ANALYSIS OF NEUTRON FLUX
IN THE SHIELDING OF THE
SODIUM REACTOR EXPERIMENT

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ABSTRACT

The development of a matrix method of solving multigroup diffusion equations in nonmultiplying regions is described. The method is applied to a three-region shielding problem, and comparison is made with experimental results. Equations obtained by this technique can be solved with a desk calculator.

I. INTRODUCTION

The problem under consideration is the calculation of flux levels in the thermal shield and concrete shield of the Sodium Reactor Experiment. This information was needed for reactor instrumentation and shield design purposes. At the time these calculations were started, multigroup machine codes were not available for use. Hence, an approximate method of obtaining a sufficiently reliable flux calculation in the shield was developed. Six-group diffusion theory fluxes were calculated by a method which considers two materials at a time. The solution proceeds stepwise through the shield, one region at a time, using the currents calculated for the interface between two materials in one problem, as a boundary condition for the first region of the next problem.

The derivation of the calculational method used is presented, along with the data used in the calculations. A comparison of the experimentally obtained thermal flux with the calculated flux is shown. The general applicability of this method to reactor problems is discussed.

II. DESCRIPTION OF THE SRE RADIAL SHIELDING

The SRE core is 6 ft high by approximately 6 ft in diameter, and is surrounded by a graphite reflector approximately 2 ft thick. A drawing of the SRE and its shielding arrangement is shown in Fig. 1. The reflector is surrounded radially by an inner liner of 1/4-in. thick stainless steel and an inert sodium blanket 2-1/2 in. thick. The sodium blanket is contained in a 1-1/2-in. thick core tank of type 304 stainless steel. A 1/4-in. thick outer tank of low alloy steel 12-1/2 ft in diameter contains the thermal shield and a 2-in. gap of helium at 3 psig. The thermal shield is composed of low carbon steel blocks 5-1/2 in. thick.

The outer tank is surrounded by one foot of insulation block consisting of calcined diatomaceous silica and asbestos, coated with sodium silicate to decrease dusting. The blocks are contained in the cavity liner, a 1/4-in. thick steel tank. Extending radially outwork from the cavity liner is a final 3-ft thick shield of ordinary reinforced concrete.

(A)

Fig. 1. Sodium Reactor Experiment

III. DERIVATION OF THE FORMULAS

The multigroup diffusion theory equations are solved by a matrix method.^{1,2,3} For nonmultiplying regions and for slab geometry, the computation of the resulting matrices can be arranged so that only diagonal and triangular matrices are involved. The eigenvalues and eigenfunctions of the problem are then easily found, the computations being suitable for solution by a desk calculator.

In view of the type of problem to be solved, certain simplifying assumptions may be made. They are:

- a) No extraneous neutron source exists.
- b) Only nonmultiplying regions are considered.
- c) Inelastic scattering is neglected.
- d) Averaged cross sections are taken to be constant over each group and in each region.
- e) Absorption cross sections have $1/v$ dependence.

The multigroup diffusion equation for the i^{th} group can now be written in the following form:

$$D_i \nabla^2 \phi_i - \Sigma_i \phi_i + p_{i-1} \Sigma_{i-1} \phi_{i-1} = 0 \quad \dots (1)$$

$$i = 1, 2, \dots, n,$$

where

n = the number of groups,

$$\Sigma_i = \Sigma_{a_i} + \Sigma_{sl_i},$$

$$\Sigma_{sl_i} = \xi \Sigma_{s_i} / \ln(E_1/E_2),$$

$$D_i = 1/3 \Sigma_{i(\text{total})} (1 - \bar{\mu}) \left[1 - 4/5 \frac{\Sigma_i}{\Sigma_{i(\text{total})}} \right], \text{ and}$$

$$\Sigma_{i(\text{total})} = \Sigma_{a_i} + \Sigma_{s_i} + \Sigma_{sl_i}$$

$$p_i = \exp \left(- \int_{E_2}^{E_1} \frac{\sigma_a}{\xi \sigma_s + \gamma \sigma_a} \frac{dE}{E} \right)$$

where the i^{th} group is bounded by the energies E_1 and E_2 .

