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LAWRENCE LIVERMORE LABORATORY

University of California/Livermore, California

REVIEW OF LATERAL PRESSURE CRITERIA FOR STEMMING MATERIALS

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August 18, 1972

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One of the problems in the containment and the emplacement hardware design is concerned with the transverse pressure of stemming material on a vertically oriented buried pipe. This parameter may be of importance in routine calculations as well as computer solutions aimed at more precise estimates of axial pipe response in a dynamic environment.

The problem of lateral pressure in underground tests is not a new one and the only reason for compiling this report is to present a critical review of the state-of-the-art knowledge in this particular area. The existing design rules state that the pressure on the pipe wall or in a silo is compatible with the weight of a column of granular material not exceeding five to ten pipe diameters in length, depending on the unit weight and other variables. Quite often the limits are specified as two to five diameters for pipe stemming in underground tests. These limits are not particularly rigid and are subject to the usual degree of engineering judgment.

The theoretical analysis of the problem is normally based on a free-body diagram such as that shown in Fig. 1. It is customary in the problems of this nature to follow the fundamental approach of soil engineering (Ref. 1). The selection of representative variables for developing the equilibrium criteria is guided by the relevant principles of soil mechanics (Ref. 2).

The following variables are identified in the derivation of basic formulas and the subsequent general discussion:

P = Pressure due to the column of stemming material, psi.

γ = Unit weight of stemming, lb./in.³

d = Outer diameter of pipe, in.

D = Inner diameter of well casing, in.

x = Distance from surface of fill to the point of reference, in.

L = Length of fill, in.

μ = Coefficient of friction between pipe wall and stemming.

K_0 = Coefficient of lateral pressure at rest.

$k = \frac{D}{d}$ Ratio of well casing to pipe diameter

θ = Angle of intergranular friction, degs.

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Utilizing the sign convention adopted in Fig. 1 the following equilibrium of forces can be postulated:

$$\frac{(P + dP) \pi (D^2 - d^2)}{4} + PK_0 \mu \pi (D + d) dx - \frac{P \pi (D^2 - d^2)}{4} - \frac{\gamma \pi (D^2 - d^2) dx}{4} = 0 \quad \dots \dots (1)$$

Simplifying and rearranging various terms, gives:

$$\frac{dP}{dx} + \lambda P = \gamma \quad \dots \dots (2)$$

The solution of Eq. (2), is:

$$P = \frac{\gamma}{\lambda} (1 - e^{-\lambda x}) \quad \dots \dots (3)$$

where

$$\lambda = \frac{4K_0 \mu}{d(k-1)} \quad \dots \dots (4)$$

Eqs. (3) and (4) are essentially the same as those obtained by Norris (Ref. 3) in the development of theory of load release, and Spangler (Ref. 1) in the study of vertical pressures on ditch conduits. The exponential function involved in Eq. (3) shows that even for moderate values of $x = L$, the pressure levels off rather quickly at a constant value equal to γ/λ . With the admission of an arbitrary error smaller than, say, one percent the exponent $\lambda L \cong 5$. Hence using Eq. (4), gives:

$$\lambda L = \frac{4K_0 \mu L}{d(k-1)} = 5 \quad \dots \dots (5)$$

The limiting theoretical ratio L/d , follows then directly from Eq. (5):

$$\frac{L}{d} = \frac{5(k-1)}{4\mu K_0} \quad \dots \dots (6)$$

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For the purpose of the calculations the limiting theoretical length of stemming fill can be assumed to be directly proportional to the hydraulic radius R_h for the annular cross section, and inversely proportional to the frictional effects. This yields:

$$L_{\max} = 5R_h / \mu K_0 \quad (7)$$

The concept of the coefficient of lateral pressure at rest K_0 , entering the above formula, is generally well recognized (Refs. 1, 2 and 4), and it is normally employed in the study of the retaining walls, conduits and similar buried structures. The coefficient defines the ratio of the horizontal stress to the axial stress at a point and it is therefore a convenient parameter in the process of setting up the equations of equilibrium for soil-structure interaction. Unfortunately its numerical values must be open to some debate since they depend on the angle of inter-granular friction (Ref. 5).

The theoretical and experimental work of Cramer (Ref. 6) and Hamilton (Ref. 7) in the past was hampered largely by the limited information on frictional effects in the open literature. Additional unknowns included the effects of arching, settling, in-situ unit weights and lateral forces. Recent increased emphasis on the theoretical and experimental investigations of stemming materials properties at the Nuclear Test Engineering Division should go a long way towards the resolution of this problem.

