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LECTURE NOTES ON REACTOR SHIELDING

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PREFACE

These lectures on Reactor Shielding include the basic information necessary to enable one to compute neutron and gamma-ray flux distributions in an unperturbed shield. The object of the series is to impart the information necessary to design shielding for stationary reactors by hand methods. The lectures have been gradually developed over a period of four years at the International School of Nuclear Science and Engineering at Argonne National Laboratory for the Argonne employees, Two-Year Appointees, as well as the International participants. Prior to that time these notes existed in shorter form as the basis of a two to three-hour lecture series for Laboratory employees and for specific Laboratory training commitments. The lectures now constitute a one-semester hour course, but could be extended to a two-semester hour course by additional emphasis on the problems involving coolant activity, heat generation, voids, irregularities, etc., as they arise in later design and construction stages. Prerequisites for this course are Physics through Applied Nuclear Physics, and Mathematics through Mathematics for Engineers and Physicists.

FOREWORD

There are at least four types of people presently involved in shielding work: the theoretical physicist, the experimental physicist, the engineering physicist, and the engineer. These individuals play rather obvious, as well as necessary, roles in the realm of shielding. Incidentally, such roles are quite analogous to those in other areas of scientific and engineering endeavors, with the possible exception of those of the engineering physicist. The field of Shielding is not in the engineering stage to the extent that, for example, is refrigeration; as a result the engineering physicist has a rather definite role in shield design. He does work that clearly may be labeled more as engineering than as physics, but such work is also not yet in the true engineering category because it just has not been developed sufficiently and is not quite out of the pure science stage.

It appears that when one speaks of Shielding it is not always clear to all concerned whether the term includes the closely associated engineering problems. The people who must be concerned with the solution of these problems should know enough about the physics of shielding so that they can at least utilize the radiation distributions, if not determine them.

The following introduction is more likely to be called Physics or Mathematics rather than Engineering, at least by an engineer. It should ultimately be called Engineering. The area that is now called Engineering (that includes stress analysis, heat removal, thermal shield design, etc.) probably is in need of attention, or perhaps publicity. Among the purposes of the present lectures is to inform people that these problems, if they are not an integral part of shielding, are at least related to shielding, and also to point out the areas in which fundamental work is needed. Probably the best example of where fundamental work is needed is in the treatment of concrete as a shielding material.

Until the present time not much coursework in Shielding has been formally presented in Universities; thus, if a shield designer were to be employed, an engineer would be hired and given some extra physics and possibly mathematics training, or a physicist who would be subjected to some engineering training. Most of this training has to be done by on-the-job experience, usually supplemented by somewhat informal lectures. With the passage of time, it is anticipated that this engineering-physicist category will disappear. At this time it is hoped that formal training in Universities will include the subject of Shielding, since appropriate handbooks and texts will be available. Basic work will still be undertaken by theoretical and experimental physicists, since fundamental knowledge is needed, although not necessarily under the category of Shielding. There will, no doubt, be a continuation of engineering research, since such appears to be required. The field of Shielding will then be another phase of Engineering.

The type of person that is required for this intellectual guessing game called Shielding is basically an engineer, that is, a person who does practical things with science. It turns out that all types of engineers are needed for what is commonly referred to as Nuclear Engineering. Rather than being a person trained for a new engineering area, the Nuclear Engineer requires know-how in any or all of the previous engineering disciplines, together with some extra basic knowledge or training. This extra training, briefly, is Mathematics and Nuclear Physics. The Nuclear Engineer that intends to design shields should have a minimum of ordinary Differential Equations plus a year of Advanced Mathematics for Engineers. Probably a year in Applied Nuclear Physics will be needed to complete the picture.

Historically, the first problems concerned with shieldings were solved by an engineer, who lacked the physics to be independent, in co-operation with a physicist, who lacked the engineering. Gradually, the physicist will become necessary only for the fundamental research; a Shielding Handbook takes the place he occupied. At the present time we do not have this handbook. This means that the engineer needs a liberal measure of insight along with the formulas for radiation attenuation.

The field of Shielding may be split according to the reactor type, i. e., mobile or stationary. The approach to shielding problems is quite dependent upon the reactor type of concern. In the case of shielding for a mobile reactor, weight is a crucial factor and, depending on the importance of the reactor, it may of necessity virtually be shielded with pure money. In practical terms, this means the use of exotic materials, or at least more or less exotic arrangements of ordinary materials. Money must not be a major concern in the design of the shield for the mobile reactor or the project is not likely to continue. In contrast, stationary reactors may be shielded with ordinary materials, usually ordinary concrete. In specific instances a heavy concrete may be economically advisable; for example, beam holes in research reactors may be more easily utilized if they are kept as short as possible. Further, the costs may be reduced by shortening penetrations and by reducing the size of the containing building. Speaking of costs, it should be noted that there are commercial enterprises based on the construction of reactors and therefore it is not enough that "an extra measure of concrete may remove doubts"; it may also remove too much money. This lecture series is based primarily on shield design for stationary reactors, although, of course, basic information generated by mobile reactor shield programs is utilized and reference made to it as classification will permit.

Since these lectures are intended for people who, for one reason or another, will not always find the services of "mechanical brains" available, a large emphasis has been placed on the computational techniques that do not require these services. This availability, or the lack thereof, may be dependent upon the attitude within the country in which the individual will do his work, but it is more often dependent upon the completeness or thoroughness of design required, or possible, under the circumstances. It is believed that there will always be a place for an ordinary desk computer in what is commonly referred to as hand calculations.

The problem to overcome is, then, twofold. First, to get the physics and mathematics of shield design accepted as engineering, and secondly, to secure recognition for the fact that many other engineering problems associated with reactor design have at their root a necessity for being aware of and knowing shielding-type calculations.

ACKNOWLEDGMENT

The accumulation of material for a lecture series is always the result of much helpful advice and assistance. The most copious source of information is the student who reads and studies the preliminary notes and makes comments. There have been many of those helpful individuals, too many to list individually. Their comments are gratefully acknowledged.

Almost all the basic information contained herein was evolved from the years of work with J. W. Butler. This association, together with his comments and discussions, without doubt had the greatest influence upon the contents of these lectures. The author is also indebted to A. E. McArthy and A. D. Rossin for helpful discussions and proofreading.

NOMENCLATURE

A	Area	cm^2
A(t)	Active atoms per cubic centimeter in the system	$atoms/cm^3$
A_1	Active atoms that leave the reactor	
a	Total shield thickness, $a = \sum_{i=1}^{n} a_i$	cm
a_i	Thickness of i-th shield material	cm
α	Constant in exponential buildup function, or	cm^{-1}
	buckling correction for thermal column neutrons, or	
	transfer coefficient of impurities from water to steam	
$B(\underline{x}, \xi)$	Beam attenuation kernel	
$B(\mu a)$	Buildup factor	
β	Buckling term for thermal column neutrons in rectangular and cylindrical geometry, or	cm^{-1}
	Reflection coefficient, (albedo), or	
	Transfer factor of fission products from water steam, or	
	Angle	
D_f	Fast neutron diffusion constant	cm
D_i	Intermediate neutron diffusion constant	cm
D_s	Thermal neutron diffusion constant	cm
E_n	Neutron energy	Mev
$E\gamma$	Gamma-ray energy	Mev

NOMENCLATURE

$E_n(x)$	Exponential integral, see Section 3	
$Ei(x)$	Exponential integral, see Section 3	
η	$k/\sigma_s(1 - \alpha)$	
$F_n(x)$	Exponential integral, see Section 3	
f	Fraction	
$f_n(x)$	Function for computation of exponential integrals	
f_1	Flow rate water through the filter	gm/sec
f_2	Flow rate of steam	gm/sec
G	Constant for fission product gamma-ray power	
$\Gamma(t)$	Fission product power after shutdown	mev/sec fission
γ	Euler's constant, see Section 3, or	
	Buckling constant for thermal column neutrons, or	cm^{-1}
	Photon energy	$h\nu - mc^2$
H	Heat generation, or	Mev/cm^3sec
	Height of cylindrical source	cm
J_f	Fast neutron current	n/cm^2sec
J_s	Thermal neutron current	n/cm^2sec
$Ki_n(x)$	Mathematical function, see Section 3	
$K_0(x)$	Bessel function	

NOMENCLATURE

k	Reciprocal relaxation length for gamma ray source strength, or	cm^{-1}
	Corrected thermal column thermal neutron attenuation constant, or	cm^{-1}
	Constant for gamma-ray buildup function	
κ_s	Reciprocal of thermal neutron diffusion length	cm^{-1}
κ_i	Reciprocal of intermediate neutron diffusion length	cm^{-1}
λ	Relaxation length, or	cm
	Disintegration constant,	sec^{-1}
M	Mass	gm
m	Integer	
μ	$\cos \theta$	
μ_i	Gamma-ray total cross section for i-th shield material	cm^{-1}
μ_0	Cosine of the maximum limit of the angle θ	
μa	Total gamma-ray mean free paths in the shield, $= \sum_{i=1}^{n} \mu_i a_i$	
μ_e	Gamma-ray energy absorption coefficient	cm^{-1}
$N(\rho)$	Plane collimated kernel for fission neutrons in water	
N(t)	Atoms per cubic centimeter of material being irradiated at time t	$atoms/cm^3$

NOMENCLATURE

N_γ	Number of gamma rays emitted per event	
N_0	Atoms per cubic centimeter of material, or	atoms/cm^3
	Avogadro's number	atom/gm atom
n	Integer	
ν	k/σ_s	
Ω	Solid angle	
P	Reactor power, or	watts
	probability of escape, or	
	fission product power	Mev/sec
P_1	Power release of fission products that leave the core	Mev/sec
Φ_f	Fast neutron flux	n/cm^{2}sec
Φ_i	Intermediate neutron flux	n/cm^{2}sec
Φ_s	Thermal neutron flux	n/cm^{2}sec
Φ_γ	Gamma-ray flux	γ/cm^{2}sec
$\Phi(t)$	Neutron flux as a function of time	n/cm^{2}sec
Q_f	Fast-neutron source strength	n/cm^{3}sec
Q_s	Thermal neutron source strength	n/cm^{3}sec
Q_i	Intermediate neutron source strength	n/cm^{3}sec
Q_0	Fast neutron or gamma-ray point source strength	n or γ/sec
Q_1	Fast neutron or gamma-ray line source strength	n or γ/cm sec
Q_2	Fast neutron or gamma-ray plane source strength	n or γ/cm^{2}sec

NOMENCLATURE

Q_3	Fast neutron or gamma-ray volume source strength	n or γ/cm^3sec
R	Radius of shield, or	cm
	distance from source to measuring point	cm
R_s	Spherical source radius, or	cm
	Disk source radius, or	cm
	Cylindrical source radius	cm
r	Rate of entry of corrosion or erosion products into coolant	atom/sec
ρ	Optical path length for fast neutrons in water	cm
$seci(x_1\theta)$	Secant integral, see Section 3	
σ	Cross section for shield material, or	cm^{-1}
	Constant adjusted for continuity of fast neutron flux for thermal neutron source strength	cm^{-1}
σa	Total number of mean free paths in shield, $= \sum_{i=1}^{n} \sigma_i a_i$	
σ_a	Neutron absorption cross-section, or	cm^{-1}
	Neutron activation cross section	$cm^2/atom$
σ_{as}	Thermal neutron absorption cross-section	cm^{-1}
σ_{af}	Fast-neutron absorption cross section	cm^{-1}
σ_{ai}	Intermediate neutron absorption cross section	cm^{-1}
σ_1 & σ_2	Constants for $N(\rho)$ when represented as a sum of two exponentials	cm^{-1}

NOMENCLATURE

σ_f	Fission cross section	cm^{-1}
σ_r	Effective removal cross section	cm^{-1}
σ_i	Neutron inelastic scattering cross section, or	cm^{-1}
	Cross section for i-th shield material	cm^{-1}
σ_s	Source cross section, or	cm^{-1}
	Neutron scattering cross section	cm^{-1}
T	Time of operation	sec
t	Time, or	sec
	Thickness	cm
t_r	Time spent in active zone	sec
t_c	Time spent in complete cycle	sec
τ	Fermi age	cm^2
θ_0	Maximum value of the angle θ	
u_0	Secant of the maximum value of the angle θ	
V	Volume	cm^3
ξ	Average logarithm of the energy loss of the neutron	
Z	Atomic number	

LECTURE NOTES ON REACTOR SHIELDING

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1. INTRODUCTION

The design of a shield for a reactor includes many complex problems concerning the distribution of the associated radiation. There are many fine theoretical analyses indicating what radiation will do in more or less ideal situations. For example, the distribution of neutrons in water from a monoenergetic source is well known both theoretically[1] and experimentally.[2] The application of these methods to a laminated shield of water and iron becomes less exact; for more complicated, though common, combinations of iron, water, concrete and boron, the problem is solved only by quite doubtful methods. Similarly the moments method[3] can solve the problem of the distribution of gamma rays in a shield composed of a single material quite accurately; however, a shield composed of more than one material may still be solved only by approximations. No universal method for computing buildup through multi-layer shields exists, although some progress is being made in this direction.[4] Solving all these problems in a solid unperturbed shield still does not answer all the questions, since there will be a multitude of penetrations and voids that will change the radiation distributions in ways that at best may be only estimated. As a result, shield designers have be educated guessers to a large extent and must depend quite strongly on experimental data and semi-empirical approaches. Although high-speed computing machines are becoming more useful all the time, they still cannot be used to solve a complete shield problem, even disregarding the void and penetration problems that inevitably appear.

These lectures will be an attempt to discuss the design of a reactor shield and the solution of the associated shielding problems by simple hand-computing methods. These include the calculation of the radiation distributions throughout the shield in order to determine the choice of materials as well as the location and thickness. Problems associated with the radioactivities of coolants and components, radiation heating of structural materials, and handling of spent fuel will also be discussed.

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[1] Aronson, R., J. Certaine, H. Goldstein and S. Preiser, "Penetration of Neutrons from a Point Isotropic Fission Source in Water," NYO-6267, (Sept 22, 1954).

[2] Bulk Shielding Facility Staff, "Attenuation in Water of Radiation from the Bulk Shielding Reactor; Measurement of Gamma-Ray Dose Rate, Fast Neutron Dose Rate and Thermal Neutron Flux," ORNL-2518, (July 8, 1958).

[3] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma Rays," NYO-3075, U. S. Govt. Printing Office, (June 30, 1954).

[4] Goldstein, H., "The Attenuation of Gamma Rays and Neutrons In Reactor Shields," U. S. Govt. Printing Office, (June 30, 1957) p. 192.

In addition to the simple hand-computing procedure, there are other approaches, none of which are really independent. These include comparison methods, utilizing data from bulk shield facilities, specific experimental determinations, and, of course, computing machine methods which are of great value for particular parts of the shield design problem, or for the more ideal problems that will aid the thinking done prior to making an educated guess. The choice of method, as well as being a somewhat personal choice of the designer, depends on the type of shield desired, the accuracy required, and the time and money available. Since virtually every approach contains approximations, this fact must be kept in mind so that the method used in designing one shield is not used on the next design without reconsideration of the basic assumptions.

As far as time is concerned, a shield design may be accomplished in one day, one week, one month, one year, or one decade. Obviously, a project cannot wait for the latter design time. Similarly, the time of one day is likely not to permit consideration of adequate detail, although it may be quite sufficient for preliminary estimates. Somewhere between these extremes there is a time which is both practical and adequate. This is usually somewhere between a month and a year, but depends on the results required. In general, the longer the time spent the more detailed the design. The output per unit time, however, has a limiting factor, or an asymptote, above which the accuracy is limited by the knowledge of fundamental constants and ability to apply fundamental procedure. For shields that must be optimized to a very high degree, for example, those of mobile reactors, the detailed calculations must be verified with experimental mock-ups. Since a full scale mock-up is the only real test, either a facility that is essentially the total shield must be built, or else a degree of uncertainty will exist until the actual plant is operated. A practical approach that may be utilized at times is to leave a portion of the shield flexible until full-power operation indicates the final answer.

The choice of the basic shield material or materials is one that must be evaluated in terms of the reactor involved. For a power reactor with few shield penetrations the basic material may well be ordinary concrete. If the reactor is for research and will contain many beam holes, a thinner shield will no doubt be worth consideration. Concrete which is made more dense by the use of metal punchings and/or a heavy aggregate, is then desirable, as the penetrations will be shorter and less expensive. When a dense concrete is utilized, the building itself will be smaller and a saving may be realized in construction costs. This is particularly true if the building is gas-tight. For extremely efficient shields, such as for a mobile reactor, more exotic materials, such as heavy metal hydrides, or, at least, more expensive arrangements of ordinary materials may be necessary.

Some thought must go in the direction of the arrangement of the shield materials in order to accomplish dual functions whenever possible. A unit shield may, for example, be the main support for much associated reactor equipment. The use of compartment or divided-type shielding may save space and materials. The use of higher radiation levels or shadow shields in relatively unaccessible areas may be quite possible as well as economical.

A detailed description of the radiation considered in reactor shielding problems will be found in the pertinent chapters. In general, the radiations considered in reactor shielding problems may be limited to neutrons and gamma rays. Alpha and beta rays will, for example, be present in large quantities, but may be shielded so easily that they are not considered part of the reactor shielding problem.

Neutrons will be found in the reactor core as a direct result of the fission process. There are, on the average, 2.5 neutrons produced per fission of uranium-235, with an energy distribution according to the well-known fission spectrum. In addition there will be delayed neutrons from fission, usually not significant in shielding problems, and neutrons from photo neutron reactions in materials such as beryllium or deuterium that may be utilized for their neutron-reflecting and moderating properties. The latter source of neutrons may be significant in cases where the reactor is in a shut-down condition. The specific activity of N^{17}, formed from the O^{17} (n,p) N^{17} reaction, includes a neutron that may well be of importance in determining the access to the coolant pipes in the case of a water-cooled reactor.

Gamma rays may originate essentially anywhere in the reactor system that neutrons are found. The reactor core is a copious source of gamma rays that have their origin in the fission process, the decay of fission products, the inelastic scattering of neutrons, or the capture of neutrons in structural or coolant materials located in the core. The latter gamma rays may be further classified as the immediate result of neutron capture, "capture gamma rays," or of the decay of the unstable nuclei created by neutron capture, "radioactive decay gamma-rays." The gamma rays born in the core are deep within the shield and usually of lower effective energy, so that they rarely determine the shield thickness. However, they are an important source of heat generation within the inner shield layers and may also figure prominently in instrumentation problems.

Gamma rays that originate outside the core are essentially all due to neutron capture or inelastic scattering in the structural and shield materials. The capture gamma rays are of higher effective energy and in greater quantity than those resulting from the fission process, decay of radioactive nuclei or inelastic scattering, and, in addition, are created nearer the outer extremity of the shield. These facts explain why capture

gamma rays are very often the controlling influence in determining the shield thickness and may never be disregarded in shield-design problems. It is to control the generation of capture gamma rays that high cross-section materials, such as boron, are incorporated into a shield. In such a material the capture gamma ray is of low energy, the remaining portion of the energy being given off as an alpha particle. Coupled with the importance of the capture gamma ray is the difficulty of predicting the location at which it will be created. This problem is essentially the heart of the reactor shield-design problem.

Shielding a reactor consists primarily of moderation and then absorption, at thermal or near-thermal energy, of the fast neutrons produced in the reactor core, plus the attenuation of the secondary products involved, chiefly gamma rays. Other radiation from the core usually produces local effects, such as gamma-ray heating in materials in or near the core. There are no extremely efficient materials for stopping fast neutrons that can compare to boron in the case of thermal neutrons. Fast neutrons must be reduced in energy by elastic or inelastic scattering to thermal or near-thermal energies at which they may be readily absorbed. Probably the most efficient way to reduce the neutron energy is by a combination of inelastic and elastic scattering. This may be considered convenient from the standpoint of including structural materials and, as well, gamma-ray shield materials, both of which will be needed and both of which require the heavy material necessary for efficient inelastic scattering. The elastic scattering will be provided by lighter materials, preferably of a hydrogenous nature.

In general, an efficient shield should have a neutron attenuation that is nearly the same as the gamma-ray attenuation. Since neutrons are attenuated by a reduction in energy followed by absorption they require a material of low mass. Gamma rays are more rapidly attenuated by a material of high mass or high Z. These materials are conveniently efficient for fast-neutron attenuation also, by virtue of inelastic scattering, if followed by hydrogenous material. In order to follow the above attenuation requirements this means that a homogeneous mixture of, perhaps, uranium and water would be an ideal shield, except for the additional complication of having fissions in the shield. Including reasonable cost, as well as structural requirements, the heavy material usually turns out to be lead and/or steel. Since a homogeneous compound such as a metal hydride is expensive, not always stable, and, in addition, does not necessarily contain the most efficient proportion of metal to hydrogen, this is not a solution at the present time. A laminated structure, the next approximation to homogeneous, is nice, but quite expensive. This is exemplified by water-filled metal honeycomb structures, or by laminated water and metal, or masonite and metal. The next best homogeneous material is concrete, and it turns out to be quite well balanced if the density is increased by including heavy

aggregate, metal punchings, or both. The heavy concrete is more expensive, but the extra expense may well be justified in reduction of building costs or an increase in accessibility. Concrete is somewhat in question because of lack of detailed knowledge on the amount of water retained or the absolute minimum amount that must be contained for adequate moderation of the fast neutrons. This lack of knowledge is paid for in terms of the extra thickness of concrete required to guarantee results.

Portions of a reactor shield may not be chosen on the option of a shield designer alone. For example, the pressure vessel may be a few inches of steel and thereby an appreciable part of the shield. Likewise, the reflector may be a material that affords a considerable amount of shielding value. Neither of these materials are subject to alteration for strictly shielding considerations, since the primary function is containment and neutron reflection, respectively.

The procedure generally followed in determining the radiation distributions in the shield is to calculate in order the neutron fluxes and then the gamma-ray fluxes. Because shield design does not follow the design of the other parts of the reactor, but must progress with it, these calculations must be done more than once, each time progressing a little more toward the final design. As an example, before the vessel containing the reactor can be designed, shielding calculations must be made to determine radiation heating. The vessel may have to be substantially altered, or a thermal shield may have to be incorporated into the design, which in turn renders the preliminary calculations obsolete. Similarly, before a thickness of concrete can be established, the heating must again be considered. A common result is that three or four inches of lead may be required to reduce gamma-ray heating in the concrete. Calculations will again have to be revised. After the total shield thickness has been decided, a look at the largest contributions to the leakage from the shield may indicate a particular capture gamma-ray source could be reduced with proper location of boron.

Other problems arise during and after this process, such as shielding of radioactive coolant or air, shielding of spent fuel elements, and shielding of the radioactive components that may have to be removed for repair. Portions of the shield will contain alterations or penetrations to permit access to the reactor core for instrumentation or research. There will be plugs which can be removed for fuel recharging. Control rod drives will have shafts that go through all or part of the shield. There will be corners and cracks introduced by virtue of the method of construction of layers of shield materials. Few, if any, of these problems have a neat solution. Careful study of the construction drawings, knowledge of the origin and characteristics of the radiation associated with reactors, and intuitive guesses will be necessary to insure a radiation-proof shield. A common conclusion is that shielding has been, and probably still is, to some extent as much an art as a science.

2. DEFINITIONS

In order to use terminology and symbols with mutual understanding it is helpful to define these concepts prior to their use. While some terms are defined because they are more or less peculiar to shielding, others may be defined to eliminate the confusion that sometimes results from use in other fields.

Probably the most prominent term is that of particle flux or flux density. Radiation or particle flux, "Φ," is a scalar quantity which, when multiplied by the macroscopic cross section for a particular event, yields the number of such events occurring per cm^3 sec. It is also the total particle path length per cm^3 sec. The definition of neutron flux is commonly given as the neutron density "n" (neutron/cm^3), times the neutron velocity, "v" (cm/sec), or "nv" (neutrons/cm^2 sec). The physical dimensions are properly neutron cm per cm^3 sec. "Φ" is the symbol for flux utilized throughout this discussion and also quite prominently elsewhere. The dimensions of flux are quite commonly written as particles per cm^2 sec, which, while correct, conceals the nature of the quantity and results in confusion with current, a vector quantity which has the same dimensions but represents the number of particles passing through a square centimeter each second. Very often the magnitudes of these two quantities are quite comparable; indeed we shall make that approximation at times. A use of flux is exemplified in the case of thermal neutron absorption;

$$\sigma_{as}\left(\frac{\text{abs}}{\text{n cm}}\right) \times \Phi_s\left(\frac{\text{n cm}}{\text{cm}^3\ \text{sec}}\right) = Q_s\left(\frac{\text{abs}}{\text{cm}^3\ \text{sec}}\right) \quad . \tag{2.1}$$

This is a term necessary in determining the activity of materials irradiated by a neutron flux, the capture gamma-ray source term in a shield material, or the gamma-ray heat generation in a material. It is to be noted that the term flux, as defined here, is not the same as the flux of light photons in optics. The latter term is actually the same as current, which is to be defined.

Fast-neutron flux as used in these notes is the flux of neutrons that have an energy greater than one Mev. It is, therefore, not the same fast flux as spoken of in the usual two-group diffusion theory. That fast flux includes all neutrons above thermal energy and actually, through the use of appropriate constants, τ and D_f, is a quantity which serves as a source term for the thermal neutrons. Because of the energy range involved, particularly as it applies to dose rate per neutron, and attenuation lengths at great distances in the shield, it is believed incorrect to compute the fast-neutron flux for shielding purposes by diffusion techniques; therefore, it is computed by Ray Theory, utilizing removal cross sections.

Ray Theory is a name, not in common use, applied to the type of attenuation appropriate for neutrons and gamma rays. The radiation is assumed to travel from source to detector along the optical path connecting them, the attenuation due to each material being apportioned according to the thickness of that material traversed by the optical path. The attenuation is assumed to be exponential with modifications for specific purposes, such as buildup of gamma rays.

Particle current "J" is a vector quantity that indicates the net number of particles per cm^2 sec in a particular direction, usually normal to the surface of the intercepting medium. In the diffusion approximation, it may be calculated by Fick's law,

$$J = -D \text{ grad } \Phi \text{ neut}/cm^2 \text{ sec}, \qquad 2.2$$

which is familiar to the users of diffusion theory as it occurs in neutron-distribution problems in reactors. This law will be applied on occasions where diffusion theory is applicable, namely, for thermal neutrons. However, it is not appropriate for high energy neutrons (> 1 Mev) unless proper precautions are taken, for instance, as with a particular definition of constants such as occurs in two-group diffusion theory. Fick's law is not proper for the high-energy neutrons that are of major concern in shielding problems. According to Ray Theory, to be discussed in more detail in the next section, the particle flux from a plane source is given by

$$\Phi = \int_A dA \, Q_2 \frac{e^{-\sigma R}}{4\pi R^2} \text{ part.}/cm^2 \text{ sec.} \qquad 2.3$$

By comparison, the current at the same location is,

$$J = \int_A dA \, Q_2 \frac{e^{-\sigma R}}{4\pi R^2} \cos\theta \text{ part.}/cm^2 \text{ sec.} \qquad 2.4$$

It is common to hear of neutron energy in terms of "hard," "fast," "soft," "thermal," "cadmium," etc. We shall be concerned here with fast neutrons (those of 1 Mev and higher energies, as they occur in fission[1]), intermediate neutrons, those between thermal energy and 1 Mev, and thermal neutrons (those of 0.025-ev energy or having an energy, corresponding to the thermal temperature of the reactor if other than 0.025 ev).

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[1] Watt, B. E., "Energy Spectrum of Neutrons from Thermal Fission of U^{235}", Phys. Rev. 87 1037 (1952).

Gamma-ray energy also is often referred to in the relative terms "hard" or "soft." For reactor shielding purposes the practical range of energies is from 1 Mev to about 8 Mev. Reference to hard, or penetrating, gamma rays then means energies closer to 8 Mev and soft means closer to 1 Mev. The gamma-ray spectrum may conveniently be divided into representative mono-energetic lines, the number of lines depending on the desired accuracy, the time available, the type of source represented, or the available knowledge of the details of the spectrum. While any given problem may require only one portion of the spectrum, or maybe one line, to be adequately represented, a reactor shield design usually requires the complete range at one time or another.

Gamma rays are known by several names, each of which basically refers to the process by which they are created. Gamma rays, in fact, are fundamentally the same as X rays and differ from X rays only in that their origin is nuclear rather than atomic, and that their energy is usually greater. The names gamma ray, photon, and X ray, are commonly used somewhat interchangeably and with good reason, since fundamentally they are in the same range of the electromagnetic spectrum and may be described with the same constants. For sake of clarity the terms applied to gamma rays in radiation shielding considerations will be defined.

Prompt fission gamma rays are those which arise from the fission event in the same instant that the fission occurs. As fission ceases these gamma rays will cease also. There has been considerable effort expended to determine the quantity and energy of these gamma rays. At the present time the spectrum is rather well known,[2] at least for shielding purposes.

Fission product decay gamma rays are those which are given off during radioactive decay of the unstable fission products. Even though fission ceases, these gamma rays continue to be emitted according to the normal laws of radioactive decay.[3,4,5] The spectra of these gamma rays are again rather well known, at least for shielding considerations, and are commonly grouped into line spectra for use in shielding problems.

Capture gamma rays are created by the absorption of neutrons in a nucleus and are released at the instant of absorption to relieve an unstable energy situation in the nucleus. These gamma rays will be found wherever neutrons are present, in quantity proportionate to the flux of neutrons and the absorption cross section of the material. The numbers and energies of these gamma rays that are emitted per capture is quite an important

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[2] Maienschein, F. C., et al., "Gamma Rays Associated With Fission," Paper P/760 of the International Conference on the Peaceful Uses of Atomic Energy held at Geneva, Switzerland, in June 1958.

[3] Way, K., and E. P. Wigner, "Decay of Fission Product Gamma Rays," Phys. Rev. 70, 115, (1946).

[4] Clark, F. H., "Decay of Fission Product Gamma Rays," NDA-27-39 (December 30, 1954).

[5] Moteff, J., "Fission Product Decay Gamma Ray Spectrum," APEX-134.

quantity to know for shielding purposes, and several summaries of this information exist.[6,7,8] The energies of these gamma rays are particularly important, since they are quite penetrating and their origin may be quite near the outer extremity of the shield.

Inelastic scatter gamma rays are formed by the inelastic scattering of neutrons with nuclei and are, therefore, found in quantity proportionate to the neutron inelastic scatter cross section and the neutron flux. It is worthy to know that these gamma rays may be created only by high-energy neutrons (energies greater than 1 Mev) and, further, they are usually of lower energies than possessed by the capture gamma rays. This information is useful because of the relative lack of knowledge and the relative complexity of the process involved,[9] which leads to the fact that the sources of inelastic scatter gamma rays are more difficult to predict than any other of the gamma rays. Fortunately, it is usually easy to prove that these gamma rays are less important than others.

Radioactive decay gamma rays are those which are emitted by unstable nuclei created by neutron absorption. These gamma rays, however, in contrast to capture gamma rays or inelastic scatter gamma rays, will not be emitted instantaneously, but with a measurable half-life. While their numbers and energies are ordinarily less than the gamma-rays discussed above, they continue to be emitted after the neutron flux has been removed and, therefore, are important for consideration after the reactor has been shut down. This applies, for example, in the case of removing components from the reactor or in considering the activities of coolants that have circulated through the reactor and then are conducted to the extremities of the shield.

Neutron cross sections may be defined in terms of the effective area that a nucleus presents to the neutron causing a particular reaction, or may be defined as in connection with the flux definition that was given in a preceding paragraph. Among the neutron cross sections there are the obvious ones: absorption, scattering, fission, etc.[10] These refer to the relative probabilities of occurrences of the particular events. The number of such events that occur per cm^3 sec is given by the product of the macroscopic cross section times the neutron flux. The removal cross section does not have such an obvious meaning, since it does not refer to a process that occurs at the point of collision, but to the ultimate

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[6] Mittleman, P. S., and R. A. Liedtke, "Gamma Rays Resulting from Thermal Neutron Captures," Nucleonics 13, #5, 50 (1955).

[7] Deloume, F. E., "Gamma-Ray Energy Spectra From Thermal Neutron Capture," APEX 407 (August, 1958).

[8] Bartholomew, G. A., and L. A. Higgs, "Compilation of Thermal Neutron Capture Gamma Rays," CRGP-784, Chalk River, Ontario, (July, 1958).

[9] Cranberg, L., et al., Chapter 4 in Progress in Nuclear Energy Series 1, Vol. 1, "Physics and Mathematics," McGraw Hill Book Company, Inc., (New York, 1956).

[10] Hughes, D. J., Neutron Cross Sections, Pergamon Press, (1957).

result. In other words, the collision does not remove the neutron, for example, by absorption, so that the neutron is physically gone, but reduces its energy by inelastic and elastic scattering so that it will be more quickly moderated to thermal energy and absorbed in the following hydrogenous material. The removal cross section may be approximated by all of the inelastic scattering and a fraction of the elastic scattering. If, further, it is assumed that the total cross section consists equally of elastic and inelastic scattering, and, if the fraction of the elastic scattering that applies is approximately one-half, the removal cross section may be seen to approximate three-quarters of the total cross section (please refer to section IV figure 19).[11,12] The removal cross section may be though of as the total cross section with a buildup correction subtracted to make it fit an exponential attenuation. This, of course, is only strictly true in a situation in which the material is immersed in a hydrogenous medium, or in which a specific energy group is followed, i.e., the fast neutrons of energy greater than one Mev.

Gamma-ray cross sections have the unfortunate tag of absorption, clinging traditionally from the days before reactors were known. This is a hold-over from X-ray technology, where the prevalence of narrow-beam geometry did not require the same differentiation of the many events that the broad-beam reactor gamma-ray problems do. At any rate the term gamma-ray absorption coefficient, or cross section, should properly be called the total gamma-ray cross section or the total gamma-ray attenuation coefficient. It is composed of several component cross sections[13,14] of which the most well-known are Compton scattering, photo-electric effect, and pair production. For the purpose of determining heat generation due to gamma-ray flux there is defined an energy-absorption cross section.[15] This is essentially the total cross section less the scattering, or the portion of the total cross section that is essentially true absorption. The total gamma-ray cross section is often referred to as the total linear attenuation coefficient, a term which is comparable to the macroscopic cross section in that the dimension is cm^{-1}. Gamma-ray cross sections are commonly given in terms of the mass attenuation coefficient. The term "mass" refers to the fact that it is per unit of mass or density. The dimensions are then cm^2/gm. This quantity must be multiplied by the appropriate density in order to get the linear attenuation coefficient.

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[11] Goldstein, H., "The Attenuation of Neutrons and Gamma Rays in Reactor Shields," p. 268 U. S. Govt. Printing Office, (May, 1957).

[12] Blizard, E. P., "The Shielding of Nuclear Reactors," Paper P/2162 of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[13] Fano, U., "Gamma-ray Attenuation," Nucleonics 11, (August, 1953), and Nucleonics 11, #9, 55, (September, 1953).

[14] Heitler, W., The Quantum Theory of Radiation, Oxford University Press, (London, 1950).

[15] Goldstein, H., "The Attenuation of Gamma rays and Neutrons in Reactor Shields," U. S. Govt. Printing Office, (May 1, 1957), p. 129.

The mean-free-path (mfp) of a particle in a medium is the average distance the particle will go in that medium before it undergoes a collision. This distance is the reciprocal of the macroscopic total cross section for that particle in the material in question. The quantity, of course, is a function of energy, as is the cross section. The symbol commonly employed is "λ."

The relaxation length, or attenuation length, of particles in a medium is the distance in which the particle flux is attenuated by a factor of e. This quantity is also commonly denoted by the symbol "λ." The equation

$$\Phi(x) = \Phi(o) \exp(-x/\lambda) \qquad 2.5$$

may be regarded as a definition of the relaxation length. It is clear that even if the initial flux is monoenergetic there will be a changing spectrum and, therefore, λ will vary with distance, although it usually approaches an asymptotic limit.

The difference between the mean-free-path and the relaxation length is that the former constant, mean-free-path, utilized in equation 2.5 will yield an uncollided flux, whereas the latter constant, relaxation length, will yield the total flux if the conditions are proper. It may be noted that the reciprocal of the macroscopic removal cross section is very nearly the same as the measured relaxation length for many materials.

A concept useful in treating gamma-ray attenuation is that of build-up. The assumption of exponential attenuation of gamma rays according to the total cross section results in a flux of gamma rays that have not had a collision, thereby constituting the uncollided flux. Since the total cross section contains scattering contributions, the extent of which depend on the medium and the gamma-ray energy, there will certainly be some scattered gamma rays present. In fact, for the broad-beam attenuation problem presented by a reactor shield the scattered radiation may exceed the uncollided radiation by an order of magnitude. To facilitate such calculations a build-up factor that is essentially the factor[16] which converts the uncollided flux to the correct total flux, has been defined. Actually there are also such factors appropriate for numbers of particles, doses and energy. Values may be found from results of both measurements[17,18] and calculations.[19]

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[16] Goldstein, H., "The Attenuation of Gamma-rays and Neutrons in Reactor Shields," U. S. Govt. Printing Office, (May, 1957), p. 137.

[17] Dixon, W. R., "Buildup Factors For the Transmission of Co^{60} Gamma Rays through Concrete and Lead," Phys. Rev. 85 498 (1952).

[18] Garrett, C., and G. N. White, "Buildup Measurements on Co^{60} Gamma Radiation in Iron and Lead," Phys. Rev. 95, 889 (1954).

[19] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma Rays," NYO-3075, U. S. Govt. Printing Office (June 30, 1954).

Since monochromatic sources are available at but few of the desired energy levels, computations must serve as the best guide, at least for the energies of gamma rays pertinent to reactor shield design. Many attempts have been made to describe the build-up factor by simple and convenient explicit analytical functions. These will be mentioned in detail in the section dealing with gamma-ray attenuation.

Gamma-ray shield thicknesses are often given graphically or in tabular form in terms of half-value layers (HVL), or tenth-value layers. These of course, are shield thicknesses that reduce the intensity of the radiation to one-half or one-tenth its initial value, respectively. Many of these tabulations, particularly those five or so years old, do not have build-up factors included, so that this must be allowed for when using attenuation information of this type. Such a table or graph is a convenient way of estimating required shield thicknesses, which may then be calculated in detail later if deemed necessary.

Dose units and the problems associated with getting them clearly in mind may be given short treatment in most shielding work. The subject cannot be completely ignored, however, since the shield thickness is determined quite often by the requirements for human occupancy. A brief discussion will give the necessary information without getting involved in the various units and the associated definitions. For the reader who would like to study these definitions in more detail there are references available.[20,21,22,23,24,25,26]

Radiation that is externally incident on people is usually limited to 7.5 mr/hr based on a forty-hour week. To convert dose quantities of neutrons or gamma rays to flux units, there are several references[27,28] in which curves and tables of conversion factors may be found. Actually these curves could be used properly without even knowing the definitions of the units. Among the uncertainties in setting dose limits is the degree of

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[20] Morgan, K. Z., "Maximum Permissible Internal Dose of Radionuclides: Recent Changes in Values," Nuclear Science and Engineering, 1, (No. 6), 477, (December 1956).

[21] "Permissible Dose from External Sources of Ionizing Radiation," NBS-Handbook 59. National Bureau of Standards, Washington, (October 24, 1954).

[22] Rossi, H. H., "Maximum Permissible Radiation Levels for High Energy Installations," Conference on Shielding of High Energy Accelerators, April 11-13, TID-7545, (December 6, 1957), p. 148.

[23] "Maximum Permissible Amounts of Radioisotopes in the Human Body and Maximum Permissible Concentrations in Air and Water." NBS Handbook 52. National Bureau of Standards, (March 20, 1958).

[24] "Protection Against Neutron Radiation up to 30 Million Electron Volts," NBS Handbook 63, National Bureau of Standards, (November 22, 1957).

[25] Report of the International Commission on Radiological Units and Measurements (ICRU), NBS Handbook 62, National Bureau of Standards, (April 10, 1957).

[26] Failla, G., "Basic Concepts of Radiation Protection," Nuclear Engineering, C. F. Bonilla, Ed., McGraw Hill, (1957).

[27] Novak, J., "Radiation Safety Guide," AN L-5574, (June, 1956), p. 17.

[28] "Reactor Physics Constants," ANL-5800, (September, 1958), pp. 466 and 474.

linearity between the irradiation time and the total dose. While a man is permitted by the usual limits to absorb about 300 mr/week, or 600 mr/week if there is no further dose during the succeeding week, it is not clear when the lack of recoverability becomes a factor. Then, too, the age of the individual imposes requirements as to total annual dose. At Argonne National Laboratory the total accumulated dose for an individual is given by the expression

Dose = 5 (age-18) roentgen.

This dose is not to exceed 15 r in any year. Since even medical men closely associated with the field of atomic energy cannot give firm answers to these and related problems, and since the values are subject to a continuing review, the degree to which the shielding designer gets involved, beyond the brief outline given and the tables of conversion factors in the neutron and the gamma-ray sections (Table 4.1 and Table 5.1), is perhaps one more of personal interest rather than a technical requirement. It is worthy of note to the shield designer that the occupancy of a location by humans may not be the most stringent radiation limitation. Instruments often set the maximum permissible radiation levels.

Internal dose limits are tabulated[29,30,31] also and need only be checked as to value. These limits, for instance, are usually given in microcuries per liter of air or water ingested. One can readily figure out the activity of air or water and decide whether dilution is necessary on the basis of the values tabulated. Again, a knowledge of the reasons for setting the various levels is perhaps more of personal interest rather than technical necessity for the shield designer.

In the process of shield design one also encounters the thermal shield. This portion of the shield is inserted to absorb radiation and convert it to heat in a location from which it may be easily removed and/or in which it will cause no excessive stresses. The material of which the thermal shield is constructed depends upon the most prominent radiation to be shielded, the location in which it must perform, and the money available for the purpose. Thermal shields are commonly located so that they may reduce radiation heating in the pressure vessels and/or the concrete portions of the shield.

Biological shield is a term applied to a shield that reduces radiation to a biologically safe level. Actually, all portions of the reactor outside of the core serve this function, but the name usually refers to

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[29] Morgan, K. Z., "Maximum Permissible Internal Dose of Radio-nuclides: Recent Changes in Values," Nuclear Science and Engineering, 1, (No. 6), 477, (December, 1956).

[30] "Maximum Permissible Amounts of Radioisotopes in the Human Body and Maximum Permissible Concentration in Air and Water," NBS Handbook 52, National Bureau of Standards, (March 20, 1958).

[31] Novak, J., "Radiation Safety Guide," ANL-5574, (June, 1956).

that portion of the shield which serves no other essential function. A unit shield, which, as the name implies, consists of a single mass, may serves as the support for considerable equipment that otherwise would require separate support structure. Similarly, by incorporating the heat exchangers and other components containing radioactive materials within the biological shield to form a compartment type shield[32] a considerable reduction in total shield material and space may be effected. Under certain circumstances the operating location may require better shielding than other areas around the reactor. If, then, part of the shield is located around the operators, a divided shield might be economical. Under similar circumstances the operating crew may work in the radiation shadow created by shielding the reactor more on one side than another, that is, employing a shadow shield.[33,34]

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[32] Hungerford, H. E., and R. F. Mantey, "Shielding the Enrico Fermi Fast Breeder Reactor," Nucleonics 16 120 (November, 1958).

[33] Nowstrup, E. I., L. A. Beach, and W. R. Faust, "Shadow Shields," NRL-4275, (November 16, 1953).

[34] Horton, C. C., et al., "Some Observations on Gamma-Ray Scattering Round a Shadow Shield in Water," AERE-RP/M67, Harwell, Berks, (November 29, 1955).

3. MATHEMATICAL FUNCTIONS AND FORMULAS

The radiations associated with a reactor may be treated by different theories of attenuation. Neutrons are quite commonly assumed to follow ordinary diffusion theory. This is appropriate for neutrons of lower energies, and, with the proper constants, high-energy neutrons may be treated similarly, i.e., by two-group diffusion theory. Fast neutrons, as well as gamma rays, however, are more properly considered to travel in a straight path, or ray, until a collision occurs. Thus, at least the uncollided fast neutron or gamma-ray flux may be treated by what will be referred to as Ray Theory. With proper adjustments this will represent the attenuation of both fast neutrons and gamma rays. Ray Theory leads to several mathematical functions which, while not peculiar to shielding, or to nuclear engineering for that matter, are not as widely known as, for example, the trigonometric functions. It is considered of value to include a brief discussion of these functions, as well as the somewhat confused nomenclature. In addition, formulas for particle flux are derived for the basic geometries.

The first of the mathematical functions to be considered is the exponential integral, or E function:

$$E_n(x) = \int_1^\infty du\, e^{-xu}\, u^{-n} = x^{n-1} \int_x^\infty du\, e^{-u}\, u^{-n} = \int_0^1 d\mu\, e^{-x/\mu}\, \mu^{n-2}, \quad 3.1$$

where

$$E_0(x) = \frac{e^{-x}}{x}$$

This function is described thoroughly in the Canadian reports MT-1 and MT-131, [1,2] and the National Bureau of Standards compilations.[3] Note that the E function described in the Reactor Handbook[4] and Glasstone[5] is not quite the same. The definition there employed is

$$E_n(x) = \int_x^\infty du\, e^{-u}\, u^{-n}, \quad 3.2$$

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[1] Placzek, G., "The Functions $E_n(x) = \int_1^\infty du\, e^{-xu}\, u^{-n}$," National Research Council of Canada #1547, MT-1.

[2] Lecaine, J., "A Table of Integrals Involving the Functions $E_n(x)$," National Research Council of Canada #1553, MT-131.

[3] Placzek, G., "The Functions $E_n(x) = \int_1^\infty du\, e^{-xu}\, u^{-n}$," Tables of Functions and Zeros of Functions, U. S. Dept. of Commerce, National Bureau of Standards, MT-37, (November, 1954).

[4] Hagerton, J. F., and R. C. Grass, The Reactor Handbook, Vol. I, Physics, AECD-3645, McGraw-Hill Book Co., (1955).

[5] Glasstone, S., Principles of Nuclear Reactor Engineering, D. VanNostrand, New York, (1955).

which differs from equation 3.1 by the factor x^{n-1}. The E function is also known as the F function, $F_n(x)$, in some work in the United States, and also as the first-order function, $E_1(x)$, as the exponential integral $-Ei(-x)$ of Jahnke-Emde.[6] The following identities explain the relationships between these functions,

$$E_n(x) \equiv F_{n-1}(x)$$

$$E_1(x) \equiv -Ei(-x) \quad .$$

A useful formula for calculating the E functions is

$$E_n(x) = \frac{e^{-x}}{x + n - 1 + f_{n-1}(x)} \quad , \qquad 3.3$$

where $f_{n-1}(x)$ is given in graphical form in Figure 3.1, taken from the old Project Handbook.[7] Rather complete tables of $E_1(x)$, or $-Ei(-x)$, are given in tables[8,9] of exponential integrals. Curves of the same function are given in the Reactor Shielding Design Manual.[10] (See also Figure 3.2)

The first-order function may be represented by the infinite series,

$$E_1(x) = -\gamma - \ln x + T_1(x) - T_2(x) + \cdots \quad , \qquad 3.4$$

where

$$T_n(x) = \frac{x^n}{n!n} \quad ,$$

and γ is Eulers' constant, 0.577216. Useful asymptotic expansions are:

$$E_n(x) = \frac{e^{-x}}{x}\left[1 - \frac{n}{x} + \frac{n(n+1)}{x^2} - \frac{n(n+1)(n+2)}{x^3} + \cdots\right] , \; n > 0, \qquad 3.5$$

and

$$E_n(x) = \frac{e^{-x}}{x+n}\left[1 + \frac{n}{(x+n)^2} + \frac{n(n-2x)}{(x+n)^4} + \frac{n(6x^2 - 8nx + n^2)}{(x+n)^6} + \cdots\right] \quad . \qquad 3.6$$

The second expansion is more accurate and convenient.

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[6] Jahnke, E., and F. Emde, Tables of Functions, Dover Publications, (New York, 1945).

[7] "Project Handbook," CL-697, Chapter V.

[8] Tables of Sine, Cosine, and Exponential Integrals, Vol. I and II, U. S. Department of Commerce, National Bureau of Standards, MT-5 and MT-6, (1940).

[9] Placzek, G., "The Functions $E_n(x)$ $\int_1^\infty du\, e^{-xu}\, u^{-n}$," Tables of Functions and Zeros of Functions, U. S. Dept. of Commerce, National Bureau of Standards, MT-37, (November, 1954).

[10] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill & D. Van Nostrand, (March, 1956).

