DESIGN GUIDE
FOR
CALCULATING NATURAL FREQUENCIES OF STRAIGHT
AND CURVED BEAMS ON MULTIPLE SUPPORTS

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
d_i	Tube inside diameter, in.
d_o	Tube outside diameter, in.
E	Young's modulus of elasticity, lb/in. ²
f_n	Vibration frequency of the nth mode, Hz
G	Shear modulus of elasticity, lb/in. ²
I	Area moment of inertia, in. ⁴
J	Polar area moment of inertia, in. ⁴
l	Span of beam, in.
m	Total mass per unit length of vibrating beam including added mass, lb-sec ² /in. ²
m_f	Added mass per unit length of tube, lb-sec ² /in. ²
m_t	Beam mass per unit length of tube, lb-sec ² /in. ²
M_j	Bending moment at jth support, lb-in.
p	$ \operatorname{Re}(\mu) /\pi$
r	Stiffness ratio
R	Radius of curvature, in.
T	Axial tension in straight beam, lb
V	Axial flow velocity, ft/sec
α	Subtended angle of a span in curved beam, degrees or radians
Γ	Dimensionless axial tension
λ	Frequency factor
λ_n	Frequency factor of the nth mode
μ	Dimensionless propagation constant
ν	Poisson's ratio
ρ_f	Fluid density, lb-sec ² /in. ⁴
ρ_t	Beam density, lb-sec ² /in. ⁴

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ABSTRACT

A graphical method is presented for calculating natural frequencies of straight and curved beams with multiple spans of equal length. The method is simple, and its degree of accuracy is directly related to the accuracy with which frequency factors are read from the graphs.

I. INTRODUCTION

There is little doubt that research in support of nuclear reactor component designs must be increasingly concerned with the dynamic nature of the loading and response of component structures. The work described here is concerned with a problem inherent in every vibration analysis: the determination of component natural frequencies. It is intended to be a first step in the development of a unified design procedure to avoid detrimental vibrations of beam-like components on multiple supports.

Many reactor components (e.g., heat exchanger tubes, fuel rods, and pipings) can be represented as continuous beams on multiple supports. In the design stage, a rigorous method for calculating precise natural frequencies is usually not warranted; rather a simple method to estimate the frequencies is desirable. Therefore, in this design guide, a simple graphical method is presented for predicting the frequencies of continuous, straight and curved beams on periodic, multiple supports.

II. STRAIGHT BEAMS

A. Single-Span Beams

The natural frequencies of a uniform beam are determined from:¹

$$f_n = \left(\frac{1}{2\pi} \right) \frac{\lambda_n}{L} \left(\frac{EI}{m} \right)^{1/2}, \quad (1)$$

where n denotes the mode number, and λ_n is a frequency factor which depends on the end conditions and is a function of the dimensionless axial tension given by

$$\Gamma = \frac{T L^2}{EI} \quad (2)$$

Values of λ_n for the first five frequencies ($n = 1, 2, 3, 4, 5$) are given in Tables 1, 2, and 3 for $\Gamma = -10$ to 10 , and three types of end conditions.*

B. Multiple-Span Beams

Several methods of analysis can be used to study the frequency of continuous beams: finite-element method, transfer-matrix technique, Rayleigh-Ritz procedure, wave-propagation approach, iterative procedure, and conventional method of solving the equation of motion directly. In this design guide, the frequencies of multiple-span beams are obtained using the dynamic three-moment equation,^{2,3} which has several advantages over the other methods.

Consider a continuous beam as shown in Fig. 1.** The intermediate supports are assumed to prevent vertical deflection. The bending moment at the j support is denoted by M_j . The dynamic three-moment equation, which can be derived using the equation of motion and boundary conditions, relates the bending moments at three supports of any two consecutive spans. In an infinite, periodically supported beam, the bending moment M_j at the j support is related to the moment at the preceding support $j-1$ by

$$M_j = M_{j-1} \exp(i\mu) \quad , \quad (3)$$

where μ is a dimensionless propagation constant. The real part $\text{Re}(\mu)$ represents the phase difference between moments at adjacent supports, and the imaginary part $\text{Im}(\mu)$ represents the exponential decay rate of the bending wave as it propagates from one support to the next. The propagation constant μ depends on the parameters λ and Γ ; Fig. 2 shows values of μ for $\Gamma = 0$. When $\text{Im}(\mu)$ is zero, bending waves can propagate without attenuation; these regions are called propagation bands. When $\text{Im}(\mu)$ is not zero, bending wave cannot propagate; these regions are called stop bands. There exist alternate bands of free propagation and attenuation.

In a finite structure, natural frequencies are in the propagation bands. In calculating the frequency, only the real part of μ is of interest; $\text{Re}(\mu)$ is zero or $\pm \pi$ in the stop band, and is between 0 and $\pm \pi$ in the propagation band. Values of p ($p = |\text{Re}(\mu)|/\pi$) are given in Figs. 3 to 6 for the first five propagation bands and for various values of Γ .

The p - λ curves are used to find the frequency of a finite structure. Consider an N -span periodically supported beam. At the two extreme ends, the supports may be hinged-hinged, hinged-clamped or clamped-clamped. Use of the theory of determinants,³ or the concept of wave propagation,⁴ yields an extremely simple graphical method to calculate the frequency. The procedure is summarized as follows:

(1) For a continuous beam with N spans and hinged at two extreme ends, divide the ordinate over the range 0 to 1 into N equal parts, and draw horizontal lines separating the parts. The projections on the abscissa (λ axis) of the points of intersection of the p - λ curve and horizontal lines corresponding to

* Tables begin on page 14.

** Figures begin on page 27.

$k = 1, 2, \dots N$ (odd-numbered propagation bands) or $k = 0, 1, 2, \dots N-1$ (even-numbered propagation bands) will give the frequency factors.

(2) For a continuous beam with N spans and hinged at one extreme end and clamped at the other, divide the ordinate over the range 0 to 1 into $2N$ equal parts, and draw horizontal lines separating the parts. The projections on the abscissa of the points of intersection of the $p-\lambda$ curve and the horizontal lines corresponding to $k = 1, 3, 5, \dots 2N-1$ will give the frequency factors.

(3) For a continuous beam with N spans and clamped at two extreme ends, divide the ordinate over the range 0 to 1 into N equal parts, and draw horizontal lines separating the parts. The projections on the abscissa of the points of intersection of the $\lambda-p$ curve and horizontal lines corresponding to $k = 0, 1, 2, \dots N-1$ (odd-numbered propagation bands) or $k = 1, 2, 3, \dots N$ (even-numbered propagation bands) give the frequency factors.

Figure 7 illustrates the graphical method used to find the frequency factors for a four-span beam hinged at the two extreme ends. Having the frequency factor λ_n , the frequency can be calculated by Eq. (1).

III. CURVED BEAMS

Whenever the central line of the unstressed beam is a plane curve, and its plane is a principal plane of the beam at each point, the vibrations of a curved beam fall into two classes: flexural vibrations of the beam in its plane, and flexural vibrations involving both displacement at right angle to the plane of beam and twist. In most curved beam-like reactor components (e.g., heat exchanger tubes), the in-plane and out-of-plane motions are uncoupled.