The system defined by eq. (1) now is reduced to a first order system by making the following substitutions:

$$y_i = \phi_i$$

$$y_{n+i} = D_i \phi_i'$$

where

$$i = 1, \dots, n,$$

then

$$y_i' = D_i^{-1} y_{n+i} \quad \dots (2)$$

The diffusion equation for the i^{th} group may be written as

$$y_{n+i}' + -p_{i-1} \sum_{i-1} y_{i-1} + \sum_i y_i \quad \dots (3)$$

where

$$i = 1 \dots n.$$

The system of equations given by (2) and (3) is written in the matrix notation,

$$y' = My \quad \dots (4)$$

where y is the one-column matrix $y_1 \dots y_{2n}$, which are functions of x , and where M is the $2n \times 2n$ matrix of constants:

$$M = \begin{pmatrix} 0 & . & . & . & . & . & . & . & . & 0 & D_1^{-1} & 0 & . & . & . & . & . & . & 0 \\ 0 & . & . & . & . & . & . & . & . & 0 & 0 & D_2^{-1} & 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . & . & 0 & 0 & . & . & . & . & . & . & 0 & D_n^{-1} \\ \Sigma_1, & 0 & . & . & . & . & . & . & . & 0 & 0 & . & . & . & . & . & . & . & 0 \\ -p_1 \Sigma_1, \Sigma_2 & 0 & . & . & . & . & . & . & . & 0 & 0 & . & . & . & . & . & . & . & 0 \\ 0 & -p_2 \Sigma_2, \Sigma_3 & 0 & 0 & . & . & . & . & . & . & 0 & . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & . & 0, -p_{n-1} \Sigma_{n-1}, \Sigma_n & . & . & . & . & . & . & 0 & . & . & . & . & . & . & . & 0 \end{pmatrix}$$

Equation (4) can be solved by looking for a set of scalar functions which satisfy eq. (4) and have the property

$$\eta' = \lambda \eta \quad \dots (5)$$

where

$$\lambda = \text{a constant scalar.}$$

Substitution of this into eq. (4) yields

$$\lambda I \eta = M \eta$$

where I is the identity matrix. This results in the characteristic equation

$$\left| M - \lambda I \right| = 0$$

It will be assumed that all 2n roots of this equation are distinct, since the problem can always be formulated in this manner. To each value of λ there

corresponds a function η given by eq. (5) as

$$\eta_i = e^{-\lambda_i x},$$

where

$$i = 1 \dots 2n.$$

The solution of eq. (4) can now be expressed as linear combinations of this linearly independent set of $2n$ functions. In matrix form this is represented by

$$y = S\eta; \quad \eta = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_{2n} \end{pmatrix} \quad \dots (6)$$

where S is a constant matrix, and y and η are linear sets.

The $4n^2$ elements of S cannot all be arbitrary, since the general solution of eq. (4) can contain only $2n$ arbitrary constants. The relationship among the elements of S is determined by imposing the conditions that eq. (6) must satisfy eq. (4):

$$y' = S\Lambda\eta = MS\eta,$$

where

$$\Lambda = \begin{pmatrix} \lambda_1 & & & 0 \\ & \ddots & & \\ 0 & & \ddots & \\ & & & \lambda_{2n} \end{pmatrix}$$

Since η is a linear set, this requires that

$$MS = S\Lambda, \text{ or} \\ S^{-1}MS = \Lambda$$

Therefore, finding S is equivalent to diagonalizing M . The coupling among the multigroup equations allows the determination of the $2n$ elements of S .

It will be shown presently that for the nonmultiplying problem being considered, the roots λ_i of the characteristic equation can be easily found. If Λ is known, S can be found by solving the system of linear equations which is obtained for each root λ_i of the characteristic equation. One of the resulting elements S_{ij} so determined is arbitrary. We will assign the value unity to S_{1j} , thereby obtaining a particular solution of eq. (4). The remaining elements of S have mutual relationships required by eq. (4).

