ANALYSIS OF PRESSURE PULSE PROPAGATION RESULTING FROM
SODIUM-WATER REACTION IN HYDRAULIC NETWORKS

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Components Technology Division

BASE TECHNOLOGY

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CT/AM0282

TASK A-4: LARGE LEAK ANALYSIS
MEMORANDUM REPORT ON PRELIMINARY DEVELOPMENT SUBTASK
3.1.2 NETWORK RESPONSE ANALYSIS

LMFBR STEAM GENERATOR DEVELOPMENT PROGRAM

CONTRACT AT(04-3)-189

P.A. #61

COMPONENTS TECHNOLOGY DIVISION

ARGONNE NATIONAL LABORATORY

JULY 18, 1973

NOMENCLATURE:

a	sonic velocity
A	net flow cross section area
b	exponent in the time-dependent rupture model
C^+, C^-	positive, negative characteristics
D	inside diameter of pipe
f	Darcy-Weisbach friction coefficient
i	index representing node within a branch
j	index representing branch
m	mass of leak water
$\dot{m}_0$	leak rate under choked flow condition
n	number of branch connections at a junction
p	absolute pressure
p_0	system pressure existing before the pressure-transients
r	rupture radius
r_i	initial rupture radius
$r_{\max}$	maximum rupture radius
t	time
u	fluid velocity
u_0	system fluid velocity existing before the pressure-transients
ρ	density
τ	time constant in the time-dependent rupture model

NETWORK RESPONSE ANALYSIS

1. Introduction

Two independent analysis methods, the arithmetic method of the Zaker and Salmon¹ code and the method of characteristics of the NAHAMMER² code were studied in detail and their relative merits are discussed. The NAHAMMER code, originally written for studying the molten fuel-sodium interaction, was converted to a sodium-water reaction code. The converted NAHAMMER incorporates the same product-zone growth model calculating the source pressure as developed by Zaker and Salmon.

It was observed that Zaker and Salmon provided velocity increments at the source point such that the pressure increment becomes the full amplitude of the pressure wave. Although this seems unlikely to be physically the condition existing at the source point, the same assumption concerning velocity increments was incorporated in the converted NAHAMMER code so that the two network analysis methods can be compared for a consistent source of momentum. The EBR-II secondary sodium pipe network was used for the comparison study for the assumed instantaneous leak rate of 1 lbm/sec.

In the simple model of Zaker and Salmon, the rupture area was assumed to open instantaneously to its maximum value. The sensitivity of this assumption was examined by way of a postulated time-dependent rupture model:

$$\frac{r}{r_{\max}} = \frac{r_i}{r_{\max}} + \left(1 - \frac{r_i}{r_{\max}}\right) \left(\frac{t}{\tau}\right)^b \quad \text{for } t \leq \tau$$

$$\frac{r}{r_{\max}} = 1 \quad \text{for } t > \tau \quad (1.1)$$

$$\dot{m}_0 = 18.9 (\pi r^2) \text{ lbm/sec}$$

The converted NAHAMMER was used and the effects of the controlled leak rate were studied by varying the parameters appearing in the leak rate equation.

The net flow cross sectional area of a steam tube in the steam generator is 0.3723 in^2 and hence the maximum leak rate due to a complete rupture of a single tube is 7 lbm/sec. The analysis here is restricted to a partial rupture corresponding to a leak rate of 1 lbm/sec.

2. The Arithmetic Method

The network analysis used in the Zaker and Salmon code is identified as the arithmetic method³. It essentially requires the knowledge of the complete sequence of events beforehand and it is necessary to keep track of all previous wave emissions and reflections throughout the network. The theory is simple and the method is regarded suitable for use as a supplement to other methods. The use of the method of characteristics is often necessary in determining boundary conditions appropriate for use with the arithmetic method.

Friction and gravity effects were neglected and constant sonic velocity was assumed. The simple equation was solved relating changes in pressure and velocity across the wave front:

$$\Delta p = \rho a \Delta u \quad (2.1)$$

Simple boundary conditions neglecting the energy losses and assuming a continuity in pressure at the junction are*:

$$\Delta p = \sum_{j=1}^n B_j \Delta p_j \quad (2.2)$$

where

$$B_j = \frac{2A_j/a_j}{\sum_{i=1}^n A_i/a_i}$$

Here, Δp represents the pressure increment at any junction due to incident pressure waves of amplitude Δp_j along the connected branch j , A_i and a_i , respectively, are the net flow cross section area and the wave velocity in the connected branch i and n is the number of connected branches at the junction. n can be theoretically any non-zero integer. $n = 1$ represents a dead end, $n = 2$ refers to the two pipe connection, $n = 3$ the Tee-junction, etc.

