FINAL REPORT ON THE
EXPERIMENTAL AND THEORETICAL INVESTIGATION OF AN
INERTIAL, ELECTROSTATIC CONFINEMENT DEVICE

Final Report

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Edward H. Klevans and James W. Robinson

The Pennsylvania State University
University Park, Pennsylvania

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ABSTRACT

The possibility of a fusion reaction among charged particles trapped in a potential well has been the motivation for experimental and theoretical studies of well formation in spherically symmetric systems. Microwave cavity resonance shifts, electron beam deflection measurements, and spectral line intensity data have provided information as to electron distribution, temperature and well formation. With ion injection at pressures from 0.5 to 5mTorr, shallow wells which trap electrons (up to $4 \times 10^9 \text{ cm}^{-3}$) are formed with accelerating voltages of 0.5 to 10kV and currents of 10 to 20mA. These wells are consistent with theoretical calculations which indicate also that deep wells are feasible at lower pressures where charge exchange is not prevalent. When electrons are injected at pressures from 0.5 to 5mTorr, densities up to $7 \times 10^9 \text{ cm}^{-3}$ are measured and data trends are indicative of well formation in the range from 60 to 100mA where accelerating voltage varies from 100 to 400. At pressures near 1μTorr, theoretical predictions of well formation establish a criterion based on perveance (≥ 0.1) which is found to hold in practice. Instabilities that may be present are considered and a water-bag theory to study these instabilities is discussed. Various theories of Hirsch's ion injection experiments are examined relative to their appropriateness for explaining his neutron outputs.

PREFACE

This report is divided into experimental and theoretical sections, with frequent cross references. It is deliberately brief in certain sections with the understanding that the interested reader will consult other reports and publications issued under this contract which are much more detailed. The most recent results are presented by Grush¹ and Imel², who have to a considerable extent filled the gaps in the previous work. Other numbered COO reports include theses by McKeon,³ Hu,⁴ and Black⁵ as well as a progress report⁶ of March 15, 1972, a submitted publication related to spectroscopic studies,⁷ and a conference paper on the theory of the electron injection device.⁸ Three additional publications^{9,10,11} are in rough draft form and will be submitted in about one month.

Contributions to this report have been made by W. H. Grush, M. Ham, K. Hu, and G. Imel. Their assistance in this writing and in the project as a whole are much appreciated. W. M. Black and R. McKeon have also contributed significantly during their past association with the project. The work was made possible through the interest and support of the U. S. Atomic Energy Commission whose help is gratefully acknowledged.

I. INTRODUCTION

The concept of purely electrostatic confinement of a fusion plasma has received much less attention than magnetic confinement systems and justifiably. Earnshaw's theorem excludes the possibility of confining charged particles in an electrostatic configuration.¹² Nevertheless, one need only trap ions in a non-equilibrium configuration to satisfy conditions for fusion yield, and Hirsch's work suggested that such an ion trapping did occur in his experimental device.¹³ Trapping has ramifications in other experiments as well. Electron beams, for example, will create a potential well in which ions are trapped.¹⁴ Stix has suggested the generation of highly-ionized heavy nucleons during a period of confinement in a potential well.¹⁵ Pankrat'yev et.al,¹⁶ Ware and Faulkner,¹⁷ Moir et.al.,¹⁸ and Strijland¹⁹ have examined the use of electrostatic stoppers in magnetic cusps. Justification for a study of electrostatic confinement thus rests on several areas of activity in fusion research.

The phenomena of ion trapping appears essential to the development of a fusion power source. Neutrons may be produced in experiments where trapping does not occur as, for example, in the passage of a beam of deuterons through a background gas of the same particles. However, considerations of efficiency dictate that scattered particles must be retained in the reaction zone for many collisions if a net yield of power is to occur. Thus much of the work on the contract has been related to the conditions under which ion trapping might occur.

The work has been restricted mainly to conditions of spherical symmetry. A particle trapped in a spherical well has no easy path of escape and can have a much longer lifetime than, say, a particle in a cylindrical geometry with end losses. The spherical geometry is amenable to theoretical studies, and it provides a greater focusing effect than other geometries. One is forced to an experimental asymmetry in the manner of mounting electrodes (supporting posts required for concentric spheres) though it is felt that the consequences are negligible. Some theoretical work on stability was more conveniently done in plane than spherical geometry.

The research techniques were oriented toward the formation of local extrema in electrical potential. These could be either positive or negative so as to trap either electrons or ions respectively. Symmetry required that these extrema or "wells" lie at the origin or about the origin as an onion layer. The technique for forming wells was that of injecting charged particles radially toward the center so as to generate a localized space charge field that would attract and hold charges of the opposite sign. The injection of particles occurred at a spherical surface, a "transparent" grid, which accelerated particles (generated external to the grid) toward the center of the system such that wells might form.

Theoretically, one can work with an idealized system where monoenergetic particles have purely radial motion. However, in practice, energy spread and non-radial motion are unavoidable,

though subject to some control. Background neutral gas was an ever present reality in ion injection experiments but was, of course, not required in theoretical modeling. Theory and experiment came together when the nonidealistic effects were included in the theoretical work. In general, potential wells are degraded by the various effects with the presence of background gas being highly detrimental for ion injection.

The sources of charged particles are of several kinds. First are those injected which from a theoretical view may be assigned energy and angular momentum at the point of injection. In practice they are produced outside of the injection site by appropriate arrays of emitting filaments and concentric grid structures. Another source is the secondary emission of electrons from the innermost grid when it is biased so as to accelerate and inject ions. These electrons escape quickly and do little more than increase the current drain on the grid supply source. A source of slow ions (and a sink for injected ions) is the process of charge exchange. Ionizing collisions serve as a source of both electrons and ions. These various sources feed the plasma and influence the formation of the potential profile as it exists in steady state. The sources may be controlled experimentally through the injection system and through control of background gas pressure, though these two controls are partially linked for the ion injection case.

A commonly used parameter in both experimental and theoretical work is the perveance defined in terms of injected particles by:

$$\text{Perv} = \frac{\alpha I}{\sqrt{3/2}} \left(\frac{2m}{e} \right)^{1/2} (\text{esu})$$

where α represents the number of cycles each particle makes through the grid. In general, the likelihood of well formation increases with perveance, though other factors are important.

Wells have been formed experimentally and confirmed theoretically, though they were not of the type expected from Hirsch's work.¹³ With ion injection shallow wells are formed where electrons are trapped. These electrons maintain a circulating current many times the ion injection current, while a large circulating ion current is unrealistic at background pressures in the mTorr range.

Several possible explanations for Hirsch's results are proposed, though none of them are completely satisfactory. A number of interesting problems remain in the interpretation of his results. Without a well, the ion injection device does not seem promising as a fusion machine, although it does relate to problems of turbulence and instability.

In the electron injection case, Hirsch's wells²⁰ observed at base pressures of 10^{-7} Torr were theoretically confirmed. Microwave cavity resonance data¹ strongly suggests the existence of wells at high perveance operation in the mTorr range. At the least, a shift in the electron distribution occurs. This shift may be interpreted in various ways.

II. EXPERIMENTAL WORK

II.1 INTRODUCTION

A model of the plasma formed by ion injection has been developed to account for the experimental data accrued from various diagnostic procedures. The measurements themselves are summarized in Section II.2 while the model features and typical numerical parameters are given in II.3. Further sections summarize related work with ion source development and probes. Measurements of electron injection systems and correlation with theoretical work are given in II.6. A discussion of experimental techniques concludes the chapter. In many cases, the work reported here has appeared previously, but it is included for completeness. The details of the ion injection model are drawn together from various sources to integrate the various aspects of the work.

The concept of a multilayered potential structure, even of a single deep well, has been discarded for ion injection systems operating at pressures of approximately 10^{-3} Torr. However, the model developed here does include a well of a few tens of volts and electrons are trapped by it. Deep well formation still appears feasible with ion injection at 10^{-6} Torr or lower pressure. The pressure requirement is imposed to reduce the high charge exchange and ionization rates below those obtained in present systems. A heavy metal ion injection system appears very likely to meet the required conditions, and it may be developed independently of the contract.