In terms of the angle of intergranular friction θ , Eq. (6) can be restated as follows:

$$\frac{L}{d} = \frac{5 (k-1) (1 + \sin\theta)}{4\mu (1 - \sin\theta)} \quad (8)$$

The above working formula may be recommended for practical design considerations provided the relevant bracketing values of k , μ and θ can be established on a rational basis. The maximum overburden pressure at the

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bottom of the pipe due to the stemming material alone is then:

$$P_{\max} = \frac{\gamma (k-1) d}{4K_0 \mu} \dots \dots (9)$$

The theoretical model described by Eq. (8) indicates that the design ratio L/d depends strongly on the aspect ratio of the annular enclosure and the angle of intergranular friction. This relationship is illustrated in Fig. 2 for a constant value of μ . The annular geometry and the relevant dimensions are usually dictated by the overall size and proportions of the test hardware. The dependence of the L/d ratio on the frictional characteristics however should be established experimentally. The theoretical limits of the coefficient of lateral pressure at rest, K_0 , are indicated in Fig. 3. When the angle of intergranular friction is required in the sandy and gravel like soils the angle of repose may be substituted (Refs. 2 and 4). Using this technique for instance some of the Russian Building Codes specify various practical ranges of θ shown in Fig. 3. This is probably one of the oldest soil mechanics procedures which may be satisfactory in dealing with relatively dry, granular materials under very small overburden pressures. However, significant differences in the numerical values may develop when the equilibrium pressures are increased. This seems reasonable since the angle of repose is determined by the least stable grains. The true angle of intergranular friction should increase with the overburden pressure, due to the additional grain interlocking. If cohesion is present, there may be an additional discrepancy between the friction values determined by the angle of repose technique and other experimental methods.

One of the best practical methods for finding the true angle of intergranular friction is based on the classical approach utilizing the Mohr's envelope of failure (Ref. 5). The peak-point-friction angle, however, further depends on the void ratio and the overburden pressure, making the tests mandatory for each new material. Fortunately for the soil engineers and designers the tests are relatively inexpensive and the differences between the angles of friction are not too large when determined by the various types

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of test apparatus. It is also well to bear in mind that uniform granular materials tend to have smaller friction angles than those with a wide range of particle size. This finding seems to be also confirmed by our own research experience at the Nuclear Test Engineering Division.

When water-saturated sands are tested (Ref. 2), the angle of intergranular friction appears to differ only slightly from that in a perfectly dry granular material provided the rate of loading is slow. This is based on the premise that slow rate of loading allows water to escape as the void changes take place. The experience here is contrary to an old belief that water acts as a lubricant. Where other dynamic phenomena are involved the experience with soil testing indicates that the difference between the static and dynamic friction angles is relatively small and of no real importance.

It may be well to reemphasize that two types of friction are involved in the model described by Eq. (8). The design chart given in Fig. 2 is only good for the case of Overton sand, with the coefficient of friction between the pipe wall and the sand equal to about 0.35. The maximum observed values of μ in the past tests on Overton sand and pea gravel were 0.7 and 0.5 respectively (Ref. 5). The experience with pile driving in industry (Ref. 2) is not too helpful in shedding more light on the skin-type friction, represented by μ in Eq. (8). The reason for this is that piles are seldom used in dry sands and gravel-like soils. Other reason for certain lack of information in this field is that loading tests on piles and buried pipes are relatively expensive, often inconvenient and time consuming. In pile driving industry therefore, frequent use is made of the dynamic formulas despite their inherent crudeness. The theoretical limits of μ are of course zero and one. In metal to metal contact some of the static values were reported to exceed 1.0 (Ref. 8), but these cases must have been due to the local welding and shearing effects at surface irregularities. In the case of pipe surface to granular material friction interactions, it is unlikely that the values greater than unity will be encountered.