* Using the method of characteristics, it has been shown that equation (2.2) results upon combining the compatibility relations along connected branches under continuous pressure condition at the junction and the continuity relation at the junction.

The system pressure and velocity existing in the network before the sodium-water reaction starts were assumed to be 60 psia and zero, respectively. At the source point, where the leak occurs and the reaction takes place, the temporal increment of pressure was taken as the amplitude of the emitted pressure wave. This implies that a velocity increment also exists where the velocity increment Δu is obtained by equation (2.1) for the reaction-produced pressure increment Δp . This particular treatment of velocity at the product-zone may not be justifiable on physical grounds. It appears that a more natural velocity at the source point is the existing velocity at the point which initially is the zero system velocity.

3. Method of Characteristics

The governing equations for one-dimensional unsteady flows are obtained by combining the conservation equations of mass, momentum and energy (neglecting gravity effects) and the isentropic relation as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + f \frac{u|u|}{2D} = 0 \quad (3.1)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \rho a^2 \frac{\partial u}{\partial x} = 0 \quad (3.2)$$

Here, f denotes the Darcy-Weisbach coefficient of pipe friction, x is the coordinate measured along pipe length, and the friction effects are included by means of the skin friction on pipe walls.

The set of equations as given above is classified as hyperbolic and the method of characteristics can advantageously be used as the solution

procedure. Following the theory of the method of characteristics⁴, the partial differential equations (3.1) and (3.2) can be converted to ordinary differential equations which can be more easily integrated. The resulting ordinary equations, called the compatibility equations and the associated characteristic equations, are:

$$\frac{du}{dt} + \frac{1}{\rho a} \frac{dp}{dt} + \frac{f u |u|}{2D} = 0 \quad (3.3)$$

$$\text{along the positive characteristic, } C^+ : \frac{dx}{dt} = a + u \quad (3.4)$$

$$\frac{du}{dt} - \frac{1}{\rho a} \frac{dp}{dt} + f \frac{u |u|}{2D} = 0 \quad (3.5)$$

$$\text{along the negative characteristic, } C^- : \frac{dx}{dt} = -a + u \quad (3.6)$$

Equations (3.3) through (3.6) may be solved numerically by finite difference as they stand. However, the non-linear friction terms appear in the compatibility relations and the characteristic equations involve the unknown fluid velocity. Hence the solution procedure requires some iterative schemes for the non-linearities and also some interpolation routines are necessary because of the non-constant nature of the characteristics.

The equations are greatly simplified if the viscous term and the convective terms are neglected. The NAHAMMER code considered these simplified equations and their validity holds only when the fluid velocity u is small in comparison to the sonic speed a . The simplified governing equations are:

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad (3.7)$$

$$\frac{\partial p}{\partial t} + \rho a^2 \frac{\partial u}{\partial z} = 0 \quad (3.8)$$

A combination of equations (3.7) and (3.8) yields the wave equations of both the pressure and the velocity and the solutions can be written in the form:

$$u - u_0 = \frac{1}{\rho a} [F(x - at) - f(x + at)] \quad (3.9)$$

$$p - p_0 = [F(x - at) + f(x + at)] \quad (3.10)$$

where $F(x - at)$ is the rightward (direction of increasing x) wave and $f(x + at)$ is the leftward (direction of decreasing x) wave traveling at the wave speed a . The compatibility and characteristic relations in a finite difference form are for the two adjacent nodes i and $i + 1$ (or i and $i - 1$) in the pipe branch j :

$$F_{j,i}^t = F_{j,i-1}^{t-\Delta t} \quad \text{along } C^+ : dx = a \Delta t \quad (3.11)$$

$$f_{j,i}^t = f_{j,i+1}^{t-\Delta t} \quad \text{along } C^- : dx = -a \Delta t \quad (3.12)$$

where the internodal distance Δx is taken as $\Delta x = a \Delta t$. Various boundary conditions describing interactions of pressure and velocity waves at various pipe junctions, together with the compatibility and characteristic relations, constitute the solution procedure by the method of characteristics.

