**THREE-DIMENSIONAL SUBSURFACE DELINEATION
VIA A NOVEL METHOD FOR DETERMINING
THE SUBSURFACE ELECTRICAL PROFILE**

R. J. Lytle
R. M. Bevensen
D. L. Lager
October 18, 1974

MASTER

Prepared for U.S. Atomic Energy Commission under contract No. W-7405-Eng-48

**LAWRENCE
LIVERMORE
LABORATORY**

University of California/Livermore

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability of responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights.

Printed in the United States of America
Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$ 2.25; Microfiche \$1.45

<u>*Pages</u>	<u>NTIS Selling Price</u>
1-50	\$4.00
51-150	\$5.45
151-325	\$7.60
326-500	\$10.60
501-1000	\$13.60

LAWRENCE LIVERMORE LABORATORY
University of California / Livermore, California / 94550

UCRL-51685

**THREE-DIMENSIONAL SUBSURFACE DELINEATION
VIA A NOVEL METHOD FOR DETERMINING
THE SUBSURFACE ELECTRICAL PROFILE**

R. J. Lytle

R. M. Bevensee

D. L. Lager

MS. date: October 18, 1974

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

Fig

Contents

Abstract	1
Introduction	1
Method of Analysis	4
Overview	4
Analysis via Probabilistic Potential Theory	4
Computation Time Estimates	10
PPT Formation for the Four-Probe Technique	11
Proposed Validation Procedure for PPT	12
Inversion Methods	13
Overview	13
Scale-Modeling for Further Validation	15
Benefits and Application	15
Summary	16
References	17

THREE-DIMENSIONAL SUBSURFACE DELINEATION VIA A NOVEL METHOD FOR DETERMINING THE SUBSURFACE ELECTRICAL PROFILE

Abstract

The combination of a standard experimental procedure (four-probe electrical resistivity), a novel analytical technique (probabilistic potential theory), and a powerful inversion algorithm (optimization/generalized linear inverse) is proposed as a method for determining the subsurface electrical profile. Four-probe electrical measurements can yield electrical profiles which, in the light of known geologic information about the site, can be related directly to subsurface structure. However, the reduction of field data from such measurements heretofore has generally been successful only for the mathematically nice situations (e. g., horizontal or vertical stratification). For the more complicated case of general subsurface structure, the presently available analytical methods rapidly become unwieldy, computer-

limited, or cost-ineffective. The analytical procedure proposed here should overcome these limitations and permit the efficient solution of complicated problems of subsurface structure. It is proposed that the feasibility of the procedure be tested by analysis of field experiments in conjunction with scale-model laboratory experiments to validate the algorithms for general structure. The procedure should have great practical value in a wide variety of applications: hydrology, location and definition of the shape and extent of underground resources (e. g., geothermal reservoirs, ore bodies, or deposits of coal, sand, or gravel), faultline definition, and monitoring of changes in subsurface conditions (e. g., as in earthquake studies, in situ coal gasification, and burn-front studies of oil-shale gasification).

Introduction

The definition of subsurface geologic structure by means of measurements performed on the surface has extensive applications and thus has been much studied. A standard technique for such surface measurements¹ is four-probe electrical resistivity. In this technique, four probes about 4 to 6 in. long (two connected as transmitters and two as

receivers) are driven into the ground. The transmitter probes inject current into and remove current from the ground, and this current induces a potential difference between the two receiver probes. With the four probes in any one of a number of possible configurations (examples are shown in Fig. 1), D , the effective depth of probing for homogeneous

ground can be shown to be approximately equal to $(0.3 \text{ to } 0.5) \cdot L$, where L is the total length of the array. The measurement procedure consists of determining the ratio between the induced voltage V and the drive current I (refer to Fig. 1). From the ratio V/I and the known geometric arrangement of the probes, one can determine ρ_a , the apparent resistivity of the ground. For a fixed geometric arrangement of probes, one can detect subsurface anomalies by noting the change

in ρ_a as the center of the four-probe array is moved along the ground surface. Further, by varying both the center of the array and the relative positions of the probes, one can in principle obtain both the depth profile and the lateral profile of subsurface electrical resistivity. In many cases, by making use of available information about the site, one can then directly relate subsurface electrical resistivity to geologic structure. Unfortunately, the reduction of field data to specific models has been inhibited by lack of an adequate analysis procedure for general structure.