II.2 EXPERIMENTS WITH ION INJECTION SYSTEMS

In the progress of the work, several different electrode configurations and diagnostic facilities were utilized. However, the diameter of the inner sphere of about 3 in. was maintained (at least for the principal data taking), and most of the work was done in helium except as noted. Pressures ranged from 0.5 to 5 mTorr with the lower limit arising from the ignition threshold for ion production. The appearance of the plasma glow (visible only at higher pressure) and the current-voltage characteristics of the device were generally insensitive to the design details. Techniques of spectroscopy, microwave resonance, and electron beam deflection were the most useful in collecting data.

II.2.a Spectroscopy

Results summarized here have been submitted for publication elsewhere.⁷ They fall in two categories, the mapping of electron distribution through a measurement of spectral line intensities and the measurement of ion velocities by Doppler shift methods.

An Abel inversion of helium line intensity data corresponds to the radial distribution of electrons which excite the background neutrals. With the sensing equipment being calibrated against a tungsten ribbon standard of radiance, absolute measurements of line intensities were obtained. Thus, with data from microwave cavity measurements giving a reference point, electron density could be determined as a function of radius and electron

temperature could be calculated. Density could be represented fairly well by an expression of the form $K/(r^2 + a^2)$.

McKeon³ has considered the errors in an Abel inversion which are introduced by terminating a scan at some point within the radiating field of the plasma. If the measured intensity is I_m at some terminal distance R_m from the center line, then the result of the Abel inversion of the data ($I(x) - I_m$) will be too small but by no more than

$$\frac{I_m}{\pi} \frac{1}{(R_m^2 - r^2)}$$

as long as I is monotone decreasing beyond R_m . Such corrections were minor in the work described here.

Doppler shifts were not observed for helium lines, presumably because the signals were too small in magnitude. However, in hydrogen, several Balmer lines exhibited Doppler effects. The shift pattern was consistent with the applied potential, and thus one may conclude that the average grid spacing of about 1 cm was not so large as to poorly define a potential surface for accelerating ions. If one supposes (see Section II.3) an electron density at the grid of 10^7 cm^{-3} and a temperature of 14.5 eV, then the Debye length is 0.9 cm, which is compatible with the grid spacing. Probe data indicates (Section II.5) a sheath thickness of 2.5 cm.

II.2.b Microwave Resonance

The inner electrode, or spherical grid, for most experiments

has been naturally resonant in the TM_{101} mode at approximately 3.14 GHz. The shift in resonant frequency is a measure of the electron density inside the cavity. An experiment was designed whereby microwave energy could be coupled into the grid structure while the grid was at a potential of up to 5 kV so that densities could be measured in the usual operating mode. The principal experimental problem was to insure that the measurement of resonant frequency was not influenced by the coupling mechanism. Detailed procedures and results are given in a separate report¹ with some highlights being summarized here.

Interpretation of data depended upon some assumption as to electron distribution in the cavity. The spectroscopic data provided a distribution function $K/(r^2 + a^2)$, the parameter a defining the distribution. Figure 1 relates resonant frequency shift to peak density at the center for various values of a , infinity corresponding to a uniform distribution. Values of a/R are also shown where R is the effective cavity radius. For a series of experiments in helium at various pressures, values of peak electron density typically exceeded 10^9 cm^{-3} for moderate to high perveance injection. A summary of results is shown in Fig. 2. Letting background pressure P , accelerating voltage V , and current I be parameters, one finds that density increases with P and I while decreasing with V .

II.2.c Potential Mapping

The beam deflection method for mapping potential profiles is being submitted for publication.⁹ The principal observation

Fig. 1. Electron density at the center of the spherical cavity as a function of resonance shift. The density distribution is assumed to be $n = n(0) \frac{a^2}{(a^2 + r^2)}$.

Fig. 2. Resonant frequency shifts at various cathode drives for discharge gauge pressures in helium as shown. The straight lines were determined by least mean square techniques.

is that wells do not form for a wide range of experimental conditions, at least wells of any depth relative to applied potentials. The experimental procedures restricted the operating potentials to the range above 2 kV while background pressures of at least 0.5 mTorr were required for ion production. The theoretical studies of Black⁵ and Imel² both indicate that background gas pressure leading to significant charge exchange and ionization will inhibit well formation. Moderate asymmetries appear to be tolerable if pressure is low. The theoretical models do support the observations, and they also suggest that a system for producing a deep well might be designed for operation at a pressure well below 1 mTorr. The ions would preferably be of large atomic mass so as to increase perveance.

II.3 MODEL OF ION INJECTION PLASMA

The features of Black's model⁵ are very similar to those given here, though the following description integrates the contributions of microwave resonance studies and spectroscopy into the previous work. Numerical values are given for one particular example with parameters as follows:

Helium pressure	4 mTorr (1.4×10^{14} atoms/cm ³)
Cathode drive	16 mA at 11 kV
Cathode grid diam	3 in.
Ion injection rate	10 mA (estimated)

Typical cross sections pertinent to this example are:

Charge exchange (10^4 eV)	10^{-15} cm ²
Electron ionization (100 eV)	3×10^{-17} cm ²
Excitation of 5016 Å line (electrons at 100 eV)	10^{-18} cm ²

The model consists of several features which are inter-related. A Coulombic potential distribution is assumed between the inner and the outer sphere such that ions are injected into the central plasma more or less along radial lines. The inner plasma consists of ions and electrons, both of low energy, as well as the fast ions which were injected. The low-energy particle density greatly exceeds that of the fast ions as the electrons are trapped in a shallow potential well and the slow ions are not accelerated rapidly. The electron gas derives its energy for exciting and ionizing neutrals from the flux of high energy ions, though the mechanism is obscure.

II.3.a Electron Density

Two diagnostics were brought to bear on this subject. Spectroscopy gave a distribution while microwave resonance gave a scale reference. Extrapolation of microwave data (1-5 kV) provides a point for center electron density of $1. \times 10^9$ cm⁻³. With a function $K/(r^2 + a^2)$ for density, one finds a value for a of 1.1 cm. Electron density is assumed to be related directly to potential well shape through the Boltzmann factor. Thus potential varies as

$$\phi = \frac{-kT}{e} \ln \left(\frac{r^2}{a^2} + 1 \right)$$

II.3.b Electron Temperature

The spectral measurements were compared with a standard lamp so that absolute radiation intensities were attained. With such information and with temperature, electron density can be calculated directly and compared with microwave results. For the case under review, temperature was found to be 14.5 eV.

II.3.c Well Characteristics

Wells are certainly not deep as they would then be measurable with beam deflection techniques. However, they should have a depth several times the temperature if they are to confine electrons as indicated by the measurements. The well depths must be great enough to confine electrons which excite and ionize neutrals as unconfined electrons would be lost too quickly to be effective agents. (Note that the mean free path at 100 eV for excitation of $5016 \text{ \AA}^0$ radiation is 7000 cm.) From the results of the two previous sections, one may calculate the well depth relative to the potential at $R = 3.75 \text{ cm}$ to be 37 volts. This result is unrealistically low as it is approximately the threshold for excitation and ionization processes. The discrepancy lies in the use of the assumed density distribution near $r = R$, where measured densities fall below that given by the function.

II.3.d Particle Generation Rates

Electrons will be generated at the cathode by secondary emission at some rate on the order of half the ion arrival rate.⁵ However, few of them will be trapped in the well as they will have

energies above escape energy. Electrons generated in the well by ionizing collisions will be the major contributors to the well population, and they will contribute also to measured electrode current.

Slow ions will be generated by ionizing collisions as well as from charge exchange of fast neutrals. As the loss rate of slow charge exchange products will equal the supply rate associated with the fast ion flux, the total ion density from that source alone will exceed considerably the density of fast ions. This is so as the mean free path for charge exchange at 4 mTorr is only 7 cm and a large fraction of the fast ions will be lost during their first transit through the spheres.