The general solution can then be written in the following form:

$$y = SYa \quad \dots (7)$$

where $\underline{a}$ is a constant vector to be determined by the boundary conditions, and Y is the diagonal matrix

$$\begin{pmatrix} \eta_1 & & 0 \\ & \ddots & \\ 0 & & \eta_{2n} \end{pmatrix}$$

The inverse of S will be required, and it can be found by solving the equation:

$$S^{-1}M = \Lambda S^{-1}$$

To find the roots λ_i of the characteristic equation, M is partitioned into four $n \times n$ submatrices as follows:

$$M = \begin{pmatrix} 0 & D^{-1} \\ M_{21} & 0 \end{pmatrix}$$

$$M - \lambda I = \begin{pmatrix} 0 & D^{-1} \\ M_{21} & 0 \end{pmatrix} - \begin{pmatrix} \lambda I & 0 \\ 0 & \lambda I \end{pmatrix} = \begin{pmatrix} -\lambda I & D^{-1} \\ M_{21} & -\lambda I \end{pmatrix}$$

Hence,

$$\begin{aligned} \left| M - \lambda I \right| &= \begin{vmatrix} -\lambda I & D^{-1} \\ M_{21} & -\lambda I \end{vmatrix} = - \left| M_{21} D^{-1} - \lambda^2 I \right| \\ &= - (\mathcal{H}_1^2 - \lambda^2) (\mathcal{H}_2^2 - \lambda^2) \dots (\mathcal{H}_n^2 - \lambda^2) = 0 \end{aligned}$$

where

$$\mathcal{H}_i^2 = \frac{\sum_i}{D_i}$$

The roots of this equation are therefore

$$\lambda_i = \mathcal{H}_i$$

$$\lambda_{i+n} = -\mathcal{H}_i$$

where

$$i = 1 \dots n.$$

Thus the diagonal matrix Y can now be written

$$Y(x) = \begin{pmatrix} \eta_1 & 0 \\ \cdot & \cdot \\ 0 & \eta_{2n} \end{pmatrix} = \begin{pmatrix} \lambda_{1x} & 0 \\ e & \cdot \\ \cdot & \cdot \\ 0 & e \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1^x & & 0 \\ e & \cdot & \mathcal{H}_n^x \\ & e & \cdot \\ 0 & & e \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1^x & & 0 \\ e & \cdot & \mathcal{H}_n^x \\ & e & \cdot \\ 0 & & e \end{pmatrix}$$

(11)

The matrix S and its inverse can now be found in terms of the properties of the material. The results are:

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \quad S^{-1} = \begin{pmatrix} (S^{-1})_{11} & (S^{-1})_{12} \\ (S^{-1})_{21} & (S^{-1})_{22} \end{pmatrix}$$

where

$$S_{11} = S_{12} = D^{-1}PD$$

$$S_{21} = -S_{22} = PD\mathcal{K}$$

$$(S^{-1})_{11} = (S^{-1})_{21} = 1/2D^{-1}P^{-1}D = 1/2S_{11}^{-1}$$

$$(S^{-1})_{12} = -(S^{-1})_{22} = -1/2\mathcal{K}^{-1}D^{-1}P^{-1} = -1/2S_{21}^{-1}$$

and where

$$D = \begin{pmatrix} D_1 & & 0 \\ & \ddots & \\ 0 & & D_n \end{pmatrix} \quad \mathcal{K} = \begin{pmatrix} \mathcal{K}_1 & & 0 \\ & \ddots & \\ 0 & & \mathcal{K}_n \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & 0 & & 0 & & \dots & 0 \\ A_{21}^1 & & 1 & & 0 & & \dots & 0 \\ A_{21}^1 A_{31}^2 & & A_{32}^2 & & 1 & & \dots & 0 \\ A_{21}^1 A_{31}^2 A_{41}^3 & & A_{32}^2 A_{42}^3 & & A_{43}^3 & & \dots & 0 \\ \vdots & & & & & & \dots & \vdots \\ \dots & & \dots & & \dots & & \dots & 1 \ 0 \\ \vdots & & & & & & \dots & \vdots \\ A_{21}^1 \dots A_{n1}^{n-1} & & A_{32}^2 \dots A_{n2}^{n-1} & & A_{43}^3 \dots A_{n3}^{n-1} & & \dots & A_{n,n-1}^{n-1} \ 1 \end{pmatrix}$$

and where

$$A_{\beta\gamma}^a = \frac{P_a \mathcal{H}_a^2}{\mathcal{H}_\beta^2 - \mathcal{H}_\gamma^2} \quad a, \beta, \gamma = 1, \dots, n$$

The boundary conditions for various types of problems will now be considered. Three cases which are of use in the applications are considered below.

