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EFFICIENCIES AND PHOTOFRACTIONS FOR GAMMA RADIATION
ON SODIUM IODIDE (THALLIUM ACTIVATED) CRYSTALS

by

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August, 1958

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ABSTRACT

The efficiencies and photofractions for monoenergetic gamma rays incident on sodium iodide (thallium activated) crystals have been calculated by the Monte Carlo method. The calculations have been carried out for broad parallel-beam sources, point sources on the crystal axis, and disc sources, all for photon energies of 0.279, 0.661, 1.33, 2.62, and 4.45 Mev. The crystals considered were right circular cylinders with radii of 1.0, 2.0, 4.0, 8.0, 12.0, and 16.0 inches, and for each radius, heights of 1.0, 2.0, 4.0, and 8.0 inches. The effect of the escape of annihilation gammas from the crystal was taken into account in the Monte Carlo calculation; however, no correction was made for escape of bremsstrahlung from secondary electrons and positrons. This latter correction is of importance only at the largest energies and for the smallest crystals where it affects the photofractions by a few per cent. This report includes the results reported by the authors in *The Review of Scientific Instruments*, 28 717 (1957).

I. INTRODUCTION

Knowledge of the detection efficiencies and photofractions for gamma rays incident on various kinds of scintillating crystals is often of considerable aid in the design of gamma-ray counters. As an aid in such design work these two quantities have been calculated for monoenergetic gamma rays from different kinds of sources and for various sizes of sodium iodide (thallium activated) crystals. The crystals considered were all right circular cylinders of different radii and heights. The three sources were: 1) a broad parallel-beam source of radiation, incident normally to one of the plane surfaces of the crystal, the breadth of the beam extending over the face of the crystal, 2) a point source located on the axis of the crystal, at various heights, and 3) a disc source with radius equal to the crystal radius. Figure 1 shows schematically the three types of sources. All sources were assumed to emit monoenergetic gamma rays; the energies considered were 0.279, 0.661, 1.33, 2.62, and 4.45 Mev.

Figure 1. Schematic diagram showing sources employed in calculations.

The detection efficiency, or simply the efficiency, refers to the ratio of the number of gammas that interact with the crystal at least once to the total number of gammas that are emitted by the source (in 2π geometry). This quantity is distinguished from what is herein called the interaction ratio, the ratio of the number of gammas that interact at least once to the number which strike the face of the crystal. Thus, if F is the interaction ratio and G is that fraction of all emitted gammas in the hemisphere which strike the crystal face, then the efficiency, E , is given by

$$E = FG \quad ; \quad (1)$$

for a broad parallel beam $G = 1$, so that $E = F$. The photofraction is taken to be the ratio of the number of gammas that are completely absorbed, including all secondary radiations, to the number of gammas that interact at least once, that is, the fraction of interacting gammas that is completely absorbed.

The calculations employed the Monte Carlo method and were carried out on Argonne's computers, AVIDAC and GEORGE.

II. PROCESSES CONSIDERED

The elementary processes which were taken into account as primary events were Compton scattering, the photoelectric effect, and pair production. All other primary processes were neglected, since their cumulative cross section was insignificant in the energy region under consideration.

The secondary radiations were treated as follows. It was assumed that all electrons or positrons which were scattered or created would be stopped in the crystal and all their kinetic energy absorbed by the crystal. This means that all secondary photons generated in the slowing down of the electrons and positrons (for example, bremsstrahlung photons) were considered to be absorbed. However, Compton-scattered photons and annihilation gammas were taken into account and were traced through the crystal in the same manner as the primary gammas. Thus for Compton scattering the kinetic energy of the recoil electron is assumed to be completely absorbed by the crystal, but the scattered photon is traced on through the crystal. In the photoelectric effect the entire energy of the incident photon was counted as being absorbed by the crystal, that is, the photoelectron would dissipate all its energy in the crystal and any fluorescent X rays would be completely absorbed. In the case of pair production all the kinetic energy of both the electron and the positron was considered to be dissipated in the crystal. In fact, it was assumed that the positron was stopped and annihilated at the same point at which pair production took place.¹ The annihilation would then take place at zero velocity, so that the momenta of the annihilation gammas would be uniformly distributed and oppositely directed. The annihilation gammas were then traced on through the crystal. When a gamma ray was degraded in energy below one-tenth of an electron rest mass, i.e., about 51 kev, it was assumed that the gamma would then be completely absorbed by the crystal. This assumption is quite reasonable in view of the fact that the photoelectric cross section becomes very large at such low energies.

III. CROSS SECTIONS EMPLOYED

The cross section for Compton scattering was taken to be given by the Klein-Nishina cross section. At the photon energies of interest herein the binding energies of the atomic electrons are sufficiently small to be neglected so that no Rayleigh scattering was considered. The photoelectric and pair production cross sections were taken from the work of Gladys White Grodstein.² These data were fitted with simple algebraic expressions by dividing the energy range into regions convenient for the fitting. The approximations were required to fit the data to within one per cent.

¹Ann Nelms, N.B.S. Circular No. 577 (July, 1956). From the range-energy data for electrons and positrons in sodium iodide as given by this circular one can easily show that for the highest energies considered the reciprocal stopping power range is about 0.14 inch. The smallest crystals considered were of 2-inch diameter and 1.0-inch height. For larger crystals and lower photon energies the error of this approximation would be quite small.

²Gladys White Grodstein, "X-ray Attenuation Coefficients from 10 kev to 100 Mev," U.S. National Bureau of Standards Circular No. 583, (April, 1957).

