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LECTURE NOTES ON HEAT EXTRACTION FROM
BOILING WATER POWER REACTORS

by

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INTRODUCTION

Considerable interest has been focused on the boiling water reactor as a possible reactor type from which economically competitive electric power can eventually be derived. An important part of the reactor development program is an intensive research program to obtain basic data on natural and forced-circulation boiling systems.

One underlying objective of such a program is to develop sound calculation procedures for the thermal and hydraulic performance of boiling reactors such that any given design can readily be evaluated and to make development programs for individual reactors unnecessary. The purpose of these notes is to present the calculation procedures developed thus far at Argonne National Laboratory, to indicate the way they are applied to the design of boiling water reactors, and to outline the remaining areas of uncertainty, where development work is still necessary. The design calculation procedures outlined in these notes are the result of the experience in calculating boiling systems at Argonne. The calculations have been applied to diverse systems with good results.

The notes thus outline the design objectives to be met, present the individual areas where the results of research programs are necessary to the calculation procedures, and then indicate the calculation procedures to be used. The research results represent the opinions of the authors as to the applicable data available that can be used as a basis for engineering calculations. No effort has been made to include all of the diverse information available in the open literature on any specific subject. Where possible, some of the many excellent literature surveys available have been referred to and can be used as a starting point for a more intense investigation of any specific area than can be covered in the allotted time for these lectures.

SECTION I
REACTOR HEAT TRANSFER

P. A. Lottes

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A. NOMENCLATURE

A	area, ft^2
a	coefficient in Eq. (I.25), Btu/hr ft F
b	coefficient in Eq. (I.25), Btu/hr ft F^2
C_1	specific heat of liquid, Btu/lb F
C_{sf}	coefficient in Eq. (I.3)
D	pipe diameter
D_e	equivalent diameter, ft
D_o	outer diameter, ft
G	mass flow rate, $\text{lb}_m/\text{hr ft}^2$
g	acceleration of gravity, $4.17(10)^8 \text{ ft/hr}^2$
g_c	conversion factor, $4.17(10)^8 \text{ lb}_m \text{ ft/hr}^2 \text{ lb}_f$
h	heat transfer coefficient, $\text{Btu/hr ft}^2 \text{ F}$
h_b	boiling coefficient, $\text{Btu/hr ft}^2 \text{ F}$
h_c	condensing coefficient, $\text{Btu/hr ft}^2 \text{ F}$
h_{fg}	latent heat of vaporization, Btu/lb
k	thermal conductivity, Btu/hr ft F
k_l	thermal conductivity of liquid, Btu/hr ft F
k_v	thermal conductivity of vapor, Btu/hr ft F
L	length, ft
P	pressure, lb_f/ft^2
P_1	vapor pressure at saturation, lb_f/ft^2
p_v	vapor pressure at maximum superheat, lb_f/ft^2
q	power, Btu/hr
q''	heat flux, Btu/hr ft^2
q'''	power density, Btu/hr ft^3
r	radius, ft
T, t	temperature
$\Delta T, T_x, T_w - T_{sat}$	heating surface temperature minus saturation temperature, $^{\circ}\text{F}$
v_f	specific volume of fluid, ft^3/lb

v_{fg}	specific volume of vaporization, ft^3/lb
x	wall thickness, ft
θ	angle
λ	heat of vaporization, Btu/lb_m
μ	viscosity, $\text{lb}_m/\text{ft hr}$
μ_1	viscosity of liquid, $\text{lb}_m/\text{ft hr}$
ρ	density, lb_m/ft^3
σ	surface tension of liquid-vapor interface, lb_f/ft

B. BASIC IDEAS AND CONCEPTS IN BOILING HEAT TRANSFER

The process of boiling is defined as heat transfer with a change in phase from liquid to vapor. The means by which energy is transferred from a solid heat transfer surface to vapor is not easily understood. A few facts about the process have been determined by experiment.

It is true that for an equilibrium condition a change in phase from liquid to vapor occurs at constant temperature. In forming a bubble of vapor, the liquid-vapor interface is supposed to remain at a constant temperature. In reality, the formation of a bubble takes place in a violent manner and may be considered as a minor underwater explosion.

In the first place, a force balance on a bubble shows that the vapor pressure inside the bubble is greater than the liquid pressure outside the bubble, and the difference is related to the liquid surface tension, σ , and the bubble radius, r , according to the equation

$$\Delta P = 2\sigma/r \quad . \quad (I.1)$$

As r approaches zero, the required pressure difference approaches infinity. Since fluids have as one of their thermodynamic properties an increase in boiling point with pressure, it now becomes necessary to have a "superheated" liquid before boiling can occur. Assume, for example, that a bubble is present with radius " r ." For an equilibrium condition without heat transfer, the bubble vapor temperature will be equal to the surrounding liquid temperature. Due to surface tension, the bubble pressure will be higher than the surrounding liquid pressure. For the bubble to exist at all, the vapor temperature must be equal to or higher than the saturation temperature corresponding to the bubble pressure. The liquid will therefore be at this same temperature and will therefore be at a temperature higher than the saturation temperature corresponding to the liquid pressure. This state of the liquid is referred to as liquid superheat.

The presence of liquid superheat has been demonstrated in the laboratory. B. R. Mead⁽⁷⁾ and others at UCLA measured liquid superheats as high as 145°F at one atmosphere. Bubble pressures at the onset of boiling could well have reached 145 psia.

The sudden explosive growth of bubbles in superheated liquid causes a very high turbulence level in the liquid. It is this turbulence or violent mixing and agitation of the liquid that accounts for the high heat transfer rates. The effect of boiling from a heated solid surface is that the layers of liquid near the solid surface are disrupted by the mixing action of the bubbles.⁽¹¹⁾ This in effect causes the bulk of the liquid to move in next to solid surface. The overall result is a very efficient scouring action of the liquid upon the surface, causing a high heat transfer coefficient.

There are several types of boiling heat transfer to be considered. These types may be classified as follows:

1. Local boiling, or surface boiling as it is sometimes called, occurs when the hot solid surface is above the liquid saturation temperature, while the liquid is below saturation temperature.
2. Net boiling, or nucleate boiling, occurs when the liquid is at or above saturation temperature.
3. Film boiling occurs when the hot surface is completely covered with a blanket of vapor, and the heat is transferred by conduction through and radiation across the vapor blanket.
4. Transition boiling is a combination of 2 and 3 above.

FIG. I.1
BOILING REGIONS

Local boiling has the unique condition that there is no net formation of vapor. The bubbles may or may not leave the hot surface. If they do leave, they are very quickly condensed. Bubble lifetimes range from a few microseconds to several seconds, depending on the amount of subcooling in the liquid phase. The higher the subcooling, the shorter the lifetime.⁽⁴⁾

Figure I.1 shows the various types of boiling that can occur. It is also worthy to note that the effect of velocity on boiling heat transfer is negligible, even though forced

convection heat transfer coefficients vary as the 0.8 power of the mass flow rate. The reason for the independence of boiling heat transfer coefficients from flow rate is due to the "turbulence promoter" action of the bubbles. The turbulence level is relatively unaffected by flow rate and therefore the heat transfer coefficient is relatively unaffected.

It is also of interest to notice that (a) the temperature differences between surface and liquid are rather small for nucleate boiling and (b) rather large for film boiling. The part of the curve of Fig. I.1 that has zero slope is the maximum point for nucleate boiling. It is this value which is of greatest concern to designers in the boiling field. In a boiling reactor, for example, this value is the maximum value that would occur just before the element jumps into film boiling. The onset of film boiling at a heat flux equal to the maximum in the nucleate boiling range almost always means that the temperature of the solid surface will rise to the melting point of the material. This condition is referred to as "burnout."

C. BOILING HEAT TRANSFER EQUATIONS

In general, the evaluation of the heat transfer characteristics during steady-state boiling in a reactor is not of great importance. The reasons will be shown by an example in a later section of this chapter. It is of interest to list at this point the various equations that may be used to calculate boiling heat transfer, together with references for those who wish to pursue the subject in detail.

(1) McAdams,⁽⁶⁾ for pool boiling (saturated water):

$$q/A = a_1 (T_w - T_{sat})^n \quad (I.2)$$

where the value of a_1 depends upon the liquid, surface condition, and pressure, while the value of n ranges from 3 to 4.

(2) Rohsenow,⁽¹²⁾ for pool boiling (saturated water):

$$\frac{C_1 T_x}{\lambda} = C_{sf} \left(\frac{q/A}{\mu_1 \lambda} \sqrt{\frac{g_c \sigma}{g(\rho_l - \rho_v)}} \right)^{0.33} \left(\frac{C_1 \mu_1}{k_l} \right)^{1.7}, \quad (I.3)$$

where the values of C_{sf} are as follows:

<u>Fluid-heating surface</u>	<u>C_{sf}</u>
Water-platinum	0.013
Benzene-chromium	0.010
Ethyl alcohol-chromium	0.0027
n-pentane-chromium	0.015
Water-brass	0.006

(3) Forster-Zuber,⁽³⁾ pool boiling (saturated water):

$$h = \frac{0.0012 \Delta T^{0.24} (p_v - p_l)^{0.75} C_1^{0.45} \rho_l^{0.49} k_l^{0.79}}{\sigma^{0.5} \lambda^{0.24} \mu^{0.29} \rho_v^{0.24}} \quad (I.4)$$

This form of the equation is presented and discussed by Perkins and Westwater.⁽⁹⁾ Comparisons are also given to the Rohsenow equation and to experimental data with a boiling methanol system.

(4) Jens-Lottes,⁽⁵⁾ subcooled boiling (subcooled water):

$$\Delta T = 1.9 (q/A)^{0.25} / e P / 900 \quad (I.5)$$

for pressures of 100 to 2500 psia, water temperatures from 229 to 636°F, mass flow rates from 0.008 (10)⁶ to 7.7 (10)⁶ lb per hr ft², and heat fluxes to 4 (10)⁶ Btu per hr ft².

(5) Mumm,⁽⁸⁾ forced circulation of boiling saturated water inside tubes:

$$\frac{h D_e}{k} = 4.3 + 5.0 (10)^{-4} \left(\frac{v_{fg}}{v_f} \right)^{1.64} \left(\frac{q/A}{G h_{fg}} \right)^{0.464} \left(\frac{G D_e}{\mu_f} \right) \quad (I.6)$$

for pressures of 45 to 200 psia, heat fluxes from 50,000 to 250,000 Btu per hr ft², and mass flow rates from 70 to 280 lb per hr ft².

(6) Bromley,⁽²⁾ film boiling:

$$h = 0.62 \left[\frac{k_v^3 \rho_v (\rho_l - \rho_v) \lambda g}{\mu_v D_o \Delta T} \right]^{0.25} \quad (I.7)$$

In addition to the above heat transfer equations there are a number of burnout equations in the literature for various types of heated channels. McAdams⁽⁶⁾ summarizes and discusses the equations. It is sufficient at this time to mention that the range of heat flux for burnout under most design conditions is from 500,000 to 1,500,000 Btu per hr ft².

The calculation of temperature drop during boiling in a nuclear reactor is of secondary importance. The accurate evaluation of heat transfer coefficients in evaporator-condenser units may also be unnecessary. An example will illustrate the two cases.

Case 1 - Boiling Reactor Heat Transfer

Let us suppose that it is desired to evaluate the fuel plate surface temperature at design conditions such that

System pressure 600 psia
Average heat flux 100,000 Btu/hr ft²

Using Equation (I.5) the film drop is calculated to be 17°F. Since the film drop is proportional to the 0.25 power of the heat flux, we may note that even if the heat flux were as high as 1,000,000 Btu/hr ft², the film drop under these conditions would only be

$$\Delta T = 17(10)^{0.25} = 30^\circ\text{F}$$

Accordingly, the surface temperature at 100,000 Btu/hr ft² would be

$$T_w = T_{\text{sat}} + 17 = 503^\circ\text{F},$$

while the surface temperature at 1,000,000 Btu/hr ft² would be

$$T_w = T_{\text{sat}} + 30 = 516^\circ\text{F}.$$

It is evident, therefore, that in a system where heat flux is the independent variable the calculation of surface temperature does not require great accuracy. Equation (I.5) is recommended for general use, since it can be evaluated quite easily. Figure I.2 shows a comparison of Equations (I.3), (I.4), and (I.5) at 600 psia.

FIG. I.2
COMPARISON OF BOILING HEAT TRANSFER EQUATIONS AT 600 PSIA

Referring to the same temperature drops listed on preceding page, it is important to notice that in a system where temperature is the controlled variable, an increase in surface temperature from 503°F to 516°F will cause an increase in heat flux by a factor of 10. This increase may not occur if there are other resistances to heat flow in the system. The following example will show these effects.