The measured emission rate for photons of the $5016 \text{ \AA}$ line was $5.6 \times 10^{12} \text{ cm}^{-3} \text{ sec}^{-1}$. With the previously given cross section data, one can estimate the ionization rate to be 30 times as great or $1.7 \times 10^{14} \text{ cm}^{-3} \text{ sec}^{-1}$ (This assuming similar curves for cross section versus electron energy). Using the function $K/(r^2 + a^2)$ and integrating over a 7.5-cm-diam sphere, one obtains an ionization rate of $6.0 \times 10^{15} \text{ sec}^{-1}$ or .97 mA which is nearly comparable with the charge exchange current. If the maximum electron density is 10^9 cm^{-3} , the total electron charge in the sphere is $5.7 \times 10^{-9} \text{ C}$. Thus the electron lifetime in the trap will be approximately 6 μsec good for 1300 cm of travel.

The above calculations may be generalized. Consider first an ionization cross section characterized by the Born approximation with parameters chosen to match data

$$\sigma = \frac{H}{E} \ln \left(\frac{E}{E_0} \right)$$

where H is in Vm^2 , E is electron energy in volts, and E_0 is the minimum energy for ionization. With a Maxwellian electron gas and $x_0 = \frac{eE_0}{kT}$, one obtains

$$\langle \sigma v \rangle = \frac{4He}{\sqrt{2\pi mkT}} \int_1^\infty \frac{e^{-x_0 y}}{y} dy$$

where the integral is tabulated in standard reference tables. With a generalized rate process or density

$$Y(r) = \frac{Y(0) a^2}{r^2 + a^2}$$

One may show that the total integrated over a sphere of radius R is

$$\Gamma = \int_0^R Y(r) 4\pi r^2 dr = 4\pi Y(0) a^3 \left[\frac{R}{a} - \arctan \frac{R}{a} \right]$$

This may be abbreviated as $\Gamma = Y(0) V_{\text{eff}}$.

With the above definitions one may write that the electron generation rate R_e is

$$R_e = n_e(0) n_n \langle \sigma v \rangle V_{\text{eff}}$$

where electron and neutral densities appear in the expression.

The total number of electrons N_e is

$$N_e = n_e(0) V_{\text{eff}} = R_e \tau_e$$

where the electron lifetime in the trap is τ_e . As a result, one finds that

$$\tau_e = \frac{1}{n_n \langle \sigma v \rangle}$$

Further calculations regarding ions are appropriate. The

ion generation rate R_i will be the same as for electrons but with an additional component R_{ce} from charge exchange. A form is suggested for R_{ce} when one considers that fast ions are lost primarily by grid collision at low pressure and by charge exchange at high pressure. With an injected ion-beam current of I_b and some reference density n_0 , one has

$$R_{ce} = \frac{I_b}{e} \frac{n_n}{n_n + n_0}$$

This expression appears from data to dominate R_e at high pressures and thus should also dominate at lower levels. These slow ions from charge exchange create a positive charge density which should be comparable to the electron density since strong fields do not exist. Contributions from fast ions are secondary. To the extent that fast ion density can be neglected, one expects a drop in pressure to cause a drop in R_{ce} , ion density, and also electron density. Microwave data confirms the drop of the electron density.

II.3.e Electron Energetics

There exists an energy loss through excitation and ionization of neutrals as well as through the escape of high energy electrons and recombinations. Assuming a loss rate of 50 eV per ionization and the previously given rate of $6.0 \times 10^{15} \text{ sec}^{-1}$, one has a power loss from the electron gas of $3.0 \times 10^{17} \text{ eV/sec}$.

The energy losses are compensated in steady state by gains with the most likely source being the injected ions. If one uses Spitzer's "slowing down" formula²¹ for fast ions in an electron

gas, the available energy is too low by a factor of about 10^4 . Thus some other coupling mechanism will predominate. Turbulence created by counter-streaming ions may provide the necessary power transfer to balance the electron losses. Possible interaction mechanisms are discussed in Section III.3. Note that for the sample calculation, the ion power is 7×10^{20} eV/sec.

Observe that if one increases the fast ion accelerating voltage at constant current, he will not change the charge exchange rate appreciably. However, it is plausible that the electron temperature and the well depth will increase while electron lifetime decreases according to the calculations of II.3.d. In the deeper well, ions will be expelled more readily with a consequence that ion and electron densities will decrease in a manner consistent with microwave data.

II.4 ION SOURCE DEVELOPMENT

Several ion source configurations were used with the principal conclusion being that the choice of configuration was not important. However, one extremely asymmetric source was tried fairly early in the program and with that source, the usual "glowing ball" pattern at the center was highly distorted. All sources have been of a diffuse nature rather than consisting of a few discrete focused beams. Efforts to develop a beam system were dropped for reasons to be given below. Some rather unconventional ion sources were also investigated.

The principal problem with ion sources as related to well

formation is that background gas is necessary for the operation of the source while it is detrimental to well formation. One may devise differential pumping schemes to overcome the problem, but these can be cumbersome and expensive, especially if a system is designed with several independent guns. An alternative scheme is to develop an ion source that will operate say at 10^{-6} Torr rather than the more customary levels above 5×10^{-4} Torr. Both Black⁵ and Imel² show the need for low pressure if deep wells are to form. However, obtaining the low pressure is difficult.

One reason for using a set of discrete guns rather than a diffuse source is that Hirsch¹³ had success in producing neutrons at pressures of 1 mTorr. However, he did not measure potential wells. The previously mentioned theoretical arguments and alternative explanations of his observed data (Section III.5) have raised such serious doubts about the existence of wells at mTorr pressures that the initial reason for building and testing a gun system seems no longer valid. Furthermore, theoretical arguments show that even at reduced pressure, wells will still not form at the low values of perveance associated with Hirsch's high applied voltages. Gun systems may still be desirable, however, for studies of beam-plasma interactions and for turbulence investigations. Some prototype guns were constructed with output being comparable to that obtained by Hirsch.²²

Numerous modifications of diffuse ion sources were used, all operating at pressures above 0.5 mTorr. In all cases, electrons emitted from filaments at ground potential were collected by a grid at approximately 80-100 V. Ions produced by

collisions were collected by various schemes and accelerated toward the inner grid at a negative potential of several kV. In certain cases, filaments were behind baffles which diffused the electron flow and which prevented illumination of the spheres by filament radiation. Often filaments were located about 3 cm from the outermost sphere, without baffles. They were placed on the 8 corners of a cube and were operated in series groups of four. The electron flow was affected by even slight changes in the balance of the filament emissions. However, the characteristics of the plasma within the inner sphere could be described in terms of inner sphere voltage and current without regard to the details of the ion-producing dynamics near the outer sphere.

The most controllable ion generation system used 3 concentric spheres of 3, 6, and 8 in. diameters with filaments placed outside. The inner sphere was negatively biased to attract ions and the outer sphere was positively biased to collect electrons. The grounded center sphere was included to define rather carefully the ion generation zone between the 6 and 8 in. spheres, though in practice, it might as well have been eliminated. Particle trajectories are indicated in Fig. 3, and also performance graphs are included. Ion generation rate is equal to the difference between electron emission from filaments and electron collection on the outer grid. This rate was maximized at outer grid voltages corresponding with the maximum ionization cross section (80-100 eV). Of ions generated, a certain fraction were collected by the inner grid, this fraction

Fig. 3.a. Schematic diagram of an ion injection system showing particle trajectories.

Fig. 3.b. Inner grid current as related to available ions.

Fig. 3c. Maximum inner grid current with properly adjusted outer grid voltage (80-100 V). Points below the line represent less than optimum adjustment.

being rather independent of the actual current magnitudes. Inner grid current at -5 kV is approximately $1/3$ of the ion production rate with the inner grid current including of course the secondary emissions as well as the primary ions. The last portion of Fig. 3 indicates the relation of inner grid current to outer grid current with the optimum performance occurring when the outer grid voltage was properly adjusted.

The three-sphere system was designed so as to increase the effective interaction region for ion generation and to enhance the collection of ions. However, it was no more effective than the simpler system with two spheres. Apparently, the ion generation is effectively constrained to a region close to the positively biased outer sphere. The loss rate of ions through the mesh of the outer sphere is then quite high regardless of the arrangement of inner spheres.

Application of magnetic fields would increase the ion production rate, but with a diffuse source, the fields would be quite extensive and thus likely to perturb the inner region of plasma under study.