IV. PHOTON HISTORIES

The calculations of the efficiencies and photofractions were carried out by means of the Monte Carlo method, wherein the physical processes which take place in the crystal were simulated by the computer. The initial distribution for the three sources were handled as follows: 1) For the broad parallel beam it was assumed that every photon in the beam would strike the crystal face and, furthermore, that the rays were distributed uniformly throughout the cross section of the beam. Therefore one had only to select a point on the face of the crystal from a uniform distribution and consider that point as the point of impact of the incident gamma. The momentum of the gamma is normal to the crystal surface at that point. 2) For the point source the direction of emission was randomly chosen from vectors uniformly distributed in the solid angle subtended by the front face of the crystal. The fraction of the 2π solid angle which the crystal face subtends (which equals the factor G) is easily computed separately from the Monte Carlo program. 3) For the disc source the direction of emission was selected from a solid angle that included the front face of the crystal. The cone sampled was a right circular cone with apex angle $\left[2 \arctan \left(\frac{2r}{z} \right) \right]$. One sees immediately that many of the rays emitted in this selected solid angle do not strike the crystal face. However, one can easily sample a very large number of rays in this solid angle in an extremely short time and compute the fraction which strike the crystal. Then the product of this fraction with the ratio of this solid angle to the 2π solid angle is just the factor G in Equation (1). For the broad beam source, $G = 1$; for the point source G is easily calculated from

$$\frac{1}{2\pi} \int_0^{\arctan\left(\frac{r}{z}\right)} \sin \theta \, d\theta \int_0^{2\pi} d\phi = G_{\text{point}} \quad (2)$$

(for $Z > 0$);

and for the disc source

$$\frac{H}{2\pi} \int_0^{\arctan\left(\frac{2r}{z}\right)} \sin \theta \, d\theta \int_0^{2\pi} d\phi = G_{\text{disc}} \quad (3)$$

(for $Z > 0$),

where H is the fraction of those gammas emitted in the designated cone which strike the crystal face.

We shall designate the total number of photons emitted as N ,³ the number which strike the crystal face as NG , the number which experience at least one interaction as I , and the number which experience total absorption including absorption of all secondary radiation as A . Then the interaction ratio, F , is given by I/NG , and the photofraction, P , is given by A/I .

Having selected an initial photon from the source distribution, the position of the first interaction within the crystal is determined by sampling the exponential distribution using the total attenuation coefficient

$$x = -\frac{1}{\mu} \ln R \quad , \quad (4)$$

where R is a random number uniform $(0, 1)$ and μ is the total attenuation coefficient (See Figure 2). If x exceeds the length of the ray in the crystal, then the photon has escaped without interaction and that history is terminated. If the initial path length terminates in the crystal a count is made in the I counter. Then the type of collision is determined by sampling the relative probabilities of photoelectric, Compton, and pair production collisions. If a photoelectric collision is determined, complete absorption is assumed, a count is made in the A counter, and the history is terminated. If a Compton collision is determined, the direction of the scattered photon is determined by sampling the Klein-Nishina distribution, and the energy of the scattered photon is then determined by the conservation laws. The history of this Compton-scattered photon is continued exactly as that of an initial photon.

If a pair production event is determined the following procedure is carried out. As explained in Part II, it is assumed that the resulting electron and positron are stopped within an extremely short distance from the point of creation and that all their kinetic energy is absorbed by the crystal. The positron is then assumed to experience an annihilation at zero velocity and the two annihilation gammas are distributed uniformly throughout all solid angle and in opposite directions. Each gamma has an energy equal to one electron rest mass. For both Compton and pair

Figure 2

Path Length Sampling

$X = 1/\mu \ln R$, for R uniform $(0, 1)$. The photon escapes when $X \geq Y$.

³Clearly, N and the related quantities represent numbers of gammas in some finite time interval.

production collisions all secondary gammas are traced until the gamma escapes the crystal, a photoelectric collision occurs, or the energy of the gamma falls below one tenth of an electron rest mass (all gammas with energy less than $1/10 m_0 c^2$ are assumed to be completely absorbed). If neither the primary nor any secondary photons escape the crystal a count is made in A.

The schematic diagram in Figure 3 shows how the calculation proceeds.

V. COMPARISON WITH EXPERIMENTAL AND THEORETICAL RESULTS

Some supplementary calculations were made for a narrow collimated beam for purposes of comparison with the calculations of Berger and Doggett⁴ and the experimental work of Foote and Koch.⁵

In the work of Berger and Doggett pair productions were treated as complete absorptions and subsequently corrections were made for the escape of annihilation gammas as well as for bremsstrahlung. In the present calculations no corrections were made for escape of bremsstrahlung since this effect is of importance only for small crystals and large energies. The present program was altered to treat pair production in the same manner as Berger and Doggett treated pair production and the results compared with their results before correction. The two calculations agree to within the probable errors in all cases tested.

Table I shows the results of the present calculations for a narrow collimated beam when pair production was included; also shown are the results of Berger and Doggett with their corrections included. (The effect of the escape of annihilation quanta and bremsstrahlung is to give a negative correction.) Table I shows that the correction is greater (in magnitude) in the present calculations than that calculated by Berger and Doggett.

Comparison with the experimental work of Foote and Koch, for a narrow collimated beam on a crystal of 2.5-in. radius and 4-in. height, is quite good. The experimental value of the photofraction for an energy of 0.661 Mev was 0.8, whereas the present calculations yielded $0.819 \pm .006$.

Table II gives a comparison of the present calculations with the experimental results obtained by Kreger⁶ for a crystal of 2-in. radius and 4-in. height. A small source was located on the crystal axis and the radiation was collimated by a 6-in. thick conical baffle which had a

⁴Martin J. Berger and J. Doggett, Rev. Sci. Instr. 27 269 (1956); J. Research Natl. Bur. Standards 56 355 (1956).

⁵R. S. Foote and H. W. Koch, Rev. Sci. Instr. 25 746 (1954).