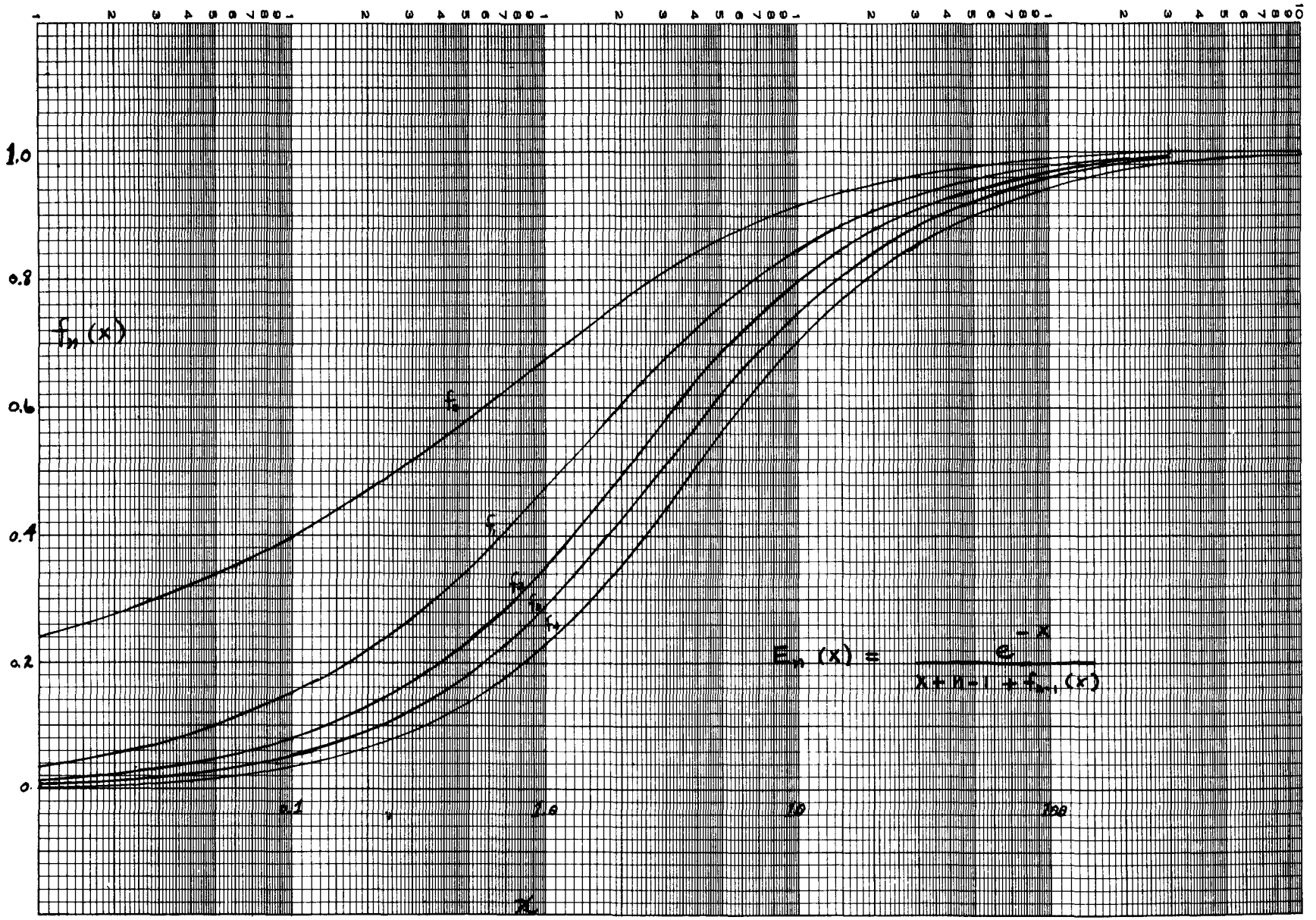


Figure 3.1. The Function $f_n(x)$

Figure 3.2

The Function $E_n(b)$

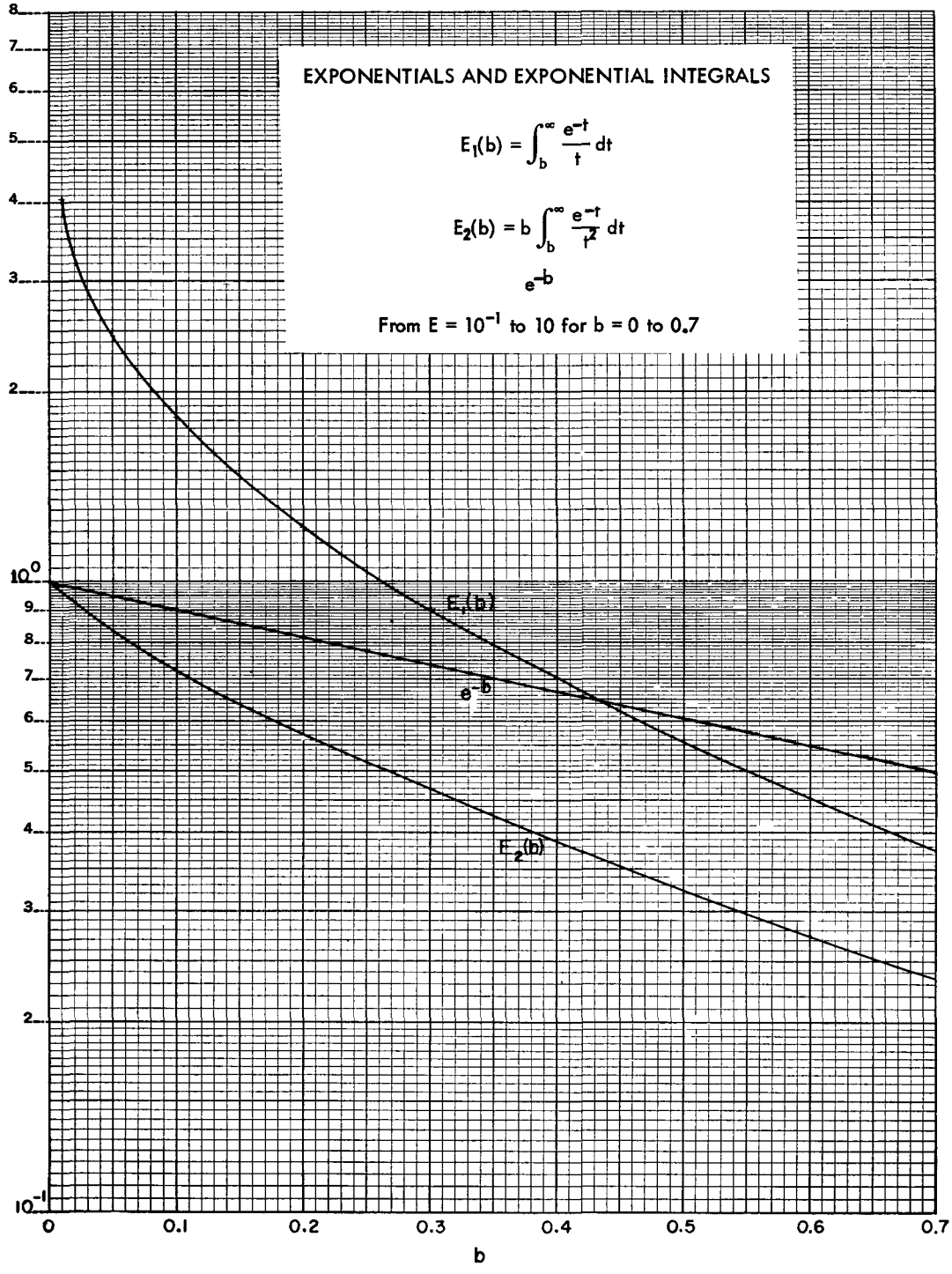


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

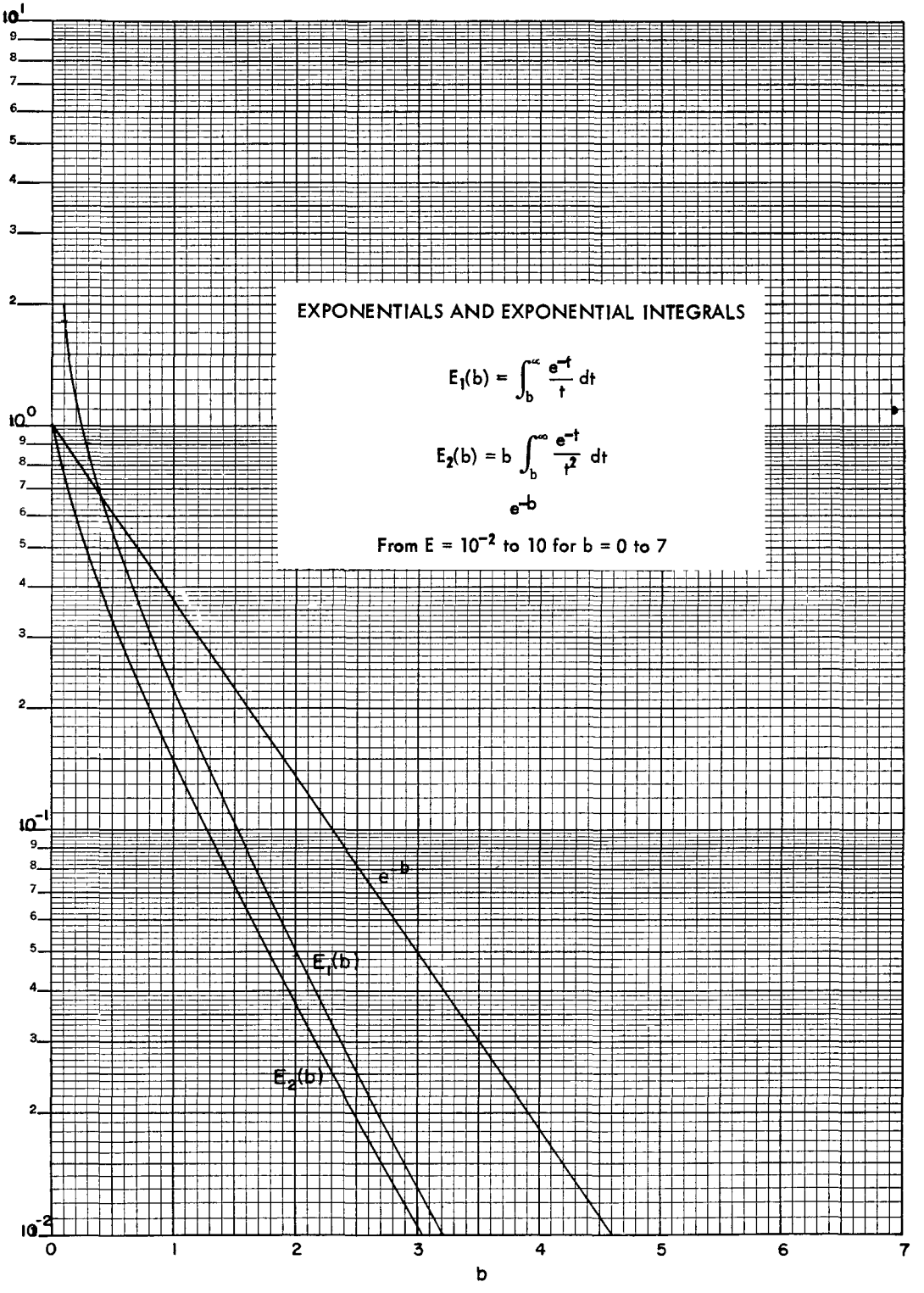


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

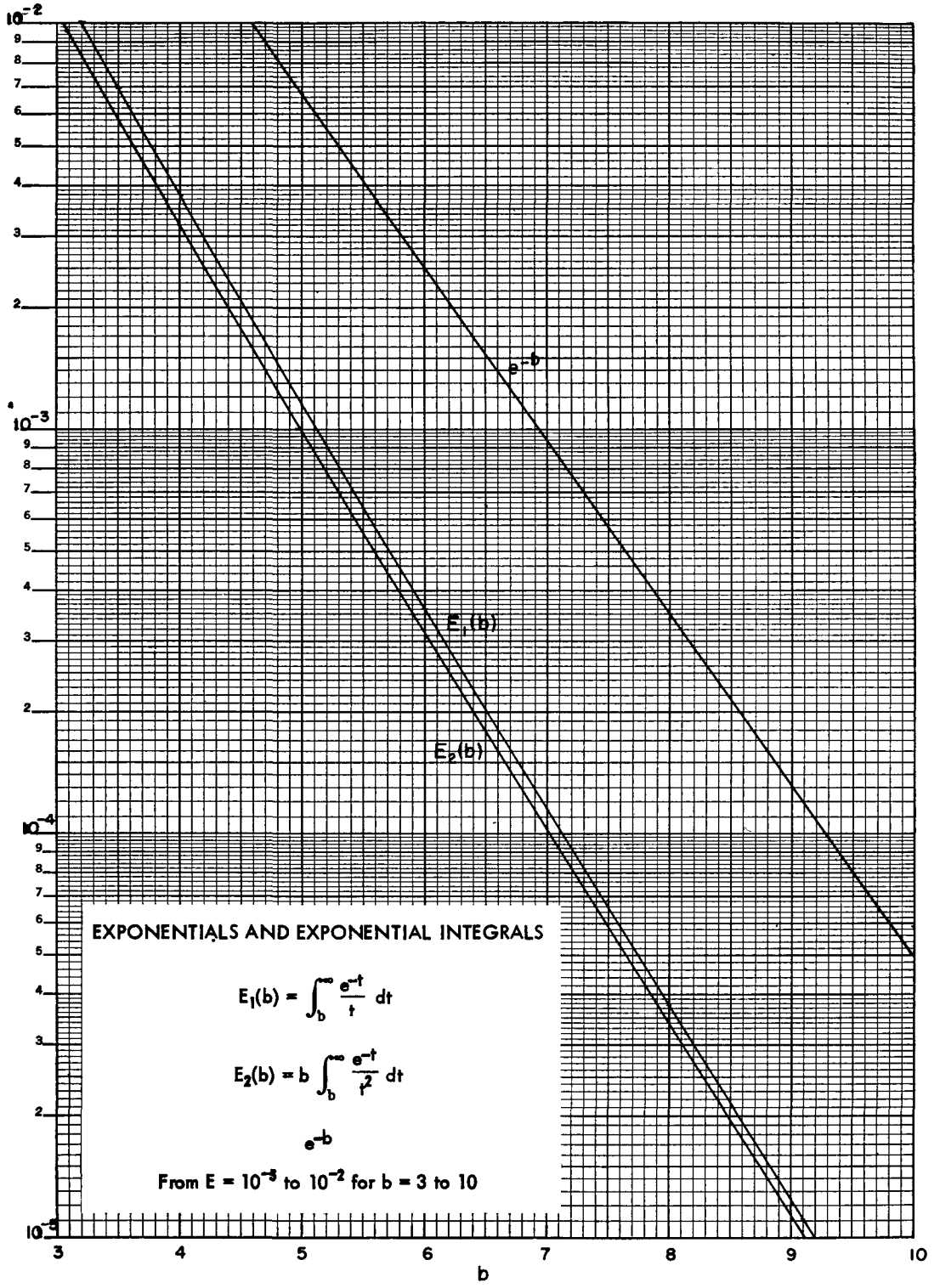


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

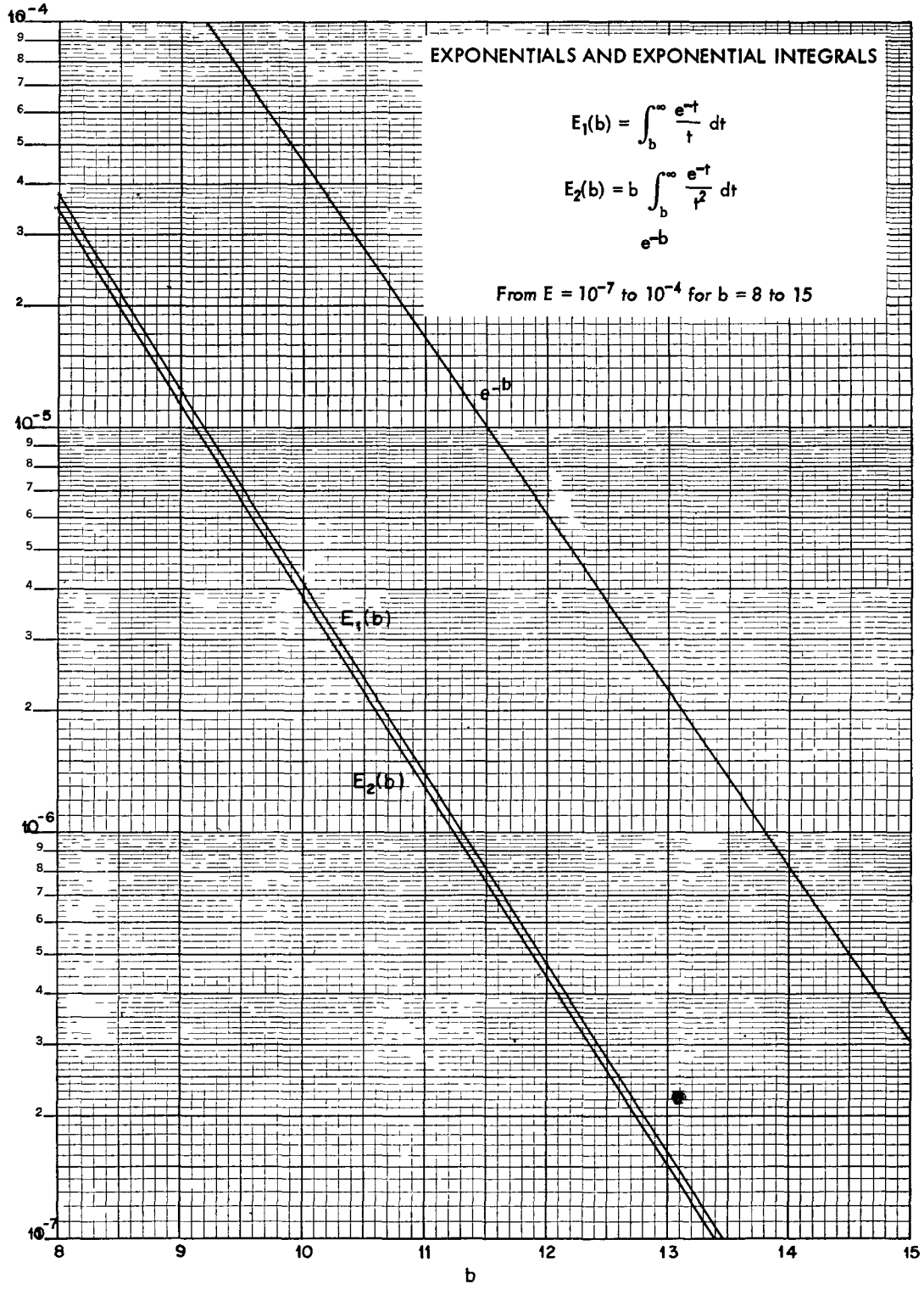


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

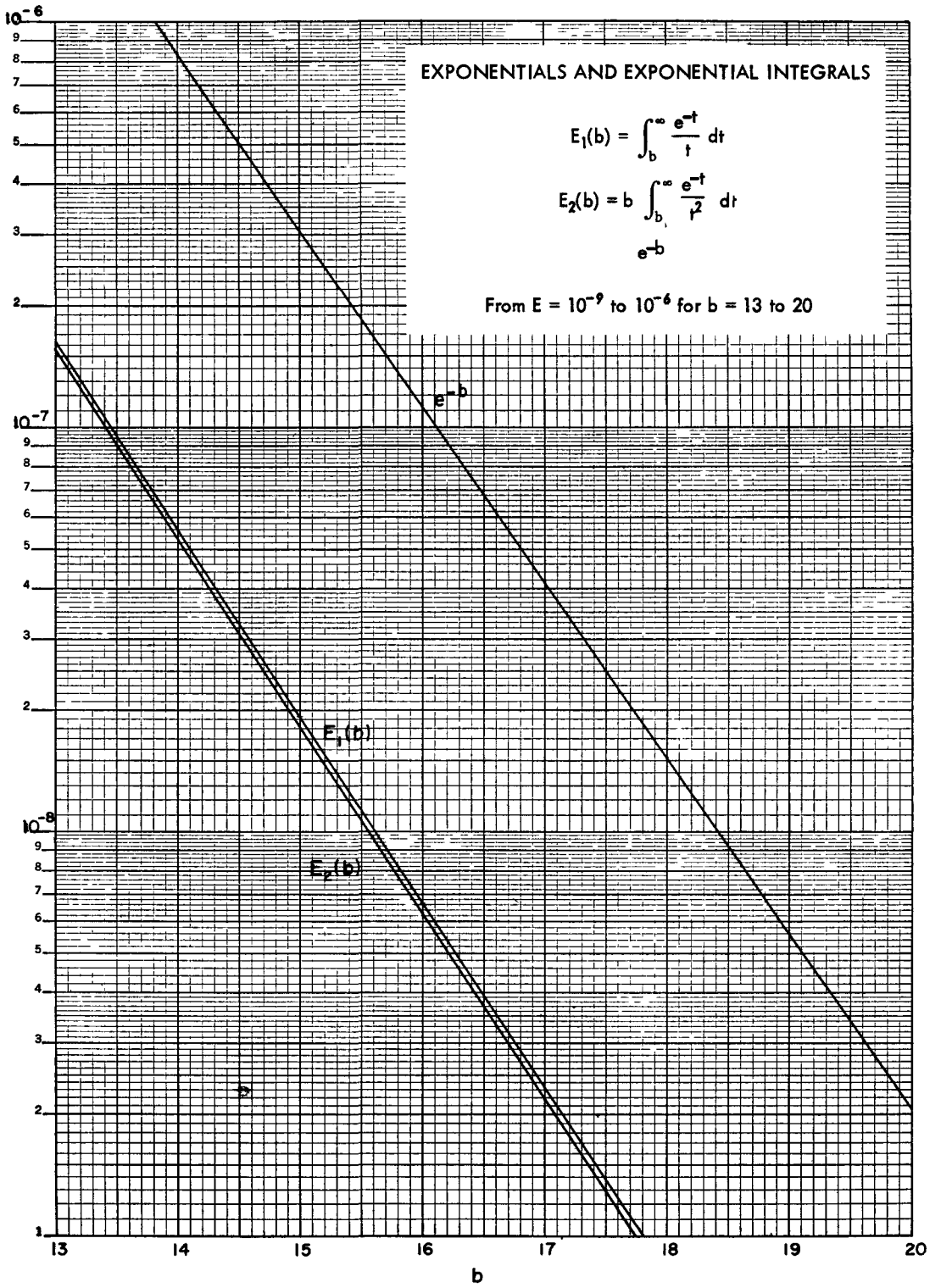


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

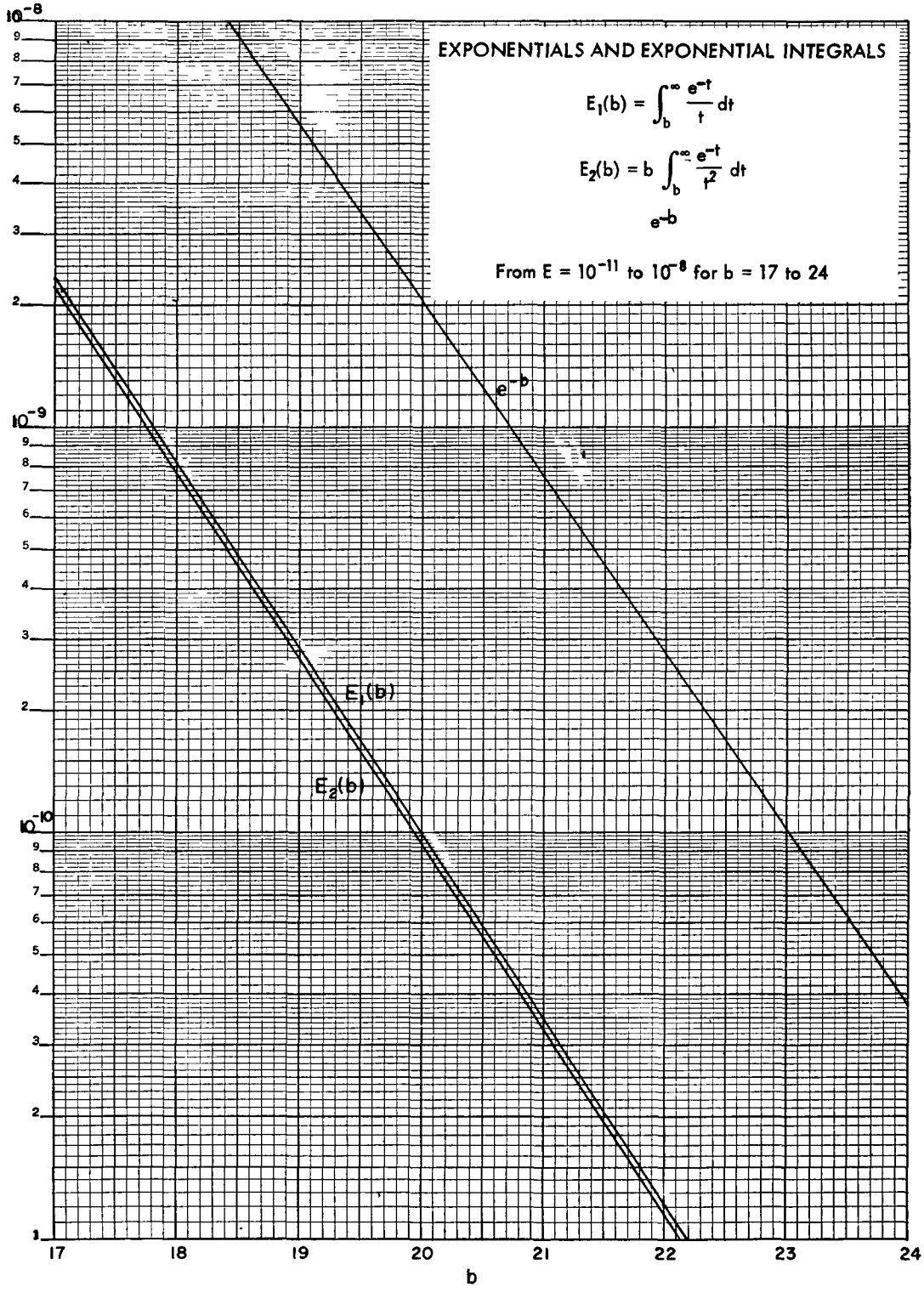


Figure 3.2 (Cont'd.)

The Function $E_n(b)$

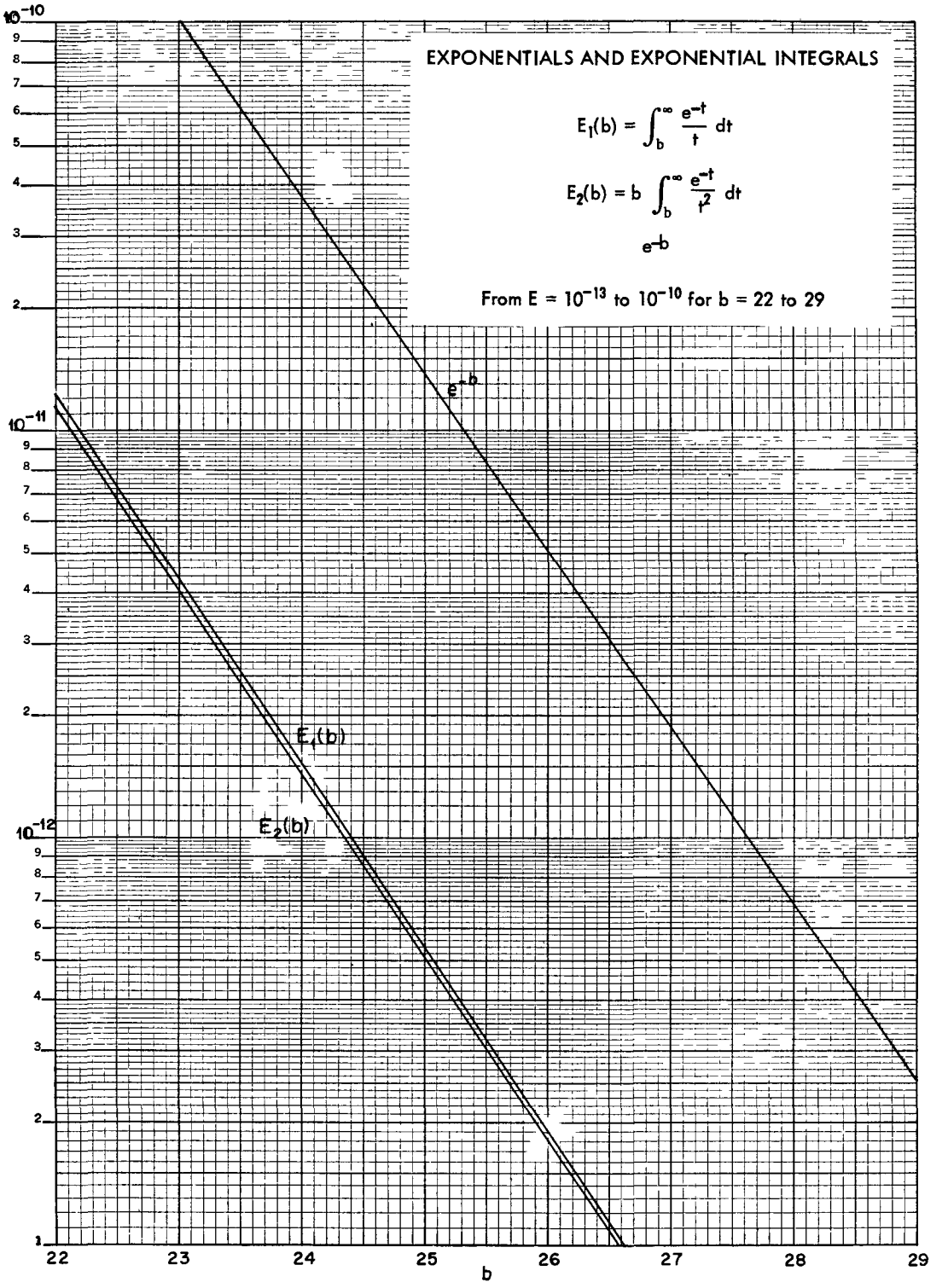
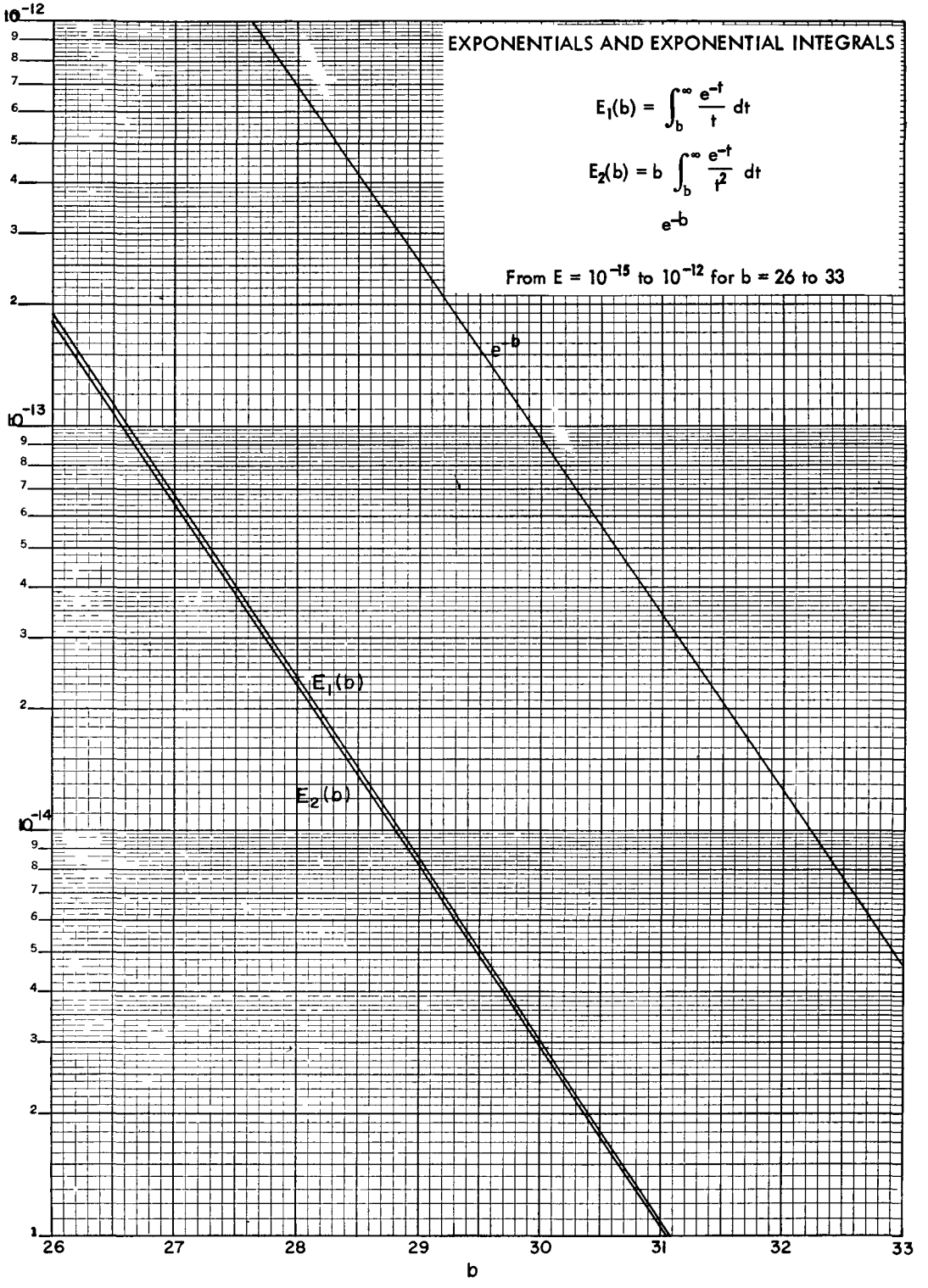


Figure 3.2 (Cont'd.)

The Function $E_n(b)$



Recurrence relations are:

$$E_n(x) = \int_x^\infty E_{n-1}(x')dx' \quad , \tag{3.7}$$

$$E_n'(x) = -E_{n-1}(x) \quad , \tag{3.8}$$

and

$$E_n(x) = \frac{1}{n-1}\left\{e^{-x} - x\,E_{n-1}(x)\right\} \quad (n>1). \tag{3.9}$$

The following expansion is equivalent to the definition, equation 3.1:

$$E_n(x) = \sum_{\substack{m=0 \\ m\neq n-1}} \frac{(-x)^m}{(n-1-m)m!} + \frac{(-1)^n x^{n-1}}{(n-1)!}\left\{\gamma + \log x - A_n\right\} \quad (n>0) \tag{3.10}$$

where

$$\gamma = 0.577216;\ A_1 = 0;\ A_n = \sum_{m=1}^{n-1} \frac{1}{m} \quad (n>1) \quad .$$

A closely related integral:

$$Ei(x) = \int_{-\infty}^{x} du\, \frac{e^{+u}}{u} \quad , \tag{3.11}$$

is described in Jahnke-Emde[11] and is tabulated in the MT tables.[12] Graphs of this function are also given in the Reactor Shielding Manual,[13] (see also Figure 3.3), under the nomenclature of $E_1(-x)$. This function may be represented by the infinite series

$$Ei(x) = \gamma + \ln x + T_1(x) + T_2(x) + \text{---} \quad , \tag{3.12}$$

or by the asymptotic series

$$Ei(x) = \frac{e^x}{x}\left[1 + \frac{1}{x} + \frac{2!}{x^2} + \frac{3!}{x^3} + \text{---}\right] \quad . \tag{3.13}$$

Some other functions that are often useful are the Ki functions.

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[11] Jahnke, E., and F. Emde, Tables of Functions, Dover Publications, (New York, 1945).

[12] Tables of Sine, Cosine, and Exponential Integrals, Vol. I and II, MT-5 and MT-6, Government Printing Office, Washington 25, D.C., (1940).

[13] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill & D. Van Nostrand, (March, 1956).

Figure 3.3

The Function $E_1(-b)$

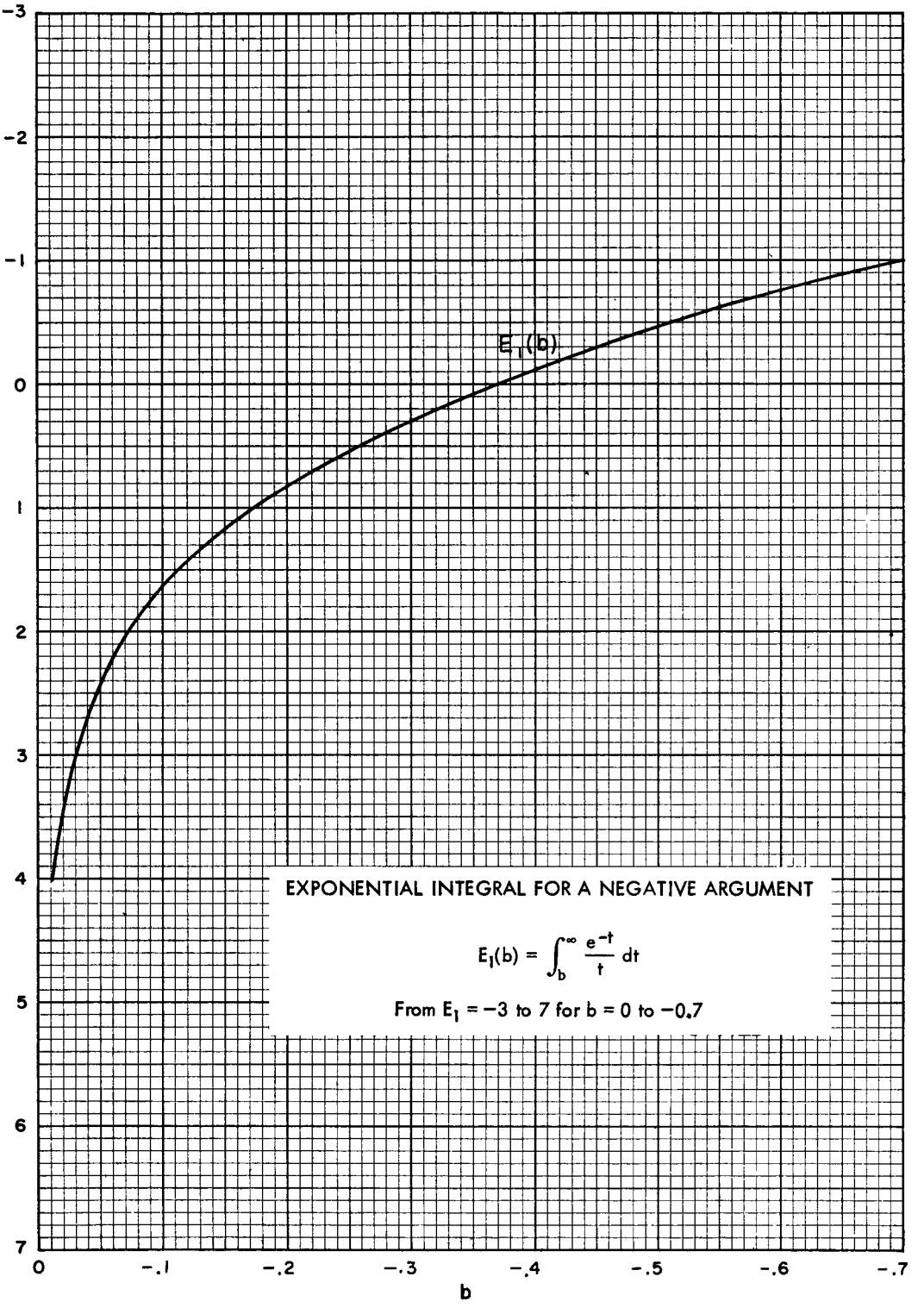


Figure 3.3 (Cont'd.)

The Function $E_1(-b)$

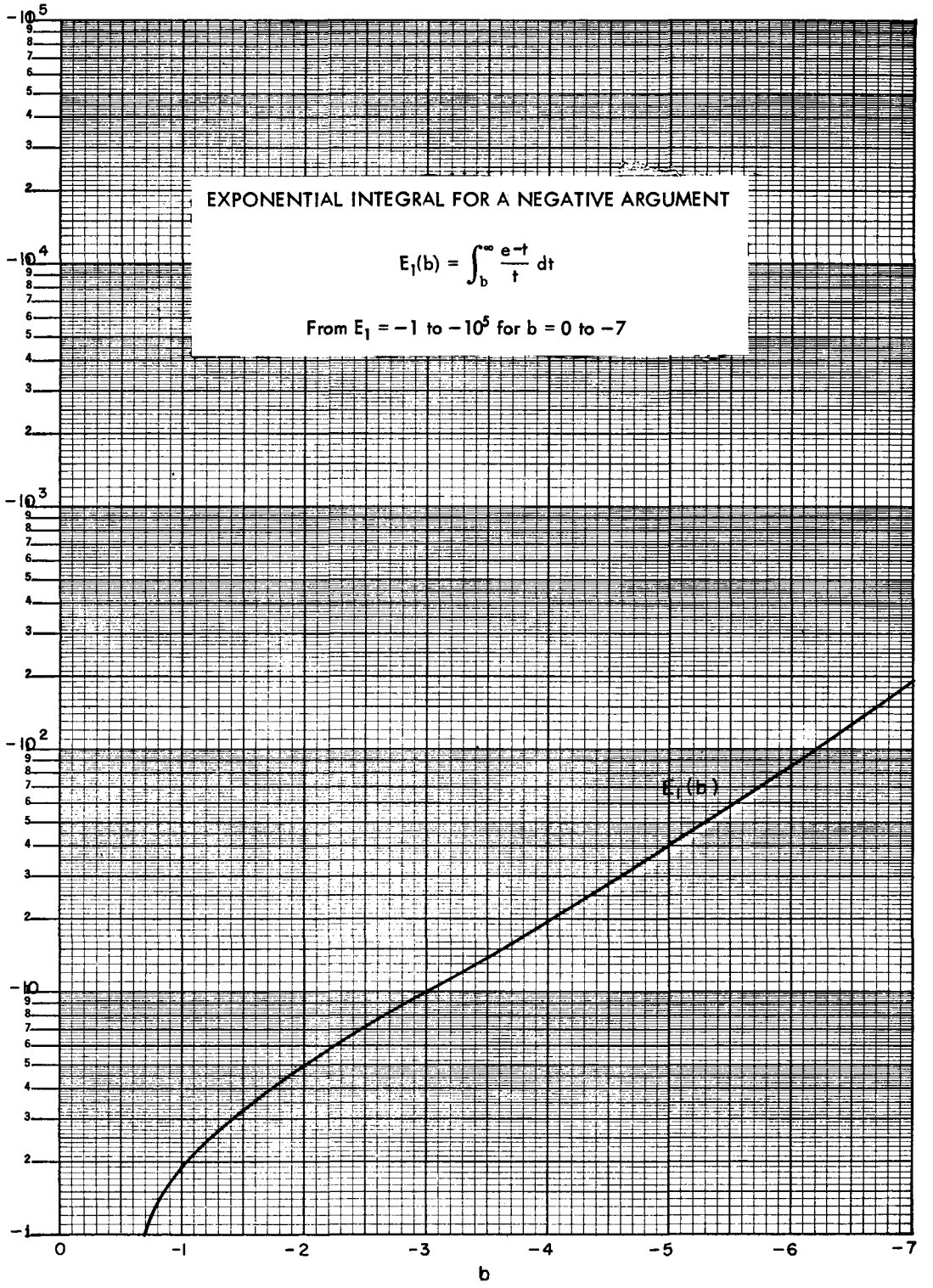


Figure 3 3 (Cont'd)

The Function $E_1(-b)$

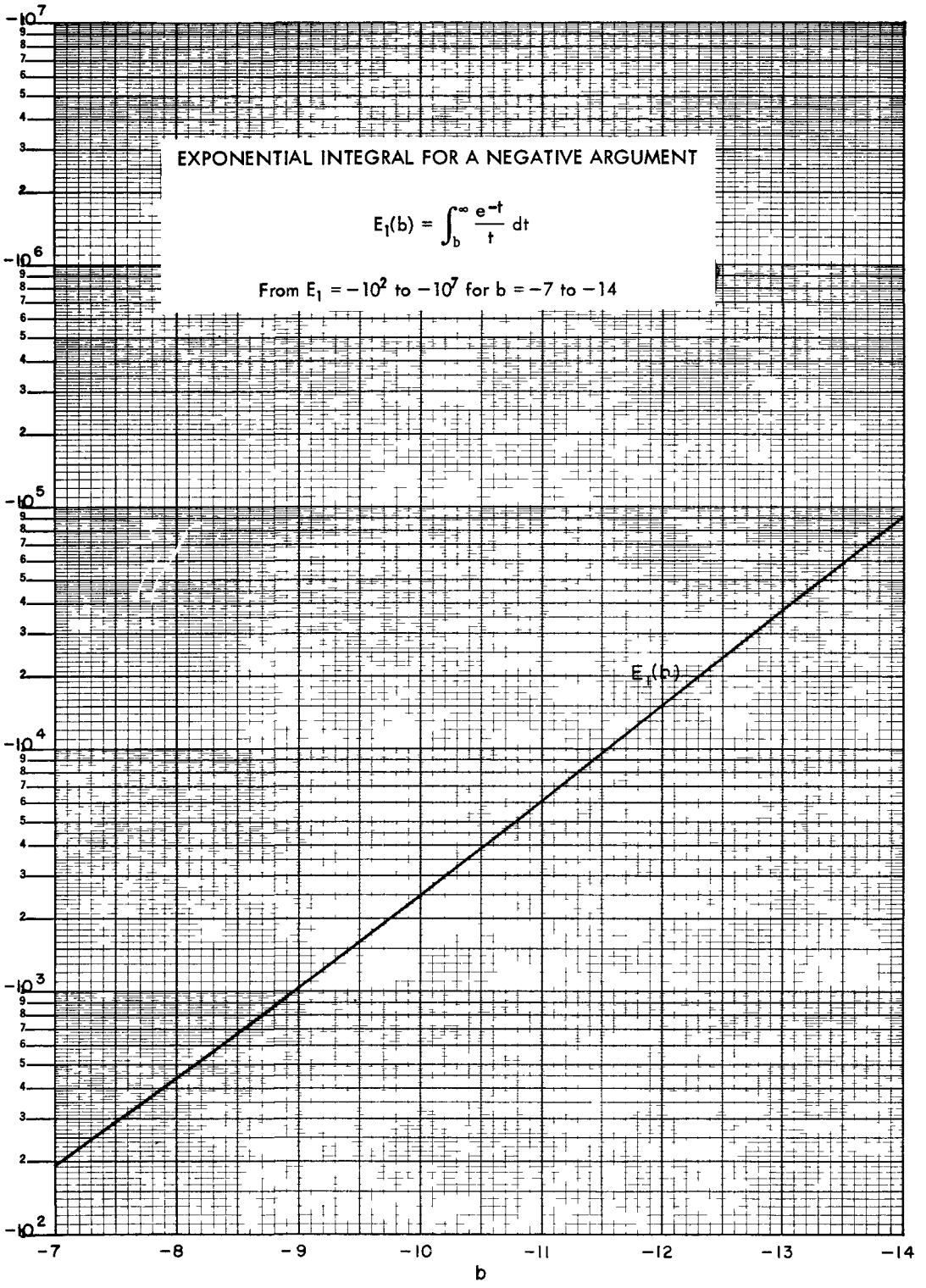


Figure 3.3 (Cont'd.)

The Function $E_1(-b)$

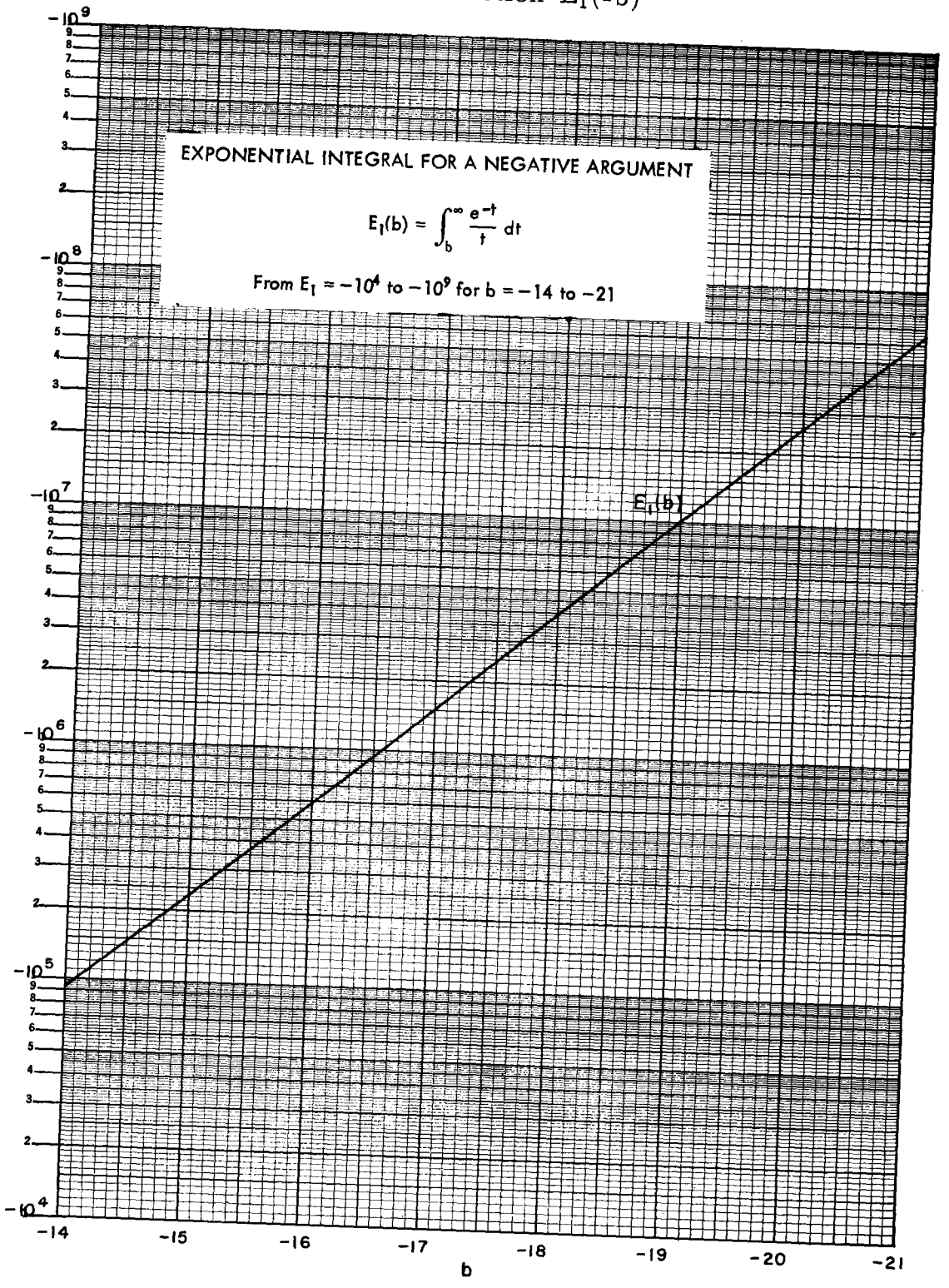
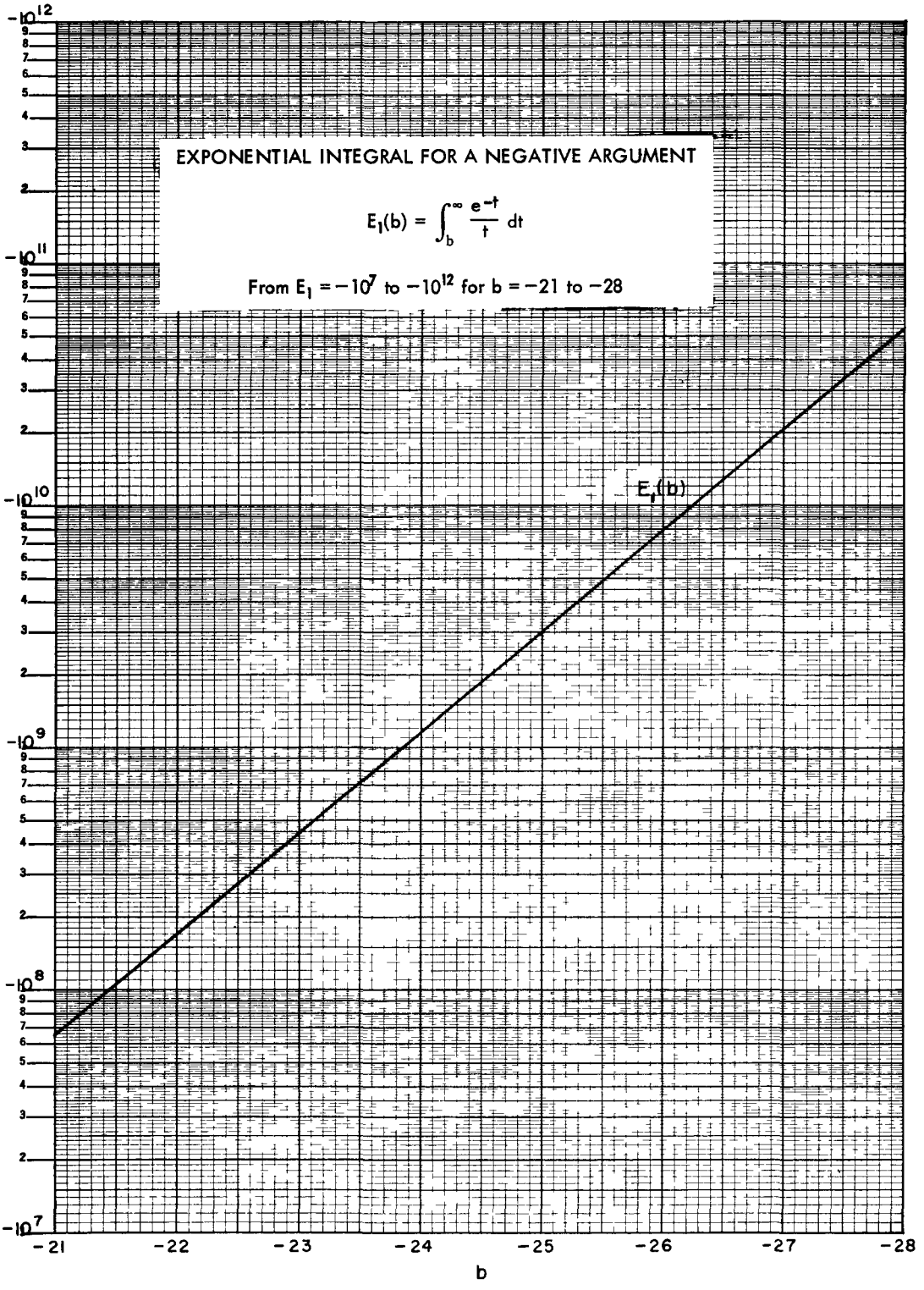


Figure 3 3 (Cont'd)

The Function $E_1(-b)$



They are defined by the integrals

$$Ki_n(x) = \int_0^\infty du \frac{\exp(-x \cosh u)}{\cosh^n u} \quad , \tag{3.14}$$

$$Ki_n(x) = \int_0^{\pi/2} d\theta \frac{\exp(-x \sec \theta)}{\sec^{n-1} \theta} \quad , \tag{3.15}$$

or

$$Ki_n(x) = \int_x^\infty du \; Ki_{n-1}(u) \quad , \; Ki_0(x) \equiv K_0(x) \quad .$$

The first-order function

$$Ki_1(x) = \int_0^{\pi/2} d\theta \; \exp(-x \sec \theta) \tag{3.16}$$

is also known as the secant integral, seci(x). The zero-order function, $Ki_0(x)$, is the Bessel function $K_0(x)$. These functions are discussed in detail by Bickley and Naylor.[14]

The secant integral may be made a bit more general:

$$seci(x,\theta) = \int_0^\theta d\phi \; \exp(-x \sec \phi) \quad . \tag{3.17}$$

Curves of the secant integral, equation 3.17, are available[15] (see also Figure 3.4). An expansion due to J. W. Butler is,

$$seci(x,\theta) = \theta \, e^{-x} \, A(x,\theta) \quad , \tag{3.18}$$

where,

$$A(x,\theta) = 1 + \frac{\theta^2}{6} + \left(5 - \frac{4x}{3}\right) \frac{\theta^4}{120} + \left[61 - 40x + \frac{(4x)^2}{3}\right] \frac{\theta^6}{5040} + \text{---} \quad .$$

For small values of θ a good approximation is,

$$seci(x,\theta) = \theta \, e^{-x} \quad , \qquad \theta > 5^\circ \quad , \tag{3.19}$$

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[14] Bickley, W. G., and J. Naylor, "A Short Table of the Functions $Ki_n(x)$, "Phil. Mag., 20, 343 (1935).

