of pion self energy diagrams. For a fuller development, we refer the reader to the original papers.^{6,7}

In diagrammatic language, the pion self energy $\pi(q)$ consists of all linked irreducible graphs which have two external pion legs. In perturbation theory, the simplest graph is the single "bubble" shown in Fig. 1:

Fig. 1

Physically, this process corresponds to the excitation of a particle-hole state¹⁷ by pion absorption (or emission), followed by propagation of this p-h excitation without further interactions with other particles and finally the re-emission (or absorption) of a pion. Higher order graphs involve a number of internal pion or heavy meson exchanges, which renormalize the single bubble, as in Fig. 2.

Fig. 2

or meson exchanges between different bubbles, as in Fig. 3

Fig. 3

One can, of course, also have an infinite number of more complicated graphs which involve bubbles within bubbles and other complicated beasts, as in Fig. 4:

Fig. 4

In Figs. 3 and 4, the double line refers to the full nucleon-nucleon t -matrix, minus the one pion contribution. Note that in Fig. 2, we draw explicitly graphs which would normally be included in the definition of the renormalized mass of the nucleon. However, the graphs in Figs. 2-4 represent excitations with respect to a filled Fermi sea, not free space. So the solid lines in Figs. 2-4 correspond to the density dependent nucleon propagator of Eq. (2.5). The result is that the series of graphs which define the renormalized mass (and also the renormalized vertex) now depend on the density of the system. One must thus carry out a new

renormalization program for the pion-nucleus problem, which involves the definition of a density dependent nucleon mass and πN coupling constant. Divergences in the self energy graphs which arise because of divergences in the free space mass renormalization graphs are then absorbed in the definition of these new renormalized quantities. The consistent formulation of this program is a difficult problem. Also, it is not feasible to try to sum all the self energy diagrams in closed form. One can approximately sum all pion interactions within a single bubble via a generalization of the Chew-Low method¹¹ and one can also sum repeated bubbles as in Fig. 3, which is reminiscent of the usual Random Phase Approximation. However, it is difficult to assess the reliability of summing selected subsets of graphs to all orders.

Some of these difficulties become more manageable if we expand the self energy $\pi(q)$ in a functional Taylor series in the density ρ . The empirical form (1.2) of the potential suggests such an expansion.⁶ This density expansion has the form

$$\begin{aligned} \pi(q) &= \sum_k n(k) \left. \frac{\delta \pi(q)}{\delta n(k)} \right|_{\rho=0} + \frac{1}{2} \sum_{kk'} n(k)n(k') \left. \frac{\delta^2 \pi(q)}{\delta n(k) \delta n(k')} \right|_{\rho=0} + \dots \\ &= \pi^{(1)}(q) + \pi^{(2)}(q) + \dots \end{aligned} \quad (3.1)$$

In Eq. (3.1), the symbol $(\delta \pi(q)/\delta n(k))_{\rho=0}$ implies that the zero density limit is to be taken after the functional derivative $\delta/\delta n(k)$ or any graph has been calculated. If the derivatives $(\delta \pi(q)/\delta n(k))_{\rho=0}$, etc., are independent of the nucleon momenta, then the terms $\pi^{(1)}(q)$, $\pi^{(2)}(q)$, etc. in Eq. (3.1) are of order ρ , ρ^2 and so forth, as in Eq. (1.2). Of course, these derivatives will in general exhibit some momentum

dependence, so Eq. (3.1) is not strictly speaking a power series in ρ ,⁶ although its convergence is of course related to low density.

The density expansion in the form (3.1) offers several significant advantages: If one stops with the n th term, only graphs with $1, 2, \dots, n$ bubbles need be considered. This is because the n th derivative of a graph with $n+1$ or more bubbles is zero in the zero density limit. If we stop with the second term in Eq. (3.1), we need only write down graphs with one or two bubbles. This constitutes an enormous simplification. In addition, one does not have to face the difficult renormalization problem outlined above, since one always takes the zero density limit of $\delta\pi(q)/\delta n(k)$, etc., which then involves only the free space coupling constants and masses.

We now show how the density expansion (3.1) is able to relate terms of order ρ and ρ^2 in the pion optical potential to the fundamental pion-nucleon and nucleon-nucleon scattering amplitudes in free space, in general off-shell.

The first term in Eq. (3.1) corresponds to the usual first order result of multiple scattering theory

$$\begin{aligned}\pi^{(1)}(q) &= 4\pi \sum_k n(k) f_{\pi N}(kq; kq) \\ &\approx 4\pi\rho \langle f_{\pi N} \rangle\end{aligned}\quad (3.2)$$

where $\langle f_{\pi N} \rangle$ is the Fermi-averaged free space pion-nucleon amplitude. To see this, we note that functional differentiation $\delta/\delta n(k)$ is related graphically to the cutting of a nucleon line. For the lowest order graph of Fig. 1, snipping of a line yields the two Born graphs for πN scattering, as per Fig. 5

Fig. 5

This connection between the functional derivative of the pion self energy and the πN amplitude is preserved in higher orders of perturbation theory, i.e., we find

$$[\delta\pi(q)/\delta\pi(k)]_{\rho=0} = 4\pi f_{\pi N}(k,q;k,q) \quad (3.3)$$

and hence we obtain Eq. (3.2).