Case II - Evaporator-Condenser Heat Transfer

In this case, we are faced with determining the size of an exchanger to transfer a given or predetermined amount of power. Unlike the reactor where power was an independent variable, we now have the overall temperature difference as the independent variable. For this case, we may write a heat balance on the element of heat exchanger surface such as is shown in Fig. I.3.

FIG. I.3

TEMPERATURE GRADIENT THROUGH HEAT EXCHANGER TUBE

Ignoring scale resistances to heat flow for this example, and assuming that the inner and outer tube areas are very nearly equal, we may write

$$q/A = h_c(T_1 - T_2) = (k/\Delta x)(T_2 - T_3) = h_b(T_3 - T_4) \quad (I.8)$$

where T_1 is the steam condensing temperature, T_2 the outer tube wall temperature, T_3 the inner tube wall temperature, T_4 the boiling fluid temperature, and x the tube wall thickness. Rearranging the above equation and solving for the required area (for this example, it is sufficient to neglect the tube curvature),

$$A = \frac{1}{h_c} + \frac{\Delta x}{k} + \frac{1}{h_b} \frac{q}{(T_1 - T_4)} \quad (I.9)$$

Suppose it is given that $(T_1 - T_4)$ is 50°F and that the overall power or heat transfer, q , is 50,000,000 Btu per hr. We must now evaluate A to determine the size or capacity of a heat exchanger that will perform the required job. We are faced with the problem of selecting an exchanger that is adequate, but not oversized by too great an amount. It may be noticed from Eq. (I.9) that a given per cent error in the bracket term will cause the same per cent error in A . Now, let us select an example to show the relative size of these errors.

Let

$$\begin{aligned} h_c &= 5000 \text{ Btu/hr ft}^2 \\ k &= 30 \text{ Btu/hr ft } ^\circ\text{F} \\ \Delta x &= \frac{1}{16} \text{ in.} = 0.0052 \text{ ft} \\ P &= 600 \text{ psia} \end{aligned}$$

Let us first assume that $h_b = 5000 \text{ Btu/hr ft}^2 ^\circ\text{F}$.

$$A = \frac{1}{5000} + \frac{0.0052}{30} + \frac{1}{5000} \frac{50(10)^6}{50} = 573 \text{ ft}^2$$

From Eq. (I.5) and from the definition of h_b ,

$$h_b = \frac{q}{A \Delta T} = \left(\frac{q}{A} \right)^{0.75} \frac{eP/900}{1.9} \quad (I.10)$$

Combining Eq. (I.9) and (I.10),

$$A = \frac{1}{h_c} + \frac{\Delta x}{k} + \frac{1.9 A^{0.75}}{(q)^{0.75} eP/900} \frac{q}{T_1 - T_4} \quad (I.11)$$

Solving this by trial and error, the correct value of A is

$$A = 551 \text{ ft}^2$$

The correct value of h_b is 8,460 Btu/hr ft² °F, the correct $(T_3 - T_4)$ is 10.7°F and the average heat flux is 90,600 Btu/hr ft². Our first calculation based on $h_b = 5000$ gave a value of $(T_3 - T_4)$ equal to 17.5°F. Our error in boiling coefficient was

$$\frac{8,460 - 5000}{8,460} \times 100 = 41\%.$$

The error in $(T_3 - T_4)$ was

$$\frac{17.5 - 10.7}{10.7} \times 100 = 64\%.$$

However, the error in the area was small:

$$\frac{573 - 551}{551} \times 100\% = 4\%.$$

The error in area is directly related to the error in boiling coefficient from Eq. (I.9). Assuming no scale resistance,

$$dA = d\left(\frac{1}{h_b}\right) \frac{q}{(T_1 - T_4)},$$

or

$$\frac{dA}{A} = \frac{d\left(\frac{1}{h_b}\right)}{\frac{1}{h_c} + \frac{\Delta x}{k} + \frac{1}{h_b}}.$$

Assume, for example,

$$\frac{1}{h_c} = 0.0002 \text{ (} h_c = 5000 \text{ Btu/hr ft}^2 \text{F)}$$

$$\frac{\Delta x}{k} = 0.0002 \left(\frac{k}{\Delta x} = 5000 \text{ Btu/hr ft}^2 \text{F} \right)$$

$$\frac{1}{h_b} = 0.0001 \text{ (} h_b = 10,000 \text{ Btu/hr ft}^2 \text{F)}$$

Since

$$d\left(\frac{1}{h_b}\right) = \frac{1}{h_b} \left(\frac{dh_b}{h_b} \right)$$

then

$$\begin{aligned} \left(\frac{dA}{A} \right) &= \frac{- \left(\frac{d h_b}{h_b} \right)}{h_b \left(\frac{1}{h_c} + \frac{\Delta x}{k} + \frac{1}{h_b} \right)} = \frac{- \frac{d h_b}{h_b}}{10,000 (0.0002 + 0.0002 + 0.0001)} \\ &= - 0.2 \left(\frac{d h_b}{h_b} \right) \end{aligned}$$

The error in the area determination is only 20% of the error in the boiling heat transfer coefficient.

If a scale resistance of 0.001 ($h = 1000$) is included on both sides of the heat exchanger tube, the errors are still less, and may be shown to be

$$\left(\frac{dA}{A} \right) = 0.04 \left(\frac{d h_b}{h_b} \right)$$

D. FUEL ELEMENT HEAT TRANSFER

Case 1 - Fuel Plate

FIG. I.4

TEMPERATURE GRADIENT IN
PLATE-TYPE FUEL ELEMENT

The heat transfer from the clad surface to the coolant has already been described in the previous section.

The temperature drop in the cladding is (see Fig. I.4)

$$T_1 - T_2 = q'' (x_2 - x_1) / k_{\text{clad}} \quad (\text{I.12})$$

For uniform heat generation in the fuel and constant thermal conductivity, the fuel temperature drop is

$$T - T_1 = q'' (x_2) / 2 k_{\text{fuel}} \quad (\text{I.13})$$

It may be noticed that the temperature distribution through the fuel is parabolic and that the temperature difference is exactly one-half of the value obtained for the case where all of the heat generated is passed through a slab of equal thickness.

In the case of variable thermal conductivity the temperature difference can be found as follows from Fig. I.5:

FIG. I.5

ELEMENT OF FUEL PLATE

$$q_1'' A + q''' A dx = q_2'' A \quad (\text{I.14})$$

$$-kA \left(\frac{dt}{dx} \right)_x + \left(k + \frac{dk}{dx} dx \right) A \left(\frac{dt}{dx} \right)_{x+dx} + q''' A dx = 0 \quad (\text{I.15})$$

Dividing by $A dx$ and rearranging

$$\frac{k \left[\left(\frac{dt}{dx} \right)_{x+dx} - \left(\frac{dt}{dx} \right)_x \right]}{dx} + \frac{dk}{dx} \left(\frac{dt}{dx} \right)_{x+dx} + q''' = 0 \quad (\text{I.16})$$

or

$$k \frac{d^2 t}{dx^2} + \frac{dk}{dx} \left(\frac{dt}{dx} \right) + q''' = 0 \quad (\text{I.17})$$

This is identical to

$$\frac{d}{dx} \left(k \frac{dt}{dx} \right) + q''' = 0 \quad (\text{I.18})$$

For constant values of k , Eq. (I.18) may be integrated to give

$$\int_0^x d \left(k \frac{dt}{dx} \right) = - \int_0^x q''' dx \quad (\text{I.19})$$

where

$$\phi = k \frac{dt}{dx}$$

Then

$$k \frac{dt}{dx} = -q''' x \quad (I.20)$$

or

$$\int_{T_{\infty}}^{T_1} k dt = - \int_0^{x_1} q''' x dx \quad (I.21)$$

Thus

$$k(T_1 - T_{\infty}) = -q''' \frac{x_1^2}{2} \quad (I.22)$$

or

$$T_{\infty} - T_1 = q''' x_1^2 / 2k \quad (I.23)$$

By definition,

$$q''' = (q'') \text{ Area/Volume} = q'' A / Ax_1 = q'' / x_1 \quad (I.24)$$

Equation (I.23) becomes identical to Eq. (I.13)

$$T_{\infty} - T_1 = q'' x_1 / 2k$$

For variable k , we can usually approximate k by the expression

$$k = a + bT \quad (I.25)$$

Then Eq. (I.21) becomes

$$\int_{T_{\infty}}^{T_1} (a + bT) dT = - \int_0^{x_1} q''' x dx \quad (I.26)$$

or

$$a(T_1 - T_{\infty}) + \frac{b}{2} (T_1^2 - T_{\infty}^2) = -q''' \frac{x_1^2}{2} \quad (I.27)$$

or

$$a(T_{\infty} - T_1) + \frac{b}{2} (T_{\infty}^2 - T_1^2) = q''' \frac{x_1^2}{2} \quad (I.28)$$

so that

$$(T_{\infty} - T_1) = - \left(T_1 + \frac{a}{b} \right) \pm \sqrt{\left(T_1 + \frac{a}{b} \right)^2 + \frac{q''' x_1^2}{b}} \quad (I.29)$$

Equation (I.13) is generally used, since the effect of a variable k is not very great, as shown by the following example.

Example

$$q''' = 5(10)^8 \text{ Btu/hr ft}^3$$

$$x = 0.12 \text{ in.} = 0.01 \text{ ft}$$

$$T_1 = 500^\circ\text{F}$$

$$k = 10 + 0.01 T$$

The heat flux is

$$\begin{aligned} q'' &= q''' x = 5(10)^8 (0.01) \\ &= 500,000 \text{ Btu/hr ft}^2. \end{aligned}$$

From Eq. (I.29), with $a = 10 \text{ Btu/hr ft}^\circ\text{F}$ and $b = 0.01 \text{ Btu/hr ft F}^2$,

$$(T_{\infty} - T_1) = -(500 + 1000) \pm \sqrt{(1500)^2 + \frac{5(10)^8 (0.01)^2}{0.01}} = 160^\circ\text{F}$$

If we approximate this case by assuming an average k over the interval, say

$$k_{\text{avg}} = 15,$$

then from Eq. (I.2)

$$(T_{\infty} - T_1) = \frac{(500,000)(0.01)}{2(15)} = 166^\circ\text{F}$$

Therefore in flat plate elements, it is sufficient to assume constant properties for metallic fuels.

Case 2 - Fuel Rods

A heat balance on the element in Fig. I.6 gives

$$q_1'' + q'''(\text{Volume}) = q_2''$$

FIG. I.6

ELEMENT OF FUEL ROD

Then

$$-kr\theta L\left(\frac{dt}{dr}\right)_r + q''' r\theta Ldr = -\left[k + \left(\frac{dk}{dr}\right)dr\right](r+dr)\theta L\left(\frac{dt}{dr}\right)_{r+dr} \quad (\text{I.30})$$

Rearranging, dividing by $r\theta Ldr$, and neglecting second-order differentials,

$$\frac{k}{dr} \left[\left(\frac{dt}{dr}\right)_{r+dr} - \left(\frac{dt}{dr}\right)_r \right] + \frac{k}{r} \left(\frac{dt}{dr}\right)_{r+dr} + \left(\frac{dk}{dr}\right) \left(\frac{dt}{dr}\right)_{r+dr} = q''' = 0 \quad (\text{I.31})$$

This may be written (as $dr \rightarrow 0$)

$$k \frac{d^2t}{dr^2} + \frac{k}{r} \frac{dt}{dr} + \left(\frac{dk}{dr}\right) \left(\frac{dt}{dr}\right) + q''' = 0 \quad (\text{I.32})$$

Another, more useful form of Eq. (I.32) is

$$\frac{1}{r} \frac{d}{dr} \left(kr \frac{dt}{dr} \right) + q''' = 0 \quad (\text{I.33})$$

Rearranging and integrating,

$$\int_0^\phi \frac{d}{dr} \left(kr \frac{dt}{dr} \right) = - \int_0^r q''' r dr \quad (I.34)$$

where

$$\phi = kr \frac{dt}{dr} .$$

Thus

$$kr \frac{dt}{dr} = - \frac{q''' r^2}{2} . \quad (I.35)$$

Rearranging again and integrating,

$$\int_{T_G}^{T_0} k dt = - \int_0^{r_0} \frac{q''' r dr}{2} \quad (I.36)$$

$$\int_{T_0}^{T_G} k dt = \frac{q''' r_0^2}{4} . \quad (I.37)$$

It is interesting to note that the heat generation per unit length of rod is

$$\frac{Q}{L} = \frac{q''' \pi r_0^2 L}{L} = q''' \pi r_0^2 . \quad (I.38)$$

The right-hand side of Eq. (I.37) multiplied by 4π will give the heat released per unit rod length.