A. Single-Span Beams

The frequencies of a uniformly curved beam are obtained from:⁵

$$f_n = \frac{\lambda_n}{2\pi} \left(\frac{1}{R\alpha} \right)^2 \left(\frac{EI}{m} \right)^{1/2}, \quad (4)$$

where R is the radius of curvature, α is the subtended angle (in radians), and λ_n is the frequency factor.

(1) In-plane Motion

The beam is considered to be inextensible, and inextensional theory predicts that the fundamental mode is asymmetric. In reality, this is true for a large subtended angle. When α is small, the fundamental frequency is associated with the first symmetric mode, and the frequency factor may be approximated by that for a corresponding straight beam whose length is the same as the arc length of the curved beam.

The frequency factors for the in-plane motion based on the inextensional theory are given in Figs. 8, 9, and 10 for three types of end conditions.

(2) Out-of-plane Motion

The frequency factor for out-of-plane motion depends on the subtended angle α , stiffness ratio r , and support conditions. The stiffness ratio is given by

$$r = \frac{GJ}{EI} , \quad (5)$$

where J is the polar area moment of inertia of the beam, G is the shear modulus of elasticity, and other parameters are the same as those of straight beams. For a circular or square cross section,

$$r = \frac{1}{1 + \nu} ; \quad (6)$$

therefore r depends on Poisson's ratio ν only. The frequency factors for $\nu = 0.3$ and three types of end conditions are given in Figs. 11, 12, and 13.

B. Multiple-Span Beams

The graphical method to determine the frequency of a continuous curved beam is exactly the same as that for a straight beam, provided $p-\lambda$ curves for curved beams are used.

The $p-\lambda$ curves for in-plane motion and out-of-plane motion are given in Figs. 14 to 23. The in-plane vibration is based on the inextensional theory and out-of-plane motion is presented for $\nu = 0.3$.

The procedures to determine the frequency factors for an N -span curved beam are exactly the same as those for straight beams (see Section II.B). In addition to the three types of end conditions at the two extreme ends, a continuous curved beam may be a periodically supported ring. In this case, frequency factors also can be obtained graphically, using the following procedure:

The ordinate over the range 0 to 1 is divided into $(N-1)/2$ (N odd) or $N/2$ (N even) equal parts, and horizontal lines are drawn to separate the parts. The projections on the abscissa of the points of intersection of the $p-\lambda$ curve and horizontal lines corresponding to $k = 0, 1, 2, \dots (N-1)/2$ (N odd) or $k = 0, 1, 2, \dots N/2$ (N even) will give the frequency factors.

IV. EXAMPLE OF APPLICATION

Consider the case of a curved tube which conveys fluid and is submerged in fluid. The tube has four equal spans and is fixed at the two extreme ends as shown in Fig. 24. System dimensions and material properties are given in Table 4. The problem is to find the frequency of out-of-plane vibration.

Internal flow can have several effects:⁶ inducing an in-plane deformation, reducing the frequencies, and exciting the tube motion from the turbulent boundary pressure fluctuation. Since the flow velocity is relatively small, its effect on out-of-plane vibration frequency is small and is neglected here. The following steps are required to find the frequency:

(1) Beam mass per unit length of tube (m_t)

$$\begin{aligned} m_t &= \rho_t \frac{\pi}{4} (d_o^2 - d_i^2) \\ &= 5.27 \times 10^{-5} \text{ lb-sec}^2/\text{in.}^2 \end{aligned}$$

(2) Added mass per unit length of fluid (m_f)

$$\begin{aligned} m_f &= \rho_f \frac{\pi}{4} (d_o^2 + d_i^2) \\ &= 3.01 \times 10^{-5} \text{ lb-sec}^2/\text{in.}^2 \end{aligned}$$

(3) Total mass per unit length (m)

$$\begin{aligned} m &= m_t + m_f \\ &= 8.28 \times 10^{-5} \text{ lb-sec}^2/\text{in.}^2 \end{aligned}$$

(4) Moment of inertia (I)

$$\begin{aligned} I &= \frac{\pi}{64} (d_o^4 - d_i^4) \\ &= 1.81 \times 10^{-3} \text{ in.}^4 \end{aligned}$$

(5) Frequency factors (λ_n)

The frequency factors are obtained from Fig. 18. For illustrating this example, the curves for the first and second propagation bands and $\alpha = 40^\circ$ are replotted in Fig. 25. Since the tube is clamped at the two extreme ends the procedure for determining the frequency factors, as outlined in Section II.B, is to first divide the ordinate over the range 0 to 1 into 4 equal parts and draw horizontal lines separating the parts. In the first propagation band the projections on the abscissa of the points of intersection of the λ - p curve and horizontal lines corresponding to $k = 0, 1, 2$, and 3 give the frequency factors corresponding to the first four natural frequencies. The second "set" of four frequency factors is found in the second propagation band as the projection on the abscissa of the intersection of the λ - p curve and the horizontal lines corresponding to $k = 1, 2, 3$ and 4. The frequency factors associated with the first eight modes are given below:

$\lambda_1 = 11.0$	$\lambda_5 = 42.2$
$\lambda_2 = 14.9$	$\lambda_6 = 49.5$
$\lambda_3 = 19.5$	$\lambda_7 = 57.1$
$\lambda_4 = 21.9$	$\lambda_8 = 61.1$

(6) Natural Frequencies (f_n)

The following natural frequencies are obtained in the first two propagation bands, using Eq. (4) along with information given in Table 4 and that calculated in steps (1) to (5) above:

$f_1 = 8.9 \text{ Hz}$	$f_5 = 34.1 \text{ Hz}$
$f_2 = 12.0 \text{ Hz}$	$f_6 = 40.0 \text{ Hz}$
$f_3 = 15.8 \text{ Hz}$	$f_7 = 46.1 \text{ Hz}$
$f_4 = 17.7 \text{ Hz}$	$f_8 = 49.4 \text{ Hz}$

V. DISCUSSION

There are several salient features in the design of finite continuous beams on multiple supports:

- (1) All natural frequencies are in the propagation bands.
- (2) In a periodically supported N-span continuous beam, there are N frequencies in every propagation band.
- (3) There are N-1 discrete values of p which are the same whether the two extreme ends are hinged or clamped; consequently, the N-1 frequencies which do not correspond to $p = 0$ or 1 are also identical in both cases.
- (4) There are $(N+1)/2$ (N odd) or $(N+2)/2$ (N even) frequencies in every propagation band for a periodically supported N-span ring.