The second term in Eq. (3.1) involves the cutting of two lines in any self energy graph. Physically, it corresponds to the absorption and dispersion of pions on two nucleon subsystems within the nucleus. As stated above, we need only consider graphs with one or two bubbles. The graphs of this type which are linear in the nucleon-nucleon t-matrix are displayed in Figs. 6 and 7. In Fig. 6, the pion lines are attached to the same bubble

Fig. 6

while in the two bubble contribution to Fig. 7 the pion lines are attached to two separate bubbles

Fig. 7

The double lines in Figs. 6 represent the full nucleon-nucleon amplitude t , defined as the solution of the Lippmann-Schwinger equation

$$t(E) = V + V(E^{(+)} - H_0)^{-1} t(E) \quad (3.4)$$

where $E^{(+)} = E + i\eta$; $\eta \rightarrow 0^+$ and H_0 is the kinetic energy operator, and V is the nucleon-nucleon potential. For some processes, such as the two bubble graph in Fig. 7, we must subtract the one pion exchange contribution. In general, the internal pion exchanges (long range correlations) are treated separately using retarded propagators rather than a static Yukawa potential; one must then correct the t -matrix calculation to avoid double counting.^{6,7} The use of a t -matrix enables us to include the short range two body correlations to all orders in the potential. This illustrates a central feature of this approach: one tries to relate the pion potential to elementary amplitudes for πN and NN scattering; certain rescattering effects are then included to all orders. For the off-shell amplitudes, we would like to use a field theory for both the πN and NN cases. The absence of a reliable theory for the NN case forces us to proceed via the intermediary of a potential, as per Eq. (3.4).

Now let us consider the density expansion of the graphs of

Figs. 6 and 7. The term $\pi^{(2)}(q)$ involves cutting two nucleon lines in each graph. Typical cuttings are shown in Fig. 8:

Fig. 8

We see that in general off-shell nucleon-nucleon amplitudes enter; in the top graph of Fig. 8, for instance, for $q \rightarrow 0$, the t -matrix element is of the form $\langle p | t(E) | p \rangle$, with $E = \mu + p^2/m$ and $p = (k - k')/2$, i.e., the incident pion deposits a large amount of energy but very little momentum on the absorbing nucleon. By explicit calculation, the contribution from the graphs of Fig. 6 to the ρ^2 term of the p -wave part of the optical potential in the form (1.2) is found to be $b_p^A + b_p^B$, where

$$b_p^{A,B} = -9f^2/4\pi \mu^4/p_F^2 \omega_q^2 (I_{01}^{(1,+)} + I_{10}^{(1,+)}), \quad (3.5)$$

where $f^2/4\pi = 0.08$ is the renormalized pseudoscalar coupling constant,

$w_q^2 = \mu^2 + q^2$, and the integrals

$$I_{ST}^{(1,\pm)}(y) = \int dx P(x,y) \tilde{t}_{ST}(x,x; E_1^{(\pm)}) \quad (3.6)$$

are weighted averages of the NN t-matrix $\tilde{t}_{ST}$ for spin S and isospin T over the distribution P(x,y) of NN relative momenta $p = xp_F$ for a given pion momentum $q = yp_F$. In Eq. (3.6), we have included only the $l = 0$ part of the NN interaction (3S_1 and 1S_0). The distribution function P(x,y) is given by

$$P(x,y)/x^2 = \begin{cases} 1-3y/4+y^3/16+(y/2-1/y)x^2+x^4/5y & \text{for } 0 < x < y/2 \\ 1+y^2/8x(y^2/20-1)+(y^2/4-3/2)x+x^3/2 & \text{for } y/2 < x < 1-y/2 \\ 1/10xy[1-5(x-y/2)^2+5(x-y/2)^3-(x-y/2)^5] & \text{for } 1-y/2 < x < 1+y/2 \\ 0 & \text{for } x > 1+y/2 \end{cases} \quad (3.7)$$

in the region $0 < y < 1$. The energy variable $E_1^{(\pm)}$ in Eq. (3.6) is given by

$$E_1^{(\pm)} = x^2 + y^2/2 \pm \gamma(1+y^2\delta^2)^{\frac{1}{2}} \\ \gamma = \mu m/p_F^2; \quad \delta = \mu/p_F \quad (3.8)$$

The t-matrix is normalized so that the on-shell value is $\tilde{t}(x,x;x^2) = -(4\pi p_F/mx)e^{i\delta} \sin\delta$, where δ is the NN phase shift.

The density expansion of the graphs of Fig. 7 yields a contribution b_p^c given by

$$b_p^c = 3f^2/4\pi \mu^4/p_F^2 \omega_q^2 (I_{01}^{(2,+)} + I_{10}^{(2,+)} + I_{01}^{(2,-)} + I_{10}^{(2,-)} - 2I_{01}^{(3)} - 2I_{10}^{(3)}) \quad (3.9)$$

where

$$I_{ST}^{(2,\pm)}(y) = \int dx P(x,y) \tilde{t}_{ST}(x, <x-y>; E_1^{(\pm)}) \quad (3.10)$$

and

$$I_{ST}^{(3)}(y) = \int dx P(x,y) \tilde{t}_{ST}(<x+y/2>, <x-y/2>; E_2) \\ E_2 = x^2 + y^2/4 \quad (3.11)$$

In obtaining Eqs. (3.9) and (3.10), we have used angle-averaged relative momentum variables in $\tilde{t}$ in order to carry out the angular integrations explicitly. We have

$$<x-y> = \begin{cases} xy^2/3x & \text{for } y < x \\ yx^2/3y & \text{for } y > x \end{cases} \quad (3.12)$$

For any pion momentum q , the total p -wave ρ^2 term has a strength

$$h_p = b_p^A + b_p^B + b_p^C \quad (3.13)$$

The above expressions enable us to study the energy dependence of the strength of the ρ^2 term. The $q \rightarrow 0$ limit was studied in detail in Ref. 7, for a variety of NN potential models. Numerical consequences will be explored in Sec. 4.