[15] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, p. 385, McGraw-Hill & D. Van Nostrand, (March, 1956).

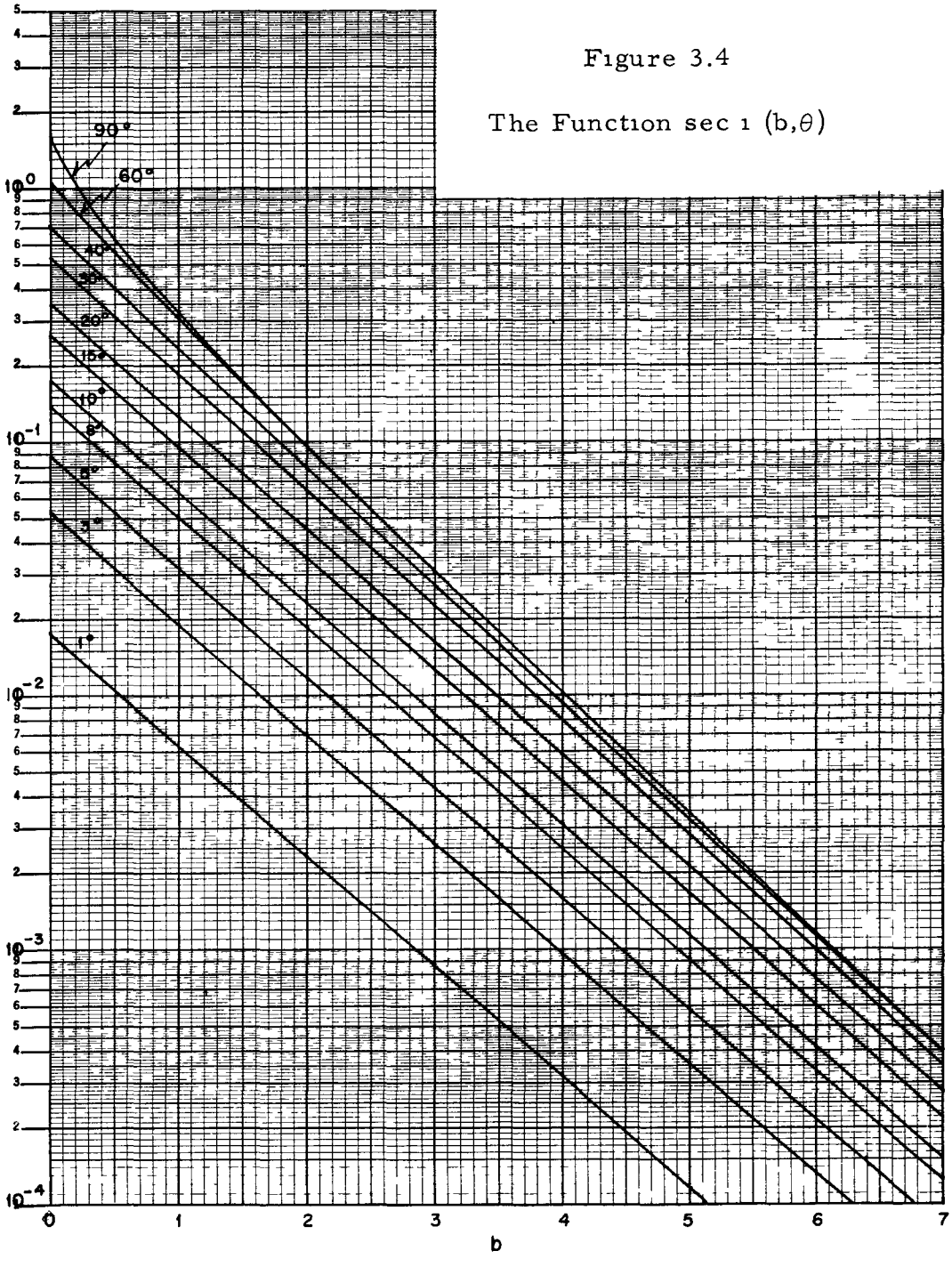


Figure 3.4

The Function sec 1 (b,θ)

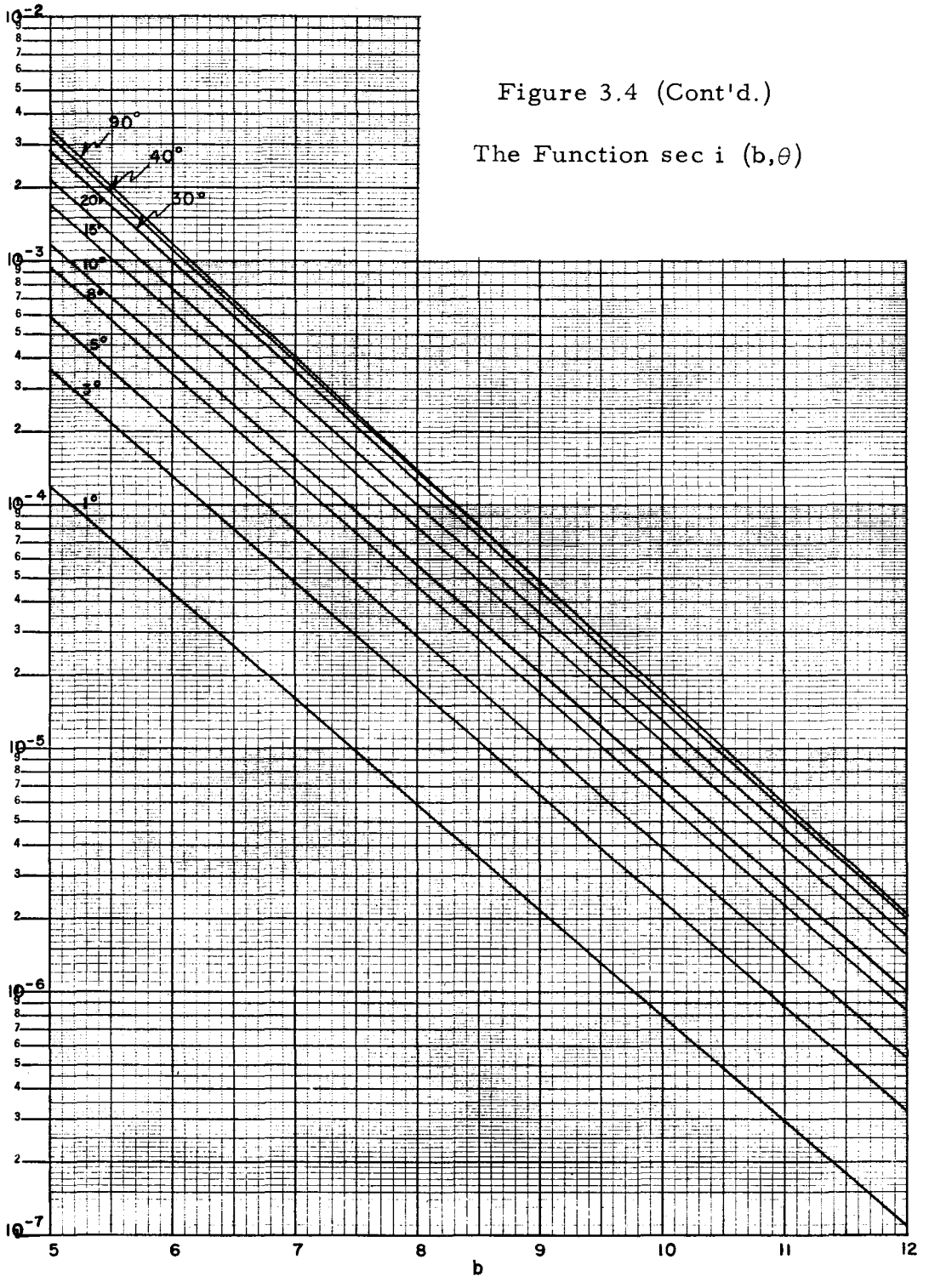


Figure 3.4 (Cont'd.)

The Function sec i (b,θ)

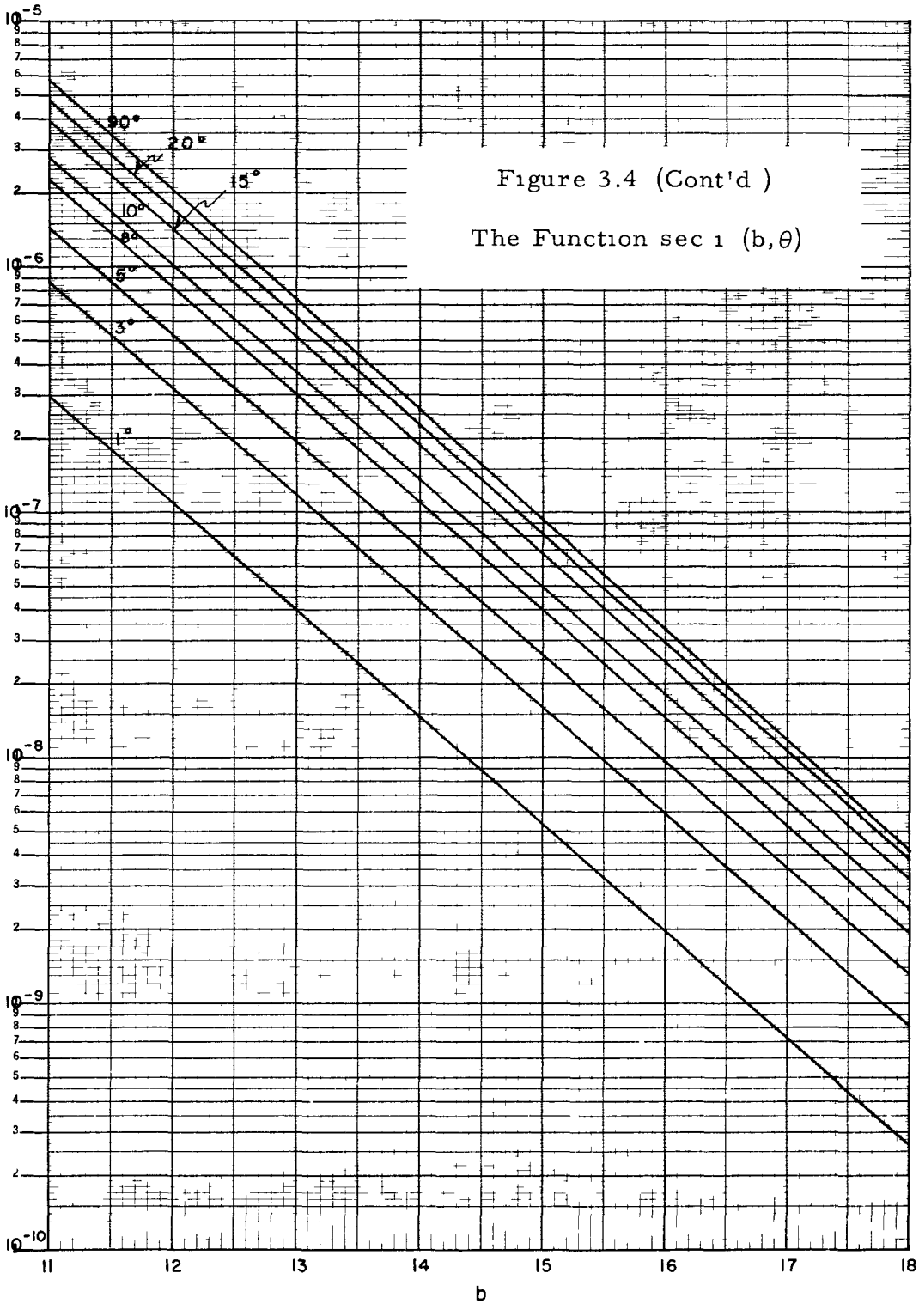


Figure 3.4 (Cont'd)

The Function sec ı (b, θ)

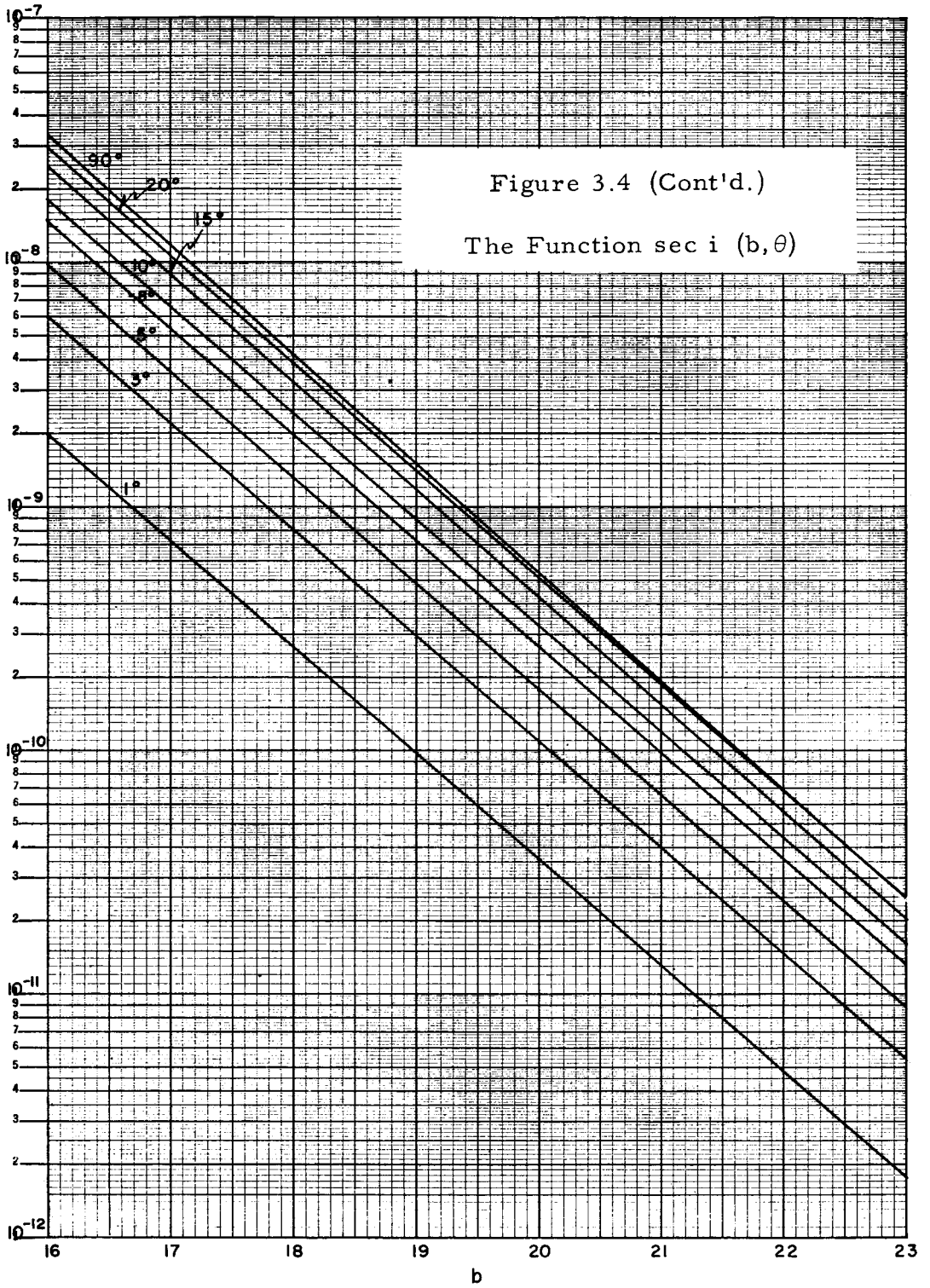


Figure 3.4 (Cont'd.)

The Function sec i (b, θ)

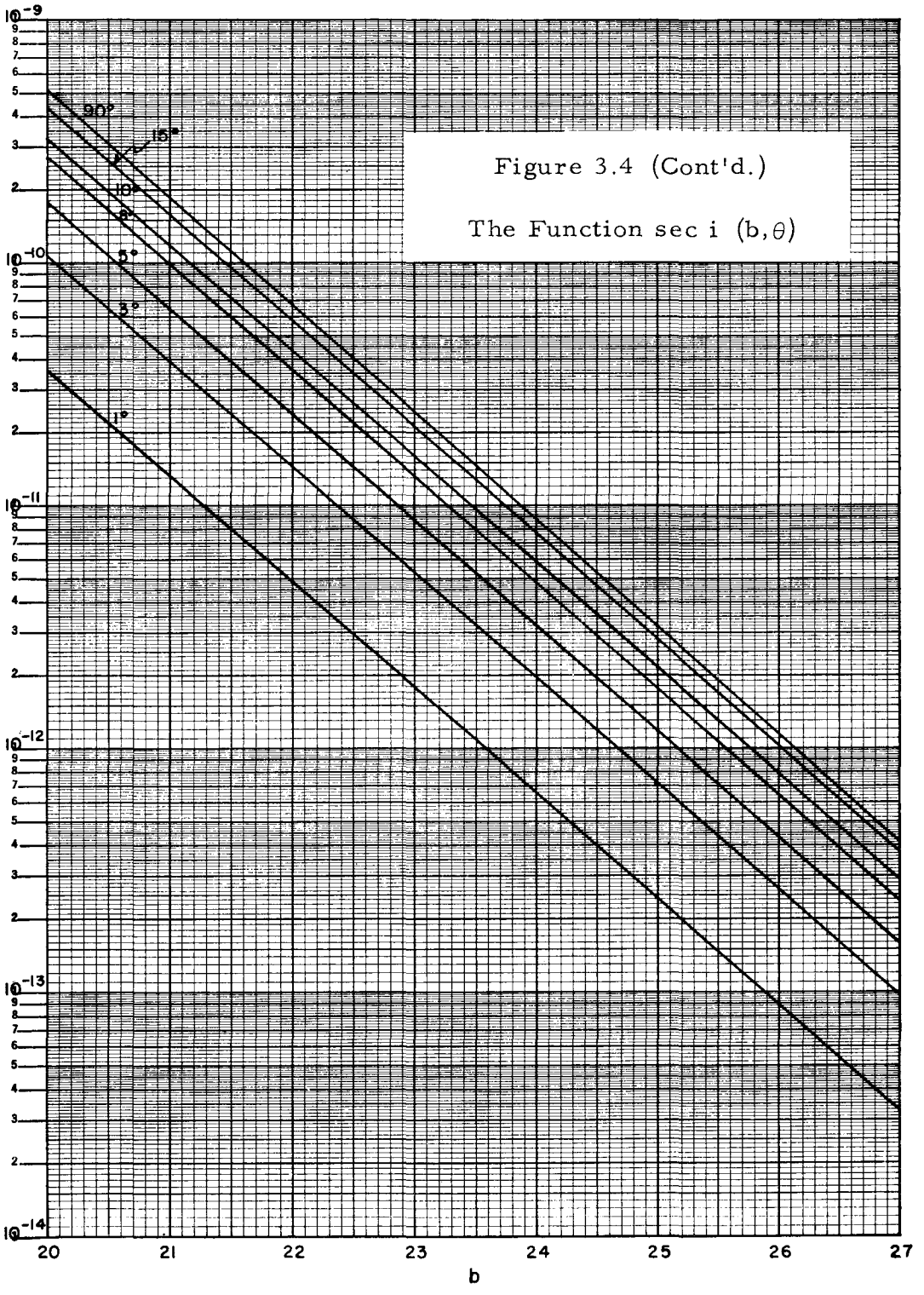


Figure 3.4 (Cont'd.)

The Function sec i (b,θ)

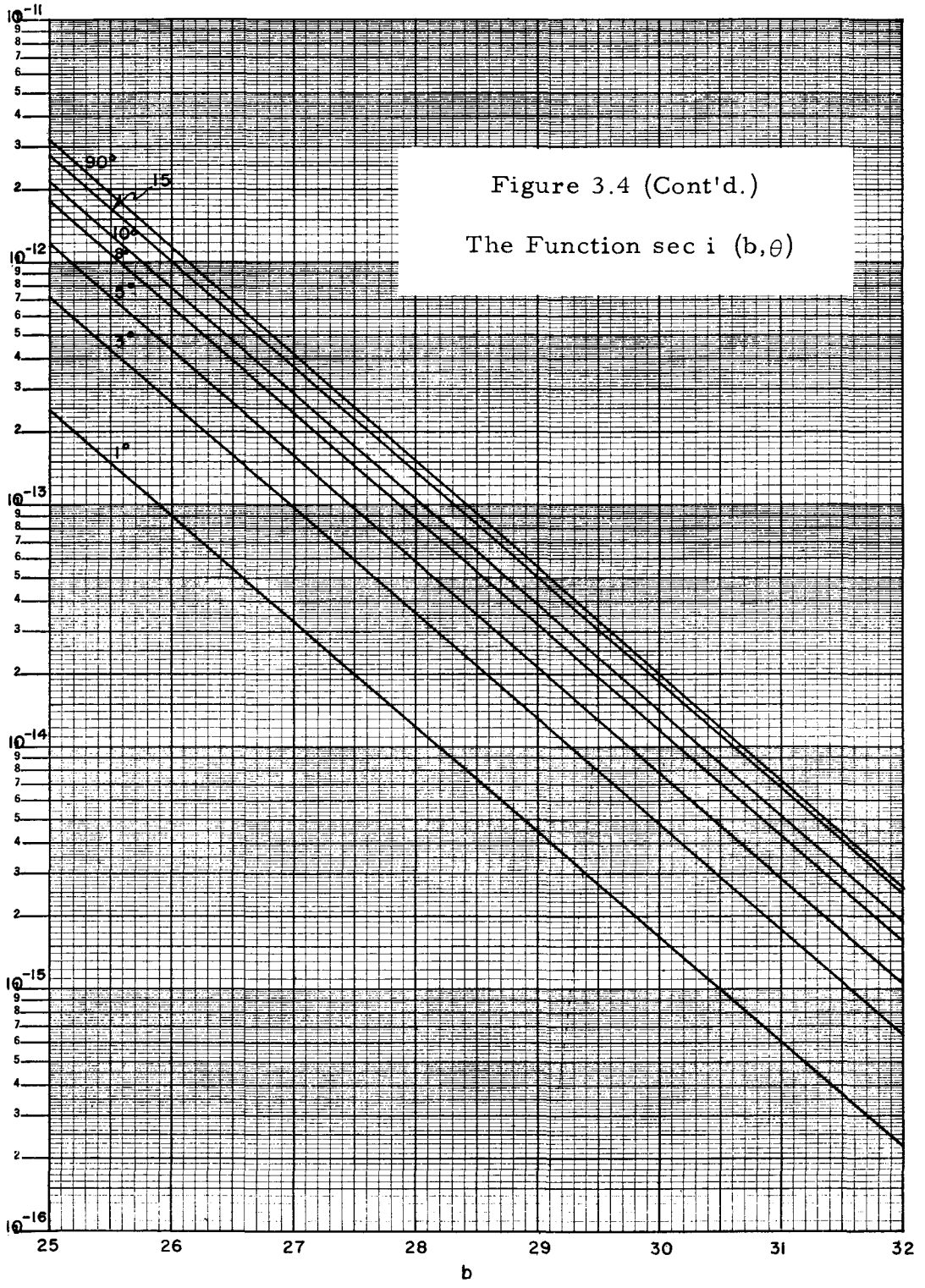


Figure 3.4 (Cont'd.)

The Function sec i (b,θ)

and for large values of x,

$$\text{seci}(x) = e^{-x}\left(\frac{\pi}{2x}\right)^{1/2}\left(1 - \frac{3}{8x}\right), \; x > 6 \quad . \tag{3.20}$$

In order to derive formulas for the radiation from various geometrical configurations the assumption of straight propagation, Ray Theory, is employed. This may be done by assuming that the radiation travels from source to detector along the optical path connecting them, the attenuation due to each material being apportioned according to the thickness of that material traversed by the optical path.

The attenuation of fast neutrons in a water component is calculated by making use of the measured attenuation for that thickness of water; the attenuation in the other components is taken into account by use of removal cross sections. Once the principle of straight-ray propagation has been accepted, the radiation from sources of complex shape surrounded by appropriate shield configurations can be calculated by integration of the contributions from different parts of the source. While this is simple in principle, very complicated integral expressions may result unless the geometry is simplified as much as possible. In a general situation, such as in Figure 3.5,

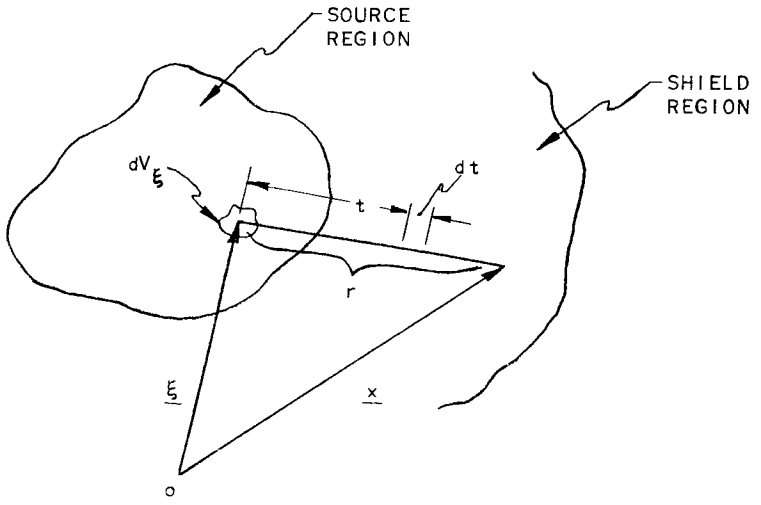


Figure 3.5

Illustration for Fast Neutron Flux Calculation

the fast neutron flux is given by

$$\Phi_f(\underline{x}) = \int_V dV\xi \; \frac{B(\underline{x},\underline{\xi})}{4\pi r^2} Q_3(\underline{\xi}) \quad \text{neut/cm}^2 \text{ sec} \quad , \tag{3.21}$$

where $Q_3(\underline{\xi})$ is the neutron source strength in neutrons/cm^3 sec and $B(\underline{x},\underline{\xi})$ is the beam-attenuation kernel for the optical path connecting the points $\underline{x}$ and $\underline{\xi}$. If the shield consists principally of water, this kernel is given by

$$B(\underline{x},\underline{\xi}) = N(\rho) \exp\left\{-\int_0^r dt\, \sigma_r(t)\right\} \quad , \tag{3.22}$$

where

$N(\rho)$ is the measured beam-attenuation kernel for water

ρ is the effective thickness of water, at unit density, between the points $\underline{x}$ and $\underline{\xi}$,

and

$\sigma_r(t)$ is the removal cross section of the other materials traversed by the ray.

A similar expression,

$$J_f(\underline{x}) = \int_S dV\xi \; \frac{(\underline{x}-\underline{\xi})}{r} \frac{B(x\,\xi)}{4\pi r^2} Q_3(\underline{\xi}) \quad \text{neut/cm}^2 \text{ sec}, \tag{3.23}$$

may be written for the current of fast neutrons. Since, in any case, the fast neutron flux must be calculated in order to evaluate the effects of fast neutrons penetrating the shield, it is usually sufficiently accurate to utilize only equation 3.21 and to assume that the fast current is equal in magnitude to the fast flux and has the direction of its gradient. This procedure tends to slightly overestimate the fast neutron current; however, it saves a considerable amount of numerical work. The fast current is needed in order to compute the thermal neutron flux distribution in the shield. If there is no water in the shield, equation 3.22 becomes

$$B(\underline{x},\underline{\xi}) = \exp\left[-\int_0^r dt\, \sigma_r(t)\right] \quad . \tag{3.24}$$

This is not as firmly grounded as equation 3.22, because the removal cross sections are measured in a water medium.

The assumption of straight-ray propagation is again introduced for gamma radiation; the dose outside of the shield is determined by an integral expression that is identical in form to equation 3.21. If the gamma-ray attenuation along the ray is taken to be exponential with linear buildup, this will mean that the kernel B in equation 3.21 is given by

$$B(\underline{x},\underline{\xi}) = \left[1 + \int_0^r dt\ \sigma(t)\right] \exp\left[-\int_0^r dt\ \sigma(t)\right] \quad , \qquad 3.25$$

where $\sigma(t)$ is now the total photon collision cross section for the material traversed by the ray. The uncollided gamma-ray flux would be obtained by utilizing equation 3.24 with the total gamma-ray cross section. The dose contribution is calculated for each energy in the gamma-ray source spectrum and the results added.

The discussion of the derivations following will be based on a general type of radiation which follows the attenuation function given in equations 3.21 and 3.24. It is usually practical to assume exponential attenuation for both fast neutrons and gamma rays, and, if a total cross section is used, the resulting flux is the uncollided flux. Appropriate adjustments, e.g., the gamma-ray build-up factor, may convert this to the appropriate gamma-ray flux, or the fast neutron-removal cross section will likewise convert this to the appropriate fast neutron flux.

For all geometries the source of radiation is considered to be a point, or an array of points, each emitting Q_0 particles per second. The radiation is generally assumed to be emitted isotropically, although other distributions may be used. For example, a cosine distribution may be introduced in a simple fashion for many cases.

In the simple point-source geometry, the uncollided flux is

$$\Phi(R) = Q_0 \frac{e^{-\sigma R}}{4\pi R^2} \quad \text{part/cm}^2\,\text{sec} \quad . \qquad 3.26$$

Throughout this discussion σ denotes the total macroscopic particle cross section for the shield material. The function

$$\frac{e^{-\sigma R}}{4\pi R^2}$$

is sometimes referred to as the point kernel. In practice, a point-source geometry may be realized in the case of a small radioactive piece of material to be shielded by a relatively thick shield. Basically, all formulas may be derived considering the source to be an array of points and the attenuation as given by the point kernel.

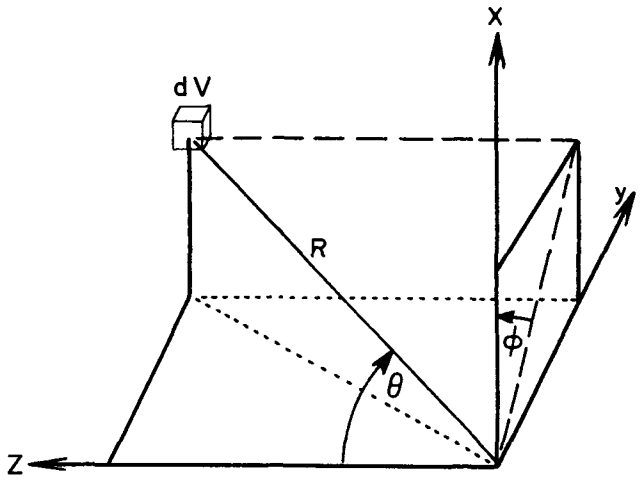


Figure 3.6

Coordinate System

The coordinate system used is that shown in Figure 3.6. The detector location, or the point at which the flux is to be calculated, is at x = 0, y = 0, z = 0. This is usually more convenient for shielding calculations, though perhaps not conventional.

The point-source strength in geometries other than a point is the source strength Q_1 particles/cm sec, Q_2 particles/cm^2 sec, and Q_3 particles/cm^3 sec, multiplied by the appropriate differential element of the geometry dL, cm, dA, cm^2, or dV, cm^3. In these geometries the point-source strength, Q_0, is then Q_1 dL, Q_2 dA, and Q_3 dV, respectively. The source distribution is usually considered to be a constant throughout the medium. Other source distributions are considered in pertinent cases, such as an exponential source distribution for capture gamma rays in a shield medium, the effect of the complete distribution being obtained by integration of the resulting point kernel over the source region.

Sources which may be considered as lines are realized frequently in radiation problems. For example, a long fuel element or a pipe full of radioactive coolant may be considered a line source of radiation. In this case the problem is set up for an element of length, dL, and the source strength is then Q_1 dl, Q_1 being the number of particles emitted per centimeter of source. The geometry is given in Figure 3.7.

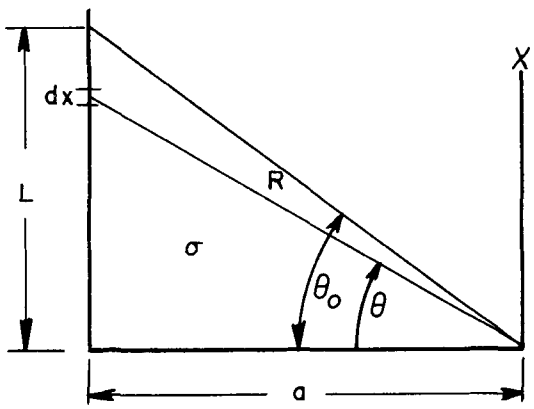


Figure 3.7

Line-Source Geometry

The centerline flux from a line source 2L centimeters long is

$$\Phi(a) = \int_{-L}^{L} dx\, Q_1 \frac{e^{-\sigma R}}{4\pi R^2} \text{ part/cm}^2\text{sec} \quad . \qquad 3.27$$

In case the shield consists of two or more regions, R is broken up accordingly, so that the number of mean-free-paths traversed by the particle is given by

$$\sigma R = \sigma_1 a_1 \sec\theta + \sigma_2 a_2 \sec\theta,$$

where

$$a = a_1 + a_2 \quad .$$

The flux from a plane source of radiation (Figure 3.8) may be similarly expressed:

$$\Phi(a) = \int_{A} dA\; Q_2 \frac{e^{-\sigma R}}{4\pi R^2} \text{ part/cm}^2 \text{ sec} \quad . \qquad 3.28$$

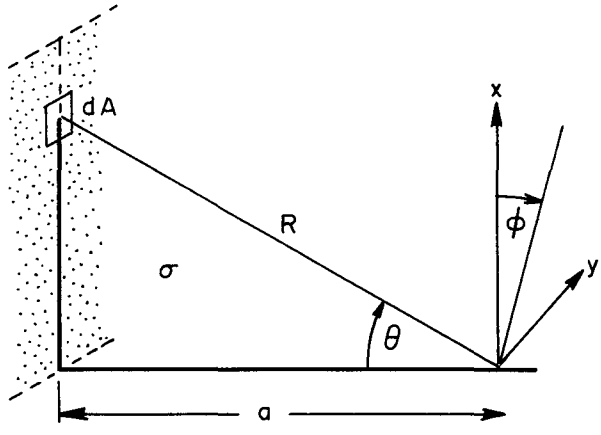


Figure 3.8

Plane Surface Source Geometry

The source Q_2 is in particles per square centimeter per second, and the point source Q_0 is then equal to Q_2 dA.

Finally, from a volume source (Figure 3.9) of Q_3 per cubic centimeter per sec, the flux is

$$\Phi(a) = \int_{V} dV\; Q_3 \frac{\exp\left[-\sigma_s (R - a\sec\theta) - \sigma a \sec\theta\right]}{4\pi R^2} \text{ part/cm}^2\text{sec.} \qquad 3.29$$

The subscript "s" refers to source cross section.

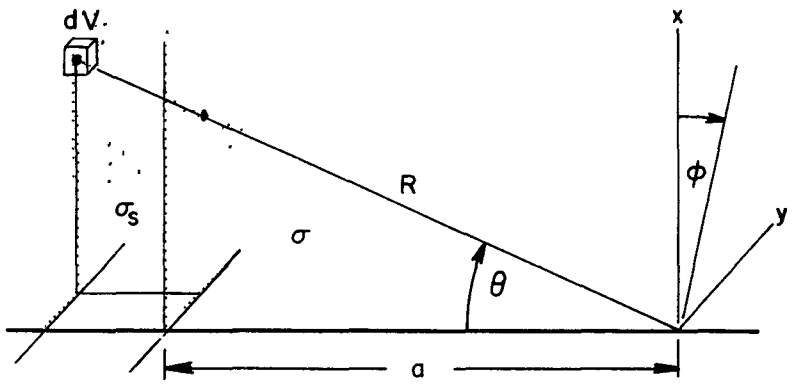


Figure 3.9

Plane Volume Source Geometry

Going back to equation 3.27 and substituting

$$R = a \sec\theta$$

and

$$x = a \tan\theta \quad ,$$

we obtain

$$\Phi(a) = \frac{Q_1}{2\pi a} \int_0^{\theta_0} d\theta \; e^{-\sigma a \sec\theta} \; \text{part/cm}^2 \text{ sec} \quad , \qquad 3.30$$

or

$$\Phi(a) = \frac{Q_1}{2\pi a} \sec_1(\sigma a, \theta_0) \; \text{part/cm}^2 \text{ sec} \quad , \qquad 3.31$$

where

$$\theta_0 = \arctan \frac{L}{a} \quad .$$

Here, as well as later,

$$\sigma a = \sum_{i=1}^{n} \sigma_i a_i \quad .$$

"n" is the number of shield materials. In case there is no shield material, i.e., $\sigma = 0$, the expression for the flux from a line source is

$$\Phi(a) = \frac{Q_1}{2\pi a} \theta_0 \quad . \qquad 3.32$$

As indicated previously a line source of radiation may be realized in a practical situation by a used fuel element or a pipe full of radioactive fluid. In each case the source must be long compared to the diameter. Whether the line is infinitely long may be judged from the curve of the secant integral. As 90° corresponds to an infinitely long line, the difference between the secant integral at 90° and the secant integral at the angle considered, σ a remaining constant, will give an estimate of the error involved in assuming that the source is an infinitely long line. To convert the volume source, Q_3, which would represent the pipe full of radioactive material, to the line source, Q_1, multiply Q_3 by the cross-sectional area. Depending upon the accuracy required, a self-absorption correction, a distributed source correction, and perhaps others, could be included.

Equation 3.28 may be transformed to

$$\Phi(a) = \int_0^{2\pi} d\phi \int_a^{\infty} dR\, Q_2 \frac{e^{-\sigma R}}{4\pi R} \quad \text{part/cm}^2 \text{ sec} \quad . \tag{3.33}$$

Noting that

$$R = a \sec\theta = a\, u \quad ,$$

equation 3.33 becomes

$$\Phi(a) = \frac{Q_2}{2} \int_1^{\infty} du \frac{e^{-\sigma a u}}{u} \quad \text{part/cm}^2 \text{ sec} \quad , \tag{3.34}$$

or

$$\Phi(a) = \frac{Q_2}{2} E_1(\sigma a) \quad \text{part/cm}^2 \text{ sec} \quad . \tag{3.35}$$

While physically there is no source that is an infinite plane, practically speaking there are sources which may be so represented or approximated. For example, a spherical source may be represented as an infinite plane source with a correction to account for the geometrical effect.

Consider a spherical surface source with a slab shield, such as in Figure 3.10.

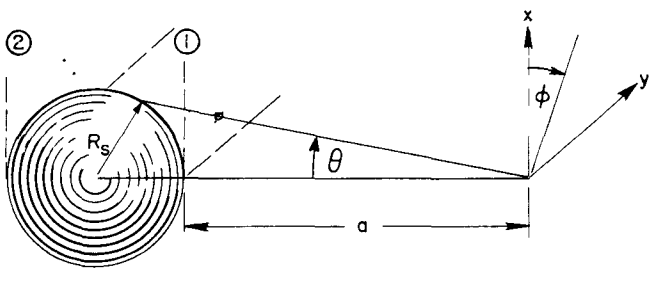


Figure 3.10

Spherical Surface Source Geometry

The flux in such a case is given by

$$\Phi(R) = \int_A dA\, Q_2 \frac{e^{-\sigma R}}{4\pi R^2} \quad \text{part/cm}^2 \text{ sec} \tag{3.36}$$

$$= \frac{Q_2}{2} \int_a^{a+2R_S} \frac{dR}{R} \frac{R_S}{(R_S + a)} e^{-\sigma R} \quad \text{part/cm}^2 \text{ sec} \tag{3.37}$$

$$= \frac{Q_2}{2} \frac{R_S}{(R_S + a)} \left\{ E_1 [\sigma a] - E_1 [\sigma(a + 2R_S)] \right\} \text{ part/cm}^2 \text{ sec.} \tag{3.38}$$

Equation 3.38 is equivalent to the flux from a plane source located at the near surface of the sphere (position 1, Figure 3.10) less the flux from a plane surface located at the far side of the sphere (position 2, Figure 3.10). If the sphere is thick, i.e., $2R_S$ is large compared to a, the second term of equation 3.38 is negligible, and that formula becomes

$$\Phi(a) = \frac{Q_2}{2} \frac{R_S}{(R_S + a)} E_1 (\sigma a) \quad \text{part/cm}^2 \text{ sec} \quad . \tag{3.39}$$

If, further, we consider that the source is really distributed throughout the sphere, the value of the surface source may be given approximately as $Q_3/2\sigma_S$ and the result is then

$$\Phi(a) = \frac{Q_3}{2\sigma_S} \frac{R_S}{(R_S + a)} E_1 (\sigma a) \quad \text{part/cm}^2 \text{ sec} \quad . \tag{3.40}$$

The flux from a shielded infinite half space, equation 3.29, may be expressed as

$$\Phi(a) = \int_0^{2\pi} d\phi \int_1^{\infty} \frac{du}{u^2} \int_{au}^{\infty} dR \frac{Q_3 \exp [-\sigma_S(R - au) - au]}{4\pi} \text{ part/cm}^2 \text{ sec,} \tag{3.41}$$

where $u = \sec \theta$.

Integrating over ϕ and R, this becomes

$$\Phi(a) = \frac{Q_3}{2\sigma_S} \int_1^{\infty} du \frac{e^{-\sigma a u}}{u^2} \quad \text{part/cm}^2 \text{ sec,} \tag{3.42}$$

or

$$\Phi(a) = \frac{Q_3}{2\sigma_S} E_2 (\sigma a) \quad \text{part/cm}^2 \text{ sec.} \tag{3.43}$$

If the source is of a finite thickness "t," the resulting flux is .

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\left\{E_2(\sigma a) - E_2(\sigma a + \sigma_s t)\right\} \text{ part/cm}^2 \text{ sec} \quad . \qquad 3.44$$

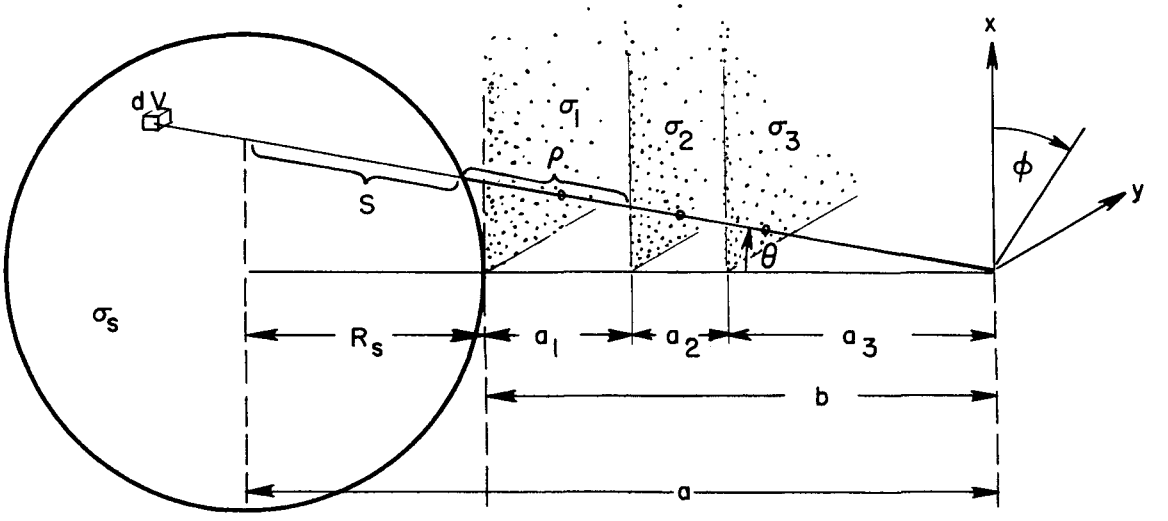
This reduces to

$$\Phi(t) = \frac{Q_3}{2\sigma_s}\left\{1 - E_2(\sigma_s t)\right\} \text{ part/cm}^2 \text{ sec} \quad , \qquad 3.45$$

where there is no shield, i.e., $\sigma = 0$.

Finally, the flux from a spherical volume source with slab shields (Figure 3.11) may be expressed as

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\int_{\mu_0}^{1} d\mu \exp\left[-\frac{1}{\mu}\sum_{i=1}^{n}\sigma_i a_i\right] \quad . \qquad 3.46$$



$$s^2 = R_s^2 - a^2 + a^2\mu^2$$

$$\rho(\mu) = a\mu - s - \left(\frac{b - a_1}{\mu}\right)$$

$$\mu = \cos\theta$$

Figure 3.11

Spherical Volume Source with Slab Shields

An assumption included here is that contributions from the farthest portion of the sphere are negligible. The flux at the surface of the sphere is

$$\Phi(R_s) = \frac{Q_3}{2\sigma_s}\left\{1 - \frac{1}{2\sigma_s R_s}\left[1 - e^{-2\sigma_s R_s}\right]\right\} \text{ part/cm}^2 \text{ sec} . \qquad 3.47$$

In order to use the water attenuation function for fission neutrons as measured and adjusted for plane-collimated geometry, this function must be included in formula 3.46 as in equation 3.22. Including the water attenuation in the form of $N(\rho)$[16] we arrive at the following;

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\int_{\mu_0}^{1} d\mu N(\rho) \exp\left[-\frac{1}{\mu}\sum_{i=2}^{n}\sigma_i a_i\right] \text{ neut/cm}^2 \text{ sec} . \qquad 3.48$$

This requires a numerical integration, as does equation 3.46. It is also possible to fit the curve, $N(\rho)$, to a sum of two exponentials,[17,18] which may in some cases permit the integral in equation 3.48 to be evaluated. If, for example, the approximation for $N(\rho)$ is the sum of two exponentials:

$$N(\rho) = A_1 e^{-\sigma_1 \rho} + A_2 e^{-\sigma_2 \rho} , \qquad 3.49$$

and the spherical approximation as in equation 3.40 is employed, the result is

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\frac{R_s}{(R_s + a)}\left\{A_1 E_1[\sigma_1 a_1 + \sigma a] + A_2 E_1[\sigma_2 a_1 + \sigma a]\right\} \qquad 3.50$$
$$\text{part/cm}^2 \text{ sec.}$$

Useful approximations may be obtained from the formulae already derived. The integral expression for the flux from a volume source,

$$\Phi(a) = \int_V dV\, Q_3 \frac{e^{-\sigma_s R}}{4\pi R^2} \text{ part/cm}^2 \text{ sec} , \qquad 3.51$$

may be integrated in a fashion that will lead to simplified formulae. For example, the integral may be expressed as

$$\Phi(a) = \int d\Omega \int dR\, Q_3 \frac{e^{-\sigma_s R}}{4\pi R^2} \text{ part/cm}^2 \text{ sec} , \qquad 3.52$$

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[16] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, p. 55ff, McGraw-Hill & D. Van Nostrand, (March, 1956).

[17] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, p. 69, McGraw-Hill & D. Van Nostrand, (March, 1956).

[18] Duncan, D. S., and H. O. Whittum, Jr., "Application of Fast Neutron Removal Theory to the Calculation of Thermal Neutron Flux Distributions in Reactor Shields," NAA-SR-2380 (July 1, 1958).

instead of as in equation 3.41. The assumption that the source region is opaque and that the source Q_3 is independent of the angle Ω leads to the result

$$\Phi = \frac{Q_3}{4\pi\sigma_s}\ \Omega \quad \text{part/cm}^2\ \text{sec} \tag{3.53}$$

for an unshielded source such as in Figure 3.12. Here Ω is the solid angle subtended by the source at the observer. This formula may be applied under circumstances that do not require as much accuracy as speed, and for sources that are not large compared to the distance of the observer from the source. It can be enlarged to include the shield by multiplying by an exponential:

$$\Phi(t) = \frac{Q_3}{4\pi\sigma_s}\ \Omega\ e^{-\sigma t} \quad \text{part/cm}^2\ \text{sec}, \tag{3.54}$$

which may be regarded as including build-up in a linear fashion.