In this design guide, the beam is assumed to be periodically supported, and the end conditions are either hinged or clamped. In actual systems, these conditions are rarely satisfied. However, the proposed method is still useful in the design stage. In this stage, the exact support conditions and other parameters are only roughly known, and an estimate of the natural frequencies is satisfactory. The graphical method can be used to establish frequency bounds. For example, consider the following two cases:

- (1) In a periodically supported, five-span beam, the two ends are elastically restrained in rotation. The upper frequency bound can be established by assuming a clamped end condition, and the lower frequency bound by assuming a hinged end condition.
- (2) Consider a continuous beam with five spans whose lengths are 20 in., 20 in., 20 in., 18 in., and 18 in. The lower frequency bound may be obtained by considering a beam with 20 in. in every span, and the upper frequency bound by considering a beam with equal spans of $19.2 \left(= \frac{20 + 20 + 20 + 18 + 18}{5} \right)$ inches.

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Table 1. Frequency Factors of a Single-span Beam Hinged at Both Ends

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
10.0	14.003	44.196	93.693	162.837	251.690
9.9	13.968	44.152	93.646	162.788	251.642
9.8	13.933	44.107	93.598	162.739	251.593
9.7	13.897	44.062	93.551	162.691	251.543
9.6	13.862	44.018	93.503	162.642	251.495
9.5	13.826	43.973	93.456	162.595	251.445
9.4	13.791	43.928	93.409	162.546	251.396
9.3	13.754	43.882	93.361	162.497	251.347
9.2	13.719	43.838	93.313	162.448	251.298
9.1	13.682	43.793	93.265	162.399	251.249
9.0	13.647	43.747	93.217	162.352	251.200
8.9	13.610	43.703	93.170	162.303	251.151
8.8	13.574	43.658	93.122	162.254	251.102
8.7	13.538	43.612	93.075	162.205	251.053
8.6	13.502	43.567	93.028	162.156	251.004
8.5	13.465	43.522	92.979	162.108	250.954
8.4	13.428	43.476	92.931	162.059	250.905
8.3	13.392	43.430	92.884	162.011	250.856
8.2	13.354	43.386	92.835	161.962	250.807
8.1	13.317	43.339	92.788	161.914	250.758
8.0	13.281	43.295	92.741	161.865	250.708
7.9	13.243	43.249	92.693	161.815	250.659
7.8	13.206	43.203	92.644	161.766	250.610
7.7	13.168	43.157	92.596	161.718	250.560
7.6	13.131	43.111	92.548	161.669	250.511
7.5	13.093	43.066	92.501	161.620	250.463
7.4	13.055	43.019	92.453	161.572	250.412
7.3	13.017	42.974	92.405	161.522	250.364
7.2	12.979	42.928	92.357	161.474	250.315
7.1	12.941	42.881	92.309	161.425	250.265
7.0	12.904	42.836	92.260	161.376	250.215
6.9	12.865	42.790	92.212	161.326	250.167
6.8	12.827	42.743	92.164	161.278	250.118
6.7	12.788	42.697	92.116	161.229	250.068
6.6	12.750	42.651	92.067	161.181	250.019
6.5	12.711	42.605	92.019	161.131	249.969
6.4	12.672	42.558	91.971	161.082	249.919
6.3	12.633	42.512	91.923	161.033	249.870
6.2	12.594	42.465	91.874	160.984	249.821
6.1	12.554	42.419	91.825	160.935	249.772
6.0	12.515	42.373	91.778	160.886	249.723
5.9	12.475	42.326	91.729	160.836	249.673
5.8	12.436	42.279	91.681	160.787	249.624
5.7	12.396	42.232	91.632	160.738	249.574
5.6	12.357	42.186	91.584	160.690	249.524
5.5	12.316	42.139	91.535	160.640	249.475
5.4	12.276	42.092	91.487	160.592	249.426
5.3	12.236	42.045	91.438	160.542	249.377
5.2	12.196	41.998	91.389	160.493	249.327
5.1	12.155	41.951	91.341	160.443	249.277
5.0	12.114	41.904	91.292	160.394	249.228

Table 1. Frequency Factors of a Single-span Beam Hinged at Both Ends (Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
5.0	12.114	41.904	91.292	160.394	249.228
4.9	12.073	41.857	91.244	160.346	249.178
4.8	12.032	41.810	91.195	160.296	249.128
4.7	11.992	41.763	91.147	160.246	249.079
4.6	11.950	41.716	91.097	160.197	249.029
4.5	11.909	41.668	91.048	160.148	248.981
4.4	11.867	41.620	90.999	160.098	248.931
4.3	11.826	41.573	90.951	160.050	248.880
4.2	11.784	41.526	90.902	160.000	248.831
4.1	11.741	41.478	90.854	159.951	248.782
4.0	11.700	41.431	90.805	159.901	248.732
3.9	11.657	41.382	90.756	159.852	248.682
3.8	11.616	41.335	90.707	159.803	248.633
3.7	11.573	41.287	90.658	159.754	248.583
3.6	11.530	41.239	90.609	159.703	248.534
3.5	11.488	41.191	90.559	159.655	248.484
3.4	11.444	41.143	90.511	159.605	248.435
3.3	11.401	41.095	90.461	159.555	248.385
3.2	11.357	41.047	90.412	159.506	248.335
3.1	11.314	41.000	90.363	159.456	248.286
3.0	11.270	40.951	90.313	159.406	248.235
2.9	11.226	40.903	90.265	159.357	248.186
2.8	11.182	40.854	90.216	159.308	248.136
2.7	11.138	40.806	90.166	159.259	248.087
2.6	11.093	40.757	90.118	159.208	248.037
2.5	11.049	40.709	90.068	159.159	247.988
2.4	11.004	40.661	90.018	159.110	247.937
2.3	10.959	40.612	89.969	159.059	247.888
2.2	10.914	40.563	89.920	159.010	247.837
2.1	10.869	40.514	89.870	158.960	247.788
2.0	10.823	40.466	89.821	158.911	247.738
1.9	10.778	40.417	89.771	158.861	247.688
1.8	10.732	40.369	89.722	158.812	247.639
1.7	10.686	40.319	89.672	158.761	247.589
1.6	10.639	40.270	89.623	158.712	247.540
1.5	10.593	40.221	89.574	158.662	247.490
1.4	10.546	40.173	89.524	158.613	247.439
1.3	10.500	40.123	89.474	158.563	247.389
1.2	10.452	40.074	89.424	158.512	247.340
1.1	10.405	40.025	89.375	158.463	247.289
1.0	10.357	39.975	89.325	158.413	247.239
0.9	10.310	39.926	89.276	158.363	247.190
0.8	10.262	39.877	89.225	158.313	247.140
0.7	10.213	39.827	89.176	158.264	247.090
0.6	10.166	39.778	89.126	158.214	247.040
0.5	10.117	39.728	89.076	158.164	246.990
0.4	10.068	39.678	89.027	158.114	246.940
0.3	10.019	39.628	88.977	158.064	246.890
0.2	9.969	39.578	88.926	158.014	246.840
0.1	9.919	39.528	88.876	157.964	246.790
0.0	9.870	39.479	88.826	157.914	246.740

Table 1. Frequency Factors of a Single-span Beam Hinged at Both Ends (Contd.)