If the surface is homogeneous, inversion of the results for apparent resistance yields an exact value for the conductivity of the ground. The ground thickness is in essence infinite, since it is homogeneous. However, the case of a homogeneous subsurface is not especially interesting in the geophysical sense.

If the subsurface is not homogeneous, inversion of the apparent resistance measurements to obtain a conductivity profile of the ground is usually not a simple exercise. The influence of a single anomaly in a homogeneous subsurface can be handled by standard methods. More complicated subsurface structures, however, lead to analytical difficulties. In practice, subsurface inhomogeneities i.e., complicated subsurface structure, can reflect any of a variety of anomalies (Fig. 2): for example, a number of horizontal layers that are individually homogeneous, a number of vertical layers that are individually homogeneous, or a number of individually homogeneous regions of arbitrary shape. Four-probe measurements

Fig. 1. Some possible arrangements of the four-probe array. I , drive current; V , induced voltage; L , total length of array. In each case, the transmitter points are those at which current is entering and leaving the ground.

can indicate the presence of such complex anomalies, but it is not an easy task to determine their shapes and dimensions from the experimental results. The difficulty lies not only in the inversion procedures (which vary the dimensions and conductivities of the various homogeneous regions) to find a "best-fit" to the experimental data, but even in the basic analysis problem for the general medium, the third illustrated in Fig. 3. In fact,

Fig. 2. Basic types of subsurface anomalies. Top, individually homogeneous horizontal layers. Middle, individually homogeneous vertical layers. Bottom, individually homogeneous regions of arbitrary shape (general structure).

Fig. 3. Closed region of space bounded by equipotential surfaces S_0 at potential 0 and S_1 at potential 1, with S_1 at intermediate potential $0 \leq V_1 \leq 1$.

to our knowledge, schemes involving inversion plus analysis so far have been successfully used only for the multiple anomaly situations depicted as the first two examples in Fig. 2.

The analytical procedure proposed here combines a powerful inversion algorithm with a novel method of analysis (probabil-

istic potential theory, PPT). It should overcome the previous limitations and permit the efficient solution of complicated problems of subsurface electrical profile. Thus, it should lead to a significant improvement in the state of the art for determining subsurface geologic structure.

Method of Analysis

OVERVIEW

To determine the interaction of the four probes with the ground, one must first solve Laplace's equation for the inhomogeneous regions diagrammed in Fig. 2. The equation describes the equilibrium state of the potentials that exist as a consequence of the presence of the two transmitter probes, and, since its solution for an arbitrarily inhomogeneous medium is not a simple problem, it has usually been avoided.

The analysis procedures used in the past to determine the potential difference between the two receivers for a given transmitter current suffered either from unsuitability for general inhomogeneities or from excessive requirements for calculation (e.g., time, storage, ease of programming). (They are based upon Bessel functions, finite difference equations, finite element methods, or surface charge integrals.) Our proposed method exploits a novel analysis method (probabilistic potential theory, PPT), which permits rapid, accurate determination of the potential difference for the two receivers for an arbitrarily inhomogeneous medium. PPT is based on

an aspect of duality (discussed further below) between the probabilistic theory of random processes and potential theory,² namely, that the mathematical processes involved are essentially the same. So far, PPT has not been applied to problems of geophysical inhomogeneity like those depicted in Fig. 2. But it is particularly appropriate for such problems for several reasons:

- It is relatively easy to program on the computer, even for extremely inhomogeneous regions.
- When the region of interest is subdivided into a very large number of cells and the potential is sought in relatively few of them, PPT becomes more efficient computationally than techniques based on ordinary matrix inversion.
- The computing time can be adjusted according to the accuracy desired.

ANALYSIS VIA PROBABILISTIC POTENTIAL THEORY

We now proceed to demonstrate our exploitation of the duality between the probabilistic theory of random processes and potential theory, by solving the Laplace equation by probabilistic potential theory (PPT). The equilibrium electrical

potentials in the ground set up by the two transmitter probes of the four-probe system are sampled at the sites of the two receiver probes. These equilibrium electrical potentials are governed physically by Laplace's equation for the electrical potential V . We will solve for the potential V via PPT.