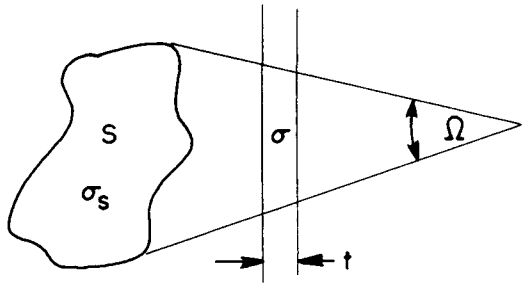

Figure 3.12

Volume Source Geometry

If the integral in equation 3.51 has the limits appropriate for the flux at the surface of an infinite half space,

$$\Phi(R) = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta\ \sin\theta \int_0^{\infty} dR\ R^2\ Q_3 \frac{e^{-\sigma R}}{4\pi R^2} \quad \text{part/cm}^2\ \text{sec}\ ; \tag{3.55}$$

the result is

$$\Phi = \frac{Q_3}{2\sigma_s} \quad \text{part/cm}^2\ \text{sec}\ . \tag{3.56}$$

This may be obtained by substituting the proper limits in equations 3.29 or 3.41, or by letting "a" go to zero in equation 3.43. Note that the flux at the surface of a finite slab and at the surface of a sphere are, respectively,

$$\Phi(t) = \frac{Q_3}{2\sigma_s}\left[1 - E_2(\sigma_s t)\right] \quad \text{part/cm}^2\ \text{sec} \tag{3.57}$$

and

$$\Phi(R_S) = \frac{Q_3}{2\sigma_S}\left\{1 - \frac{1}{2\sigma_S R_S}\left[1 - e^{-2\sigma_S R_S}\right\}\right. \text{ part/cm}^2 \text{ sec} \quad . \qquad 3.58$$

It can be seen that these formulas represent the flux from an infinite half space less a corrective term that is due, respectively, to the difference in source size between the infinite half space, and the infinite slab of finite thickness, or the sphere. As $\sigma_S t$ or $\sigma_S R_S$ become large, the slab or sphere may be represented by an infinite half space. If, for instance, the slab is 3 mean-free-paths (mfp) thick, i.e., $\sigma_S t = 3$, the correction for the finite thickness is

$$1 - E_2(3) = 1 - 0.0103 = 0.99.$$

Similarly, the ratio of the flux at the surface of a sphere of radius 3 mfp to that at the surface of an infinite half space is

$$1 - \frac{1}{6}(1 - e^{-6}) = 0.83 \quad .$$

Thus in many cases the flux at the surface of a reactor may be adequately given by 3.56. A reactor is usually somewhat cylindrical in shape. Since this geometrical configuration is somewhere between those corresponding to the slab and the sphere, the same logic applies. This may be extended further by noting that the flux in an infinite medium is

$$\Phi = \frac{Q_3}{\sigma_S} \quad \text{part/cm}^2 \text{ sec} \quad , \qquad 3.59$$

or simply twice that at the surface of an infinite half space, given in equation 3.56. If, then, the reactor, or the source, is reasonably large, the flux at the center may be quickly approximated by equation 3.59.

The foregoing discussion on Ray Theory forms the essential basis for calculating the probability of escape of particles from a medium. The probability of escape may be defined by the expression

$$P = \frac{JA}{Q_3 V} \quad , \qquad 3.60$$

or as the total number of particles escaping divided by the total number of particles being born in the source. In equation 3.60 "J" is the particle current as defined in equation 3.23, "A" is the surface area of the source, "Q_3" is the source strength, and "V" is the volume of the source. Note that the current is given by the same expression as the flux, multiplied by the cosine of the angle of incidence (see equations 3.21 and 3.23).

The following formulae assume a constant isotropic source with no build-up correction. A source distribution could be included as could a build-up factor. The absence of the latter may be justified to some extent

on the basis that most particles come from within one mean-free-path of the surface and, therefore, the error is not as great as might be supposed. It should usually be less than a factor of two.

For an infinite slab of thickness "t" (refer to Figure 3.9) the probability of escape is

$$P = \frac{A}{Q_3 V}\int_V dV\, Q_3 \frac{e^{-\sigma R}}{4\pi R^2} \cos\theta \quad . \tag{3.61}$$

Integration yields the result

$$P = \frac{1}{2\sigma t}\left\{1 - [1 - \sigma t]\, e^{-\sigma t} - (\sigma t)^2\, E_1(\sigma t)\right\} \quad . \tag{3.62}$$

For a thick slab, $\sigma t >> 1$, this reduces to

$$P \cong \frac{1}{2\sigma t} \quad . \tag{3.63}$$

The probability of escape for a sphere is, by the same considerations (refer to Figure 3.11),

$$P = \frac{3}{4\sigma R}\left\{1 - \frac{1}{2(\sigma R)^2}[1 + \sigma R]\, e^{-2\sigma R}\right\} \quad . \tag{3.64}$$

This, in turn, for a large sphere, $\sigma R >> 1$, reduces to

$$P \cong \frac{3}{4\sigma R} \quad . \tag{3.65}$$

This information is displayed graphically in Figure 3.13. More detailed information on this and related subjects may be found in the literature.[18a]

All the foregoing formulae were expressions for uncollided fluxes. In order to adjust these for the scattered radiation that is always present, a correction, or build-up factor, is required. For the sake of convenience we shall assume a linear form of build-up, additive for more than one material:

$$B(\sigma x) = 1 + \sigma_1 x_1 + \sigma_2 x_2 + \ldots \quad . \tag{3.66}$$

[18a] Storm, M. L., H. Hurwitz, Jr., and G. M. Roe, "Gamma-Ray Absorption Distribution from Plane, Spherical and Cylindrical Geometries," KAPL-783 (July 24, 1953).

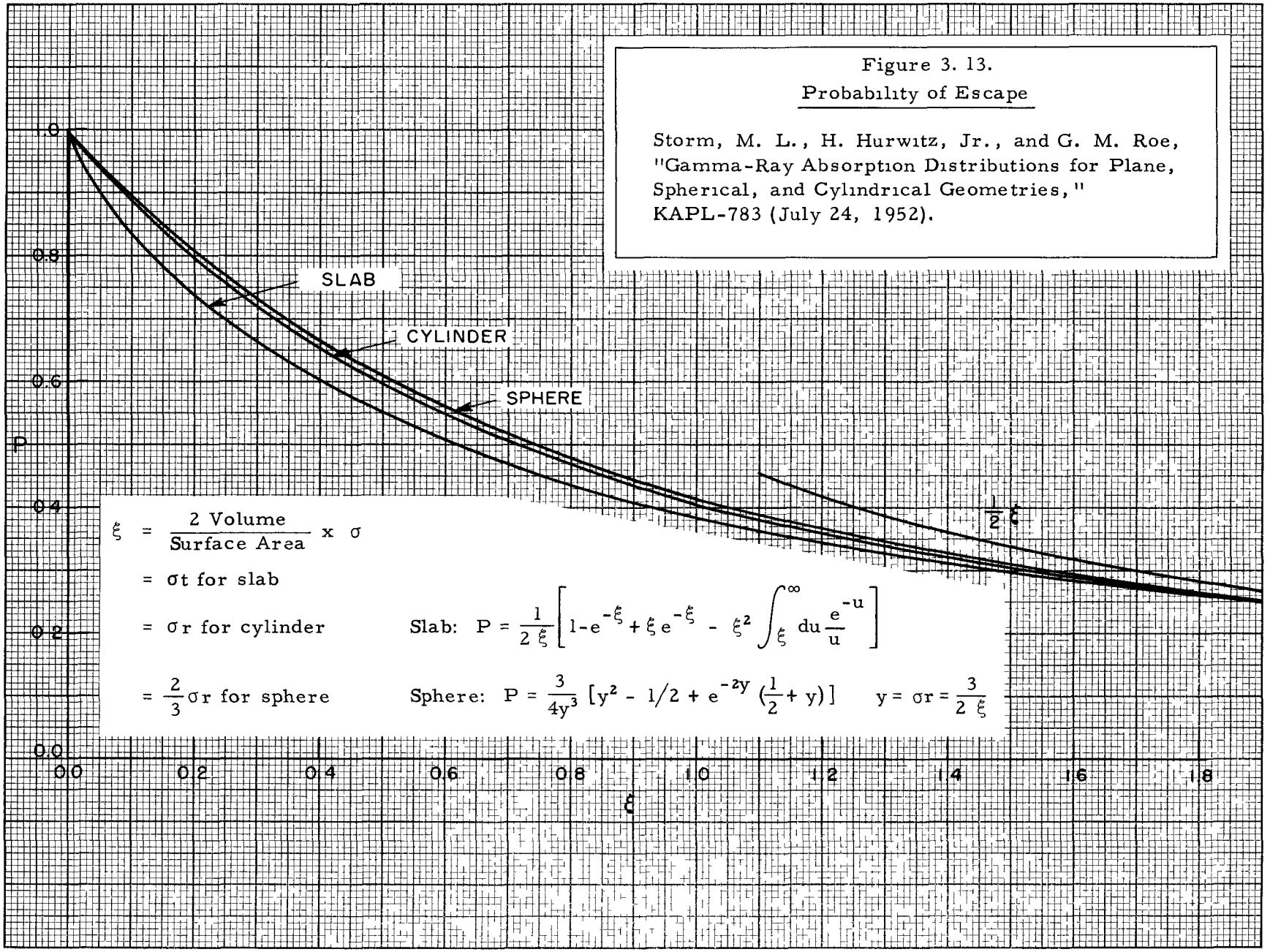

Figure 3. 13.
Probability of Escape

Storm, M. L., H. Hurwitz, Jr., and G. M. Roe, "Gamma-Ray Absorption Distributions for Plane, Spherical, and Cylindrical Geometries," KAPL-783 (July 24, 1952).

This is usually accurate enough for the case of gamma-ray calculations in a predominantly concrete shield. Equations 3.26 through 3.29 then become, respectively,

$$\Phi(R) = Q_0 \frac{e^{-\sigma R}}{4\pi R^2} (1 + \sigma R) \quad \text{part/cm}^2 \text{ sec}, \quad 3.67$$

$$\Phi(a) = \int_{-L}^{L} dx \frac{e^{-\sigma R}}{4\pi R^2} (1 + \sigma R) \quad \text{part/cm}^2 \text{ sec}, \quad 3.68$$

$$\Phi(a) = \int_{A} dA\, Q_2 \frac{e^{-\sigma R}}{4\pi R^2} (1 + \sigma R) \quad \text{part/cm}^2 \text{ sec}, \quad 3.69$$

and

$$\Phi(a) = \int_{V} dV\, Q_3 \frac{\exp[-\sigma_s(R - a\,u) - \sigma a\,u]}{4\pi R^2}$$

$$x[1 + \sigma_s(R - a\,u) + \sigma a\,u)] \quad \text{part/cm}^2 \text{ sec}. \quad 3.70$$

Upon integration, the latter three equations become

$$\Phi(a) = \frac{Q_1}{2\pi a} \left\{ Ki_1(\sigma a) + \sigma a\, Ki_0(\sigma a) \right\} \quad \text{part/cm}^2 \text{ sec} \quad 3.71$$

(this represents an infinite line source, $\theta = \pi/2$),

$$\Phi(a) = \frac{Q_2}{2} \left\{ E_1(\sigma a) + \sigma a\, E_0(\sigma a) \right\} \quad \text{part/cm}^2 \text{ sec}, \quad 3.72$$

and

$$\Phi(a) = \frac{Q_3}{2\sigma_s} \left\{ 2\, E_2(\sigma a) + \sigma a\, E_1(\sigma a) \right\} \quad \text{part/cm}^2 \text{ sec}. \quad 3.73$$

If the plane volume source is of finite thickness "t," then equation 3.73 becomes,

$$\Phi(a) = \frac{Q_3}{2\sigma_s} \left\{ 2\, E_2(\sigma a) + \sigma a\, E_1(\sigma a) - 2\, E_2(\sigma a + \sigma_s t) \right\}$$

$$- (\sigma a + \sigma_s t)\, E_1(\sigma a + \sigma_s t) \Big\} \quad \text{part/cm}^2 \text{ sec}. \quad 3.74$$

If equation 3.70 is rewritten for an exponential source (Figure 3.14), the following integral may be set up:

$$\Phi(a) = \int dV \frac{Q_3(v)}{4\pi R^2} \exp[-\sigma_s(R - a \sec\theta) - \sigma a \sec\theta\ ]$$

$$x[1 + \sigma_s(R - a \sec\theta) + \sigma a \sec\theta\ ] \quad \text{part/cm}^2 \text{ sec}, \quad 3.75$$

where

$$Q_3(v) = Q_3(0)\, e^{kv}, \qquad \text{part/cm}^3 \text{ sec} \qquad 3.76$$

and for capture gamma rays

$$Q_3(0) = \sigma_a N_\gamma \Phi(0). \qquad \text{part/cm}^3 \text{ sec.} \qquad 3.77$$

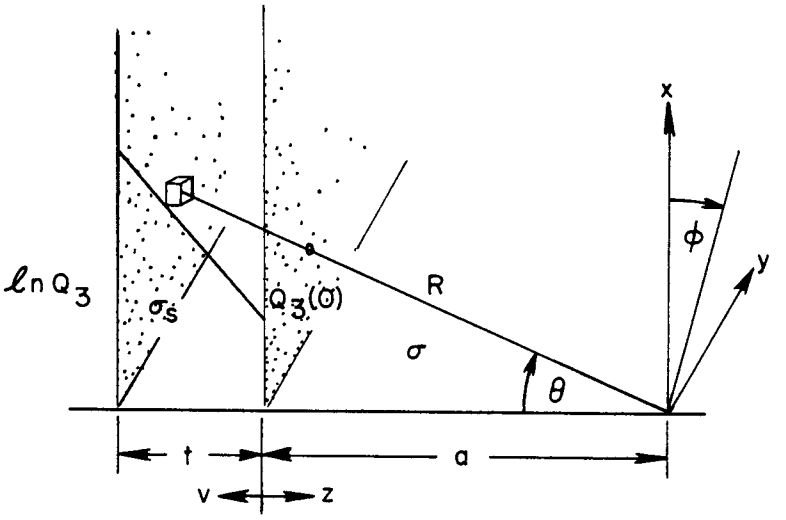

Figure 3.14

Exponential Plane Volume Source Geometry

Here σ_a is the neutron absorption cross section, N_γ is the number of gamma rays emitted per neutron capture, and $\Phi(0)$ is the neutron flux at the boundary. In the process of integrating equation 3.75, care must be maintained that the relative size of k and σ_s is noted. The results are:

$$\Phi(a) = \frac{Q_3(0)}{2k}\left\{e^{+kt}E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + \frac{k}{(k - \sigma_s)}\, e^{-\sigma a}[e^{+(k - \sigma_s)t} - 1]\right.$$
$$\left. + e^{-\nu\sigma a}\left[E_1\left\{\sigma a(1 - \nu)\right\} - E_1\left\{(\sigma_s t + \sigma a)(1 - \nu)\right\}\right]\right\} \qquad 3.78$$
$$\text{part/cm}^2 \text{ sec}, \ \nu < 1,$$

$$\Phi(a) = \frac{Q_3(0)}{2k}\left\{e^{+kt}\, E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + \frac{k\, e^{-\sigma a}}{(R - \sigma_s)}\,[e^{+(k - \sigma_s)t} - 1]\right.$$
$$\left. + e^{-\nu\sigma a}\left[Ei\left\{(\sigma_s t + \sigma a)(\nu - 1)\right\} - Ei\left\{\sigma a\,(\nu - 1)\right\}\right]\right. \qquad 3.79$$
$$\text{part/cm}^2 \text{ sec}, \ \nu > 1,$$

and

$$\Phi(a) = \frac{Q_3(0)}{2k}\left\{e^{+kt}\,E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + kt\,e^{-\sigma a} + e^{-\nu\sigma a}\ln\frac{\sigma_s t + \sigma a}{a}\right\} \quad \text{part/cm}^2\,\text{sec},\ \nu = 1 \qquad 3.80$$

where

$$\nu = \frac{k}{\sigma_s} .$$

If the shield thickness "a" is reduced to zero, these formulae reduce to

$$\Phi(t) = \frac{Q_3(0)}{2k}\left\{e^{+kt}\,E_1(\sigma_s t) - \ln(1-\nu) + \frac{k}{(k-\sigma_s)}\left[e^{+(k-\sigma_s)t} - 1\right] - E_1\left\{\sigma_s t\,(1-\nu)\right\}\right\} \quad \text{part/cm}^2\,\text{sec},\ \nu<1, \qquad 3.81$$

$$\Phi(t) = \frac{Q_3(0)}{2k}\left\{e^{+kt}\,E_1(\sigma_s t) - \ln(\nu-1) + \frac{k}{(k-\sigma_s)}\left[e^{+(k-\sigma_s t)} - 1\right] + Ei\left\{\sigma_s t(\nu-1)\right\}\right\} \quad \text{part/cm}^2\,\text{sec},\ \nu>1, \qquad 3.82$$

and

$$\Phi(t) = \frac{Q_3}{2K}\left\{e^{+kt}\,E_1(\sigma_s t) + \ln kt + \gamma + \sigma_s t\right\} \quad \text{part/cm}^2\,\text{sec},\ \nu = 1. \qquad 3.83$$

In case the flux distribution is desired throughout the medium of the source either a combination of the previous formulae, or formulae derived for the purpose may be utilized.[18b] The difference in the two sets of formulae is that no build-up is included in the latter. In cases where this is important the build-up effect must be estimated using basic information on build-up.[19]

The formulae which give the flux in the source medium with no build-up are:

$$\Phi(t) = \frac{Q_3}{2k}\,e^{-kt}\left\{e^{+kt}\,E_1(\sigma_s t) + Ei\left[\sigma_s t(\nu-1)\right] + \ln\left(\frac{\nu+1}{\nu-1}\right)\right\} \qquad 3.84$$

$$\text{part/cm}^2\,\text{sec},\ \nu>1,$$

[18b] Hagerton, J. F., and M. C. Glass, The Reactor Handbook, Vol. I, Physics AECD-3645, McGraw-Hill Book Co., (1955) p. 706.

[19] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma-Rays," NYO-3075, U. S. Govt. Printing Office (May 1, 1957).

$$\Phi(t) = \frac{Q_3}{2k} e^{-kt} \left\{ e^{+kt} E_1(\sigma_s t) + E_1[(\sigma_s t)(1-\nu)] + \ln\left(\frac{1+\nu}{1-\nu}\right) \right\} \quad 3.85$$

part/cm^2 sec $\nu < 1$,

and

$$\Phi(t) = \frac{Q_3}{2k} e^{-\sigma st} \left\{ e^{+\sigma st} E_1(\sigma_s t) + \ln \sigma_s t + \ln 2\gamma \right\} \quad 3.86$$

part/cm^2 sec, $\nu = 1$

where

$$\ln 2\gamma = 1.27 \quad .$$

These formulae, along with similar formulae representing linear source terms, may also be found in The Design Manual.[20]

Additional equations not previously encountered are summarized at the end of this section. The basic formula involved is

$$\Phi(a) = \int_S dS\, Q(R)\, B(R) \cos^n\theta \frac{e^{-\sigma R}}{4\pi R^2} \qquad \text{part/cm}^2 \text{ sec.} \quad 3.87$$

In the summary $B(\sigma R)$ takes two forms:

$$B(\sigma R) = 1 + \sigma R,$$

and

$$B(\sigma R) = K e^{+\alpha\sigma R} \quad .$$

Similarly, the source distribution takes on two forms:

$$Q(R) = Q \quad (\text{a constant})$$

and

$$Q(R) = Q e^{+kR} \quad .$$

The term "$\cos^n\theta$" is inserted to illustrate the general effect of this function. It may represent current or it may represent a source distribution when n = 1. When "n" has other values it may represent other combinations of the cos θ or $\cos^2\theta$ distribution, current, or perhaps some other phenomena represented by a cosine.

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[20] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, p. 397. (McGraw-Hill & D. Van Nostrand). (March, 1956).

A. Line source,

$$\Phi(R) = 2\int_0^L dx\, Q_1(x)\, B(\sigma R)\, \cos^n\theta\, \frac{e^{-\sigma R}}{4\pi R^2} \qquad 3.89$$

1. Infinite line source, $Q_1(x)$ = constant, $B(\sigma R) = 1 + k\sigma R$, (Figure 3.7).

$$\Phi(a) = \frac{Q_1}{4\pi a}\left\{Ki_{n+1}(\sigma a) + k\sigma a\, Ki_n(\sigma a)\right\} \qquad 3.90$$

a) n = 0

$$\Phi(a) = \frac{Q_1}{4\pi a}\left\{seci(\sigma a) + k\sigma a\, K_0(\sigma a)\right\} \qquad 3.91$$

b) n = 0, $\sigma = 0$

$$\Phi(a) = \frac{Q_1}{4a} \qquad 3.92$$

2. Infinite line source, $Q_1(x)$ = constant, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = \frac{Q_1}{4\pi a} A\, Ki_{n+1}\left[\sigma a(1 - \alpha)\right] \qquad 3.93$$

3. Finite line source center line flux, $Q_1(x)$ = constant, $B(\sigma R) = 1 + k\sigma R$).

$$\Phi(a) = \int_0^{\theta_0} d\theta\, (1 + k\sigma R)\, \cos^n\theta\, e^{-\sigma a \sec\theta} \qquad 3.94$$

a) n = 0, k = 0

$$\Phi(a) = \frac{Q_1}{2\pi a}\, seci(\sigma a, \theta_0) \qquad 3.95$$

b) n = 0, $\sigma = 0$

$$\Phi(a) = \frac{Q_1}{2\pi a}\, \theta_0 \qquad 3.96$$

4. Finite line source center line flux, $Q_1(x)$ = constant, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = \int_0^{\theta_0} d\theta \cos^n\theta \, A \exp[-(1 - \alpha)\sigma a \sec\theta] \qquad 3.97$$

a) n = 0

$$\Phi(a) = \frac{Q_1 A}{2\pi a} \operatorname{seci}[\sigma a(1 - \alpha)\theta_0] \qquad 3.98$$

5. Finite line source, $Q_1(x)$ = constant, $B(\sigma R) = 1 + k\sigma R$ (Figure 3.15).

a) n = 0, k = 0

$$\Phi(a) = \frac{Q_1}{4\pi a} \left\{\operatorname{seci}(\sigma a, \theta_2) - \operatorname{seci}(\sigma a, \theta_1)\right\} \qquad 3.99$$

b) n = 0, σ = 0

$$\Phi(a) = \frac{Q_1}{4\pi a} \left\{\theta_2 - \theta_1\right\} \qquad 3.100$$

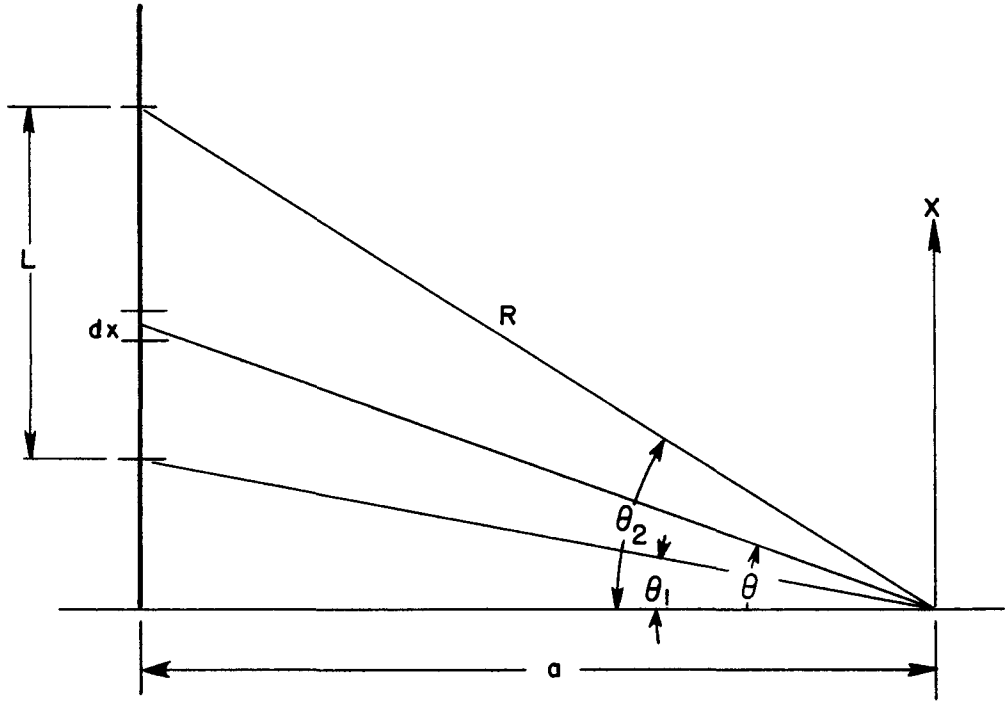

Figure 3.15

Line Source Geometry

6. Finite line source, $Q_1(x)$ = constant, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

a) $n = 0$

$$\Phi(a) = \frac{Q_1 A}{4\pi a}\left\{\text{seci}[\sigma a(1-\alpha),\ \theta_2] - \text{seci}[\sigma a(1-\alpha),\theta_1]\right\} \qquad 3.101$$

B. Plane Surface Source (Figure 3.8).

$$\Phi = \int dA\ Q_2(R)\ B(\sigma R)\ \cos^n\theta\ \frac{e^{-\sigma R}}{4\pi R^2} \qquad 3.102$$

1. Infinite plane source, $Q_2(R) = Q_2$, $B(\sigma R) = 1 + k\sigma R$.

$$\Phi(a) = \frac{Q_2}{2}\left\{E_{n+1}(\sigma a) + k\sigma a\ E_n(\sigma a)\right\} \qquad 3.103$$

a) $n = 1, k = 1$

$$\Phi(a) = \frac{Q_2}{2}\ e^{-\sigma a} \qquad 3.104$$

b) $\sigma = 0$

$$\Phi(a) = \frac{Q_2}{2}\frac{1}{n} \quad , \qquad n \neq 0 \qquad 3.105$$

2. Infinite plane source, $Q_2(R) = Q_2$, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = \frac{Q_2}{2} A\ E_{n+1}[\sigma a(1-\alpha)] \qquad 3.106$$

3. Finite plane, disk of radius R_s, $Q_2(R) = Q_2$, $B(\sigma R) = 1 + k\sigma R$.

$$\Phi(a) = \frac{Q_2}{2}\left\{E_{n+1}(\sigma a) + k\sigma a\ E_n(\sigma a) - u_0^{-n}\ E_{n+1}(\sigma a\ u_0)\right.$$
$$\left. - k\sigma a\ u_0^{1-n}\ E_n(\sigma a\ u_0)\right\} \qquad u_0 = \sec\theta \quad . \qquad 3.107$$

a) $\sigma = 0$

$$\Phi(a) = \frac{Q_2}{2}\frac{1}{n}\left[1 - u_0^{-n}\right] \qquad n \neq 0 \quad . \qquad 3.108$$

b) $\sigma = 0, n = 0$

$$\Phi(a) = \frac{Q_2}{2}\ \ln u_0 \qquad 3.109$$

4. Finite plane, disk of radius R_s, $Q_2(R) = Q_2$, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = A\left\{E_{n+1}[\sigma a(1-\alpha)] - u_0^{-n} E_n[\sigma a(1-\alpha)]\right\} \quad 3.110$$

C. Plane Volume Source.

$$\Phi(a) = \int dV\, Q_3(R)\, B(\sigma R) \cos^n\theta \frac{e^{-\sigma R}}{4\pi R^2} \quad 3.111$$

1. Infinite plane volume source, $Q_3(R) = Q_3$, $B(\sigma R) = 1 + k\sigma R$, (Figure 3.9).

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\left\{(1+k)\, E_{n+2}(\sigma a) + k\sigma a\, E_{n+1}(\sigma a)\right\} \quad 3.112$$

2. Infinite plane volume source, $Q_3(R) = Q_3$, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\frac{A}{(1-\alpha)} E_{n+2}[\sigma a(1-\alpha)] \quad 3.113$$

3. Finite plane volume source, $Q_3(R) = Q_3$, $B(\sigma R) = 1 + k\sigma R$.

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\Big\{(1+k)\, E_{n+2}(\sigma a) + k\sigma a\, E_{n+1}(\sigma a) - (1+k)\, E_{n+2}(\sigma a + \sigma_s t) - k(\sigma a + \sigma_s t)\, E_{n+1}(\sigma a + \sigma_s t)\Big\} \quad 3.114$$

$\sigma = 0$

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\left\{\frac{(1+k)}{(n+1)} - (1+k)\, E_{n+2}(\sigma_s t) - k\sigma_s t\, E_{n+1}(\sigma_s t)\right\} \quad 3.115$$

4. Finite plane volume source, $Q_3(R) = Q_3$, $B(\sigma R) = Ae^{+\alpha\sigma R}$.

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\frac{A}{(1-\alpha)}\left\{E_{n+2}[\sigma a(1-\alpha)] - E_{n+2}[\sigma a(1-\alpha)]\right\} \quad 3.116$$

$\sigma = 0$

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\frac{A}{(1-\alpha)}\left\{\frac{1}{(n+1)} - E_{n+2}[\sigma a(1-\alpha)]\right\} \quad 3.117$$

5. Infinite plane volume source, $Q_3(R) = Q_3\, e^{+k\nu}$, $B(\sigma R) = 1 + k\sigma R$, $\nu = \frac{k}{\sigma_s}$, (Figure 3.14).

$$\Phi(a) = \frac{Q_3}{2} \int_0^\infty d\nu e^{+k\nu} \left\{ E_{n+1}(\sigma_s \nu + \sigma a) + k(\sigma_s \nu + \sigma a)\, E_n(\sigma_s \nu + \sigma a) \right\} \qquad 3.118$$

a) $n = 0$, $k = 0$

$$\Phi(a) = \frac{Q_3}{2k} \left\{ e^{-\nu \sigma a}\, E_1[\sigma a(1 - \nu)] - E_1(\sigma a) \right\} \quad , \qquad \nu < 1 \qquad 3.119$$

b) $n = 0$, $k = 1$

$$\Phi(a) = \frac{Q_3}{2k} \left\{ e^{-\nu \sigma a}\, E_1[\sigma a(1 - \nu)] - E_1(\sigma a) + \frac{k}{(k - \sigma_s)} e^{-\sigma a} \right\} \qquad 3.120$$

$\nu < 1$

c) $n = 1$, $k = 1$

$$\Phi(a) = \frac{Q_3}{2} \frac{e^{-\sigma a}}{(k - \sigma_s)} \qquad 3.121$$

6. Infinite plane volume source, $Q_3(R) = Q_3\, e^{+k\nu}$, $B(\sigma R) = Ae^{+\alpha\sigma R}$, $\eta = \dfrac{k}{\sigma_s(1 - \alpha)}$.

$$\Phi(a) = \frac{Q_3}{2} A \int_0^\infty d\nu e^{+k\nu}\, E_{n+1}[(\sigma_s \nu + \sigma a)(1 - \alpha)] \qquad 3.122$$

a) $n = 0$

$$\Phi(a) = \frac{Q_3}{2} \frac{A}{k} \left\{ e^{-\nu \sigma a}\, E_1[\sigma a(1 - \alpha)(1 - \eta)] - E_1[\sigma a(1 - \alpha)] \right\} , \qquad 3.123$$

b) $\eta = 1$ $\qquad \eta < 1$

$$\Phi(a) = \frac{Q_3}{2} \frac{A}{k} \left\{ \frac{e^{-\nu \sigma a}}{\eta} E_1[\sigma a(1 - \alpha)(\eta - 1)] - \frac{1}{\eta} E_1[\sigma a(1 - \alpha)] \right.$$

$$\left. - E_2[\sigma a(1 - \alpha)] \right\} \quad , \qquad \eta > 1 \qquad 3.124$$

7. Finite plane volume source, $Q_3(R) = Q_3\, e^{+k\nu}$, $B(\alpha R) = 1 + k\sigma R$, $\nu = \dfrac{k}{\sigma_s}$.

$$\Phi(a) = \frac{Q_3}{2}\int_0^t d\nu e^{+k\nu}\left\{ E_{n+1}(\sigma_s\nu + \sigma a) + k(\sigma_s\nu + \sigma a)\, E_n(\sigma_s\nu + \sigma a)\right\} \qquad 3.125$$

a) $n = 0, k = 0$

$$\Phi(a) = \frac{Q_3}{2k}\left\{e^{+kt}\, E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + e^{-\nu\sigma a}[E_1\left\{\sigma a(1 - \nu)\right\} - E_1\left\{(\sigma_s t + \sigma a)(1 - \nu)\right\}]\right\} \quad , \qquad \nu<1 \qquad 3.126$$

$$\Phi(a) = \frac{Q_3}{2k}\left\{e^{+kt}\, E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + e^{-\nu\sigma a}\left[\mathrm{Ei}\left\{(\sigma_s t + \sigma a)(1 - \nu)\right\} - \mathrm{Ei}\left\{\sigma a(\nu - 1)\right\}\right]\right\} \quad , \qquad \nu>1 \qquad 3.127$$

$$\Phi(a) = \frac{Q_3}{2k}\left\{ e^{+kt}\, E_1(\sigma_s t + \sigma a) - E_1(\sigma a) + e^{-\sigma a}\ln\left(\frac{\sigma_s t + \sigma a}{\sigma a}\right)\right\}, \qquad \nu = 1 \ . \qquad 3.128$$

b) $n = 0, k = 1$ (See equations 3.78, 3.79, and 3.90.)
c) $n = 0, k = 1, a = 0$ (See equations 3.81, 3.82 and 3.83.)

Note that $k = 1$ introduces the term

$$\frac{k}{(k - \sigma_s)} e^{-\sigma a}\left[e^{+(k - \sigma_s)t} - 1\right] \qquad 3.129$$

d) $n = 1, k = 1$

$$\Phi(a) = \frac{Q_3}{2}\frac{e^{-\sigma a}}{(k - \sigma_s)}\left[e^{+(k - \sigma_s)t} - 1\right] \quad , \qquad k \neq \sigma_s \ , \qquad 3.130$$

$$\Phi(a) = \frac{Q_3}{2} t\, e^{-\sigma a} \quad , \qquad k = \sigma_s \ , \qquad 3.131$$

e) $n = 0, k = 0, a = 0$

$$\Phi(t) = \frac{Q_3}{2k}\left\{e^{+kt}\, E_1(\sigma_s t) - \ln(1 - \nu) - E_1\left\{(\sigma_s t)(1 - \nu)\right\}\right\} \quad , \qquad \nu<1 \qquad 3.132$$

$$\Phi(t) = \frac{Q_3}{2k}\left\{e^{+kt}\, E_1(\sigma_s t) - \ln(\nu - 1) + \mathrm{Ei}\left\{\sigma_s t(\nu - 1)\right\}\right\} \quad , \qquad \nu>1 \qquad 3.133$$

$$\Phi(t) = \frac{Q_3}{2k}\left\{e^{+kt}\, E_1(\sigma_s t) + \ln\sigma_s t + \gamma\right\}, \qquad \nu = 1 \qquad 3.134$$

f) $n = 1$, $k = 1$, $a = 0$

$$\Phi(t) = \frac{Q_3}{2}\,\frac{1}{(k - \sigma_s)}\left[e^{+(k - \sigma_s)t} - 1\right] \qquad 3.135$$

8. Finite plane volume source, $Q_3(R) = Q_3\, e^{+k\nu}$, $B(\sigma R) = Ae^{+\alpha\sigma R}$, $\eta = \nu/(1 - \alpha)$.

$$\Phi(a) = \frac{Q_3}{2} A \int_0^t d\nu e^{+k\nu}\, E_{n+1}\left\{(\sigma_s\nu + \sigma a)(1 - \alpha)\right\} \qquad 3.136$$

a) $\eta = 0$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{e^{+kt}\, E_1\left\{(\sigma_s t + \sigma a)(1 - \alpha)\right\} - E_1\left\{\sigma a(1 - \alpha)\right\} + e^{-\eta\sigma a}\left[E_1\left\{\sigma a(1 - \alpha)(1 - \eta)\right\} - E_1\left\{(\sigma_s t + \sigma a)(1 - \alpha)(1 - \eta)\right\}\right]\right\} \qquad \eta < 1 \qquad 3.137$$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{e^{+kt}\, E_1\left\{(\sigma_s t + \sigma a)(1 - \alpha)\right\} - E_1\left\{\sigma a(1 - \alpha)\right\} + e^{-\eta\sigma a}\left[Ei\left\{(\sigma_s t + \sigma a)(1 - \alpha)(\eta - 1)\right\} - Ei\left\{\sigma a(1 - \alpha)(\eta - 1)\right\}\right]\right\} \qquad \eta > 1 \qquad 3.138$$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{e^{+kt}\, E_1\left\{(\sigma_s t + \sigma a)(1 - \alpha)\right\} - E_1\left\{\sigma a(1 - \alpha)\right\} + e^{-\sigma a} \ln\left(\frac{\sigma_s t + \sigma a}{\sigma a}\right)\right\}, \qquad \eta = 1 \qquad 3.139$$

b) $n = 0$, $a = 0$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{e^{+kt}\, E_1\left\{\sigma_s t\,(1 - \alpha)\right\} - \ln(1 - \eta) - E_1\left\{\sigma_s t(1 - \alpha)\right\}\right\}, \qquad \eta < 1 \qquad 3.140$$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{e^{+kt}\, E_1\left\{\sigma_s t(1 - \alpha)\right\} - \ln(\eta - 1) + Ei\left\{\sigma_s t(1 - \alpha)\right\}\right\}, \qquad \eta > 1 \qquad 3.141$$

$$\Phi(a) = \frac{Q_3}{2k} A \left\{ e^{+kt} E_1 \left\{ \sigma_s t(1 - \alpha) \right\} + \ln[\sigma_s t(1 - \alpha)] + \gamma \right\}_{\eta = 1} \qquad 3.142$$

9. Finite plane volume source, cylinder viewed on the longitudinal centerline, $Q_3(R) = Q_3$, $B(R) = 1 + k\sigma R$, (Figure 3.16).

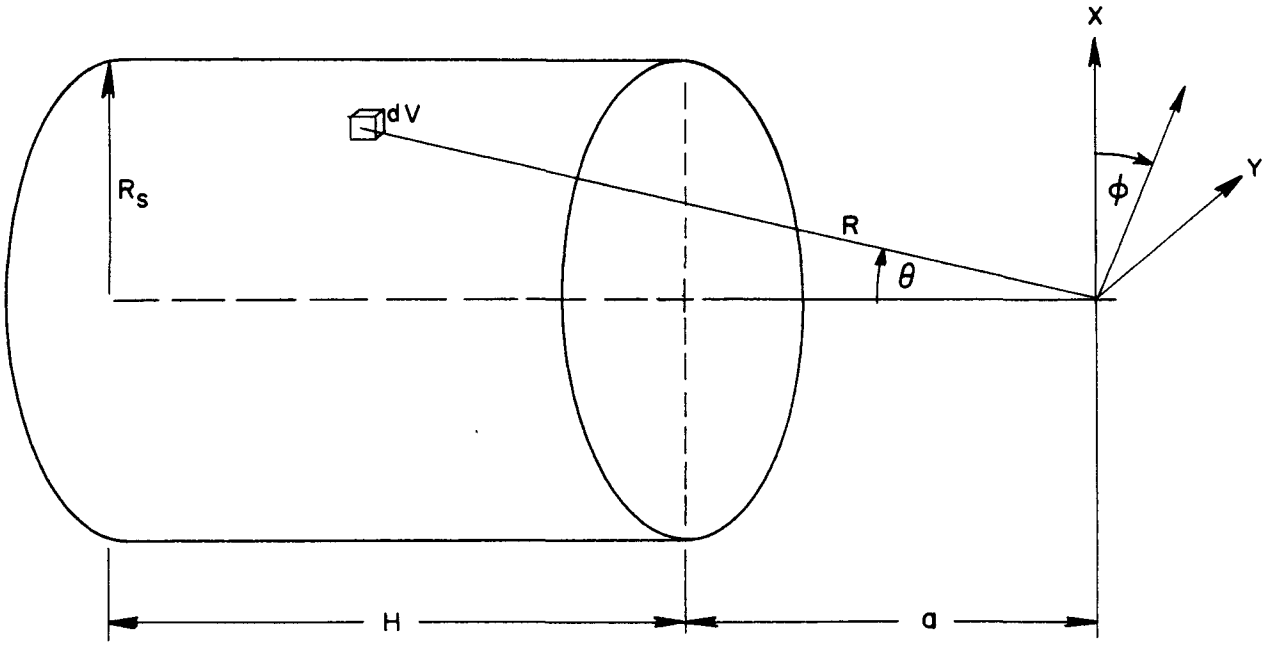

Figure 3.16

Cylindrical Volume Source Geometry

$$\Phi(a) = \frac{Q_3}{2} \int_1^{u_0} du\, u^{-n-1} \int_{au}^{(a+H)u} dR[1 + k\sigma_s(R - au) + k\sigma au]$$

$$e^{-\sigma_s(R-au) - \sigma au} \qquad 3.143$$

An upper limit of the flux is obtained when

$$u_0 = \left[1 + \left(\frac{R_s}{a} \right)^2 \right]^{1/2} ,$$

and a lower limit is obtained when

$$u_0 = \left[1 + \left(\frac{R_s}{a + H} \right)^2 \right]^{1/2} .$$

The result is,

$$\Phi(a) = \frac{Q_3}{2\sigma_s}\Big\{(1+k)\,[E_{n+2}(\sigma a) - u_0^{-n-1}E_{n+2}(\sigma a u_0)]$$
$$+ k\sigma a\,[E_{n+1}(\sigma a) - u_0^{-n}E_{n+1}(\sigma a\, u_0)]$$
$$- (1+k)\,[E_{n+2}(\sigma a + \sigma_s H) - u_0^{-n-1}E_{n+2}(\sigma a\, u_0 + \sigma_s H\, u_0)]$$
$$- k(\sigma a + \sigma_s H)\,[E_{n+1}(\sigma a\ \sigma_s H) - u_0^{-n}E_{n+1}(\sigma a\, u_0 + \sigma_s H\, u_0)]\Big\} \qquad 3.144$$

D. Cylindrical surface source, cylindrical well viewed on centerline, Q_3 = constant, $B(\sigma R) = 1 + k\sigma R$; (Figure 3.17).

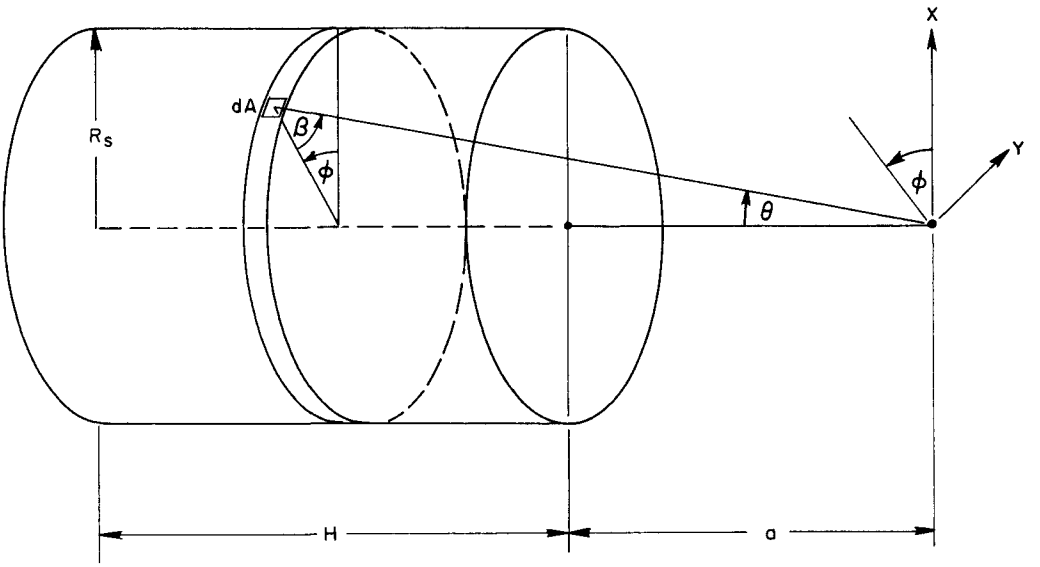

Figure 3.17

Cylindrical Surface Source Geometry

$$\Phi(a) = \frac{Q_2}{2}\int_{\beta_1}^{\beta_2} d\beta\ (1 + k\sigma R_s \sec\beta)\sin^n\beta\cos^m\beta\ e^{-\sigma R_s \sec\beta} \qquad 3.145$$

1. $n = 0, m = 0, k = 0$

$$\Phi(a) = \frac{Q_2}{2}\Big\{\sec i(\sigma R_s, \beta_2) - \sec i(\sigma R_s, \beta_1)\Big\} \qquad 3.146$$

2. $n = 0, m = 0, k = 0, a = 0$

$$\Phi(a) = \frac{Q_2}{2}\sec i(\sigma R_s, \beta_2) \qquad 3.147$$

Chapter 3 Problems

1. Derive the formula for:

   a) the uncollided flux from a shielded infinite half space.
   b) the uncollided flux from a shielded line source.
   c) the uncollided flux from a shielded plane source.
   d) the uncollided flux from a shielded plane volume source.

2. Repeat the derivation in 1-b, c, and d.

   a) after multiplying by $\cos \theta$
   b) after multiplying by $e^{-k\sigma R}$
   c) after multiplying by $1 + k\sigma R$
   d) after multiplying by $1 + k_1 \sigma R + k_2 (\sigma R)^2$

3. Derive the formula for the flux from a plane volume source with an exponential distribution (equation 3.78).

4. NEUTRON ATTENUATION*

The predominant sources of neutrons that occur in a reactor come, of course, from the fission process. There are approximately 2.5 neutrons per fission of uranium-235, emitted with energy described by the well-known fission spectrum.[1] The distribution of the neutron source is the same as the fission or power distribution which, in a thermal reactor, is the same as the thermal neutron flux distribution. The magnitude of the neutron source is given by

$$Q_3 \left(\frac{\text{neutron}}{\text{cm}^3\ \text{sec}}\right) = 3.1 \times 10^{10}\ \frac{P}{V} \left(\frac{\text{fission}}{\text{cm}^3\ \text{sec}}\right)\ 2.5 \left(\frac{\text{neutron}}{\text{fission}}\right) \quad . \quad 4.1$$

Quite often it is sufficient to assume that the neutron source strength is uniformly distributed over the core volume as given by equation 4.1 For deep penetration problems, it is only those neutrons with energies above one Mev that are significant. Such neutrons constitute about two-thirds of the total number of neutrons emitted in the fission process. Those with energies equal to or greater than six Mev contribute only two or three per cent to the total number of neutrons. This may be taken into account by adjusting the number of neutrons per fission in equation 4.1 to suit a particular situation for which it applies.

The main source of thermal neutrons arises from the process of moderation of the fast neutrons. In case the thermal neutron flux at the core reflector interface is desired, it may be obtained from the core physics calculations or else estimated from the known power and fission cross section, that is, from the solution of the equation

$$\frac{P}{V} \left(\frac{\text{watts}}{\text{cm}^3}\right)\ 3.1 \times 10^{10} \left(\frac{\text{fissions}}{\text{watt sec}}\right) = \Phi_s \left(\frac{\text{neut cm}}{\text{cm}^3\ \text{sec}}\right) \sigma_f \left(\frac{\text{fissions}}{\text{neut cm}}\right)$$

4.2

for $\overline{\Phi}_s$, the average thermal neutron flux in the core. This expression, incidentally, would have to be equal to an integral of the product, $\Phi(E)\sigma_f(E)$ for a reactor in which fissions caused by neutrons other than those of thermal energy predominate.

Other sources of neutrons may assume importance in the shield design under certain circumstances. One of these is the source of one-Mev neutrons generated by the reaction $O^{17}(n,p)N^{17}$. This is a high threshold

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*While specific references to constants and data necessary for the calculation of neutron attenuation are given in the text, a complete summary of constants is given in Section 7 of ANL-5800, Reactor Physics Constants. In addition, the bibliography will contain literature that is not always specifically referred to in the text.

[1] Watt, B.E., "Energy Spectrum of Neutrons from Thermal Fission of U^{235}," Phys. Rev. 87, 1037 (1952).

reaction, ~10 Mev, and the unstable product has a 4.2-second half-life, but it still may contribute to the neutron dose when a rapidly circulating coolant containing oxygen is brought near inhabited areas. The cross section and procedures for calculating this activation have been described.[2,3,4,4a]

Other neutrons may originate from photo-neutron reactions[5,6] in D_2O or Be that may be located around the reactor core. This reaction usually does not contribute to the shielding problem. Similarly the delayed neutrons from the fission process do not ordinarily receive consideration in reactor shielding problems.

There are several reasons for needing to know the neutron distributions in a shield. The most obvious reason, although usually not the one of primary importance, is to know what shield thickness is necessary to limit the biological dose of neutrons (see Table 4.1) to required levels at the exterior of the shield. The neutron distribution in the shield will be required to determine heating effects and radiation damage in the shield materials. The range of operation of instruments may be predicted, or their locations determined, by these neutron distributions. More importantly, at least from a strict shielding viewpoint, is that the capture gamma-ray source term is dictated by the neutron absorption density in the shield, and, of course, this is a function of the neutron distribution in terms of space and energy. These capture gamma rays are usually the most predominant source of radiation escaping the shield, and, therefore, their description is a critical calculation for shield thickness as well as for heating, instrumentation, etc. This is particularly true of a gamma-ray-controlled shield such as concrete. Inasmuch as the strength of capture gamma-ray sources is a function of neutron energy by virtue of the energy dependence of the neutron absorption cross section, the capture gamma-ray source distribution may be quite complex.

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[2] Rockwell, T., III, Editor, TID-7004, Reactor Shielding Design Manual, McGraw-Hill & D. Van Nostrand, (March, 1956) p. 37.

[3] Roys, P. A., and K. Shure, "Production Cross Section of N^{16} and N^{17}," Nucl. Sci. Eng., 4(#6) 536, (October 1958).

[4] Henderson, W. J., and P. R. Tunnicliffe, "The Production of N^{16} and N^{17} in the Cooling Water of the NRX Reactor," Nucl. Sci. Eng. 3(2) 145 (February 1958).

[4a] Rocklin, R. S., "Data Sheet #28, Fission Neutron Cross Sections for Threshold Reactions," Nucleonics 17 (#1) 54, (January 1959).

[5] Hogerton, J. F., and R. C. Grass, The Reactor Handbook, Vol. 1, Physics, AECD-3645, McGraw-Hill Book Co., (1955) p. 158.

[6] deSaussure, G., "Calculation of the Photo-neutron Flux in the Water Near the Bulk Shielding Reactor," ORNL-2545 (July, 1958).

Table 4.1

NEUTRON FLUX - TO - DOSE CONVERSION FACTORS
FIRST COLLISION RBE DOSE

For Neutrons at Energy E, Multiply Neutron Flux, $\Phi(E)$, neutron/cm^2 - sec by Conversion Factor, F, to Obtain Dose in mrem/hr

Neutron Energy, E	Conversion Factor, F, mrem/hr neut/cm^2-sec	Neutron Energy, E	Conversion Factor, F, mrem/hr neut/cm^2-sec
10 Mev	0.13	0.025 Mev	0.010
5 Mev	0.11	0.01 Mev	0.0036
2.5 Mev	0.094	0.001 Mev	0.0026
1.0 Mev	0.097	100 ev	0.000054
0.5 Mev	0.068	10 ev	0.000072
0.25 Mev	0.047	1.0 ev	0.00016
0.1 Mev	0.030	0.1 ev	0.00043
0.05 Mev	0.019	0.025 ev	0.00086

From "Protection Against Neutron Radiation up to 30 Million Electron Volts," NBS Handbook 63, National Bureau of Standards (April 10, 1957)

The problem of calculating the neutron attenuation in a shield is one of calculating both spatial and energy distributions. In principle this problem can be solved; for specialized situations, such as a point isotropic source of fission neutrons in water, much work has been done.[7,8] While the results agree well with the experimental observations, the practical significance is not as great as it should be inasmuch as a reactor shield is more complicated in both geometry and material disposition.