Ion gun design was carried forward to a point but then discontinued as the significance of background pressure became known. A source of metallic ions has been developed which does meet the requirements for a deep well as they are presently understood. The source produces 1 mA of Zn ions without any requirement of background gas. Several such sources appropriately spaced around a spherical accelerating grid will provide an ion

injection rate of sufficient perveance that wells are expected to form. The choice of Zn was mainly dependent upon the problems of stabilizing the low-level arc discharge which produces the ions. Though heavy metals are not appropriate for fusion, they provide a means for establishing and investigating well formation in the ion injection mode. Such work, if carried to completion, will lie outside the coverage of this contract.

II.5 PROBE MEASUREMENTS

Several electrical probe measurements were made in an attempt to measure space potential of the plasma directly. Theories related to the electrical probes are very complicated physically and mathematically. However, there exist numerous theoretical calculations for various types of electrical probes.²³ Most of the useful theories use either electron temperature or the ratio of probe radius to Debye length as a normalizing parameter of the solution of the related equations. This means that one needs some knowledge about the plasma before he can relate experimental data with one of the theories. Most importantly, the plasma which we want to investigate should not be disturbed by inserting the probes into the plasma. Thus, the main probe experiments were to find out whether the electrical probe can be used as a diagnostic tool or not. Two experimental results showed that electrical probe data is unreliable.

II.5.a Sheath Size

Two identical probes of small size were used to estimate sheath thickness formed on the probe. Using one of two probes

as sensing probe (which was located at the center of the system) and the other as a disturbing probe (which was moved in toward the center to meet the sensing probe), and abrupt change in current of the sensing probe was observed when the disturbing probe was about 2.5 cm away from the sensing probe, which implies that sheath thickness formed on the sensing probe is approximately 2.5 cm. This size is comparable to the radius of the inner grid which contains the plasma. Thus the probe will have a strong perturbing effect on the plasma being examined.

II.5.b Floating Probes

Two probes were inserted into the plasma from diametrically opposite sides. One probe was left floating (unconnected) and the other was biased so as to draw zero current. Floating potential was measured on the latter.

Two cases were considered with: 1) the unconnected probe being inserted only as far as the inner grid radius; ii) the probes being equidistant from the center. The results shown in Fig. 4 indicate the existence of a potential well, but the depth of the well apparently increases when the second probe is inserted. Interpretation of the data is difficult, but one cannot argue for the existence of a well because of the perturbing effects. An extension of case 1) where a single probe extends across the center is also shown in Fig. 4. The asymmetric response is an indication of probe effects rather than plasma characteristics. The plasma is shown to be symmetric in that symmetric measuring techniques yielded symmetric data.

Fig. 4. Floating probe voltages for ion injection at 5 kV and 13.5 mA cathode current. Pressure was 0.5 mTorr in H_2 gas.

In other work, floating probes were found to affect the spectral distribution of the light emissions. Also, in certain regions of the plasma, the floating probe was heated to incandescence. This heating effect occurred sometimes near the grid structure and sometimes near the center of the plasma. These above effects were linked to transitions in operating mode which were influenced by the presence of the probe. These transitions occurred in He at about 5 mTorr in one of the earlier device configurations, though behavior was not easily controlled or even reproducible on later versions.

II.6 ELECTRON INJECTION EXPERIMENTS

Much of the results have been reported previously by Black.⁵ At the time, the data from beam deflection studies were interpreted as meaning that shallow, distorted wells were present. However, irregularities in the data-collecting procedures were identified at a later time so as to cast doubt on the results. Imel's calculations² show that Black's conditions were near the threshold leading to well formation. Hirsch's results²⁰ indicate the formation of wells at perveances somewhat higher than where Black worked. Furthermore, Verdeyen²⁴ reports wells at higher perveances, these being altered by raising pressure above the 10^{-5} Torr range.

At higher pressures, background ionization becomes important, and low-energy ions and electrons become prevalent. A microwave cavity resonance technique has been applied to the

measurement of electron density within the inner spherical grid at pressures from 0.5 to 5 mTorr.¹ As shown in Fig. 5, the resonant shift increased with increasing current (perveance) to some extent and then decreased with a further increase of current.

The reversal in trend implies at least a significant change in the electron density distribution pattern. Presuming a distribution $K/(r^2 + a^2)$ at low current levels, a likely shift with increasing current is for the region of maximum electron density to move outward from the center because of electrostatic effects. For the TM_{101} mode, the most sensitive location for an electron is at the center. Thus the formation of an electron space charge leading to a migration of particles away from the center can explain the drop in frequency shift.

An interesting but unexplained phenomena is that at a given pressure, the current corresponding to peak frequency shift does not depend on voltage.

II.7 EXPERIMENTAL TECHNIQUES

Several problems arose during the course of the work where special techniques or procedures were found necessary. These are summarized here.

II.7.a Power Supplies

When 60 Hz was used for heating filaments, all system voltages carried 60 Hz modulation. Thus regulated DC power

Fig. 5. Cavity resonant frequency shift as a function of anode drive for electron injection with a background gas of helium.

supplies were used for filament heating. Regulated or well-filtered supplies were used for supplying power to the spherical grid electrodes. Buffer resistors were usually connected in series with the power supplies so as to prevent instabilities associated with the coupling of regulated supplies through the discharge phenomena.

II.7.b Vacuum Systems

An oil diffusion pump and a turbo-molecular pump were used at various times. Base pressures were always less than 5×10^{-6} Torr and often in the 10^{-7} range. Gas was bled in to maintain desired pressure at full pumping speed. Often after exposure of the chamber to air, the data would be unreliable until the chamber had been pumped for several hours and the discharge operated for several minutes.

II.7.c Grid Structures

Over the course of the work, many spheres were fabricated. Concern over sphericity, uniformity of wire spacing, and transparency varied considerably from time to time with the final impression being that tolerances need not be close. Angular momentum of injected particles is acceptable in moderate amounts. Transparency remains desirable. The fabrication of spheres using ribbon techniques⁶ is relatively easy and precise compared to the process of spotwelding wires around a spherical mold. An approximate solution of Laplace's equation for a gridded sphere inside a spherical shell shows that distortion of potentials

near the grid wires is consistent with the experimental measurements by Black.⁵ He applied the beam probe technique to electrostatic configurations and found a drop typically of 5 to 10% in potential inside the gridded sphere.

III. THEORETICAL PROGRAM

III.1 INTRODUCTION

The theoretical program has, from the beginning, had two primary objectives: (1) to understand the experimental results obtained by R. Hirsch,¹³ and (2) to understand the results obtained in our own experiments at Penn State.

The first part of the program has focused primarily on steady state solutions.⁴ This work represented a generalization of the work of Porter,²⁵ who had studied steady state solutions based on having only beam electrons and ions. Further extensions of the steady state theory with application to electron injection experiments has recently been completed by Imel.² The results are summarized in Section III.2 under the heading "Steady State Solutions."

While many steady state solutions are possible, only those which are stable are of interest for interpreting the experimental results. The initial stability study of Porter and Klevans²⁶ focused on a two-stream startup instability. The formulation of a more general kinetic theory based on the water-bag model is now

in progress. The results to this time are presented in Section III.3 titled "Stability Theory." The primary objective is to examine the possible instabilities that could be initiated in a bounded plasma by the presence of injected electron or ion beams.

In the above steady state theory, the form of the particle distribution functions was assumed. A more complete theory requires the derivation of the distribution function by consideration of atomic processes and particle balance. A model appropriate for the experiments discussed (Section III.4) has been developed by Black⁵ to accomplish this task. Discussion follows under the heading "Theory of Diffuse Ion Source."

Finally, neutron reaction rate calculations from various possible sources have been performed to try to shed further light on the origin of the neutrons in Hirsch's experiments. These are presented under the title "Neutron Production" in Section III.5. Several possibilities are discussed, none of them completely satisfactory, but each having credibility.

III.2 STEADY STATE THEORY

This theory is based on the self-consistent solution of the Vlasov equation and Poisson's equation²⁷ with boundary conditions appropriate to nearly radially injected current at the boundary. The results of such calculations include the potential and particle densities, both of which are assumed to depend only on the radius r . The choices of the particle distribution functions are not unique. They have been chosen to

represent the particle behavior in the system--or some aspect of this behavior--and have been modified as our understanding of the physics has increased.