The problem of the s -wave ρ^2 term in Eq. (1.2) is more difficult. The fundamental s -wave πN coupling is certainly non-linear in the meson field, so the s -wave self energy graphs have a different structure than

the p-wave graphs. For a quadratic coupling, for instance, typical graphs are shown in Fig. 9:

Fig. 9

However, the true nature of the s-wave coupling is not well established, so we adopt a simpler ansatz which preserves the analogy to the p-wave πN coupling. We represent the s-wave interaction by a linear charged scalar coupling $\lambda \delta_{\sigma', \sigma} (\tau_{\alpha})_{\tau', \tau}$, even though this is inconsistent with the negative intrinsic parity of the pion. We adjust λ to produce the empirical s-wave πN scattering length in Born approximation; this yields $\lambda^2/4\pi = 0.05$. With this crude approximation, we obtain expressions analogous to Eqs. (3.5) and (3.9) for the s-wave ρ^2 terms:

$$\begin{aligned}
 b_s^{A,B} &= -9 \lambda^2/4\pi \mu^4/p_F^2 \omega_q^2 (I_{01}^{(1,+)} + I_{10}^{(1,\pm)}) \\
 b_s^C &= -3\lambda^2/4\pi \mu^4/p_F^2 \omega_q^2 (I_{01}^{(2,+)} - 3I_{10}^{(2,+)} + I_{01}^{(2,-)} - 3I_{10}^{(2,-)} \\
 &\quad - 2I_{01}^{(3)} + 6I_{10}^{(3)}) \\
 b_s &= b_s^A + b_s^B + b_s^C
 \end{aligned} \tag{3.14}$$

The expressions (3.13) and (3.14) for b_s and b_p involve weighted averages of various off-shell NN t-matrices. Before proceeding further, it will be useful to establish contact between the very formal development presented here and more intuitive notions of the pion absorption

mechanism. Consider for instance the first graph shown in Fig. 6. If we examine the part of the t-matrix which is of second order in V (dashed line), we have Fig. 10

Fig. 10

To evaluate such a graph, we must integrate over the intermediate momentum p , which is how high relative momenta enter the theory, even though the initial NN relative momentum is rather low ($|k-k'| < 2p_F$). The absorptive part corresponds to cutting the graph in two between the V lines as indicated in Fig. 10 and requiring that both intermediate state nucleons be on the energy shell, i.e., that $p = (p, p_0)$ satisfies $p_0 = \epsilon_p$ and $k_0 + q_0 - p_0 + k_0' = \epsilon_{k+q+k'-p}$. Using $k_0 = \epsilon_k$, $k_0' = \epsilon_{k'}$, and $q_0 = \omega_q$, and eliminating p_0 , we find the energy conservation condition

$$\epsilon_p + \epsilon_{k+q+k'-p} = \epsilon_k + \epsilon_{k'} + \omega_q \quad (3.15)$$

Now consider a slow pion ($q \rightarrow 0$) absorbed on a pair of nucleons essentially at rest ($k \approx k' \approx 0$). Equation (3.15) then becomes

$$\epsilon_p + \epsilon_{-p} = \mu \quad (3.16)$$

This equation has a natural interpretation: a pion absorbed at rest on a two nucleon system results in two back-to-back nucleons being preferentially emitted, with each one taking one-half of the rest mass μ of the pion as kinetic energy. This corresponds rather closely to what is¹⁸ seen experimentally in the (π, NN) process. Thus the graph in Fig. 10 corresponds to real pion absorption; note that all the graphs which correspond to real pion absorption (π, NN) involve t-matrix elements which are far off the energy shell, i.e., by an amount $\mu \approx 140$ MeV. As shown in Ref. 7, these graphs only make a rather small contribution to the imaginary part of the ρ^2 term, since the t-matrix elements $t(p, p; E)$ are essentially real for large E.

The main contribution to the absorption comes from the graphs like the second one in Fig. 8, which involve on-shell t-matrix elements for $q \rightarrow 0$. If we cut such a graph between two V lines, we obtain Fig. 11

Fig. 11

If we require that the intermediate nucleons be on shell to obtain the absorptive part, we obtain the energy balance equation

$$\epsilon_p + \epsilon_{k+k'-p} = \epsilon_k + \epsilon_{k'}, \quad (3.17)$$

Note that the pion is present in both the initial state and the intermediate state in Fig. 11, and hence the graph no longer corresponds to real pion absorption. In a finite nucleus, such a graph would correspond to pion inelastic scattering (if both intermediate nucleons remain in bound states) or one or two nucleon knockout processes ($\pi, \pi N$) or ($\pi, \pi NN$) if one or both of the intermediate nucleons are in continuum states.

Since k and k' are in the Fermi sea, Eq. (3.17) can only be satisfied if either p or $k+k'-p$ or both are also in the Fermi sea. Since the structure of the density expansion places no restriction on p , the contribution from Fig. 11 is non-zero; in Ref. 7, such on-shell contributions are found to generate a large part of the calculated absorption. However, at threshold, the only channel which is physically open is the real absorption process (π, NN). In a finite nucleus, the inelastic channels only open up above the excitation energy of the first excited state. Also, the knockout processes ($\pi, \pi N$) etc. require that the pion supply at least the binding energy of the last nucleon; thus knockout channels are also closed at threshold. These considerations illustrate several difficulties:

1) the nuclear matter calculation generates a continuum of particle-hole states; thus the inelastic channel opens at zero energy. Any absorptive contribution from inelastic scattering produced by the nuclear matter calculation should then be disregarded below the finite nucleus inelastic threshold.

2) our simple estimates have also neglected binding energy effects. A strict density expansion requires that the single particle energy ϵ_k which characterizes nucleon propagation be replaced by the free kinetic

energy $k^2/2m$ in the zero density limit. One could modify the structure of the expansion slightly to include a single particle potential $U(k)$ on each nucleon line. Note that since $U(k)$ is proportional to the density in first order, such a procedure is no longer a strict density expansion. Including binding effects via $U(k)$, the energy balance (3.17) becomes

$$k^2/2m + k'^2/2m + U(k) + U(k') = p^2/2m + (\tilde{k} + \tilde{k}' - p)^2/2m + U(p) + U(k + k' - p) \quad (3.18)$$

The effect of $U(k)$ can be seen qualitatively if we assume a simple model in which the potential operates only in hole states (as in Brueckner theory):

$$U(k) = \begin{cases} -|U_0| & \text{for } k < p_F \\ 0 & \text{for } k > p_F \end{cases} \quad (3.19)$$

If p , $\tilde{k} + \tilde{k}' - p$ corresponds to a particle-hole excitation, then (3.18) cannot be satisfied at all if $|U_0| \geq \epsilon_F = p_F^2/2m$. Thus the effect of binding corrections is to decrease the absorptive part near threshold.