r	λ_1	λ_2	λ_3	λ_4	λ_5
0.0	9.870	39.479	88.826	157.914	246.740
-0.1	9.820	39.429	88.776	157.864	246.690
-0.2	9.769	39.378	88.726	157.814	246.640
-0.3	9.719	39.329	88.676	157.764	246.590
-0.4	9.667	39.278	88.626	157.714	246.540
-0.5	9.616	39.228	88.576	157.664	246.490
-0.6	9.565	39.177	88.526	157.614	246.440
-0.7	9.514	39.127	88.476	157.564	246.390
-0.8	9.461	39.076	88.425	157.513	246.340
-0.9	9.409	39.026	88.375	157.463	246.290
-1.0	9.356	38.975	88.326	157.413	246.240
-1.1	9.304	38.924	88.275	157.362	246.190
-1.2	9.250	38.874	88.225	157.312	246.140
-1.3	9.196	38.823	88.174	157.263	246.090
-1.4	9.143	38.773	88.124	157.213	246.039
-1.5	9.089	38.721	88.073	157.162	245.989
-1.6	9.034	38.670	88.023	157.112	245.939
-1.7	8.979	38.619	87.973	157.061	245.889
-1.8	8.924	38.567	87.921	157.011	245.839
-1.9	8.869	38.517	87.871	156.961	245.788
-2.0	8.813	38.465	87.820	156.911	245.738
-2.1	8.757	38.414	87.770	156.860	245.688
-2.2	8.701	38.363	87.720	156.810	245.638
-2.3	8.643	38.311	87.669	156.759	245.587
-2.4	8.586	38.260	87.618	156.710	245.537
-2.5	8.529	38.208	87.568	156.659	245.488
-2.6	8.470	38.156	87.516	156.608	245.437
-2.7	8.411	38.105	87.466	156.558	245.387
-2.8	8.353	38.052	87.415	156.508	245.336
-2.9	8.294	38.000	87.364	156.457	245.286
-3.0	8.234	37.949	87.314	156.406	245.235
-3.1	8.173	37.897	87.263	156.356	245.185
-3.2	8.114	37.844	87.212	156.306	245.134
-3.3	8.052	37.793	87.161	156.256	245.085
-3.4	7.990	37.740	87.110	156.205	245.035
-3.5	7.929	37.688	87.058	156.154	244.984
-3.6	7.866	37.636	87.008	156.103	244.933
-3.7	7.803	37.583	86.957	156.053	244.883
-3.8	7.740	37.530	86.906	156.002	244.833
-3.9	7.675	37.477	86.855	155.952	244.783
-4.0	7.612	37.425	86.803	155.901	244.733
-4.1	7.546	37.372	86.753	155.850	244.681
-4.2	7.480	37.320	86.701	155.800	244.631
-4.3	7.414	37.266	86.650	155.748	244.581
-4.4	7.347	37.214	86.599	155.698	244.531
-4.5	7.280	37.160	86.547	155.647	244.480
-4.6	7.211	37.107	86.496	155.597	244.429
-4.7	7.143	37.054	86.444	155.546	244.379
-4.8	7.074	37.001	86.393	155.495	244.329
-4.9	7.004	36.947	86.341	155.444	244.278
-5.0	6.933	36.893	86.290	155.394	244.227

Table 1. Frequency Factors of a Single-span Beam Hinged at Both Ends (Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
-5.0	6.933	36.893	86.290	155.394	244.227
-5.1	6.861	36.841	86.239	155.343	244.177
-5.2	6.789	36.787	86.187	155.292	244.126
-5.3	6.716	36.733	86.136	155.241	244.076
-5.4	6.641	36.679	86.084	155.190	244.025
-5.5	6.567	36.625	86.033	155.140	243.974
-5.6	6.491	36.572	85.981	155.088	243.924
-5.7	6.416	36.517	85.929	155.037	243.874
-5.8	6.337	36.463	85.878	154.986	243.823
-5.9	6.259	36.409	85.826	154.935	243.772
-6.0	6.180	36.355	85.774	154.885	243.722
-6.1	6.099	36.301	85.722	154.834	243.671
-6.2	6.018	36.246	85.671	154.782	243.620
-6.3	5.936	36.191	85.619	154.732	243.570
-6.4	5.852	36.137	85.567	154.681	243.519
-6.5	5.767	36.083	85.514	154.630	243.469
-6.6	5.681	36.028	85.463	154.578	243.418
-6.7	5.593	35.972	85.411	154.528	243.367
-6.8	5.504	35.917	85.359	154.476	243.317
-6.9	5.413	35.862	85.307	154.426	243.265
-7.0	5.322	35.807	85.254	154.374	243.215
-7.1	5.228	35.753	85.203	154.323	243.164
-7.2	5.133	35.698	85.150	154.272	243.114
-7.3	5.036	35.642	85.099	154.220	243.063
-7.4	4.937	35.587	85.046	154.169	243.013
-7.5	4.836	35.532	84.994	154.118	242.962
-7.6	4.734	35.476	84.942	154.067	242.911
-7.7	4.627	35.419	84.889	154.016	242.860
-7.8	4.520	35.364	84.837	153.965	242.809
-7.9	4.409	35.308	84.784	153.912	242.759
-8.0	4.295	35.252	84.732	153.862	242.707
-8.1	4.179	35.196	84.679	153.810	242.655
-8.2	4.060	35.140	84.627	153.759	242.606
-8.3	3.936	35.084	84.574	153.707	242.555
-8.4	3.808	35.027	84.522	153.656	242.504
-8.5	3.676	34.971	84.470	153.605	242.453
-8.6	3.540	34.915	84.417	153.554	242.402
-8.7	3.398	34.858	84.364	153.502	242.352
-8.8	3.249	34.802	84.311	153.450	242.300
-8.9	3.094	34.745	84.259	153.399	242.249
-9.0	2.929	34.688	84.206	153.348	242.198
-9.1	2.756	34.631	84.153	153.296	242.147
-9.2	2.570	34.573	84.101	153.245	242.096
-9.3	2.371	34.517	84.048	153.193	242.045
-9.4	2.153	34.460	83.995	153.142	241.995
-9.5	1.910	34.402	83.943	153.090	241.943
-9.6	1.632	34.344	83.889	153.039	241.893
-9.7	1.293	34.288	83.836	152.987	241.841
-9.8	0.830	34.230	83.783	152.935	241.791