Although thermal conductivities of oxide fuels are difficult to determine accurately, Eq. (I.37) gives a convenient way of comparing experimental test data on rods of various diameters, enrichment, and composition. J. A. L. Robertson⁽¹⁰⁾ et al. show that satisfactory performance without melting was achieved with oxide fuel provided that

$$\int k dt \leq 39 \text{ watts/cm} .$$

Oxide fuels appear to have conductivities that obey the law⁽¹⁾

$$k = d/T ,$$

where T is given in degrees absolute. An example is given: Find the maximum heat generation rate and heat flux that will produce a centerline temperature of 4000°F for the following conditions:

$$r_0 = 0.24 \text{ in.} = 0.02 \text{ ft}$$

$$T_{\text{CL}} = 4000^{\circ}\text{F}$$

$$T_1 = 700^{\circ}\text{F}$$

$$k = \frac{3300}{T + 460} \text{ (T in. } ^{\circ}\text{F)}$$

From Eq. (I.37)

$$q''' = \frac{4}{r_0^2} \int_{1000}^{4000} \left(\frac{3300}{T + 460} \right) dt = \frac{4(3300)}{(0.02)^2} \ln \left(\frac{4460}{1460} \right) = 4.45(10)^7 \text{ Btu/hr ft}^3$$

The heat flux is

$$\begin{aligned} q'' &= \frac{q''' (\text{Volume})}{(\text{Area})} = \frac{q''' \pi r_0^2 L}{2 \pi r_0 L} = q''' \frac{r_0}{2} = 4.45(10)^7 \frac{(0.02)}{2} \\ &= 445,000 \text{ Btu/hr ft}^2 \end{aligned}$$

The errors introduced in assuming a constant value of k can be shown as follows:

$$\text{Assume } k = 1.17 \text{ Btu/hr ft } ^{\circ}\text{F (value at } 2350^{\circ}\text{F)}$$

From Eq. (I.37)

$$q''' = \frac{4}{r_0^2} \int_{700}^{4000} 1.17 dT = \frac{4(1.17)(3300)}{(0.02)^2} = 3.86(10)^7 \text{ Btu/hr ft}^3$$

and

$$q'' = 3.86(10)^7 \frac{(0.02)}{2} = 386,000 \text{ Btu/hr ft}^2$$

which is a per cent decrease of

$$\frac{445,000 - 386,000}{445,000} \times 100\% = 13\%$$

Conversely, if we assume a heat flux of 445,000 Btu/hr ft² and calculate the centerline temperature by using a constant k value,

$$T_{CL} = 700 + \frac{4.45(10)^7 (0.02)^2}{4(1.17)} = 700 + 3800 = 4500^\circ\text{F} ,$$

which is in error by 500°F.

E. STEAM HEAT TRANSFER

1. Saturated Steam

Heat transfer coefficients to flowing saturated steam are low compared to boiling coefficients, but not very much lower than the coefficients to flowing water when compared at the same mass flow rate. For either water or steam, we may write

$$\frac{hD}{k} = 0.023 \left(\frac{GD}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.4} . \quad (\text{I.39})$$

Rearranging,

$$h = 0.023 \frac{G^{0.8} C_p^{0.4} k^{0.6}}{D^{0.2} \mu^{0.4}} . \quad (\text{I.40})$$

The ratio of the heat transfer coefficient for water and steam is therefore

$$\frac{h_{\text{steam}}}{h_{\text{water}}} = \frac{\left[\left(\frac{C_p}{\mu} \right)^{0.4} k^{0.6} \right]_{\text{steam}}}{\left[\left(\frac{C_p}{\mu} \right)^{0.4} k^{0.6} \right]_{\text{water}}} . \quad (\text{I.41})$$

A comparison is given in Fig. I.7. The minimum ratio is about 0.3 and this ratio occurs at a temperature of about 320°F.

2. Superheated Steam

Equation (I.39) may be used to calculate the heat transfer coefficients for superheated steam. Rearranging Eq. (I.39) or Eq. (I.40),

$$h = 0.023 (G^{0.8}/D^{0.2}) \phi , \quad (\text{I.42})$$

where

$$\phi = \left(\frac{C_p}{\mu} \right)^{0.4} k^{0.6} . \quad (\text{I.43})$$

FIG. I.7
COMPARISON OF STEAM AND WATER AS COOLANTS

This function is shown in Fig. I.8. The values given are approximate and should not be used for design equations. Figure I.8 is given to show the trend in the heat transfer coefficient with temperature and pressure.

FIG. I.8
EFFECT OF PRESSURE AND TEMPERATURE
ON STEAM HEAT TRANSFER

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SECTION II
TWO-PHASE PRESSURE DROP

John F. Marchaterre

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A. NOMENCLATURE

a	Parameter defined in text
C_p	Specific heat, Btu/lb °F
D	Diameter of pipe, ft
$\left(\frac{dP}{dL}\right)_1$	Isothermal liquid pressure gradient at average bulk water temperature
f	Fanning friction factor
G	Mass velocity, lb/hr ft ²
g_c	Gravitational force per unit mass 32.2 lb ft/lb sec ²
h_{fg}	Heat of vaporization, Btu/lb
L	Length, ft
L_t	Total channel length, ft
P	Pressure, lb/in ²
ΔP	Finite difference of static pressure in length ΔL
Q	Total heat input, Btu/hr
q''	Heat flux, Btu/hr ft ²
R	Two-phase friction factor multiplier
T, t	Temperature, °F
$(\Delta T_{sub})_0$	Subcooling at point where local boiling starts
w	Weight flow, lb/hr
X	Parameter discussed in text
x	Quality, lb steam/ lb mixture
μ	Viscosity
ρ	Density, lb/ft ³
ϕ	Parameter discussed in text
ψ	Parameter defined in Eq. (II.10)

Subscripts

B	Refers to boiling
e	Refers to exit conditions
g	Refers to gaseous phase

Subscripts (Cont'd.)

LB	Refers to local boiling
l	Refers to liquid phase
TP	Refers to two-phase flow
TPF	Refers to two-phase friction
tt	Refers to turbulent-turbulent flow

B. INTRODUCTION

Numerous engineering systems require that the pressure drop associated with the flow of a two-phase mixture be evaluated. This problem is of particular importance in the design of boiling water reactors where the pressure drop associated with the flow of a two-phase mixture under both boiling and nonboiling conditions in vertical flow must be evaluated. The earliest papers on this subject assumed that the two phases formed a uniform homogeneous mixture with a calculable specific volume, and the frictional pressure drop was evaluated in the usual manner. This method was, in general, unsatisfactory because of the difficulty in defining a Reynolds number.

A more easily used method of calculation is that due to Martinelli et al., (10,11,12,13) which uses the concept of a friction factor multiplier:

$$R = \frac{(dP/dL)_{TP}}{(dP/dL)_l} ,$$

which is defined as the ratio of the two-phase friction pressure gradient to the single-phase friction pressure gradient. Using this concept, the two-phase pressure drop in a two-phase adiabatic system will be

$$\Delta P_{TPF} = R \frac{f_l G^2 L}{2g_c \rho_l D} ,$$

and the pressure drop in a boiling channel will be

$$\Delta P_{TPF} = \frac{f_l G^2 L}{2g_c \rho_l D} \int_0^{L_B} R dL = \bar{R} \frac{f_l G^2 L_B}{2g_c \rho_l D} .$$

Several literature surveys giving complete bibliographies of work done in this field have been compiled. (1,6,9,15,21)

A majority of the work has been done on adiabatic, two-component (gas-oil, air-water) flow in horizontal pipes. Very little pressure drop data has been reported for heated vertical channels in which a change of phase is occurring, which is the region of interest for reactor design.

The mechanism of two-phase flow and the resulting pressure drop have all but defied analysis because of the complex interaction between phases. There are, however, methods of estimation that can be used for engineering calculations.

C METHOD OF MARTINELLI

The Martinelli correlations for the calculation of two-phase pressure drops represent the best general method yet proposed. It should be pointed out, however, that serious discrepancies between the Martinelli method and later data have developed, and a modified method is recommended for engineering calculations. This method will be discussed in Section E.

The initial two-phase pressure drop studies on which the Martinelli model was based were made by L. M. K. Boelter and R. H. Kepner⁽²⁾ who determined pressure gradients for the two-phase flow of air and water and air and oil through horizontal and inclined pipes. In a series of four papers^(10,11,12,13) Martinelli and coworkers presented a correlation of the experimental data and extended it to the prediction of the pressure drop of steam-water mixtures. Two basic assumptions were made to establish the basis of the correlations:

1. the static pressure drop for the gaseous phase is equal to the liquid phase static pressure drop, regardless of the details of the flow patterns;
2. the volume occupied by the liquid plus that occupied by the gas equals the volume of the pipe.

Using these assumptions, Martinelli arrived at a semiempirical correlation of two-phase frictional pressure drop. The two-phase friction pressure drop was expressed as

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{TPF}} = \left(\frac{\Delta P}{\Delta L}\right)_g \phi_g^2,$$

where $(\Delta P/\Delta L)_g$ is the pressure drop per unit of length if the gas alone were flowing in the pipe at a flow rate of x times the total flow rate of the two-phase mixture, where the weight fraction of the gas (quality) is x . An alternate expression was derived using ϕ_g^2 and $(\Delta P/\Delta L)_1$:

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{TPF}} = \phi_1^2 \left(\frac{\Delta P}{\Delta L}\right)_1,$$

where $(\Delta P/\Delta L)_1$ represents the pressure drop per unit length if only liquid were flowing in the pipe at a flow rate $(1 - x)$ times the total flow.

The quantities ϕ_1^2 and ϕ_g^2 were then correlated as a function of a parameter of X , where

$$X = \frac{(\Delta P/\Delta L)_1}{(\Delta P/\Delta L)_g}.$$

This parameter was defined for four types of flow, depending on whether the phases were in viscous or turbulent flow. This discussion will be confined to the case where both phases are in turbulent flow, as indicated by the subscript *tt*.

The correlation was then extended to include conditions in which the quality would vary directly with flow length such as might be typical for forced-circulation boiling of water. The method of development was to use the previous correlations of

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{TPF}} = \left(\frac{\Delta P}{\Delta L}\right)_1 \phi_1^2$$

for point conditions:

$$\left(\frac{dP}{dL}\right)_{\text{TPF}} = \left(\frac{dP}{dL}\right)_1 \phi_1^2$$

The value of $(dP/dL)_1$ and ϕ_1 varies with quality and, assuming that the quality varies directly with length, graphical integrations were made by mean pressure levels. The data of Davidson⁽⁵⁾ were used to establish an empirical pressure dependence, since it was discovered that applying the correlation directly definitely overestimated the experimental tests.

The important features of the Martinelli correlations shown in Fig. II.1, II.2, II.3, and II.4. Figure II.1 shows ϕ_1 as a function of the parameter X , where

$$X_{\text{tt}} = \frac{w_1}{w_g} \left(\frac{\rho_g}{\rho_1}\right)^{0.5} \left(\frac{\mu_1}{\mu_g}\right)^{0.1}.$$

FIG. II.1
CORRELATION OF LOCKHART AND MARTINELLI GIVING THE DEPENDENCE OF ϕ_L ON X_{tt}

FIG. II.2
LOCAL TWO-PHASE FRICTION FACTOR VS QUALITY

FIG. II.3
LOCAL TWO-PHASE FRICTION FACTOR VS QUALITY

FIG. II.4
AVERAGE TWO-PHASE FRICTION MULTIPLIER
(UNIFORM HEAT INPUT)

This curve was revised to give it an empirical pressure dependence and re-plotted as a function of quality. Figures II.2 and II.3 show ϕ_1^2 as a function of quality at various pressures. Here,

$$\phi_1^2 = \left(\frac{dP}{dL} \right)_{\text{TPF}} / \left(\frac{dP}{dL} \right)_1 = R \quad ,$$

defines the two-phase friction factor multiplier. The results of the graphical integration are shown in Fig. II.4, where

$$\bar{R} = \Delta P_{\text{TPF}} / \Delta P_1$$

is the average two-phase friction factor multiplier for vaporization, based on exit conditions.

D. EXTENSION OF THE MARTINELLI-NELSON WORK FOR A COSINE HEAT INPUT

Cohen⁽⁴⁾ has extended the Martinelli-Nelson work for a cosine heat input. Since the curves of Martinelli for $\Delta P_{\text{TPF}} / \Delta P_1$ as based on values of the quality at the exit were drawn by integrating the local R values with respect to length, assuming a linear relationship exists between the quality, X , and the boiling length, L , this condition would exist for a uniform heat flux with respect to length. It is generally true in reactor design, however, that the neutron flux, and thus the power input, are more closely represented by a cosine function with respect to core length. The maximum heat input exists at the center of the core. Thus the development by Martinelli-Nelson was extended for a cosine heat flux, the curves of $(dP/dL)_{\text{TPF}} / (dP/dL)_1$ vs X_{tt} with constant pressure parameters in Ref. (12) being used. The curves represent conditions at points along the length and therefore are applicable to any type of heat flux.