A Laplace-type equation³ for the potential V in a three-dimensional cartesian coordinate system is

$$\nabla \cdot [f(\vec{r}) \nabla V] = 0,$$

where the function $f(\vec{r})$ is the constant of proportionality between the flow of a physical quantity, such as electric flux, and the potential V . For problems involving electrical potential, $f(\vec{r}) = \sigma(\vec{r})$, the conductivity of the ground. If the space is divided into innumerable small cubes with sides of area Δ , then the equation for one of them, in difference form, reads

$$\begin{aligned} f(\vec{r} + \vec{a}_x \Delta/2) \frac{V(\vec{r} + \vec{a}_x \Delta) - V(\vec{r})}{\Delta} \\ - f(\vec{r} - \vec{a}_x \Delta/2) \frac{V(\vec{r}) - V(\vec{r} - \vec{a}_x \Delta)}{\Delta} \\ + \dots + f(\vec{r} + \vec{a}_z \Delta/2) \frac{V(\vec{r} + \vec{a}_z \Delta) - V(\vec{r})}{\Delta} \\ - f(\vec{r} - \vec{a}_z \Delta/2) \frac{V(\vec{r}) - V(\vec{r} - \vec{a}_z \Delta)}{\Delta} = 0, \end{aligned} \quad (1)$$

where $\vec{a}_x$, etc., are the unit vectors along the axes. After cancellation of factor Δ , this is rearranged into the form

$$\begin{aligned} V(\vec{r}) = \frac{f(\vec{r} + \vec{a}_x \Delta/2)}{F} V(\vec{r} + \vec{a}_x \Delta) \\ + \frac{f(\vec{r} - \vec{a}_x \Delta/2)}{F} V(\vec{r} - \vec{a}_x \Delta) \\ + \dots + \frac{f(\vec{r} + \vec{a}_z \Delta/2)}{F} V(\vec{r} + \vec{a}_z \Delta) \\ + \frac{f(\vec{r} - \vec{a}_z \Delta/2)}{F} V(\vec{r} - \vec{a}_z \Delta), \end{aligned} \quad (2)$$

with

$$\begin{aligned} F = f(\vec{r} + \vec{a}_x \Delta/2) + f(\vec{r} - \vec{a}_x \Delta/2) \\ + \dots + f(\vec{r} + \vec{a}_z \Delta/2). \end{aligned}$$

We observe that the sum of all the factors of V on the right side of Eq. (2) is exactly 1, by definition of F .

Now, consider a random process of the Markov type in which a large number N_1 of noninteracting particles is released at some point $\vec{r}_1$ and subsequently performs Brownian motion in steps of length Δ in each unit of time. At some later time when a few arrive at point $\vec{r}$ in Fig. 3, we define a probability $P(\vec{r})$ that any of them will reach surface S_1 at potential 1 at any subsequent time in the future. Whenever one of these particles later does arrive on S_1 , it is counted and removed from the system. Since $P(\vec{r})$ is defined over an infinite time interval of the diffusion process, it follows that the probability that any particle will leave $\vec{r}$ and arrive on S_0 of potential 0 at some future time is just $1 - P(\vec{r})$. Whenever a particle arrives on S_0 , it too is removed from the system. The rule for handling particles striking surface S_1 at an intermediate potential is explained below.

Having defined probability $P(\vec{r})$ for an arbitrary $\vec{r}$, we have, of course, defined $P(\vec{r} + \vec{a}_x \Delta)$, $P(\vec{r} - \vec{a}_x \Delta)$, etc., for neighboring points. Since the process is Markovian, meaning that a particle jumps from a point to a neighbor with no memory of the past, $P(\vec{r})$ can be expressed as

$$\begin{aligned} P(\vec{r}) = & p(\vec{r} + \vec{a}_x \Delta | \vec{r}) P(\vec{r} + \vec{a}_x \Delta) \\ & + p(\vec{r} - \vec{a}_x \Delta | \vec{r}) P(\vec{r} - \vec{a}_x \Delta) \\ & + \dots + p(\vec{r} - \vec{a}_z \Delta | \vec{r}) P(\vec{r} - \vec{a}_z \Delta), \end{aligned} \quad (3)$$

in which $[p(\vec{r} + \vec{a}_x \Delta | \vec{r})]$, for example, is the conditional probability of jumping to $\vec{r} + \vec{a}_x \Delta$, given that the particle is at $\vec{r} + \vec{a}_x \Delta$. Equation (3) evaluates $P(\vec{r})$ as the sum of the probabilities of reaching surface S_1 at some future time by the various routes through the six neighboring points around $\vec{r}$. We notice that the sum of all p is precisely 1, since a particle at $\vec{r}$ must jump to a neighboring point during the next time interval.