Table 2. Frequency Factors of a Single-span Beam Hinged at One End and Clamped at the Other

r	λ_1	λ_2	λ_3	λ_4	λ_5
10.0	18.760	54.083	108.664	182.837	276.687
9.9	18.730	54.045	108.621	182.791	276.642
9.8	18.699	54.006	108.578	182.747	276.596
9.7	18.669	53.965	108.534	182.701	276.549
9.6	18.638	53.926	108.491	182.656	276.503
9.5	18.608	53.886	108.448	182.611	276.457
9.4	18.577	53.847	108.404	182.566	276.410
9.3	18.547	53.807	108.360	182.520	276.365
9.2	18.516	53.768	108.318	182.476	276.319
9.1	18.486	53.727	108.274	182.430	276.272
9.0	18.455	53.688	108.230	182.385	276.226
8.9	18.425	53.647	108.187	182.340	276.179
8.8	18.393	53.608	108.143	182.294	276.133
8.7	18.363	53.567	108.100	182.250	276.087
8.6	18.332	53.528	108.057	182.204	276.040
8.5	18.300	53.487	108.013	182.159	275.995
8.4	18.270	53.447	107.970	182.113	275.948
8.3	18.238	53.408	107.926	182.069	275.902
8.2	18.208	53.367	107.882	182.023	275.855
8.1	18.176	53.327	107.839	181.978	275.809
8.0	18.145	53.287	107.796	181.932	275.763
7.9	18.114	53.247	107.752	181.887	275.717
7.8	18.082	53.206	107.708	181.842	275.670
7.7	18.050	53.166	107.664	181.797	275.624
7.6	18.018	53.126	107.621	181.751	275.577
7.5	17.988	53.086	107.577	181.706	275.531
7.4	17.956	53.044	107.533	181.661	275.484
7.3	17.924	53.004	107.490	181.615	275.438
7.2	17.893	52.964	107.446	181.570	275.392
7.1	17.861	52.924	107.402	181.524	275.345
7.0	17.828	52.883	107.359	181.479	275.299
6.9	17.796	52.842	107.315	181.434	275.252
6.8	17.764	52.802	107.270	181.387	275.207
6.7	17.733	52.761	107.226	181.342	275.160
6.6	17.700	52.721	107.182	181.297	275.113
6.5	17.668	52.680	107.140	181.251	275.067
6.4	17.636	52.639	107.096	181.206	275.021
6.3	17.603	52.599	107.052	181.160	274.974
6.2	17.572	52.557	107.008	181.115	274.928
6.1	17.539	52.517	106.964	181.069	274.881
6.0	17.506	52.476	106.919	181.024	274.834
5.9	17.474	52.435	106.876	180.978	274.789
5.8	17.441	52.395	106.831	180.933	274.741
5.7	17.408	52.353	106.787	180.887	274.695
5.6	17.375	52.313	106.743	180.842	274.648
5.5	17.342	52.272	106.699	180.795	274.602
5.4	17.309	52.231	106.656	180.750	274.556
5.3	17.276	52.190	106.611	180.705	274.509
5.2	17.243	52.149	106.567	180.659	274.462
5.1	17.210	52.108	106.523	180.613	274.416
5.0	17.177	52.067	106.479	180.568	274.369

Table 2. Frequency Factors of a Single-span Beam Hinged at One End
and Clamped at the Other (Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
5.0	17.177	52.067	106.479	180.568	274.369
4.9	17.143	52.024	106.435	180.523	274.323
4.8	17.110	51.984	106.390	180.477	274.276
4.7	17.077	51.943	106.347	180.430	274.230
4.6	17.043	51.901	106.302	180.385	274.183
4.5	17.010	51.860	106.258	180.339	274.136
4.4	16.976	51.818	106.214	180.294	274.089
4.3	16.943	51.777	106.169	180.247	274.043
4.2	16.909	51.736	106.125	180.202	273.996
4.1	16.875	51.695	106.081	180.156	273.949
4.0	16.840	51.653	106.036	180.111	273.904
3.9	16.807	51.611	105.992	180.065	273.856
3.8	16.773	51.570	105.948	180.018	273.810
3.7	16.739	51.528	105.903	179.973	273.764
3.6	16.705	51.487	105.858	179.927	273.717
3.5	16.671	51.445	105.815	179.881	273.670
3.4	16.636	51.404	105.771	179.836	273.623
3.3	16.601	51.362	105.725	179.790	273.577
3.2	16.567	51.320	105.681	179.744	273.530
3.1	16.533	51.278	105.637	179.699	273.483
3.0	16.497	51.236	105.592	179.652	273.437
2.9	16.463	51.195	105.547	179.606	273.390
2.8	16.428	51.152	105.503	179.560	273.343
2.7	16.394	51.111	105.458	179.514	273.297
2.6	16.358	51.069	105.414	179.469	273.249
2.5	16.323	51.027	105.370	179.423	273.203
2.4	16.287	50.984	105.324	179.377	273.155
2.3	16.253	50.942	105.280	179.330	273.109
2.2	16.218	50.901	105.235	179.285	273.062
2.1	16.182	50.858	105.190	179.239	273.015
2.0	16.147	50.817	105.146	179.192	272.969
1.9	16.110	50.774	105.101	179.146	272.921
1.8	16.075	50.732	105.056	179.100	272.875
1.7	16.040	50.690	105.011	179.054	272.828
1.6	16.004	50.647	104.967	179.008	272.781
1.5	15.968	50.605	104.922	178.962	272.735
1.4	15.932	50.563	104.877	178.916	272.688
1.3	15.896	50.521	104.832	178.870	272.641
1.2	15.859	50.477	104.788	178.824	272.594
1.1	15.823	50.434	104.742	178.778	272.547
1.0	15.787	50.393	104.698	178.732	272.500
0.9	15.750	50.350	104.653	178.685	272.453
0.8	15.714	50.307	104.608	178.640	272.406
0.7	15.677	50.264	104.563	178.593	272.360
0.6	15.640	50.221	104.518	178.547	272.312
0.5	15.604	50.179	104.473	178.501	272.266
0.4	15.567	50.137	104.428	178.455	272.218
0.3	15.529	50.093	104.383	178.408	272.171
0.2	15.493	50.051	104.337	178.362	272.125
0.1	15.456	50.008	104.292	178.316	272.077
0.0	15.418	49.965	104.248	178.269	272.031

Table 2. Frequency Factors of a Single-span Beam Hinged at One End and Clamped at the Other (Contd.)