The NAHAMMER code, originally written for studying the molten fuel-coolant interaction, has been converted to one that is suitable for analyzing the sodium-water reaction and the subsequent propagation of the pressure and velocity waves. The sodium-water reaction and the acoustically constrained

growth model of the reaction product zone characterizing the source pressure used in the converted NAHAMMER is the same as developed by Zaker and Salmon and they are reproduced in the following section. The converted NAHAMMER code now can handle the following specific junction conditions:

- a. Diameter discontinuity
- b. Elbow
- c. Tee (joining from 3 up to 6 branches)
- d. Partial blockage or orifice-like restriction
- e. Acoustic impedance discontinuity
- f. Dummy junction
- g. Dead end
- h. Free surface
- i. Far end
- j. Rupture disk

In the diameter discontinuity, the energy loss due to the sudden area change was accounted for and in general the pressure can jump across a junction. However, in the Tee junction, simplifications were made by neglecting the energy loss and assuming continuous pressure across the junction. The rupture disk condition is that it acts the same as the dead end until it is exposed to a critical over-pressure of 100 psi, at which instant it ruptures and opens to the system pressure 60 psia and subsequently behaves as the free surface condition.

The method of characteristics is a convenient and accurate analysis procedure for handling hyperbolic equations. With this method, the

sequential events of wave propagation do not have to be known in advance, but rather obtained as the result of the solution. The method can accommodate sophisticated and physically meaningful boundary conditions that other methods may fail to. A demerit may be that it requires an extensive coding effort especially in cases of multi-dimensional characteristics^{5,6} or problems where shock wave formation* and their subsequent movement is encountered. When shock waves are present, the finite difference technique using artificial viscosity may be preferred despite the inherent inaccuracy of treating shock discontinuities in a smoothed-out fashion.

4. Source Pressure Calculation

The one-dimensional model as developed by Zaker and Salmon for the sodium-water reaction and the product-zone growth characterizing the source pressure is reproduced and discussed here. An instantaneous reaction was assumed and the break was assumed to occur at a junction connecting two pipe branches. Mass and energy balances taken for a control volume enclosing the product zone at the junction lead to the following differential equations in pressure P and product-zone volume V :

$$\frac{dV}{dt} = B \frac{dm}{dt} + \sum_{i=1}^n u_i A_i \quad (4.1)$$

* For the present problem of sodium-water reaction, the source pressure is relatively low and it is unlikely to observe any shock waves. The waves are considered to be essentially the acoustic waves.

$$\frac{dp}{dt} = \left[(E - C B p) \frac{dm}{dt} - (1 + C) p \sum u_i A_i \right] / CV \quad (4.2)$$

where $\frac{dm}{dt}$ is the leak rate which depends on the steam reservoir condition, rupture area and the back pressure in the reaction-product zone, C and E are constants, and u_i and A_i are fluid velocity and flow cross-sectional area, respectively, of the connected branch i. For the steam reservoir state of 1325 psia and 580°F, the critical pressure at the break is calculated to be 790 psia and for these conditions the leak rate $\frac{dm}{dt}$ is written:

$$\begin{aligned} \frac{dm}{dt} &= \dot{m}_0 && \text{for } p < 790 \\ \frac{dm}{dt} &= \dot{m}_0 \left[1 - \left(\frac{p-790}{535} \right)^2 \right]^{1/2} && \text{for } 790 \leq p \leq 1325 \quad (4.3) \\ \frac{dm}{dt} &= 0 && \text{for } p > 1325 \text{ psia} \end{aligned}$$

The initial conditions considered are at $t = 0$:

$$\begin{aligned} \frac{dp}{dt} &= 0 \\ V &= 0 \\ u_i &= \frac{p - p_1}{\rho a_i} \end{aligned} \quad (4.4)$$

where p_1 is the fluid pressure in the system before the leak starts. Thus the initial pressure is determined by equation (4.2):

$$p = \frac{(E - CBP) \frac{dm}{dt}}{(1 + C) \sum u_i A_i} \quad (4.5)$$

These equations and initial conditions as given above, (4.1) through (4.5), are used in the converted version of the NAHAMMER code.

Some discussion follows on the initial conditions. The above initial conditions imply that at the instant of the leak the pressure suddenly rises to a value higher than the system pressure p_1 while the rate of pressure change $\frac{dp}{dt}$ is maintained zero. These conditions on p and $\frac{dp}{dt}$ appear to be an inconsistent set of initial conditions on physical grounds. That is, a discontinuity in p will have to result in a large value of $\frac{dp}{dt}$. Also, the condition $V = 0$ indicates an apparent singularity initially in the expression for $\frac{dp}{dt}$, equation (4.2).