Several possible potential profiles are shown in Figs. 6a, b, and c. For all of these, one type of charge (electrons or ions) is produced at the outer grid and is accelerated radially inward by the potential difference between inner and outer grids. These particles are referred to as beam particles. For example, in Hirsch's ion injection experiment,¹³ the outer grid is an anode and the inner grid a cathode. Beam electrons are produced with zero kinetic energy when fast ions collide with the cathode. In electron injection experiments one usually has only beam electrons and no beam ions.

In addition to the beam particles, there may also be trapped particles which result from charge exchange, ionization, collisions, or electric field fluctuations. These particles oscillate in the potential wells until they are lost by collision or turbulent diffusion out of the well or until they intersect a grid wire. Note that what is a well for one charge is a hill for the other.

Initially,²⁵ our attention was focused on the beam aspect of the system. This led to potential solutions with virtual anodes and cathodes already found by Hirsch.¹³ However, it was also found that the ion current dropped by at least 90% across a virtual anode, so that the current amplification of the injected beam--considered necessary by Hirsch to explain his data--could not be accounted for.

Fig. 6a. Potential profile with radial motion only. Beam behavior for both species is dominant. (Porter and Hu)

Fig. 6b. Potential profile including angular momentum on the beam (or beams). Trapped particles of both species may be present in the center.

Fig. 6c. Potential profile with injected beam having angular momentum. No beam behavior for species not radially injected. (Imel)

One means of increasing the amplification and also of moving the virtual anode from near the center out to a distance suggested by experiment was to include trapped particles in the potential wells. Hu⁴ investigated this case, assuming pure radial motion, i.e., no angular energy on the beams. He showed that beam current amplification was indeed possible if there was a large trapped ion current oscillating between the anode and virtual anode through the cathode. This did not seem possible for Hirsch's experiment since most of his cathode was solid. Hu also found that the trapped particles could strongly effect the locations of the virtual anode and cathode. The virtual cathode could be moved to the center (Fig. 6b) while the virtual anode could be moved outward,

In the real experimental situation, the injected particles are subject to non-radial motion as a result of finite beam width, grid asymmetries, and deflection at the grid wires. The most recent theoretical work² has been concerned with the effects of this angular momentum. Also included is the finite energy spread on the beam. It has been found that it is still possible to have solutions of the form shown in Fig. 6b although the potential at R_{v0} will not drop all the way down to form a full virtual electrode (i.e., to reflect all the injected charge). However, for the small injected current used by Hirsch,¹³ a potential of the form shown in Fig. 6c seems much more likely, especially the shallow well solution. Imel² has found that even with beam ion injection alone, the angular momentum on the beams should have been sufficient to prevent formation of a deep well.

Imel has studied in detail the condition needed for the different potential shapes shown in Fig. 6c, especially for electron injection experiments. Trapped ions were included only to the degree that they did not destroy the potential well. Excellent agreement has been found between the theoretical predictions on well formation and experimental results (Section II.6).

Trapped electrons were not included, but could allow oscillating potential solutions (Fig. 6b) as found recently at the University of Illinois.²⁴ It is hoped that this case can be investigated in the near future.

A summary of the different theoretical treatments is given in Table I.

III.3 STABILITY

The electrostatic confinement experiments may, in one sense, be viewed as a special class of counterstreaming beam and beam-plasma experiments. The stability of the system, with or without potential wells, is crucial to understanding system behavior. In addition, the plasma produced in these experiments could be potentially interesting for instability and turbulence studies. Finally, while the steady state theory has led to some proposed particle distribution functions, these can be considered possible and reasonable only if they are stable.

In this section we discuss the stability problem for both ion and electron injection experiments. Our approach has been

TABLE I

Summary of conditions for which steady state solutions were investigated.

Inner Sphere	{	Cathode	X	X	X	
		Anode				X
Ions	{	Beam	X	X	X	
		Ang. Mom.			X	
		Energy Spread			X	
		Trapped		X		X
		Ang. Mom.				X
		Energy Spread		X		X
Electrons	{	Beam	X	X	X	X
		Ang. Mom.			X	X
		Energy Spread			X	X
		Trapped		X		
		Ang. Mom.				
		Energy Spread		X		
Reference			Porter	Hu	Klevans and Hu	Imel
Reference No.			25	4	6	2

to treat the system as a bounded diode with counterstreaming beams. With the exception of the startup instability discussed by Porter and Klevans,²⁶ essentially all theoretical analyses have been in plane geometry. The reported new work will also be a plane geometry representation of the system. It should be noted that the use of the bounded diode model to represent the system is also found in the computer simulation studies of Barnes.²⁸

First consider ion injection experiments. If we assume, as found by Black,⁵ Imel,⁸ and Barnes,²⁸ that there is not a deep potential well, but at most a shallow well, then we are led to consider ion beam injection into an electron neutralizing background or into a plasma background.

Indeed, one possible explanation for Hirsch's results is that a background plasma was present and the ions were heated by turbulence. A possible mechanism here is the following: Counterstreaming ion beams produce an instability which couples to the ion-acoustic waves of the background plasma. This could heat high energy ions through Landau damping and heat the bulk of the ions by scattering in turbulent pulsations.²⁹ Another possibility is that within the background, electrons and ions are moving relative to each other, and this could lead to a relative-streaming instability. Finally, direct excitation of the background is possible. However, because of the beam energy, it is the electron waves, and not the ion waves, which are driven unstable. While any of these mechanisms are questionable,

instability-oriented experiments in this configuration might be quite interesting, and we have begun to examine them theoretically. Experiments which emphasize the second mechanism have resulted in weak ion heating.³⁰

The injection of an ion beam into a neutralizing electron background has been studied by Barnes²⁸ by modifying the Faulkner and Ware theory³¹ for injection of electrons into a cold ion background. However, the electron background was treated as being cold, an assumption of questionable validity. Computer simulations were performed with finite electron temperature.

Barnes²⁸ shows by computer simulation that a finite energy spread is very important in the stability of the system, for both ion and electron injection experiments. We have not found any theoretical treatments of bounded plasmas which take this effect into account.

Turning now to electron injection experiments, we find that several different potential profiles have been observed, including a shallow well,⁵ a deep well,^{20,24} and an oscillating potential well.

The latter result is rather interesting in light of the fact that oscillating potential well equilibria for the diode problem are presumed to be unstable.³² However, an examination of the theoretical work on the stability of the bounded plasma diode indicates that substantial gaps exist. In particular:

(1) Cold fluid models are usually assumed for both equilibrium

and stability analyses, i.e. the beams have zero temperature.³¹⁻⁴⁰

(2) While the stability of unneutralized cold beams³⁵⁻³⁸ and of completely neutralized cold beams have been studied,^{31,39,40} considerably less has been done for the partially neutralized beam. Barnes²⁸ has developed a stability criteria for the case of electron beams into a partially neutralizing homogeneous background of infinitely massive ions, but no work has appeared for the inhomogeneous background, nor have growth rates been computed, even for the homogeneous case. But the experiments at the University of Illinois were precisely the case of the partially neutralized plasma with the ion background inhomogeneous. Thus, while oscillating potential equilibria for unneutralized beams are expected to be unstable,³² the stability properties for the inhomogeneous, partially neutralized diode has received no attention, especially if trapped particles are present in the potential wells. (3) Except for the work of Faulkner and Ware in the neutralized diode,³¹ little work has been done to examine the excitation of low frequency ion wave instabilities. Finite temperature on the electron beam could significantly alter the results of Faulkner and Ware, as could a combination of beam and low energy background electrons. (Even in a neutralized diode, one expects some cold electrons produced by ionization.) These problems remain unexplored.

Because of our interest in finite beam energy and/or finite background temperature, a kinetic theory model is felt to be preferable in treating the instabilities described above. Although a number of kinetic theory techniques have been developed

in the last decade for investigating the stability of bounded plasmas, the techniques tend to be specialized to the particular problem. For example, if the plasma is homogeneous and the walls are perfectly reflecting, the problem is equivalent to the homogeneous plasma with special periodicity of the perturbation.^{41,42}

Fourier transform methods, which are so convenient and powerful for linearized theory in the infinite plasma, are much less so in the bounded plasma because the different Fourier modes are coupled at the boundary. Nonetheless, Cheng and Harris,⁴³ using a Fourier series expansion, were able to obtain a tractable dispersion relation for a homogeneous plasma with a magnetic field perpendicular to perfectly reflecting walls. Dobrowolny et. al.⁴⁴ studied a plasma diode with absorbing (and emitting) walls. They obtained an infinite matrix for the eigenfrequencies of the perturbed system, and were able to solve it only for two simplified special cases.