r	λ_1	λ_2	λ_3	λ_4	λ_5
0.0	15.418	49.965	104.248	178.269	272.031
-0.1	15.380	49.922	104.203	178.223	271.984
-0.2	15.344	49.879	104.157	178.177	271.938
-0.3	15.306	49.836	104.113	178.131	271.890
-0.4	15.268	49.793	104.068	178.084	271.844
-0.5	15.230	49.749	104.023	178.039	271.797
-0.6	15.192	49.706	103.976	177.992	271.749
-0.7	15.155	49.663	103.932	177.946	271.703
-0.8	15.117	49.620	103.886	177.899	271.655
-0.9	15.078	49.577	103.841	177.853	271.609
-1.0	15.040	49.534	103.796	177.807	271.561
-1.1	15.002	49.491	103.750	177.760	271.515
-1.2	14.963	49.448	103.705	177.714	271.467
-1.3	14.924	49.404	103.659	177.668	271.419
-1.4	14.886	49.360	103.615	177.622	271.373
-1.5	14.847	49.316	103.569	177.575	271.325
-1.6	14.808	49.273	103.523	177.528	271.278
-1.7	14.769	49.229	103.478	177.482	271.231
-1.8	14.730	49.186	103.432	177.435	271.184
-1.9	14.691	49.143	103.388	177.388	271.137
-2.0	14.652	49.099	103.341	177.343	271.090
-2.1	14.611	49.054	103.296	177.296	271.043
-2.2	14.572	49.011	103.250	177.249	270.996
-2.3	14.532	48.968	103.205	177.202	270.948
-2.4	14.493	48.923	103.160	177.156	270.902
-2.5	14.453	48.879	103.114	177.110	270.854
-2.6	14.412	48.836	103.069	177.063	270.807
-2.7	14.372	48.792	103.022	177.016	270.760
-2.8	14.332	48.748	102.977	176.970	270.713
-2.9	14.292	48.704	102.932	176.924	270.666
-3.0	14.251	48.660	102.886	176.877	270.619
-3.1	14.210	48.616	102.840	176.830	270.571
-3.2	14.169	48.571	102.795	176.783	270.524
-3.3	14.129	48.528	102.748	176.737	270.476
-3.4	14.088	48.484	102.703	176.690	270.429
-3.5	14.046	48.438	102.657	176.643	270.382
-3.6	14.005	48.394	102.611	176.596	270.335
-3.7	13.963	48.350	102.566	176.550	270.287
-3.8	13.922	48.306	102.519	176.503	270.240
-3.9	13.880	48.261	102.473	176.456	270.193
-4.0	13.839	48.218	102.427	176.410	270.146
-4.1	13.797	48.173	102.382	176.363	270.098
-4.2	13.754	48.128	102.336	176.316	270.051
-4.3	13.712	48.083	102.290	176.269	270.004
-4.4	13.670	48.038	102.243	176.223	269.956
-4.5	13.627	47.994	102.198	176.176	269.909
-4.6	13.585	47.949	102.152	176.129	269.862
-4.7	13.542	47.904	102.105	176.082	269.815
-4.8	13.499	47.860	102.060	176.035	269.768
-4.9	13.455	47.814	102.013	175.989	269.719
-5.0	13.413	47.770	101.967	175.942	269.672

Table 2. Frequency Factors of a Single-span Beam Hinged at One End and Clamped at the Other (Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
-5.0	13.413	47.770	101.968	175.942	269.673
-5.1	13.369	47.725	101.921	175.896	269.625
-5.2	13.326	47.679	101.875	175.848	269.577
-5.3	13.282	47.634	101.830	175.801	269.530
-5.4	13.238	47.590	101.783	175.755	269.483
-5.5	13.194	47.545	101.736	175.707	269.436
-5.6	13.150	47.500	101.691	175.660	269.387
-5.7	13.105	47.454	101.645	175.614	269.341
-5.8	13.061	47.410	101.598	175.566	269.292
-5.9	13.016	47.364	101.551	175.520	269.246
-6.0	12.972	47.318	101.505	175.472	269.198
-6.1	12.927	47.273	101.458	175.426	269.151
-6.2	12.882	47.228	101.413	175.378	269.103
-6.3	12.836	47.183	101.366	175.332	269.056
-6.4	12.790	47.137	101.319	175.284	269.008
-6.5	12.745	47.092	101.273	175.238	268.961
-6.6	12.700	47.046	101.227	175.190	268.914
-6.7	12.653	46.999	101.180	175.144	268.866
-6.8	12.607	46.954	101.134	175.096	268.819
-6.9	12.561	46.908	101.088	175.050	268.771
-7.0	12.514	46.863	101.040	175.003	268.723
-7.1	12.468	46.816	100.995	174.955	268.676
-7.2	12.420	46.772	100.948	174.907	268.628
-7.3	12.374	46.725	100.902	174.861	268.581
-7.4	12.326	46.680	100.855	174.813	268.533
-7.5	12.279	46.633	100.808	174.766	268.485
-7.6	12.231	46.587	100.761	174.719	268.438
-7.7	12.183	46.540	100.715	174.672	268.390
-7.8	12.135	46.495	100.668	174.625	268.342
-7.9	12.087	46.449	100.622	174.577	268.295
-8.0	12.038	46.402	100.574	174.530	268.247
-8.1	11.989	46.357	100.528	174.483	268.199
-8.2	11.940	46.310	100.481	174.436	268.152
-8.3	11.892	46.263	100.434	174.389	268.104
-8.4	11.843	46.217	100.387	174.342	268.056
-8.5	11.793	46.171	100.341	174.294	268.009
-8.6	11.743	46.124	100.293	174.247	267.962
-8.7	11.693	46.077	100.247	174.199	267.913
-8.8	11.643	46.031	100.199	174.152	267.866
-8.9	11.593	45.985	100.153	174.105	267.818
-9.0	11.541	45.938	100.106	174.057	267.771
-9.1	11.491	45.891	100.059	174.010	267.722
-9.2	11.440	45.845	100.011	173.963	267.675
-9.3	11.389	45.797	99.964	173.914	267.627
-9.4	11.336	45.751	99.918	173.867	267.579
-9.5	11.285	45.703	99.870	173.820	267.531
-9.6	11.232	45.656	99.823	173.772	267.484
-9.7	11.180	45.610	99.776	173.725	267.436
-9.8	11.127	45.562	99.729	173.677	267.389
-9.9	11.075	45.515	99.682	173.630	267.340
-10.0	11.021	45.468	99.635	173.582	267.293

Table 3. Frequency Factors of a Single-span Beam Clamped at Both Ends

r	λ_1	λ_2	λ_3	λ_4	λ_5
10.0	24.957	65.290	124.925	204.107	302.945
9.9	24.934	65.259	124.886	204.064	302.900
9.8	24.909	65.222	124.845	204.022	302.857
9.7	24.884	65.187	124.806	203.980	302.814
9.6	24.860	65.150	124.767	203.938	302.770
9.5	24.836	65.117	124.726	203.896	302.726
9.4	24.810	65.079	124.688	203.854	302.683
9.3	24.786	65.046	124.648	203.812	302.639
9.2	24.761	65.009	124.608	203.770	302.595
9.1	24.737	64.977	124.569	203.728	302.552
9.0	24.713	64.938	124.529	203.686	302.508
8.9	24.687	64.907	124.489	203.644	302.465
8.8	24.663	64.867	124.449	203.602	302.422
8.7	24.638	64.835	124.410	203.560	302.378
8.6	24.614	64.800	124.369	203.517	302.334
8.5	24.589	64.762	124.330	203.475	302.290
8.4	24.564	64.728	124.290	203.433	302.247
8.3	24.539	64.690	124.251	203.391	302.202
8.2	24.515	64.657	124.211	203.349	302.159
8.1	24.489	64.622	124.171	203.306	302.116
8.0	24.465	64.586	124.131	203.264	302.072
7.9	24.440	64.551	124.092	203.222	302.027
7.8	24.415	64.513	124.052	203.179	301.984
7.7	24.390	64.479	124.012	203.137	301.941
7.6	24.365	64.445	123.972	203.096	301.896
7.5	24.340	64.408	123.932	203.054	301.853
7.4	24.315	64.373	123.892	203.010	301.810
7.3	24.289	64.339	123.852	202.969	301.765
7.2	24.265	64.300	123.813	202.927	301.722
7.1	24.239	64.263	123.772	202.884	301.679
7.0	24.215	64.228	123.732	202.842	301.635
6.9	24.189	64.194	123.693	202.800	301.590
6.8	24.163	64.161	123.652	202.757	301.547
6.7	24.139	64.125	123.612	202.715	301.502
6.6	24.113	64.086	123.572	202.672	301.460
6.5	24.088	64.052	123.532	202.630	301.416
6.4	24.062	64.013	123.493	202.587	301.371
6.3	24.036	63.979	123.452	202.546	301.328
6.2	24.012	63.944	123.413	202.504	301.284
6.1	23.986	63.907	123.373	202.461	301.240
6.0	23.961	63.873	123.332	202.418	301.197
5.9	23.935	63.837	123.293	202.376	301.152
5.8	23.909	63.799	123.252	202.333	301.109
5.7	23.884	63.765	123.211	202.292	301.065
5.6	23.858	63.730	123.172	202.249	301.021
5.5	23.833	63.693	123.132	202.206	300.978
5.4	23.808	63.657	123.091	202.164	300.933
5.3	23.781	63.620	123.051	202.122	300.889
5.2	23.756	63.586	123.011	202.079	300.846
5.1	23.730	63.547	122.971	202.037	300.802
5.0	23.704	63.513	122.931	201.994	300.758