The correspondence between potential $V(\vec{r})$ in Eq. (2) and probability $P(\vec{r})$ in Eq. (3) becomes an identity if we define the conditional probabilities as

$$p(\vec{r} + \vec{a}_x \Delta | \vec{r}) \equiv f(\vec{r} + \vec{a}_x \Delta / 2) / F, \text{ etc.}, \quad (4)$$

and if we also force $V(S_1) \equiv P(S_1) = 1$; $V(S_0) \equiv P(S_0) = 0$. Both the potential V and the probability P tend to be continuous functions in the space as the grid length Δ approaches zero, and both are well-behaved as $\vec{r}$ approaches a boundary point. A particle starting on S_1 , where the potential is 1, has a probability 1 of arriving there, and one starting on S_0 ,

where the potential is zero, is immediately removed with no probability of arriving on S_1 .

If a particle arrives on surface S_1 at intermediate potential V_1 (see Fig. 3), it can perform only one of two jumps during the next time-step - either to S_1 with probability $P_1 \equiv V_1$ or to S_0 with probability $1 - P_1$. Thus the probabilistic interpretation of V_1 is consistent. One can easily verify the validity of the following general formula for $V(\vec{r}) \equiv P(\vec{r})$:

$$V(r) \equiv P(r) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_i N_{\infty}(S_i | \vec{r}) V_i, \quad (5)$$

where N is the number of particles starting from point r and $N_{\infty}(S_i | \vec{r})$ is the number eventually - after infinite time - arriving on surface S_1 at potential V_i . The sum includes surface S_1 and (trivially) S_0 .

This general formula (Eq. 5) is the aspect of PPT that we wish to exploit in geophysical modeling. To illustrate this application, we now consider a specific example.

Let us assume the situation depicted in Fig. 4, in which a voltage of 1 V is impressed at a piece of metal at the ground/air interface. The ground is divided into N regions, each of which is individually homogeneous, and has conductivity σ_i where $i = 1, 2, \dots, N$. We desire to determine the potential in the k th region only, when the potential is known to be 1 V at the metal region and zero at infinity. It can be shown that the voltage in the k th region is equivalent to the probability that a particle starting at the k th cell and wandering in a random manner (the random walk or drunkard's walk) would end

up at the metal region.²⁻⁶ The particle travels in a random manner from cell to cell; the transition probabilities in going from one cell to another are as defined below for a two-dimensional problem. The three-dimensional analog follows directly from the two-dimensional result;

the method is basically the same, but because the underlying physical processes can be easily obscured by the mathematical details of the three-dimensional problem, it is not suitable for presentation here.

If a portion of the ground is subdivided into cells as in Fig. 5, some of the

Fig. 4. Illustration of the procedure for subdividing the subsurface into a number of cells.

Fig. 5. Possible number sequences within a cell and between cells; this example is the basis for the discussion in the text. The fact that a boundary is common to two cells means that point 3 in cell $i, j-1$ corresponds to point 1 in cell i, j . This is designated notationally as $(i, j-1, 3) = (i, j, 1)$.

transition probabilities p_{kl} in going from point k to point l (with each jump parallel to a cartesian axis) are the following:

1. For the broken-outlined cell around point 5 in cell i, j :

$$\begin{aligned} p(i, j, 2; i, j, 5) &= p(i, j, 4; i, j, 5) \\ &= p(i, j, 6; i, j, 5) \\ &= p(i, j, 8; i, j, 5) = \frac{1}{4}. \end{aligned}$$

2. For the broken-outlined cell around point 4 in cell i, j :

$$\begin{aligned} p(i, j, 1; i, j, 4) &= p(i, j, 7; i, j, 4) = \frac{1}{4}. \\ p(i, j-1, 5; i, j, 4) &= \frac{\sigma_{i, j-1}}{2(\sigma_{i, j-1} + \sigma_{i, j})}, \end{aligned}$$

where $\sigma_{i, j}$ is the conductivity of cell i, j .