3) a more subtle problem has to do with the treatment of the Pauli principle. In the original uncut graphs of Figs. 6 and 7, the Pauli principle requirements are satisfied exactly, by use of the Fermi gas propagator (2.5). When we expand each graph in powers of ρ , however, the Pauli restrictions only operate on the external lines, i.e., $|k| < k_F$, $|k'| < k_F$; it is consistent with the structure of the density expansion that the Pauli principle does not operate on internal nucleon lines, since we are instructed to take the zero density limit after

cutting two nucleon lines. It seems that this type of expansion may not be appropriate for the region very close to threshold, at least for the absorptive part. An alternative form of expansion, which organizes graphs according to the number of hole lines rather than powers of the density, retains a Pauli restriction on internal particle lines. However, this hole line expansion also has some difficulties, as we discuss later. It is not clear which form of expansion, if any, is appropriate near threshold. The strict density expansion overestimates the absorption near threshold, while the hole line expansion underestimates the absorption, since as we shall see later, it also suppresses the real absorption channel near threshold.

These diseases of density expansions in nuclear matter are only expected to be critical for the threshold region. Thus it is still of some interest to present predictions of the many body theory for the entire low energy region 0-100 MeV. As we have seen above, the many body graphs contain complete information about the processes which contribute to the optical potential. One can identify specific graphs with certain physical processes, such as real absorption, inelastic scattering, knockout processes, etc. The separate model dependence of each of these processes can then be explored; the many body theory provides a common framework in which all contributions can be calculated simultaneously.

The preceding development included all graphs which contain one t-matrix. There are also contributions of order ρ^2 in the density expansion from graphs involving two or three t-matrices. Typical cuttings are shown in Fig. 12:

FIG. 12

All cuttings of self-energy graphs containing two t-matrices give rise to one of the four basic graphs $\Delta\pi_A$, $\Delta\pi_B$, $\Delta\pi_C$ or $\Delta\pi_D$ of Fig. 13

$$\Delta\sigma_C =$$

$$\Delta\sigma_D =$$

FIG. 13

There is another set of graphs identical to those in Fig. 13, except that the external pion lines are crossed; the density expansion retains the crossing symmetry characteristic of the exact self energy. A complete set of distinct cut graphs can be generated by first drawing the skeleton graph with two nucleon lines connected by two t-matrices, and then adding the external pion lines in all combinations such that at least one pion line intercedes between the two t-matrices. This last point is important, since a graph with two consecutive t-matrices

with no pion in between is redundant, i.e., the t-matrix is already a ladder sum of interactions V.

There are also two basic cut graphs with three t-matrices, shown in Fig. 14:

FIG. 14

Note, however, that there are no graphs containing more than three t-matrices; any such graph would be redundant, since there are only two external pion lines to be distributed within the graph. Thus the number of graphs which contribute to the potential of order ρ^2 is finite. No discussion of the convergence of the series in powers of t is necessary, since only graphs of order t, t-t and t-t-t contribute.

The contribution of the graphs of Figs. 13 and 14 to $\pi(q)$ can easily be evaluated. For the effective charged scalar coupling $\lambda\tau\phi$ discussed previously for s-wave pions, we find

$$\begin{aligned}
 \Delta \pi_A(q) &= \frac{\lambda^2}{1} \sum_{kk'} \sum_{\text{spins}} \sum_{i=n,p} n(k)n(k') G_0^f(k+q) \\
 &\int \frac{d^4 p}{(2\pi)^4} G_0^f(p+q) G_0^f(p) G_0^f(k+k'-p) \varepsilon_{n1;n1}(k+q, k'; p+q, k'+k-p) \\
 &(\varepsilon_{p1;p1}(p, k+k'-p; k, k') - \varepsilon_{p1;i1}(p, k+k'-p; k', k)) \\
 \Delta \pi_B(q) &= \frac{\lambda^2}{1} \sum_{kk'} \sum_{\text{spins}} \sum_{\substack{i,j,\delta=n,p \\ \tau_\delta^2 = \tau_1^2 + \tau_j^2 + \frac{1}{2}}} n(k)n(k') \\
 &\int \frac{d^4 p}{(2\pi)^4} (G_0^f(p))^2 G_0^f(p+q) G_0^f(k+k'-p) \varepsilon_{ij;p\delta}(k, k'; p, k'+k-p) \\
 &\times [\varepsilon_{p\delta;ij}(p, k+k'-p; k, k') - \varepsilon_{p\delta;j1}(p, k+k'-p; k', k)] \\
 \Delta \pi_C(q) &= \frac{\lambda^2}{1} \sum_{kk'} \sum_{\text{spins}} \sum_{j=n,p} n(k)n(k') G_0^f(k+q) \\
 &\int \frac{d^4 p}{(2\pi)^4} G_0^f(p) G_0^f(k+k'-p) G_0^f(k+k'+q-p) \varepsilon_{nj;jn}(k+q, k'; p, k'+k+q-p) \\
 &\times (\varepsilon_{jp;pj}(p, k+k'-p; k, k') - \varepsilon_{jp;jp}(p, k+k'-p; k', k)) \\
 \Delta \pi_D(q) &= \frac{\lambda^2}{1} \sum_{kk'} \sum_{\text{spins}} \sum_{\substack{ij \\ \tau_i^2 + \tau_j^2 = 0}} n(k)n(k') \\
 &\int \frac{d^4 p}{(2\pi)^4} G_0^f(p) G_0^f(p+q) G_0^f(k+k'-p) G_0^f(k+k'-q-p) \varepsilon_{ij;pn}(k, k'; p, k+k'-p) \\
 &\times [\varepsilon_{np;ij}(p+q, k+k'-p-q; k, k') - \varepsilon_{np;j1}(p+q, k+k'-p-q; k', k)] \quad (3.20)
 \end{aligned}$$