To obtain the average value of the frictional pressure drop along the length of the boiling section, or in this case of $\Delta P_{\text{TPF}} / \Delta P_1$, it was necessary to integrate these curves over the boiling section length. For a constant heat flux this integration is a relatively simple matter, since there exists a linear relationship between quality and length, and thus the integration can be performed over the quality, from zero quality to exit quality, X_e . Therefore, it follows:

$$\frac{\Delta P_{\text{TPF}}}{\Delta P_1} = \frac{1}{X_e} \int_0^{X_e} \left[\frac{(dP/dL)_{\text{TPF}}}{(dP/dL)_1} \right] dX \quad . \quad (\text{II.1})$$

The analysis is not quite as simple for a heat flux which is other than constant. For the case under consideration, the integration must be performed over the length of the boiling section, and therefore a relationship must be found between length and quality for the particular type of heat flux desired, in this case a cosine heat flux. Therefore, in general,

$$\frac{\Delta P_{\text{TPF}}}{\Delta P_1} = \frac{1}{L_B} \int_{L_{\text{NB}}}^{L_t} \left[\frac{(dP/dL)_{\text{TPF}}}{(dP/dL)_1} \right] dL \quad . \quad (\text{II.2})$$

For a cosine heat flux, the derivation of a relationship between the quality and the length is straightforward. At any point along the length of the heated section the power input can be expressed as

$$Q = Q_{\text{max}} \sin (\pi L / L_t) \quad . \quad (\text{II.3})$$

Therefore, the heat input in the boiling section up to any length L is

$$Q_B = \int_{L_{\text{NB}}}^L Q_{\text{max}} \sin (\pi L / L_t) dL = \frac{L_t}{\pi} Q_{\text{max}} \left[\cos \frac{L_{\text{NB}}}{L_t} - \cos \frac{\pi L}{L_t} \right] \quad . \quad (\text{II.4})$$

The total heat input over the entire boiling length is

$$Q_{B_t} = \int_{L_{\text{NB}}}^{L_t} Q_{\text{max}} \sin (\pi L / L_t) dL = \frac{L_t}{\pi} Q_{\text{max}} \left[1 + \cos \frac{L_{\text{NB}}}{L_t} \right] \quad . \quad (\text{II.5})$$

Dividing Eq. (II.4) by (II.5) gives

$$\frac{Q_B}{Q_{B_t}} = \frac{\cos \frac{L_{\text{NB}}}{L_t} - \cos \frac{\pi L}{L_t}}{1 + \cos \frac{L_{\text{NB}}}{L_t}} \quad . \quad (\text{II.6})$$

Also the following relationships are true:

$$Q_B = w_t X h_{fg} \quad ; \quad (\text{II.7})$$

$$Q_{B_t} = w_t X_e h_{fg} \quad . \quad (\text{II.8})$$

Dividing Eq. (II.7) by (II.8):

$$Q_B/Q_{B_t} = X/X_e \quad (II.9)$$

Therefore, setting Eq. (II.9) equal to Eq. (II.6), we have a relationship between X and L :

$$\frac{X}{X_e} = \frac{\cos \frac{\pi L_{NB}}{L_t} - \cos \frac{\pi L}{L_t}}{1 + \cos \frac{\pi L_{NB}}{L_t}}$$

It should be noticed that the quality at any point is strongly dependent upon the length of the subcooled region, or the ratio of the nonboiling length to the total length, L_{NB}/L_t . Therefore, we will have a distinct series of curves for $\Delta P_{TPF}/\Delta P_0$ for each L_{NB}/L_t chosen.

The boiling length was broken up into ten equal increments, from $\left(\frac{L}{L_t}\right)_B = 0$ to $\left(\frac{L}{L_t}\right)_B = 1$. The integration was performed using Simpson's rule to evaluate the following integral

$$\frac{\Delta P_{TPF}}{\Delta P_0} = \int_0^1 \left[\frac{\left(\frac{dP}{dL}\right)_{TPF}}{\left(\frac{dP}{dL}\right)_1} \right] d\left(\frac{L}{L_t}\right)_B$$

The final curves are shown in Figures (II.5) through (II.14) for different values of L_{NB}/L_t .

E. MODIFIED MARTINELLI METHOD FOR USE IN ENGINEERING CALCULATIONS

Several investigators have reported an effect of mass flow rate on the two-phase friction factor multiplier. Petrick⁽¹⁶⁾ has reported results for the vertical flow of air-water mixtures. A strong effect of mass flow rate on the friction factor multiplier was found. Moen⁽¹⁴⁾ and Rodriguez⁽¹⁸⁾ have also reported an effect of flow rate on the friction factor multiplier in the horizontal flow of steam-water mixtures. Sher⁽²⁰⁾ has reported results over a wide range of flow rates and qualities at 2000 psia. A strong effect of mass flow rate on ϕ_1^2 was found. He suggested that the local two-

FIG. II.5
 $\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS
 (COSINE HEAT INPUT)

FIG. II.6
 $\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II.7

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II.8

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II 9

$\frac{\Delta P_{TPF}}{\Delta P_0}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II.10

$\frac{\Delta P_{TPF}}{\Delta P_0}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II.11

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II.12

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II. 13

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

FIG. II. 14

$\frac{\Delta P_{TPF}}{\Delta P_o}$ versus PRESSURE EXIT QUALITY PARAMETERS

phase friction factor multiplier for pressures lower than 2000 psia might be calculated by using their values of ϕ_1^2 for the corresponding mass velocity and quality and applying a pressure correction obtained from the Martinelli-Nelson curve.

The experimental data of several investigators have been examined and a revised method of estimating two-phase friction pressure drops has been formulated. Since the data of Martinelli-Nelson appear to closely represent the value of ϕ_1^2 at a mass flow rate of 1×10^6 lb/hr ft², the following procedure is recommended:

1. Obtain the value of ϕ_1^2 at the conditions of interest, using the correlation of Martinelli-Nelson.
2. Correct this value to the mass flow rate in question, using the values obtained by Sher at 2000 psi, by taking the ratio of ϕ_1^2 at the mass flow rate in question to ϕ_1^2 at $G = 1.0 \times 10^6$ at the same steam volume fraction.

This correction seems a more valid method of correcting the data since the same mass flow rate effect has been found by several investigators at widely differing qualities at varying pressures, indicating that the correct method of comparison should be on a steam volume fraction basis.

Figure II.15 shows an error plot of the data of several investigators compared with the values computed using this method. These data represent the available data with steam-volume fraction measurements in the range of variables of interest to reactor designers.

Figures II.16 and II.17 show the mean lines through the data of Sher and the correction factors for the Martinelli-Nelson correlation based on this data.

CONCLUSIONS

The corrected Martinelli-Nelson method of predicting two-phase friction pressure drops represents an adequate method for engineering calculations. Additional work however remains to be done to establish the effect of mass flow rate for all pressures, to check geometry effects, and effects of flow pattern.

FIG. II.15

COMPARISON OF THE MODIFIED MARTINELLI METHOD
WITH THE EXPERIMENTAL DATA OF FIVE INVESTIGATORS.

FIG. II.16

WESTINGHOUSE TWO-PHASE PRESSURE DROP DATA

FIG. II.17

CORRECTION FACTORS BASED ON α FROM WESTINGHOUSE DATA
BASED ON SLIP = 1.0

F. LOCAL BOILING PRESSURE DROP

Another aspect of the problem of predicting the frictional pressure drop in two-phase flow is the prediction of pressure drop during subcooled boiling. If we consider a channel with liquid entering below saturation temperature and if the heat flux is large enough, subcooled boiling, with bubbles growing and collapsing on the walls, will take place. Thus, even though the bulk of the fluid is not at saturation temperature, the average steam density will decrease, and there will be an acceleration component in addition to the frictional loss. Very little experimental work has been done on the measurement of frictional pressure drops in the subcooled boiling region; where experimental measurements have been made, the total pressure drop, rather than just the frictional component, has been correlated.

Some local boiling pressure drop data have been reported by Buchberg et al.⁽³⁾ for pressures from 100 to 2000 psia, velocities from 5 to 30 fps and heat fluxes up to 2,000,000 Btu/hr ft² in a 0.226-in. tube, 24.6 inches long. Rohde⁽¹⁹⁾ correlated these data by means of the following equation:

$$\left(\frac{dP}{dL}\right)_{LB} \bigg/ \left(\frac{dP}{dL}\right)_1 = \left(\frac{q''_{LB}}{q''_0} - 1\right) e^{384/P}$$

Reynolds⁽¹⁷⁾ correlated his data taken at pressures from 45 to 100 psia and heat fluxes from 130,000 to 304,000 Btu/hr ft² with the following equation:

$$\Delta P_{LB} = \left(\frac{dP}{dL}\right)_1 \frac{GDC_p (\Delta T_{sub})_0}{4aq''} \sinh \left[\frac{4aq''L}{GDC_p (\Delta T_{sub})_0} \right]$$

where

$$a = 4.6 \left(\frac{q''}{10^6} \right) + 1.2$$

The dimensional units of the parameters used are

$$G - \text{lb/hr ft}^2$$

$$\Delta P_{LB} - \text{in. of H}_2\text{O}$$

$$\left(\frac{dP}{dL}\right)_1 - \text{in. of H}_2\text{O/ft}$$

$$q'' - \text{Btu/hr ft}^2$$

$$\Delta T_{\text{sub}} - ^\circ\text{F}$$

$$C_p - \text{Btu/lb } ^\circ\text{F}$$

$$D - \text{ft}$$

Reynolds compared this correlation with the UCLA data⁽³⁾ and found that it predicted the local boiling pressure drop to within $\pm 50\%$ for heat fluxes up to 500,000 Btu/hr ft² over the entire pressure range.

Sher⁽²⁰⁾ has reported a design equation for rectangular channels at 2000 psi. For these conditions he recommends the use of the following equation:

$$\frac{\left(\frac{dP}{dL}\right)_{\text{LB}}}{\left(\frac{dP}{dL}\right)_1} = (1 - 0.0025 \Delta T^*) \left[1 + 0.76 \left(\frac{10^6}{G}\right)^{\frac{2}{3}} \psi \right] ,$$

$$\text{where } \psi = 1 - \frac{\Delta T^*}{\Delta T_f} \quad (\text{II.10})$$

and

$$\Delta T^* = T_{\text{sat}} + \frac{60 \left(\frac{q''}{10^6}\right)^{0.25}}{e^{P/90}} - T_B .$$

Of these, perhaps the best general equation for estimating frictional losses in subcooled boiling is that of Reynolds.

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SECTION III

THE PREDICTION OF STEAM VOLUME FRACTIONS
IN BOILING SYSTEMS

John F. Marchaterre and M. Petrick

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A. NOMENCLATURE

C_s/C_l	Slip ratio
e	Base of natural logarithms
G	Mass flow rate, lb mixture/hr ft ²
L	Length, ft
L_B	Boiling length
L_{NB}	Nonboiling length
L_t	Total length
Nu	Nusselt number
P	Pressure, psia
Pr	Prandtl number
Q_B	Heat input to boiling section
Q_t	Total heat input
q''	Heat flux, Btu/hr ft ²
Re	Reynolds number
ΔT_{sat}	Wall superheat, $T_{wall} - T_{saturation}$
V_f	Water velocity, ft/sec
V_g	Steam or gas velocity, ft/sec
V_0	Superficial velocity (defined as G/ρ_f), ft/sec
x	Quality, lb steam/lb mixture
x_c	Quality at inception of local boiling
x_{tt}	Parameter of Martinelli
α	Steam volume fraction
ρ_f	Density of liquid, lb/ft ³
ρ_g	Density of gas, lb/ft ³

B. INTRODUCTION

The ability to accurately predict the steam volume fractions in boiling systems is a prerequisite for a competent evaluation of the performance characteristics of such systems. This is especially true in boiling water reactors where the nuclear aspects are intimately inter-related to the two-phase flow characteristics of the system. The reactor recirculation rate, coolant moderator density, core neutron kinetics, and reactor stability are all dependent upon the steam volume fraction.

Because of the complexity of the boiling process, the approach to the prediction of the density of steam-water mixture is of necessity almost completely empirical. As a result, considerable effort has been expended during the past few years by a number of investigators on the experimental measurement of steam volume fractions. Some of the pertinent information derived from these studies is described briefly.