Probably the best summary of the more sophisticated ways of determining neutron distributions for shielding purposes is given by Goldstein.[9] A more recent summary, although not as detailed, is given by E. P. Blizard.[10]

[7] Aronson, R., J. Certaine, H. Goldstein and S. Preiser, "Penetration of Neutrons from a Point Isotropic Fission Source in Water," NYO-6267 (September 22, 1954).

[8] Aronson, R., J. Certaine, and H. Goldstein, "Penetrations of Neutrons from Point Isotropic Monoenergetic Sources in Water," NYO-6269 (December 15, 1954).

[9] Goldstein, H., "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," U.S. Govt. Printing Office (May 1, 1957).

[10] Blizard, E. P., "The Shielding of Nuclear Reactors," Paper P/2162 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June 1958.

The British,[11] French[12] and Russians[13] all presented approaches to the problem at the 1958 Geneva Conference. They include experimental work to support the theory and to enable application to shield design problems. The shielding work presented at this conference has been summarized by Jones.[14]

It is possible to determine shield materials and thicknesses from observations of an already existing reactor shield. This approach may result in gross overestimates and lack of detailed knowledge of the neutron or, for that matter, all radiation, distributions. The result is likely to be a more expensive shield than necessary and, perhaps, an inadequate shield in one or several respects, although this certainly does not mean that experimental data on existing shields should be ignored.

There is an approach which is between the two extremes. This involves the calculations of the distributions by approximate methods that are not too time-consuming, but will still yield useful results in terms of radiation distributions. A comparison of these results with values derived from existing shields and other experimental data will indicate whether the calculations are seriously in error. It will be the goal of this section to present those approximate methods which are of value in predicting neutron flux levels throughout the shield.

A useful engineering concept in neutron-attenuation analysis is that of "removal theory." It applies to the attenuation of fast neutrons, i.e., those of energies equal to or greater than 1 Mev as they occur in the fission spectrum. This concept was introduced by Albert and Welton[15,16] and has since been given a firmer foundation and a somewhat different conceptual basis by the work of Blizard[17,18,19] and others in the Oak Ridge

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[11] Cooper, C., J. D. Jones and C. C. Horton, "Some Design Criteria for Hydrogen-Metal Reactor Shields," Paper P/84 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[12] Bourgeois, P., et al., "Methods and Experimental Coefficients Used in the Computation of Reactor Shielding," Paper P/1190 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[13] Broder, D. L., et al., "The Study of Spatial and Energetic Distribution of Neutrons in Various Medium," Paper P/2147 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[14] Jones, J. D., "Radiation Shielding," Engineering, 186 568 (October 31, 1958).

[15] Albert, R. D., and T. A. Welton, "A Simplified Theory of Neutron Attenuation and Its Application to Reactor Shield Design," WAPD-15 (November 30, 1950).

[16] Rockwell, T., III, Reactor Shielding Design Manual, TID-7004, McGraw-Hill & D.van Nostrand (March, 1956) pp. 6-48.

[17] Blizard, E. P., Ann. Rev. Nucl. Sci. 5, 73 (1955)

[18] Chapman, G. T., and C. L. Storrs, "Effective Neutron Removal Cross-Sections for Shielding," AECD-3078 (ORNL-1843) (September 19, 1955).

[19] Hogerton, J. F., and R. C. Grass, The Reactor Handbook, Vol. I, Physics, AECD-3645, McGraw-Hill Book Co., (1955) p. 671.

Shielding Group. The value of this work has also been demonstrated at the Brookhaven Shielding Facility[20,21] and in England.[22]

The central concept of the removal theory is the removal cross section. This quantity can best be defined by describing an ideal experiment for measuring it. Consider, as in Figure 4.1 an infinite medium composed of material #1 in which there is a plane source of neutrons with a certain energy spectrum, usually a fission spectrum. At some distance from the source, depending on the nature of the medium and upon the energy and angular distribution of the source, the neutron distribution will have a characteristic shape that is independent of the energy response of the detecting instrument used to measure it.

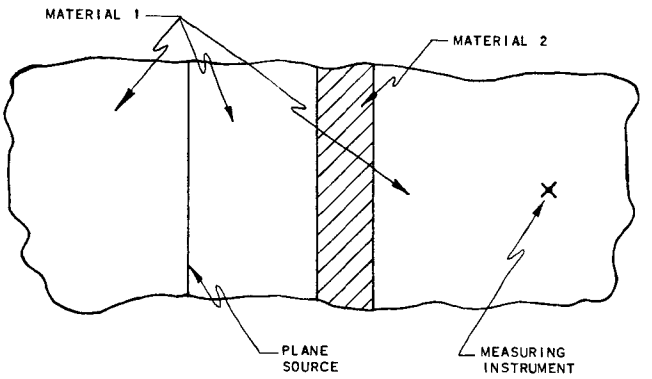

Figure 4.1

Arrangement for Removal Cross-Section Measurement

A slab of another material, called material #2, is now introduced into the medium, and the neutron flux in the medium of material #1 is measured at a sufficient distance from the slab so that the characteristic flux shape is again established. If the slab of material #2 is now removed, leaving a void in the position which it occupied, the flux as measured at this same point will increase. If this experiment is repeated for a range of slab thicknesses and positions, and if it is found that the attenuation caused by the slab of material #2 can be described by an exponential function of a constant of proportionality times its thickness, this constant is then defined to be the removal cross section of material #2 with respect to material #1.

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[20] Schamberger, R. D., and F. J. Shore, "A Review of the Brookhaven Shielding Program," BNL-2713 (September 1, 1955).

[21] Schamberger, R. D., and F. J. Shore, "The Effective Removal Cross Section of Iron," BNL-2714 (November 22, 1955).

[22] Avery, A. F., and R. A. Dugdale, "Attenuation Studies in the Lids Pool," AERE-R/R-2558 (April, 1958).

In cases in which this method is directly applicable, the neutron attenuation of the shield is accomplished principally by one dominant material, usually water or other hydrogenous material, and the effect of introducing other materials, structure, etc., is described by use of the removal cross section. The experiment described above is, of course, highly idealized, and in both measurement and application geometrical compromises are made. Thus, most removal cross sections have been measured in the Lid Tank Facility[23] at Oak Ridge, which consists of a large tank of water containing a 28-inch diameter disc source of fission neutrons. Hence, the values[24] so obtained are removal cross sections with respect to water as the principal neutron attenuating substance (see Tables 4.2, 4.3 and Figure 4.2).

The concept of removal cross section came about because of the attempt to express the attenuation of fission neutrons within a material located in a hydrogenous medium by a simple exponential. Knowledge of the rapid moderation of the once-collided fission neutrons made this a reasonable assumption, and experimental measurements soon proved that

Table 4.2

REMOVAL AND TOTAL (at 8 Mev) CROSS SECTIONS FOR VARIOUS ELEMENTS

(A = atomic weight; N = Avogadro's number; σ_R = microscopic removal cross section; σ_T = microscopic total cross section; Σ_R = macroscopic removal cross section; Σ_T = macroscopic total cross section; ρ = density; Σ/ρ = mass attenuation coefficient)

Element	A	$\frac{N}{A} = \frac{0.6023 \times 10^{-24}}{A}$	σ_R (barns)	$\frac{\Sigma_R}{\rho} = \frac{N\sigma_R}{A}$ ($cm^2\ g^{-1}$)	σ_T at 8 Mev (barns)	$\frac{\Sigma_T}{\rho}$ at 8 Mev ($cm^2\ g^{-1}$)
Hydrogen[a]	1.008	5.98×10^{-1}	1.00 ± 0.05	$(5.98 \pm 0.3) \times 10^{-1}$	1.10	6.58×10^{-1}
Lithium	6.94	8.70×10^{-2}	1.01 ± 0.04	$(8.79 \pm 0.35) \times 10^{-2}$	1.80	1.57×10^{-1}
Beryllium	9.01	6.70×10^{-2}	1.07 ± 0.06	$(7.17 \pm 0.43) \times 10^{-2}$	1.70	1.14×10^{-1}
Boron[a]	10.82	5.57×10^{-2}	0.97 ± 0.10	$(5.40 \pm 0.54) \times 10^{-2}$	1.50	8.36×10^{-2}
Carbon	12.01	5.02×10^{-2}	0.81 ± 0.05	$(4.07 \pm 0.24) \times 10^{-2}$	1.60	8.03×10^{-2}
Oxygen[a]	16.06	3.76×10^{-2}	0.99 ± 0.10	$(3.72 \pm 0.37) \times 10^{-2}$	1.20	4.51×10^{-2}
Fluorine[a]	19.00	3.17×10^{-2}	1.29 ± 0.06	$(4.09 \pm 0.20) \times 10^{-2}$	1.60	5.07×10^{-2}
Aluminum	26.98	2.23×10^{-2}	1.31 ± 0.05	$(2.92 \pm 0.12) \times 10^{-2}$	1.80	4.01×10^{-2}
Chlorine[a]	35.46	1.70×10^{-2}	1.2 ± 0.8	$(2.0 \pm 1.4) \times 10^{-2}$	2.40	4.08×10^{-2}
Iron	55.85	1.08×10^{-2}	1.98 ± 0.08	$(2.14 \pm 0.09) \times 10^{-2}$	3.20	3.46×10^{-2}
Nickel	58.69	1.03×10^{-2}	1.89 ± 0.10	$(1.90 \pm 0.10) \times 10^{-2}$	3.45	3.55×10^{-2}
Copper	63.54	9.50×10^{-3}	2.04 ± 0.11	$(1.94 \pm 0.11) \times 10^{-2}$	3.65	3.47×10^{-2}
Tungsten	183.92	3.27×10^{-3}	2.51 ± 0.55	$(8.21 \pm 1.81) \times 10^{-3}$	4.90	1.60×10^{-2}
Lead	207.21	2.91×10^{-3}	3.53 ± 0.30	$(1.03 \pm 0.09) \times 10^{-2}$	5.20	1.51×10^{-2}
Bismuth	209.00	2.88×10^{-3}	3.49 ± 0.35	$(1.01 \pm 0.10) \times 10^{-2}$	5.10	1.47×10^{-2}
Uranium	238.07	2.53×10^{-3}	3.6 ± 0.4	$(9.1 \pm 1.0) \times 10^{-3}$	6.10	1.54×10^{-2}

[a] Derived σ_R values from measurements behind compounds of the elements

From Chapman, G. T., and C. L. Storrs, "Effective Neutron Removal Cross Sections for Shielding," AECD-3978 (ORNL-1843)(September 19, 1955)

[23] Clifford, C. E., "The ORNL Shield Testing Facility," ORNL-402 (November 4, 1949)

[24] Chapman, G. T., and C. L. Storrs, "Effective Neutron Removal Cross Sections for Shielding," AECD-3978 (ORNL-1843)(September 19, 1955).

Table 4.3

REMOVAL CROSS SECTIONS FOR VARIOUS COMPOUNDS

Compound	R, Microscopic Removal Cross Section (barns)		Compound	R, Microscopic Removal Cross Section (barns)	
Boric Oxide, B_2O_3	4 30	0.41	Lithium Fluoride, LiF	2 43	0 34
Boron Carbide, B_4C	4 3	0 4[a]	Oil, CH_2	2 84	0 11
	5 1	0 4[b]	Paraffin, $C_{30}H_{62}$	80 5	5 2
Fluorothene, C_2F_3Cl	6.66	0 8	Perfluoroheptane, C_7F_{16}	26 3	0.8
Heavy Water, D_2O	2 76	0 11			

[a]Twelve 1-in -thick B_4C slabs against source plate (Exp 42)
[b]Solid slab against source plate (Exp 33)

From Chapman, G. T , and C L Storrs, "Effective Neutron Removal Cross Sections for Shielding," AECD-3978 (ORNL-1843)(September 19, 1955)

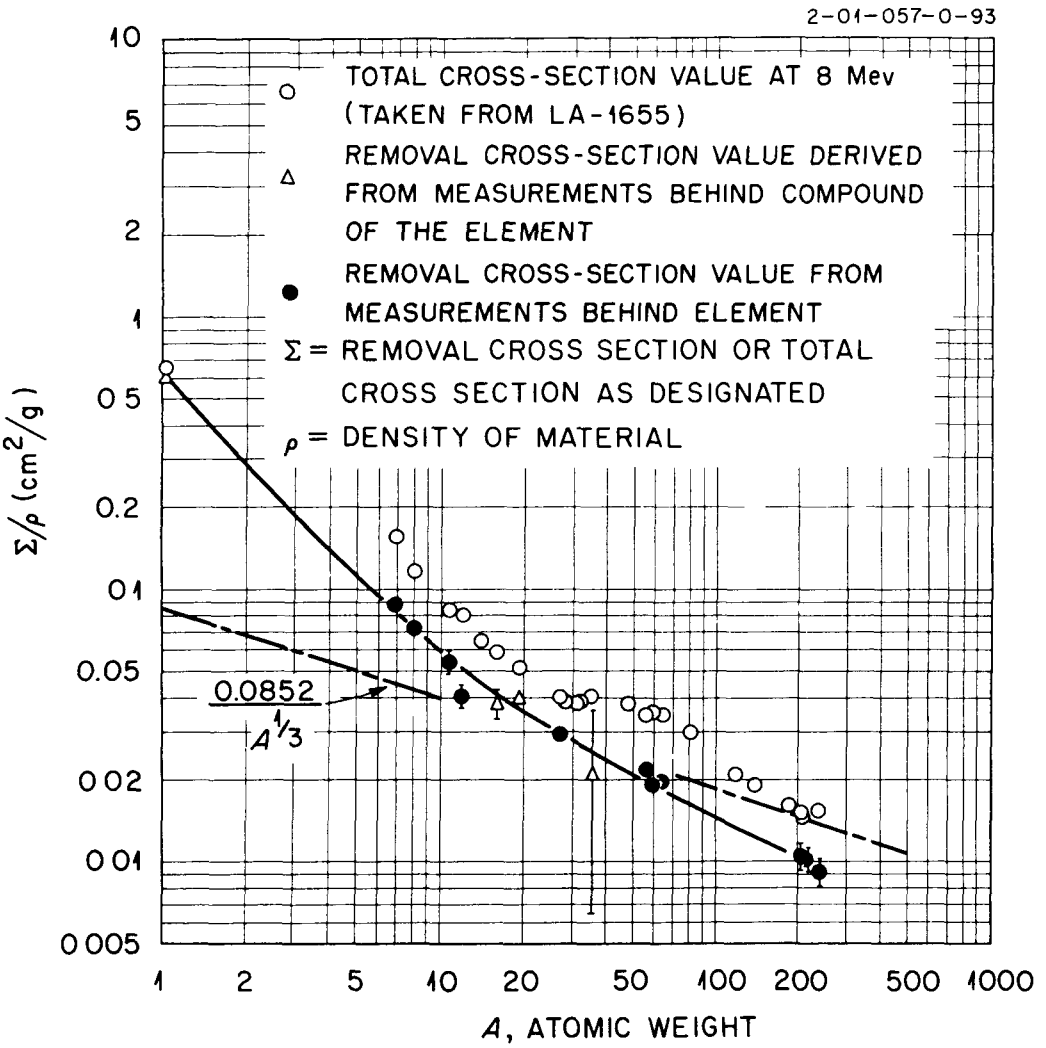

Figure 4. 2 Neutron Shielding Ability per Unit Weight of a Material as a Function of Atomic Weight.

this was a practical thing to do. It was recognized that there were limitations, since this removal cross section was not really a constant and, therefore, could be used only under limited circumstances. Subsequent investigations did substantiate the use of the removal cross section under certain limitations and with limited accuracy. For a more elaborate discussion of the theoretical and experimental implications of removal theory the reader is referred to the summary by Goldstein.[25] Some of the more obvious limitations will be briefly given here.

In the first place, the use of removal theory requires a fission neutron source and a hydrogenous medium, the layers of other materials being immersed in the hydrogenous medium and near the source. There have been measurements on the effect of the slab location on the removal cross section[26,27] which indicate that this is not a serious problem except when the slab is near the detector. In this case the once-collided neutrons will not have had ample hydrogen in which to be moderated and absorbed; as a result, there will be an inordinate number of intermediate-energy neutrons to give erroneous readings for removal cross-section measurements. A layer of material, for example, iron, located too near the shield exterior surface would result in the leakage of a large number of intermediate-energy neutrons. Additional, more difficult, calculations would be required in order to describe adequately such a situation.[28] To a limited degree this same effect would be noticeable for the removal cross section of a material distributed uniformly throughout the hydrogenous medium. The difference in the removal cross section due to this distributed effect has been measured[29] and found to be relatively small. This does, however, require some consideration when removal cross sections are utilized to describe attenuation in concrete. The fact that measured relaxation lengths in concrete are very nearly the reciprocal of the calculated removal cross sections (please refer to Table 4.4) indicates that the concept of a removal cross section is applicable, but assurance that there is adequate hydrogen present is also necessary to ensure adequacy of the shield.

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[25] Goldstein, H., "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," U. S. Govt. Printing Office (May 1, 1957) p. 255 ff.

[26] Schamberger, R. D., and F. J. Shore, "The Effective Removal Cross-Section of Iron," BNL-2714 (November 22, 1955).

[27] Cooper, C., J. D. Jones and C. C. Horton, "Some Design Criteria for Hydrogen-Metal Reactor Shields," Paper P/84 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[28] Wood, D. E., "Intermediate Energy Neutron Leakage through Iron," Nucl. Sci. and Eng. 5 (#1) 45 (January, 1959).

[29] Trubey, D. K., and G. T. Chapman, "Effective Neutron Removal Cross Sections of Carbon and Oxygen in Continuous Media," Applied Nuclear Physics Division Annual Report, ORNL-2081, (November, 1956) Chap. 41, p. 167.

Table 4.4

NEUTRON CONSTANTS FOR CONCRETES

Concrete[a]		Density, gm/cm³	κ_s, cm⁻¹	D_s, cm	σ_r, cm⁻¹ Calc.	σ_r, cm⁻¹ Exptl.	λ, cm (Exptl.)	τ, cm²	D_f, cm
01-	a	2.33	0.0744	1.00	0.078	0.083[b]		349	1.40
	b				0.072			581	
	c							1370	
02-	a	2.30	0.130	0.531	0.088		11.1[c]	101	1.14
	b	2.22	0.102	0.707	0.081			205	1.51
	c							1530	2.04
03-	a	2.39	0.127	0.558	0.087	0.086[e]		119	1.23
	b							222	1.39
	c							1250	1.65
	-	2.47			0.087[f]		11.7[f]		
	-	2.47					11.8[d]		
04-	a	2.35	0.108	0.715	0.088			179	1.31
	b							316	1.57
	c							1450	1.73
FP-	a	4.68	0.474	0.348	0.13	0.15[b]		51.6	
	b	4.57	0.405	0.463	0.12			92.4	
	c							275	
BA-	a	3.46	0.200	0.484	0.095		8.0[c]	86.9	1.10
	b	3.35	0.154	0.722	0.084			179	1.29
	c							1650	1.62
M-	a	3.62	0.379	0.418	0.11			75.4	
	b	3.52	0.322	0.559	0.10			141	
	c							708	
M		3.29					8.75[d]		
BR-	a	4.26	0.477	0.387	0.12		6.3[c]	79.1	.889
	b							153	
	c							925	
I-1		3.62	0.581	0.315	0.13				
2		3.62	0.454	0.442	0.11				
MS1-	a	4.5	0.504	0.365	0.13			69.0	
	b	4.4	0.436	0.472	0.12			131	
	c							713	
MS2-	a	4.74	0.504	0.356	0.13			56.0	0.814
	b	4.64	0.436	0.458	0.12			113	0.932
	c							958	1.00
LS-	a	4.64	0.586	0.269	0.14	0.13[b]			
	b	4.54	0.528	0.324	0.13				
LS		4.24					7.4[d]		
Fe		7.85			0.17	0.19[b]			

[a] Assumed water retention: a = 100%; b = 50%; c = 0%

[b] Unpublished results from the collimated source ANL Shield Facility, "Shielding Experiments," Chapter 6.2 p. 172, Nuclear Reactor Experiments (Princeton: D. Van Nostrand Co., 1958).

[c] Hogerton, J. F., and R. C. Glass, "Reactor Handbook," AECD-3645, Vol. 1, Physics, (McGraw Hill Book Co., March, 1955) p. 674,

[d] Hanford data, Table 5.

[e] Zobel W., and T. A. Love, "Applied Nuclear Physics Division Report," ORNL-2081 (September, 1956) p. 161.

[f] Blizard, E. P., and J. M. Miller, "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 29, 1958).

There should be a variation of removal cross section with the thickness of the slab, but this has not been observed to date. Furthermore, the effect of geometry on the removal cross section must be properly accounted for. This includes the fact that the slab thickness changes the path geometry of the neutrons in water, as well as the fact that the neutron sources in practice are finite and isotropic rather than constituting the ideal infinite plane collimated source. Measurements of collimated source removal cross sections for iron have been made[30,31,32] and the results are cross sections that are approximately 10% higher than the usual values. These cannot be utilized in the usual calculations involving removal cross sections (see formula 3.21) but, no doubt, could give results that would indicate the relative merits of two materials, or could be used in equations adjusted to be used with these cross sections.

Many of the preceding effects are due to the removal process occurring at different distances from the source and are to some extent a measurement of the energy dependence of the removal cross section. Attempts to define a theoretical removal cross section that is energy-dependent have been made[33,34] for application to neutron-distribution calculations. The results have been very satisfactory, but at the present time it is not clear just how much accuracy was gained by the energy dependency of the removal cross section. This may result in a refinement of the use of removal cross sections, but probably the greatest difference will be noted in the case of a material with a rapidly varying cross section in the energy range of interest. As previously discussed, there is not an appreciable variation of the removal cross section of iron with the slab location, even though there is a reasonable spectrum change. This merely points out the fact that, in general, there is not much variation in the total neutron cross section of iron in the high-energy region, nor is there much variation for most materials. The variation of the fission neutron spectrum as filtered through water is given in the Shielding Design Manual.[35]

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[30] Schamberger, R. D., and F. J. Shore, "A Review of the Brookhaven Shielding Program" BNL-2713 (September 1, 1955).

[31] Schamberger, R. D., and F. J. Shore, "Cross-Sections with a Collimated Fission Neutron Source," Reactor Shielding Information Meeting, WASH-292 Pt. 3 (May, 1955).

[32] Unpublished results from ANL Shield Facility.

[33] Cooper, C., J. D. Jones and C. C. Horton, "Some Design Criteria for Hydrogen-Metal Shields," Paper P/84 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[34] Bourgeois, J., et al., "Methods and Experimental Coefficients Used in the Computation of Reactor Shielding," Paper P/1190 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[35] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill & D. Van Nostrand, March, 1956) p. 55.

Studies have been made of the relative values of the removal cross section as compared to the total neutron cross section.[36,37,38] The results indicate that the neutrons which are important are those of about 8 Mev (please refer to Figure 4. 2). The decrease in number of fission neutrons with increase in energy, coupled with the more rapid attenuation of neutrons in water with decrease in energy, tends to define an effective energy band at about eight Mev in a situation where fission neutrons are attenuated by water. This, in addition to the relatively small variation of the total cross section for most materials at high energy, gives the reason that the concept of a constant removal cross section is relatively valuable.

In addition to the removal cross section, an attenuation kernel[39] for fission neutrons in water has been determined. This has been demonstrated to match very nicely a theoretically determined function:

$$N(x) = 5.4(\theta x)^{0.29} \exp\left[-0.928\,(\theta x)^{0.58} - \theta\sigma_0 x - (1-\theta)\,\sigma_r x\right] \quad , \qquad 4.3$$

where N(x) gives the neutron flux at x centimeters from an infinite-plane source emitting 2.5 neutrons per cm^2 sec in a parallel beam. The macroscopic oxygen removal cross section is denoted by σ_0 and the macroscopic removal cross sections of the other heavy elements in the shield are symbolized as σ_r. The volume fraction of water in the shield is denoted as θ. While this formula is somewhat clumsy for analytical purposes, it may be quite easily utilized by numerically integrating it in an expression such as equation 3.48. For convenience to this end, note Figure 4.3.

Another concept frequently utilized in fast neutron calculations is that of the measured attenuation length. Presumably, if the fast neutron relaxation length, or attenuation length, i.e., the distance in which the fast neutron flux is reduced by a factor of e, is measured for a concrete shield, it may be applied in the calculations for another concrete shield of the same composition. Appropriate corrections must be made. Of these the geometrical correction will not be much of a problem, but a spectrum change would be rather complex, and extrapolation to slightly different concretes may not be easy to do properly. There has not been very much reliable data in the past,[40,41,42] but there are increasing efforts in this

[36] Chapman, G. T., and C. L. Storrs, "Effective Neutron Removal Cross Sections for Shielding," AECD-3978, (ORNL-1843)(September 19, 1955).

[37] Blizard, E. P., "The Shielding of Nuclear Reactors," Paper P/2162 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[38] Goldstein, H., "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," U. S. Govt. Printing Office (May 1, 1957), p. 275.

[39] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill & D. van Nostrand, March, 1956), p. 52.

[40] Hogerton, J. F., and R. C. Grass, The Reactor Handbook, Vol. I, Physics, AECD-3645 (McGraw-Hill Book Co., 1955) p. 674.

[41] Clifford, C. E., "The ORNL Shield Testing Facility," ORNL-402 (November 4, 1949).

[42] Pratt, W. W., et al., "The Attenuation Characteristics of Brookhaven Concrete," ANL-145 (December 17, 1951).

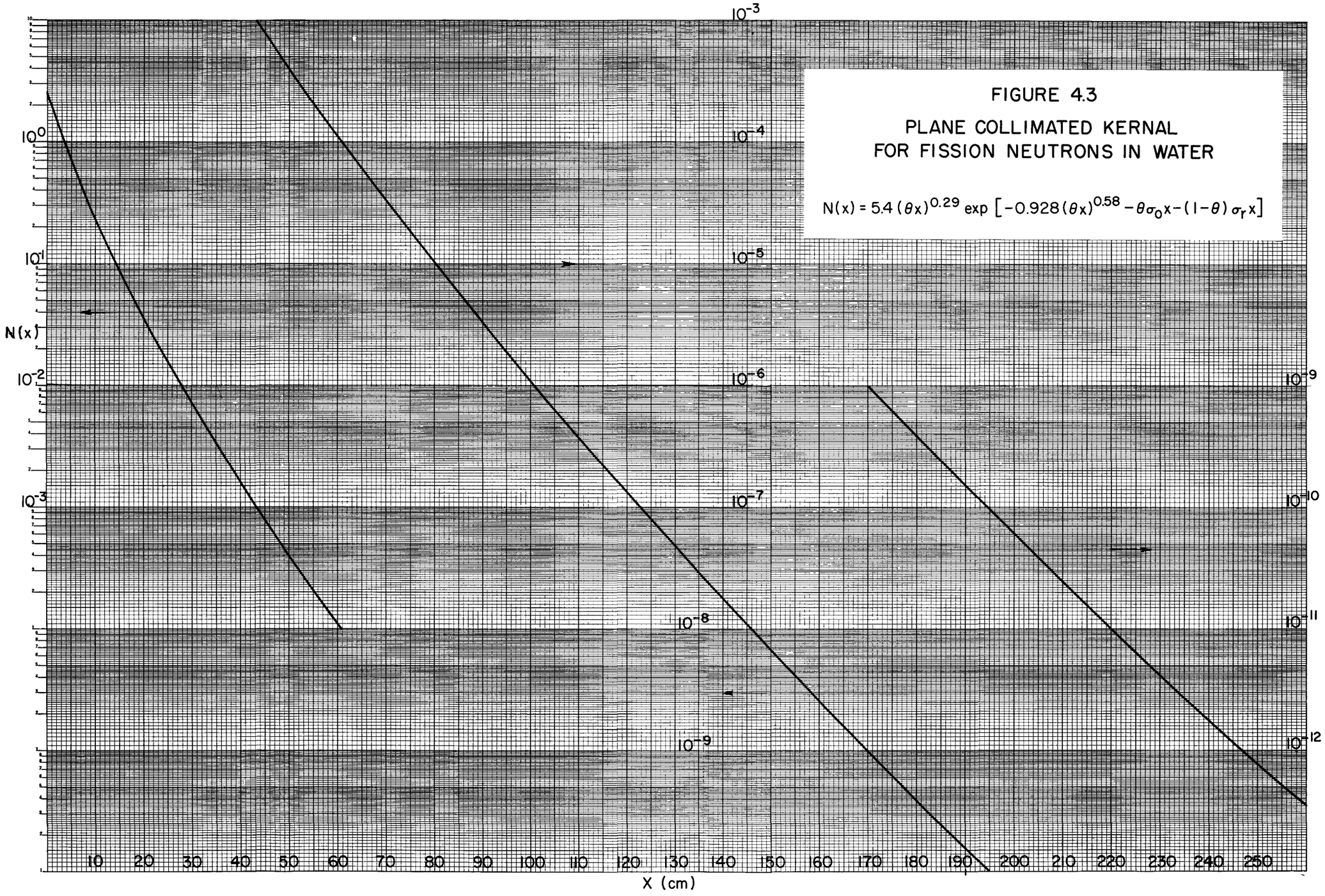

FIGURE 4.3

PLANE COLLIMATED KERNAL
FOR FISSION NEUTRONS IN WATER

$$N(x) = 5.4\,(\theta x)^{0.29} \exp\left[-0.928\,(\theta x)^{0.58} - \theta\sigma_0 x - (1-\theta)\,\sigma_r x\right]$$

direction[43,44,45,46,47,48,49,50] some of which include the effects of removing water by successive heating (see Table 4.5). Preliminary work indicates that the removal cross section is quite nearly the value of the reciprocal of the measured attenuation length for fast neutrons[51,52] and may be applied in a similar fashion.

Table 4.5

MEASURED RELAXATION LENGTHS FOR SEVERAL CONCRETES AS A FUNCTION OF CONCRETE TEMPERATURE
(Sulfur Foil Activation Method)

Concrete Type	Density, gm/cm³	Relaxation Length, cm Temperature				Reference
		Unheated	100° C	185° C	320° C	
Iron-Limonite	4.231	7.4	7.9	9.0	10 0	b
Magnetite-Limonite	3.39	8 9	9.7	10.5	10.5	b
Magnetite	3.59(?)	8.75	9 52	9.75	10.1**	a,c,d
Ordinary	2.47	11.8*	-	-	-	a,c,d

*11.8 cm for first 2 ft; 12.4 cm for next 2 ft Measurements at higher temperatures in progress.

**10.1 cm for first 28 in.; 11 1 after 28 in.

[a]Wood, D.E., "Neutron Attenuation of Magnetite Concrete Heated to 100° C," HW-53395 (October, 1957)

[b]Bunch, W. L., "Attenuation Properties of High-Density Portland Cement Concretes as a Function of Temperature," HW-54656 (January 22, 1958), p. 41.

[c]Wood, D.E., and E. G. Peterson, "Neutron Attenuation in Magnetite Concrete Heated to 200° C," HW-55372 (March 25, 1958)

[d]Wood, D.E., and E.G. Peterson, "Neutron Attenuation of Magnetite Concrete Heated to 100° C, 200° C, and 300° C," Trans. Amer. Nuclear Society, 1 (#1) 67 (June, 1958).

[43] Davis, H.S., "Thermal Considerations in the Design of Concrete Structures for Shielding Atomic Power Plants," Paper #9, Nuclear Congress, Chicago (March 17-24, 1958).

[44] Bunch, W.L., "Attenuation Properties of High Density Portland Concretes As a Function of Temperature," HW-54656 (January 22, 1958).

[45] Blizard, E.P., and J. M. Miller, "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 22, 1958).

[46] Peterson, E.G., "Neutron Attenuation in Ordinary Concrete," HW-55938 (1958).

[47] Blosser, T.V., "A Study of the Nuclear and Physical Properties of the ORNL Graphite Reactor Shield," ORNL-2195 (September 8, 1958).

[48] Wood, D.E., and E.G. Peterson, "Neutron Attenuation Characteristics of Magnetite Concrete Heated to 100°C, 200°C, and 300°C," Transactions of the American Nuclear Society, 1, #1, 67 (June 1958)

[49] Fryar, R.M., and E.G. Peterson, Private Communication

[50] Dyson, J.A., and J.R. Harrison, "The Dependence of Fast Neutron Attenuation in Portland Concrete on Its Hydrogen Content," AERE-RP/R-1942, Harwell, England (1956).

[51] Blizard, E.P., "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 29, 1958).

[52] ANL-5800 "Reactor Physics Constants"(1952) Table 7-18, p. 477.

In applying any of the concepts to the design of concrete shields, one must be cautious regarding the water content. While the high-energy flux will no doubt be fairly well represented by the removal cross section and/or the attenuation length, the prediction of the total neutron absorption density must be considered also. If there is an abundance of water in the shield the neutrons that leave the fast range will be rapidly moderated and absorbed. In such a case the total neutron absorption density may be adequately represented by the thermal neutron absorption density and the total neutron dose will be represented by the fast neutron dose. If, however, no hydrogen is present, the flux of neutrons in the intermediate energy range will tend to increase and may well be the main contributor to the total neutron dose and the total neutron absorption density. Examples of materials in which this may occur are thick layers of sodium,[52a] iron[53] or concrete with a low water content. The water content has recently been determined to be adequate at 7%,[54,55] although more water would cause an increase in the neutron attenuation.

The concepts of "removal theory" and fast neutron relaxation length are not sufficiently complete to do a shield design. In order to predict neutron absorption distributions in the shield materials, the distribution of fluxes of the neutrons of lower energy must first be predicted. Perhaps the first method one would think of is the usual two-group diffusion method of calculating reactor fluxes:

$$D_f \nabla^2 \Phi_f - \sigma_{af} \Phi_f + Q_f = 0 \qquad 4.4$$

$$D_s \nabla^2 \Phi_s - \sigma_{as} \Phi_s + Q_s = 0 \qquad 4.5$$

The solution of these equations would yield a thermal neutron flux which, in turn, would give a first approximation to the neutron absorption distribution when multiplied by the thermal neutron absorption cross section. These equations would differ from the two-group equations for reactor core calculations in that the source term of fission neutrons would be zero in the shield. There are at least two discrepancies in this use of the two-group equations. First, this fast neutron flux would not necessarily be the same as the fast neutron flux determined by removal theory and, therefore, could not be used to determine neutron dose rate or radiation damage.

52a Grotenhuis, M., H. I. Kraig, and A. E. McArthy, "EBR-II Shield Design Calculations" Reactor Engineering Division Quarterly Report, ANL-5571, Section I, p. 146, (July, 1956).

53 Wood, D. E., "Intermediate Energy Neutron Leakage through Iron," Nuc. Sci. and Eng. 5, (#1) 45 (January, 1959).

54 Blizard, E. P., "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 29, 1958).

55 Dyson, J. A., and J. R. Harrison, "The Dependence of Fast Neutron Attenuation in Portland Concrete on Its Hydrogen Content," AERE-RP/R-1942 (1956).

It really is a quantity which, with properly determined constants, serves as a source term of the thermal neutrons, and it includes all neutrons above thermal energy. This, of course, is proper in the type of calculation for which it was intended, namely, core calculations. Secondly, the quantity σ_{af}/D_f is not properly defined for such use in that it refers to the reciprocal of the age of fission neutrons in a material when the fission neutrons are born in that material. As the neutrons are not born in shield materials the spectrum would be different from that in the core. This spectrum, of course, differs more and more from the core spectrum as the distance from the core becomes greater.

An extension of this method to six energy groups has been used[56] to determine the neutron distributions in the SRE. Measurements are now being made[57] to test the validity of the calculations. Other analyses have been made with up to twenty neutron energy groups.[58,59,60]

In order to avoid some of the difficulties of the two-group method, such as the determination of τ and D_f in shield materials, an adjusted version may be adopted. This probably is more correctly called a one-group problem:

$$D_s \nabla^2 \Phi_s - \sigma_{as} \Phi_s + Q_s = 0 \quad , \tag{4.6}$$

where Q_s is determined from the loss of fast neutrons, and the other constants are the usual ones. The fast neutrons in this case are those having an energy of one Mev and greater as determined by removal theory. It is these neutrons which are the controlling influence in deep shield penetrations.

Assuming local slowing down, the source term "Q" may be expressed as the negative divergence of the fast current:

$$Q_s = -\text{Div } J_f \quad , \tag{4.7}$$

since this is the number of fast neutrons lost per unit volume per sec.

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[56] Fillmore, F. L., and R. J. Doyas, "Analysis of Neutron Flux in the Shielding of the Sodium Reactor Experiment," NAA-SR-2953 (October 15, 1958)

[57] Anderson, F. D., "An Experimental Method of Measuring Neutron Streaming at Reactor Vessel Pipe Penetrations," Trans. Amer. Nucl. Soc. 1 (#2) 70 (December, 1958).

[58] Hungerford, H. E., and R. F. Manley, "Shielding the Enrico Fermi Fast Breeder Reactor," Nucleonics 16 (#11) 120 (November, 1958).

[59] Grotenhuis, M., H. I. Kraig, and A. E. McArthy, "EBR-II Shield Design Calculations," Reactor Engineering Division Quarterly Report, Section I, ANL-5571 (July 1956) p. 146.

[60] Abrams, M. J., and J. Agresta, "Multi-group Treatment of the 2.8-Kev Sodium Resonance," Trans. Amer. Nucl. Soc. 1 (#2) 134 (December, 1958).

It is pessimistic to assume that the fast neutron current is equal in magnitude to the fast neutron flux:

$$|J_f| = \Phi_f \quad ,$$

so that the source term is given approximately by

$$Q_s = -\text{Div}\,(\underline{\Omega}\ \Phi_f) \quad . \tag{4.8}$$

Here $\underline{\Omega}$ is a unit vector having the direction of the gradient of the flux. For a fast neutron flux which is exponential in plane geometry,

$$\Phi_f(x) = \Phi_f(o)\, e^{-\sigma x} \text{ neut/cm}^2 \text{ sec} \quad ; \tag{4.9}$$

the source term becomes,

$$Q_s(x) = \sigma \Phi_f(o)\, e^{-\sigma x} \text{ neut/cm}^3 \text{ sec} \quad , \tag{4.10}$$

and the thermal neutron flux may be expressed in slab geometry as,

$$\Phi_s(x) = A\, e^{\kappa_s x} + B\, e^{-\kappa_s x} + \frac{\sigma \Phi_f(o)}{D_s(\kappa_s^2 - \sigma^2)}\ e^{-\sigma x} \text{ neut/cm}^2 \text{ sec.} \tag{4.11}$$

The coefficients A and B are to be determined by boundary conditions as in similar problems that occur in reactor theory or diffusion theory. The boundary conditions include the usual continuity of flux and current at the interfaces of regions, and a known thermal neutron flux at the initial boundary. At black boundaries the flux may be set equal to zero at the black surface, or at the extrapolated boundary, and in the last material, which is usually quite thick, the flux may be set equal to zero at infinity. Note that "σ" need not be a cross section. It is the negative slope of the logarithm of the fast neutron flux in the region under consideration:

$$\sigma = -\frac{d}{dx} \ln \Phi_f(x) \text{ cm}^{-1} \quad . \tag{4.12}$$

Often the fast neutron flux is not exponential. It may then be approximated by an exponential, where σ is determined by (refer to Figure 4.4)

$$\sigma = \frac{1}{a} \ln \frac{\Phi_f(o)}{\Phi_f(a)} \text{ cm}^{-1} \quad . \tag{4.13}$$

This makes possible the use of equation 4.11, which makes the calculation a relatively simple one by hand computing methods. The fast neutron flux in the source term, equation 4.10, may also be represented as a sum of two exponentials in circumstances where this would yield the necessary

accuracy. Of course, if a computing machine is available, the source term may be put in point by point and this approximation may be avoided. The use of a machine will also make possible a somewhat more representative geometry.

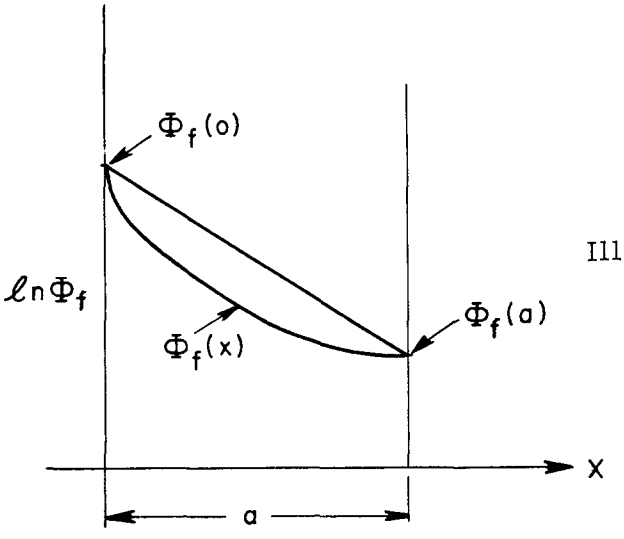

Figure 4.4

Illustration of Fast Neutron Flux for the Determination of σ.

The application of Fick's law,

$$J_f(x) = - D_f \text{ grad } \Phi_f(x) \text{ neut/cm}^2 \text{ sec} \quad , \qquad 4.14$$

in equation 4.7 leads to a change in equation 4.11:

$$\Phi_s(x) = A\, e^{\kappa_s x} + B\, e^{-\kappa_s x} + \frac{\sigma^2 D_f \Phi_f(o)}{D_s(\kappa_s^2 - \sigma^2)}\, e^{-\sigma x} \text{ neut/cm}^2 \text{ sec.} \qquad 4.15$$

While this leads to more accurate representation of the thermal neutron flux[61] in a specific case, it is believed to be somewhat fortuitous since both approaches give results reasonably close to the measured values. The magnitude of the change is not a very great one and since the value of D_f to be used is not too obvious in all materials, the gain in accuracy is somewhat in doubt.

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[61] Duncan, D. S., "Application of Fast Neutron Removal Theory to the Calculation of Thermal Neutron Flux Distributions in Reactor Shields," NAA-SR-2380 (July 1, 1958).

If the procedure given by equations 4.6 and 4.10 is applied to the calculation of the thermal neutron flux in a thermal column, formulas for the thermal neutron flux that will be useful for shielding calculations and, on a comparative basis, for other studies, may be obtained. The usual diffusion equation with no source term is

$$\nabla^2 \Phi_s - \kappa_s^2 \Phi_s = 0 \quad , \tag{4.16}$$

where the flux may be represented as

$$\Phi_s(x,y,z) = X(x)\, Y(y)\, Z(z) \quad \text{neut/cm}^2 \text{ sec.} \tag{4.17}$$

Assuming the variables to be separable, it follows that

$$X''YZ + XY''Z + XYZ'' - \kappa_s^2 XYZ = 0 \tag{4.18}$$

and

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} - \kappa_s^2 = 0 \quad . \tag{4.19}$$

Defining the constants α^2, β^2, and γ^2 properly yields

$$\frac{X''}{X} - \kappa_s^2 + \frac{Y''}{Y} + \frac{Z''}{Z} = \alpha^2 - \beta^2 - \gamma^2 = 0 \tag{4.20}$$

which may be broken into parts that give the following solutions, assuming that the thermal neutron flux is zero for $y = \mp b$ and $z = \pm c$, (see Figure 4.5),

$$X(x) = A \exp\left[-(\kappa_s^2 + \alpha^2)^{1/2} x\right] \quad \alpha^2 = \beta^2 + \gamma^2 \tag{4.21}$$

$$Y(y) = B \cos \beta y, \qquad \beta = \frac{\pi}{2b}$$

and,

$$Z(z) = C \cos \gamma z, \qquad \gamma = \frac{\pi}{2c} \quad .$$

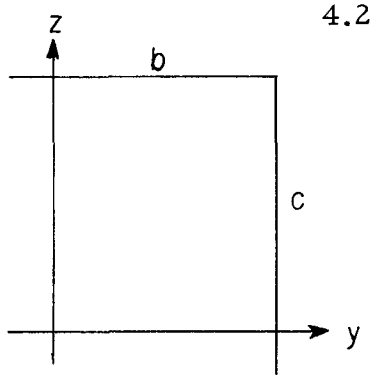

Figure 4.5

Geometry for the Rectangular Thermal Column

The thermal neutron flux in the absence of a source term is then

$$\Phi_s(x,y,z) = K \exp[-(\kappa_s^2 - \alpha^2)^{1/2} x] \cos\beta y \cos\gamma z \text{ neut/cm}^2\text{-sec} \quad . \quad 4.22$$

Note that the above expression, except for the cosine terms, corresponds to the approximate formula given[62] for thermal neutron attenuation in a thermal column. An approximate solution for the thermal column centerline thermal neutron flux in the presence of an exponential source term is

$$\Phi_s(x) = Ae^{kx} + Be^{-kx} + \frac{\sigma \Phi_f(o)\, e^{-\sigma x}}{D_s(k^2 - \sigma^2)} \text{ neut/cm}^2\text{sec} \quad , \quad 4.23$$

where the use of

$$k^2 = \kappa_s^2 + \alpha^2$$

instead of κ_s^2 accounts for the transverse leakage.

In the case of a cylindrical thermal column similar considerations yield the following thermal neutron flux (see Figure 4.6):

$$\Phi(r,x) = \Phi_s(o)\, J_0(\beta r) \exp[-(\kappa_s^2 + \beta^2)^{1/2} x] \text{ neut/cm}^2\text{sec} \quad , \quad 4.24$$

where

$$\beta = \frac{2.4048}{r_0} \quad .$$

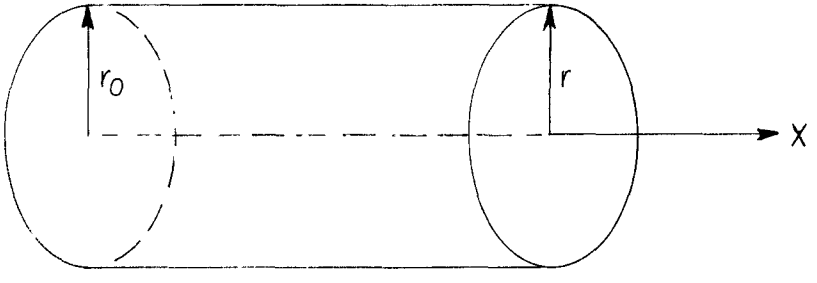

Figure 4.6. Geometry for a Cylindrical Thermal Column

The centerline thermal neutron flux with a source term becomes

$$\Phi(x) = A\, e^{kx} + Be^{-kx} + \frac{\sigma \Phi_f(o)\, e^{-\sigma x}}{D_s(k^2 + \sigma^2)} \text{ neut/cm}^2 \text{ sec} \quad , \quad 4.25$$

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[62] Hogerton, J. F., and R. C. Glass, Reactor Handbook Vol. I, Physics AECD-3645 (McGraw-Hill Book Co., March, 1955) p. 18

where, similarly,

$$k^2 = \kappa_s^2 + \beta^2 \quad .$$

While the one-group method described on pages 99 and 100 is not a "handbook" method that may be indiscriminately used, it has been checked against EBR-I data,[63,64] compared to other calculations for EBR-II design work,[65] and used as a basis for shield designs.[66,67] A comparison of this procedure to BSF data[68] indicates the accuracy of the approach in a water medium.

If, for some type of reactor, such as a fast reactor, or, for some region of a shield, such as a thick layer of iron, the thermal neutron flux is not an adequate representation of the absorption density of neutrons, an intermediate-energy group may be inserted:

$$D_i \nabla^2 \Phi_i - \sigma_{ai} \Phi_i + Q_i = 0 \quad . \qquad 4.26$$

Now Q_i is the previous Q_s in equation 4.6, and the Q_s there becomes

$$Q_s = \sigma_{ai} \Phi_i \quad \text{neut/cm}^3\,\text{sec} \quad . \qquad 4.27$$

Equation 4.11 now becomes

$$\Phi_i(x) = A\, e^{+\kappa_i x} + B\, e^{-\kappa_i x} + C\, e^{-\sigma_i x} \quad \text{neut/cm}^2\,\text{sec} \quad , \qquad 4.28$$

where

$$C = \frac{-\sigma \Phi_f(o)}{D_i \sigma^2 - \sigma_{ai}} \quad , \qquad 4.29$$

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[63] Butler, J. W., M. Grotenhuis, "Analysis of Results of EBR Shielding Measurements," Reactor Engineering Division Quarterly Report, ANL-5297 (July 1954) p. 63

[64] Grotenhuis, M., "Analysis of Results of EBR Shielding Measurements," Reactor Engineering Division Quarterly Report, ANL-5371 (January 1954) p. 134

[65] Grotenhuis, M., H. I. Kraig, and A. E. McArthy, "EBR-II Shield Design Calculations," Reactor Engineering Division Quarterly Report ANL-5571, Section I, (July, 1956) p. 146

[66] Grotenhuis, M., and J. W. Butler, "Experimental Boiling Water Reactor (EBWR) Shield Design," ANL-5544 (August, 1956)

[67] Duncan, D. S., "Results of Preliminary Shield Analysis in the 45.5 MW OMR," NAA-SR-2234 (November 15, 1958)

[68] Duncan, D. S., "Application of Fast Neutron Removal Theory to the Calculation of Thermal Neutron Flux Distributions in Reactor Shields," NAA-SR-2380 (July 1, 1958)

and the thermal neutron flux is

$$\Phi_s(x) = D\, e^{+\kappa_s x} + E\, e^{-\kappa_s x} + F\, e^{+\kappa_i x} + G\, e^{-\kappa_i x} + H\, e^{-\sigma x} \text{ neut/cm}^2\text{sec} \qquad 4.30$$

where

$$F = \frac{-\sigma_{ai} A}{D_s \kappa_i^2 - \sigma_{as}} \quad , \qquad 4.31$$

$$G = \frac{-\sigma_{ai} B}{D_s \kappa_i^2 - \sigma_{as}} \quad , \qquad 4.32$$

and

$$H = \frac{-\sigma_{ai} C}{D_s \sigma^2 - \sigma_{as}} \quad . \qquad 4.33$$

This is not too cumbersome to do by hand methods and results may be obtained[69] which can at least serve as a guide to thinking.

The use of diffusion theory with a fast group determined by removal theory has been extended to three groups[70,71] with good results. In both cases the materials were iron and water, the fast neutron group was based on removal theory, and the intermediate constants were strongly dependent on experimental results. The paper by Cooper et al.,[71] shows a most startling agreement between calculated and measured thermal neutron fluxes in rather extreme arrangements of iron and water. If machine services are available and the problem is of sufficient magnitude, the same type of approach may be used with many energy groups.[72]

In order to discuss the approach to be used in calculating neutron distributions, let us refer to specific types of shields and specific energy ranges of neutrons. Since fast neutrons are subject to a separate type of theory and are significant in that they are the chief contributors to the direct neutron dose, as well as the source for all other energy neutrons far out in the shield, consider them first.

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[69] Grotenhuis, M., H. I. Kraig, and A. E. McArthy, Reactor Engineering Division Quarterly Report, ANL-5571, Section I (March, 1956) P. 146.