$$p(i, j, 5; i, j, 4) = \frac{\sigma_{i, j}}{2(\sigma_{i, j-1} + \sigma_{i, j})}.$$

3. For the broken-outlined cell around point 1 in cell i, j :

$$\begin{aligned} p(i-1, j, 4; i, j, 1) &= \frac{\sigma_{i-1, j-1} + \sigma_{i-1, j}}{T}, \\ p(i, j, 2; i, j, 1) &= \frac{\sigma_{i-1, j} + \sigma_{i, j}}{T}, \\ p(i, j, 4; i, j, 1) &= \frac{\sigma_{i, j-1} + \sigma_{i, j}}{T}, \\ p(i, j-1, 2; i, j, 1) &= \frac{\sigma_{i, j-1} + \sigma_{i-1, j-1}}{T}, \end{aligned}$$

where

$$\begin{aligned} T &= 2(\sigma_{i-1, j-1} + \sigma_{i-1, j} + \sigma_{i, j-1} \\ &\quad + \sigma_{i, j}). \end{aligned}$$

Air has a conductivity $\sigma = 0$; thus, no particles pass from the ground into the air, and this transition probability is zero. The conductivity of the probe is assumed to be $\sigma = \infty$; thus, the transition probability is 1.0 for a particle passing to the metal from any cell adjacent to it. The metal is an absorber; particles enter but do not leave.

The transition probability between two cells that are not immediately adjacent to each other is zero. That is, one-movement transitions between nonadjacent cells are forbidden; such transitions can occur, however, in two or more movements. Also, note that generally $p_{i, j} \neq p_{j, i}$. The transition probabilities indicate that although particle movement is random, there is greater probability that a particle will move into a region of higher conductivity. In this sense, the preferred (or more probable) direction of particle flow is similar to actual current flow in the medium.

The probability that a particle starting in the k th region will be absorbed in the metal region can be calculated in two ways³ — by a Monte Carlo method and by a state-transition method (a Markov chain).^{5, 7, 8} In the Monte Carlo method, a large number of particles is emitted; these are moved in a random manner determined by the transitional probabilities; then, the percentage of these particles absorbed in the metal region is calculated.⁵⁻⁸ In the state-transition method, the state-transition matrix of conditional probabilities is multiplied by the probability that the particle will be in region i , where $i = 1, 2, \dots, k, \dots, N$.

If initially the particle is in cell k , the initial state matrix P_0 is

$$P_1 = P P_0, P_2 = P P_1$$

$$= P^2 P_0, \dots, P_M = P^M P_0.$$

The answer desired is the probability of starting from the k th cell and being in the N th cell (at potential 1.0 V) after enough movements that additional movements do not affect the probability of being in this cell. That is, we are interested in the probability of traveling to the metal (at potential 1.0 V) after M transitions, where M is large enough that additional movements do not significantly affect the cumulative probability of traveling to the metal.

Computation Time Estimates

How much computer time, how many regions (N), and how many multiplications (M) are feasible? P is an $N \times N$ sparse matrix. Thus, it can be shown that for a CDC 7600 computer, the calculation time for evaluating $P^M P_0$ is 2.2 NM microseconds for a two-dimensional model and 3.4 NM microseconds for a three-dimensional model.

We arbitrarily assume that $P^M P_0 = P^{\alpha} P_0$ for M equal to five uninterrupted transits across a finite-sized model. Thus,

$$M = 5\sqrt{N} \text{ for a } \sqrt{N} \times \sqrt{N} \text{ two-dimensional model, and}$$

$$M = N^{1/3} \text{ for a } N^{1/3} \times N^{1/3} \times N^{1/3} \text{ three-dimensional model.}$$

With these assumptions and the stipulation that the calculation time be one second, one can model a two-dimensional

structure with 2022 cells (with 44×44 partitioning) and a three-dimensional structure with 3777 cells (with $15 \times 15 \times 15$ partitioning).

With the capability for attaining in one second an accurate potential with a subsurface structure of several thousand cells, it should be possible to model most of the physical situations of interest.