Martinelli et al.⁽¹⁷⁾ obtained liquid holdup data at atmospheric pressure and correlated this data as a function of quality and a physical property parameter, x_{tt} . They also presented an extrapolation of their atmospheric pressure correlation to regions of higher pressure. Dengler⁽³⁾ reported measurements of steam volume fractions at atmospheric pressure, using a radioactive tracer in the water for volume fraction measurements. Data were correlated using the Martinelli x_{tt} parameter. Isbin et al.^(5,10,11,13) obtained volume fraction data for both horizontal and vertical flow over a wide pressure range and found that the velocity ratios were a function of mixture quality as well as pressure.

Lottes and Flinn,⁽¹⁴⁾ Cook,⁽²⁾ Petrick,⁽¹⁸⁾ Richardson⁽²¹⁾ and Marchaterre⁽¹⁵⁾ have reported results from studies at ANL with air-water systems and steam-water systems. The superficial velocity and quality were found to affect the velocity ratio; significant differences between horizontal and vertical flow were also found.

Egen, Dingee and Chastain⁽⁴⁾ have reported results from narrow rectangular channels at 2000 psi. The volume fractions were found to be heat flux dependent well into the net quality region. The velocity ratio was independent of quality and velocity over the range of variables tested. Hughes⁽⁹⁾ presented volume fraction measurements obtained from large equivalent diameter adiabatic systems at 1200, 1400, 1800, and 2400 psia. The velocity ratio and velocity difference were shown to be a function of the flow rate and quality. Recently Asyee⁽¹⁾ has reported results for a cosine-heated annulus over a considerable pressure range. He also found velocity ratios to be a function of pressure, quality and circulation velocity. Schwartz⁽²²⁾ conducted investigations in horizontal and vertical boiler tubes and observed that the volume fractions were different for the two cases.

Other investigations have been performed and the work is discussed in several extensive literature surveys on the subject of two-phase flow. (6,10,16,24)

This paper summarizes all the data that has been obtained to date at Argonne National Laboratory. Several possible correlations are discussed and compared with data of other investigators for vertical up-flow of steam-water mixtures. Also working curves are presented which may be used for the prediction of the density of steam-water mixtures.

C. EXPERIMENTAL APPARATUS

A number of experimental systems have been used for studying the relative velocity of the phases in two-phase mixtures. Data were taken in both horizontal and vertical flow air-water systems, a 600-psi single-channel boiler, a 600-psi multichannel boiler, and a 2000-psi single-channel boiler.

The apparatus used for the vertical up-flow air-water studies is shown in Fig. III.1. Air and demineralized water were mixed in a mixing section and the mixture then passed upwards through the test section. Five test sections of 4-ft length and 2-in. width with spacings of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 in. were constructed from Lucite to allow visual observations and photographic studies.

A schematic diagram of a representative boiling loop is illustrated in Fig. III.2. Tests were run with both natural and forced circulation. Boiling took place in the active test section and the two-phase mixture flowed upwards through a riser section. Steam was separated at the steam-water interface and discharged to a condenser. The water returned through a crossover, where makeup water was added, and flowed down the downcomer to the test section inlet.

Steam volume fractions were measured by a gamma-attenuation technique. The equipment and instrumentation consisted basically of a Tm^{170} gamma source, a scintillation crystal photomultiplier tube arrangement, amplifier, differential analyzer, count rate meter and recorder. The attenuation of the gamma beam passing through the steam-water mixture was measured and calibrated against the attenuation observed when the section was full of water and when it was full of steam. The gamma-attenuation measurement technique has been described in detail by Petrick⁽¹⁹⁾ and by Hooker and Popper.⁽⁸⁾ Figure III.3 shows the source and photomultiplier tube arrangement in place on a test section.

FIG. III.1 ATMOSPHERIC AIR-WATER SYSTEM

FIG. III.2 SCHEMATIC DIAGRAM OF TYPICAL BOILING LOOP

FIG. III.3 SOURCE AND PHOTOMULTIPLIER TUBE ARRANGEMENT FOR DENSITY STUDIES

D. CORRELATION OF DATA

In the analysis of experimental data several choices of dependent variable for correlation can be made. The steam volume fraction can be correlated directly as a function of the various parameters or may be reduced to phase velocities. A correlation of the steam volume fraction vs. system parameters is perhaps the easiest to use but has the objectionable feature that extrapolation is difficult and may be misleading. For example, Martinelli's data, which was presented in this manner, predicts steam volume fractions that do not occur in vertical upflow systems, i.e., the steam velocity is less than the water velocity.

A more useful parameter is obtained by using an expression involving the velocity of each phase. The parameters most frequently used are the phase velocity difference, defined by the following equation:

$$V_g - V_f = V_0 \left[\frac{x}{\alpha} \frac{\rho_f}{\rho_g} - \frac{1-x}{1-\alpha} \right],$$

and the velocity ratio:

$$V_g/V_f = \left(\frac{x}{1-x} \right) \left(\frac{1-\alpha}{\alpha} \right) \frac{\rho_f}{\rho_g}.$$

These parameters have the advantage that they are more sensitive to the variation in other variables such as superficial velocity and quality. Also they have the additional advantage that they are more basic parameters,

and, using them, arguments concerning postulated mechanisms can be made more readily. Since the velocity ratio is the more convenient term, it was chosen for correlation. It is recognized that this ratio represents a mean velocity for each phase and it is not necessarily the correct velocity to be used in momentum and kinetic energy terms.

It is of interest to see how the phase velocities vary in a typical boiling channel. Figure III.4 shows the variation of the liquid velocity,

steam velocity, velocity ratio, and velocity difference for an experimental run in a $\frac{1}{4} \times 2 \times 60$ in. rectangular channel. It can be seen that both the relative velocity and the velocity ratio vary markedly with channel length and hence mixture quality.

The data of four experimental systems have been used in developing the correlations. Extensive data were taken on air-water systems to establish the effects of the various parameters at atmospheric pressure. Additional volume fraction measurements of steam-water mixtures were made on $\frac{1}{4} \times 2$, $\frac{1}{2} \times 2$ and $\frac{7}{16} \times 4$ in. rectangular channels in boiling, and 1 in. and 2 in. circular channels under adiabatic flow, conditions to extend the range of variables such as pressure, quality, etc.

E. EXPERIMENTAL RESULTS

1. Air-Water Tests

Tests were run in both horizontal and vertical flow with atmospheric air-water systems. These results have been reported by Petrick⁽¹⁸⁾ and Richardson.⁽²¹⁾ Since the tests indicated significant differences between horizontal and vertical flows, only the vertical flow tests will be considered here.

FIGURE III.5
REPRESENTATIVE RESULTS FROM
AIR-WATER STUDIES

Figure III.5 shows representative data for the velocity ratio as a function of the superficial velocity at a constant quality. The data show the relative velocity of the two phases is a function of both the water velocity and quality. The slope of the quality parameters changed only slightly over the range of qualities studied. The studies also indicated that geometrical arrangements may affect the phase velocities because of changing flow patterns, but that this effect is small.

Studies were made of the distribution of the air phase in the water by traversing the channel with a narrow gamma beam. These data showed that the distribution of the air phase in the water was parabolic in nature, but that the ratio of $\alpha_{\max}/\alpha_{\text{avg}}$ which characterizes the distribution varied at random.

The results of these studies were used as a guide in analyzing the steam-water data at higher pressures.

2. Steam Volume Fraction Data

The steam-water volume fraction data were analyzed, using the air-water results as a guide. When the slip ratios were plotted as a function of the same parameters, similar effects of flow rate, quality, pressure and geometrical arrangement were observed (see Figs. III.5 to III.16).

The data shown are typical of the great quantity of data accumulated during the past few years on a number of experimental loops. It can be seen that each of the parameters by itself does not affect the velocity ratio greatly; however, several such parameters in combination could produce a substantial effect. It is possible, therefore, that the differences in results reported in the literature by the various investigators are not due as much to differences in experimental technique as they are to the aforementioned effects. Effects of this type can be found only if experimental measurements are such that the velocity ratios can be calculated to within $\pm 10\%$, which is extremely difficult experimentally.

FIGURE III.6
EFFECT OF GEOMETRY ON VELOCITY RATIO AT 600 psig

As an example, effects of channel geometry have not been clearly established. Figure III.6 shows a representative difference between data taken on a 2-in. diameter circular channel and data taken on a 0.622 in. diameter circular channel. This is a representative trend but the effect has not been noted in all cases.

Since the effect cannot be predicted on the basis of present data, it has not been included in the data correlation.

3. Quality Effects

The effect of quality on the slip ratio is shown in Figs. III.6 to III.12. The data encompass a superficial water velocity range from 1.4 to 2.5 ft/sec (the average velocity of approximately 2 ft/sec), a pressure range from 150 to 1500 psi, and a flow channel equivalent diameter range from 0.44 to 2 in. These parameters are indicated on the figures. The lines drawn through the data represent a least squares fit of a first degree polynomial. It can be seen that the slopes of the lines, and hence effect of quality on the slip ratio, decrease with the pressure for the range of variables specified.

FIG. III.7
THE EFFECT OF QUALITY ON VELOCITY RATIO AT 150 psig

FIG. III.8
THE EFFECT OF QUALITY ON VELOCITY RATIO AT 250 psi - $V_o \approx 1.5 \text{ ft/sec}$

FIG. III.9

EFFECT OF QUALITY ON VELOCITY RATIO AT 600 psig

FIG. III.10

THE EFFECT OF QUALITY ON VELOCITY RATIO AT 1000 psi;
 $V_0 \approx 2.5$ ft/sec

FIG. III.11
VELOCITY RATIO AS A FUNCTION OF QUALITY AT 1500 psig

FIG. III.12
THE EFFECT OF SUPERFICIAL VELOCITY ON VELOCITY RATIO AT 1200 psia - DATA OF HUGHES⁽⁹⁾

Data of other investigators were analyzed in an analogous manner to obtain a comparison. These data are also plotted in Fig. III.13, and, as can be seen, tend to check the ANL data. The quality effects on the slip ratio also appears to diminish with increasing mass velocity. This has been observed in recent data obtained in the higher mass velocity range. However, sufficient data have not been accumulated to quantitatively establish the magnitude of effect. It appears that a family of curves of the type shown in Fig. III.13 may be obtained.

FIG. III.13
SLOPE OF $\frac{V_g}{V_f}$ QUALITY LINES AS A FUNCTION OF PRESSURE

4. Mass Velocity Effect

The magnitude of velocity effect on the slip ratio is illustrated in Figs. III.7, III.12 and III.14 for a steam-water system and in Fig. III.5 for an air-water system.

It can be seen that the slip ratio shows the same dependency on the superficial velocity for both systems. As the superficial velocity increases, the slip ratio decreases. The magnitude of the velocity effect appears to decrease with decreasing quality and increasing pressure.

FIG. III. 14

VELOCITY RATIO AS A FUNCTION OF SUPERFICIAL VELOCITY AT 150 & 600 psig
 $X \approx 0.03$

From Figs. III.7 and III.12 it can be seen that for a constant quality of the slope of the V_g/V_f vs. V_0 line decreases with increasing pressure. It is interesting to note, however, that the velocity effect is greater at 150 psi than that observed at atmospheric pressure (see Figs. III.5 and III.14). This apparent incompatibility is probably due to the quality effect. Extrapolation of the quality range of the air-water data to coincide with the quality range of the steam-water data would make the velocity effect greater than that observed at 150 psi.

The decrease in the velocity effect with decreasing quality can be seen in the data from ANL for 150 psi (Fig. III.7) and from the data of Hughes⁽⁹⁾ at 1200 psi (Fig. III.12). The slopes of the slip ratio-quality lines increase with decreasing velocity. Thus at low mixture qualities there is very little effect of the mass flow rate on the slip ratio, whereas in the higher quality range the velocity effect is substantial. It appears that the mass velocity effect also decreases with pressure (see Figs. III.7 and III.12).

5. Pressure Effects

As expected, the slip ratio decreases with increasing pressure; this is illustrated in Fig. III.15. The lines shown represent a least square fit of the data at pressures of 150, 200, 600, 1000 and 1500 psi at approximately the same superficial velocity and in similarly sized channels. The dashed line shown represents a series of data points obtained from a 1 in. pipe at 1500 psig, whereas the solid line at the same pressure represents

data taken from a 2-in. pipe. No conclusive explanation can be given for the inconsistencies between the two sets of data. However, the velocity measurements for the 1-in. pipe were subject to question due to probable errors in the venturi measurement. This in turn would be reflected in the calculated slip ratio through the mixture quality. A new venturi was installed prior to running the several series of data points in the 2-in. pipe.