[70] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, (McGraw-Hill & D. van Nostrand, March, 1956) p. 66

[71] Cooper, C., J. D. Jones, and C. C. Horton, et al., "Some Design Criteria for Hydrogen-Metal Shields," Paper P/84 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[72] Butler, M., and Cook, J., "A Reactor Shielding Code for the IBM 704," ANL-5859, unpublished.

Probably the best known neutron distribution is that of fast neutron dose in water as measured in the BSF[73] (Figure 4.7) and Lid Tank[74] (Figure 4.8) at Oak Ridge National Laboratory. This distribution has also been established theoretically; as a result the neutron distribution from a fission neutron source in water is well known. The theoretical plane collimated kernel, N(x), has been discussed earlier in this section (please refer to equation 4.3 and Figure 4.3). Taking formulas 3.21 and 3.22 and integrating for a shield consisting of water only, there is obtained

$$\Phi_f(R) = \frac{Q_3}{2\sigma_s} \int_{\mu_0}^{1} d\mu \; N(\rho), \text{ neut/cm}^2\text{sec}. \qquad 4.34$$

In effect this takes the data, which has been converted to plane collimated geometry, and integrates it in spherical geometry. It is important to note that the measurements were made of dose rate and are, therefore, somewhat more difficult to normalize to absolute neutron flux than the theoretical curve.

In a practical situation the fast neutrons from the core may be easily shielded by water, but the thickness of water that shields the fast neutrons will not be nearly thick enough to shield the associated gamma rays. Except for a swimming pool-type reactor, which is not efficiently shielded, some heavy material must be added to the water shield. The associated attenuation of the fast neutrons due to this added heavy material will be taken into account by utilizing removal cross section in the same formula:

$$\Phi_f(a) = \frac{Q_3}{2\sigma_s} \int_{\mu_0}^{1} d\mu \; N(\rho) \exp\left[-\frac{1}{\mu} \sum_{i=1}^{n} \sigma_i a_i \right] \text{neut/cm}^2\text{sec} \quad . \qquad 4.35$$

In order to make this concept valid, the heavy material must not be the last layer of the shield; the last layer must be water, or at least hydrogenous, and a minimum of about six inches thick. This will insure that the intermediate-energy neutrons do not assume too great an importance. Since formula 4.35 cannot be further integrated, a numerical integration is necessary in order to obtain numerical values of the flux.

For a simpler calculation an approximation for the plane collimated kernel may be made:

$$N(a) = A_1 e^{-\sigma_1 a} + A_2 e^{-\sigma_2 a} \quad . \qquad 4.36$$

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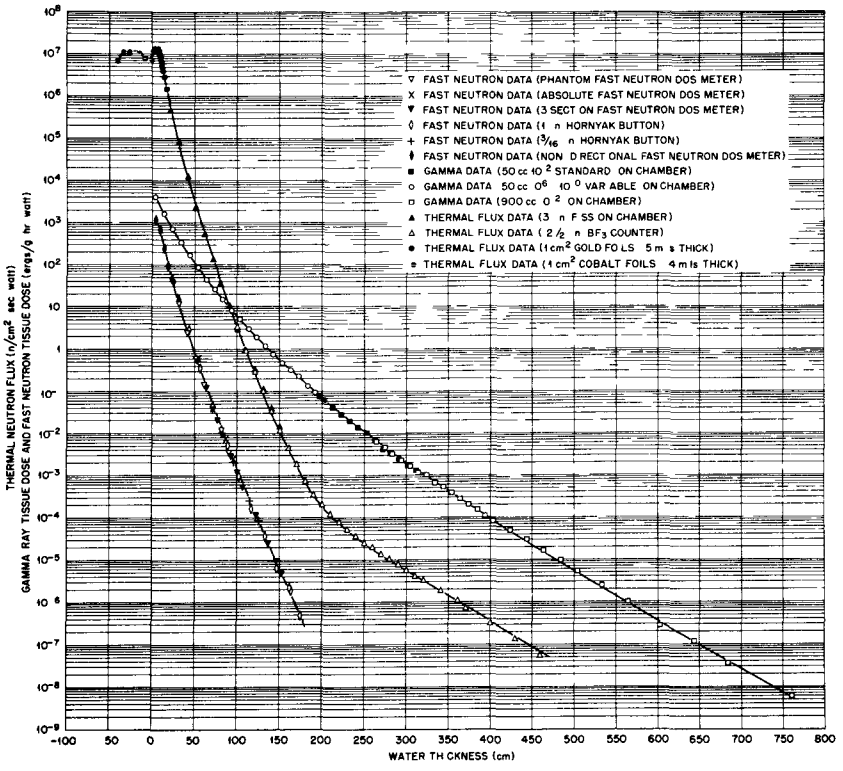
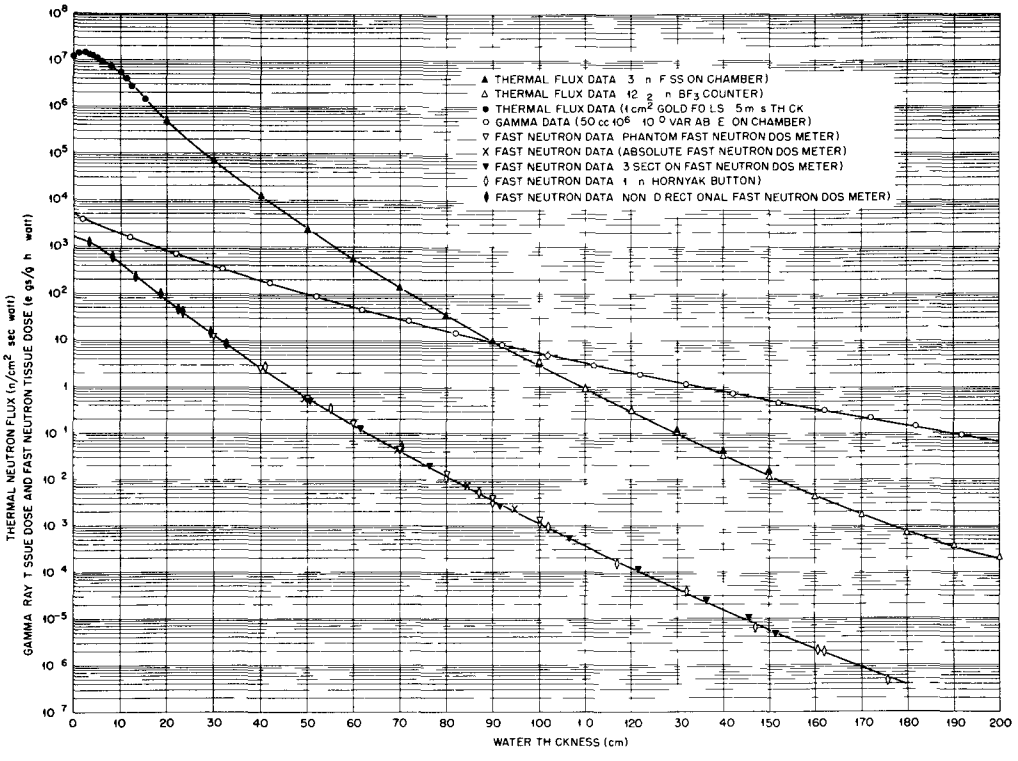
[73] "Attenuation In Water of Radiation from the Bulk Shielding Reactor: Measurements of the Gamma-Ray Dose Rate, Fast-Neutron Dose Rate, and Thermal Neutron Flux," prepared by the staff of the Bulk Shielding Facility, ORNL-2518 (July 8, 1958).

[74] Cady, D. W., and E. A. Warman, "Radiation Attenuation Measurements in Plain Water, Borated Water, and Oil Media in the Lid Tank Shielding Facility," Applied Nuclear Physics Division Annual Report ORNL-2389 (September 1, 1957).



NEUTRON AND GAMMA RAY ATTENUATION IN WATER AT
CENTERLINE OF BULK SHIELDING FACILITY



NEUTRON AND GAMMA RAY ATTENUATION IN WATER AT CENTERLINE
OF BULK SHIELDING FACILITY (EXPANDED SCALE)

Figure 4.7

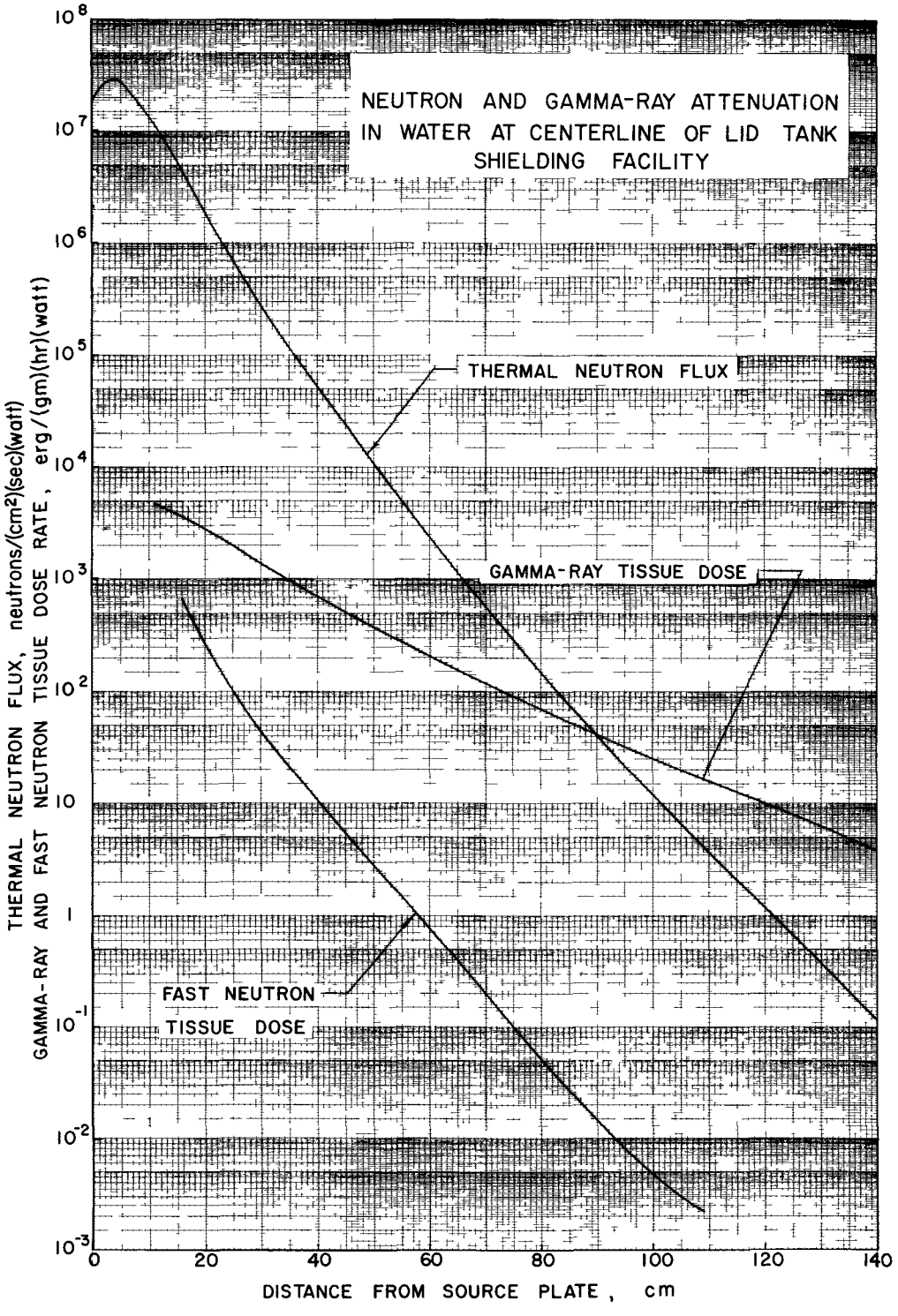

NEUTRON AND GAMMA-RAY ATTENUATION
IN WATER AT CENTERLINE OF LID TANK
SHIELDING FACILITY
THERMAL NEUTRON FLUX
GAMMA-RAY TISSUE DOSE
FAST NEUTRON
TISSUE DOSE
THERMAL NEUTRON FLUX, neutrons/(cm²)(sec)(watt)
GAMMA-RAY AND FAST NEUTRON TISSUE DOSE RATE, erg/(gm)(hr)(watt)
DISTANCE FROM SOURCE PLATE, cm
10⁸
10⁷
10⁶
10⁵
10⁴
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
0
20
40
60
80
100
120
140

Figure 4.8

The values of σ_1 and σ_2 to be used here can be found in available references[75,76] or obtained by refitting the data. Note that in the Design Manual the value of A_1/A_2 should be 35, not A_2/A_1. If the spherical approximation, formula 3.40 is also included, the resulting formula is

$$\Phi_f(a) = \frac{Q_3}{2\sigma_s} \frac{R_s}{(R_s + a)} [A_1 E_1(\sigma_1 R_1 + \sigma a) + A_2 E_1(\sigma_2 R_1 + \sigma a)] \text{ neut/cm}^2\text{ sec.} \qquad 4.37$$

This formula represents the fast neutron flux from a spherical source of fission neutrons shielded by water and metal. The choice of formula is determined by the accuracy of other quantities, such as source geometry, as well as time available and the accuracy that is required.

If the shield is all material other than water, or at least a minimum of water, such as a foot of reflector, formula 4.35 becomes

$$\Phi(a) = \frac{Q_3}{2\sigma_s} \int_{\mu_0}^{1} d\mu \exp\left[-\frac{1}{\mu} \sum_{i=1}^{n} \sigma_i a_i\right] \text{ neut/cm}^2\text{ sec.} \qquad 4.38$$

In this case the water attenuation will be expressed by a removal cross section which, while somewhat improper, and although the removal cross section is not a constant value, is accurate enough over a limited thickness of water. If the same plane approximation for a sphere is utilized, formula 4.37 becomes

$$\Phi(a) = \frac{Q_3}{2\sigma_s} \frac{R_s}{(R_s + a)} E_1(\sigma a) \text{ neut/cm}^2\text{ sec} \quad . \qquad 4.39$$

The subject of whether a substance like concrete may be treated by removal theory is often raised. As far as the fast neutrons are concerned, this theory should be adequate. This statement is supported by the fact that the measured fast neutron attenuation lengths are very nearly the same as the reciprocal of the calculated removal cross sections,[77,78] or, in the case of ordinary concrete, the measured removal cross section[79] (please refer back to Table 4.4). The real question, however, is whether

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[75] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill & D Van Nostrand, March, 1956) p. 69.

[76] Duncan, D. S., and H. O. Whittum, Jr., "Application of Fast Neutron Removal Theory to the Calculation of Thermal Neutron Flux Distributions in Reactor Shields," NAA-SR-2380.

[77] Bourgeois, J., et al., "Methods and Experimental Coefficients Used in the Computation of Reactor Shielding," Paper P/1190 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[78] Hanford work - personal communication.

[79] Blizard, E. P., and J. M. Miller, "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 29, 1958).

the fast neutrons that are removed are adequately removed, i.e., reduced to thermal energy rapidly. In order to be sure of this the concrete must contain an adequate amount of water.[80] For example, while the use of removal cross sections for calculating fast neutron attenuation in gravel will yield a fast neutron flux which is representative, it will make necessary detailed consideration of the neutrons in the intermediate energy range, which will tend to stream through the shield in large numbers. If adequate water is present, the intermediate neutron flux may be virtually ignored, particularly in cases where it is the product of $\sigma_a \Phi_n$ that is to be considered, such as in the determination of the capture gamma-ray source strength. In this case the absorption cross section decreases with energy faster than the neutron flux increases, and the resulting product decreases with neutron energy.

The next neutron energy to be considered is that of thermal neutrons. In situations where there is adequate hydrogen present, a simplified picture, such as given by equation 4.11 will be adequate to determine the thermal neutron flux and, subsequently, the neutron absorption density in a reasonably pessimistic fashion. The slab geometry of equation 4.11 is not a serious deviation from the truth in the farther portions of the shield, and even close to the core does not introduce great errors, usually less by a factor of two.

The value of σ to be used must be obtained from the fast neutron calculations, as indicated by equation 4.12 or 4.13. This number should be somewhat larger than the removal cross section for the region, since the true curve is always convex down from the exponential approximation. This deviation, which is at least in part due to the spherical geometry applied in calculating the fast neutron flux, should decrease with increasing distance from the core. Care must be taken that, in fitting the exponentials, the fast neutron flux does not become discontinuous at a boundary. The thermal neutron flux may not be as representative.

In order to apply equation 4.11 the boundary conditions must be set. At the core surface the thermal neutron flux must usually be specified. If this value is not available from core physics calculations it may be estimated by equation 4.2 or some other suitable method. The usual boundary conditions at shield interfaces are based on the requirements of continuity of the flux and of the current. At black boundaries or at an outer shield surface the flux may be set equal to zero at the extrapolated distance. The flux may also be permitted to go to zero at a boundary which is a black surface. This will of course result in a somewhat higher current into the black surface than if the flux is zero at the extrapolation distances. Care must be exercised in the case of a low-absorbing reflector followed by a

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[80] Blizard, E. P., and J. M. Miller, "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 29, 1958).

strong absorber such as boral. The boundary conditions given will no doubt be satisfactory as far as the low-absorbing reflector flux is concerned, but may over-estimate the thermal neutron flux attenuation by the boral, and, as a result, under-estimate the thermal neutron flux in the next region. It may be best to estimate a new boundary flux for the region following the boral using better estimates of the attenuation properties of boral.[81] It is convenient to use as a boundary condition for the last region that the flux goes to zero after an infinite thickness. This will require that "A" in equation 4.11 be zero and will simplify the problem for hand calculations without undue loss of accuracy.

A convenient device for solving the determinant that results for the thermal neutron flux for the entire shield is to work the problem in pairs of regions, the last of which is always infinite. As long as the regions are thick, in terms of units of diffusion length, the flux at the boundary of regions one and two will remain unchanged when region two is reduced in thickness from infinity to the proper finite thickness. This flux will then become the boundary flux for the next problem, which involves region two at its proper thickness and an infinite region three. The use of slab geometry will permit the coordinate of each region to begin at zero. A typical set of boundary equations will be:

$$\Phi_{s1}(o) = \text{a predetermined number},$$

$$\Phi_{s1}(a_1) = \Phi_{s2}(o),\ J_{s1}(a_1) = J_{s2}(o) \quad ,$$

and

$$\Phi_{s2}(\infty) = 0 \qquad (\therefore A_2 = 0) \quad .$$

The resulting determinant is

A_1	B_1	B_2	
1	1	0	$-N$
$e^{\kappa_{s1}a_1}$	$e^{-\kappa_{s1}a_1}$	-1	$C_2-C_1e^{-\sigma_1a_1}$
$-D_{s1}\kappa_{s1}e^{\kappa_{s1}a_1}$	$D_{s1}\kappa_{s1}e^{-\kappa_{s1}a_1}$	$-D_{s2}\kappa_{s2}$	$D_{s2}\sigma_2C_2-D_{s1}\sigma_1C_1e^{-\sigma_1a_1}$

A little thought in advance will indicate the regions which are not important. For example, a one-eight-inch aluminum tank will not affect the thermal neutron flux appreciably and, therefore, may be omitted from a calculation of the neutron flux. It may be worth considering later as a capture gamma-ray source, and at that time the thermal neutron flux may be assumed to be that which was calculated at the position of the aluminum.

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[81] Burris, W. R., "How Channeling Between Chunks Raises Neutron Transmission Through Boral," Nucleonics 16 (#1) 91 (January, 1958).

In case there is a region which is too important to be ignored, but still is only of the order of one diffusion length in thickness, the same procedure may be applied as before but in a three-region unit. This requires that a five-by-five determinant be solved. One must be careful to note, however, any regions of heavy materials in which this approach will not represent the thermal neutron flux adequately. If, for example, there is an iron region which is thick, that is, greater than something like two inches, the thermal neutron flux will not be correct. Judgment as to the validity must be based on the exact situation, that is, by noting the point where the primary thermal neutron flux becomes dominated by the thermal neutron flux created by moderation in iron. Since the true moderation of neutrons in iron is quite small compared to that predicted by this theory, that is to say, the theory is not applicable in this situation, the point at which the primary thermal neutron flux becomes dominated is the location beyond which the thermal neutron flux is not to be given credence. If it is not possible to guess pessimistically the thermal neutron flux in such a situation so that the resulting capture gamma rays are not a dominating source, then further steps must be taken. A similar sort of error exists in cases where this approach is used to represent the thermal neutron flux in thick layers of graphite or heavy water, except that the error is a pessimistic one rather than optimistic.

The further steps to be taken could include the insertion of an intermediate energy group (see equation 4.30). This problem may still be done by hand calculations. Another step to be taken is the utilization of existing data on iron-water shields[82,83] or iron-masonite shields.[84] This, of course, would be a wise move in any situation. The next step would be to go to multi-group machine calculations, which is certainly a labor-saving device but should not be regarded as a cure-all.

The last neutron energy to be discussed is that of the wide range of intermediate-energy neutrons, those with energies above thermal and below one Mev. As indicated in a previous paragraph these neutrons may be treated as a group if constants are available. These constants could be, under certain circumstances, τ and D_f, or their modified values.[85] The knowledge of the neutron spectrum necessary in order to determine the modified value of τ is usually not known; indeed, if it were, the problem

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[82] Rockwell, T., III, Editor TID-7004, Reactor Shielding Design Manual, McGraw-Hill & D. Van Nostrand, March, 1956.

[83] Cooper, C., J. D. Jones, and C. C. Horton, "Some Design Criteria for Hydrogen-Metal Reactor Shields," Paper P/84 of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[84] Fryar, R. M., and E. G. Peterson, Private Communication

[85] Grotenhuis, M., H. I. Kraig, and A. E. McArthy "EBR-II Shield Design Calculations," Reactor Engineering Division Quarterly Report ANL-5571 Section I, (July, 1956) p. 146.

would be solved. It is possible at times to estimate the spectrum and to adjust τ accordingly. It is also possible that τ may not be radically changed when the upper energy limit is reduced from the six Mev or so necessary for two-group calculations to the one Mev appropriate for the intermediate neutrons. In any case it is wise to note the effect of a change in τ on the problem, since it may well be that the uncertainty in τ does not seriously effect the knowledge of the neutron flux. Actually the use of τ is merely a way of determining the averaging process of the constants. By separating the intermediate range into a number of energy intervals the averaging process becomes simpler, but the solution of the problem becomes much more tedious, or it becomes necessary to rely on the assistance of a machine.

The constants for an intermediate group may be determined experimentally. For the particular case of iron in hydrogenous medium accurate work has been done.[86,87] It is true that a large amount of data may be necessary before such constants may be known for all materials, but it is also true that the same accuracy may not be required of some materials because of the type of reactor for which they are more likely to be used.

In order to improve upon the previously mentioned procedures it is necessary to mock-up shields or at least parts of shields in experimental facilities. It is still likely that the final result will not be known with certainty until the plant is built. This requires that measured conservatism be employed wherever possible. A common and practical procedure is to permit a portion of the shield to remain flexible until the preliminary operational tests are complete. This is in reality a bulk shield facility, but of course need not be carried to an extreme in order to be valuable, nor need it be an uneconomical procedure.

The problem of neutron transport in rare media is one for which the rough-and-ready formulas are not as ready as in, for example, the same problem for gamma rays. There are some formulas from the work of Biram and Tait[88] that will prove useful on occasions. The flux of neutrons returning to the source after reflection from a plane non-capturing earth is

$$\Phi_f(d) = \frac{Q_0}{10\pi d^2} \quad , \qquad 4.40$$

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[86] Cooper, C., J. D. Jones and C. C. Horton, "Some Design Criteria for Hydrogen-Metal Shields," Paper P/84, of the International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, June, 1958.

[87] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill & D. VanNostrand, March, 1956)

[88] Biram, M. B., and J. H. Tait, "The Scattering of Neutrons by the Walls of a Laboratory," AERE-T/R-563 (1950).

where d is the distance from the source to the earth scatterer. The same source placed midway between two non-capturing semi-infinite media, placed a distance 2 d apart, will produce a scattered flux at the source of magnitude

$$\Phi_f(d) = A \frac{2Q_0}{10\pi d^2} \quad , \qquad 4.41$$

where A is a factor between 1.5 and 4, depending on the angular distribution of the reflected neutrons. If this source is placed at the center of a spherical room whose inner wall is of radius "a" and whose outer wall is of radius "b" the flux of returning neutrons is

$$\Phi_f \cong \frac{3Q_0}{4\pi ab} \quad \frac{b-a}{l} \quad , \qquad 4.42$$

where l is the scattering mean free path. There are other references[89,90] containing basic information that will be of use in problems of neutron scattering in air. At the present time, however, the literature does not have a great deal of information on neutron scattering in air since there has been no great necessity for this type of calculation in the power and research reactor programs to date.

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[89] Weller, G. S., and B. J. Workman, "Single Scattering of Neutrons in Air," CVAC-211T (January 22, 1954).

[90] Weller, G. S., and B. J. Workman, "Angular Neutron Scattering Cross-Sections for Air," CVAC-251T (December 4, 1953).

Chapter 4 Problems

1. Calculate the fast neutron flux for the Oak Ridge Bulk Shielding Reactor. Check results for equation 4.34 against equation 4.38 or 4.39. Compare to data given in ORNL-2518.

2. Calculate the thermal neutron flux for the Oak Ridge Bulk Shielding Reactor by the one-group method. Compare to data in ORNL-2518 (Figure 4.7).

3. Calculate the neutron distributions for a reactor such as EBWR (see ANL-5544 and ANL-5607). What must be done in the thick iron region above the core?

4. Calculate the neutron fluxes in a graphite-reflected reactor such as CP-5. Include a thermal column.

5. GAMMA-RAY ATTENUATION*

Gamma rays may originate wherever neutrons are found. The sources of gamma rays that are important in shielding considerations are fission, neutron absorption, and neutron inelastic scattering. All of these processes occur in the reactor core. The largest contributor to the total gamma-ray energy release in the core is the fission process, which includes the prompt fission gamma rays as well as those released by the fission product decay. While the capture gamma rays are not usually a great proportion of the total gamma-ray energy release in the core, they are of higher energy and their more penetrating characteristics may give them greater importance at some distance from the core. The fission gamma rays with their lower energy normally prove to be the major source of radiation heating near the core. All of the gamma-ray-producing processes, excepting fission, also occur outside the core in the shield materials. Gamma rays produced in the shield are not only born closer to the exterior surface, but also are in general more energetic, so they commonly determine the shield thickness, (see Table 5.1 for the gamma-ray flux to dose conversion factors) as well as prove to be a factor in cooling requirements and instrumentation.

Table 5.1

GAMMA-RAY FLUX - TO - DOSE CONVERSION FACTORS

For gamma rays of energy E Multiply gamma-ray flux, $\Phi(E)$ in photons/cm^2-sec by conversion factor N, or multiply gamma-ray flux $\Phi(E)$ in Mev/cm^2-sec by conversion factor M, to obtain dose in mr/hr.

Gamma-Ray Energy E (Mev)	Conversion Factor N (-/cm^2-sec mr/hr)	Conversion Factor M (Mev/cm^2-sec mr/hr)
10	10.8×10^{-3}	1.08×10^{-3}
9	9.8	1.09
8	8.8	1 1
7	8.0	1.14
6	7.1	1 18
5	6.2	1.24
4	5.3	1.32
3	4.3	1.43
2	3.25	1.62
1	1.93	1.93
0.5	1.02	2.06

From Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill & D. Van Nostrand, March, 1956).

*While specific references to constants and data necessary for the calculation of gamma-ray attenuation are given in the text, a complete summary of constants is given in Section 7 of ANL-5800, Reactor Physics Constants. In addition, the Bibliography will contain literature that is not always specifically referred to in the text.

The fission process releases gamma rays at the instant of fission. These prompt fission gamma rays are not as energetic as the capture gamma rays, but are a much more copious source in the core and, therefore, must receive consideration. The major area of importance of these gamma rays is in heating materials in and near the reactor core. Recent studies[1,2,3] of gamma rays associated with fission indicate the spectrum involved. For shielding considerations it is more convenient to represent this spectrum by equivalent lines. Representative line spectra of the prompt fission gamma rays are reproduced here for convenience (Table 5.2).

The fission process is the origin of many unstable fission products. The radioactive decay of these fission products represents another source of gamma rays in the core, which is of about the same magnitude in total energy release per unit time as the prompt fission gamma rays, but has an energy spectrum of a lower maximum value. While it is possible to calculate the fission product gamma-ray spectrum from the known fission products, the task is rather forbidding unless the detail is really appropriate

Table 5.2

PROMPT FISSION GAMMA-RAY SPECTRA

Spectrum A*			Spectrum B		Spectrum C	
E (Mev)	N(e) (γ/fission)	M(E) (Mev/fission)	E (Mev)	M(E) (Mev/fission)	E (Mev)	M(E) (Mev/fission)
0.5	3.1	1.55				
1.0	1.9	1.90	1.0	3.450	1.0	3.45
1.5	0.84	1.26				
2.0	0.55	1.10	2.0	2.360	2.0	3.085
2.5	0.29	0.725				
3.0	0.15	0.450	3.0	1.175		
3.5	0.062	0.217				
4.0	0.065	0.260	4.0	0.477	4.0	1.035
4.5	0.024	0.108				
5.0	0.019	0.095	5.0	0.203		
5.5	0.017	0.094				
6.0	0.007	0.042	6.0	0.136	6.0	0.256
6.5	0.004	0.026	7.0	0.026		
	7.028	7.827		7.827		7.827

*Reproduced from Reactor Shielding Design Manual, Table 3.2[6] from data of R. L. Gamble.[5]

Note: In addition, Bertini, et al.,[4] estimate the effective prompt capture gamma-ray source from U^{235} in a thermal reactor to be 1.18 Mev per fission, based on a value of 0.184 for alpha (ratio of capture to fission) of U^{235}.

[1] Maienschien, F. C., et al., "Gamma-Rays Associated with Fission," Paper P/670, presented at the International Conference on Peaceful Uses of Atomic Energy held at Geneva, Switzerland (June, 1958).

[2] Gamble, R. L., "Prompt Fission Gamma-Rays from Uranium-235," Reactor Shielding Information Meeting, WASH-292, Pt. 3 (May, 1955).

[3] Peele, R. W., "The Energy Spectrum of Gamma-Rays Accompanying the Fission of Uranium-235," Reactor Shielding Information Meeting, WASH-292, Pt. 3 (May, 1955).

[4] Bertini, H. W., et al., "Basic Gamma-Ray Data for the ART Heat Deposition Calculations," ORNL-2113 (October, 1956).

[5] Gamble, R. L., "Prompt Fission Gamma-Rays from Uranium-235," WASH-292 Pt. 3 (September, 1955), p. 28.

and necessary. As a consequence, the first estimate of the fission product gamma-ray power behavior after shutdown[7] was a more macroscopic consideration based on early experiments. Subsequent experimental work and analysis indicated that the simple expression of Way and Wigner:

$$\Gamma(t) = 1.26\ t^{-1.2}\ \text{Mev/sec fission} \quad , \qquad 5.1$$

was quite accurate and very often as accurate as necessary. Refinements have been included[8] and comparisons have been made with more detailed compilations[9,10,11] of the fission product gamma-ray power after shutdown, so that the behavior of the fission product gamma rays may be fairly well predicted. A relatively simple, but informative, analysis[12] of the fission product power behavior after shutdown from cyclic or burst-type operation is based on equation 5.1. Representative line spectra during operation are tabulated (see Table 5.3) and shown graphically as a function of time after shutdown.[13]

Table 5.3

EQUILIBRIUM FISSION PRODUCT GAMMA-RAY SPECTRA

	Spectrum A				Spectrum B		
			M(E)			M(E)	
Energy Group	Energy Range (Mev)	Effective Energy (Mev)	Mev per Fission	Mev/(sec)(watt)* of Reactor Power	Effective Energy (Mev)	Mev per Fission	Mev/(sec)(watt) of Reactor Power
1	0.1 -0.4	0.4	0.645	2.0×10^{10}	1	5.16	1.6×10^{11}
2	0.4 -0.9	0.8	3.87	1.2×10^{11}			
3	0.9 -1.35	1.3	0.645	2.0×10^{10}	2	1.737	5.38×10^{10}
4	1.35-1.8	1.7	1.06	3.3×10^{10}			
5	1.8 -2.2	2.18	0.677	2.1×10^{10}	3	0.322	1.0×10^{10}
6	2.2 -2.6	2.5	0.290	9.0×10^{9}			
7	2.6	2.8	0.032	1.0×10^{9}			
Total			7.219	2.24×10^{11}			

*Data from Reactor Shielding Design Manual, Table 3.5,[6] 3.1 fissions/(watt)(sec) assumed.

[6]Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956).

[7]Way, K., and E. P. Wigner, "Decay of Fission Product Gamma-Rays," Phys. Rev. 70, 115 (1946).

[8]Untermyer, S., and J. T. Wells, "Heat Generation in Irradiated Uranium," ANL-4790 (AECD-3454) (February 25, 1952).

[9]Clark, F. H., "Decay of Fission Product Gamma-Rays," NDA-27-39 (December 30, 1954).

[10]Moteff, J., "Fission Product Decay Gamma-Ray Spectrum," APEX-134.

[11]Perkins, J. F., and R. W. King, "Energy Release from the Decay of Fission Products," Nucl. Sci. Eng. 3, (#6) 726 (June, 1958).

[12]Butler, J. W., and M. Grotenhuis, "The Decay of Fission Product Activity for Cyclic Operation of a Reactor," Nucl. Sci. Eng. 3 (#6) 47 (June 1958).

[13]Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004 (McGraw-Hill and D. Van Nostrand, March, 1956) p. 34.

The integral of equation 5.1,

$$P = \int_t^{T+t} d\tau \Gamma(\tau) \text{ Mev/sec} \quad , \tag{5.2}$$

is

$$P = P_0 \frac{KG}{0.2} [t^{-0.2} - (T + t)^{-0.2}] \text{ Mev/sec} \quad . \tag{5.3}$$

This may be rewritten as

$$P = P_0 \frac{KG}{0.2} \left[1 - \left(1 + \frac{T}{t}\right)^{-0.2}\right] t^{-0.2} \text{ Mev/sec} \tag{5.4}$$

It is clear that as T becomes large this expression reduces to

$$P = P_0 \frac{KG}{0.2} t^{-0.2} \text{ Mev/sec} \tag{5.5}$$

A convenient graph of the two portions of equation 5.4 is given in Figure 5.1. The function

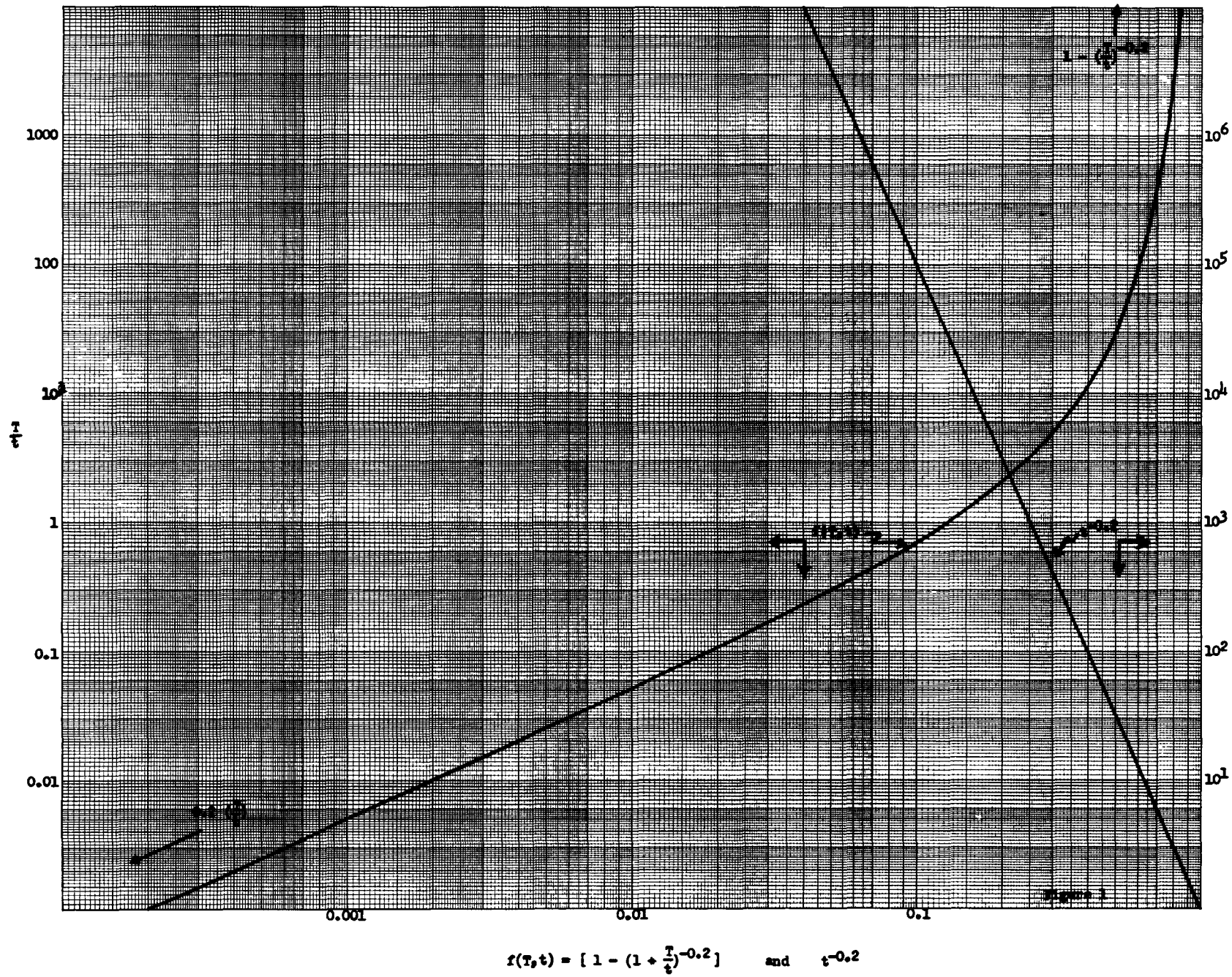
$$f(T,t) = 1 - \left(1 + \frac{T}{t}\right)^{-0.2} \tag{5.6}$$

represents a correction term in equation 5.5 to adjust the fission product power after an infinite operating time to that for a finite operating time. The total prompt and delayed fission gamma-ray power is about six or seven per cent of the total fission power. The source strength may be determined by the equation

$$Q_3 \left(\frac{\gamma}{cm^3 \text{ sec}}\right) = 3.1 \times 10^{10} \frac{P}{V} \left(\frac{\text{fiss}}{cm^3 \text{ sec}}\right) N\gamma \left(\frac{\gamma}{\text{fiss}}\right) \quad , \tag{5.7}$$

in which it is assumed the source strength is constant over the core volume. The power distribution would be a more accurate representation of the source distribution; for thermal reactors the thermal neutron distribution would be equally accurate. The values of $N\gamma$ are the intensities of the various lines chosen to represent the spectrum as given in Tables 5.2 and 5.3. The total energy as given in Tables 5.2 and 5.3 is actually 15 Mev rather than the later value of 13 Mev given in reference 1.

Neutron absorption gives rise to an unstable nucleus. The instability is relieved to some extent by the practically instantaneous emission of a capture gamma ray. The source of these capture gamma rays may be represented by the equation



$f(T,t) = [\,1 - (1 + \frac{T}{t})^{-0.2}\,]$ and $t^{-0.2}$

Figure 5.1

Power From Fission Products After Shutdown

$$Q_3\left(\frac{\gamma}{cm^3\ sec}\right) = \sigma_a\left(\frac{abs}{neut\ cm}\right)\Phi\left(\frac{neut\ cm}{cm^3\ sec}\right) N_\gamma\left(\frac{\gamma}{abs}\right) \quad . \qquad 5.8$$

In case the capture gamma rays being considered are created within the reactor core the source could also be represented by applying a multiplicative factor to equation 5.2:

$$Q_3\left(\frac{\gamma}{cm^3\ sec}\right) = 3.1 \times 10^{10}\,\frac{P}{V}\left(\frac{fiss}{cm^3\ sec}\right)\frac{\sigma_a}{\sigma_f}\left(\frac{abs}{fiss}\right) N_\gamma\left(\frac{\gamma}{abs}\right) , \qquad 5.9$$

where σ_a is the macroscopic absorption cross section of the material from which the capture gamma rays are being emitted and σ_f is the macroscopic fission cross section for the core. In any case, the gamma-ray source takes the spatial distribution of the neutrons that produce it. In the core all gamma-ray sources may be considered constant over the core volume or, a little more accurately, to follow the power distribution. In the case of capture gamma rays outside the core, the flux "Φ," in equation 5.3, which is usually thermal in energy, is often exponential in character; therefore, the source term Q_3 must also be exponential. The values of N_γ, the capture gamma-ray emission per capture in the elements have been summarized in several references[14,15,16] and in Table 5.4. Values of N_γ for some of the concretes used in reactor shields may be found in Table 5.5.

The gamma rays caused by neutron inelastic scattering are not as well known, nor are the cross sections for their creation.[17] These gamma rays are, as a rule, not as energetic as those from neutron capture and are caused by high-energy neutrons which are generally not as abundant nor do they have as large a cross section as the neutrons of lower energy. A combination of uses of these facts can lead to approximations that serve to make pessimistic estimates of the inelastic scattering gamma rays and may show them to be negligible.

The unstable nucleus may remain unstable after the emission of a capture gamma ray. This radioactive nucleus may then decay by the emission of gamma rays, beta rays, etc. The gamma rays thus given off are known as radioactive decay gamma rays.[18] While they rarely dominate the fission gamma rays or capture gamma rays, those due to decay may

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[14] Bartholomew, G. A., and L. A. Higgs, "Compilation of Thermal Neutron Capture Gamma Rays," CRGP-784, Chalk River, Ontario (July, 1958).

[15] Mittleman, P., and R. A. Liedtke, "Gamma Rays Resulting from Thermal Neutron Captures," Nucleonics 13 (#5) 50 (1955).

[16] Deloume, F. E., "Gamma-Ray Energy Spectra from Thermal Neutron Capture," APEX-407 (August, 1958).

[17] Cranberg, et al., Physics and Mathematics, Chapter 4 Progress in Nuclear Energy Series 1, Vol. 1, McGraw-Hill Book Co. (New York, 1956).

[18] Bopp, C. D., and O. Sisman, "How to Calculate Gamma Radiation Induced in Reactor Materials," Nucleonics, 14 (#1) 46 (January 1956).

Table 5.4

PROMPT GAMMA RAYS RESULTING FROM NEUTRON CAPTURE

Capture Gamma-Ray Energy, in Mev per Neutron Capture

Element	Atomic Number	Energy of Emitted Gamma-Rays, Mev										References
		0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	
Aluminum	13				1.29	1.29	.51	.42	1.90			B,P
Antimony	51				.78	.64	.45	.28				K
Arsenic	33				1.01	.93	.65	.64	.17			K
Barium	56				1.89	1.08	.69	.10	.01	.01	.01	L
Beryllium	4				.85			5.12				C,G,T
Bismuth	83					4.17						A
Boron (B^{10})*	5					3.51		.19		.05		
Cadmium	48	.46	.34	.40			.52	.13	.11	.03	.02	L,N,O,P
Calcium	20	.07	.94	.41	1.08	1.96	1.28	2.32	.07			D,P,S
Carbon	6				1.10	3.47						C,G
Chlorine	17	.40		1.57	1.07	1.57	1.45	1.89	1.06	.12		D,P,T
Chromium	24				.26	.29	.62	.75	1.87	3.93	1.11	E,P
Cobalt	27				.77	.87	1.57	1.70	.57			F
Copper	29					.61	.62	.94	3.13	.016		F
Fluorine	9					1.83	3.20	2.82				B
Gadolinium	64				.64	.30	.15	.06				L
Gold	79				1.01	1.53	1.31	1.10	.01			K
Hafnium	72					.50	.51	.17	.026			
Hydrogen	1			2.23								
Indium	49				.76	.55	.26					K
Iron	26	.05	.30	.20	.37	.45	.84	.67	2.66	.25	.20	E,Q
Lead	82								7.40			A,N
Lithium (Li^6)**	3							.51	.94			
Magnesium	12			.62	2.68	.25	.31	.27	.01	.26	.05	B,I,P
Manganese	25					1.24	.98	1.11	1.78			F
Mercury	80	.21	.50	.56	1.35	1.94	1.91	.63	.04			L,N
Molybdenum	42				1.93	1.41	.90	.66	.16	.05		L
Nickel	28	.07	.06	.04	.20	.51	.81	1.32	1.35	4.57	.98	E,P,S
Niobium	41				1.22	.88	.68	.20	.04			
Nitrogen***	7				1.15	.72	5.06	1.07	.66	.35		C
Phosphorous	15				2.17	1.51	.66	1.04	.58			D,P
Platinum	78				1.07	.71	.75	.08	.06			L
Potassium	19	.24	.61	.83	1.80	1.86	1.99	.24	.39	.01		D,J,P,S
Praseodymium	59				.76	.59	.45					K
Rhodium	45				.82	.68	.45	.17				K
Samarium	62	.53	.21	.77	1.21	.55	.28	.08	.07			L,N
Silicon	14		.37	1.61	2.14	3.89	.47	.92	.62	.14		B,P,Q
Silver	47			.24	1.52	1.28	.81	.23	.04			K
Sodium	11	.80	.39	1.44	1.05		.33	.83				B,O,P,T
Strontium	38				1.46	1.03	.91	2.18	.79	.29	.02	L
Sulphur	16	.40		1.77	1.62	1.92	4.64	.29	.22	.09		D,P,T
Tantalum	73					.05	.10	.04				K
Thallium	81				.94	2.15	2.68	.96				K
Tin	50				3.64	1.89	1.24	.65	.25		.04	L
Titanium	22	.14	1.59	.08	.37	.72	.09	6.58	.11	.02	.01	E,P,Q
Tungsten	74				1.28	.77	.58	.33	.03			L
Uranium (U^{238})†	92	.30		.43								R
Vanadium	23				.39	.50	1.33	2.16	1.07	.008		F,P
Zirconium	40				2.52	1.71	1.07	1.15	.15	.11		L

*$\sigma_{n,\gamma}$ B^{10} = 0.5 ± .2b **$\sigma_{n,\gamma}$ Li^6 = 0.023 ± .008b ***$\sigma_{n,\gamma}$ N = 0.080 ± .020b

†Added to original Deloume compilation

From F. E. Deloume, "Gamma-Ray Energy Spectra from Thermal Neutron Capture," DC-58-1-30 (January 2, 1958).

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A "Neutron Capture Gamma Rays from Lead and Bismuth," B B Kinsey, G A Bartholomew, and W H Walker, Physical Review 82, 380, 1951

B "Neutron Capture Gamma Rays from Fluorine, Sodium, Magnesium, Aluminum, and Silicon," B B Kinsey, G A Bartholomew, and W H Walker, Physical Review 83, 519, 1951

C "Gamma Rays Produced by Slow Neutron Capture in Beryllium, Carbon, and Nitrogen," B B Kinsey, G A Bartholomew, and W H Walker, Canadian Journal of Physics 29, 1, 1951

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I "Note on Some Neutron Capture Gamma Rays from Magnesium," B B Kinsey and G A Bartholomew, Canadian Journal of Physics 31, 901, 1953

J "The Neutron Capture Gamma Rays from Potassium," G A Bartholomew and B B Kinsey, Canadian Journal of Physics 31, 927, 1953

K "Neutron Capture Gamma Rays from Heavy Odd-Charge Nuclei," G A Bartholomew and B B Kinsey, Canadian Journal of Physics 31, 1025, 1953

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P "Neutron-Capture Gamma Rays from Various Elements," T H Braid, Physical Review 102, 1109, 1956

Q "An Investigation of Neutron-Capture Gamma Rays from Titanium, Iron, and Silicon," B P Adyasevich, L V Groshev, and A M Demidov, The Soviet Journal of Atomic Energy, Vol 1, No 2, pp 258-272, October, 1956

R Bartholomew, G. A., and L. A Higgs, "Compilation of Thermal Neutron Capture Gamma Rays," CRGP-784 (July 1958).

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CAPTURE GAMMA-RAY SPECTRA OF CONCRETES

Concrete*	Σ_a** cm^{-1}	N(E) Effective Number of Capture Gamma Rays of Energy E Emitted per Neutron Absorption			
		2 Mev	4 Mev	6 Mev	8 Mev
01	0.00553	0.655	0.832	0.699	0.066
02-a	0.0089	0.830	0.678	0.211	0.168
02-b	0.0074	0.788	0.953	0.296	0.236
03	0.00895	0.693	0.512	0.454	0.077
04	0.00836	0.745	0.834	0.335	0.129
FP-a	0.0781	0.154	0.281	0.230	0.444
b	0.0759	0.129	0.289	0.237	0.457
BA-a	0.0193	0.663	0.555	0.301	0.120
b	0.0171	0.620	0.624	0.339	0.135
M-a	0.0600	0.384	0.252	0.374	0.349
b	0.0581	0.365	0.260	0.386	0.360
BR	0.0879	0.137	0.243	0.232	0.448
I-1a	0.0888	0.807	0.299	0.758	0.163
2a	0.0941	0.710	0.287	0.686	0.204
MS1-a	0.0939	0.225	0.247	0.289	0.435
b	0.0897	0.208	0.251	0.295	0.445
MS2-a	0.0962	0.182	0.248	0.265	0.453
b	0.0942	0.215	0.243	0.260	0.444
LS-a	0.0923	0.176	0.232	0.216	0.451
b	0.0903	0.158	0.237	0.221	0.461

*Assumed water retention. a = 100%; b = 50%

**Σ_a = Thermal neutron absorption cross section at 20°C corrected (0.8862) for 1/v absorption in a Maxwell distribution.

create a shielding problem when a material is removed from the reactor shield environment, either for replacement or repair, or by a coolant stream. These gamma rays will be dealt with in the next section.

Since all of the gamma rays arise from processes involving neutrons it becomes clear that, not only must the neutron distributions be treated first, but that the accuracy of these distributions controls the accuracy of the value of the gamma-ray source. It is quite often true for gamma-ray problems that the attenuation methods are more accurate than the source representation. An evaluation of the relative accuracies involved in a problem should be made to determine whether inconsistencies exist and, is so, how they should be eliminated.

The basic character of gamma-ray attenuation is exponential in nature. For the narrow beam, thin sample, or "good," geometry this is exactly true since any scattered photons cannot reach the detector (Figure 5.2).