⁶W. E. Kreger, Phys. Rev. 96 1554 (1954).

Figure 3

Photon History Schematic Flow Diagram

1.25-in. diameter at the crystal surface and a 1-in. diameter on the source side. In the present calculations this source was approximated by a point source located 30 in. from the crystal surface, along the axis, and collimated so that the gammas would strike within a circle of 1.25-in. diameter on the crystal face. For all cases, except the one of highest energy, the experimental photofractions are smaller than the theoretical ones. This systematic discrepancy may be due to the fact that, as pointed out by Berger and Doggett,⁴ in the experiment the effect of background radiation would be to lower the photofraction.

Table I

PHOTOFRACTIONS FOR NaI CRYSTALS OF 2.5-IN.
RADIUS FOR A NARROW COLLIMATED BEAM

Energy, Mev	8-in. height		4-in. height	
	Present results	Berger and Doggett	Present results	Berger and Doggett
0.661	0.887 ± 0.005^a	0.884 ± 0.003	0.819 ± 0.006^a	0.821 ± 0.004
1.33	0.747 ± 0.006^a	0.758 ± 0.006	0.651 ± 0.007^a	0.667 ± 0.007
2.62	0.632 ± 0.009^b	0.643 ± 0.009	0.510 ± 0.010^b	0.531 ± 0.010
4.45	0.556 ± 0.015^a	0.608 ± 0.015	0.447 ± 0.020^a	0.491 ± 0.020

^aFor 5000 histories.

^bFor 6000 histories.

Table II

PHOTOFRACTIONS FOR A NaI CRYSTAL OF 2-IN.
HEIGHT FOR A COLLIMATED POINT SOURCE

Energy, Mev	Present calculation	Kreger	Berger and Doggett
0.279	0.965	0.93	(0.90)
0.661	0.777	0.725	0.74
1.33	0.599	0.54	(0.58)
2.62	0.453	0.47	(0.50)

VI. RESULTS OF PRESENT CALCULATIONS

It should be pointed out that in the tables showing the results of the present calculations all probable errors quoted refer only to the statistical fluctuations inherent in the Monte Carlo method and, of course, do not include any systematic errors inherent in the model chosen or the cross sections employed.

A. Broad Parallel Beam.

For the broad parallel beam the efficiency can be calculated simply from the exponential law:

$$P(E, x) = 1 - e^{-\mu E x}, \quad (5)$$

where $P(E, x)$ is the probability that the gamma ray of energy E will experience an interaction in traversing the distance x . Since for the broad parallel beam the rays were all normal to the crystal face, one need only substitute the crystal height for x and the total attenuation coefficient for μE . A comparison of the analytical calculation of efficiencies and the Monte Carlo values serves as one simple check on the Monte Carlo calculation. Table III gives this comparison and shows good agreement between the two calculations.

Table III
EFFICIENCIES FOR NaI CRYSTALS FOR A BROAD PARALLEL BEAM

Energy, Mev	1-in. height		2-in. height		4-in. height		8-in. height	
	Monte Carlo ^a	Analytical	Monte Carlo ^a	Analytical	Monte Carlo ^a	Analytical	Monte Carlo ^a	Analytical
0.279	0.795 ± 0.004	0.798	0.957 ± 0.004	0.959	0.997 ± 0.003	0.998	1.000 ± 0.003	1.000
0.661	0.496 ± 0.005	0.498	0.749 ± 0.005	0.748	0.937 ± 0.004	0.936	0.995 ± 0.004	0.996
1.33	0.374 ± 0.004	0.370	0.613 ± 0.005	0.604	0.847 ± 0.005	0.843	0.976 ± 0.005	0.975
2.62	0.297 ± 0.004	0.298	0.509 ± 0.003	0.507	0.761 ± 0.004	0.757	0.942 ± 0.004	0.941
4.45	0.276 ± 0.001	0.278	0.484 ± 0.002	0.478	0.734 ± 0.001	0.728	0.931 ± 0.002	0.926

^aFor 5000 histories.

Table IV gives the photofractions from the present calculations for the broad parallel beam.