FIG. III.15
VELOCITY RATIO AS A FUNCTION OF QUALITY AT VARIOUS PRESSURES

The pressure effect on the slip ratio is perhaps more vividly demonstrated in Fig. III.16 for constant parameters of velocity ($V_0 \approx 1-3$ ft/sec) and quality ($x \approx 0.05$). Data of other investigators at the higher pressures are also compared with the ANL data and, as can be seen, the data check quite well. The extrapolation into the very high pressure range as indicated in the figure was based on the variation of the density difference of the two phases. The density difference is a measure of the buoyancy force which is a factor affecting the relative velocity of the two phases. The data of Hughes,⁽⁹⁾ and Egen *et al.*,⁽⁴⁾ which are shown on Fig. 16, were also used as a guide; however, their data are subject to question. The data of Hughes may be in error since only centerline void measurements were taken and multiplied by a constant $\alpha_{\max}/\alpha_{\text{avg}}$ to obtain the average voids at any cross section. Data taken at ANL⁽¹⁸⁾ have indicated that this multiplier varied considerably. The data of Egen *et al.* were obtained from a very narrow rectangular channel (0.1 x 1 in.) and, as a result, the low slip ratio

may be due to the geometrical effect as pointed out previously. Also, it should be noted that the accuracy of void measurements tends to decrease with increasing pressure, especially for small-width channels.

FIG. III.16

VELOCITY RATIO AS A FUNCTION OF PRESSURE;
 $V_0 \approx 1$ to 3 ft/sec and $X \approx 0.05$

Curves using this method of extrapolation have been drawn and are presented in Fig. III.20 for pressures of 1000, 1500 and 2000 psia. Extrapolation beyond these quality ranges should probably not be made on a linear basis, since there are indications that over a wide quality range slip is not a linear function of quality.

6. Working Curves for the Prediction of Steam Volume Fractions

Working curves for predicting slip ratios at any pressure can be made by extrapolating the data presented. Initially, working curves were prepared at 150, 250, and 600 psig by smoothing the velocity ratio data with respect to quality and velocity at each of the pressures. These curves represent an attempt to get the best smooth fit of the data at each pressure and are shown in Figs. III.17, III.18 and III.19. Comparisons of the working curves with actual data at 600 psig and various qualities are shown in Figs. III.22 to III.24. Also, an error plot of all the ANL data at 600 psi is shown in Fig. III.21, which serves to demonstrate the accuracy of the working curve.

FIG. III.17

WORKING CURVE FOR PREDICTION OF
VELOCITY RATIOS AT 150 psig

FIG. III.18

WORKING CURVE FOR PREDICTION OF
VELOCITY RATIOS AT 250 psig

FIG. III.19

WORKING CURVE FOR THE PREDICTION OF
VELOCITY RATIOS AT 600 psig

FIG. III.20

WORKING CURVES FOR THE PREDICTION OF VELOCITY RATIOS
AT 1000, 1500, AND 2000 psig.

FIG. III.21
ERROR PLOT OF ARGONNE 600-psig
DATA USING WORKING CURVE

FIG. III.22
VELOCITY RATIO AS A FUNCTION OF SUPERFICIAL VELOCITY
AT $X=0.022$ AT 600 psig

FIG. III.23
VELOCITY RATIO AS A FUNCTION OF
SUPERFICIAL VELOCITY AT 600 psig

FIG. III.24
VELOCITY RATIO AS A FUNCTION OF SUPERFICIAL VELOCITY
AT $X=0.06$ AT 600 psig

The 600-psig working curve (Fig. III.19) and Fig. III.16, which gives the variation of the velocity ratio as a function of pressure at $x \approx 0.05$ and $V_0 \approx 2$ ft/sec, can be used as a basis for predicting velocity ratios at the high pressures. From Fig. III.16 the slip ratio is obtained at any pressure for $x \approx 0.05$ and $V_0 \approx 2$ ft/sec. This value represents the intercept of a velocity ratio-quality curve at these conditions. The slope of the curve can then be estimated from Fig. III.13. The slope of the velocity effect can be assumed to vary with the density difference ($\rho_f - \rho_g$) and can be extrapolated from the 600-psig data. Since the slopes are quite small (< 0.25), a linear approximation may be adequate. Using extrapolated slopes for several qualities and the derived slip ratio-quality curve, a working curve at the pressure of interest can be drawn. An alternate, simpler method of drawing working curves would be possible if sufficient data were available to establish a family of curves of the type shown in Fig. III.13.

7. Steam Volume Fractions in Subcooled Boiling

Steam volume fractions have been measured locally in the subcooled boiling region in $\frac{1}{4} \times 2$ -in. and $\frac{1}{2} \times 2$ -in. vertical channels. The volume fraction was found to be a function of the bulk water temperature, velocity, heat flux, pressure and channel dimensions. The effects of heat flux and velocity in a $\frac{1}{4}$ -in. rectangular channel are shown in Figs. III.25 and III.26. It can be seen that the steam volume fraction is dependent upon heat flux well into the net quality region. Because of this effect the data have been plotted as a function of $x + 0.02$ to give a continuous function through $x = 0$.

Several methods of correlating the data have been attempted, but as yet no adequate correlation for the data has been found. One of the most promising is shown in Figs. III.27 and III.28. The steam volume fraction is correlated using $(x - x_c)$ as a parameter, where x_c is the negative quality at the inception of local boiling.

relation, (12) These qualities were evaluated using the Jens and Lottes

$$\Delta T_{\text{sat}} = \frac{60 (q''/10^6)^{0.25}}{P_e/900},$$

and the Colburn relation,

$$\text{Nu} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4}.$$

The data of Egen, Dingee and Chastain have also been plotted in this fashion and found to be adequately correlated (Fig. III.28). Since these volume fractions vary considerably with channel dimensions, data from a number of geometrical arrangements must be obtained to establish

FIG. III.25
EFFECT OF VELOCITY ON STEAM VOLUME FRACTIONS
IN LOCAL BOILING IN $\frac{1}{4} \times 2$ IN. CHANNEL

FIG. III.26
EFFECT OF HEAT FLUX ON STEAM VOLUME FRACTIONS IN
LOCAL BOILING IN $\frac{1}{4} \times 2$ IN. CHANNEL

FIG. III.27
TENTATIVE METHOD OF CORRELATION OF STEAM VOLUME
FRACTIONS IN LOCAL BOILING AT 600 psig,
 $q'' = 59,000 \frac{\text{BTU}}{\text{hr-ft}^2}$

FIG. III.28
DATA OF EGEN, DINGEE AND CHASTAIN AT 2000 psia

this effect. The assumption of a constant void thickness was found to be inadequate in a comparison of data at identical conditions from $\frac{1}{4} \times 2$ -in. and $\frac{1}{2} \times 2$ -in. rectangular channels.

Griffith, Clark and Rohsenow⁽⁷⁾ have presented a model for steam volume fractions in subcooled boiling. The model breaks down at low flow rates and low heat fluxes as predicted by the authors, and indeed was found to deviate by several orders of magnitude for the range covered by these investigations.

Poletavkin and Shapkin⁽²⁰⁾ have also reported results for subcooled boiling in a vertical 5.6-mm-inside diameter test section. Steam volume fractions were measured using a tracer technique; the steam volume fraction was correlated by the following empirical relationship:

$$\alpha = 1 - 12 \left(\frac{V \Delta T}{q''} \right)^{0.25} P^{-0.07},$$

where the units used are:

ΔT is in $^{\circ}\text{C}$

q'' is in million kg-cal/meter²

P is in atmospheres

V is in meters/sec.

Additional geometrical arrangements were not tried, so no generalizations can be made regarding the data.

F. PREDICTION OF VOID PROFILE

The working curves and method of extrapolation presented in these notes can be used to predict the variation of steam volume fractions with channel length. A representative run showing the variation of steam volume fraction with length in a $\frac{1}{4} \times 2 \times 60$ -in. rectangular channel is shown in Fig. III. 29. The dashed line represents the calculated values of steam volume fraction using the slip ratios obtained from the working curves. The local boiling steam volume fractions were estimated from the correlation for local boiling volume fractions for the particular channel (Figs. III.25 and III.26).

FIG. III.29

COMPARISON OF PREDICTED AND MEASURED STEAM VOLUME FRACTIONS AT 600 psig

G. PREDICTION OF AVERAGE STEAM VOLUME FRACTION

The average density in a channel in which vaporization is taking place may be found as a function of pressure, slip ratio and exit steam volume fraction. For the case of a constant heat flux along the channel (if it is assumed that the slip ratio is constant along the heated length and that there is a uniform input),

$$\frac{\bar{\rho}_B}{\rho_f} = 1 - \frac{1 - \frac{\rho_g}{\rho_f}}{1 - x} \left\{ 1 - \left(\frac{1}{e^{(1-x)}} - 1 \right) \left[\ln \frac{1}{1 - \frac{1}{e^{(1-x)}}} \right] \right\}$$

Where $x = V_g \rho_g / V_f \rho_f$. For an axial cosine heat input,

$$\frac{\bar{\rho}_B}{\rho_f} = 1 - \left(1 - \frac{\rho_g}{\rho_f} \right) \left[\frac{b'}{b} + \frac{a'b - ab'}{b(a^2 - b^2)} \left(\frac{L_t}{L_B} \right) \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\sqrt{a^2 - b^2} \tan(\pi L_{NB}/2L_t)}{a + b} \right) \right]$$

for $a > b$, and

$$\frac{\bar{\rho}_B}{\rho_f} = 1 - \left(1 - \frac{\rho_g}{\rho_f} \right) \left[\frac{b'}{b} - \frac{a'b - ab'}{\pi b \sqrt{b^2 - a^2}} \left(\frac{L_t}{L_B} \right) \ln \frac{\sqrt{b^2 - a^2} \tan(\pi L_{NB}/2L_t) + a + b}{b^2 - a^2 \tan(\pi L_{NB}/2L_t) - a - b} \right]$$

for $a < b$

where

$$a = \frac{2}{\alpha_e} + b$$

$$a' = 2 + b$$

$$b = [1 - x] b'$$

$$b' = -\frac{Q_t}{Q_B} = -\left[\frac{2}{1 - \cos \frac{\pi L_B}{L_t}} \right]$$

These equations are shown graphically in Figs. III. 30, III. 31 and III. 32.

Figure III. 33 shows a comparison between calculated and measured average volume fractions. These curves are slightly in error due to the assumption of a constant velocity ratio over the channel length but are adequate for design calculations, providing that the exit steam volume fraction of the boiling channel does not exceed 0.75. The exit steam volume fraction of the channel is estimated by using the working curves for velocity ratio presented in this paper, and the average steam volume fraction is then read off Figs. III. 30, III. 31 and III. 32. These curves are also in error since they neglect the volume fraction in local boiling. A more accurate density can be obtained for irregular flux distributions by calculating the steam volume fraction as length curve and graphically integrating.

H. CONCLUSIONS

The working curves presented for the calculation of velocity ratios and hence steam volume fractions are adequate for engineering calculations and will predict steam volume fractions to within $\pm 15\%$. Additional work is needed to determine effects of geometry, differences between adiabatic flow and boiling at high heat fluxes, and means of predicting steam volume fractions in the local boiling region.

FIG. III.30

RATIO OF MEAN DENSITY IN VERTICAL BOILING CHANNEL
TO LIQUID DENSITY FOR UNIFORM HEAT GENERATION AND
SLIP RATIO OF UNITY ($C_s = 1$)

FIG. III.31

RATIO OF MEAN DENSITY IN VERTICAL BOILING CHANNEL
TO LIQUID DENSITY FOR UNIFORM HEAT GENERATION, FOR
PRESSURES OF 500 AND 2000 PSIA, SLIP RATIO OF 1 TO 3.

FIG. III.32
MEAN DENSITY RATIO IN BOILING SECTION OF VERTICAL
CHANNEL AS FUNCTION OF BOILING LENGTH RATIO FOR A
COSINE HEAT INPUT

FIG. III.33
COMPARISON OF CALCULATED AND
MEASURED AVERAGE VOLUME FRACTIONS

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SECTION IV
CRITICAL HEAT FLUX OR "BURNOUT"

J. F. Marchaterre

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A. NOMENCLATURE

C	Constant
$\bar{c}_l$	Mean specific heat of liquid
c_p	Specific heat
D	Diameter
e	Base of natural logarithms
G	Mass velocity
g	Acceleration due to gravity
g_c	Gravitational constant
h	Heat transfer coefficient
h_b	Bulk enthalpy
h_{bo}	Enthalpy at burnout point
h_f	Enthalpy of liquid
h_{fg}	Latent heat of vaporization
h_g	Enthalpy of vapor
h_t	Total enthalpy
K	Thermal conductivity
L	Length
P	Pressure
P_c	Critical pressure
Pr	Prandtl number
q''	Heat flux
q_{bo}	Heat flux at burnout point
q_p''	Peak heat flux
r	Radius
S	Spacing between plates in a rectangular channel
T, t	Temperature
t_b	Bulk temperature
t_s	Saturation temperature
u	Velocity

x	Quality
α	Volume fraction of vapor
μ	Viscosity
ρ_b	Bulk density
ρ_g	Vapor density
ρ_l	Liquid density
σ	Surface tension

B. INTRODUCTION

One of the major problems in reactor design is the prediction of the maximum heat flux or "burnout" point. If boiling takes place on an electrically heated wire immersed in a pool of water and the temperature difference between the wire and the water is measured, a characteristic curve of the type shown in Fig. I.1 will be obtained.