An entirely different approach to the problem of absorbing walls was treated earlier by Rosenbluth et. al., who used a variational technique to study the stability of various possible equilibria in a slab with electron-emitting boundary planes and a uniform, infinite mass ion background. While this technique can indicate that a certain equilibrium state is unstable to the $\text{Re } w = 0$ mode and provide a lower bound on the growth rate, it does not provide information on growth rates or on overstable modes.

One of the more complete analyses of a beam-plasma instability

in an inhomogeneous plasma is the work of Leavens and Love.⁴⁶ They used a numerical technique to obtain the eigenfrequency spectrum and eigenoscillations of the plasma by analyzing the frequency dependence of driven oscillations.^{47,48} They then coupled these normal modes to the external electron beam and analyzed the stability characteristics for four modes.

Unfortunately, extensive numerical work is required to obtain the eigenfrequency spectrum and the eigenmodes. Recently, a much simpler, though less accurate, approach has been developed to analyze the stability of electrostatic oscillations in bounded plasma⁴⁹ and stellar systems.⁵⁰ This method, which is based on the "water-bag" model, is especially well suited to study instabilities where counterstreaming is present.

In the water bag model, first introduced by DePackh,⁵¹ the particle distribution functions are written as a succession of step functions, as shown in Fig. 7 for a three-contour representation.

Figure 7: Distribution Function $F(x,v)$ in the Water Bag Model

The velocities $v_1^\pm, v_2^\pm, \dots$ represent contour lines in a phase space plot along which F is constant. (See Fig. 8) While the distribution function is admittedly approximate (especially for two or three bag distributions), the main physical features of many problems are retained, and the mathematical simplification (as compared with the use of continuous distributions) for stability analysis is considerable.

The water-bag model has been used for numerical studies of two-stream instabilities⁵²⁻⁵⁴ for the analytical determination of eigenfrequencies of a bounded, one-dimensional electron plasma,⁵⁵ for a variety of astrophysical applications,^{50,56,57} and, quite recently, for the analytical study of linear instabilities in inhomogeneous plasmas which are either periodic⁵⁸ or non-periodic.⁴⁹

Fig. 8. Phase Space Diagram of a Water Bag Distribution

In the latter paper and in a stellar stability paper,⁵⁰ methods have been developed for analyzing electrostatic counter-streaming instabilities in certain kinds of bounded plane geometry systems. In the paper by Bertrand et.al.,⁴⁹ criteria for a marginal mode analysis ($w = 0$) are established for distribution functions symmetric in velocity space. A simple example of a plasma passing through two grids is studied in detail, including obtaining eigenfunctions and eigenvalues for the unstable modes. The simplifying features were the homogeneity within the grids and the grid boundary conditions. Ions were treated as an infinite mass neutralizing background.

In the more general inhomogeneous system analysis,⁵⁰ Sturm-Liouville equations are obtained which describe the perturbed electric field in each bag. The modes of each bag are obtained by computer solution. A separate equation couples the modes and results in a dispersion matrix for the modes of the system.

We have applied these methods to some diode stability problems, as sketched in the appendix. These analyses apply to electron injection experiments in plane, rather than spherical, geometry. Nonetheless, numerical calculations for the experimental parameters of the electron injection experiments are planned. Extensions of this work will include consideration of finite ion mass and ion temperature and the examination of the effects of using an absorbing boundary instead of a reflecting boundary. Also, ion injection problems will be considered.

III.4 THEORY OF DIFFUSE ION SOURCE

A system model was developed to enhance the understanding of the experimental ion injection devices such that improvements in design might be incorporated.³ This model provided potential profiles and charged particle densities within the inner grid. Dominant interaction phenomena were included in the model, and angular momentum of particles was incorporated. Charge exchange was significant for ions while collisional ionization produced trapped electrons which could escape by recombination or by thermalization with the high energy component leaving the trap. The conclusions of the theory rest upon the extent of trapped electron accumulation which is a problem still not completely resolved. However, at the relatively high trapping levels which are consistent with the spectroscopic studies of Section II.2., only very shallow wells are possible. One finds that for deep wells to form, he must have a system with a high degree of radial symmetry and/or a system where the electrons are not overly prevalent. This work is being submitted for publication.¹⁰

III.5 NEUTRON PRODUCTION

One of the main goals of the research on electrostatic confinement has been to understand the ion injection experiments of Hirsch.¹³ In this section, we will discuss possible models to explain the results.

The main results are presented in three graphs, Figs. 9 a,b,c, which show (1) total neutron production vs. applied voltage with

Neutron outputs vs applied potential at various pressures (ion current = 10 mA).

The results of a neutron collimation study at -90 kV, 20 mA, and 8 μ of deuterium.

The results of the gamma collimation experiment

Fig. 9. Results of the experiments of R. L. Hirsch. See Reference 13.

background pressure as a parameter; (2) a neutron collimation profile; and (3) an x-ray profile. The latter does not pass through the center. Most of the neutron output data were obtained for D-D reactions.

From the results, it appears that one needs a model which explains the following dependences:

- 1) Neutron output increases as pressure decreases,
- 2) Neutron output increases with applied cathode voltage.
- 3) The perveance decreases rapidly as V increases, suggesting that perveance is not a significant parameter in defining the behavior of the system.
- 4) Neutron output increases like I_{input} and not I^2 .
If the ion density $n_i \propto I_{\text{in}}$, the reaction rate would go like n_i^2 , i.e., like I_{in}^2 . There are a number of reasons why n_i might not be proportional to I , such as beam focusing effects with increasing I , or that n_i is not a beam quantity and hence has a more complicated relation to I .
- 5) Neutrons are being produced in the center with sufficient reaction rate to be observed in a collimation scan.
- 6) Neutrons are produced only when the beams are well-focused. Misalignment leads to a failure of the device to operate.

Hirsch suggested a model in which the potential had a central well for trapping ions. He proposed that the neutron and x-ray scans indicated the presence of a virtual anode at approximately 1.26 cm. He showed that the existence of the virtual anode and the central well were possible solutions of Poisson's equation, based on a bipolar beam model for electrons and ions.

The difficulties with this model have been discussed in the section on steady state models. The primary problem is the low input current (20 mA) which would predict a virtual anode at 10^{-4} cm with no angular energy on the input beam, and no virtual anode when reasonable angular energy is included. Only by having an oscillating current of 10^4 the input current could the virtual anode be brought out to 1.26 cm. In addition, Barnes,²⁸ using computer simulation, was not able to find a stable equilibrium of the form suggested by Hirsch.

An alternative potential profile is the shallow well shown in Fig. 6c. This profile was found in the experiments of Black and in the computer simulations of Barnes with low perveance inputs. The problem here is accounting for any neutron output. Interaction of the beams among themselves could not possibly explain the results because the input beam currents are much too low.