The preceding development assumes that straight matrix multiplication was performed, using our knowledge of the sparseness of the matrix and the locations of the non-zero matrix elements. An alternative matrix method might be of some use. If P is a diagonalizable matrix, then it can be expressed as

$$P = B^{-1} \lambda B,$$

where B is some nonsingular matrix and λ is a diagonal matrix. The powers of the matrix P can be calculated readily via

$$P^2 = B^{-1} \lambda B B^{-1} \lambda B = B^{-1} \lambda^2 B$$

$$P^M = B^{-1} \lambda^M B.$$

Hence, if the matrices B , B^{-1} , and λ can be determined, then the calculation of P^M for large M is not at all difficult. The difficulties in determining B , B^{-1} , and λ for the large sparse matrix P remain to be investigated.

Another computational aspect worthy of some investigation is the use of parallel processing rather than serial processing. Certain aspects of parallel processing are available on the CDC STAR system and the ILLIAC system. Both the Monte Carlo and the state-transition

approaches would benefit from the use of parallel processing. The extent of the possible advantage remains to be investigated.

PPT Formulation for the Four-Probe Technique

The four-probe arrangement for analysis is shown in Fig. 7. A current source of magnitude I with internal conductance G injects a current I_+ into the ground, with the current I_+ passing through the surface S_+ . The current I_+ is assumed to establish a potential difference of 1 V between the two transmitter probes. Thus, by Ohm's law, $I = 1 \cdot G + I_+$.

The open circuit potentials V_1 and V_2 are experimentally measured at the positions 1 and 2. The potential V_2 at position 2 is determined by PPT by releasing particles at position 2 and measuring the fraction collected on the surface S_+ at the 1-V probe of the transmitter. This fraction of particles is identically equal^{2, 3, 5} to potential V_2 . Potential V_1 is computed similarly.

It is necessary also to compute the potential near the transmitter surface S_+ , in order to determine the current I_+ injected into and retracted from the ground

by the 1-V source. The current I_+ is evaluated by

$$I_+ = \int_{\text{Surface enclosing } S_+} \sigma \mathbf{E} \cdot d\mathbf{S}.$$

The electrical field $\mathbf{E}$ is determined by its basic definition (i.e., the difference in potential between two closely spaced points near the surface S_+ is divided by the distance separating the two points). These potentials near S_+ are very easy to determine, because PPT converges extremely rapidly for evaluation locations very near the 1-V source.⁷

A mathematical description of the preceding development is now given. Let

$p(S_+, 2)$ = probability that a particle will diffuse from point 2 to the surface S_+ .

$p(S_+, 1)$ = probability that a particle will diffuse from point 1 to the surface S_+ .

$p(S_+, S_+ + \Delta S_+) =$ probability that a particle located at distance Δ from the surface S_+ will diffuse to the surface S_+ .

Fig. 7. The four-probe configuration for PPT analysis.

Then, by duality,

$$V_2 - V_1 = p(S_+, 2) - p(S_+, 1)$$

$$I_+ = I - G = \int_{\text{Surface enclosing } S_+} \sigma \mathbf{E} \cdot d\mathbf{S}$$

$$= \sum_{\text{Cells about } S_+} \sigma_{\text{cell}} \left[\frac{1 - p(S_+, S_+ + \Delta S_+)}{\Delta} \right] \Delta S.$$

The result from the four-probe measurement in terms of apparent resistance R_a

$$R_a = \frac{V_2 - V_1}{I_+}.$$

The formula for apparent resistance is described above for the situation in which all four probes are on the surface of the ground (Fig. 7). There is, however, no requirement in the formula that the probes be at the ground surface; one or more could be subsurface, e. g. in one or more drill holes. In fact, this formula could be generalized to include multiple current sources and/or multiple voltage pickups. An array of such transmitter/receiver probes could have application in monitoring detailed changes in subsurface conditions as a function of time. For example, such an array might be useful in monitoring the progression of the burn front in in situ coal gasification.

Proposed Validation Procedure for PPT

There is need for demonstration of the capability of the PPT approach for analyzing geophysically interesting subsurfaces. What is needed is a simple

hierarchy of problems (arranged from easiest to most difficult) that can be investigated by numerical modeling. The experience gained in solving these problems would permit the establishment of rules of thumb for the calculation time required to achieve a given degree of accuracy for a problem of a specific degree of complexity. Where possible, PPT results for a specific model should be compared with results obtained by alternative techniques. If alternative results are not readily available, comparison of PPT with results from experimental simulation should be considered. It is necessary, of course, that the experimental procedure be validated before complex experimental simulations are considered.