Table IV

PHOTOFRACTIONS FOR NaI CRYSTALS FOR A BROAD PARALLEL BEAM
AS CALCULATED BY MONTE CARLO METHOD^a

Radius, inches	Height, inches	Energy, Mev				
		0.279	0.661	1.33	2.62	4.45
1	1	0.816 ± 0.004	0.399 ± 0.007	0.233 ± 0.007	0.147 ± 0.006	0.103 ± 0.006
1	2	0.855 ± 0.003	0.481 ± 0.005	0.286 ± 0.006	0.199 ± 0.005	0.141 ± 0.005
1	4	0.869 ± 0.003	0.530 ± 0.005	0.334 ± 0.005	0.232 ± 0.005	0.172 ± 0.004
1	8	0.869 ± 0.003	0.548 ± 0.005	0.367 ± 0.005	0.258 ± 0.004	0.192 ± 0.004
2	1	0.845 ± 0.004	0.450 ± 0.007	0.286 ± 0.007	0.191 ± 0.007	0.139 ± 0.006
2	2	0.894 ± 0.003	0.576 ± 0.005	0.385 ± 0.006	0.282 ± 0.006	0.213 ± 0.006
2	4	0.913 ± 0.003	0.663 ± 0.005	0.477 ± 0.005	0.361 ± 0.005	0.295 ± 0.005
2	8	0.915 ± 0.003	0.701 ± 0.004	0.543 ± 0.005	0.419 ± 0.005	0.335 ± 0.005
4	1	0.864 ± 0.004	0.485 ± 0.007	0.335 ± 0.007	0.208 ± 0.007	0.184 ± 0.007
4	2	0.919 ± 0.003	0.631 ± 0.005	0.452 ± 0.006	0.346 ± 0.006	0.282 ± 0.006
4	4	0.943 ± 0.002	0.752 ± 0.004	0.592 ± 0.005	0.474 ± 0.005	0.409 ± 0.006
4	8	0.944 ± 0.002	0.811 ± 0.004	0.694 ± 0.005	0.582 ± 0.005	0.519 ± 0.005
8	1	0.869 ± 0.004	0.502 ± 0.007	0.353 ± 0.007	0.222 ± 0.007	0.176 ± 0.007
8	2	0.927 ± 0.003	0.664 ± 0.005	0.490 ± 0.006	0.383 ± 0.007	0.309 ± 0.006
8	4	0.953 ± 0.002	0.802 ± 0.004	0.653 ± 0.005	0.547 ± 0.005	0.479 ± 0.006
8	8	0.954 ± 0.002	0.878 ± 0.003	0.792 ± 0.004	0.693 ± 0.005	0.630 ± 0.005
12	1	0.872 ± 0.004	0.510 ± 0.007	0.358 ± 0.008	0.231 ± 0.007	0.198 ± 0.007
12	2	0.931 ± 0.003	0.674 ± 0.005	0.503 ± 0.006	0.401 ± 0.007	0.319 ± 0.006
12	4	0.958 ± 0.003	0.823 ± 0.004	0.679 ± 0.005	0.569 ± 0.005	0.502 ± 0.006
12	8	0.960 ± 0.002	0.902 ± 0.003	0.828 ± 0.004	0.733 ± 0.004	0.667 ± 0.005
16	1	0.872 ± 0.004	0.511 ± 0.007	0.363 ± 0.008	0.246 ± 0.008	0.202 ± 0.007
16	2	0.934 ± 0.002	0.679 ± 0.005	0.510 ± 0.006	0.407 ± 0.007	0.333 ± 0.007
16	4	0.960 ± 0.002	0.831 ± 0.005	0.692 ± 0.005	0.576 ± 0.005	0.518 ± 0.006
16	8	0.962 ± 0.002	0.912 ± 0.003	0.848 ± 0.004	0.752 ± 0.004	0.687 ± 0.005

^aFor 5000 histories.

B. Point Source on Crystal Axis:

Table V gives extra calculations which were made for purposes of comparison with the calculations of Vegors, Marsden, and Heath,⁷ who used numerical integration to calculate these efficiencies as compared to our Monte Carlo method. The values in Table V agree within the statistical deviations in all cases.

Table V

EFFICIENCIES FOR NaI CRYSTALS FOR A POINT SOURCE*

Crystal Radius, in.	Crystal Height, in.	Source Height, cm.	Energy, Mev	Present Calculation	Vegors, Marsden, and Heath
3	4	5	0.332	0.1836	0.1844
3	4	1	3.8	0.2543	0.2564
3	4	1	0.152	0.4348	0.4325
0.5	0.5	0.5	0.204	0.0408	0.0409
1	2	10	0.566	0.0090	0.0090

*For 5000 histories and 4π geometry.

⁷Stanley H. Vegors, Louis L. Marsden, and R. L. Heath, Calculated Efficiencies of Cylindrical Radiation Detectors, Phillips Petroleum Co. Idaho Falls, Idaho.

Table VI
G FACTORS FOR POINT SOURCE CALCULATIONS
(for 2π geometry)

$$G = \frac{1}{2\pi} \int_0^{\tan^{-1}(r/z)} \sin \theta \, d\theta \int_0^{2\pi} d\phi$$

Crystal Radius, in.	Z = 30 in.	Z = 18 in.	Z = 12 in.	Z = 6 in.
1	0.00055	0.00154	0.0035	0.0136
2	0.00222	0.00610	0.0136	0.0513
4	0.00888	0.02383	0.0513	0.1680
8	0.03387	0.08610	0.1680	0.4000
16	0.11760	0.25260	0.4000	0.6490

Tables VII, VIII, IX, X, XI give efficiencies and photofractions from the present calculation for a point source. All efficiencies refer to 2π geometry. The errors in Tables VII to XI in all cases are practically the same as the statistical deviations shown for the broad parallel beam in Table IV.

Table VII
EFFICIENCIES AND PHOTOFRACTIONS FOR A POINT SOURCE*
Point Source Height = 0 in.

Crystal Radius, in.	Efficiency				Photofraction				Energy, Mev
	Crystal Height				Crystal Height				
	1 in.	2 in.	4 in.	8 in.	1 in.	2 in.	4 in.	8 in.	
1	0.838	0.867	0.869	0.869	0.824	0.842	0.844	0.844	0.279
2	0.920	0.970	0.977	0.977	0.854	0.884	0.892	0.892	
4	0.940	0.991	0.999	0.999	0.868	0.905	0.911	0.911	
8	0.941	0.991	1.000	1.000	0.865	0.903	0.911	0.911	
16	0.945	0.991	1.000	1.000	0.862	0.903	0.910	0.910	
1	0.546	0.595	0.607	0.608	0.425	0.467	0.486	0.488	0.661
2	0.688	0.792	0.824	0.828	0.486	0.575	0.620	0.628	
4	0.746	0.894	0.955	0.963	0.533	0.662	0.746	0.763	
8	0.758	0.909	0.986	0.998	0.547	0.687	0.789	0.820	
16	0.766	0.917	0.985	1.000	0.548	0.693	0.793	0.825	
1	0.411	0.449	0.464	0.465	0.246	0.286	0.309	0.313	1.33
2	0.546	0.655	0.700	0.708	0.313	0.406	0.449	0.466	
4	0.623	0.778	0.870	0.892	0.362	0.492	0.585	0.633	
8	0.659	0.829	0.944	0.984	0.386	0.542	0.686	0.764	
16	0.663	0.832	0.956	0.994	0.376	0.536	0.700	0.794	
1	0.320	0.360	0.375	0.378	0.180	0.209	0.223	0.227	2.62
2	0.453	0.558	0.608	0.622	0.208	0.271	0.320	0.339	
4	0.528	0.690	0.797	0.826	0.262	0.390	0.486	0.546	
8	0.573	0.761	0.900	0.955	0.267	0.415	0.574	0.674	
16	0.591	0.780	0.918	0.983	0.273	0.425	0.600	0.728	
1	0.312	0.352	0.367	0.370	0.096	0.123	0.136	0.138	4.45
2	0.439	0.529	0.576	0.588	0.154	0.219	0.263	0.282	
4	0.516	0.678	0.778	0.813	0.182	0.298	0.398	0.449	
8	0.552	0.736	0.875	0.949	0.202	0.340	0.494	0.595	
16	0.570	0.752	0.905	0.981	0.212	0.354	0.533	0.670	

