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SPACE CHARGE LIMITING OF
BOUNDED RELATIVISTIC ELECTRON BEAMS

Thomas J. Fessenden

May 30, 1974

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Introduction

It has been known for some time that several processes contribute to the limiting of the total current that can be caused to flow within conducting walls. One of the most fundamental of these results from the space charge depression in an unneutralized electron stream. Clearly this potential depression cannot exceed the total energy of the beam electrons. It is this space charge limiting process that is re-examined in this note for relativistic electron beams.

Past theories¹⁻⁴ have considered an electron stream of uniform current density inside a conducting pipe. This is most reasonable because the electron stream was presumably generated by a grid in front of a hot cathode which then produced a constant current density. Since no motion is allowed except in the direction of the stream (by virtue of a sufficiently strong axial magnetic field) the current density is constant, independent of position. This leads to a nonuniform charge density in the beam.

In this note we consider a uniform charge density in the relativistic stream. This is partially justified because the theory is directed toward understanding the current flow from relativistic E-beam type devices. Here the current is produced at relativistic energies by applying potentials

of megavolts across a single gap and producing current by field emission. Since the details of how the cold cathode operates are not well understood, it seems that a uniform charge density assumption is equally as reasonable as a uniform current density assumption and that neither is likely to correspond to reality. The uniform charge density assumption allows an easy solution to Poisson's equation and, consequently, this hypothesis was used in this note. In the ultrarelativistic limit, the current density also becomes uniform with the uniform charge density hypothesis since all electrons move at the speed of light ($J = \rho c$).

The magnetic field produced by the relativistic current plays a strong role in determining how the current reaches its maximum value. However, that maximum value is determined solely by the electrostatic effects. This is in sharp contrast to the case of a spinning beam in a magnetic field as examined by Langdon et al.⁵ For this reason, the self magnetic field of the relativistic beam is not considered.

Theory

We shall assume that, because of a sufficiently strong axial magnetic field, the electrons are constrained to move only in the direction of beam propagation. Thus $\beta_1 = 0$ and $\beta_2 = \beta = \sqrt{1 - 1/\gamma^2}$. Here β is v/c and γ is the ratio of the electron mass to its rest mass. We further assume that the charge density of the beam (ρ) is uniform in the region $0 \leq r \leq a$ and that the beam is inside a conducting pipe of radius b .

With these hypotheses, Poisson's equation is easily solved and in mks units the radial electric field E is given by

$$\begin{aligned} \epsilon &= \frac{\rho r}{2\epsilon_0} & 0 \leq r \leq a \\ &= \frac{nea^2}{2\epsilon_0 r} & a \leq r \leq b. \end{aligned} \quad (1)$$

Thus the space charge potential inside the cylinder is given by

$$\begin{aligned} \phi_{sc} &= \frac{\rho a^2}{2\epsilon_0} \ln \frac{r}{b} & a \leq r \leq b \\ &= -\frac{\rho a^2}{4\epsilon_0} \left[2 \ln \frac{b}{a} + \left(1 - \left(\frac{r}{a} \right)^2 \right) \right] & 0 \leq r \leq a \end{aligned} \quad (2)$$

Here we have chosen the potential to be zero on the conducting wall.

The electron beam is assumed to have been produced in a cathode anode gap by a potential V_b . Thus if space charge were neglected each electron would have an energy eV_b or in terms of mass ratio

$$\gamma_0 = 1 + \frac{eV_b}{mc^2}. \quad (3)$$

Thus assuming all electrons have the same total energy $\gamma_0 mc^2$ we find the mass ratio γ varies with radius according to

$$\gamma = \gamma_0 + \frac{\phi_{sc} e}{mc^2}. \quad (4)$$

Or from Eq. (2) we have

$$\frac{\gamma}{\gamma_0} = 1 - \frac{\rho a^2 e}{4\epsilon_0 mc^2 \gamma_0} \left[2 \ln \frac{b}{a} + \left(1 - \left(\frac{r}{a} \right)^2 \right) \right] \quad (5)$$

inside the beam. Define a normalized charge

$$Q = \frac{n_b e^2 a^2}{4\epsilon_0 mc^2 \gamma_0} = \left(\frac{\omega_p a}{2c} \right)^2. \quad (6)$$

Here n_b is the beam electron density (assumed uniform).