The following hierarchy of problems (illustrated in Fig. 8) is proposed:

1. Homogeneous ground (least interesting but simplest to model).
2. Two homogeneous horizontal layers (useful in hydrology and definition of resources).
3. Two homogeneous vertical layers (useful in definition of resources).
4. Skewed layer (useful in fault-line definition and in earthquake studies).
5. Single anomaly (useful in geothermal studies).
6. Multiple horizontal layers (useful in studies of burn front in coal gasification).
7. Multiple vertical layers (useful in earthquake studies).
8. Multiple anomalies (useful in definition of ore bodies).
9. Multiple skewed layers (useful in hydrology, determination of permafrost depth, and fault-line studies).

(1) Homogeneous ground

(2) Two homogeneous horizontal layers

(3) Two homogeneous vertical layers

(4) Skewed layer

(5) Single anomaly

(6) Multiple horizontal layers

(7) Multiple vertical layers

(8) Multiple skewed layers

(9) Multiple anomalies

Fig. 8. Possible hierarchy of problems from which one might determine the validity of the PPT analytical method.

Inversion Methods

OVERVIEW

It is a difficult problem to determine the subsurface electrical structure from a set of four-probe experimental measurements. Nevertheless, the importance of the problem has led many

investigators to try a variety of methods. The classical procedure is to attempt to curve-match the experimental data to a specific theoretical curve from a series of previously prepared master curves for different subsurface conditions. This technique has had some success, being

particularly helpful for uncomplicated subsurface conditions.

A modification of this procedure is to discern the subsurface profile by the use of auxiliary curves.⁹

One method that has had notable success involves the use of a generalized linear inverse.¹⁰ This technique was applied to seismic data and clearly demonstrated its usefulness.¹¹ This application of generalized linear inverse has since been extended to four-probe experimental results with gratifying results. Advantages of the approach are that the significant model parameters and their sensitivities are identified, and that an a priori experimental design can be constructed so that the experimental measurements are concentrated on the most significant parameters in the theoretical model. A disadvantage is that it is a linear theory, whereas the model parameters to be determined do not satisfy a first-order linear relation (however, the differential behavior of the unknowns is linear). Hence, a good first guess of the model probably will converge to an accurate answer, whereas a bad first guess may present convergence difficulties. Also, constraints cannot be imposed on the results; hence, unreal results can be generated.

Optimization techniques also have been used to discern subsurface profiles. There is a variety of such techniques: those that brute-force a solution (e.g., the simplex method), those that use gradient information,¹² and those that do not utilize

gradient information.¹³ Optimization techniques are helpful in that constraints can be imposed on the results, thus eliminating unreal answers. A variety of starting points can be tried and various convergence results can be obtained. The degree of consistency of the results is a measure of the sensitivity of the model.

The method thought to be most helpful is to combine the generalized linear inverse method with the optimization method. That is, a reasonably constrained model can be obtained by optimization, and then, with a good initial answer, the generalized linear inverse can be used to further define the answer. This approach enables one to take advantage of the strong points and to avoid the weak points of both techniques.

This overview of data inversion techniques is not intended to be exhaustive. However, the classical method and the methods thought to be most useful when used in conjunction with PPT analysis of complex subsurface structure have been covered. Additional inversion methods might include, among many others, the kernel function procedure,¹³ the method of geometric progression of depths,¹⁴ and the automatic interpretation of resistivity sounding curves by the use of modified Dar Zurrouk functions.¹⁵ A common disadvantage of these methods is that they are tied specifically to a layered ground composed of individually homogeneous parallel structures. The generalized linear inverse and optimization methods do not suffer from these limitations.

Scale-Modeling for Further Validation

A potentially fruitful method for the modeling of realistic ground situations is to combine PPT analysis with the (generalized inverse/optimization) inversion procedure. The PPT algorithm can be validated for simple structures, since analytical results are available or can be generated for the multiple horizontal, multiple vertical, and dipping bed problems. For more complicated structures, however, analytical results are not always available for checking; results from scale-model laboratory experiments would be helpful for substantiating the PPT results.