Another possibility is that beams--both electron and ion--are responsible for producing a plasma in the center. Fusions could then result from several sources:

- 1) Beam-plasma interaction
- 2) Fusions in the background plasma

The plasma density needed to account for the neutron output by beam-plasma interaction has been computed by assuming a mono-energetic beam with radial motion and a plasma with a Maxwellian distribution. To avoid divergence at the origin, the r^{-2} spherical focusing factor has been replaced by $(r^2 + r_a^2)^{-2}$ where r_a is a constant. This qualitatively brings in the effect of angular momentum. The expression for the reaction rate is found to be

$$R_{BP} = \frac{I_B n_1}{2\pi e (r^2 + r_a^2) v_B^2} K(v_B, T_1)$$

where

$$K = \left(\frac{n_1}{2\pi K T_1} \right)^{1/2} \int_0^\infty dv_r v_r^2 \sigma_{DD}(v_r) \left[e^{-\frac{m_1}{2KT_1} (v_r - v_B)^2} - e^{-\frac{m_1}{2KT_1} (v_r + v_B)^2} \right]$$

In the above expressions, I_B is the beam current, n_1 the ion density, v_B the beam velocity in the center, and σ_{DD} the fusion cross section. If V_c is the voltage difference between anode and cathode, and if $K T_1 \ll e V_c$, the integral can be approximated by the cold plasma result.

$$K = v_B^2 \sigma(v_B)$$

If the plasma density varies like

$$n_1 = \frac{n_1(o) r_b^2}{r^2 + r_b^2}$$

where r_b is a constant, and assuming $r_b \gg r_a$, we obtain for the total reaction rate

$$T_{BP} \approx \int_0^R d^3r R_{BP} = \pi^2 r_b^3 R_{BP}(o) = \frac{\pi r_b}{2e} I_B n_1 \sigma_{DD}(v_B)$$

From Fig. 9, we see that for $I_B = 10$ mA, $V_c = 100$ kV, and $P = 1.1$ microns, $T = 10^7$ /sec. Using these values, we obtain for $r_b = 1$ cm that

$$n_i(o) \approx 10^{16}/\text{cm}^3$$

This does not seem realistic for the kind of system being discussed. Even if we required only 10% of the fusion from the center, the density is extremely high.

The second source would require a very hot plasma, and one wonders how such a hot plasma could be produced. The only possible answer is that instabilities are present which ultimately heat the ions. A number of beam-plasma instabilities are possible, almost all of which lead to electron heating. (Some of these instabilities have been discussed in Section III.3 on stability.) The energy would then have to be transferred to the ions. Since the processes involved are complex and have not been explored in the present context, it is not possible to suggest current or pressure dependence of neutron output. For example, while a decrease in pressure could lead to lower plasma densities, it could also lead to a hotter plasma, thus producing an increase in neutron output. The increase of neutrons with applied voltage could lead to higher density or hotter temperature.

An estimate of the necessary densities has been obtained by assuming a Maxwellian distribution for the ions. The results are as follows:

Temp-Kev	r_b -cm	n_1 -cm ⁻³
20	1.0	10^{12}
	0.1	3×10^{13}
40	1.0	5×10^{11}
	0.1	10^{13}

Assuming density profiles of $nr_b^2/(r^2 + r_b^2)^{-1}$, the thermal energy in such a plasma at 20 Kev would be $E \approx 3 \times 10^{-2}$ joules. Using a power balance and assuming that 20% of the injected energy goes into thermal energy, we take

$$0.2 P = 0.2 IV_c \approx \frac{E}{\tau}$$

For $V_c = 1.5 \times 10^5$ V and $I = 10$ mA, we find $\tau = 10^{-4}$ sec. Without a potential well to trap ions, the density, ion temperature, and confinement time all appear excessively large.

We have also made calculations of fusions at the wall. Deuterium atoms absorbed on the cathode wall could serve as the targets. Surface densities are difficult to estimate; however, neutron outputs comparable to those observed by Hirsch are feasible. While the neutron output pressure dependence could be related to a reduction in beam loss with decreasing pressure, the neutron collision results are not easily explained. However, if we added the effect of beam-neutral interactions, we could also obtain a central fusion output. The pressure dependence could possibly be accounted

for by having a beam which was a mixture of D_3^+ , D_2^+ , and D^+ as the pressure is reduced, the beam changes from primarily D_3^+ to D^+ . The average ion kinetic energy changes by a factor of two to three. An examination of the D-D cross section indicates that a factor of two or three increase in energy could result in a cross section increase of more than an order of magnitude. This would compensate for the neutral density reduction.

However, both wall interactions and beam-neutral interactions should occur even if the beams are not precisely focused. With good focusing, the wall interactions could be enhanced because of Coulomb scattering--if a cold plasma of density 10^{12} were in the center, there would be 10^7 scatterings/sec--but even this is not sufficient to account for the dramatic change that occurs between the "poor" focusing and the "good" focusing experiments.

In summary, we have no satisfactory theory which we feel adequately accounts for all the observations of Hirsch. Further observations on a similar device are needed to provide additional information on densities, temperatures, and potential.

IV. SYNOPSIS

During the period June 1969-June 1973, experiments and theoretical analysis have been performed on spherical electrostatic confinement systems operating in steady state. Both ion and electron injection have been studied.

For ion injection, ions were produced by background ionization near the anode. These ions were radially accelerated by

applying high voltage (0.5 to 15 kV) on an inner spherical grid (cathode). The behavior of the plasmas was studied by means of an electron beam probe to measure potential profile, a microwave cavity resonance technique to measure electron density, and spectroscopy to measure the electron density profile and ion velocity.

It was originally anticipated that deep potential wells would be produced. However, for operating pressures of 0.5 to 5 mTorr, input currents of 10-30 mA and applied voltages as specified above between anode and cathode, it has been found that only shallow wells were formed. Electron densities in the range 1 to $4 \times 10^9/\text{cm}^3$ have recently been measured, and the electron temperature is estimated to be 14.5 eV. An energy balance suggests that electrons were heated by electrostatic turbulence.

A theoretical model was developed to describe the above experiments. The behavior of the system is in agreement with predictions. However, uncertainties in the treatment of electrophysics make predictions of electron density and temperature difficult. Conditions needed to produce deep wells have been developed, but it is unlikely that these conditions can be met unless one goes to high vacuum (10^{-6} Torr).

Electron injection experiments have been performed under high vacuum and at pressures of 0.5 to 5.0 mTorr. For the former case, electron densities are too low to be measured by the microwave method. Beam deflections studies indicated that no wells were formed. For the higher pressure case densities up to

$7 \times 10^9/\text{cm}^3$ have been observed. The possible existence of potential wells at the higher pressures has been noted,

A steady state theory of electron injection experiments which includes angular momentum on the beam (input) electrons has been developed. Criteria for deep well formation in high vacuum systems has been developed, and predictions are in reasonable agreement with experiments at Penn State and elsewhere.

Steady state analyses to model the ion injection experiments of Hirsch have also been performed. The conditions needed to produce a virtual anode and to trap large ion densities in a central well were studied, and it was found that very large circulating ion currents, oscillating between the anode and virtual anode, would be needed. This seemed unlikely for Hirsch's experiment. Recent work, including beam angular momentum, suggests that no well was formed at all.

Various mechanisms have been studied to try to account for the neutron emission data of Hirsch. These mechanisms include fusions at the walls, beam-neutral fusions, beam interactions with a cold plasma, and hot plasma fusions. Each of the mechanisms can account for part of the results, but none of the models or mechanisms can explain all the results.

Finally, the possibility of turbulent heating through the generation of an instability has indicated the need for stability theory. A water-bag model has been developed which has been applied to some plane geometry problems where counterstreaming electrons

are present. Further work on stability of bounded inhomogeneous systems is planned.

APPENDIX A

Stability of Bounded Plasma with Reflecting Boundaries

I. Neutralized Homogeneous Plasma

The bounded plasma is pictured in Fig. 1A, with boundaries at $\pm L$.

Figure 1A. Phase Space Diagram for Bounded Plasma

Figure 2A. Electron Distribution Function

Electrons are represented by a two bag distribution function as shown in Fig. 2A. The boundary velocities are also shown in Fig. 1A as straight lines in x - v space. This distribution describes counterstreaming beams with a low energy background. Ions form an infinite mass neutralizing background.

Writing

$$\begin{aligned}
 v_j^+(x, t) &= a_j + w_j^+(x, \omega) \exp(i\omega t) \\
 v_j^-(x, t) &= -a_j + w_j^-(x, \omega) \exp(i\omega t) \\
 E(x, t) &= E^{(0)}(x) + E^{(1)}(x, \omega) \exp(i\omega t)
 \end{aligned}$$

where a_j is the unperturbed contour velocity, $E^{(0)}$ is the unperturbed electric field (and equals zero), and $w_j^+(x, \omega)$, $w_j^-(x, \omega)$ and $E^{(1)}(x, \omega)$ are the velocity contour and electric field perturbations. It is assumed that the perturbations vary as $\exp(i\omega t)$. As shown by Bertrand et.al.⁴⁹

$$\frac{dE^{(1)}}{dx} = \sum_{j=1}^2 \frac{dE_j^{(1)}}{dx} = \sum_{j=1}^2 \left(\frac{e}{\epsilon_0} \right) A_j (w_j^+ - w_j^-) \quad (1)$$

and

$$\omega^2 E_j^{(1)} = \omega_j^2 E_j^{(1)} - a_j^2 \frac{d^2 E_j^{(1)}}{dx^2} \quad (2)$$

where A_j are shown in Fig. 2A and $\omega_j^2 = (e^2/\epsilon_0 n) 2A_j a_j(x)$ is the local plasma frequency. (Note that $A_2 < 0$.)