*For 5000 histories.

Table VI

EFFICIENCIES AND PHOTOFRACTIONS FOR A POINT SOURCE*

Point Source Height = 6 in.

Crystal Radius, in.	Efficiency $\times 10^2$				Photofraction				Energy, Mev
	Crystal Height				Crystal Height				
	1 in.	2 in.	4 in.	8 in.	1 in.	2 in.	4 in.	8 in.	
1	0.978	1.12	1.14	1.14	0.802	0.843	0.852	0.853	0.279
2	3.69	4.24	4.34	4.34	.834	0.889	0.904	0.904	
4	12.8	14.5	14.8	14.8	0.868	0.918	0.932	0.932	
8	33.0	37.0	37.8	37.8	0.878	0.931	0.945	0.946	
16	58.2	63.1	63.8	63.8	0.890	0.930	0.942	0.942	
1	0.607	0.831	0.949	0.976	0.419	0.487	0.536	0.549	0.661
2	2.27	3.16	3.64	3.73	0.445	0.555	0.635	0.662	
4	7.51	10.9	12.6	12.9	0.499	0.621	0.726	0.766	
8	21.6	28.8	33.3	34.1	0.518	0.670	0.786	0.834	
16	41.0	54.0	60.8	62.2	0.555	0.708	0.835	0.875	
1	0.428	0.646	0.808	0.853	0.253	0.298	0.339	0.354	1.33
2	1.68	2.51	3.13	3.30	0.273	0.384	0.461	0.503	
4	5.90	8.89	11.0	11.7	0.309	0.440	0.570	0.641	
8	16.2	24.2	30.2	32.1	0.347	0.489	0.646	0.740	
16	31.9	46.7	56.9	60.0	0.367	0.547	0.714	0.821	
1	0.364	0.564	0.718	0.786	0.133	0.179	0.215	0.232	2.62
2	1.35	2.10	2.70	2.95	0.201	0.271	0.353	0.401	
4	4.86	7.46	9.73	10.7	0.237	0.339	0.447	0.532	
8	13.5	20.8	27.2	30.2	0.248	0.375	0.511	0.637	
16	26.8	40.2	51.9	57.8	0.258	0.419	0.600	0.735	
1	0.337	0.505	0.662	0.736	0.105	0.135	0.157	0.175	4.45
2	1.26	1.99	2.59	2.89	0.148	0.214	0.278	0.324	
4	4.50	6.96	9.16	10.3	0.166	0.270	0.372	0.453	
8	12.1	19.1	25.5	28.9	0.197	0.324	0.471	0.600	
16	24.9	38.0	50.6	56.7	0.198	0.347	0.530	0.679	

*For 5000 histories.

Table IX

EFFICIENCIES AND PHOTOFRACTIONS FOR A POINT SOURCE*

Point Source Height = 12 in.

Crystal Radius, in.	Efficiency $\times 10^2$				Photofraction				Energy, Mev
	Crystal Height				Crystal Height				
	1 in.	2 in.	4 in.	8 in.	1 in.	2 in.	4 in.	8 in.	
1	0.265	0.308	0.316	0.316	0.813	0.853	0.856	0.866	0.279
2	1.03	1.20	1.23	1.23	0.831	0.892	0.911	0.911	
4	3.93	4.57	4.69	4.69	0.859	0.918	0.935	0.937	
8	13.4	15.4	15.8	15.8	0.871	0.931	0.949	0.950	
16	34.3	36.2	38.8	38.9	0.879	0.936	0.951	0.952	
1	0.164	0.229	0.274	0.284	0.398	0.461	0.500	0.518	0.661
2	0.638	0.917	1.07	1.12	0.448	0.565	0.656	0.682	
4	2.39	3.47	4.11	4.28	0.499	0.621	0.741	0.781	
8	8.43	12.0	14.1	14.6	0.515	0.669	0.802	0.850	
16	22.6	31.4	36.2	36.2	0.531	0.693	0.824	0.879	
1	0.121	0.185	0.239	0.239	0.226	0.260	0.322	0.346	1.33
2	0.473	0.718	0.944	1.03	0.285	0.379	0.460	0.508	
4	1.86	2.87	3.66	4.00	0.339	0.442	0.583	0.661	
8	6.20	9.61	12.5	13.5	0.337	0.478	0.639	0.759	
16	17.1	26.2	33.0	35.5	0.356	0.524	0.701	0.816	
1	0.0973	0.160	0.219	0.247	0.168	0.200	0.228	0.246	2.62
2	0.358	0.592	0.823	0.949	0.209	0.278	0.354	0.400	
4	1.49	2.31	3.17	3.62	0.217	0.337	0.447	0.549	
8	5.17	8.30	11.1	12.6	0.243	0.361	0.521	0.654	
16	14.2	22.3	30.0	34.1	0.255	0.403	0.571	0.726	
1	0.0886	0.147	0.240	0.237	0.097	0.140	0.164	0.176	4.45
2	0.369	0.586	0.804	0.926	0.127	0.197	0.269	0.317	
4	1.34	2.21	3.05	3.55	0.171	0.280	0.400	0.479	
8	4.75	7.74	10.7	12.4	0.210	0.321	0.469	0.600	
16	13.5	21.4	28.8	33.6	0.212	0.341	0.519	0.672	

*For 5000 histories.