The total current flowing in the beam is

$$I = nec \int_0^a \beta(r) 2\pi r dr. \quad (7)$$

With the aid of Eqs. (5) and (6) we find

$$i = \frac{I}{I_a} = \frac{1}{\beta_0} \int_A^B \frac{\sqrt{x^2 - 1/\gamma_0^2}}{x} dx \quad (8)$$

where

$$A = 1 - Q(1 + 2 \ln b/a)$$

$$B = 1 - Q 2 \ln b/a$$

$$\text{and } I_a = 17,000 \beta_0 \gamma_0.$$

The integral can be performed with the result

$$i = \frac{1}{\sqrt{\gamma_0^2 - 1}} \left[\sqrt{\gamma_0^2 x^2 - 1} - \tan^{-1} \sqrt{\gamma_0^2 x^2 - 1} \right]_A^B. \quad (9)$$

The maximum current that can flow is found by differentiating Eq. (8) with respect to Q and setting the resulting equation to zero. In general, this leads to lots of algebra and the resulting equation is transcendental and eludes a simple solution. Two simple cases exist where a solution can be obtained without difficulty. The first case to be considered is that of a uniform beam filling the conducting pipe. The second case is the thin beam limit in that the beam diameter is assumed small compared to the pipe diameter.

Case 1 - Uniformly Filled Pipe

The above equations simplify when the beam fills the pipe (i.e., $b = a$). We then have the lower limit of the integral of Eq. (8) $A = 1-Q$ and the upper, $B = 1$. Setting the result of differentiating Eq. (8) with respect to Q to zero yields

$$\frac{di}{dQ} = 0 = - \frac{\sqrt{(1-Q)^2 - 1/\gamma_o^2}}{(1-Q)} \quad (10)$$

or

$$Q = 1 - 1/\gamma_o. \quad (11)$$

From Eq. (5) we find

$$\gamma(r) = 1 + \left(\frac{r}{b}\right)^2 (\gamma_o^2 - 1). \quad (12)$$

Thus the particles on axis stop ($\gamma=1$), and the total particle energies increase parabolically to the peak energy at the wall radius. The total current given by Eq. (9) is

$$i = \frac{I_b}{I_a} = 1 - \frac{\tan^{-1} \sqrt{\gamma_0^2 - 1}}{\sqrt{\gamma_0^2 - 1}} \quad (13)$$

Taking Eq. (13) to the nonrelativistic limit by letting $\sqrt{\gamma_0^2 - 1} = \beta_0$ become small yields

$$i = \frac{\beta_0^2}{3} = \frac{I_b}{17,000 \beta_0} \quad (14)$$

We then find $I_b = KV^{3/2}$ where

$$K = \epsilon_0 \frac{8\pi}{3} \sqrt{\frac{2e}{m}} = 43.988 \times 10^{-6}.$$

This is the well known nonrelativistic result for the case of a uniform charge density filling the beam pipe.