Table 3. Frequency Factors of a Single-span Beam Clamped at Both Ends
(Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
5.0	23.704	63.512	122.931	201.994	300.758
4.9	23.679	63.476	122.891	201.951	300.714
4.8	23.653	63.436	122.850	201.909	300.670
4.7	23.626	63.403	122.810	201.867	300.626
4.6	23.601	63.365	122.770	201.824	300.582
4.5	23.575	63.328	122.729	201.782	300.539
4.4	23.549	63.296	122.690	201.739	300.495
4.3	23.522	63.259	122.649	201.696	300.451
4.2	23.497	63.221	122.609	201.655	300.406
4.1	23.471	63.182	122.568	201.612	300.362
4.0	23.444	63.150	122.528	201.569	300.318
3.9	23.419	63.110	122.488	201.527	300.275
3.8	23.392	63.074	122.448	201.484	300.231
3.7	23.366	63.038	122.406	201.442	300.187
3.6	23.339	63.001	122.366	201.399	300.143
3.5	23.314	62.967	122.326	201.356	300.099
3.4	23.288	62.932	122.285	201.314	300.055
3.3	23.261	62.895	122.245	201.271	300.011
3.2	23.236	62.856	122.204	201.229	299.967
3.1	23.209	62.821	122.164	201.186	299.923
3.0	23.182	62.785	122.124	201.143	299.879
2.9	23.155	62.745	122.083	201.101	299.834
2.8	23.129	62.712	122.043	201.057	299.791
2.7	23.103	62.674	122.002	201.016	299.746
2.6	23.076	62.637	121.962	200.972	299.702
2.5	23.049	62.598	121.922	200.930	299.658
2.4	23.023	62.562	121.880	200.888	299.615
2.3	22.996	62.524	121.841	200.845	299.571
2.2	22.969	62.491	121.800	200.802	299.527
2.1	22.943	62.455	121.759	200.759	299.483
2.0	22.916	62.419	121.718	200.717	299.439
1.9	22.889	62.377	121.678	200.674	299.395
1.8	22.862	62.342	121.637	200.631	299.350
1.7	22.835	62.304	121.596	200.589	299.306
1.6	22.808	62.271	121.556	200.545	299.262
1.5	22.781	62.234	121.515	200.502	299.219
1.4	22.755	62.196	121.474	200.459	299.173
1.3	22.728	62.159	121.434	200.416	299.129
1.2	22.701	62.119	121.393	200.374	299.085
1.1	22.674	62.082	121.352	200.331	299.042
1.0	22.645	62.046	121.312	200.288	298.998
0.9	22.619	62.010	121.270	200.246	298.954
0.8	22.592	61.975	121.230	200.203	298.909
0.7	22.564	61.934	121.189	200.159	298.865
0.6	22.537	61.900	121.148	200.117	298.821
0.5	22.510	61.863	121.107	200.074	298.776
0.4	22.484	61.822	121.066	200.031	298.733
0.3	22.455	61.786	121.025	199.989	298.688
0.2	22.429	61.749	120.984	199.945	298.644
0.1	22.401	61.713	120.944	199.902	298.600
0.0	22.374	61.674	120.902	199.860	298.555

Table 3. Frequency Factors of a Single-span Beam Clamped at Both Ends
(Contd.)

Γ	λ_1	λ_2	λ_3	λ_4	λ_5
0.0	22.374	61.674	120.903	199.860	298.555
-0.1	22.346	61.640	120.862	199.816	298.511
-0.2	22.318	61.601	120.821	199.773	298.467
-0.3	22.291	61.562	120.780	199.731	298.423
-0.4	22.263	61.528	120.739	199.688	298.379
-0.5	22.236	61.490	120.699	199.644	298.334
-0.6	22.208	61.453	120.657	199.602	298.290
-0.7	22.180	61.415	120.616	199.559	298.246
-0.8	22.152	61.375	120.575	199.516	298.202
-0.9	22.125	61.340	120.534	199.473	298.158
-1.0	22.097	61.301	120.493	199.429	298.113
-1.1	22.069	61.264	120.453	199.387	298.069
-1.2	22.041	61.227	120.412	199.344	298.025
-1.3	22.012	61.189	120.370	199.301	297.979
-1.4	21.984	61.152	120.329	199.258	297.936
-1.5	21.956	61.111	120.288	199.215	297.892
-1.6	21.928	61.077	120.246	199.172	297.848
-1.7	21.900	61.036	120.206	199.129	297.803
-1.8	21.872	61.002	120.164	199.086	297.759
-1.9	21.844	60.959	120.123	199.042	297.715
-2.0	21.816	60.926	120.082	199.000	297.669
-2.1	21.788	60.886	120.041	198.956	297.625
-2.2	21.760	60.850	119.999	198.912	297.582
-2.3	21.730	60.809	119.959	198.870	297.538
-2.4	21.702	60.775	119.917	198.826	297.493
-2.5	21.674	60.737	119.876	198.783	297.448
-2.6	21.645	60.695	119.835	198.740	297.404
-2.7	21.617	60.661	119.794	198.697	297.360
-2.8	21.589	60.622	119.752	198.654	297.315
-2.9	21.560	60.584	119.710	198.610	297.271
-3.0	21.531	60.543	119.669	198.568	297.226
-3.1	21.502	60.506	119.628	198.524	297.181
-3.2	21.474	60.470	119.587	198.482	297.138
-3.3	21.445	60.428	119.546	198.438	297.093
-3.4	21.417	60.394	119.504	198.395	297.049
-3.5	21.387	60.355	119.463	198.352	297.004
-3.6	21.358	60.314	119.422	198.308	296.959
-3.7	21.329	60.280	119.380	198.265	296.915
-3.8	21.301	60.238	119.339	198.221	296.870
-3.9	21.271	60.204	119.297	198.178	296.826
-4.0	21.242	60.163	119.255	198.135	296.782
-4.1	21.213	60.125	119.214	198.092	296.737
-4.2	21.184	60.082	119.173	198.049	296.693
-4.3	21.154	60.044	119.131	198.005	296.648
-4.4	21.125	60.005	119.090	197.962	296.604
-4.5	21.096	59.968	119.048	197.918	296.559
-4.6	21.066	59.929	119.007	197.875	296.515
-4.7	21.037	59.891	118.965	197.832	296.470
-4.8	21.008	59.852	118.924	197.789	296.426
-4.9	20.979	59.813	118.882	197.744	296.382
-5.0	20.949	59.778	118.839	197.702	296.337

