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ESTIMATION OF NEUTRON SELF ABSORPTION IN RaCO_3 SAMPLES

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ESTIMATION OF NEUTRON SELF ABSORPTION IN RaCO_3 SAMPLES

Robert Keyes

The permissible sample size in neutron irradiations in nuclear reactors is limited by self absorption in the sample. Approximate solutions for spherically and cylindrically symmetric geometries are obtained here and evaluated numerically for the case of RaCO_3 surrounded by Be. →

The following conditions are assumed:

1. Small absorption in the sample.
2. Negligible scattering in the sample.
3. Only scattering in the surrounding medium.
4. Monoenergetic neutrons.

The notation used is:

R = Ratio of the activity induced in the sample to the activity without self absorption.

N = Number of atoms per unit volume in the irradiated sample.

σ = Capture cross section of the irradiated atoms.

a = Radius of the sample.

$\mu = N a$; Assumption 1 requires $\mu \ll 1$.

z = The cosine of the angle between the direction of the neutron and the radius r from the center of the sphere.

$\psi(r, z)$ = The number of neutrons per unit volume at r with direction cosine between z and $z + dz$.

The mean free path of the neutrons in the surrounding medium is used as unit of length and the neutron velocity is taken as unity.

The neutron transport equation has been solved in the spherical case by the spherical harmonic method described by Marshak.* Marshak's boundary condition for a black sphere was replaced by the condition

*R. E. Marshak, Phys. Rev. 71, 443 (1947).

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$\psi(\underline{a}, z) = e^{-2\mu z} \psi(\underline{a}, -z)$. The factor $e^{-2\mu z}$ represents the absorption of neutrons of direction cosine z in traversing the sphere. Expanding $\psi(r, z)$ in a series of Legendre polynomials

$$\psi(r, z) = \sum_{l=0}^3 \frac{1}{2} (2l+1) \psi_l(r) P_l(\mu) \quad (1)$$

and $e^{-2\mu z}$ in powers of z

$$e^{-2\mu z} = 1 - 2\mu z + 2\mu^2 z^2 - \frac{4}{3} \mu^3 z^3$$

one obtains, to first order in μ , by substituting (1) in the transport equation and by equating coefficients of corresponding powers of z in the boundary condition:

$$\psi_0 = K + \frac{3A}{a} - \frac{6k}{5} B, \quad \psi_1 = \frac{A}{r^2}$$

$$\psi_2 = \frac{6}{5} \frac{A}{r^3} + \frac{3k}{5} B Q_2(r) \frac{e^{-kr}}{r}$$

$$\psi_3 = \frac{18}{7} \frac{A}{r^4} + B Q_3(r) \frac{e^{-kr}}{r}$$

where

$$A = \frac{a^2 \mu K}{3} \left[1 - \mu a \left(1 + \frac{1}{Q_3(a)} \frac{4}{5a^2} \left(1 + \frac{6}{ka} \right) \right) \right]$$

$$B = + K \mu a e^{ka} \frac{6}{7a^2 Q_3(a)} \left[1 - \frac{4\mu a}{5} \left(1 + \frac{1}{a^2 Q_3(a)} \left(1 + \frac{6}{ka} \right) + \frac{5}{4ka} \frac{Q_2(a)}{Q_3(a)} \right) \right]$$

$$Q_2(r) = 1 + \frac{3}{kr} + \frac{3}{(kr)^2}$$

$$Q_3(r) = 1 + \frac{6}{kr} + \frac{15}{(kr)^2} + \frac{15}{(kr)^3}$$

K = neutron density far from the absorber

$$k = \frac{1}{3} \sqrt{35}$$

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Thus neutrons are absorbed in the sphere at a rate

$$4\pi A = \frac{4}{3} \pi a^2 \mu K \left[1 - \mu a \left(1 + \frac{1}{Q_3(a)} \frac{4}{5a^2} \left(1 + \frac{6}{ka} \right) \right) \right]$$