Case 2 - Thin Beam Limit

If we assume that the electron beam radius is small compared to the radius of the conducting pipe, and that the electrons all have the same energy, Eq. (5) becomes

$$\frac{\gamma}{\gamma_0} = 1 - Q(2\ln b/a + 1). \quad (15)$$

The current given by Eq. (7) is then simply

$$I = \pi a^2 n e \beta c. \quad (16)$$

From Eqs. (6), (15), and (16) one finds

$$\frac{I}{I_c} = \frac{\beta(\gamma_0 - \gamma)}{1 + 2 \ln b/a}. \quad (17)$$

Here $I_c = 4\pi m/e \epsilon_0 c^3 = 17,000 = I_a/\beta_0 \gamma_0$. Using $\beta = \sqrt{1 - 1/\gamma^2}$ and setting the result of differentiating Eq. (12) to zero yields a maximum current

$$I = \frac{I_c (\gamma_0^{2/3} - 1)^{3/2}}{1 + 2 \ln b/a} \quad (18)$$

at which point the relativistic electron mass ratio γ is simply

$$\gamma = \gamma_0^{1/3}. \quad (19)$$

These results have been obtained by Mkheidze et al (6). For further discussion of this limit see Reference 7.

Taking the result to the nonrelativistic limit yields from Eq. (18)

$$I = K V_0^{3/2}$$

where

$$K = \frac{8\pi}{3} \epsilon_0 \sqrt{\frac{2e}{3m}} = 25.3964 \times 10^{-6}. \quad (20)$$

Furthermore, from Eq. (19) we find that the kinetic energy of the particles is 1/3 the applied potential V_b . Therefore the space charge potential of the beam is 2/3 the applied potential (V_0).

General Case

The space charge limiting of the beam was investigated for arbitrary ratios of beam to pipe radius with the aid of a computer. The above procedure was followed in that the maximum current was found by differentiating Eq. (8) with respect to zero and setting the result to zero. The results of these computations are presented in Figs. 1 to 3.

Figure 1 shows the variation of the maximum space charge limited beam current normalized to the Alfven current that can be caused to flow in a conducting pipe as a function of the pipe to beam radius ratio b/a . Here the initial γ_0 of the electrons ($\gamma_0 = 1 + eV_0/mc^2$) is doubled from curve to curve. This figure is most useful for considering the effects of changes of pipe radius on a beam of constant radius a . Figure 2 shows the same data as presented in Fig. 1, however the normalized current is calculated as a function of the beam to pipe radius. Figure 3 shows the variation of electron kinetic energy as a function of normalized radius. Here the maximum space charge limited current is flowing and ten ratios of beam to pipe radii are displayed. An initial γ_0 of 10 was assumed.

REFERENCES

1. A. V. Haeff, Proc. IRE 27, 952, (1939).
2. L. P. Smith and P. L. Hartman, J. Appl. Phys. 2, 220 (1940).
3. J. R. Pierce, Theory and Design of Electron Beams, Princeton, N. J., Van Nostrand 1947.
4. W. B. Bridges, J. I. Frey, and C. K. Birdsall, IEE Trans on Electron Devices, ED-12, 264 (1965).
5. A. B. Langdon, M. E. Rensink, and T. J. Fessenden, Plasma Phys. 15, 1267 (1973).
6. G. P. Mkheidze, V. I. Pulin, M. D. Raizer, and L. E. Tsopp, JETP 36, 54 (1973).
7. T. J. Fessenden and R. J. Briggs, "Experiments on the Auto Acceleration of a Relativistic High Current Electron Beam", LLL Report No. UCID-16487, April 18, 1974.

FIGURE CAPTIONS

- Fig. 1: Plot of the maximum unneutralized current normalized to the Alfvén current that can be made to flow in a conducting pipe as a function of the ratio of the wall (b) to beam (a) radii for various initial energies.
- Fig. 2: Same data as contained in Fig. 1 plotted as a function of beam to wall radii.
- Fig. 3: Radial energy profiles of the electron beam (solid) and potential profiles (dotted) as functions of the normalized radius for various beam to wall radii at initial $\gamma = 10$.

$$\frac{I}{I_a} = \frac{I}{17,000 \beta_0 \gamma_0}$$

Figure 1

$$\frac{I}{I_a} = \frac{I}{17,000 \beta \gamma_0}$$

Figure 2

GAMMA= 10.0

Figure 3

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