Initially, the wire will be cooled only by convection currents in the water. As the heat input to the heater is increased, vapor bubbles will begin to grow on the heater surface and collapse as the bubble touches the main body of subcooled liquid. This is known as local or subcooled boiling, and is characterized by a change in slope of the heat flux-temperature difference curve. As the water temperature increases to the saturation point, vapor bubbles will grow and detach themselves from the heater surface and "net" or "bulk" boiling will occur. Both types of boiling are known as "nucleate boiling." As the heat flux is increased, a vapor film will tend to form on the heater surface. The change in slope is defined as the departure from nucleate boiling.

As the heat flux is further increased, a maximum heat flux, which is referred to as the "critical heat flux" or "burnout" point, is reached, since most heater materials will fail because of the sudden large temperature increase. Characteristics of the same type are observed in flowing channels. An excellent review of existing burnout data is contained in Ref. 6.

C. POOL BOILING

Cichelli and Bonilla⁽⁵⁾ have correlated $q''_{\max}$ in pool boiling for several organic liquids. They found they could correlate the data for many fluids by plotting the peak flux, q''_p , divided by the critical pressure as ordinate with the reduced pressure as the abscissa. Other investigators have obtained data for water in agreement with this correlation, which predicts that $q''_{\max}$ is greatest at about 1000 psia, the heat flux being approximately 1,250,000 Btu/hr ft². Kazakova⁽¹²⁾ has reported results for the upwards flow of fluids in vertical tubes at low velocities that indicate that the pressure effect in flowing systems is essentially the same as that predicted by Cichelli and Bonilla's correlation.

D. LOCAL BOILING BURNOUT

For local or subcooled boiling with forced circulation the attainable heat fluxes are much higher than for pool boiling burnout. Several investigators^(1,4,9,11) have reported results and correlations of the critical or maximum heat in the subcooled boiling region. The experimental data were obtained on long, electrically heated tubes or annuli and burnout or overheating occurred at the outlet end. The data were all found to be a function of the velocity, the subcooling ($\Delta t_{\text{sub}} = t_{\text{sat}} - t_f$) at the exit of the tube, and the pressure, though there are some unexplained discrepancies between the data of various investigators.

Jens and Lottes⁽¹¹⁾ have analyzed the data of several investigators and recommended the following equation for use in predicting burnout in the subcooled boiling region:

$$q''_{\max}/10^6 = C(G/10^6)^m \Delta t_{\text{sub}}^{0.22}$$

where $q''_{\max}$ is expressed in Btu/hr ft², G is expressed in lb_m/hr ft², and Δt is in °F.

The constants C and m are a function of pressure as follows:

<u>P(psia)</u>	<u>m</u>	<u>C</u>
500	0.16	0.817
1000	0.28	0.626
2000	0.50	0.445

This is the recommended equation for use in estimating "burnout" in subcooled conditions at pressures from 500 to 2000 psia. Additional subcooled burnout correlations are those of McAdams,⁽¹⁷⁾ Gunther,⁽⁹⁾ Buchberg⁽⁴⁾ and Bernath.⁽²⁾

E. NET BOILING BURNOUT

The problem of burnout with two-phase flow in enclosed tubes is a much more complex one. Large amounts of data have been collected for burnout with net steam generation, but as yet no adequate correlation exists for the prediction of critical heat fluxes over a wide range of parameters. The problem is further complicated by the fact that the critical heat flux may not be the same for natural and forced circulation. There are indications that the instabilities in natural circulation will lead to a premature transition to film boiling because of flow oscillations.

Numerous investigators have reported experimental measurements of the critical or "burnout" heat flux and a variety of types of correlation of the experimental data have been tried. Some are statistical correlations of raw experimental data, others are attempts of more basic approaches. These will be listed here for comparison.

R. A. DeBortoli, S. J. Green et al.⁽⁶⁾ recommend the following design equations for use at 2000 psi:

For round tubes ($0.2 \times 10^6 \leq G \leq 8.0 \times 10^6$ lb/hr ft²):

$$\frac{q_{bo}}{10^6} = 0.186 \left(\frac{h_{bo}}{10^3} \right)^{-2.5} \left(1 + \frac{G}{10^7} \right)^2 e^{-0.0012 L/D}$$

For rectangular channels ($1.6 \times 10^6 \leq G \leq 5 \times 10^6$ lb/hr ft²):

$$\frac{q_{bo}''}{10^6} = 0.240 \left(\frac{h_{bo}}{10^3} \right)^{-2.5} \left(1 + \frac{G}{10^7} \right)^2 e^{-0.012 L/S}$$

For rectangular channels ($0.2 \times 10^6 \leq G \leq 1.6 \times 10^6$ lb/hr ft²):

$$\frac{q_{bo}''}{10^6} = 0.325 \left(\frac{h_{bo}}{10^3} \right)^{-2.5} e^{-0.0012 L/S}$$

The actual correlations of the data have coefficients of 0.28, 0.37, and 0.50, respectively, instead of 0.186, 0.240, and 0.325. The reduction of 65% is an attempt to attain a high degree of confidence that burnout will be avoided. Additional conservative equations are recommended for lower pressures.

Griffith⁽⁸⁾ has presented a burnout correlation, applicable from atmospheric to 3000 psia, as follows:

$$q_{bo}'' = f[P/P_c] (F) \rho_g (h_g - h_b) \left[\frac{g(\rho_l - \rho_g)}{\mu_l} \left(\frac{K_l}{\rho_l c_l} \right)^2 \right]^{\frac{1}{3}}$$

The factor (F) is defined by

$$(F) = 1 \times 10^{-6} \left(\frac{u_b D \rho_1}{\mu_1} \right) + 0.014 \left[\frac{\rho_1 \bar{c}_1 (t_s - t_b)}{\rho_g h_{fg}} \right] + 0.5 \times 10^{-3} \left\{ \left(\frac{u_b D \rho_1}{\mu_1} \right) \left[\frac{\rho_1 \bar{c}_1 (t_s - t_b)}{\rho_g h_{fg}} \right] \right\}^{\frac{1}{2}}$$

The Griffith correlation was developed from an existing pool boiling correlation for saturated liquids by modifying the bubble growth rate term to account for the effects of fluid velocity and subcooling.

Reynolds⁽¹⁹⁾ has presented a dimensionless correlation of burnout data for water at pressures of 500 to 2000 psia.

$$\left(\frac{\delta}{r} \right)^{1.94} = 1.49 \times 10^{-23} \left(\frac{q'' D}{\sigma u_b} \right) \left(\frac{\rho_b}{\rho_1} \right) \left(\frac{GD}{\mu_1} \right)^{1.5} \left(\frac{h_{fg} g_c}{gD} \right)$$

The dimensionless film thickness (δ/r) is given by $1 - \sqrt{\alpha}$.

Additional correlations are those of Sonemann,⁽²⁰⁾ Hoe and Helve,⁽¹⁰⁾ and Lowdermilk.^(14,15) All of the correlations presented fall short of the objective of a generalized method of predicting the "burnout" point over a wide range of reactor design conditions. It is suggested that the equations be compared by the reactor designer interested in this problem at the conditions of interest to give him a method of evaluating his design flux. The actual experimental data of a variety of investigators has been reported in Ref. 6 and can be used as a guide. It will be noticed that the various correlations predict widely different burnout points.

Figures IV.1, IV.2 and IV.3 give examples of some of the data of Weatherhead⁽²¹⁾ for vertical upflow burnout studies in circular tubes and illustrate some of the effects to be found. The data in each case have been plotted as a function of the enthalpy at the "burnout" point, defined as $h_t = h_f + xh_{fg}$.

Figure IV.1 indicates the effect of mass velocity on critical heat flux, indicating a decreasing effect with increasing enthalpy.

A representative effect of channel geometry is shown in Fig. IV.2. The effect of channel geometry decreases with increasing enthalpy. Fig. IV.3 shows the effect of pressure on critical heat flux in these tests. No satisfactory explanation can be given for any of the phenomena, and no general correlation of the experimental data has been developed. In all of

these cases straight lines have been drawn through the data, though they are probably not linear relationships beyond the range of the data. Some idea of the complexity of the problem of obtaining an adequate design procedure with respect to predicting the critical heat flux can be gained from these experimental data. As can be seen, extrapolation of data of this type could be extremely misleading.

FIG. IV.1
DEPENDENCY OF CRITICAL HEAT FLUX ON MASS VELOCITY
AT 2000 PSI; $D = 0.304$ INCH

FIGURE IV.2
EFFECT OF DIAMETER ON
CRITICAL HEAT FLUX AT 2000 PSI;
 $G = 1 \times 10^6 \text{ LB/HR FT}^2$

FIGURE IV.3
EFFECT OF PRESSURE ON CRITICAL HEAT FLUX;
 $G = 0.5 \times 10^6 \text{ LB/HR FT}^2$ AND $D = 0.306 \text{ IN.}$

As a result, the reactor designer must usually incorporate large factors of safety unless specific data for the geometrical arrangement and variables in question are available, though usually other factors, such as fuel element temperature, limit the design heat flux.

It should be mentioned here for the sake of completeness that the standard methods of core analysis include the use of hot channel factors and "uncertainty" factors. The factors arise from a number of conditions, such as the nuclear flux distribution, flow distribution, differences in fuel concentration, etc. The use of these is described in detail in Ref. 13. It should be remembered in connection with this that burnout will not necessarily occur at the channel exit or the point of maximum heat flux, but at some intermediate point, usually very close to the point at which the maximum surface temperature would occur if a single phase system were considered.

One of the most important design needs of burnout information is in the case of forced-circulation reactors, where the reactor must have adequate cooling during loss of flow transients.

F. FILM BOILING

Current design practice is to not allow the heat flux at any point in the reactor to exceed the "burnout" heat flux. However, under certain conditions the attainment of the critical heat flux may not necessarily mean that the heating surface will melt, that is, the critical flux and "burnout" may not be synonymous. The temperature excursion of the fuel element will depend on the heat transfer that can be attained in film boiling. If heat transfer coefficients during transition and film boiling can be predicted, then the resulting temperature rise can be calculated. If this temperature rise is allowable, then the burnout point can be exceeded. This could be of importance in relaxing hot channel factors, particularly during transient conditions in cases where the reactor is being shut down because of a loss of flow accident, and heat fluxes are quite low.

Very little work that is specifically applicable to boiling reactor design has been done.

Bromley⁽³⁾ obtained data on film boiling outside a horizontal tube having an outside diameter of 0.35 in. His experiments were conducted at atmospheric pressure with a variety of fluids, including water. He found that the results could be predicted by a simple equation, derived in a similar manner to Nusselt's equations for condensing with $g\rho$ replaced by $g(\rho_l - \rho_g)$. The integration of the resulting differential equation gave

$$h = (\text{const.}) \left[\frac{K^2 \rho_g (\rho_l - \rho_g) g h_f g c_p}{D \Delta T \text{Pr}} \right]^{\frac{1}{4}}$$

The constant has the value of 0.512 if the liquid surrounding the tube is stagnant and 0.724 if the liquid can move freely.

The same type of analysis can be applied to a vertical plate or tube and modified for the effect of water flowing past the film, though as yet there are no experimental results to justify the validity of the analysis.

Ellion⁽⁷⁾ conducted experiments using water in the partial film boiling region. The experiments were conducted at high heat fluxes using water flowing in an annulus at subcooled conditions. He found the film boiling coefficient to be independent of water pressure, velocity and subcooling.

Some experimental results have been reported by MSA Research Corporation⁽¹⁸⁾ for partial film boiling of water at 2000 psi. Film coefficients as low as 100 Btu/hr ft² were measured. No correlation of the data could be made.

Considerable additional experimental work is needed before heat transfer coefficients in the film boiling region can be estimated with accuracy.

G. CONCLUSIONS

No adequate method now exists for predicting the critical heat flux for a wide range of design conditions without specific data for the geometrical arrangements in question. If design heat fluxes fall into a questionable range, then specific tests must be run on the geometry in question.