Perhaps the physically more meaningful set of initial conditions for the present initial value problem may be that at $t=0$, $p = p_1$ implying no jump in pressure, $V = 0$ and the branch fluid velocity u_i is the system velocity existing before the leak starts. The original form of the energy equation is more suitable as the initial singularity is removed.

$$C \frac{d}{dt} (pV) = E \frac{dm}{dt} - p \sum u_i A_i \quad (4.6)$$

The new set of equations and initial conditions indicate that $\frac{dp}{dt}$ is large since $V = 0$ but the product pV remains always finite, which appears to be consistent with the nature of the acoustic constraints. Equations (4.1) and (4.6) and the new initial conditions require some iterative schemes in the numerical solution procedure to account for the involved coupling and non-linearity.

6. Results of Calculations

Calculations were made for a partial rupture corresponding to $\dot{m}_0 = 1$ lbm/sec and for the leak location at the upper end of the evaporator (junction 1 in figure 2) in the secondary sodium circuit of the EBR-II reactor. The physical layout and the dimensions of the individual pipes are given in figure 1. Figure 2 depicts the simplified branch and junction arrangements as used for the present calculations. Junctions 36 and 37 simulate the rupture disks that act as dead ends until they rupture upon an exposure to an over-pressure 100 psi or greater and since then they are maintained at the system pressure 60 psia. Junctions 38 through 42 are the simulated far ends at which no reflected waves are allowed.

In figure 3 the result of Zaker and Salmon code is compared with the result of the converted NAHAMMER code for the instantaneous leak rate $\dot{m}_0 = 1$ lbm/sec. The over-pressures at the reaction point (junction 1) and at the lower end of the evaporator (junction 9) are plotted. The dotted curves represent the converted NAHAMMER result which, same as the Zaker and Salmon code, provides a corresponding source velocity increment Δu given by equation (2.1) for an increment of source pressure Δp . The comparison shows a good agreement. The most deviation in the junction 1 pressure is 20 psi over 350 psi and that in the junction 9 pressure is 60 psi over 450 psi. These observed discrepancies in the two results are attributed mainly to the inherent differences between the two codes in the time steps taken and the manner in which the boundary conditions are handled.

Effects of parameters, τ and b , appearing in the time-dependent rupture model equation (1.1), on the source pressure (junction 1) are given in figures 4 and 5. Figure 4 shows that the effect of the time constant τ is confined to small times $t < .01$ sec except that the long time constant $\tau = 0.05$ sec indicates effects over longer time, $t < 0.06$ sec. Outside the initial stage, in general, an increase in τ resulted in a slight increase in the obtained source pressure. Effects of the exponent b as it is varied from 1 to 5 are given in figure 5 for a fixed time constant $\tau = 0.01$ sec. It is seen that there exists practically no effects for large times $t \geq 0.02$ sec and only small effects during the initial stage $t \leq 0.02$ sec.

As discussed in section 2, it was deemed physically more compatible to regard the reaction product zone as purely the source for the pressure increments. A calculation was also made in which no velocity increments were given to the source point and its result is shown in figure 6. As expected, pronounced effects were observed for the early stage of the event, $t < 0.01$ sec. The source pressure exhibited a sharp rise from the initial pressure of ~ 120 to ~ 250 psi and it subsequently followed the same corresponding curve as in figure 3. This contrasts to the case of figure 3, where the source point velocity increments were considered, in that in this case the initial source pressure, ~ 250 psi, was the local maximum and showed only little change during the early time stage, $t \leq 0.01$ sec. As a result of this difference in the source pressure, during the early stage, some difference in the junction 9 pressure is also shown. Figure 6 indicates the first peak

junction pressure of ~ 480 psi in comparison to the corresponding pressure of ~ 400 psi for the case of figure 3. For the later stage of the event, $t > 0.01$ sec, the pressure plots of figure 6 are practically the same as those of figure 3.

7. References

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Figure 2 Branch and Junction Numbering for Sodium Network Analysis

Figure 3 Pressure vs Time for Instantaneous Leak Rate $m_0 = 1$ lb/sec

Figure 4 Effects of Time Constant τ on Product-Zone Pressure (Junction 1) for $\dot{m}_0 = 1 \text{ lbm/sec}$

Figure 5 Effect of Exponent b on Product-Zone Pressure (Junction 1) for $\dot{m}_0 = 1 \text{ lbm/sec}$

Figure 6 Pressure vs Time for Instantaneous Leak Rate $\dot{m}_0 = 1 \text{ lbm/sec.}$
(Converted NAHAMMER Result. Only the Pressure Increments Are Given at the Source Point)