Table X
EFFICIENCIES AND PHOTOFRACTIONS FOR A POINT SOURCE*
Point Source Height = 18 in.

Crystal Radius, in.	Efficiency $\times 10^2$				Photofraction				Energy, Mev
	Crystal Height				Crystal Height				
	1 in.	2 in.	4 in.	8 in.	1 in.	2 in.	4 in.	8 in.	
1	0.116	0.136	0.140	0.140	0.816	0.857	0.868	0.869	0.279
2	0.471	0.554	0.572	0.572	0.831	0.896	0.914	0.914	
4	1.85	2.17	2.23	2.23	0.859	0.920	0.937	0.939	
8	6.83	7.97	8.16	8.17	0.867	0.927	0.947	0.948	
16	21.1	24.0	24.5	24.5	0.873	0.934	0.952	0.953	
1	0.0712	0.103	0.115	0.119	0.397	0.467	0.517	0.529	0.661
2	0.293	0.425	0.512	0.531	0.449	0.561	0.650	0.680	
4	1.12	1.63	1.99	2.07	0.486	0.622	0.738	0.785	
8	4.21	6.17	7.38	7.62	0.518	0.667	0.796	0.852	
16	13.3	19.0	22.6	23.4	0.518	0.685	0.819	0.880	
1	0.0511	0.0816	0.109	0.121	0.258	0.300	0.350	0.363	1.33
2	0.217	0.345	0.453	0.495	0.269	0.371	0.460	0.518	
4	0.826	1.33	1.78	1.95	0.316	0.451	0.588	0.676	
8	3.13	4.88	6.56	7.15	0.347	0.481	0.633	0.767	
16	10.5	15.9	20.5	22.5	0.342	0.509	0.688	0.813	
1	0.0417	0.0692	0.0978	0.114	0.158	0.200	0.226	0.247	2.62
2	0.175	0.289	0.398	0.464	0.193	0.258	0.341	0.400	
4	0.712	1.17	1.60	1.85	0.223	0.327	0.453	0.548	
8	2.65	4.28	5.89	6.80	0.246	0.379	0.530	0.658	
16	8.11	13.3	18.6	21.3	0.253	0.414	0.596	0.742	
1	0.0399	0.0664	0.0942	0.112	0.102	0.137	0.163	0.174	4.45
2	0.163	0.270	0.384	0.452	0.144	0.199	0.263	0.321	
4	0.645	1.07	1.50	1.78	0.177	0.258	0.380	0.485	
8	2.38	3.98	5.57	6.60	0.172	0.292	0.436	0.582	
16	7.68	12.9	17.9	21.0	0.192	0.341	0.524	0.673	

*For 5000 histories.

Table XI
EFFICIENCIES AND PHOTOFRACTIONS FOR A POINT SOURCE*
Point Source Height = 30 in.

Crystal Radius, in.	Efficiency $\times 10^2$				Photofraction				Energy, Mev
	Crystal Height				Crystal Height				
	1 in.	2 in.	4 in.	8 in.	1 in.	2 in.	4 in.	8 in.	
1	0.0394	0.0465	0.0480	0.0481	0.814	0.857	0.868	0.868	0.279
2	0.173	0.204	0.211	0.212	0.834	0.901	0.918	0.919	
4	0.696	0.821	0.848	0.849	0.861	0.919	0.939	0.941	
8	2.68	3.16	3.26	3.27	0.869	0.933	0.953	0.955	
16	9.57	11.2	11.4	11.4	0.869	0.933	0.955	0.956	
1	0.0242	0.0356	0.0433	0.0454	0.402	0.460	0.512	0.525	0.661
2	0.107	0.157	0.192	0.201	0.461	0.571	0.664	0.692	
4	0.429	0.628	0.766	0.800	0.487	0.622	0.740	0.797	
8	1.67	2.43	2.96	3.10	0.507	0.658	0.803	0.816	
16	5.92	8.76	10.5	10.9	0.516	0.677	0.816	0.886	
1	0.0182	0.0284	0.0388	0.0436	0.230	0.246	0.339	0.355	1.33
2	0.0777	0.124	0.170	0.191	0.293	0.379	0.471	0.517	
4	0.322	0.502	0.677	0.763	0.332	0.449	0.589	0.679	
8	1.23	1.96	2.67	2.97	0.341	0.480	0.639	0.769	
16	4.42	7.08	9.51	10.5	0.343	0.513	0.688	0.821	
1	0.0144	0.0241	0.0342	0.0409	0.162	0.203	0.235	0.251	2.62
2	0.0638	0.106	0.151	0.180	0.204	0.269	0.340	0.400	
4	0.253	0.428	0.607	0.725	0.232	0.329	0.456	0.561	
8	1.02	1.67	2.38	2.79	0.235	0.368	0.521	0.669	
16	3.58	6.62	8.54	10.1	0.258	0.382	0.567	0.731	
1	0.0132	0.0227	0.0330	0.0398	0.096	0.138	0.165	0.183	4.45
2	0.0565	0.0968	0.135	0.175	0.149	0.216	0.291	0.336	
4	0.241	0.399	0.583	0.706	0.158	0.269	0.387	0.477	
8	0.923	1.56	2.28	2.74	0.189	0.296	0.449	0.592	
16	3.39	5.61	8.15	9.81	0.213	0.348	0.521	0.679	

*For 5000 histories.