The boundary conditions for the reflecting wall are $J_j^{(1)}(\pm L) = 0$ which implies $E_j^{(1)}(\pm L) = 0$.

The adiabatic marginal mode analysis is easily performed for $\omega = 0$.

Defining

$$\phi(x) = - \int_0^x dx' E^{(1)}(x')$$

equation (2) becomes, after a little algebra

$$\frac{d^2 \phi}{dx^2} - K^2 \phi = 0 \quad (3)$$

with

$$K^2 = \frac{e^2}{\epsilon_0 n} \sum_{j=1}^N \frac{2A_j}{a_j} \equiv -K_0^2$$

In general, eq.(3A) with $\phi(\pm L) = 0$ has no solution.

2.

However, replacing (3A) by the problem

$$\left[\frac{d^2}{d\lambda^2} + R_0^2 \right] \Phi = -\lambda \Phi, \quad \Phi'(\pm L) = 0 \quad (4)$$

it can be shown³¹ that $\lambda < 0$ implies instability.

The solutions are

$$\Phi = \begin{cases} C_1 \sin \sqrt{R_0^2 + \lambda} x, & \sqrt{R_0^2 + \lambda} = \frac{n\pi}{2L}, n=1,3,\dots \\ C_2 \cos \sqrt{R_0^2 + \lambda} x, & \sqrt{R_0^2 + \lambda} = \frac{n\pi}{L}, n=1,2,\dots \end{cases}$$

Noting that $(dL/d\lambda) < 0$, the marginal mode is $\lambda=0$, so that the plasma is unstable if

$$L > \frac{\pi}{2R_0} \quad \text{symmetric } E^{(1)}$$

$$L > \frac{\pi}{R_0} \quad \text{antisymmetric } E^{(1)}$$

This means that for instability

$$R_0^2 = -\frac{2e^2}{\epsilon_0 m} \left[\frac{A_1}{a_1} + \frac{A_2}{a_2} \right] > \frac{\pi^2}{4L^2} \quad (5)$$

with $A_1 < 0$ and $A_2 > 0$.

Detailed calculations of the eigenfrequencies are easily performed from eq. (2). Writing

$$\tilde{E} = \begin{bmatrix} E_1^{(u)} \\ E_2^{(u)} \end{bmatrix} \quad \gamma = \begin{bmatrix} \frac{\omega^2 - \omega_1^2}{a_1^2} & -\frac{\omega_1^2}{a_1^2} \\ -\frac{\omega_2^2}{a_2^2} & \frac{\omega^2 - \omega_2^2}{a_2^2} \end{bmatrix}$$

eq. (2) becomes

$$\frac{d^2 \tilde{E}}{dx^2} + \gamma \tilde{E} = 0, \quad \tilde{E}(\pm L) = 0 \quad (6)$$

Expanding

$$\tilde{E} = \sum_{n=1}^{\infty} \tilde{b}_n \psi_n$$

where $\{\psi_n\}$ are $\{\cos \frac{n\pi}{2L} x, n=1, 3, \dots\}$

for symmetric solutions and $\{\sin \frac{n\pi}{2L} x, n=1, 2, \dots\}$

for antisymmetric solutions, and using orthogonality, we obtain the dispersion relation.

$$\text{Det} \begin{bmatrix} \frac{\omega^2 - \omega_1^2}{a_1^2} - \lambda_n^2 & -\frac{\omega_1^2}{a_1^2} \\ -\frac{\omega_2^2}{a_2^2} & \frac{\omega^2 - \omega_2^2}{a_2^2} - \lambda_n^2 \end{bmatrix} = 0 \quad (7)$$

$$\text{where } \lambda_n = \begin{cases} \frac{n\pi}{2L}, & n = 1, 3, \dots \quad \text{symmetric} \\ \frac{n\pi}{L}, & n = 1, 2, \dots \quad \text{antisymmetric} \end{cases}$$

Detailed calculations are in process.

II. Nonneutral Inhomogeneous Plasma

If the plasma is not neutralized, the unperturbed electric field is not zero, but must be obtained from Poisson's equation. We present the analysis for the case where no ions are present. This is the counterstreaming unneutralized diode. Again we choose $E_j^{(1)}(\pm L) = 0$ for the boundary conditions.

The unperturbed potential is shown in Fig. 3A and the $x-v$ profile is in Fig. 4A.

Figure 3A. Equilibrium Potential Profile

Figure 4A. Phase Space Diagram

The equation for the perturbed electric field of the j th bag is given by³¹

$$a_j \frac{d}{dx} a_j \frac{dE_j^{(1)}}{dx} + \omega^2 E_j^{(1)} - \omega_{pj}^2(x) \sum_{l \neq j}^N E_l^{(1)} = 0 \quad (8)$$

this equation is solved by the expansion

$$E_j^{(1)} = \sum_{n=1}^{\infty} B_{jn} \psi_{jn}$$

where ψ_{jn} is a complete orthonormal set of eigenfunctions which satisfy the equation

$$a_j \frac{d}{dx} a_j \frac{d\psi_{jn}}{dx} + (\beta_{jn}^2 - \omega_j^2(x)) \psi_{jn} = 0 \quad (9)$$

with

$$\psi_{jn}(\pm L) = 0 \quad \text{and} \quad \int_{-L}^L dx \frac{\psi_{jn}(x) \psi_{jm}(x)}{a_j(x)} = \delta_{nm}$$

These eigenfunctions are obtained by computer solution.

For two bags, using Eqns. (8) and (9) we obtain

$$B_{1m} (\omega^2 - \beta_{1m}^2) - \frac{\omega_1^2}{a_1} \sum_{n=1}^{\infty} B_{2n} \gamma_{nm} = 0 \quad (10)$$

$$B_{2m} (\omega^2 - \beta_{2m}^2) - \frac{\omega_2^2}{a_2} \sum_{n=1}^{\infty} B_{1n} \gamma_{nm} = 0$$

where

$$\gamma_{nm} \equiv \int_{-L}^L dx \psi_{2n}(x) \psi_{1m}(x)$$

The bags are coupled through the coefficient γ_{nm} . We can approximately solve this coupled set of equations by terminating the sums at some value N . The set of equations can then be reduced to a matrix eigenvalue problem,

$$[M] [A] = \omega^2 [A] \quad (11)$$

where $[M]$ is a $2N \times 2N$ matrix. The value N is related to the number of accurate eigenvalues we can obtain. To get N_0 accurate eigenvalues we have found that $N > 2N_0$ is required.

We have computed the eigenvalues for some typical system parameters and the results are shown in Figures 5A and 6A. The quantities ω^2 and y_g are normalized eigenfrequencies and plasma half-thickness. Only symmetric modes are shown. A similar graph is obtained for antisymmetric modes. Instability is indicated when two modes cross. The region of y_g over which instability exists is indicated by circles without a line. The major instability occurs through the coupling of 2S2 and 1S1 modes, where the first number refers to the bag, the S indicates symmetric, and the last number indicates the mode number for that bag. Thus, the above coupling means that mode 2 of bag 2 is coupled to mode 1 of bag 1. The frequency and growth rate of this mode is shown in Fig. 6A. Note that $\gamma/\omega_r < 1$. Further studies are planned to examine the marginal instability situation and to try to find a mode with ω_r^i close to zero.

Figure 5A. Normalized Eigenfrequencies ω^2 vs. Normalized Diode Half Width y_g . Notice nSm means the m-th symmetric mode of the n-th bag. "o" represents bag 1, "x" represents bag 2, the solid line represents the eigenfrequencies of the total system as a result of the coupling of the two adjacent bags.

Figure 6A. Growth Rate and Frequency of Two Unstable Modes

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