In Eq. (3.20), G_0^f is the free nucleon propagator, i.e., the zero density limit of Eq. (2.5), and the indices i, j, δ , n (neutron), p (proton) label the z -components of isospin of the colliding nucleons. The t -matrix is normalized as in Eq. (3.4). Similarly, the contributions involving three t -matrices can be evaluated, with the result

$$\begin{aligned} \Delta \epsilon_E(q) &= \lambda^2 \sum_{kk'} \sum_{\text{spins}} \sum_{\substack{i,j,\delta=n,p \\ \tau_j^2 = \frac{1}{2} + \tau_1^2 + \tau_\delta^2}} n(k)n(k') \iint \frac{d^4 p d^4 p'}{(2\pi)^8} \\ &G_0^f(p) G_0^f(p+q) G_0^f(k+k'-p) G_0^f(p') G_0^f(p'+q) G_0^f(k+k'-p) t_{nj;nj}(p+q, k+k'-p; p'+q, k+k'-p') \\ &[t_{pj;\delta\delta}(p', k+k'-p'; k, k') - t_{pj;\delta\delta}(p', k+k'-p'; k', k)] t_{\delta\delta;pj}(k, k'; p, k+k'-p) \\ \Delta \epsilon_F(q) &= \lambda^2 \sum_{kk'} \sum_{\text{spins}} \sum_{\substack{i,j,\delta=n,p \\ \tau_j^2 = \frac{1}{2} + \tau_1^2 + \tau_\delta^2}} n(k)n(k') \iint \frac{d^4 p d^4 p'}{(2\pi)^8} G_0^f(p) G_0^f(p+q) \\ &G_0^f(k+k'-p) G_0^f(p') G_0^f(p'+q) G_0^f(k+k'-p') t_{ij;pj}(k, k'; p, k+k'-p) \\ &t_{nj;nj}(p+q, k+k'-p; k+k'-p', p'+q) \\ &\times [t_{jp;\delta\delta}(k+k'-p', p'; k, k') - t_{jp;\delta\delta}(k+k'-p', p'; k', k)] \end{aligned} \quad (3.21)$$

We now reduce these expressions to a more tractable form. In each case, the energy integral $\int dp_0/2\pi$ can be carried out immediately. For instance, for the "direct" part ($k, k' \rightarrow k, k'$) of $\Delta \epsilon_A(q)$, we obtain

$$\Delta \pi_A^{dir}(q, \omega) = \lambda^2 \sum_{\underline{k}, \underline{k}', p'} \sum_{spins} \sum_{i=n, p} n(k) n(k') \times (\epsilon_{k+q} - \epsilon_k - \omega)^{-1} (\epsilon_{k+k'-p} + \epsilon_p - \epsilon_k - \epsilon_{k-i\delta})^{-1} \times (\epsilon_{k+k'-p} + \epsilon_p + q - \epsilon_k - \epsilon_{k-i\delta} - \omega)^{-1} \times t_{ni;ni}(k+q, k'; p'+q, k+k'-p') t_{pi;pi}(p', k+k'-p'; k, k') \quad (3.22)$$

where $p_0' = \epsilon_k + \epsilon_k - \epsilon_{k+k'-p}$, and contributions from poles of t have been neglected (these correspond to renormalization processes, as will be shown elsewhere).

Now consider the limit $q \rightarrow 0$ appropriate for mesonic atoms. It is then convenient to introduce relative momentum variables

$$p_R = (\underline{k} - \underline{k}')/2 ; \quad p_R' = (2p' - \underline{k} - \underline{k}')/2 \quad (3.23)$$

In terms of p_R and p_R' , we find

$$\Delta \pi_A^{dir}(q, \omega) \xrightarrow[q \rightarrow 0]{\omega \rightarrow \mu} - \frac{8\lambda^2 m^2}{\mu} \sum_{\underline{k}, p_R, p_R'} \sum_{spins} \sum_{i=n, p} n(\underline{k}) n(\underline{k} - 2p_R) (p_R'^2 - p_R^2 - i\delta)^{-1} (p_R'^2 - p_R^2 - \mu - i\delta)^{-1} t_{ni;ni}(p_R, p_R'; \mu + p_R^2/m) t_{pi;pi}(p_R, p_R'; p_R^2/m) \quad (3.24)$$

where t is now labelled by the relative momenta p_R and p_R' and the (in general off-shell) energy. The $\underline{k}$ integral can now be done exactly using

$$\sum_{\underline{k}} n(k) n(\underline{k} - 2p_R) = \frac{\rho}{4} \left(1 - \frac{3p_R}{2p_F} + \frac{p_R^3}{2p_F^3} \right) \theta(p_F - p_R) \quad (3.25)$$

If we now assume s-waves only for t, then the angular integrations are trivial. After further summing over spin and isospin and introducing the dimensionless variables $\gamma = \mu m/p_F^2$, $x = p_R/p_F$, $y = p_R'/p_F$ and $\tilde{t} = p_F^2 t$, we find the final results ($q=0$, $\omega=\mu$)