C. Disc Source Parallel to Crystal Face:

Tables XII, XIII, XIV give the interaction ratios, efficiencies, and photofractions for disc sources at different heights. All efficiencies, refer to 2π geometry. The errors in Tables XII to XIV in all cases are practically the same as the statistical deviations shown for the broad parallel beam in Table IV.

Table XII

EFFICIENCIES, PHOTOFRACTIONS AND INTERACTION RATIOS FOR A
DISC SOURCE PARALLEL TO CRYSTAL FACE.*

Radius, in.	Interaction Ratios				Detection Efficiencies				Photofractions				Disc Source Height in.
	Crystal Height				Crystal Height				Crystal Height				
	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	
16	0.972	0.972	0.961	0.877	0.466	0.466	0.460	0.420	0.935	0.935	0.924	0.869	0.279 (Mev)
8	0.944	0.943	0.927	0.830	0.287	0.286	0.281	0.252	0.941	0.940	0.929	0.869	
4	0.897	0.896	0.880	0.773	0.123	0.122	0.120	0.106	0.924	0.924	0.913	0.854	
2	0.860	0.860	0.841	0.734	0.041	0.041	0.040	0.035	0.909	0.908	0.895	0.844	
1	0.844	0.843	0.821	0.713	0.011	0.011	0.011	0.009	0.860	0.860	0.852	0.811	
16	0.940	0.923	0.821	0.629	0.458	0.450	0.400	0.307	0.870	0.826	0.706	0.545	0.661 (Mev)
8	0.891	0.872	0.765	0.555	0.265	0.259	0.227	0.165	0.835	0.790	0.679	0.530	
4	0.796	0.779	0.674	0.493	0.111	0.109	0.094	0.069	0.771	0.737	0.639	0.498	
2	0.729	0.710	0.610	0.443	0.034	0.034	0.029	0.021	0.663	0.640	0.561	0.453	
1	0.709	0.689	0.602	0.435	0.009	0.009	0.008	0.005	0.524	0.514	0.461	0.398	
16	0.911	0.861	0.706	0.497	0.445	0.420	0.344	0.242	0.794	0.689	0.536	0.356	1.33 (Mev)
8	0.828	0.774	0.620	0.429	0.248	0.232	0.186	0.129	0.750	0.640	0.493	0.349	
4	0.727	0.677	0.546	0.373	0.100	0.093	0.075	0.051	0.636	0.567	0.444	0.314	
2	0.661	0.620	0.500	0.340	0.032	0.030	0.024	0.016	0.524	0.478	0.399	0.293	
1	0.621	0.580	0.473	0.325	0.008	0.007	0.006	0.004	0.344	0.327	0.286	0.234	
16	0.873	0.794	0.619	0.412	0.422	0.384	0.299	0.199	0.737	0.599	0.424	0.274	2.62 (Mev)
8	0.774	0.698	0.531	0.346	0.233	0.210	0.160	0.104	0.655	0.531	0.396	0.238	
4	0.667	0.603	0.455	0.291	0.095	0.086	0.065	0.041	0.565	0.477	0.367	0.244	
2	0.600	0.544	0.419	0.277	0.028	0.025	0.020	0.013	0.409	0.361	0.284	0.199	
1	0.553	0.500	0.390	0.258	0.007	0.006	0.005	0.003	0.240	0.226	0.198	0.151	
16	0.857	0.761	0.582	0.381	0.413	0.366	0.280	0.184	0.657	0.510	0.344	0.201	4.45 (Mev)
8	0.754	0.671	0.501	0.321	0.225	0.201	0.150	0.096	0.579	0.447	0.299	0.180	
4	0.646	0.574	0.437	0.277	0.089	0.079	0.060	0.038	0.476	0.402	0.275	0.150	
2	0.587	0.524	0.390	0.253	0.028	0.025	0.018	0.012	0.316	0.281	0.203	0.132	
1	0.541	0.486	0.365	0.233	0.007	0.006	0.004	0.003	0.172	0.161	0.137	0.102	

*For 5000 histories.