Table 3. Frequency Factors of a Single-span Beam Clamped at Both Ends
(Contd.)

r	λ_1	λ_2	λ_3	λ_4	λ_5
-5.0	20.949	59.778	118.839	197.702	296.337
-5.1	20.920	59.736	118.799	197.658	296.292
-5.2	20.889	59.695	118.757	197.615	296.248
-5.3	20.860	59.653	118.714	197.572	296.203
-5.4	20.831	59.614	118.673	197.528	296.159
-5.5	20.800	59.581	118.631	197.485	296.114
-5.6	20.771	59.541	118.590	197.440	296.069
-5.7	20.740	59.501	118.548	197.398	296.024
-5.8	20.711	59.460	118.506	197.354	295.980
-5.9	20.681	59.422	118.464	197.310	295.935
-6.0	20.651	59.383	118.423	197.268	295.891
-6.1	20.621	59.347	118.381	197.223	295.846
-6.2	20.592	59.309	118.339	197.180	295.801
-6.3	20.561	59.265	118.297	197.137	295.757
-6.4	20.531	59.227	118.256	197.093	295.711
-6.5	20.500	59.190	118.214	197.050	295.668
-6.6	20.471	59.153	118.172	197.006	295.624
-6.7	20.440	59.110	118.130	196.962	295.578
-6.8	20.410	59.075	118.087	196.919	295.534
-6.9	20.379	59.033	118.047	196.875	295.489
-7.0	20.349	58.997	118.004	196.832	295.444
-7.1	20.318	58.957	117.962	196.788	295.399
-7.2	20.288	58.918	117.920	196.745	295.355
-7.3	20.257	58.877	117.878	196.701	295.310
-7.4	20.227	58.838	117.836	196.657	295.266
-7.5	20.195	58.801	117.794	196.613	295.221
-7.6	20.164	58.758	117.752	196.570	295.176
-7.7	20.134	58.718	117.710	196.526	295.131
-7.8	20.103	58.679	117.668	196.483	295.086
-7.9	20.072	58.641	117.627	196.438	295.042
-8.0	20.041	58.602	117.584	196.395	294.998
-8.1	20.009	58.565	117.542	196.351	294.953
-8.2	19.979	58.524	117.500	196.308	294.907
-8.3	19.948	58.486	117.458	196.264	294.863
-8.4	19.917	58.444	117.416	196.220	294.818
-8.5	19.886	58.406	117.373	196.177	294.773
-8.6	19.854	58.367	117.331	196.132	294.728
-8.7	19.823	58.326	117.289	196.089	294.684
-8.8	19.791	58.288	117.247	196.045	294.640
-8.9	19.760	58.248	117.205	196.002	294.594
-9.0	19.729	58.204	117.163	195.957	294.549
-9.1	19.697	58.169	117.120	195.914	294.505
-9.2	19.665	58.126	117.079	195.870	294.459
-9.3	19.633	58.088	117.035	195.827	294.414
-9.4	19.602	58.050	116.994	195.782	294.370
-9.5	19.570	58.007	116.951	195.739	294.325
-9.6	19.538	57.971	116.908	195.695	294.280
-9.7	19.506	57.926	116.866	195.651	294.236
-9.8	19.475	57.889	116.824	195.607	294.190
-9.9	19.442	57.851	116.781	195.563	294.145
-10.0	19.410	57.808	116.739	195.520	294.101

Table 4. Data for Sample Problem

Tube

Outside Diameter (d_o) 0.5 in.

Inside Diameter (d_i) 0.4 in.

Subtended Angle (α) 40°

Radius of Curvature (R) 100 in.

Material: stainless steel

Density (ρ_t) $74.53 \times 10^{-5} \text{ lb-sec}^2/\text{in.}^4$

Young's Modulus (E) $28 \times 10^6 \text{ lb/in.}^2$

Poisson's Ratio (ν) 0.3

Coolant

Water

Density (ρ_f) $9.35 \times 10^{-5} \text{ lb-sec}^2/\text{in.}^4$

Flow Velocity (U) 20 ft/sec

Fig. 1. Schematic of a continuous beam with intermediate supports

Fig. 2. Propagation constant for a periodically-supported beam; $\Gamma = 0$

Fig. 3. p - λ curves for straight beams; first and second propagation bands

Fig. 4. p - λ curves for straight beams; third propagation band

Fig. 5. p - λ curves for straight beams; fourth propagation band

Fig. 6. p - λ curves for straight beams; fifth propagation band

Fig. 7. Frequency factors of a four-span beam hinged at the two extreme ends; $\Gamma = 2.0$

Fig. 8. Frequency factors of in-plane vibration of hinged-hinged beams

Fig. 9. Frequency factors of in-plane vibration of hinged-clamped beams

Fig. 10. Frequency factors of in-plane vibration of clamped-clamped beams

Fig. 11. Frequency factors of out-of-plane vibration of hinged-hinged beams;
 $\nu = 0.3$

Fig. 12. Frequency factors of out-of-plane vibration of hinged-clamped beams;
 $\nu = 0.3$

Fig. 13. Frequency factors of out-of-plane vibration of clamped-clamped beams;
 $\nu = 0.3$

Fig. 14. p - λ curves for in-plane vibration of curved beams ($\alpha = 20^\circ$ to 120°); first and second propagation bands

Fig. 15. p - λ curves for in-plane vibration of curved beams ($\alpha = 20^\circ$ to 120°); third propagation band

Fig. 16. p - λ curves for in-plane vibration of curved beams ($\alpha = 130^\circ$ to 240°); first and second propagation bands

Fig. 17. p - λ curves for in-plane vibration of curved beams ($\alpha = 130^\circ$ to 240°); third propagation band

Fig. 18. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 20^\circ$ to 120°); first and second propagation bands

Fig. 19. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 20^\circ$ to 120°); third propagation band

Fig. 20. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 20^\circ$ to 120°); fourth propagation band

Fig. 21. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 140^\circ$ to 240°); first and second propagation bands

Fig. 22. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 140^\circ$ to 240°); third propagation band

Fig. 23. p - λ curves for out-of-plane vibration of curved beams ($\alpha = 140^\circ$ to 240°); fourth propagation band

Fig. 24. Schematic of four-span curved tube conveying fluid and submerged in fluid

Fig. 25. Frequency factors for out-of-plane vibration of a four-span curved tube