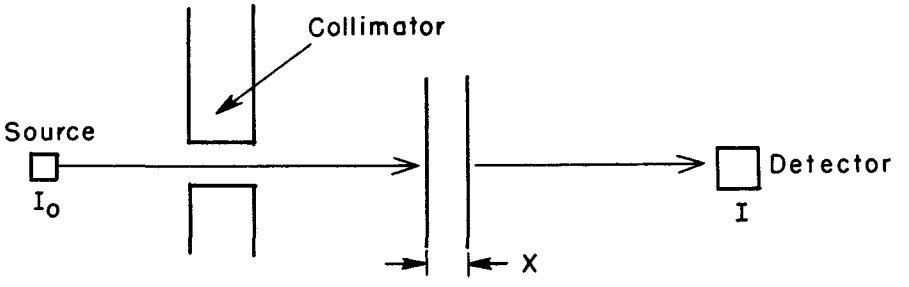

Figure 5.2

Narrow Beam Gamma-Ray Absorption

Under these circumstances the detector response will be given by

$$\Phi_\gamma(x) = \Phi_\gamma(0)\, e^{-\mu x}\ \gamma/cm^2\ sec \quad . \qquad 5.10$$

This would also be true for a plane collimated gamma-ray beam, such as in Figure 5.3,

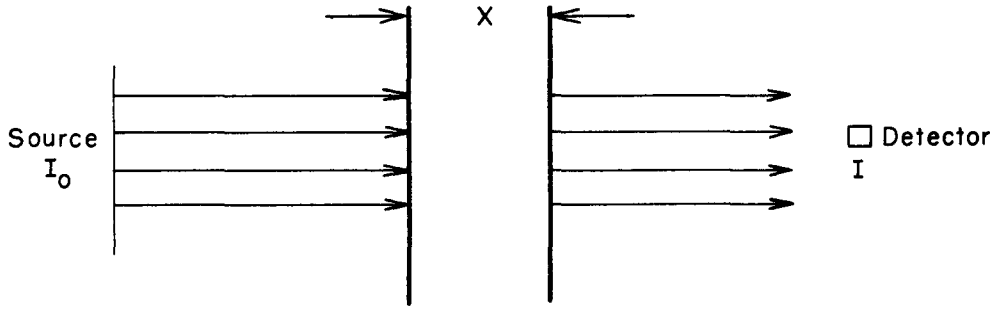

Figure 5.3

Broad Beam Gamma-Ray Absorption

if the only type of interaction were absorption. This same plane beam actually has a large portion of scattered gamma rays by the time it penetrates a shield, and thus the detector response would be much greater than equation 5.5 would predict. In a broad beam thick shield geometry with a source that is isotropic, or nearly so, which is typical of all reactor shields, the uncollided flux given by equation 5.10 could be too low by an order of magnitude or more. In fact the rapid build-up of scattered low-energy photons in light elements leads to the somewhat ridiculous appearing fact that the measured dose through a given thickness of water is larger than it would be were the water not present.[19] This phenomena occurs only at a short distance from the source and it has been experimentally verified.[20] In any case a correction factor, commonly referred to as the build-up factor, must be applied to the uncollided flux. The result is

$$\Phi_\gamma(x) = \Phi_\gamma(0)\, B\,(\mu x)\, e^{-\mu x}\ \gamma/cm^2\ sec \quad . \qquad 5.11$$

This factor is obviously a function of such things as gamma-ray energy, shield material, and shield thickness. These are conveniently lumped

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[19]Goldstein, H., "The Attenuation of Neutrons and Gamma-Rays in Reactor Shields," U. S. Government Printing Office (May, 1957) p. 178, Figure 5.24.

[20]Van Dilla, M. A. and G. J. Hine, "Gamma-Ray Diffusion Experiments in Water," Nucleonics 10, (#7) 54 (July, 1952).

together in the form of μx, the number of mean-free-paths traversed by the ray from source to detector. This factor also varies with the quantity being observed, i.e., the dose as compared to the energy flux.

It would be of obvious advantage to have an explicit function to represent the build-up. Historically one of the first attempts to represent build-up in a convenient analytical form was the linear function

$$B(\mu x) = 1 + \mu x \quad , \tag{5.12}$$

without much regard as to whether it was a build-up of dose or energy flux. Time has proven this to be not only a convenient but, comparatively, a rather accurate form that tends to be an overestimation. Additional work by the combined NDA-NBS study[21] has yielded more accurate results for thick shields containing a single material (see Table 5.6). Utilizing this information enables one to adjust equation 5.7 by the introduction of a constant:

$$B(\mu x) = 1 + k\mu x \quad , \tag{5.13}$$

which will reduce the degree of pessimism to some extent. This constant "k" is, for example, approximately one-half to one-third for a plane monodirectional source of hard gamma rays in lead.

In addition to the linear approximation, build-up has been described in a more complex fashion by Fano.[22] R. L. Ashley[23] of Atomics International has extended the linear form to a quadratic:

$$B(\mu x) + 1 + k_1 \mu x + k_2 (\mu x)^2 \quad . \tag{5.14}$$

The constants "k_1" and "k_2" are determined by the results in Goldstein.[24] J. J. Taylor[25,26] of WAPD has fit the same NDA-NBS data to the sum of two exponentials:

$$B(\mu x) = A_1 \exp[-\alpha_1 \mu x] + A_2 \exp[-\alpha_2 \mu x] \quad . \tag{5.15}$$

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[21] Goldstein, H., "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," U. S. Government Printing Office (May 1, 1957) p. 181.

[22] Fano, U., "Gamma-Ray Attenuation," Nucleonics 11 (#8) 8 (August, 1953) and Nucleonics 11 (#9) 55 (September, 1953).

[23] Personal communication

[24] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma Rays," NYO-3075, U. S. Government Printing Office (June 30, 1954).

[25] Taylor, J. J., "Application of Gamma-Ray Build-Up Data to Shield Design," WAPD-RM-217 (January 25, 1954).

[26] Rockwell, T., III, Editor, The Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956).

Table 5.6

DOSE BUILD-UP FACTOR FOR A POINT ISOTROPIC SOURCE

	$\mu_t x$	Gamma-Ray Energy, Mev							
		0.5	1.0	2.0	3.0	4.0	6.0	8.0	10.0
Water	1	2.52	2.13	1.83	1.69	1.58	1.46	1.38	1.33
	2	5.14	3.71	2.77	2.42	2.17	1.91	1.74	1.63
	4	14.3	7.68	4.88	3.91	3.34	2.76	2.40	2.19
	7	38.8	16.2	8.46	6.23	5.13	3.99	3.34	2.97
	10	77.6	27.1	12.4	8.63	6.94	5.18	4.25	3.72
	15	178	50.4	19.5	12.8	9.97	7.09	5.66	4.90
	20	334	82.2	27.7	17.0	12.9	8.85	6.95	5.98
Aluminum	1	2.37	2.02	1.75	1.64	1.53	1.42	1.34	1.28
	2	4.24	3.31	2.61	2.32	2.08	1.85	1.68	1.55
	4	9.47	6.57	4.62	3.78	3.22	2.70	2.37	2.12
	7	21.5	13.1	8.05	6.14	5.01	4.06	3.45	3.01
	10	38.9	21.2	11.9	8.65	6.88	5.49	4.58	3.96
	15	80.8	37.9	18.7	13.0	10.1	7.97	6.56	5.63
	20	141	58.5	26.3	17.7	13.4	10.4	8.52	7.32
Iron	1	1.98	1.87	1.76	1.55	1.45	1.34	1.27	1.20
	2	3.09	2.89	2.43	2.15	1.94	1.72	1.56	1.42
	4	5.98	5.39	4.13	3.51	3.03	2.58	2.23	1.95
	7	11.7	10.2	7.25	5.85	4.91	4.14	3.49	2.99
	10	19.2	16.2	10.9	8.51	7.11	6.02	5.07	4.35
	15	35.4	28.3	17.6	13.5	11.2	9.89	8.50	7.54
	20	55.6	42.7	25.1	19.1	16.0	14.7	13.0	12.4
Tungsten	1	1.28	1.44	1.42	1.36	1.29	1.20	1.14	1.11
	2	1.50	1.83	1.85	1.74	1.62	1.43	1.32	1.25
	4	1.84	2.57	2.72	2.59	2.41	2.07	1.81	1.64
	7	2.24	3.62	4.09	4.00	4.03	3.60	3.05	2.62
	10	2.61	4.64	5.27	5.92	6.27	6.29	5.40	4.65
	15	3.12	6.25	8.07	9.66	12.0	15.7	15.2	14.0
	20		7.35	10.6	14.1	20.9	36.3	41.9	39.3
Lead	1	1.24	1.37	1.39	1.34	1.27	1.18	1.14	1.11
	2	1.42	1.69	1.76	1.68	1.56	1.40	1.30	1.23
	4	1.69	2.26	2.51	2.43	2.25	1.97	1.74	1.58
	7	2.00	3.02	3.66	3.75	3.61	3.34	2.89	2.52
	10	2.27	3.74	4.84	5.30	5.44	5.69	5.07	4.34
	15	2.65	4.81	6.87	8.44	9.80	13.8	14.1	12.5
	20	2.73	5.86	9.00	12.3	16.3	32.7	44.6	39.2
Uranium	1	1.17	1.31	1.33	1.29	1.24	1.16	1.12	1.09
	2	1.30	1.56	1.64	1.58	1.50	1.36	1.27	1.20
	4	1.48	1.98	2.23	2.21	2.09	1.85	1.66	1.51
	7	1.67	2.50	3.09	3.27	3.21	2.96	2.61	2.26
	10	1.85	2.97	3.95	4.51	4.66	4.80	4.36	3.78
	15	2.08	3.67	5.36	6.97	8.01	10.8	11.2	10.5
	20			6.48	9.88	12.7	23.0	28.0	28.5

From H. Goldstein and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma-Rays - Final Report," NYO-3075 (June 30, 1954).

The build-up factor may be assumed to be a function of the perpendicular distance traversed in the shield in units of mean-free-paths. This is a fair assumption and certainly has the feature of simplicity. The uncollided gamma-ray flux from a plane isotropic source shielded by a thickness of "a" cm,

$$\Phi(a) = \frac{Q_2}{2} E_1(\mu a) \ \gamma/cm^2 \ sec \quad , \qquad 5.16$$

would then be corrected to account for scattered flux by multiplying it by $B(\mu a)$,

$$\Phi(a) = \frac{Q_2}{2} B(\mu a) \ E_1(\mu a) \ \gamma/cm^2 \ sec \quad , \qquad 5.17$$

where $B(\mu a)$ would be obtained from Table 5.6 or in reference 27. A more correct way would be to take the original integral,

$$\Phi(R) = \int_A dA \ Q_2 \frac{e^{-\mu R}}{4\pi R^2} \quad , \qquad 5.18$$

and correct it by an analytical form of the build-up function before integration:

$$\Phi(r) = \int_A dA \ Q_2 \frac{e^{-\mu R}}{4\pi R^2} B(\mu R) \quad . \qquad 5.19$$

The form of $B(\mu R)$ is probably best represented by equation 5.15 if a single material is used for the shield. The equations that develop from this form of the build-up in the various geometrical configurations are given in the Design Manual.[28] The equations that develop from linear forms of the build-up have been discussed in Section 3.

The build-up factor is also given for energy flux or dose,[29] although the difference is generally negligible unless more elegant calculations are applied. Certainly the source distribution should be better represented than is usually the case before great concern is given to these differences in build-up factors. At the present time the accuracy in the calculated build-up factors[30] is better than that of the attenuation coefficients as applied to some shield design problems.

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[27] Goldstein, H., "The Attenuation of Neutrons and Gamma Rays in Reactor Shields," U. S. Government Printing Office (May 1, 1957) p. 181.

[28] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956) p. 412.

[29] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma Rays," NYO-3075, U. S. Government Printing Office (June 30, 1954).

[30] Goldstein, H., "The Attenuation of Neutrons and Gamma Rays in Reactor Shields," U. S. Government Printing Office (May, 1957) p. 180.

Probably the most important feature of build-up is not which of the build-up factors or which of the analytical forms should be used to represent build-up, but how any of these may best be combined to represent build-up in a shield containing more than one material. In a case where the shield is predominantly one material, the use of the equation

$$B(\mu x) = B_1 (\mu_1 x_1) + B_2 (\mu_2 x_2) \qquad 5.20$$

is a reasonable assumption. The subscript refers to the number of the material. If the materials are more or less homogeneous an effective atomic number may be assigned and the build-up described as though there were one material of the assumed atomic number and appropriate density. The effective atomic number of a mixture may be estimated by computing the total gamma-ray cross section (see Tables 5.7 and 5.8) for the mixture and comparing it to the cross sections of the elements. The element that has most nearly the same cross section as the mixture in the energy range in which the Compton scattering is important determines the effective atomic number for the mixture. The build-up factors can then be determined by interpolation in Z. For details as to accuracies etc., the reader is referred to Goldstein.[31,32]

In a multi-layer shield that does not fit the above conditions one must derive effective build-up functions from the characteristics of the materials and the energy of the gamma rays. For example, if material 1 is lead and material 2 is water, and the gamma-ray energy is below 3 Mev, the total build-up may be represented as

$$B(\mu x) = B_1(\mu_1 x_1) \times B_2(\mu_2 x_2) \quad . \qquad 5.21$$

The resulting build-up is quite high and so it should be since the scattered photons from lead may still be able to penetrate through water fairly easily. In case water is the first layer, the degraded photons are easily removed by the lead in the second layer and the build-up is substantially reduced. In this case it may be represented as

$$B(\mu x) = B_2(\mu_1 x_1 + \mu_2 x_2) \quad . \qquad 5.22$$

In some specific cases of lead and water combinations studies have led to rather accurate but complex forms.[33] Of course, there are combinations of materials that have a similar enough atomic number so that the build-up of the combination would be approximately the same as the build-up for any

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[31] Goldstein, H., "The Attenuation of Gamma-Rays and Neutrons in Reactor Shields," U. S. Government Printing Office (May 1, 1957).

[32] Goldstein, H., and J. E. Wilkins, Jr., "Calculations of the Penetrations of Gamma Rays," NYO-3075, U. S. Government Printing Office (June 30, 1954).

[33] Goldstein, H., "The Attenuation of Gamma Rays and Neutrons in Reactor Shields," U. S. Government Printing Office (May 1, 1957) p. 180.

Table 5.7

TOTAL GAMMA-RAY MASS ATTENUATION COEFFICIENTS

(in cm^2/gm)

Material	Gamma-Ray Energy Mev																	
	0 1	0 15	0 2	0 3	0 4	0 5	0 6	0 8	1 0	1 25	1 5	2	3	4	5	6	8	10 0
H	295	265	243	212	189	173	160	140	126	113	103	0876	0691	0579	0502	0446	0371	0321
Be	132	119	109	0945	0847	0773	0715	0628	0565	0504	0459	0394	0313	0266	0234	0211	0180	0161
C	149	134	122	106	0953	0870	0805	0707	0636	0568	0518	0444	0356	0304	0270	0245	0213	0194
N	150	134	123	106	0955	0869	0805	0707	0636	0568	0517	0445	0357	0306	0273	0249	0218	0200
O	151	134	123	107	0953	0870	0806	0708	0636	0568	0518	0445	0359	0309	0276	0254	0224	0206
Na	151	130	118	102	0912	0833	0770	0676	0608	0546	0496	0427	0348	0303	0274	0254	0229	0215
Mg	160	135	122	106	0944	0860	0795	0699	0627	0560	0512	0442	0360	0315	0286	0266	0242	0228
Al	161	134	120	103	0922	0840	0777	0683	0614	0548	0500	0432	0353	0310	0282	0264	0241	0229
Si	172	139	125	107	0954	0869	0802	0706	0635	0567	0517	0447	0367	0323	0296	0277	0254	0243
P	174	137	122	104	0928	0846	0780	0685	0617	0551	0502	0436	0358	0316	0290	0273	0252	0242
S	188	144	127	108	0958	0874	0806	0707	0635	0568	0519	0448	0371	0328	0302	0284	0266	0255
A	188	135	117	0977	0867	0790	0730	0638	0573	0512	0468	0407	0338	0301	0279	0266	0248	0241
K	215	149	127	106	0938	0852	0786	0689	0618	0552	0505	0438	0365	0327	0305	0289	0274	0267
Ca	238	158	132	109	0965	0876	0809	0708	0634	0566	0518	0451	0376	0338	0316	0302	0285	0280
Fe	344	183	138	106	0919	0828	0762	0664	0595	0531	0485	0424	0361	0330	0313	0304	0295	0294
Cu	427	206	147	108	0916	0820	0751	0654	0585	0521	0476	0418	0357	0330	0316	0309	0303	0305
Mo	1 03	389	225	130	0998	0851	0761	0648	0575	0510	0467	0414	0365	0349	0344	0344	0349	0359
Sn	1 58	563	303	153	109	0886	0776	0647	0568	0501	0459	0408	0367	0355	0355	0358	0368	0383
I	1 83	648	339	165	114	0913	0792	0653	0571	0502	0460	0409	0370	0360	0361	0365	0377	0394
W	4 21	1 44	708	293	174	125	101	0763	0640	0544	0492	0437	0405	0402	0409	0418	0438	0465
Pt	4 75	1 64	795	324	191	135	107	0800	0659	0554	0501	0445	0414	0411	0418	0427	0448	0477
Tl	5 16	1 80	866	346	204	143	112	0824	0675	0563	0508	0452	0420	0416	0423	0433	0454	0484
Pb	5 29	1 84	896	356	208	145	114	0836	0684	0569	0512	0457	0421	0420	0426	0436	0459	0489
U	1 06	2 42	1 17	452	259	176	136	0952	0757	0615	0548	0484	0445	0440	0446	0455	0479	0511
Air	151	134	123	106	0953	0868	0804	0706	0655	0567	0517	0445	0357	0307	0274	0250	0220	0202
NaI	1 57	568	305	155	111	0901	0789	0657	0577	0508	0465	0412	0367	0351	0347	0347	0354	0366
H_2O	167	149	136	118	106	0966	0896	0786	0706	0630	0575	0493	0396	0339	0301	0275	0240	0219
Concrete	169	139	124	107	0954	0870	0804	0706	0635	0567	0517	0445	0363	0317	0287	0268	0243	0229
Tissue	163	144	132	115	100	0936	0867	0761	1683	0600	0556	0478	0384	0329	0292	0267	0233	0212

From Gladys White Grodstein, "X-Ray Attenuation Coefficients from 10 kev to 100 Mev," NBS Circular 583 (April 30, 1957), Chapts. 1-3.

Table 5.8

TOTAL GAMMA-RAY ATTENUATION COEFFICIENTS

(in cm^{-1})

		Gamma-Ray Energy, Mev																	
Material	Density	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.5	2	3	4	5	6	8	10.0
Be	1.85	.244	.220	.202	.1748	.1567	.1430	.1323	.1162	.1045	.0932	.0849	.0729	.0579	.0492	.0433	.0390	.0333	.0298
C	2.25	.335	.302	.275	.239	.2144	.1958	.1811	.1591	.1431	.1278	.1166	.0999	.0801	.0684	.0608	.0551	.0479	.0437
Na	.9712	.147	.126	.115	.099	.0886	.0809	.0748	.0657	.0590	.0530	.0482	.0415	.0338	.0294	.0266	.0247	.0222	.0209
Mg	1.741	.279	.235	.212	.185	.1643	.1497	.1384	.1217	.1092	.1975	.0891	.0770	.0627	.0548	.0498	.0463	.0421	.0397
Al	2.70	.435	.362	.324	.278	.2489	.2268	.2098	.1844	.1658	.1480	.1350	.1166	.0953	.0837	.0761	.0713	.0651	.0618
Si	2.42	.416	.336	.303	.259	.2309	.2103	.1941	.1709	.1537	.1372	.1251	.1082	.0888	.0782	.0716	.0670	.0615	.0588
P	1.83	.318	.251	.223	.190	.1698	.1548	.1427	.1254	.1129	.1008	.0919	.0798	.0655	.0578	.0531	.0500	.0461	.0443
S	2.07	.389	.298	.263	.224	.1983	.1809	.1668	.1463	.1314	.1176	.1074	.0927	.0768	.0679	.0625	.0588	.0551	.0328
K	0.87	.187	.130	.110	.092	.0816	.0741	.0684	.0599	.0538	.0480	.0439	.0381	.0318	.0284	.0265	.0251	.0238	.0232
Ca	1.55	.369	.245	.205	.169	.1496	.1358	.1254	.1097	.0983	.0877	.0803	.0699	.0583	.0524	.0490	.0468	.0442	.0434
Fe	7.86	2.704	1.438	1.085	.833	.7223	.6508	.5989	.5219	.4677	.4174	.3812	.3333	.2837	.2594	.2460	.2389	.2319	.2311
Cu	8.933	3.814	1.840	1.313	.965	.8183	.7325	.6709	.5842	.5226	.4654	.4252	.3734	.3189	.2948	.2823	.2760	.2707	.2725
Mo	9.01	9.280	3.505	2.027	1.171	.8991	.7668	.6857	.5838	.5181	.4595	.4208	.3730	.3289	.3144	.3099	.3099	.3144	.3190
Sn	7.298	11.53	4.109	2.211	1.117	.795	.6466	.5663	.4722	.4145	.3656	.3350	.2978	.2678	.2591	.2591	.2613	.2686	.2795
I	4.94	9.040	3.201	1.675	.815	.563	.4510	.3912	.3226	.2821	.2480	.2272	.2020	.1828	.1778	.1783	.1803	.1862	.1946
W	19.3	81.25	27.79	13.66	5.655	3.358	2.413	1.949	1.473	1.235	1.050	.9496	.8434	.7817	.7759	.7894	.8067	.8453	.8975
Pt	21.37	101.51	35.05	16.99	6.924	4.082	2.885	2.287	1.710	1.408	1.184	1.071	.9510	.8847	.8783	.8933	.9125	.9574	1.019
Tl	11.86	61.20	21.35	10.27	4.104	2.419	1.696	1.328	.9773	.8005	.6677	.6025	.5361	.4981	.4934	.5017	.5135	.5384	.5740
Pb	11.34	59.99	20.87	10.16	4.037	2.359	1.644	1.293	.9480	.7757	.6452	.5806	.5182	.4774	.4763	.4831	.4944	.5205	.5545
U	18.7	19.82	45.25	21.88	8.452	4.843	3.291	2.543	1.780	1.416	1.150	1.025	.9051	.8322	.8228	.8340	.8509	.8957	.9556
NaI	3.667	5.757	2.083	1.118	.568	.407	.3304	.2893	.2409	.2116	.1863	.1705	.1511	.1346	.1287	.1272	.1272	.1298	.1342
H_2O	1.00	.167	.149	.136	.118	.106	.0966	.0896	.0786	.0706	.0630	.0575	.0493	.0396	.0339	.0301	.0275	.0240	.0219
Concrete	2.35	.397	.327	.291	.251	.2242	.2045	.1889	.1659	.1492	.1332	.1215	.1046	.0853	.0745	.0674	.0630	.0571	.0538

of the individual layers. For those simple cases the build-up may be expressed by equation 5.20 or 5.22. To check the accuracy of this assumption, the build-up may be compared to that of the material which has the closest value of Z and for which the build-up has been computed; examples are beryllium, sugar, heavy water, and water. The build-ups for these materials or combinations thereof may be compared to the computed build-up for water or aluminum. The build-up in chromium, nickel, zinc, or copper may, similarly, be regarded as essentially the same as that for iron. The situation at present is that one must carefully examine each configuration, see that a combination such as leads to equation 5.21 does not unknowingly exist, and then proceed with a convenient analytic form.

Another consideration in gamma ray problems is that of slant penetration of shields.[34,35] All the previous considerations have been for radiation which is normal to the shield surface. While this is usually not a restrictive consideration, it is necessary to be aware of the inherent dangers of slant penetration. A simple illustration of slant penetration is given in Figure 5.4. Ray path A represents the incident ray which is normal to the shield slab. It is effectively shielded by σt mean-free-paths. Ray B might be supposed to be shielded by $\sigma t \sec \theta$ mean-free-paths; however, the probability for a photon to be scattered at some point S in the shield taking the shorter path to the surface of the shield represented by C is too large to discount. Like scattered gamma rays in path A the scattered component may become the dominant one and, therefore, the effective thickness of the shield for ray path B must be less than $\sigma t \sec \theta$.

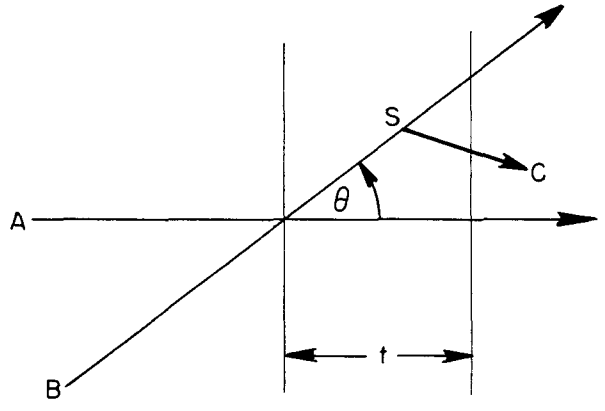

Figure 5.4

Illustration of Slant Penetration for Gamma Rays

The considerations given are for monoenergetic sources of gamma rays. While, in truth, this is seldom realized, it may be a reasonable approximation, and the information required to work problems is usually presented in a form convenient for this purpose. Often a single photon energy is assumed to be representative and this may be reasonable. It must be kept in mind, then, that this energy may be representative of one

[34] Kirn, F. S., et al., "Oblique Attenuation of Gamma Rays from Co^{60} and Cs^{137} in Polyethylene and Lead," NBS-2125 (December 23, 1952).

[35] Zerby, C. D., "Transmission of Obliquely Incident Gamma-Radiation Through Stratified Slab Barriers," ORNL-2224 (December 13, 1956).

situation and not another. For example, the same assumption would not be as true for a heating problem as for a shield penetration problem. Similarly, a three-Mev photon may be representative of a fission source shielded by lead, and this assumption may not seriously be affected by the presence of gamma rays due to neutron capture in water. However, a change would be quite in order if a gamma-ray source, due to capture of neutrons in steel, were present. This latter situation may permit the three-Mev assumption near the core with a lead shield, but requires a seven-Mev assumption farther out in the shield with no lead present. The best compromise is to use a line spectrum with as many lines as is deemed practical or necessary. The data on source spectra are presented in adequate detail for this purpose. A reasonable compromise for a complete reactor shield is to represent all source spectra by a minimum of three lines ranging from one Mev to seven Mev.

The preceding discussion, supported by the formulas that appear in Section 3, provides the necessary tools for the calculation of the gamma-ray fluxes in a reactor shield. One of the first steps is to put down in orderly fashion all the information necessary for the solution of the problem: materials, thicknesses, cross sections, geometry, etc. The selection of the appropriate formula is strongly dependent on the geometry and, because an exact representation is never possible, there may be more than one reasonable geometrical approximation. Because of the reactor physics considerations a reactor core is usually designed to be a cylinder with a ratio of height to diameter of nearly one. Since the actual core will not truly be a cylinder, but some mechanically feasible approximation, it might be equally well represented by a cylinder or a sphere. Thus the core gamma rays could be computed by cylindrical geometry[36] or by spherical geometry, formulas 3.40 and 3.46, with essentially the same results. Close to the core, before the R^2 effect becomes large, a slab geometry would no doubt also suffice. A more accurate representation could be made, but it does not usually yield correspondingly accurate results unless all ingredients of the calculation are known equally well. It is not, for example, worth the effort to carefully represent the geometry to within 10% accuracy when the build-up is not known to within 50% nor the source strength to within a factor of two.

A reasonably simple, yet often sufficiently accurate, formula is the following:

$$\Phi = \frac{Q_3}{2\mu_s}\left(\frac{R_s}{R_s + a}\right) e^{-\mu a} \ \gamma/cm^2 \ sec \quad . \qquad 5.23$$

This is the same formula as derived in Section 3, equation 3.40, with a linear build-up included. It will apply when $\mu a > 1$ and the build-up can be

[36] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956).

reasonably approximated by $B(\mu a) = \mu a$. It may be used equally well to represent prompt fission, fission product, or capture gamma rays in a core that is nearly a sphere and has a uniform power distribution.

In order to judge whether a constant-source representation is adequate for the core gamma rays, compare this constant source to the actual distribution of the fissions (see Figure 5.5). Since the major contribution of gamma rays that leak out of a source come from within the outermost mean-free-path, a look at the relative location of that distance may determine the most proper source representation. If "a" represents one mean-free-path from the outside boundary of the core, the constant-source representation would clearly be an overestimation, the magnitude of which is strictly a function of the individual reactor. If "b" is one mean-free-path, the constant source may possibly be an underestimate, a condition that is traditionally avoided by shield designers.

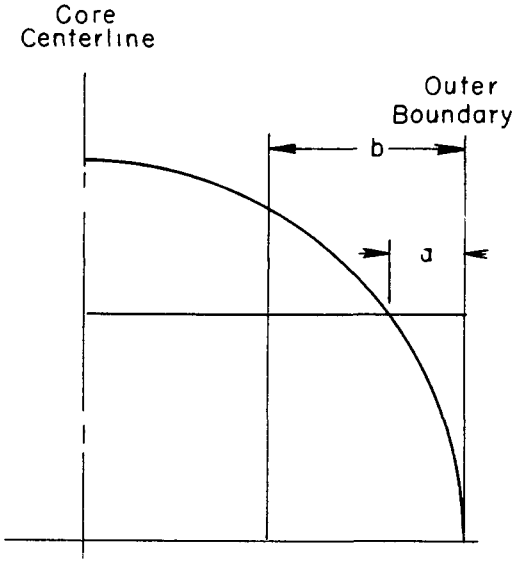

Figure 5.5

Core Gamma-Ray Source Distribution

The regions outside the core will generate capture gamma rays that may at times be approximated by a uniform source, but as a rule will tend to be more or less exponential. An exponential source some distance from the core and out in the shield may well be represented by slab geometry as given by formula 3.78. An exponential source cannot be simply represented in spherical geometry except by replacing the exponential source by a suitable constant source. For example, in slab geometry an average value for an exponential source may be represented as

$$\bar{Q}_\gamma = \frac{\int_0^t dx\, Q_0 \exp\left[-kx - \sigma_s (t - x)\right]}{\int_0^t dx \exp\left[-\sigma_s (t - x)\right]} \gamma/\text{cm}^3 \text{ sec}, \qquad 5.24$$

which, referring to Figure 5.6 yields the result

$$\bar{Q}_\gamma = \frac{Q_0 \sigma_s}{(k - \sigma_s)} \frac{\left\{1 - \exp\left[-(k - \sigma_s)t\right]\right\}}{\left[\exp\left(+\sigma_s t\right) - 1\right]} \gamma/\text{cm}^3 \text{ sec} \quad . \qquad 5.25$$

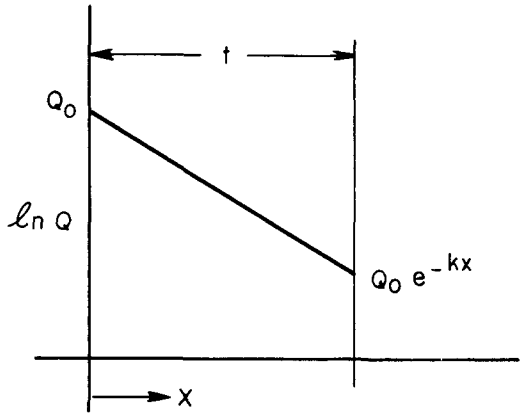

Figure 5.6

Illustration for Average of Exponential Source

Since the slab representation is a pessimistic one, but not always excessively so, it is usually employed whenever the source distribution is exponential. It is convenient, and usually there is not a serious loss in accuracy, to utilize the exponential source formulas (equations 3.81 ff) to determine the capture gamma-ray fluxes at the surface of the region in which they are produced and then to extend these fluxes throughout the remainder of the shield by equation 5.23. This is a particularly appreciable time saver when the gamma-ray distribution throughout a shield material is required in some detail.

While the considerations of fuel element transfer have not been explicitly discussed, the basic information required has been covered. The first consideration is the geometry to be selected to represent the problem. Quite often this will be a line source, as indicated in Section 3, since a fuel element is commonly long and thin. The strength of this line source may be determined by considerations of the power level, operating time and cooling time as given in equations 5.3 and 5.7. The spectrum changes with time; therefore the equilibrium spectrum will have to be adjusted to the value appropriate for the time after shutdown.[37,38,39,40] Examples of such calculations are given in the literature.[41,42]

The gamma-ray scattering that occurs within a dense media can be treated by the build-up concept as discussed previously. There are, however, regions around a reactor in which the scattering occurs in a

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[37] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956).

[38] Moteff, J., "Fission Product Decay Gamma-Ray Spectrum," APEX-176 (December 1, 1954).

[39] Ashley, R. L., "Effective-Energy Method for Spent Fuel Shielding," Nucleonics 16 (#10) 78 (October, 1958).

[40] Clark, F. H., "Decay of Fission Gamma-Rays," NDA-27-39 (December 30, 1954).

[41] Stern, H. E., "Calculations of Shielding for Burned-Out Reactor Fuel Elements," ORNL-1840 (December 22, 1953).

[42] "The EBWR Experimental Boiling Water Reactor," ANL-5607 (May, 1957) p. 215.

relatively penetrable medium, such as air or water.[43,44,45,46] Under these circumstances the scattering problem must be treated on a somewhat more microscopic basis. One must then delve a little deeper into the more basic processes and the physical laws by which they are governed.

There are several ways in which gamma rays may be scattered.[47,48] These include Compton, Rayleigh, Thomson, and nuclear resonance. The one of the most practical importance is Compton scattering, which involves a change in direction as well as energy of the photon. The photon energy after being scattered through an angle θ is given by

$$\gamma = \gamma_0 \left[1 + \gamma_0 (1 - \cos \theta)\right]^{-1} \quad , \tag{5.26}$$

where γ is $h\nu/mc^2$, or $E(\text{Mev})/0.51$.

Equation 5.21 is derived from the laws of conservation of energy and momentum as applied to the collision between a photon of energy E(Mev) and an electron. It is useful to note that a large angle scatter of high-energy photon ($\gamma >> 1$) results in a photon of energy approximately given by

$$E_\gamma = \frac{0.51}{1 - \cos \theta} \text{ Mev} \tag{5.27}$$

If θ is 90° or greater then the scattered photon energy is 0.51 Mev or less; similarly, if the angle is 180° the scattered photon energy is 0.25 Mev or less, no matter what the initial energy of the gamma ray may have been.

The change in direction is not quite so simply stated. The differential cross section per unit solid angle Ω is given by the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{1+\cos^2 \theta}{2} \frac{1}{\left[1 + \gamma_0 (1 - \cos \theta)^2\right]} \times \left\{1 + \frac{\gamma_0^2 (1 - \cos \theta)^2}{(1 + \cos^2\theta)\left[1 + \gamma_0 (1 - \cos \theta)\right]}\right\} \frac{\text{cm}^2}{\text{elect.-steradian}} \quad , \tag{5.28}$$

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[43] Horton, C. C., "Some Worked Examples in Radiation Shielding," AERE-RS/L-3A, Harwell, England (1957).

[44] Moran, H. S., "Air Scattering of Co^{60} Gamma Rays; Theory Versus Experiment," ORNL-2019 (April 17, 1956).

[45] Jones, B. L., J. W. Harris and W. P. Kunkle, "Air and Ground Scattering of Cobalt-60 Gamma Radiation," CVAC-170T (March 30, 1955).

[46] Woodruff, L. V., J. W. Harris and W. P. Kunkel, "Air and Ground Scattering of Co^{60} Gamma Rays in a Shadow Shield Geometry," CVAC-198-T (December 16, 1953).

[47] Heitler, W., "The Quantum Theory of Radiation," Oxford University Press (London, 1950).

[48] Fano, U., "Gamma-Ray Attenuation," Nucleonics, 8 (#8) (August, 1953) and Nucleonics 11 (#9) 55 (September, 1953).

where $\gamma_0 = \frac{e^2}{mc^2} = 2.818 \times 10^{-13}$ cm, the classical radius of the electron. If the substitution

$$P = \frac{\gamma}{\gamma_0}$$

is made, equation 5.28 is simplified to

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} P^2 \left(\frac{1}{P} + P - \sin^2 \theta \right) \frac{cm^2}{\text{elect. - steradian}} \qquad 5.29$$

A useful plot of the Klein-Nishina formula may be seen in reference 49 and a more detailed set of graphs is given by Nelms.[50] A study of this information will indicate that for certain energies and certain angles simplifications may be made. For example, the differential cross section for 0.51-Mev photons is nearly constant from 90° to 180° and for 5.1-Mev photons is of nearly exponential form from 0° to 60°. Under these circumstances an integral expression involving the Klein-Nishina formula may be greatly simplified since $d\sigma/d\Omega$ would be a constant, or at least a more manageable analytic representation.

A simple problem in scattering is that given in Figure 5.7. The gamma rays scattered from the thin mirror to the detector X may be given by

$$\Phi_\gamma = \frac{Q_0}{4\pi r_1^2} \frac{N_0 M Z}{A r_2^2} \frac{d\sigma}{d\Omega} \gamma/cm^2 \text{ sec} \qquad 5.30$$

Since θ_1 is 90° the photon energy at X will be 0.51 Mev or less, and since, in addition, the scatterer is small, the cross section will be a constant. If r_1 and r_2 are small compared to the mean-free-path of the gamma rays in air (~400 ft), then multiple air scattering may be neglected. Should the mirror be larger, but not thicker, so that multiple scattering in the mirror is still not important, then an integration must be performed over the extent of the mirror and the value of θ may not be considered constant. Should the scatterer be not only large, but thick and dense, then an integration must be performed over the scattering volume, and multiple scattering in the scatterer must be included. This problem would not be a simple one, probably being beyond the range of hand computation.

In the same problem scattering from the walls and ceiling must also be considered and this too involves multiple scattering. If the shield

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[49] Price, B. T., C. C. Horton, and K. T. Spinney, "Radiation Shielding," Pergamon Press, N. Y. (1957) p. 26.

[50] Nelms, A. T., "Graphs of the Compton Energy-Angle Relationship and the Klein-Nishina Formula from 10 kev to 500 Mev," NBS Circular 542 (August 28, 1953).

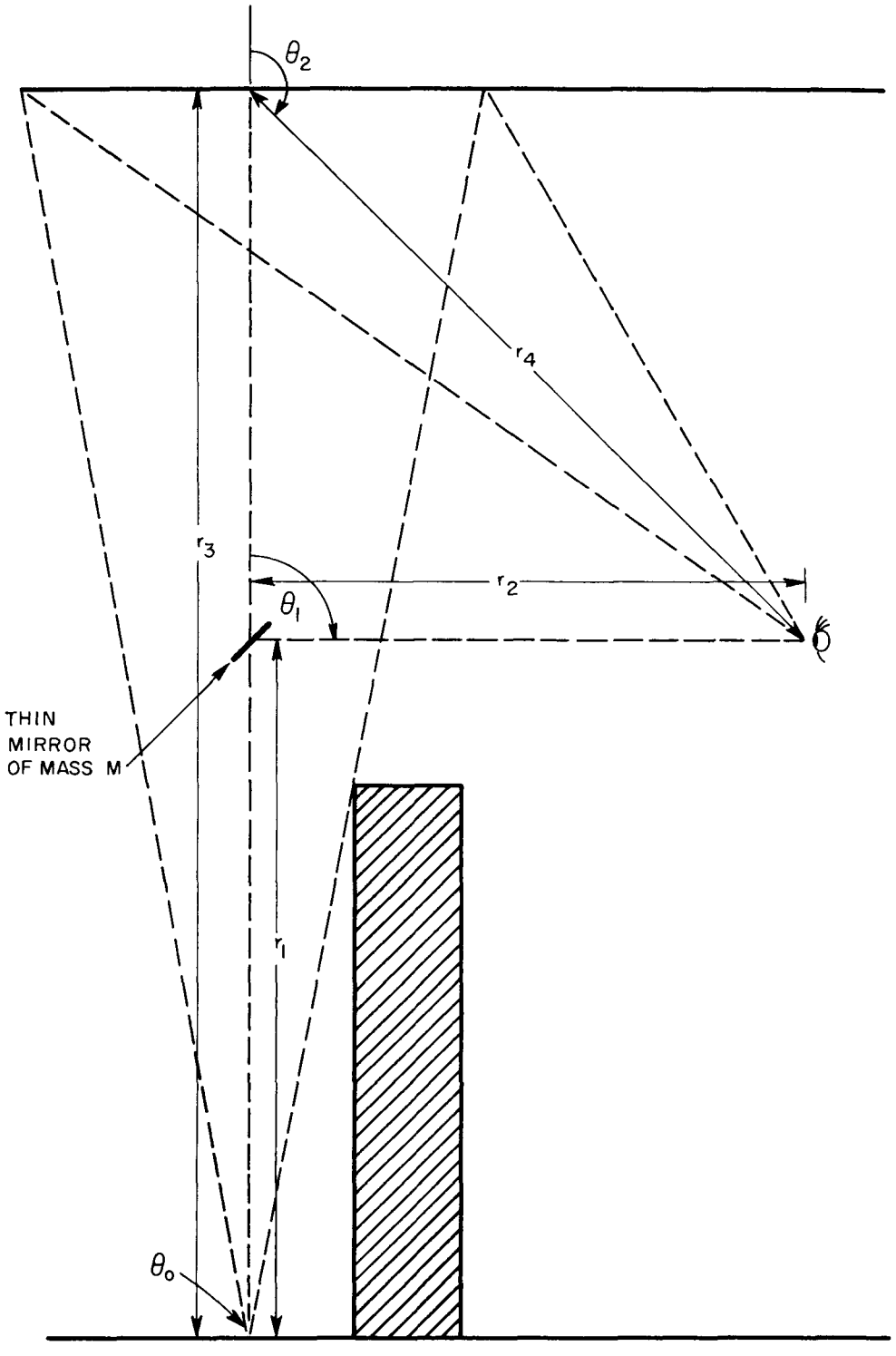

Figure 5.7

Gamma-Ray Scattering Problem

is such that the scattering area on the ceiling is circular, the problem may be easily approximated. The flux at the ceiling is nearly uniform and given by

$$\Phi_2 = \frac{Q_0}{4\pi r_3^2} \gamma/cm^2 \text{ sec} \quad . \qquad 5.31$$

The scattered flux at X is then given by

$$\Phi_3 = \int dA \frac{\Phi_2 \beta}{4\pi r_4^2} \gamma/cm^2 \text{ sec} \quad , \qquad 5.32$$

where β is the reflection coefficient, or albedo, of the material in the ceiling.[51] Equation 5.32 integrates rather easily for the centerline of a disk source (formula 3.47), which geometry may be reasonably assumed in this problem. Since θ_2 is between 90° and 180° the scattered photons are between 0.51 Mev and 0.25 Mev or less.

An example of air scattering is that of an opened reactor vessel (Figure 5.8) that emits a somewhat collimated beam of gamma rays that is reflected from the ceiling. This reflection from the ceiling could be estimated by the method of the previous paragraph; the air scattering may be estimated as follows. The gamma-ray flux in the air, Φ_γ, may be considered to be nearly constant from A to B and cylindrical in shape. The scattered source is then also cylindrical in shape and of magnitude

$$Z N_0 \Phi_\gamma \frac{d\sigma}{d\Omega} \quad ,$$

where $Z N_0$ is the electron density of the air. The observation that the angle of scatter is between 90° and 180° will indicate the energy range of the scattered photon. If $d\sigma/d\Omega$ may be approximated by a constant or at least by a more convenient analytical expression than in equation 5.28, the problem is that of computing the flux from a cylindrical volume source of radius R. For example, the source may be considered isotropic and of magnitude "$\mu\Phi_\gamma$," where "μ" is the total gamma-ray cross section for air. Multiple air scattering may be neglected unless the ceiling is 400 or more feet high. Should the scattering air volume not be so conveniently defined, it may be possible to work out the individual scattered contributions from many elementary volume sources, perhaps arranged in a convenient analytical fashion. The total scattered flux would then be the summed contributions from all such elementary volumes.

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[51] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand (March, 1956) p. 332.

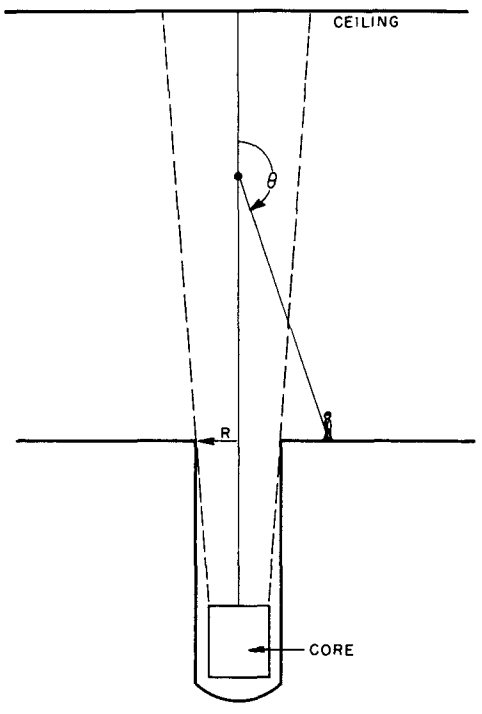

Figure 5.8

Gamma-Ray Air Scattering Problem

Another of the applications of the air scattering of gamma rays is that in hazards evaluation problems. This involves an estimation of the gamma-ray dose, both direct and by air scattering, from a building full of gaseous fission products. A recent evaluation of this type of problem was given at the Geneva Conference.[52]

[52] Geller, L., and R. Epstein, "A General Method for Evaluating Containment Shielding Under Normal and Emergency Conditions," Paper P/435, presented at the International Conference on Peaceful Uses of Atomic Energy held at Geneva, Switzerland (June, 1958).

Chapter 5 Problems

1. What should be the thickness of an EBWR fuel element transfer flask? See ANL-5607 for details on the fuel element and the predicted flask thickness.

2. What would be the shutdown radiation at the top of the reactor vessel when the EBWR reactor is shut down and the vessel head removed? What water level is required?

3. Determine whether it would be hazardous to stand at the side of the open vessel with no water above the core? How much water should there be?

6. NEUTRON-INDUCED ACTIVITY

Wherever neutron fluxes occur, neutron absorptions will take place. The nucleus that absorbs a neutron becomes excited and releases its excitation in the form of radiation of one sort or another and at a time consistent with physical laws of the nucleus. Capture gamma rays emerging at the instant of absorption are one avenue of relief for this unstable nucleus. Many materials remain in an excited state after the emission of capture gamma rays. These elements transfer to a more stable state with the emission of additional radiation according to the laws of radioactive decay. The half-life varies from a few seconds for some nuclei to many years for others. The radiation emitted varies in both energy and form according to the isotope[1,2,3] concerned (see Table 6.1). The problems caused by these radioactive nuclei depend strongly on the form of the radiation as well as the half-life. A strong gamma-ray emitter may be a shielding problem. A strong beta-ray emitter, while not generally a shielding problem, may be an ingestion hazard, the seriousness depending on the beta-ray energy, the radioactive half-life, and the body half-life (that is, the time it takes the human body to excrete one half of it.)

Table 6.1

INDUCED ACTIVITIES FROM SOME REACTOR COOLANTS

Target Isotope	Isotopic %	Cross Section at 2200 meter/sec, barn	Radioactive Product of Reaction	Half-Life	Energy of Radiation, Mev (γ)	Gammas per Disintegration
Na^{23}	100	0.53	Na^{24}	14.9 hr	2.76; 1.38	1 each
K^{41}	6.8	1.15	K^{42}	12.4 hr	1.51	0.25
O^{18}	0.204	0.00021	O^{19}	29.4 sec	1.6	0.7
O^{16}	99.8	0.019×10^{-3}[a]	N^{16}	7.4 sec	6.13; 7.10	0.76; 0.06
O^{17}	0.039	0.0052×10^{-3}[a]	N^{17}	4.1 sec	1 (neutron)	1 (neutron)

[a] Fast (n,p) reactions; cross sections averaged over fission spectrum. Data quoted is from Roys and Shure. A previous determination by Henderson and Tunnicliffe gives cross-sections for N^{16} of 0.0185×10^{-3} barn and for N^{17} of 0.0093×10^{-3} barn.

From Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand, (March, 1956).

1. Strominger, D., J. M. Hollander, and G. T. Seaborg, "Table of Isotopes," Rev. Mod. Phys., 30, 585 (1958).
2. Bopp, C. D., and O. Sisman, "How to Calculate Gamma Radiation Induced in Reactor Materials," Nucleonics, 14, (#1) 46 (January 1956).
3. Smith, G. W., and D. R. Farmelo, "Radionuclides Arranged by Gamma-Ray Energy," Nucleonics, 16, (#2) 80 (February 1958).

Neutron fluxes high enough to cause extreme activation do not usually exist outside a shielded environment. An obvious exception to this is, of course, the fluxes from an experimental beam port. Thus, normally, radioactive materials are found behind more than adequate shielding. There are, however, many ways in which these materials may get outside the reactor shield during the normal operation of a reactor. Test materials may be inserted and removed for examination; dust or just air, may drift into, and later, out of high neutron-flux zones. Coolant materials must normally be circulated from the reactor core to a location outside the reactor shield proper to a heat exchanger, or may be circulated almost anywhere in case of a leak; fuel elements must be replaced periodically, and mechanical parts may need replacement. These problems have all been met and solved one way or another. It is necessary to recognize these problems and their magnitudes in advance of the actual operation in order to prevent harmful exposure to personnel.

The radiation emitted by the activated material is rarely as energetic as the capture gamma rays in the core or shield materials, nor does it occur in as large quantities. This radiation does have the feature of getting outside the shield in one way or another; operating crews may then ingest the radioactive particles or be otherwise exposed to the radiation. The actual problem of shielding the radiations involved is essentially the same as shielding reactor gamma rays, as has been discussed, except that the source geometries may be quite different. The equations governing the build-up and decay of the radioactive nuclei are discussed in this section.

The differential equations governing the build-up of activity in a material are, neglecting the burnup of active nuclides.

$$A'(t) = N(t)\sigma_a\Phi(t) - \lambda A(t) \qquad \text{active atoms/cm}^3\,\text{sec} \qquad 6.1$$

and

$$N'(t) = -N(t)\sigma_a\Phi(t) \qquad \text{atoms/cm}^3\,\text{sec} \quad . \qquad 6.2$$

In these equations A(t) is the number of active atoms per cubic centimeter and N(t) is the number of target atoms per cubic centimeter. For many cases the burnup of the irradiated material is also negligible. Under this condition the differential equation governing the build-up of activity is

$$A'(t) = N_0\sigma_a\Phi(t) - \lambda A(t) \qquad \text{active atoms/cm}^3\text{sec} \qquad 6.3$$

The number of active atoms present after irradiation for a time "t" and cooling for a time "T" in a constant neutron flux "Φ_0" is then

$$A(t) = \frac{N_0\sigma_a\Phi_0}{\lambda}\,[1 - \exp(-\lambda t)]\exp(-\lambda T) \qquad \text{active atoms/cm}^3 \quad , \qquad 6.4$$

and the activity is

$$\lambda A(t) = N_0 \sigma_a \Phi_0 \left[1 - \exp(-\lambda t)\right] \exp(-\lambda T) \quad \text{dis/cm}^3 \text{ sec} \quad . \qquad 6.5$$

Here "N_0" is the number of atoms of the material to be irradiated at time $t = 0$, "Φ_0" is the neutron flux, "σ_a" the activation cross section, and "λ" the decay constant.