Table XIII

 EFFICIENCIES, PHOTOFRACTIONS AND INTERACTION RATIOS FOR A
 DISC SOURCE PARALLEL TO CRYSTAL FACE.*

Radius, in.	Interaction Ratios				Detection Efficiencies				Photofractions				Disc Source Height 12 in.
	Crystal Height				Crystal Height				Crystal Height				
	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	
16	0.976	0.976	0.958	0.860	0.293	0.293	0.288	0.258	0.945	0.945	0.934	0.875	0.279 (Mev)
8	0.947	0.947	0.927	0.808	0.134	0.134	0.131	0.114	0.947	0.946	0.931	0.871	
4	0.923	0.922	0.902	0.767	0.044	0.044	0.043	0.036	0.938	0.938	0.920	0.859	
2	0.913	0.913	0.884	0.753	0.012	0.012	0.011	0.010	0.915	0.913	0.897	0.845	
1	0.904	0.902	0.873	0.743	0.003	0.003	0.003	0.002	0.865	0.865	0.857	0.810	
16	0.932	0.911	0.791	0.581	0.265	0.276	0.239	0.176	0.878	0.830	0.693	0.544	0.661 (Mev)
8	0.891	0.863	0.734	0.514	0.126	0.125	0.104	0.072	0.845	0.790	0.666	0.511	
4	0.844	0.816	0.696	0.491	0.039	0.037	0.032	0.022	0.776	0.729	0.613	0.486	
2	0.828	0.794	0.667	0.461	0.011	0.010	0.008	0.006	0.690	0.659	0.578	0.464	
1	0.820	0.783	0.656	0.456	0.002	0.002	0.002	0.001	0.530	0.516	0.475	0.388	
16	0.691	0.530	0.659	0.434	0.265	0.247	0.196	0.129	0.801	0.681	0.506	0.353	1.33 (Mev)
8	0.838	0.762	0.590	0.379	0.116	0.105	0.081	0.052	0.762	0.656	0.496	0.357	
4	0.776	0.716	0.560	0.362	0.037	0.034	0.026	0.017	0.680	0.596	0.466	0.314	
2	0.747	0.684	0.524	0.347	0.010	0.009	0.007	0.004	0.500	0.451	0.374	0.279	
1	0.752	0.691	0.531	0.337	0.002	0.002	0.001	0.001	0.348	0.327	0.290	0.241	
16	0.864	0.768	0.578	0.370	0.260	0.232	0.174	0.111	0.737	0.584	0.410	0.249	2.62 (Mev)
8	0.783	0.683	0.507	0.326	0.108	0.094	0.070	0.045	0.654	0.537	0.381	0.235	
4	0.724	0.638	0.468	0.293	0.035	0.031	0.022	0.014	0.560	0.466	0.353	0.237	
2	0.699	0.614	0.445	0.276	0.009	0.008	0.006	0.003	0.418	0.369	0.289	0.199	
1	0.682	0.592	0.434	0.281	0.002	0.002	0.001	0.000	0.239	0.223	0.186	0.157	
16	0.843	0.725	0.524	0.329	0.253	0.217	0.157	0.099	0.660	0.513	0.338	0.213	4.45 (Mev)
8	0.762	0.658	0.477	0.295	0.107	0.092	0.067	0.041	0.597	0.468	0.299	0.172	
4	0.706	0.611	0.441	0.269	0.033	0.029	0.021	0.012	0.481	0.392	0.259	0.150	
2	0.682	0.583	0.407	0.245	0.008	0.007	0.005	0.003	0.318	0.271	0.197	0.133	
1	0.671	0.575	0.419	0.259	0.002	0.001	0.001	0.000	0.181	0.164	0.146	0.107	

*For 5000 histories.

Table XIV

 EFFICIENCIES, PHOTOFRACTIONS AND INTERACTION RATIOS FOR A
 DISC SOURCE PARALLEL TO CRYSTAL FACE.*

Radius in.	Interaction Ratios				Detection Efficiencies				Photofractions				Disc Source Height 18 in.
	Crystal Height				Crystal Height				Crystal Height				
	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	8 in.	4 in.	2 in.	1 in.	
16	0.965	0.964	0.944	0.826	0.192	0.192	0.188	0.165	0.951	0.951	0.936	0.876	0.279 (Mev)
8	0.949	0.949	0.926	0.792	0.074	0.074	0.072	0.061	0.944	0.953	0.957	0.881	
4	0.940	0.938	0.910	0.769	0.021	0.021	0.021	0.017	0.938	0.937	0.917	0.854	
2	0.937	0.936	0.908	0.772	0.005	0.005	0.005	0.004	0.912	0.912	0.900	0.840	
1	0.935	0.934	0.904	0.760	0.001	0.001	0.001	0.001	0.867	0.866	0.851	0.811	
16	0.935	0.903	0.777	0.545	0.187	0.181	0.156	0.109	0.883	0.821	0.686	0.520	0.661 (Mev)
8	0.898	0.864	0.722	0.504	0.067	0.064	0.054	0.037	0.864	0.801	0.666	0.508	
4	0.884	0.845	0.695	0.476	0.020	0.019	0.016	0.011	0.798	0.750	0.633	0.499	
2	0.862	0.824	0.673	0.473	0.005	0.004	0.003	0.002	0.675	0.636	0.559	0.455	
1	0.865	0.827	0.683	0.471	0.001	0.001	0.001	0.000	0.541	0.526	0.476	0.410	
16	0.898	0.821	0.629	0.410	0.180	0.165	0.126	0.082	0.818	0.691	0.519	0.354	1.33 (Mev)
8	0.847	0.770	0.590	0.375	0.065	0.059	0.045	0.028	0.770	0.661	0.506	0.341	
4	0.813	0.736	0.568	0.362	0.019	0.017	0.013	0.008	0.693	0.604	0.470	0.328	
2	0.802	0.722	0.538	0.333	0.004	0.004	0.003	0.002	0.516	0.461	0.378	0.286	
1	0.808	0.729	0.545	0.343	0.001	0.001	0.000	0.000	0.356	0.357	0.295	0.236	
16	0.866	0.755	0.545	0.336	0.175	0.153	0.110	0.068	0.730	0.563	0.399	0.250	2.62 (Mev)
8	0.805	0.691	0.494	0.307	0.062	0.053	0.038	0.023	0.663	0.524	0.365	0.226	
4	0.760	0.646	0.462	0.277	0.018	0.015	0.010	0.006	0.546	0.460	0.327	0.220	
2	0.758	0.650	0.464	0.285	0.004	0.003	0.002	0.001	0.406	0.350	0.270	0.206	
1	0.762	0.651	0.457	0.275	0.001	0.000	0.000	0.000	0.257	0.236	0.202	0.152	
16	0.847	0.725	0.519	0.317	0.167	0.143	0.102	0.062	0.661	0.509	0.337	0.204	4.45 (Mev)
8	0.781	0.655	0.462	0.274	0.062	0.052	0.036	0.021	0.608	0.465	0.307	0.183	
4	0.752	0.638	0.445	0.265	0.017	0.014	0.010	0.006	0.498	0.401	0.284	0.167	
2	0.730	0.610	0.435	0.267	0.004	0.003	0.002	0.001	0.323	0.285	0.211	0.141	
1	0.740	0.622	0.457	0.267	0.001	0.000	0.000	0.000	0.173	0.154	0.128	0.098	

*For 5000 histories.