It will be convenient to partition Y and the vector $\underline{a}$ as follows:

$$Y = \begin{pmatrix} Y_1 & 0 \\ 0 & Y_2 \end{pmatrix} \quad a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

where

$$Y_1 = \begin{pmatrix} \mathcal{H}_{1x} & & 0 \\ e & \ddots & \\ & \ddots & \mathcal{H}_{nx} \\ 0 & & e \end{pmatrix} \quad a_1 = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

$$Y_2 = \begin{pmatrix} -\mathcal{H}_{1x} & & 0 \\ e & \ddots & \\ & \ddots & -\mathcal{H}_{nx} \\ 0 & & e \end{pmatrix} \quad a_2 = \begin{pmatrix} a_{n+1} \\ \vdots \\ a_{2n} \end{pmatrix}$$

A. CASE ONE, FOR ONE REGION

Consider a single homogeneous region of thickness x_1 bounded by $x = 0$ and $x = x_1$. (This includes the extrapolation distance, which is assumed to be the same for all groups.) Take the boundary conditions to be as follows:

- 1) The neutron current is specified at $x = 0$.
- 2) The flux vanishes at x_1 .

Applying these conditions to eq. (7) and recalling that $y_{n+i} = -J_i$, we have the resultant equations:

$$\begin{pmatrix} \alpha \\ -J \end{pmatrix} = S Y(0) a$$

$$\begin{pmatrix} 0 \\ \beta \end{pmatrix} = S Y(x_1) a$$

where α and β are not specified. From these we obtain, since $Y(0) = I$,

$$\begin{cases} -J = S_{21} a_1 + S_{22} a_2 \\ 0 = S_{11} Y_1(x_1) a_1 + S_{12} Y_2(x_1) a_2 \end{cases}$$

These matrix equations are to be solved for the n-vectors a_1 and a_2 .

$$\begin{aligned} a_1 &= -Y_1^{-1}(x_1) S_{11}^{-1} S_{12} Y_2(x_1) a_2 = -Y_2(2x_1) a_2 \\ a_2 &= \left[I + Y_2(2x_1) \right]^{-1} \mathcal{K}^{-1} D^{-1} P^{-1} J \end{aligned}$$

The flux and current are now given by

$$\left. \begin{aligned} \phi(x) &= D^{-1} P D \left[Y_1(x) a_1 + Y_2(x) a_2 \right] \\ J(x) &= -P D \mathcal{K} \left[Y_1(x) a_1 - Y_2(x) a_2 \right] \end{aligned} \right\} \quad 0 \leq x \leq x_1 \quad \dots (8)$$

B. CASE TWO, FOR TWO REGIONS

Next consider the two-region problem shown in Fig. 2. Region one here is of thickness x_1 and region two is of infinite thickness. Take the boundary conditions to be as follows:

- 1) The flux is known at $x = 0$.
- 2) The flux and current are continuous at x_1 .
- 3) The flux vanishes at infinity.

Fig. 2. Two-Region Boundary Value Problem

In region one we will use the notation $D_I, \mathcal{L}_I, S, P, Y$ and $\underline{a}$, as in Case One. Then the solution in region one has the form

$$y(x) = S Y(x) \underline{a}; \quad 0 \leq x \leq x_1. \quad \dots (9a)$$

In region two, the corresponding quantities will be denoted by $D_{II}, \mathcal{L}_{II}, T, Q, Z$ and $\underline{b}$. Then the solution in region two has the form

$$z(x) = T Z(x) \underline{b}; \quad 0 \leq x \leq \infty \quad \dots (9b)$$

For the case of slab geometry, the coordinate x in the region two solution may be chosen such that $x = 0$ at the interface between regions one and two. This can be done because of the exponential form of the solution in region two.

Applying these boundary conditions to eq. (9) gives the equations

$$\left. \begin{aligned} \begin{pmatrix} \phi(0) \\ \underline{a} \end{pmatrix} &= S Y(0) \underline{a} \\ S Y(x_1) \underline{a} &= T Z(0) \underline{b} \\ \begin{pmatrix} 0 \\ \underline{\beta} \end{pmatrix} &= T Z(\infty) \underline{b} \end{aligned} \right\} \quad \dots (10)$$