$$\begin{aligned} \Delta\pi_A(\mu) &= \Delta\pi_C(\mu) = \frac{\lambda^2 m \rho}{2\pi^4} \int_0^1 dx P(x) \int_0^\infty dy y^2 \tilde{t}_{01}(y, x; x^2 + \gamma) \\ &\quad \times \tilde{t}_{01}(y, x; x^2) \left[\frac{1}{y^2 - x^2 - 1\delta} - \frac{1}{y^2 - x^2 - \gamma - 1\delta} \right] \\ \Delta\pi_B(\mu) &= \frac{3\lambda^2 m \rho}{2\pi^4 p_F^4} \int_0^1 dx P(x) \int_0^\infty dy y^2 \left(\frac{1}{y^2 - x^2 - 1\delta} \right)^2 \frac{1}{y^2 - x^2 - \gamma - 1\delta} \\ &\quad [\tilde{t}_{01}(y, x; x^2) \tilde{t}_{01}(y, x; x^2) + \tilde{t}_{10}(y, x; x^2) \tilde{t}_{10}(y, x; x^2)] \\ \Delta\pi_D(\mu) &= \frac{\lambda^2 m \rho}{2\pi^4 \mu^2} \int_0^1 dx P(x) \int_0^\infty dy y^2 [\tilde{t}_{01}^2(y, x; x^2) - 3\tilde{t}_{10}^2(y, x; x^2)] \\ &\quad \left[\frac{1}{y^2 - x^2 - \gamma - 1\delta} + \frac{1}{y^2 - x^2 + \gamma - 1\delta} - \frac{2}{y^2 - x^2 - 1\delta} \right] \end{aligned} \quad (3.26)$$

where $P(x) = x^2(1 - \frac{3x}{2} + \frac{x^3}{2})$. When we now evaluate the total contribution

$\Delta\pi_s(\mu)$ to the s-wave self energy from all graphs with two t-matrices, we find

$$\Delta\pi_s(\mu) = 4\Delta\pi_A(\mu) + 4\Delta\pi_A(-\mu) + \Delta\pi_B(\mu) + \Delta\pi_B(-\mu) + \Delta\pi_D \quad (3.27)$$

One can go through the analogous calculations for pseudoscalar coupling, to find the p-wave self energy. The same integrals appear, but with different relative weights for the 1S_0 and 3S_1 channels. If we define

the following integrals

$$\begin{aligned}
 I_{ST}(\alpha) &= \int_0^1 dx P(x) \int_0^\infty dy y^2 \frac{1}{y^2 - x^2 + i\alpha\gamma - i\delta} \times \tilde{t}_{ST}^2(y, x; x^2) \\
 J_{ST}(\alpha) &= \int_0^1 dx P(x) \int_0^\infty dy y^2 \frac{1}{y^2 - x^2 + i\alpha\gamma - i\delta} \tilde{t}_{ST}(y, x; x^2) \tilde{t}_{ST}(y, x; x^2 - \alpha\gamma) \\
 K_{ST}(\alpha) &= \int_0^1 dx P(x) \int_0^\infty dy y^2 \frac{1}{y^2 - x^2 - i\delta} \tilde{t}_{ST}(y, x; x^2) \tilde{t}_{ST}(y, x; x^2 + \alpha\gamma) \quad (3.28)
 \end{aligned}$$

We can write the contribution Δb_s and Δb_p to the ρ^2 term in Eq. (1.2) in the form

$$\begin{aligned}
 \Delta b_s &= \left(\frac{\lambda^2}{4\pi}\right) \frac{3m_\mu^2}{2\pi^2 p_F^3} (I_{01}(1) + I_{01}(-1) - 2J_{01}(1) - 2J_{01}(-1) - 2I_{01}(0) \\
 &\quad + 2K_{01}(1) + 2K_{01}(-1)) \\
 \Delta b_p &= \frac{f^2}{4\pi} \frac{3m_\mu^2}{2\pi^2 p_F^3} (I_{01}(1) + I_{01}(-1) + 4J_{01}(1) + 4J_{01}(-1) - 2I_{01}(0) - 4K_{01}(1) \\
 &\quad - 4K_{01}(-1) + I_{10}(1) + I_{10}(-1) - 2I_{10}(0)) \quad (3.29)
 \end{aligned}$$

Equation (3.29) represents the contribution for $q \rightarrow 0$ of all graphs with two t-matrices. It is to be added to the values b_p and b_s of Eqs. (3.13) and (3.14) and the terms with three t-matrices in order to obtain the full ρ^2 potential.

Several points are to be noted about Eq. (3.29)

- a) Δb_s depends only on the 1S_0 channel; 3S_1 contributions are present in individual graphs, but cancel out in the sum.
- b) Both Δb_s and Δb_p vanish if we make an on-shell approximation for

the t-matrix, or if we replace t by the potential V. Point b) indicates that these corrections will be quite sensitive to the details of the off-shell behavior of the t-matrix. Calculations are in progress to estimate the corrections Δb_s and Δb_p , as well as the terms of Eq. (3.21) involving three t-matrices. The dependence of the terms linear in the t-matrix on the choice of V has been investigated in detail in Ref. 7.

As we have just shown, the calculation of the pion optical potential to order ρ^2 involves only a finite number of graphs. If we try to extend our expansion to order ρ^3 and higher, the situation becomes more complicated, since one can have an infinite number of interactions between successively different pairs of nucleons before, after, and in between the pion absorption and emission vertices. Thus one must consider graphs such as shown in Fig. 15:

FIG. 15

where Γ is an off-shell three body amplitude. If one has a model for Γ , for example the Fadeev theory, then the term of order ρ^3 in $\pi(q)$ involves only a finite number of graphs involving Γ . Similarly, for calculating the contribution of order ρ^N , one needs a model for the N-body off-shell amplitude. In practice, this requirement limits one to calculating terms of order ρ and ρ^2 , as we have done. Of course, some special classes of terms, involving only intermediate pion

exchanges, can be summed to all orders in ρ (such as the Lorentz-Lorenz series [5]).