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In a directly related experiment tests were run at the Materials Division, Engineering Measurements Section, to get some idea as to the pressure transfer through a column of Overton sand. Although no detailed report on these tests is available, the method of loading and support can be illustrated schematically in Fig. 4. The results of pressure measurement are given in Fig. 5. The extrapolation of the test data to zero pressure gives $L/D = 5$, which indicates that the skin friction and the intergranular friction equilibrium are such that L/D ratio appears to be largely independent of the axial pressure exerted on the top of the stemming material. In a sense therefore the stemming material locks in a given position and behaves almost as a rigid plug. Since the test was conducted with only one level of pressure on the actuator and one type of material, a more general conclusion is not warranted at this time. The mechanism of pressure transfer illustrated in Fig. 5 however can be related to the compaction characteristics of the material and the general pattern of the frictional load transfer to the wall of the experimental canister, Fig. 4. In reporting on extensive field tests, Cramer (Ref. 6) points out that Monterey sand, for instance is sufficiently self compacting. It is easy to visualize then how Overton sand compacts under pressure and why it transfers a substantial portion of the downward load by means of the wall friction.

The general area of lateral pressure criteria may be closely related to the analysis of bins and silos (Ref. 9). The knowledge of pressure distribution in these structures is required for sizing of walls, circumferential stiffeners and support components. The more widely used expression for the calculation of the vertical pressure in silos is known as the Janssen's Formula:

$$P = \frac{\gamma R_h}{\mu K_0} (1 - e^{-\frac{K_0 \mu x}{R_h}}) \quad \dots \dots (10)$$

The corresponding lateral pressure is:

$$P_h = P K_0 \quad \dots \dots (11)$$

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In the case of a circular bin $R_h = D/4$, and again the expression given by Eq. (10) is essentially the same as that given by Eq. (3). Hence the majority of the available design formulas in industry, suitable for the calculation of lateral pressure in granular materials, appear to have been derived from the solution of one linear differential equation, such as Eq. (2). The differences are in the methods of application and are all subject to the lack of reliable material properties and experimental confirmation of the mathematical models involved. Janssen's formula was originally developed for circular bins and its application to other shapes may be in question. The main disadvantage of the majority of formulas used for the determination of the vertical and lateral pressures stems from the necessity of utilizing the somewhat controversial factor K_0 . The expression for K_0 , employed in the above formulas, is:

$$K_0 = \frac{1 - \sin\theta}{1 + \sin\theta} \quad \dots \dots \dots (12)$$

Essentially this equation was proposed by Rankine (Ref. 10) some sixty years ago. The aim at that time was to represent the pressure ratio for an incompressible granular material having zero cohesion strength. His formula ignores the effect of wall friction and therefore it should be verified experimentally. The research work in this area, involving full scale tests, has been rather limited. In the meantime however the theoretical model depicted by Eq. (12) may be considered to be reasonably applicable to dry stemming materials for lack of better data.

In summary this review indicates that a rational formulation of the problem of lateral pressure is likely to be of the following type:

$$\left(\frac{L}{d}\right)_{\max} = H_{(k)} H_{(\theta)} H_{(\mu)} \quad \dots \dots \dots (13)$$

For a given stemming material $H_{(\mu)}$ can be determined experimentally as a function of fill depth. The $H_{(\theta)}$ parameter depends on the angle of intergranular friction which may be derived from the Mohr's envelope. Finally the geometrical factor $H_{(k)}$ should be obtained on an empirical basis with

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special regard to the higher numerical values of k . It is certainly reasonable to postulate that with a relatively small diameter of inner pipe the influence of the bridging and other frictional effects can not extend too deeply into the surrounding material (Ref. 6). It is likely that the net effect of such a study will be to decrease the average slopes of the theoretical curves of constant k values below those given in Fig. 2.

It should be mentioned in conclusion that other pressure formulas available in industry such as those due to Marston (Ref. 1) or Airy (Ref. 9) can be adapted to our lateral pressure problems, provided suitable experimental verification and sound engineering assumptions are made. In view of the various limitations involved however there appears to be little if any advantage in using them over the theoretical and empirical models represented by Eqs. (8) and (13) respectively.

Experimental data is being developed on the samples of Area 1 stemming material for the direct use with the theoretical model given by Eq. (13).

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FIG.1. LATERAL PRESSURE EQUILIBRIUM MODEL

FIG. 2. THEORY OF LENGTH TO DIAMETER RATIO

SQUARE 10 X 10 TO 11.4 INCH AS-0007-C1

NOTE: THESE RANGES ARE BASED ON
 THE EXPERIMENTAL VALUES OF
 ANGLE OF REPOSE (REF. 4)

FIG. 3. COEFFICIENT OF LATERAL PRESSURE
 AT REST, K_0

FIG. 4. PRESSURE TRANSFER TEST APPARATUS

FIG. 5. RESULTS FROM PRESSURE TRANSFER
EXPERIMENTS

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