We eliminate a from the first two equations, then solve the remaining equations for b. The vanishing of the flux at infinity requires that $b_1 = 0$, and there results after much manipulation:

$$b_2 = \left[\frac{Y_1(x_1) + Y_2(x_1)}{2} A + \frac{Y_1(x_1) - Y_2(x_1)}{2} B \right]^{-1} P^{-1} D_I \phi(0)$$

where

$$A = P^{-1} D_I D_{II}^{-1} Q D_{II}$$

$$B = \mathcal{Y}_I^{-1} P^{-1} Q D_{II} \mathcal{Y}_{II}$$

We then find the vector a from eq. (10).

$$\begin{aligned} a_1 &= 1/2 Y_2(x_1) D_I^{-1} [A - B] b_2 \\ a_2 &= 1/2 Y_1(x_1) D_I^{-1} [A + B] b_2 \end{aligned} \quad \dots (11)$$

The fluxes and currents are given by

$$\begin{aligned} \phi_I(x) &= D_I^{-1} P D_I \left[Y_1(x) a_1 + Y_2(x) a_2 \right] & 0 \leq x \leq x_1 \\ \phi_{II}(x) &= D_{II}^{-1} Q D_{II} Z_2(x) b_2 & 0 \leq x \leq \infty \\ J_I(x) &= -P D_I \mathcal{Y}_I \left[Y_1(x) a_1 - Y_2(x) a_2 \right] & 0 \leq x \leq x_1 \\ J_{II}(x) &= Q D_{II} \mathcal{Y}_{II} Z_2(x) b_2 & 0 \leq x \leq \infty \end{aligned} \quad \dots (12)$$

In these equations, x in region two is measured from the boundary between regions one and two.

C. CASE THREE, FOR TWO REGIONS

Finally, consider the two-region problem shown in Fig. 2, and having the following boundary condition:

- 1) The current is known at $x = 0$.
- 2) The flux and current are continuous at x_1 .
- 3) The flux vanishes at infinity.

The solutions have the same form as those for Case Two, using the same notation as in Case Two. Applying the boundary conditions to eq. (9) now results in

$$\left. \begin{aligned} \begin{pmatrix} a \\ -J(0) \end{pmatrix} &= S Y(0) a \\ S Y(x_1) a &= T Z(0) b \\ \begin{pmatrix} 0 \\ \beta \end{pmatrix} &= T Z(\infty) b \end{aligned} \right\} \dots (13)$$

We again eliminate $\underline{a}$ and solve for $\underline{b}$, obtaining

$$b_2 = \left[\frac{Y_1(x_1) - Y_2(x_1)}{2} A + \frac{Y_1(x_1) + Y_2(x_1)}{2} B \right]^{-1} \mathcal{H}_I^{-1} P^{-1} J(0)$$

A and B are the same as in Case Two, and from eq. (13) we find the same expressions for a_1 and a_2 , namely eq. (11). The fluxes and currents are again given by eq. (12).

IV. SHIELDING CALCULATIONS

The application of the foregoing to a shielding calculation consists of solving the diffusion equations for two regions at a time, and proceeding stepwise through the shielding materials. The equations are solved for slab geometry. An approximate solution for the radial problem is then obtained by multiplying the slab geometry result by $\sqrt{R_0/r}$, where R_0 is the core radius. This is the value approached asymptotically by the ratio of the solutions of the diffusion equation for slab and cylindrical geometry. Six groups were chosen in order to investigate the effect of the important energy-dependent iron cross-section; and so that the approximately exponential attenuation of fast neutrons in the thermal shield could be represented more accurately than in a two-group representation.

A detailed description of the method of obtaining an approximate solution for a multiregion problem is now given. The solution proceeds in a stepwise manner through the shield, considering two regions at a time as described previously. The first problem solved consists of the two-group core-reflector criticality equation, which then allows us to determine the thermal and fast flux at the core-reflector boundary. It is then assumed that the energy distribution of the fast flux is $1/E$. The multigroup fluxes are calculated for the fast energy groups according to this distribution with the condition that their sum be the value of the fast flux previously obtained in the two-group calculation.

The next two-region problem concerns the reflector and the material adjacent to and outside the reflector, which for the top shield is sodium and for the radial shield is iron. (The small sodium and void regions were not considered, since their effect on the neutron fluxes is slight.) The thickness of the first region (the reflector in this case) is taken to be that actually occurring in the reactor. The second region is assumed to be of infinite thickness. The boundary condition at the inside of the reflector must be specified. It is taken to be the flux at the core-reflector interface, which was calculated in the first problem. The other boundary conditions are: 1) the continuity of flux and current at the interface between the two regions, and 2) the vanishing of the flux at infinity. This two-region problem is then solved and the current at the interface between the two regions is found. This current is used as the boundary condition at the entrance to the first region of the next two-region problem.