The expansion we have just discussed organizes the terms according to powers of the occupation numbers $n(k)$. The density dependence of the particle and hole parts of the propagator (2.5) are treated on the same footing. One can derive an alternative expansion in which the hole and particle occupation numbers $n_{<}(k)$ and $n_{>}(k)$ of Eq. (2.5) are treated as independent variables. This sort of hole line expansion is analogous to a Taylor expansion in x of a function $f(x,y)$ of two variables. This expansion assumes the form

$$\pi(q) = \sum_k n_{<}(k) \left. \frac{\delta \pi(q)}{\delta n_{<}(k)} \right|_{n_{<}=0} + \frac{1}{2!} \sum_{kk'} n_{<}(k) n_{<}(k') \left. \frac{\delta^2 \pi(q)}{\delta n_{<}(k) \delta n_{<}(k')} \right|_{n_{<}=0} + \dots \quad (3.30)$$

As in Eq. (3.2), the derivative $\delta \pi / \delta n$ generates a series of graphs for a pion nucleon amplitude $\tilde{f}$ in the medium. Since we are instructed to take the limit $n_{<}=0$, each internal line in a graph for $\tilde{f}$ retains a factor $n_{>}$, corresponding to propagation above the Fermi sea. Typical graphs contributing to $\tilde{f}$ are shown in Fig. 16:

$$\tilde{f}(k, q; k, q) = \begin{array}{c} \text{Diagram 1: A horizontal line with three segments labeled } n_{<}(k), n_{>}(k-q), n_{<}(k). \text{ Above the line, a wavy line with two arrows labeled } q \text{ connects the first and third segments.} \\ \text{Diagram 2: A horizontal line with three segments labeled } n_{<}(k), n_{>}(k+q), n_{<}(k). \text{ Above the line, a wavy line with two arrows labeled } q \text{ connects the first and third segments.} \\ \text{Diagram 3: A horizontal line with five segments labeled } n_{<}, n_{>}, n_{>}, n_{>}, n_{<}. \text{ Above the line, a complex wavy line structure connects the first and last segments.} \end{array} + \dots$$

FIG. 16

The first order term in the hole line expansion is thus of the form

$$\pi^{(1)}(q) = 4\pi \sum_k n_{\zeta}(k) \tilde{f}(k, q; k, q) \quad (3.31)$$

For p-waves, $\tilde{f}$ satisfies a modified Chew-Low equation of the form

$$\begin{aligned} \tilde{f}(k, q; k', q') = \tilde{f}_B(k, q; k' q') - 4\pi i \int \frac{d^4 p}{(2\pi)^4} D_0(p) G_{\zeta}(k+q-p) \\ \tilde{f}^+(k', q'; k+q-p, p) \tilde{f}(k, q; k+q-p, p) \\ + \text{crossing term} \end{aligned} \quad (3.32)$$

where G_{ζ} corresponds to the second term of Eq. (2.5), containing n_{ζ} . In Eq. (3.32), $\tilde{f}_B$ is the modified Born term, incorporating the Pauli restriction on the intermediate line shown in Fig. 16. If we average Eq. (3.32) over nucleon momenta k in the Fermi sea ($k \leq k_F$), and perform the energy integral $\int dp_0/2\pi$, we obtain an equation for the Fermi-averaged amplitude $\tilde{f}(q, q)$, now labelled only by the pion momentum q and energy ω :

$$\begin{aligned} \tilde{f}(q, q) = \tilde{f}^B(q, q) + 4\pi \sum_p \frac{1}{2\omega_p} \frac{1}{\omega - \omega_p - i\epsilon} F(q-p) \tilde{f}^+(q, p) \tilde{f}(q, p) \\ + \text{crossing term} \end{aligned} \quad (3.33)$$

where the Born term $\tilde{f}^B(q, q)$ is given by [19]

$$\begin{aligned} \tilde{f}^B(q, q) = f^B(q, q) F(q) \\ F(q) = \begin{cases} 3q/4p_F(1-q^2/12p_F^2) & 0 \leq q \leq 2p_F \\ 1 & q > 2p_F \end{cases} \end{aligned} \quad (3.34)$$

In Eq. (3.34), $f^B(q, q)$ is the free space Born amplitude and $F(q)$ is

Bethe's blocking factor [19]. If we now expand [11]

$$\tilde{f}(q, q') = \sum_{\alpha \in \{J, T\}} P_{\alpha}(q, q') \tilde{h}_{\alpha}(\omega) \quad (3.35)$$

In terms of spin-isospin channels α and projection operators P_{α} , we obtain a set of Pauli modified Chew-Low equations for $h_{\alpha}(\omega)$. If we retain only the (3,3) channel, we obtain for $\tilde{h}_{3,3}(\omega) \equiv h(\omega)$ the equation

$$\begin{aligned} \tilde{h}(\omega) &= \tilde{h}_B(\omega) + \frac{1}{\pi} \int_{\mu}^{\infty} p^3 d\omega_p \frac{|\tilde{h}(\omega_p)|^2}{\omega_p - \omega - i\epsilon} I(q, p) \\ \tilde{h}_B(\omega) &= \frac{\lambda}{\omega} F(\omega) \end{aligned} \quad (3.36)$$

In Eq. (3.36), $\lambda = 4/3 f^2/\mu^2$ and $I(q, p)$ is defined by

$$I(q, p) = \frac{1}{4q^2 p^2} \int d(\cos \theta_p) F(q-p) P_{33}(q, p) \times P_{33}(p, q) \quad (3.37)$$

$I(q, p)$ can be explicitly evaluated [20], but the result is rather complicated. Equation (3.36) is very similar to an equation discussed by Eisenberg and Weber [19], except that these authors only Fermi average the Born term and not the dispersion term; i.e., they replace $F(q-p)$ by $1-n(q-p)$ in Eq. (3.37).

The use of the hole line expansion in the threshold region has been discussed in ref. [14]. Although the Pauli principle is very effective in quenching the Born terms near threshold ($F(q) \rightarrow 0$ as $q \rightarrow 0$), it operates to a considerably lesser extent in higher order nN scattering graphs. Since the Born terms in any case do not contribute to the p-wave strength $b_p^{(1)}$ of the optical potential of

FIG. 17

FIRST ORDER POTENTIAL
ABSORPTIVE PART

Fig. 18

FIG. 19

Fig. 20

FIG. 21

Second Order Potential in g^2
(p-wave)

$$P_F = 270 \text{ MeV}/c$$

Eq. (1.2), the net effect of the Pauli principle on $b_p^{(1)}$ is found to be rather small near threshold [14]. The density and hole line expansions are thus roughly equivalent in first order.