If the irradiated material were diffuse enough for self absorption to be neglected the neutrons would be absorbed at a rate

$$\frac{4}{3} \pi a^2 \mu K$$

$$\text{hence } R = 1 - \mu a \left[1 + \frac{1}{Q_3(a)} \frac{4}{5a^2} \left(1 + \frac{6}{ka} \right) \right] = 1 - f$$

For the cylinder the ordinary diffusion equation was solved using an averaged boundary condition leading to the results to first order in μ :

for neutron density

$$n(r) = K \left[1 - \frac{3}{2} \mu a \log \frac{r}{b} \right]$$

for the rate of neutron absorption per unit length

$$\pi \mu a K \left[1 - \frac{3}{2} \mu a \log \frac{b}{a} \right]$$

where K is the neutron density at distance b. If there were no self absorption the neutrons would be absorbed at a rate

$$\pi \mu a K$$

$$\text{hence } R' = 1 - \frac{3}{2} \mu a \log \frac{b}{a} = 1 - f'$$

These solutions are good for large a , i.e., neutron m.f.p. in the scattering medium small compared to the sample radius. For m.f.p. large compared to the sample size the neutrons incident on the sample have come from far away and consequently are representative of the unperturbed flux far from the sample. Under these conditions the self absorption is*

$$f = \frac{3}{4} \mu \text{ for the sphere}$$

$$f' = \frac{4}{3} \mu \text{ for the infinite cylinder.}$$

The references also give the result for a thin slab:

$$f = \frac{3}{2} \mu \left[1 - \frac{2}{3} \ln 2 \gamma \mu \right]; \quad \ln \gamma = 0.577.$$

*W. J. C. Bartels, KAPL-336. This report gives curves for values of μ not small compared to unity.

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Figure 1 compares the two methods for the sphere. It can be seen that for large a the P_1 and P_3 approximations in the spherical harmonic method approach one another, indicating fairly rapid convergence in this region, whereas for small a the tail of the P_3 approximation lies considerably closer to the result from the isotropic flux viewpoint. The curves indicate that both methods give too low a value for the self absorption. The accompanying table was compiled using the spherical harmonic method for radii greater than the value of a at the intersection of the two curves and the isotropic flux method for radii less than this value.

Figure 2 similarly compares the two methods for a cylinder and also shows the dependence on b . The diffusion approximation used for this case is comparable to the P_1 approximation for the sphere.

Table I and Figures 3 and 4 give values of f and f' as functions of sample radius for RaCO_3 using the following data:

$$\sigma = 15 \text{ barns}$$

$$\text{Powder density} = 2.75 \text{ gm/cm}^3$$

$$\text{Mean free path in Be} = 2.61 \text{ cm}^*$$

Thanks are due to Mr. Elmer Eisner for several helpful discussions of this problem.

*CL-697, Project Handbook, Chapter IVE

FIGURE 1

Self Absorption in a Sphere. The curves illustrate the rapid convergence of the Spherical Harmonic Method for neutron mean free paths small compared to the sample radius and the slow approach to the isotropic flux result for large mean free paths.

FIGURE 2

Self Absorption in a Cylinder. The curves illustrate the relationship of the diffusion and isotropic flux solutions for an infinitely long cylinder, and the effect of the reactor size.

Table I

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Sample Radius	Percentage Sphere	Self Absorption* Cylinder
0.2 cm	1.3%	2.4%
0.3	2.0	3.5
0.4	2.6	4.7
0.5	3.3	5.8
0.6	3.9	7.0
0.7	4.6	9.2
0.8	5.2	11.6
0.9	5.9	14.3
1.0	6.6	17
1.2	7.9	23
1.4	10.3	
1.6	13.0	
1.8	16	

*Self absorption = $1-R$, where R is the ratio of the activity induced in the sample to the activity without self absorption.

FIGURE 3

Self Absorption in a Sphere of RaCO_3 . Numerical values for the percentage loss in activity as a function of sample radius.

FIGURE 4

Self Absorption in a Cylinder of RaCO_3 . Numerical values for the percentage loss in activity as a function of sample radius.