The material of the second region of the preceding two-region problem becomes the first region of the next two-region problem. Here the material has the thickness actually occurring in the reactor structure. The second region of the new problem is infinite, and consists of the next occurring material. This manner of treatment can be justified on the assumption that the alteration of shield materials far enough from the interface being studied does not appreciably affect the neutron current at that interface.

This process continues until the last region is reached. Here a one-region problem with the flux vanishing at the extrapolated outer surface of the shield is solved. The boundary condition at the entrance to this region is the current found at the interface in the preceding problem.

V. RESULTS

Calculations were made for a somewhat simplified model of the SRE shield, from which the thin sodium region and the weakly absorbing gas and insulation block regions were omitted. The steel regions were consolidated into a single region whose thickness was the sum of the thicknesses of the regions being consolidated. The data used in making the calculation are given in Table I. The resulting six-group fluxes are shown in Fig. 3 and 4.

When the results of several two-region calculations are combined to give an approximate solution to a many-region problem, small flux discontinuities occur at each material boundary in the calculated fast groups. The reason for this is that continuity of current at all boundaries was the condition used to obtain the solution of the many-region problem. The added requirement of flux continuity across interfaces would have overdetermined the problem.

In order to plot these results in such a manner that they will more closely correspond to the actual SRE shield structure, the omitted regions are reinserted in the radial dimensions. The calculated flux is plotted across those regions for which calculations were done, and connected across the thin uncalculated regions by a simple straight-line extrapolation. The flux across the small uncalculated regions is corrected for cylindrical geometry in the radial case by applying the correction factor (R_i/R_o) to the flux at R_o , where R_i is the inner radius and R_o the outer radius of the region under consideration. The results are plotted in Fig. 5 and 6. The calculated thermal fluxes are compared to the experimental thermal fluxes which are indicated by the circled points on the graphs. The experimental values were obtained, some time after the reactor went into operation, by gold foil exposure and fission chamber techniques.⁴

TABLE I
SIX-GROUP CONSTANTS FOR VARIOUS MATERIALS

Sodium						Iron				
Group	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_a(\text{cm}^{-1})$	$\Sigma_r(\text{cm}^{-1})$	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_a(\text{cm}^{-1})$	$\Sigma_r(\text{cm}^{-1})$
1	0.0003869	0.9994	3.453	0.000003	0.00133	0.0014505	0.9921	1.156	0.00005	0.00168
2	0.0027777	0.9996	2.955	0.000013	0.00821	0.0070069	0.9928	1.259	0.00024	0.00882
3	0.0013109	0.9980	3.071	0.000030	0.00403	0.0164286	0.9833	0.577	0.00054	0.00948
4	0.0028080	0.9563	5.180	0.000142	0.00135	0.0253842	0.8660	0.375	0.00260	0.00951
5	0.0005671	0.6433	4.694	0.001452	0.00266	0.0925690	0.2154	0.366	0.02727	0.03387
6	0.0013580	-	4.390	0.005962	0.00596	0.3290051	-	0.359	0.118277	0.11823
T = 468° C E _{th} = 0.082 ev ρ = 0.84 gm/cm ³						T = 400° C E _{th} = 0.074 ev ρ = 7.74 gm/cm ³				

Magnetite Concrete						Ordinary Concrete				
Group	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_a(\text{cm}^{-1})$	$\Sigma_r(\text{cm}^{-1})$	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_a(\text{cm}^{-1})$	$\Sigma_r(\text{cm}^{-1})$
1	0.01776	0.9998	1.245	0.00003	0.0220	0.00923	1.00	0.539	0.000005	0.0141
2	0.2740	0.9999	1.152	0.00014	0.2537	0.12555	1.00	1.240	0.000023	0.1557
3	0.1665	0.9996	0.772	0.00032	0.1285	0.05883	1.00	1.324	0.000052	0.0779
4	0.1026	0.9955	0.646	0.00153	0.0664	0.03239	0.999	1.215	0.000250	0.0393
5	0.1189	0.9533	0.620	0.01786	0.0737	0.03181	0.987	1.149	0.002911	0.0366
6	0.0814	-	0.600	0.04879	0.0488	0.00639	-	1.156	0.007387	0.00739
T = 70° C E _{th} = 0.038 ev ρ = 3.60 gm/cm ²						T = 70° C E _{th} = 0.038 ev ρ = 2.32 gm/cm ³				