For a situation in which the material is cycling in and out of the neutron flux, as would be the case with the reactor coolant material, the activity build-up after "n" such cycles of "t_c" sec for the complete cycle and "t_r" sec in the active zone would be

$$\lambda A(nt_c) = N_0 \sigma_a \Phi_0 \frac{\left[1 - \exp(-\lambda t_r)\right]}{\left[1 - \exp(-\lambda t_c)\right]} \left[1 - \exp(-n\lambda t_c)\right] \quad \frac{\text{dis}}{\text{cm}^3 \text{sec}} \quad . \qquad 6.6$$

This assumes no initial activity, i.e., $A(0) = 0$. The above equation may be simplified under certain circumstances; for example, after a large number of cycles,

$$\lambda A(nt_c) \longrightarrow N_0 \sigma_a \Phi_0 \frac{\left[1 - \exp(-\lambda t_r)\right]}{\left[1 - \exp(-\lambda t_c)\right]} \quad \frac{\text{dis}}{\text{cm}^3 \text{sec}} \qquad 6.7$$

as $n \longrightarrow \infty$.

Also, if λt_c (and hence λt_r) is a very small number,

$$\lambda A(nt_c) = N_0 \sigma_a \Phi_0 \frac{t_r}{t_c} \left[1 - \exp(-n\lambda t_c)\right] \quad \text{dis/cm}^3 \text{sec} \quad . \qquad 6.8$$

Finally, if n becomes large and t_c is small,

$$\lambda A(nt_c) \longrightarrow N_0 \sigma_a \Phi_0 \frac{t_r}{t_c} \quad \text{dis/cm}^3 \text{sec} \qquad 6.9$$

as $n \longrightarrow \infty$.

If it is desired to include the burnup of the original atoms, an approximate result is

$$\lambda A(nt_c) \cong N_0 \sigma_a \Phi_0 \theta \left[1 - \exp(-\lambda t_r)\right] \exp(-n\lambda t_c)$$

$$\times \frac{\left[1 - \exp(n\lambda t_c - n\sigma_a \Phi_0 t_r)\right]}{\left[\exp(-\lambda t_c - \sigma_a \Phi_0 t_r) - 1\right]} \quad \frac{\text{dis}}{\text{cm}^3 \text{sec}} \quad , \qquad 6.10$$

where

$$1 \leq \theta \leq \exp(\sigma_a \Phi_0 t_r).$$

Another treatment of this problem, including burnup, is given by Allard.[4]

Equations (6.6) through (6.10) are suitable for calculating the number of active atoms per cubic centimeter of coolant. It must be remembered that Φ_0 is assumed not to vary with time and is also assumed to be a suitable average flux value. In case of a threshold reaction[5] such as O^{16} (n,p) N^{16}[6,7,8,8a] the determination of Φ_0 is more difficult than for the thermal neutron absorptions because the average effective threshold neutron flux is not as easily determined.

Equations (6.6) through (6.10) are also suitable for the calculation of radioactive atoms in a coolant due to the irradiation of impurities in the coolant. These impurities might be due to erosion or corrosion products in the water, in which case "N" is a function of time.

The problem of the activation of corrosion and erosion problems may be worked out if some assumptions are made regarding the coolant system. Let us assume that the reactor is a boiling water reactor (Figure 6.1) such as EBWR. Corrosion and erosion products enter the coolant at a rate "r" atoms per second, and leave at a rate determined by the water flow rate through the filter, "f_1" grams of coolant per second, which returns pure water to the system. The corrosion and erosion products also leave with the steam but at a reduced rate determined by the steam flow rate, "αf_2" grams of steam per second, where "α" is the distillation factor of the water to steam, and "f_2" is the steam flow rate. This distillation factor is not a simple quantity to determine since it is a

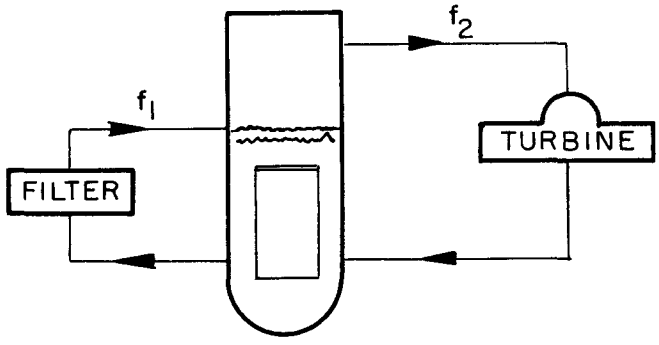

Figure 6.1
Illustration of the Reactor Coolant Cycle

[4] Allard, G. A., "Activation of Fluid Circulating Through A Reactor," KDPL-665 (1951)

[5] Rocklin, R. S., "Data Sheet No. 28-Fission Neutron Cross Sections for Threshold Reactions," Nucleonics, 17, (#1) 54 (January 1959).

[6] Rockwell, T., III, Editor, Reactor Shielding Design Manual, McGraw-Hill and D. Van Nostrand, (March, 1956) p. 87 and 229.

[7] Roys, P. A., and K. Shure, "Production Cross Section of N^{16} and N^{17}," Nucl. Sci. Eng., 4 (#6) 536 (December 1958).

[8] Henderson, W. J., and P. R. Tunnicliffe, "The Production of N^{16} and N^{17} in the Cooling Water of the NRX Reactor," Nucl. Sci. Eng., 3, 145, 1958.

[8a] Martin, H. C., "Cross-Sections for the O^{16}(n, p) N^{17} Reaction from 12 to 18 Mev," Phys. Rev. 93 498, (1954).

function of the somewhat unpredictable and certainly variable chemistry of the coolant and/or the impurities in the system.

The differential equation governing the number of atoms of corrosion and erosion products per gram of coolant in the above model is

$$N'(t) + F\,N(t) = r/M. \qquad 6.11$$

Here M is the total mass of the coolant in grams and

$$F = (f_1 + \alpha f_2)/M \quad \text{sec}^{-1} \quad .$$

The solution is

$$N(t) = \frac{r}{MF}\,[1 - \exp(-Ft)] \quad \text{atoms/gm.} \qquad 6.12$$

The differential equation governing the number of active atoms per gram of coolant in the system is

$$A'(t) + (\lambda + F)\,A(t) = \sigma_a \Phi_0 N(t), \qquad 6.13$$

where σ_a is the activation cross section in cm^2/atom. The solution is

$$A(t) = \frac{\sigma_a \Phi_0 r}{M\lambda F}\left\{\frac{\lambda}{(\lambda + F)} - \exp(-Ft) - \left[\frac{\lambda}{(\lambda + F)} - 1\right]\exp[-(\lambda + F)t]\right\} \quad \frac{\text{act. atoms}}{\text{gm}} \quad . \qquad 6.14$$

After operation for an infinitely long time, i.e., $(\lambda + F)t$ and Ft are large,

$$A(\infty) = \frac{\sigma_a \Phi_0 r}{MF}\,\frac{1}{(\lambda + F)} \quad \frac{\text{active atoms}}{\text{gm}} \quad . \qquad 6.15$$

The activity in disintegrations per gram per second is, of course, $\lambda A(t)$.

The number of active atoms in the turbine may be obtained if simplifying assumptions are made. If it is assumed that all the active atoms that leave the reactor in the steam deposit in the turbine and only in the turbine, the following may be written;

$$A_1'(t) + \lambda A_1(t) = \alpha f_2 A(t) \quad . \qquad 6.16$$

The solution is

$$A_1(t) = \frac{\alpha f_2 \sigma_a \Phi_0 r}{M\lambda F}\left\{\frac{1}{(\lambda + F)} - \frac{\exp(-Ft)}{(\lambda - F)} + \left[\frac{\lambda}{(\lambda - F)} - 1\right]\frac{\exp[-(\lambda + F)t]}{F}\right.$$

$$\left. - \left[\frac{1}{(\lambda + F)} - \frac{1}{(\lambda - F)} + \left(\frac{\lambda}{(\lambda + F)} - 1\right)\frac{1}{F}\right]\exp(-\lambda t)\right\} \text{ act atom.} \qquad 6.17$$

After an infinite time of operation, i.e., λt, $(\lambda + F)t$ and Ft becomes large,

$$A_1(\infty) = \frac{\alpha f_2 \sigma_a \Phi_0 r}{M F}\frac{1}{(\lambda + F)} \text{ act atoms.} \qquad 6.18$$

In reality this is the total number of active atoms that leave the reactor.

Similar considerations may be made for the escape of fuel into the system due to a fuel element rupture. The activity of the fuel may be represented as given in Section 5:

$$P(t,T) = \frac{P_0 KG}{0.2}[t^{-0.2} - (t + T)^{-0.2}]\frac{\text{Mev/sec}}{\text{gram fuel}} \quad . \qquad 5.3$$

Please refer to Figure 5.1 for a convenient graph of this function in the following form:

$$P(t,T) = \frac{P_0 KG}{0.2}\left[1 - \left(1 + \frac{T}{t}\right)^{-0.2}\right] t^{-0.2}\ \frac{\text{Mev/sec}}{\text{gram fuel}} \quad . \qquad 5.4$$

Equation 5.3 is the result obtained from the formula due to Way and Wigner.[9] For the symbols in equations 5.3 and 5.4 the following definitions are employed:

P_0 = steady state reactor power, watts/gram of fuel,
K = 3.1×10^{10} fiss/watt sec,
G = 1.6 (gamma-ray power only),
t = time after shutdown, sec,
T = time of operation, sec

After a long period of operation, this may be expressed as

$$P(t) = \frac{P_0 KG}{0.2} t^{-0.2}\ \frac{\text{Mev/sec}}{\text{gram fuel}} \quad . \qquad 5.5$$

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[9] Way, K. and E. P. Wigner, "Decay of Fission Product Gamma Rays," Phys. Rev. 70 115 (1946).

The amount of fuel in the system at any given time after the release of fuel is governed by the equation

$$N'(t) - F\,N(t) = 0, \qquad 6.19$$

which leads to the formula

$$N(t) = N_0 \exp[-Ft] \text{ grams fuel/gram coolant}, \qquad 6.20$$

where N_0 is the amount of fuel released into the system in a burst. The activity of the coolant may then be expressed as

$$P(t,\infty) = \frac{P_0 K G N_0}{0.2} \exp[-0.2 \ln t - Ft] \frac{\text{Mev/sec}}{\text{gram coolant}} \quad . \qquad 6.21$$

The activity from this source that gets into the turbine, or more correctly, the sum of the fission product activities in the external loop is then

$$P_1(t_1 \infty) = \frac{P_0 K G}{0.2} \frac{\beta f_2 N_0}{F} [1 - \exp(-Ft)]\, t^{-0.2} \text{ Mev/sec} \qquad 6.22$$

Here β is a transfer factor, such as α, which governs the amount of fuel activity that goes from the water to the steam.

The calculation of the activity involved in a reactor coolant usually is a fairly complex problem. Fluxes must be appropriately averaged, circulation times must be accurately known and, perhaps, most difficult of all, the chemistry of the active materials must be known. It is, however, possible to make simplifying assumptions that can guide the design of the pertinent parts of the reactor. Results of the analysis of such problems, both theoretical and experimental, have been reported.[10,11,12,13,14,15,16]

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[10] Small, W. J., J. L. Zegger, A. L. Medin, "Long-Lived Circulating Activity in the Army Package Power Reactor," APAE-20, (August 28, 1957).

[11] Pearce, W. R., "Analysis of Water Activation and Component Shielding for the ORNL Package Reactor Power Plant," CF-53-10-168, (November 10, 1954).

[12] Faull, N., "An Experimental Study of Neutron-Induced Activities in Water," AERE-R/R-1919, (1957).

[13] Meem, J. L., "Activity Buildup from Stainless Steel in Pressurized Water Reactors," Trans. Amer. Nucl. Soc., 1, (#2), 73 (December 1958).

[14] Lewis, W. B., "Data Sheet No. 19 - Radiation from Neutron-Activated Slabs and Cylinders," Nucleonics, 15, (#4), 84 (April 1957).

[15] Persiani, P. J., "The Calculation of Induced Gamma Activity in Commercial Materials," ANL-4487, (August 8, 1957).

[16] Haag, F. G., "Activity Transport in Sodium-Cooled Systems," Nucleonics, 15,(#2) 58 (February 1957).

Chapter 6 Problems

1. How much shielding is required on a 12-inch EBWR water pipe? (Assume t_c = 10 sec, t_r = 2 sec.)

2. What is the activity of the Na in the EBWR coolant water if there is 1 ppm and no loss of Na atoms to the filter?

3. Estimate the Na activity due to Al^{27} (n,α) Na^{24} if the EBWR fuel was aluminum clad. Compare with problem 2.

4. Could you safely inhale air that passed through a thermal neutron flux of 1×10^{10} nvt?

7. RADIATION HEATING

The subject of radiation heating is usually treated on a rather macroscopic basis, that is to say, the details of how the radiation energy is actually transformed to heat energy is not considered in detail. A gamma ray, for example, is assumed to be absorbed according to the energy-absorption cross section without regard to the detailed considerations of electrons, brehmsstrahlung, etc. This may be done simply because often the distributions of secondary products do not vary excessively from the absorption of the primary product, the gamma ray. In the core regions the beta rays from fission must be considered, since they are a major source of heat, but only neutrons and gamma rays constitute vehicles for transporting significant quantities of energy from the core to surrounding materials.[1,2]

High-energy neutrons may transport energy in the form of kinetic energy, which, of course, is directly related to the energy they possess. The rate of energy release by the elastic scattering of neutrons is given by

$$H = \int_0^\infty dE\, E_n\, \Phi(E) \sigma_s(E)\, g \quad \text{Mev/cm}^3\text{sec}, \qquad 7.1$$

where $\Phi(E)\, dE$ is the flux of neutrons with energies in the range E to E + dE, σ_s is the elastic cross section, and "g" is the average fractional energy loss of the neutron. This power may be approximated by

$$H = \sigma_s \Phi_f\, \xi\, E_n \quad \text{Mev/cm}^3\text{sec}, \qquad 7.2$$

where σ_s is the neutron scattering cross section, Φ_f is the fast neutron flux, ξ is the average logarithm of the energy loss, and E_n is the energy of the neutron. While this is true for neutrons of all energies above thermal, generally only those in the range of energy above one Mev have enough kinetic energy to be significant, although even these neutrons produce less heat than do the gamma rays.

High-energy neutrons may also release energy by the process of inelastic scattering, which may be estimated by the following formula:

$$H = \sigma_i \Phi_f f E_n \quad \text{Mev/cm}^3\text{sec} \qquad 7.3$$

Here σ_i is the inelastic scattering cross section[3] and fE_n is the neutron energy loss per collision. Inelastic scatter is also not a major source of

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[1] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill and D. Van Nostrand, (March 1956) p. 76

[2] Byrum, B. L., and J. A. Biggerstaff, "Nuclear Radiation Heating: Preliminary Design Considerations," Nucl. Sci. and Eng., 5, (#1) 28 (1959)

[3] Cranberg, et al., Physics and Mathematics, Chapter 4 Progress in Nuclear Energy Series 1, Vol. 1, McGraw-Hill Book Co., Inc., (New York, 1956)

heat; because of the complications, as well as the relative inadequacy of the data, it is usually only necessary to prove that this is not a major source of heat by simple considerations such as through the use of equation 7.3. It should be mentioned that the energy is liberated in the form of gamma rays so that, if it should become a significant source of energy release, the actual heat release would be via the subsequent gamma-ray absorption. As such, equation 7.3 is to be considered as referring to a source of gamma rays created by inelastic scatter. These gamma rays are usually of a lower energy than those created by neutron absorption and are thus absorbed nearer the point of creation. Simplifying assumptions regarding their absorption, such as that they are absorbed at the point of creation, make equation 7.3 adequate under many circumstances.

The most profuse source of heat brought about by neutron interactions is due to the absorption process. This interaction does not usually produce its heat directly, but indirectly in the form of gamma rays that do not give up their energy in the immediate vicinity of the absorption of the neutron. Of course, reactions leading to particle emission are a noteworthy exception, the ideal example of which is neutron absorption in boron. In this case the energy is carried off in the form of kinetic energy by an alpha particle, which is essentially absorbed at the place it was created. The neutron absorption distribution generally is a source distribution for the emitted gamma ray. This source distribution is

$$Q_\gamma = \sigma_a \Phi_n N_\gamma \ \gamma/cm^3 sec \quad . \tag{7.4}$$

Usually this is the thermal neutron absorption cross section times the thermal neutron flux times the number of gamma rays emitted per neutron absorption. This may be stated because the product $\sigma_a \Phi_n$ is usually greater for thermal neutrons than for the sum of all other energies, N_γ being considered independent of neutron energy. In the materials where intermediate neutron fluxes become significantly larger than the thermal neutron fluxes, such as in iron, the increase of the absorption cross section with decreasing energy may be off-set by the higher intermediate energy neutron fluxes. Under circumstances such as these the neutron absorption density cannot be described more accurately than is the flux of intermediate neutrons.

The release of heat by the absorption of gamma rays may be expressed as

$$H = \mu_E \Phi_\gamma E_\gamma \quad Mev/cm^3 \ sec, \tag{7.5}$$

where μ_E is the energy absorption coefficient (see Tables 7.1 and 7.2) and E_γ the gamma-ray energy. The gamma-ray flux, Φ_γ, has been previously independently determined from all sources, as discussed in section 5 using the formulas in section 3, and then treated as indicated in equation 7.5. If the gamma-ray fluxes are determined by energy lines, then equation 7.5 should be applied for each line, or for each gamma-ray energy. The total heat will then be the sum of the results obtained by considering each line.

Table 7.1

GAMMA-RAY ENERGY ABSORPTION MASS ATTENUATION COEFFICIENTS

(in cm^2/gm)

	Gamma-Ray Energy, Mev																	
Material	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.50	2	3	4	5	6	8	10.0
H	.0411	.0487	.0531	.0575	.0589	.0591	.0590	.0575	.0557	.0533	.0509	.0467	.0401	.0354	.0318	.0291	.0252	.0255
Be	.0183	.0217	.0237	.0256	.0263	.0264	.0263	.0256	.0248	.0237	.0227	.0210	.0183	.0164	.0151	.0141	.0127	.0118
C	.0215	.0246	.0267	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0237	.0209	.0190	.0177	.0166	.0153	.0145
N	.0224	.0249	.0267	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0238	.0211	.0193	.0180	.0171	.0158	.0151
O	.0233	.0252	.0271	.0289	.0296	.0297	.0296	.0289	.0280	.0268	.0257	.0238	.0212	.0195	.0183	.0175	.0163	.0157
Na	.0289	.0258	.0266	.0279	.0283	.0284	.0284	.0276	.0268	.0257	.0246	.0229	.0207	.0194	.0185	.0179	.0171	.0168
Mg	.0335	.0276	.0278	.0290	.0294	.0293	.0292	.0285	.0276	.0265	.0254	.0237	.0215	.0203	.0194	.0188	.0182	.0180
Al	.0373	.0283	.0275	.0283	.0287	.0286	.0286	.0278	.0270	.0259	.0248	.0232	.0212	.0200	.0192	.0188	.0183	.0182
Si	.0435	.0300	.0286	.0291	.0293	.0290	.0290	.0282	.0274	.0263	.0252	.0236	.0217	.0206	.0198	.0194	.0190	.0189
P	.0501	.0315	.0292	.0289	.0290	.0290	.0287	.0280	.0271	.0260	.0250	.0234	.0216	.0206	.0200	.0197	.0194	.0195
S	.0601	.0351	.0310	.0301	.0301	.0300	.0298	.0288	.0279	.0268	.0258	.0242	.0224	.0215	.0209	.0206	.0206	.0206
A	.0729	.0368	.0302	.0278	.0274	.0272	.0270	.0260	.0252	.0242	.0233	.0220	.0206	.0199	.0195	.0195	.0194	.0197
K	.0909	.0433	.0340	.0304	.0298	.0295	.0291	.0282	.0272	.0261	.0251	.0237	.0222	.0217	.0214	.0212	.0215	.0219
Ca	.111	.0489	.0367	.0318	.0309	.0304	.0300	.0290	.0279	.0268	.0258	.0244	.0230	.0225	.0222	.0223	.0225	.0231
Fe	.225	.0810	.0489	.0340	.0307	.0294	.0287	.0274	.0261	.0250	.0242	.0231	.0224	.0224	.0227	.0231	.0239	.0250
Cu	.310	.107	.0594	.0368	.0316	.0296	.0286	.0271	.0260	.0247	.0237	.0229	.0223	.0227	.0231	.0237	.0248	.0261
Mo	.922	.294	.141	.0617	.0422	.0348	.0315	.0281	.0263	.0248	.0239	.0233	.0237	.0250	.0262	.0274	.0296	.0316
Sn	1.469	.471	.222	.0873	.0534	.0403	.0346	.0294	.0268	.0248	.0239	.0233	.0243	.0259	.0276	.0291	.0316	.0339
I	1.726	.557	.260	.100	.0589	.0433	.0366	.0303	.0274	.0252	.0241	.0236	.0247	.0265	.0283	.0299	.0327	.0353
W	4.112	1.356	.631	.230	.121	.0786	.0599	.0426	.0353	.0302	.0281	.0271	.0287	.0311	.0335	.0355	.0390	.0426
Pt	4.645	1.556	.719	.262	.138	.0892	.0666	.0465	.0375	.0315	.0293	.0280	.0296	.0320	.0343	.0365	.0400	.0438
Tl	5.057	1.717	.791	.285	.152	.0972	.0718	.0491	.0393	.0326	.0301	.0288	.0304	.0326	.0349	.0354	.0406	.0446
Pb	5.193	1.753	.821	.294	.156	.0994	.0738	.0505	.0402	.0332	.0306	.0293	.0305	.0330	.0352	.0373	.0412	.0450
U	9.63	2.337	1.096	.392	.208	.132	.0968	.0628	.0482	.0383	.0346	.0324	.0332	.0352	.0374	.0394	.0443	.0474
Air	.0233	.0251	.0268	.0288	.0296	.0297	.0296	.0289	.0280	.0268	.0256	.0238	.0211	.0194	.0181	.0172	.0160	.0153
NaI	1.466	.476	.224	.0889	.0542	.0410	.0354	.0299	.0273	.0253	.0242	.0235	.0241	.0254	.0268	.0281	.0303	.0325
H_2O	.0253	.0278	.0300	.0321	.0328	.0330	.0329	.0321	.0311	.0298	.0285	.0264	.0233	.0213	.0198	.0188	.0173	.0165
Concrete	.0416	.0300	.0289	.0294	.0297	.0296	.0295	.0287	.0278	.0272	.0256	.0239	.0216	.0203	.0194	.0188	.0180	.0177
Tissue	.0271	.0282	.0293	.0312	.0317	.0320	.0319	.0311	.0300	.0288	.0276	.0256	.0220	.0206	.0192	.0182	.0168	.0160

From Gladys White Grodstein, "X-Ray Attenuation Coefficients from 10 kev to 100 Mev," NBS Circular 583 (April 30, 1957), Chapts. 1-3.

Table 7.2

GAMMA-RAY ENERGY ABSORPTION ATTENUATION COEFFICIENTS

(in cm^{-1})

Material	Density	Gamma-Ray Energy, Mev																	
		0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.25	1.50	2	3	4	5	6	8	10.0
Be	1.85	.0339	.0401	.0438	.0474	.0487	.0488	.0487	.0474	.0459	.0438	.0420	.0389	.0339	.0303	.0279	.0261	.0235	.0218
C	2.25	.0484	.0554	.0601	.0648	.0666	.0668	.0666	.0650	.0630	.0603	.0576	.0533	.0470	.0427	.0398	.0374	.0344	.0326
Na	.9712	.0281	.0251	.0258	.0271	.0275	.0276	.0276	.0268	.0260	.0250	.0239	.0222	.0201	.0188	.0180	.0174	.0166	.0163
Mg	1.741	.0583	.0481	.0484	.0505	.0512	.0510	.0508	.0496	.0481	.0461	.0442	.0413	.0374	.0353	.0338	.0327	.0317	.0313
Al	2.70	.1007	.0764	.0743	.0764	.0775	.0772	.0772	.0751	.0729	.0699	.0670	.0626	.0572	.0540	.0518	.0508	.0494	.0491
Si	2.42	.1053	.0726	.0692	.0704	.0709	.0702	.0702	.0682	.0663	.0636	.0610	.0571	.0525	.0499	.0479	.0469	.0460	.0457
P	1.83	.0917	.0576	.0534	.0529	.0531	.0531	.0525	.0512	.0496	.0476	.0458	.0428	.0395	.0377	.0366	.0361	.0355	.0357
S	2.07	.1244	.0727	.0642	.0623	.0623	.0621	.0617	.0596	.0578	.0555	.0534	.0501	.0464	.0445	.0433	.0426	.0426	.0426
K	0.87	.0791	.0377	.0296	.0264	.0259	.0257	.0253	.0245	.0237	.0227	.0218	.0206	.0193	.0189	.0186	.0184	.0187	.0191
Ca	1.55	.172	.0758	.0569	.0493	.0479	.0471	.0465	.0450	.0432	.0415	.0400	.0378	.0357	.0349	.0344	.0346	.0349	.0358
Fe	7.86	1.769	.6367	.3844	.2672	.2413	.2311	.2256	.2154	.2051	.1965	.1902	.1816	.1761	.1761	.1784	.1816	.1879	.1965
Cu	8.933	2.769	.956	.5306	.3287	.2823	.2644	.2555	.2421	.2323	.2206	.2117	.2046	.1992	.2028	.2064	.2117	.2215	.2332
Mo	9.01	8.307	2.649	1.270	.5559	.3802	.3135	.2831	.2532	.2370	.2234	.2153	.2099	.2135	.2253	.2361	.2469	.2667	.2847
Sn	7.298	10.721	3.437	1.620	.6371	.3897	.2941	.2525	.2146	.1956	.1810	.1744	.1700	.1773	.1890	.2014	.2124	.2306	.2474
I	4.94	8.704	2.752	1.284	.494	.2910	.2139	.1808	.1497	.1354	.1245	.1191	.1166	.1220	.1309	.1398	.1477	.1615	.1744
W	19.3	79.362	26.171	12.178	4.439	2.335	1.517	1.156	.8222	.6813	.5829	.5423	.5320	.5539	.6002	.6466	.6852	.7527	.8222
Pt	21.37	99.264	33.252	15.365	5.599	2.949	1.906	1.423	.9937	.8014	.6732	.6261	.5984	.6326	.6838	.7330	.7800	.8548	.9360
Tl	11.86	59.976	20.364	9.381	3.380	1.803	1.153	.8515	.5823	.4661	.3866	.3570	.3416	.3605	.3866	.4139	.4198	.4815	.5290
Pb	11.34	58.889	19.879	9.310	3.334	1.769	1.127	.8369	.5727	.4559	.3765	.3470	.3323	.3459	.3742	.3992	.4230	.4672	.5103
U	18.7	180.08	43.702	20.495	7.330	3.890	2.468	1.810	1.174	.9013	.7162	.6470	.6059	.6208	.6582	.6994	.7368	.8284	.8364
NaI	3.667	5.376	1.745	.8214	.3260	.1988	.1503	.1298	.1096	.1001	.0928	.0887	.0862	.0884	.0931	.0983	.1030	.1111	.1192
H_2O	1.00	.0253	.0278	.0300	.0321	.0328	.0330	.0329	.0321	.0311	.0298	.0285	.0264	.0233	.0213	.0198	.0188	.0173	.0165
Concrete	2.35	.0978	.0705	.0679	.0691	.0698	.0697	.0693	.0674	.0653	.0639	.0602	.0562	.0508	.0477	.0456	.0442	.0423	.0416

Since the general shape of the heat distribution will be exponential, subsequent calculations to determine stresses and temperature distributions must be on the basis of an exponentially distributed heat source. The analysis of such problems is contained in the literature.[4,5,6,7,8]

The results of such analyses must be interpreted as to permissible limits. These are, for example, the conventional limits imposed by boiler codes, etc., when applied to a reactor vessel. These limits, however, are not so obvious in the case of concrete stresses and temperatures. The results of some Hanford work[9] indicate that the temperature has a special significance as applied to water content (please refer back to Table 4.5). The neutron-attenuating properties are significantly decreased with the loss of water. On this basis a 50° limitation on the concrete temperatures would not be unreasonable. Stress conditions are quite dependent on the use of the concrete for dual purposes. The concrete biological shield may, for example, be the structural support of a considerable amount of equipment, and this would more likely set the stress limitation since small cracks in the concrete would not be a serious problem as far as radiation streaming is concerned. A discussion of this subject by Davis[10] is helpful in determining concrete stresses. Others have published information along this line for concrete and other materials.[11,12,13]

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[4] Bonilla, F., "Heat Removal," Nuclear Engineering, McGraw-Hill Book Co., (New York, 1957) Chapter IX, p. 360

[5] Kroeger, H. R., et al., "The Effect of Gamma Heating on the APPR-1 Pressure Shell," APAE-Memo-85, (September 1956)

[6] Heisler, M., and J. Wetch, "Heat Generation in Thermal Shields," NAA-SR-942 (August 15, 1954)

[7] Durham, Franklin P., "Heat Transfer and Thermal Stresses in Nuclear Reactor Shells," LA-1590 (September 1953)

[8] Lansing, F., "Determining the Geometry of Thermal Shields," Nucleonics 13 (#6) 58 (June 1955)

[9] Wood, D. E., "Neutron Attenuation in Magnetite Concrete Heated to 100°C, 200°C, 300°C," Trans. Amer. Nucl. Soc. 1 (#1) 67 (June 1958)

[10] Davis, H. S., "Thermal Considerations in the Design of Concrete Structures for Shielding Atomic Power," Nuclear Congress, Paper #9 (March 17-24, 1958)

[11] Halliday, D. B., "Heat Release in Concrete Reactor Shields," AERE-R/R-1963, SWP/P31 (1956)

[12] Bonsall, W., "Thermal Stresses in Reinforced Concrete Shields," IGE-R-3 (October 1955)

[13] Claiborne, H. C. et al., "Calculating Gamma Heating in Reactor Structures," Nucleonics, 15 (#10) 114 (October 1957)

Chapter 7 Problems

1. Compare the heating caused by a fast neutron flux due to inelastic scattering to that due to elastic scattering.

2. What is the heat release due to a thermal neutron flux of 1×10^{12} in Boral?

3. What is the heat generation in a thick slab of iron that has an incident thermal neutron flux of 1×10^{12}?

8. MATERIALS

The materials used to shield a reactor will be discussed from two somewhat different viewpoints. The first is on the basis of fundamental properties, that is, ideally, what are the desired nuclear properties of a reactor shield? The second is in a practical engineering sense, that is, what materials actually possess the properties desired and are structurally, chemically and economically feasible as well?

Shielding any single form of radiation is a relatively simple affair. Gamma rays are attenuated by processes which are a function of atomic number and mass. This can obviously be incorporated into a shield most efficiently by materials of high density or those that are high in the periodic table of the elements. Uranium is an example of such a material and actually depleted uranium is worthy of consideration for a gamma-ray shield material. Lead, gold, bismuth, and mercury are other examples. Quite often uranium is not as practical as lead, because the presence of neutrons in the uranium would produce undesirable side effects. Lead and alloys of lead that improve the structural properties are the materials most commonly used for gamma-ray shielding. They are selected from a structural viewpoint combined with a measure of economy. More extreme structural requirements often lead to iron or steel as a choice for a gamma-ray shield. The reduction in density will require additional space in the form of a thicker, as well as a heavier, shield. Even water is used as a primary shield material in gamma-ray irradiation facilities. This points out the fact that the material selected for a gamma-ray shield can be quite dependent upon the over-all objective of the facility for which it is intended, rather than simply the most efficient material on a nuclear or economic basis.

Neutrons are a more complicated form of radiation for which to select shielding materials: first, because the energy range that must be considered is extremely wide, up to eight decades as contrasted to two decades at the most for gamma rays, and secondly, because of the associated capture gamma rays. It is obvious that a material of low mass is best for moderating fast neutrons by elastic scattering; as a result hydrogen, probably in the form of water, should be employed whenever possible and as generously as possible. Since the cross section of hydrogen is quite small for the neutrons that occur at the high end of the fission spectrum, that is, for those neutrons with energies greater than one Mev, it is possible, as well as desirable, to improve the energy loss on the first collision for high-energy neutrons by making use of materials that have good inelastic scattering properties. Materials that have this effect are those of high mass, for instance, iron and lead. In order to eliminate a neutron from the scene it must be reduced in energy by elastic or inelastic scattering until it reaches thermal or near thermal energy where it can readily be absorbed. Actually it is not as efficient to reduce high-energy neutrons to thermal

energy by either elastic or inelastic scattering alone as it is by a combination of the two. Absorption does occur at high energies but not in large enough measure to be significant. Elastic and inelastic cross sections, although appreciably greater than the absorption cross sections, are also not large for high-energy neutrons, and as a result fast neutron attenuation is not accomplished easily. While there are materials that have larger high-energy cross sections than others, the relative variation in high-energy cross sections from one material to another is not nearly as great as, for example, the difference in the thermal neutron cross section in boron as compared to that of graphite. Nor is this change in fast neutron cross sections as great as the variation in intermediate energy neutron cross sections in resonance regions. Since there are no "magic" materials, such as are boron or cadmium for thermal neutrons, fast neutrons take a certain relatively unalterable amount of material in which to be attenuated. Efficient arrangements of materials may reduce this total amount of material significantly, but not radically. A view of the total problem may, however, lead to a more efficient shield for the secondary products that result from neutron absorption, with a resulting appreciable net saving in shield thickness or weight. To shield fast neutrons, which when including secondary products is essentially the total shield design problem for a reactor, one should include both inelastic and elastic scattering materials, the object being to first reduce the energy of those neutrons with highest energy by inelastic scattering and to further reduce this energy by elastic collisions to thermal energy where the neutron may be readily absorbed.

Thermal neutrons are not much of a problem per se. They may be absorbed readily enough in most materials, so that they rarely become a dose problem; however, since they produce capture gamma rays upon absorption, consideration must be given to where they are absorbed and in what materials. Neutrons absorbed in boron, which has a conveniently high thermal neutron cross section, do not produce as troublesome capture gamma rays as do those absorbed in cadmium, which also has a conveniently high thermal neutron cross section. Judicious placement of boron will often result in an improvement in the shield thickness by virtue of the reduced number of capture gamma rays.

The actual choice of a shield material must be the result of compromises between the nuclear properties, economics and general objectives of the plant. Nuclear properties include not only the ability to attenuate radiation as necessary, but to withstand this radiation for a period of time without suffering serious deterioration.[1] Economics, of course, includes not only the cost of the materials, but also the cost of materials in place.[2]

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[1] Wilson, J. C., and D. S. Billington, "Effect of Nuclear Radiation on Structural Materials," American Institute of Chemical Engineers, Preprint #91 (December 1955)

[2] Lane, J. A., "How to Design Reactor Shields for Lowest Cost," Nucleonics, 13 (#6) 56 (June 1955)

Thus transportation, fabrication, and installation must each be considered. An example is that the choice of a heavy concrete aggregate may be made more on the basis of what kind is most readily available rather than what kind has the best nuclear properties. The general objectives include such things as whether the plant is for economical nuclear power, neutron physics research or nuclear propulsion. These things very clearly influence the choice of shield materials. There are summaries of materials and their properties available in the literature.[3,4]

The above discussion leads us to the first guess as to the materials to compose a reactor shield. These materials are: a heavy material for inelastic scattering of fast neutrons, as well as the absorption of the gamma rays, which might originate in the reactor core or by neutron capture in the shield, and a light material for elastic scattering of neutrons. It is efficient to attenuate the neutrons and gamma rays at approximately the same rate, that is with a shield that has approximately the same attenuation coefficient for neutrons as for gamma rays. This is somewhat subject to alteration when total weight is important. A savings in weight may be realized, for example, by permitting a larger portion of the total dose to consist of gamma rays, since this can be accomplished, to a limited extent, by replacing heavy material by light. With this possible exception the best reactor shield would appear to be an intimate mixture of hydrogen and metal, with the final selection a compromise between nuclear properties and costs.[5]

According to the knowledge of the nuclear properties that are required to attenuate radiation from a reactor, it would appear that the best shield would be, perhaps, a heavy metal hydride. The metal hydrides can be made, but they are usually quite unstable and would no doubt be costly to procure and maintain. In addition, they do not necessarily form with the proper ratio of metal to hydrogen so that the attenuation of the neutrons will proceed at the same rate as the attenuation of the gamma rays. All things considered, hydrides are not too appealing and to date no known application of these compounds has been made in reactor shields, although they have been considered.[6]

Another interesting material arrangement that has been considered and which is also a metal-hydrogen mixture is a lead honeycomb structure filled with water. The ratio of metal to hydrogen can certainly be controlled in this situation, but the cost is still rather prohibitive, at least until efficient shields are needed to the extent that they can be mass produced. An approximation to the honey comb that has been utilized is that of alternate slabs of metal and hydrogen-containing material. Examples of this arrangement include the Hanford reactors, which utilized laminated masonite and steel, the

[3] Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill & D. Van Nostrand, (March 1956).

[4] Hogerton, J. F., and R. C. Grau, The Reactor Handbook, Vol. III, Materials, AECD-3647, (1955)

[5] Judkins, M. F., "What Price Reactor Materials? "Nucleonics, 16 (#1) 96 (January 1958).

[6] Slesser, C. E., and R. P. Gibb, Jr., "Properties of Lithium Hydride for Shielding Purposes," Shielding Information Meeting, May, 1955, WASH-292 Pt. 3, (September 1955) p. 42

CP-5 thermal column shield, which contains lead and masonite, as well as the top shield and thermal column shield of the Argonaut, which are steel and masonite. Paraffin and lead laminations could be utilized under favorable circumstances and paraffin with lead shot is very often used to shield experimental facilities. Probably the chief factor limiting the use of these arrangements is the cost of materials and construction. As a result they appear only in portions of a shield or in small shields for experimental facilities and not as a rule as the total bulk shield for a reactor.

A more common method of attaining a balanced mixture of metal and hydrogen is exemplified by the use of concrete as a bulk shield material. Due to the facts that concrete is a material having fairly desirable nuclear properties and one with which people have had much construction experience, it has been utilized quite extensively. When there are no severe performance requirements, ordinary concrete is quite satisfactory as a reactor shield. If the additional expense may be justified on the basis of the reduced building size, or on the basis of reduced length of shield penetrations, the concrete may be made more dense by the addition of heavy aggregates, such as magnetite, barytes, limonite, ferrophosphorous, etc., and/or steel punchings. All things considered the selection of the materials for a biological shield concrete is often more a function of what is locally available then virtually any other consideration. Tables 8.1 and 8.2 contain data on the contents of some concretes.

A perplexing problem for many years has been the water content of concrete. In the first place, it is not well-known just how much water normally stays with the concrete as it cures, nor is it known too well just how much should be required as a minimum amount that should be retained for shielding considerations. Recent studies have established that a minimum amount of seven percent water content by weight would be adequate for ordinary concrete.[7] There is still some question as to the minimum water content necessary in the concretes made from the heavy aggregates as well as the more detailed effects of water content on the attenuating properties of the concrete. It is interesting to note that to a certain degree concretes made for shielding purposes tend to require more water while concrete made for structural purposes tend to require less water than the amount

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[7] Blizard, E. P., "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193, (August 29, 1958).

Table 8.1

COMPOSITION AND DENSITY OF SOME CONCRETES

Concrete	*Symbol	Density gm/cm^3	Density lb/ft^3	Composition, lb/yd^3 Water	Portland Cement	Aggregate	Steel Punchings	Total	Ref.
Ordinary									
1	01	2.33	145						40
2a	02-a	2.3	144	260	318	3300	-	3878	40
2b	02-b	2.22	139			(Sand and Gravel)			
3	03	2.39	149						41
4	04	2.35	147						42
Ferrophosphorous									
a	FP-a	4.68	292	384	730	6870	-	7894	‡
b	FP-b	4.57	285			(Ferrophosphorus Slag)			
Barytes									
a	BA-a	3.46	216	370	550	4980	-	5900	40,43
b	BA-b	3.35	212			($BaSO_4$ Ore)			
Magnetite									
a (EBWR Shield)	M-a	3.62	226	330	875	4900	-	6105	44,45
b	M-b	3.52	220			(Magnetite Ore)			
Brookhaven (Limonite)	BR	4.26	266						40
Ilmenite									
1 (New York State)	I-1	3.68							46†
2 (Swedish)	I-2	3.62							47†
Magnetite and Steel Punchings									
1a (MIT Reactor Shield)	MS1-a	4.5	280	333	550	3310	3360	7553	44
1b	MS1-b	4.4	274			(Magnetite Ore)			
2a (EBWR Shield)	MS2-a	4.74	295	340	940	1900	4800	7980	44,45
2b	MS2-b	4.64	290			(Magnetite Ore)			
Limonite and Steel Punchings									
a (CP-5 Shield)	LS-a	4.64	289	347	980	1825	4680	7832	††
b	LS-b	4.54	283			(Limonite Ore)			

*Where appearing, "a" and "b" indicate 100% and 50% water retention, respectively.

†Ilmenite Ore substituted for Magnetite in M-a.

‡Victor Chemical Works, Chicago, Illinois.

††Spectrochemical Analysis by Hanford, unpublished.

Table 8.2

ELEMENTAL COMPOSITION OF SOME CONCRETES

(in gm element / cm^3 concrete)

Ordinary Concretes

Element	O1	O2-a	O2-b	O3	O4
H	0.00484	0.023	0.0144	0.020	0.013
O in water	0.0384	0.183	0.114	0.159	0.103
in dry mix	1.1106	1.037	1.036	0.957	1.062
C	0.130	0.0023		0.118	-
Mg	0.00486	0.005		0.057	0.006
Al	0.0119	0.078		0.085	0.107
Si	0.438	0.775		0.342	0.737
S	0.00192	-		0.007	0.003
K	-	0.0299		0.004	0.045
Ca	0.581	0.100		0.582	0.194
Fe	0.00726	0.032		0.026	0.029
Na	-	0.0368		-	0.400
ρ , gm/cm^3:	2.33	2.30	2.22	2.39	2.35

Heavy Concretes

Element	FP-a	FP-b	BA-a	BA-b	M-a	M-b	BR	I-1	I-2
H in water	0.0253	0.0127	0.0243	0.0122	0.0217	0.0109	0.0211	0.0217	0.0217
O in water	0.202	0.101	0.194	0.0972	0.174	0.087	0.763	0.174	0.174
in ore	-		0.828		0.894			1.037	0.916
in cement	0.156		0.118		0.187			0.187	0.187
Mg in ore	-		-		0.0263		0.008	0.280	0.0139
in cement	0.00522		0.00396		0.0063			0.0063	0.0063
Al in ore	-		-		0.0614		0.0213	-	-
in cement	0.0183		0.0139		0.0220			0.022	0.022
Si in ore	-		-		0.0339		0.0589	0.047	-
in cement	0.0465		0.0352		0.0567		-	0.0557	0.0557
P	1.206		-		0.0004		-		
S	-		0.361		0.0052		0.0035	-	-
Ca in ore	-		0.0203		0.0166		0.259	-	0.0012
in cement	0.195		0.148		0.234		-	0.234	0.234
Ti	-		-		0.192		-	0.998	0.914
V	-		-		0.0113		-	-	-
Cr	-		-		0.00575		-	-	-
Mn	-		-		0.0067		0.067	0.0247	0.0305
Fe in ore	2.814		0.150		1.661		3.055	0.583	1.001
in cement	0.0121		0.00917		0.0145		-	0.0145	0.0145
Ba	-		1.551		-		-	-	-
ρ , gm/cm^3:	4.68	4.57	3.46	3.35	3.62	3.52	4.26	3.62	3.62

Heavy Concretes with Steel Punchings

Element	MS1-a	MS1-b	MS2-a	MS2-b	LS-a	LS-b
H in water	0.0221	0.0110	0.0224	0.0112	0.0228	0.0114
in ore					0.172	
O in water	0.176	0.0882	0.0180	0.0898	0.182	0.0911
in ore	0.725		0.347		0.408	
in cement			0.201		0.209	
Mg in ore	0.0179		0.0102		-	
in cement	0.0040		0.0067		0.007	
Al in ore	0.0417		0.0239		0.0043	
in cement	0.0139		0.0236		0.0245	
Si in ore	0.0230		0.0132		0.0043	
in cement	0.0350		0.0599		0.0623	
S	-		-		-	
K	-		-		0.0043	
Ca in ore	0.0113		0.00646		-	
in cement	0.1480		0.251		0.261	
Ti	0.130		0.074		-	
P	0.0003		0.00017		-	
V	0.0046		0.0026		0.0043	
Fe in ore	1.127		0.645		0.633	
in cement	0.009		0.0156		0.0162	
in punchings	2.002		2.851		2.772	
Zn	-		-		0.0043	
ρ , gm/cm^3:	4.5	4.4	4.74	4.64	4.64	4.54

that occurs more naturally. It is clear from studies[8,9,10,11,12,13,14,14a] that the water content does have a marked effect on the fast neutron attenuation, and even if a minimum amount of water is currently established, additional water included in the concrete would improve the fast neutron attenuation characteristics. The total amount of water is limited by the subsequent loss of density and structural strength of the concrete. Serpentine concrete[15] has been examined and found to be a desirable aggregate for a high-temperature concrete in that it holds its water reasonably well at temperatures up to 900°F, or up to 800°F when in a mixed portland cement concrete.

The location of high cross-section materials, which have little or no capture gamma-ray emission associated with the neutron absorption, within the shield is a rather established procedure. Boron in one form or another is the most common substance used for this purpose. One of the first ways boron was introduced into a shield was in the form of boral.[16] This is now a commercially produced product and is quite useful though not inexpensive. Boron has been incorporated into the EBWR shield in two ways. The thermal shield consists of a one-inch thickness of one percent borated steel,[17,18,19] and boron also precedes the biological shield in the form of a plaster[20] made of ferro-boron. Boron can be simply introduced in the form of borofin, a paraffin-borax mixture.[21] This is not an expensive material, but it must be contained in an air-tight can lest the absorption of water cause the material to expand. This same mixture has been

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[8]Davis, H. S., "Thermal Considerations in the Design of Concrete Structures for Shielding Atomic Power Plants," Paper 9, Nuclear Congress, Chicago, March 17-24, 1958.

[9]Bunch, W. L., "Attenuation Properties of High Density Portland Concretes as a Function of Temperature," HW-54656 (January 22, 1958).

[10]Wood, Don E., "Neutron Attenuation of Magnetite Concrete Heated 100°C," HW-53395 (October 1957).

[11]Peterson, E. G., "Neutron Attenuation in Ordinary Concrete," HW-55938 (1958).

[12]Wood, D. E., and E. G. Peterson, "Neutron Attenuation Characteristics of Magnetite Concrete Heated to 100°C, 200°C, and 300°C," Trans. Amer. Nucl. Soc., 1 (#1) 67 (June 1958).

[13]Fryar, R. M., and E. G. Peterson, private communication

[14]Dyson, J. A., Miss, and J. R. Harrison, "The Dependence of Fast Neutron Attenuation in Portland Concrete on its Hydrogen Content," AERE-RP/R-1942, Harwell, England (1956).

[14a]Blizard, E. P., and J. M. Miller, "Radiation Attenuation Characteristics of Structural Concrete," ORNL-2193 (August 22, 1958).

[15]Hungerford, H. E., et al., "New Shielding Materials for High Temperature Application," Trans. Amer. Nucl. Soc., 1 157 (December 1958).

[16]McKinney, V. L., and T. Rockwell, III, "Boral: A New Thermal Neutron Shield," AECD-3625, (May 1954), p. 49.

[17]Balai, N. "Boron Steel for Control Rods and Thermal Shields," Nucleonics, 16, (#1) 100 (January 1958).

[18]Watanabee, H. T., and W. O. Schaffnit, "Radiation Stability Studies of Boron Stainless Steel," Trans. Amer. Nucl. Soc. 1 (#2) 157 (December 1958).

[19]"The EBWR, Experimental Boiling Water Reactor," ANL-5607, (May 1957).

[20]Saller, H. A., J. T. Stacy and H. L. Klebanow, "High Boron Steels for Reactor Shielding," BMI-1039 (September 27, 1955).

[21]Rockwell, T. R., III, "The Construction of Cheap Shields," AECD-3352 (January 16, 1950).

utilized at the CP-5 reactor top shield with the addition of lead shot to increase the density. It is canned in an air-tight steel container to prevent the above-mentioned absorption. Boron may be introduced in the form of a canned boron carbide-graphite mixture in the case that a high-temperature material is required, such as in the EBR-II or the Enrico Fermi reactor.[22] Graphite is present to dilute the more expensive boron carbide, which in its pure form has more than enough boron to serve the purpose. In addition, a cheaper boron vehicle has been studied[23] with the result that calcium borate and a borated diatomaceous earth concrete have emerged as practical materials for the Enrico Fermi Plant.

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[22]Hungerford, H. E., and R. F. Mantey, "Shielding the Enrico Fermi Fast Breeder Reactor," Nucleonics 16 (11), 120 (November 1958).

[23]Hungerford, H. E., et al., "New Shielding Materials for High Temperature Application," Trans. Amer. Nucl. Soc., 1 (#2) 157 (December 1958).

9. EPILOGUE

The basic information for the design of a reactor shield has been presented. It must be kept in mind that the objective was not to discuss shielding from a scientific view, but from that of an engineer. This is what must be done when calculating shield distributions within the present state of the art. It should have been noticed that very little emphasis was placed on the use of computing machines. This is primarily because they could not be used as tools during the course, but partly because this is not at the present time a complete answer to the problem. When machines are used they must be used by people who understand shielding problems. This course then precedes the use of computing machines.

Assuming that the information given is adequate, and for all except an extremely efficient shield such as for a mobile reactor it should be so, there are still a multitude of problems to try the patience of a shield design engineer. Nothing will be built exactly as his more ideal shield design depicted it. There will be gaps, irregularities, penetrations, and a multitude of other departures from design, all of which are, of course, necessary. It is at this point that the shield designer becomes more of an artist, or perhaps seer would be an appropriate title. His decisions are not likely to be proven conclusively right or wrong except in the most extreme variations, or unless quite deliberate experimental plans are made for this purpose.

Many of the problems have been pointed out along with their solutions in the Shielding Design Manual;[1] in fact some of them include relatively sophisticated formulas such as those for gaps or holes left by ducts penetrating the shield[2,3,4] or for structural design details[5] necessary to remedy irregularities of deviations from the solid, unperturbed shield layer. Offsets may quite ideally be determined to be somewhat greater than the gap thickness for shield plugs, but when many openings are concentrated in a small area, as is necessary for fuel removal parts or control rod penetrations, the ideal conditions are not economically satisfied, if satisfied at all. When the final design is completed and the shield may be studied as a whole, there will be angles from the core at which the reactor and associated radiations will see a quite different array of shield thicknesses than those laid out on the centerline directions. Most, if not all, of these problems must be solved by intuitive guessing. This must be done by someone who is familiar with the types of radiation, the attenuation characteristics of each, and the points of origin of each. This person is the shield engineer and it is hoped that these lecture notes will be a step toward training such individuals.

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[1]Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, Office of Technical Services (March 1956).

[2]Rockwell, T., III, Editor, Reactor Shielding Design Manual, TID-7004, McGraw-Hill & D. Van Nostrand (March 1956) p. 261.

[3]Simon, A., and C. E. Clifford, "The Attenuation of Neutrons by Air Ducts in Shields," Nucl. Sci. Eng. 1 (#6) 156 (December 1956).

[4]Fisher, E., "The Streaming of Neutrons in Shields," Nucl. Sci. Eng. 1 (#3) 222 (July 1956).

[5]Rockwell, T., III, Editor, Reactor Shielding Design Manual TID-7004, McGraw-Hill & D. Van Nostrand, (March 1956) p. 196.

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