The consistent derivation of the hole line expansion, and an assessment of its convergence properties in the (3,3) resonance region, will be presented in a forthcoming article [20]. This article will compare the density expansion [6,7], hole line expansion, and the more general Chew-Low method [11]; the latter method attempts to avoid any sort of expansion by summing the Chew-Low graphs to all orders in the density.

4. NUMERICAL RESULTS

In this section, we present some typical numerical results in the low energy region from 0-100 MeV. This is the energy region of interest for the Amsterdam accelerator. Of particular interest is the competition between terms of order ρ and ρ^2 in the optical potential as a function of energy. The energy dependence of the first order absorptive parts $\text{Im}b_s^{(1)}$ and $\text{Im}b_p^{(1)}$ is shown in Fig. 17. These values are calculated directly from free space πN phase shifts, without Fermi averaging. $\text{Im}b_s^{(1)}$ and $\text{Im}b_p^{(1)}$ display q and q^3 dependences near threshold and $\text{Im}b_p^{(1)}$ is seen to rise rapidly even in the low energy region because of the very wide (3,3) resonance near 175 MeV.

The real and imaginary parts of the ρ^2 contributions b_s and b_p to the optical potential are shown in Figs. 18-21. The results for the density expansion are obtained using Eqs. (3.13) and (3.14). These curves represent the extension of the results of ref. [7] to values of energy above threshold. In these illustrative calculations, a simple one term separable NN potential was used, the parameters of

which are given in ref. [7].

As seen from Figs. 18-21, the real part of the ρ^2 potential obtained from a calculation using the full NN t-matrix is of the same order as the imaginary part, i.e., dispersive and absorption contributions are of comparable size. Note that we have not corrected Figs. 18 and 20 to remove some contributions due to single pion exchange which correspond to reducible self energy graphs or double counting. This is discussed in detail in ref. [7]; these corrections change the sign of $\text{Re}b_p$, but the magnitude remains large.

As we discussed in Sec. 3, the strict density expansion overestimates the absorptive parts $\text{Im}b_s$, $\text{Im}b_p$ near threshold. In particular, the on-shell graph of Fig. 11 gives a large contribution near threshold, which would disappear if one were more careful and included binding and Pauli effects. One can get an idea of the true ρ^2 absorption near threshold by simply omitting the on-shell graphs from the theory. We are then left with graphs like Fig. 10 which contain no intermediate pion; this is then a rough estimate of the true contribution of the (π , NN) process to the optical potential. This estimate is labelled as "OFF-SHELL ONLY" in Figs. 19 and 21. We note that this estimate of the absorptive parts $\text{Im}b_s$ and $\text{Im}b_p$ of the ρ^2 term provides only 15-25% of the values required to fit the pi-mesic atom data [2]. This is not unreasonable, since the empirical ρ^2 term must mock up the effects of all multiparticle absorption processes. Talks by Engfer, Engelhardt and Cernigoi at this conference have emphasized that these many particle processes (more than two nucleons emitted) dominate pion absorption in most cases (except for some light nuclei).

One can also estimate the contribution of processes such as shown in Figs. 6 and 7 in the context of the hole line expansion of

Eq. (3.30). In this case, we obtain expressions analogous to Eqs. (3.5) to (3.14), except that the free-space t -matrix is replaced by the usual effective NN interaction G in the medium. G is related to t by the equation

$$G = t + t \left(\frac{Q}{e} - \frac{1}{e_0} \right) G \quad (4.1)$$

where $e = e_0 + u_1 + u_2$; e_0 is the free two particle kinetic energy and u_1 and u_2 are single particle potential energies. The Pauli operator Q guarantees that the two particles in intermediate states of the ladder sum propagate above the Fermi sea. In addition to replacing t by G , we must also include a Pauli factor $1-n$ on each internal nucleon line. A typical graph in the hole line expansion is shown in Fig. 22:

FIG. 22

The precise evaluation of such graphs as a function of q will be discussed in a future article [20]. A quick estimate of the above graph is obtained by noting that the most marked effect of the Pauli principle will come from the two external pion vertices. As $q \rightarrow 0$, the external vertices each approximately yield a factor

$$\sum_k n(k) (1-n(k+q)) = \frac{\rho}{4} F(q) \quad (4.2)$$

where $F(q)$ is the blocking factor of Eq. (3.34). If we ignore the difference $G-t$ in Fig. 22 (the Pauli effects in G will further damp the absorptive part near threshold), we may roughly write the strengths b_s^{HL} and b_p^{HL} of the ρ^2 terms in the hole line expansion as

$$\begin{aligned} b_s^{HL} &= F^2(q) b_s^t \\ b_p^{HL} &= F^2(q) b_p^t \end{aligned} \quad (4.3)$$

where b_s^t and b_p^t are the strengths calculated in the density expansion with a free t -matrix. The results for b_s^{HL} and b_p^{HL} are shown in Figs. 18-21, labelled as "HOLE LINE EXP."

Several observations are in order: The hole line expansion, if we work only to first order in G , yields $b_p^{HL} \rightarrow 0$ and $b_s^{HL} \rightarrow 0$ as $q \rightarrow 0$, due to the factor $F^2(q)$ in Eq. (4.3). It is thus clear that we overestimate the effect of the Pauli principle in this case, since we also kill the graphs which give the physical (π, NN) process at threshold. It thus appears that both density and hole line line expansions have some difficulties in calculating the ρ^2 terms at threshold. Note, however, that these two expansions agree rather well in their first order terms [$b_s^{(1)}$ and $b_p^{(1)}$]; see ref. [14]. A more detailed study of the threshold problem is clearly in order.

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