Pure Graphite					Axial Reflector				Radial Reflector			
Group	$\Sigma_r(\text{cm}^{-1})$	$\kappa^2(\text{cm}^{-2})$	D(cm)	p	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_r(\text{cm}^{-1})$	$\kappa^2(\text{cm}^{-2})$	p	D(cm)	$\Sigma_r(\text{cm}^{-1})$
1	0.00788	0.00684	1.153	1.00	0.006554	1.00	1.153	0.00755	0.007043	1.00	1.153	0.00812
2	0.05490	0.05917	0.928	1.00	0.056680	1.00	0.928	0.05260	0.06094	1.00	0.928	0.05654
3	0.02620	0.02823	0.928	1.00	0.027050	1.00	0.928	0.02510	0.02908	1.00	0.928	0.02699
4	0.01310	0.01412	0.928	1.00	0.013530	1.00	0.928	0.01255	0.01454	1.00	0.928	0.01349
5	0.01274	0.01372	0.928	0.998	0.013530	0.99	0.928	0.01210	0.01413	0.997	0.928	0.01312
6	0.0002135	0.0002334	0.914	-	0.000800	-	0.914	0.000731	0.0003666	-	0.914	0.000335
ρ = 1.635 gm/cm ³ T = 20° C					T = 400° C E _{th} = 0.074 ev τ = 357 cm ² L ² = 1250 cm ²				T = 468° C E _{th} = 0.082 ev τ = 322 cm ² L ² = 2728 cm ²

Fig. 3. Calculated Radial Flux Normalized to Unit Thermal Flux at Center of Core

Fig. 4. Calculated Axial Flux Above Reactor Normalized to Unit Thermal Flux at Center of Core

Fig. 5. Radial Thermal Flux in SRE

Fig. 6. Axial Thermal Flux in SRE

VI. DISCUSSION

It will be noted that the calculated radial thermal flux agrees reasonably well with the experimental points except in the concrete, see Fig. 5, which turns out to be more strongly attenuating than was expected. A two-group multiregion calculation, using the Wanda IBM program, was performed after the experiments were completed. The cross-sections used were consistent with those used in the above calculations. The results showed much the same discrepancy for the flux in the concrete. Agreement with experiment was obtained only after substantially increasing the concrete cross-sections. The experimental fast relaxation length was found to be 7.1 cm, while that used in the calculation was 10.4 cm. A possible explanation for the high removal cross-section of the concrete is that, during its cure, the concrete retained a high percentage of water. This could be due to partial submergence of the reactor below the water table of the immediate area. Also, the steel reinforcement in the concrete was not accounted for in calculating the concrete cross-sections.

The axial flux is shown in Fig. 6. The results of the calculation are given by the solid lines. The thermal flux is compared with experimental measurements, which are represented by the circles. The discrepancy in the magnitude of the calculated and experimental fluxes can be attributed to two causes:

- 1) The axial reflectors were assumed, for the calculation, to be pure graphite. Actually, however, the process tubes contain considerable steel and sodium, both of which substantially increase the reflector absorption cross-section.
- 2) The experimental thimble in which the measurements were taken was surrounded by a ganged configuration of four control rods throughout the upper sodium plenum. The effect of these rods was not accounted for in the calculation.

A report⁵ by A. R. Vernon discusses more fully the work done on the top biological shield of the SRE.

The method developed in this report gives satisfactory results for shielding flux studies, provided the cross-sections employed approximate those of the actual case being studied. Obtaining multigroup cross-sections, particularly for concrete, was a problem. Machine codes now available could calculate these cross-sections, providing the exact composition of the concrete were known. It is believed that fewer energy groups, perhaps three, would yield a sufficiently accurate description of the shielding fluxes.

The calculation, though it can be made by a desk calculator, was tedious to perform. Few-group machine codes, such as Wanda, are much less time-consuming, and would now be used for problems of this type.

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