The inversion algorithms can also be validated for simple structures. However, it would be helpful to substantiate those for complicated structures as well. Scale-model laboratory experiments would be helpful to validate the combined use of PPT and inversion in converging to

actual physical configurations. This would permit clear identification of such factors as degree of horizontal and vertical resolution, and the dependence of resolution on the spacing of probes, both at the surface and subsurface (as in drill holes for field experiments).

To determine the validity of the method, we propose a three-pronged approach:

- PPT for analysis of field data for complex structures.
- Inversion algorithm for complex structure via generalized linear inverse and optimization procedures.
- Scale-model laboratory experiments to determine the feasibility and utility of using four-probe measurements to resolve subsurface structure.

This approach offers a good possibility of permitting the delineation of complicated subsurface geologic structure — a heretofore intractable problem.

Benefits and Application

The proposed method would directly benefit LLL programs in that it would permit further development and refinement of schemes for subsurface probe measurements and associated procedures for analysis. Such subsurface probing techniques are applicable to hydrology (of interest near reactor sites and waste storage areas), location and definition of underground resources (e.g., geothermal, coal, sand, and gravel, ore bodies),

fault-line definition, monitoring of changes in underground conditions (e.g., earthquake studies, in situ coal gasification).

It should be noted that as Laplace's equation describes many physical phenomena, the use of PPT is not limited to the four-probe problem. For example, it can also be applied to problems which arise in the fields of magnetostatics, electrostatics, gravitation, fluid flow, and heat conduction, among others.

Summary

A three-faceted approach is proposed for resolving subsurface structure. It combines a standard experimental method (four-probe electrical resistivity) with a novel analysis technique (probabilistic potential theory) and powerful inversion algorithms (optimization and generalized linear inverse). The computational efficiency for computing electric potentials via the probabilistic potential algorithm

permits the realistic modeling of complex ground situations; heretofore this has been an intractable problem – the computational costs have been prohibitive. It also permits the use of presently available iterative inversion algorithms to reduce the experimental data to specific subsurface models. The method is applicable either to surface or subsurface (drill-hole) experiments.

References

1. R. J. Lytle, "Measurement of Earth Medium Electrical Characteristics: Techniques, Results, and Applications," IEEE Trans. Geosci. Electronics GE-12, 81-101 (1974).
2. R. Hersh and R. J. Griego, "Brownian Motion and Potential Theory," Sci. Am. 220 (March), 87-74 (1969).
3. R. M. Bevensee, "Probabilistic Potential Theory Applied to Electrical Engineering Problems," Proc. IEEE 61, 423-437 (1973).
4. G. Gamow, One Two Three . . . Infinity, Mentor Book (New American Library of World Literature, Inc., New York, 1947).
5. F. Spitzer, Principles of Random Walk (Van Nostrand Co., Inc., New York, 1964).
6. G. M. Royer, "A Monte Carlo Procedure for Potential Theory Problems," IEEE Trans. Microwave Theory Technol. MTT-19, 813-818 (1971).
7. W. Feller, An Introduction to Probability Theory and Its Applications, Vol. 1 (John Wiley and Sons, Inc., New York, 1957).
8. E. Parzen, Modern Probability Theory and Its Applications, (John Wiley and Sons, Inc., New York, 1960).
9. A. A. R. Zohdy, "The Auxiliary Point Method of Electrical Sounding Interpretation and Its Relationship to the Dar Zarrouk Parameters," Geophysics 30, 644-660 (1965).
10. J. R. Inman, Jr., J. Ryu, and S. H. Ward, "Resistivity Inversion," Geophysics 38, 1088-1107 (1973).
11. G. Backus and F. Gilbert, "Numerical Applications of a Formalism for Geophysical Inverse Problems," Geophys. J. Roy. Astronom. Soc. 13, 247-276 (1967).
12. D. W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," J. Soc. Ind. Appl. Math. 2, 432-441 (1963).
13. C. L. Pekeris, "Direct Method of Interpretation in Resistivity Prospecting," Geophysics 5, 31-52 (1940).
14. D. Marsden, "The Automatic Fitting of a Resistivity Sounding by a Geometrical Progression of Depths," Geophys. Prospect. 21, 266-280 (1973).
15. A. A. R. Zohdy, "Automatic Interpretation of Resistivity Soundings Using Modified Dar Zurrouk Functions," Geophysics 38, 196-197 (1972). (Abstract)