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SECTION V

CALCULATION PROCEDURES FOR BOILING SYSTEMS

M. Petrick

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A. NOMENCLATURE

A	Area
A_p	Area of pipe
D	Equivalent diameter
F	Friction factor
F_{fg}	Force expended by gas phase in overcoming friction
F_{fl}	Force expended by liquid phase in overcoming friction
F_y	Force of the y'th variety
f	Moody friction factor
G	Mass flow rate
g	Acceleration of gravity
g_c	Conversion factor
h	Enthalpy
K_c	Contraction coefficient
K_E	Expansion coefficient
k	Thermal conductivity
L	Length
m	Mass
P	Pressure
Pr	Prandtl Number
q	Power
q''	Heat flux
Re	Reynold's number
r	Acceleration multiplier of Martinelli and Nelson
T	Temperature
T_w	Wall temperature
t	Time
V, v	Velocity
V_g	Velocity of gas phase
V_l	Velocity of liquid phase

W_g	Weight flow of gas
W_l	Weight flow of liquid
x_e	Quality at exit
α	Volume fraction of vapor
ρ	Density
τ_w	Shear stress

B. NATURAL CIRCULATION SYSTEM ANALYSIS

A natural circulation system is one in which fluid flow is derived through a density potential existing between vertical segments of the system, commonly referred to as the riser and downcomer. The density differential may be induced by a variety of methods, such as maintaining a sufficient temperature difference by heating or cooling one of the legs, etc.

Of particular interest with respect to boiling reactors is the system where sufficient heat is added to one leg to cause boiling of the fluid.

Analysis of such a natural circulation system is complicated by the interrelationship existing between various system parameters such as power, steam volume and weight fraction, circulation velocity, subcooling, etc., and plagued by inadequacy of information on two-phase flow concerning the relative velocity between the steam and water phases, two-phase friction factors, expansion and contraction losses of two-phase fluids, etc. The basic research in the field of two-phase flow continues in an attempt to provide sufficient information on the above-mentioned problems that can be utilized in making a competent hydrodynamic analysis on natural circulation systems.

An analytical procedure for calculating the performance of a natural circulation system is presented, whose development was aided by comparing numerous data obtained from laboratory tests on natural circulation systems with varying geometries. The laboratory tests were required to establish the validity of a number of assumptions made in the analysis.

An analysis of a basic simple natural circulation system has been given by Lottes and Flinn.* However, the authors did not account for various riser heated section geometries which tend to complicate the problems considerably,

*Lottes, P. A., and W. S. Flinn, A Method of Analysis of Natural Circulation Boiling Systems, Nuc. Sci. Eng. 1, 461 (1956).

nor did they account for the affect of subcooling which becomes increasingly important at higher pressures. Therefore, a very general analysis was made that could be applicable to the large majority of systems. The analysis for the natural circulation system is then modified for application in a forced circulation system.

1. Hydrodynamic Analysis

In a natural circulation system

$$\sum \Delta P = 0 \quad , \quad (V.1)$$

which states that the summation of the pressure drops, ΔP , around the closed loop, consisting of riser and downcomer, must be equal to zero. The total pressure drop over a differential length may be obtained as follows:

From a force or momentum balance,

$$\sum F_y = m \frac{dv}{dt} \quad . \quad (V.2)$$

Applying Eq. (V.2) over a differential pipe length depicted below,

$$PA - A(P + dP) - A\rho dL \left(\frac{g}{g_c} \right) - \tau_w \pi D dL = m \frac{dv}{dt}$$

or

$$PA - PA - AdP - A\rho dL \left(\frac{g}{g_c} \right) - \tau_w \pi D dL = \rho \frac{AdL}{g_c} \frac{dv}{dt} \quad .$$

Simplifying

$$- dP - \rho dL \left(\frac{g}{g_c} \right) - \tau_w \frac{\pi D dL}{\pi D^2} = \rho \frac{dL}{g_c} \frac{dv}{dt}$$

By definition

$$\text{Friction factor} = F = \tau_w / \rho \frac{v^2}{2g}$$

Also, substituting $\frac{dL}{dt} = v$,

$$- dP - \rho dL \frac{g}{g_c} - 4F \rho \frac{v^2}{2g} \frac{dL}{D} = \rho \frac{v dv}{g_c}$$

or

$$- dP - \rho dL \frac{g}{g_c} - f \rho \frac{v^2}{2g} \frac{dL}{D} = \rho \frac{v dv}{g_c},$$

where f is the Moody friction factor

The total pressure drop over a differential length therefore may be written as a summation of the frictional, hydrostatic head and acceleration components:

$$- dP_T = dP_f + dP_A + dP_{acc} \quad (V.3)$$

A basic natural circulation system is schematically illustrated in Fig. V.1. The system is broken down into various segments which lend themselves conveniently to analysis. Equation (V.3) is applied to each section individually and then a summation is made to satisfy Eq. (V.1). Starting at point UP (Fig. V.1) the loop may be divided into the following sections:

a. Contraction from Upper Plenum

$$\begin{aligned} - \int_{UP}^{DT} dP &= \int_{UP}^{DT} dP_{\text{Head}} + \int_{UP}^{DT} dP_{\text{acc}} + \int_{UP}^{DT} dP_{\text{Friction}} \\ &= \frac{g}{g_c} \rho_f \int_{UP}^{DT} dL + \int_{UP}^{DT} \rho_f \frac{v dv}{g_c} + \int_{UP}^{DT} dP_{\text{Friction}} \end{aligned}$$

Key

UP	Upper Plenum
DT	Downcomer Top
DB	Downcomer Bottom
LPB	Lower Plenum Bottom
LPT	Lower Plenum Top
HSB	Heated Section Bottom
HST	Heated Section Top
RB	Riser Bottom
RT	Riser Top
D	Downcomer
LB	Inception of Local Boiling
sat	Inception of Nucleate Boiling
HS	Heated Section
A _D	Area of Downcomer
A _p	Area of Upper Plenum
L _p	Lower Plenum

FIGURE V.1

Basic Natural Circulation System

By definition for a contraction,

$$\int_{UP}^{DT} dP_{\text{Friction}} = K_{cUP} \rho_f \frac{v_{DT}^2}{2g_c},$$

so that

$$- \left[P \right]_{UP}^{DT} = \frac{g}{g_c} \rho_f \left[L \right]_{UP}^{DT} + \rho_f \left[\frac{v^2}{2g_c} \right]_{UP}^{DT} + K_{cUP} \rho_f \frac{v_{DT}^2}{2g_c}$$

or, since $L_{DT} = L_{UP}$,

$$P_{UP} - P_{DT} = \rho_f \left[\frac{v_{DT}^2}{2g_c} - \frac{v_{UP}^2}{2g_c} \right] + K_{cUP} \rho_f \frac{v_{DT}^2}{2g_c}$$

Simplifying, since $V_{UP} \ll V_{DT}$,

$$P_{UP} - P_{DT} = \rho_f \frac{v_{DT}^2}{2g_c} \left[1 + K_{cUP} \right] \quad (V.4)$$

Also from the continuity equation

$$v_{DT} A_D \rho_{DT} = v_{sat} A_{HS} \rho_f$$

Thus

$$v_{DT}^2 = v_{sat}^2 \left(\frac{A_{HS}}{A_{DT}} \right)^2 \left(\frac{\rho_f}{\rho_{DT}} \right)^2$$

Since $\rho_f = \rho_{DT}$,

$$v_{DT}^2 = v_{sat}^2 \left(\frac{A_{HS}}{A_D} \right)^2$$

Equation (V.4) thus becomes

$$P_{UP} - P_{DT} = \frac{v_{sat}^2}{2g_c} \left(\frac{A_{HS}}{A_D} \right)^2 \rho_f \left[1 + K_{cUP} \right]$$

or

$$P_{UP} - P_{DT} = \frac{v_{sat}^2}{2g_c} [N_1] \quad (V.5)$$

where $N_1 \equiv (A_{HS}/A_D)^2 \rho_f (1 + K_{cUP})$

b. Downcomer

$$\begin{aligned} - \int_{DT}^{DB} dP &= \int_{DT}^{DB} dP_{Head} + \int_{DT}^{DB} dP_{acc} + \int_{DT}^{DB} dP_{Friction} \\ &= \int_{DT}^{DB} \frac{g}{g_c} \rho_D dL + \int_{DT}^{DB} \rho_D \frac{v dv}{g_c} + \int_{DT}^{DB} f_D \rho_D \frac{v_D^2}{2g_c} \frac{dL}{D_D} \end{aligned}$$

Integrating,

$$- \left[P \right]_{DT}^{DB} = \frac{g}{g_c} \rho_D \left[L \right]_{DT}^{DB} + \rho_D \left[\frac{v^2}{2g_c} \right]_{DT}^{DB} + f_D \rho_D \frac{v_D^2}{2g_c D_D} \left[L \right]_{DT}^{DB}$$

Since $v_{DB} = v_{DT} = v_D$,

$$P_{DT} - P_{DB} = - \rho_D \left[L_{DT} - L_{DB} \right] \frac{g}{g_c} + f_D \frac{\rho_D v_D^2}{D_D 2g_c} \left[L_{DT} - L_{DB} \right]$$

Substituting

$$v_D^2 = v_{sat}^2 \left(\frac{A_{HS}}{A_D} \right)^2 \left(\frac{\rho_f}{\rho_D} \right)^2,$$

there is obtained

$$P_{DT} - P_{DB} = - \rho_D \left[L_{DT} - L_{DB} \right] \frac{g}{g_c} + f_D \frac{v_{sat}^2}{2g_c} \left(\frac{A_{HS}}{A_D} \right)^2 \left(\frac{\rho_f}{\rho_D} \right)^2 \left[L_{DT} - L_{DB} \right]$$

or

$$P_{DT} - P_{DB} = - \rho_D \left[L_{DT} - L_{DB} \right] \frac{g}{g_c} + \frac{v_{sat}^2}{2g_c} [N_2] \quad (V.6)$$

with $N_2 \equiv f_D (A_{HS}/A_D)^2 (\rho_f^2 / \rho_D D_D) (L_{DT} - L_{DB})$.

c. Expansion from Downcomer to Lower Plenum

$$\begin{aligned} - \int_{DB}^{LPB} dP &= \int_{DB}^{LPB} dP_{Head} + \int_{DB}^{LPB} dP_{acc} + \int_{DB}^{LPB} dP_{Friction} \\ &= \int_{DB}^{LPB} \rho_D \frac{g}{g_c} dL + \int_{DB}^{LPB} \rho_D \frac{v dv}{g_c} + \int_{DB}^{LPB} dP_{Friction} \end{aligned}$$

As for the contraction, we define

$$\int_{DB}^{LPB} dP_{Friction} \equiv K_{E_{LPB}} \rho_D \frac{v_{DB}^2}{2g_c}$$

and, noting that $L_{LPB} = L_{DB}$,

$$- \left[P \right]_{DB}^{LPB} = \rho_D \left[\frac{v^2}{2g_c} \right]_{DB}^{LPB} + K_{E_{LPB}} \frac{v_{DB}^2}{2g_c} \rho_D$$

or

$$P_{DB} - P_{LPB} = \rho_D \left[\frac{v_{LPB}^2}{2g_c} - \frac{v_{DB}^2}{2g_c} \right] + K_{E_{LPB}} \frac{v_{DB}^2}{2g_c} \rho_D$$

Substituting

$$v_{LPB}^2 = v_{DB}^2 \left(A_D / A_{LP} \right)^2$$

and

$$v_{DB}^2 = v_{sat}^2 \left(\frac{A_{HS}}{A_D} \right)^2 \left(\frac{\rho_f}{\rho_D} \right)^2,$$

there is obtained

$$P_{DB} - P_{LPB} = \frac{v_{sat}^2}{2g_c} [N_3] \quad (V.7)$$

$$\text{with } N_3 \equiv (A_{HS}/A_D)^2 (\rho_f/\rho_D)^2 \left[\rho_D (A_D/A_{LP})^2 - \rho_D + K_{E_{LPB}} \right]$$

d. Lower Plenum

$$\begin{aligned} -\int_{LPB}^{LPT} dP &= \int_{LPB}^{LPT} dP_{Head} + \int_{LPB}^{LPT} dP_{acc} + \int_{LPB}^{LPT} dP_{Friction} \\ &= \int_{LPB}^{LPT} \frac{g}{g_c} \rho_D dL + \int_{LPB}^{LPT} \rho_D \frac{v dv}{g_c} + \\ &\quad + \int_{LPB}^{LPT} f_{LPB} \rho_D \frac{v^2 dL}{2g_c D_{LP}} \end{aligned}$$

Since there is no change in area, and hence fluid velocity, and the diameter of the lower plenum is so very large

$$P_{LPB} - P_{LPT} = (g/g_c) \rho_D [L_{LPT} - L_{LPB}] \quad (V.8)$$

e. Contraction from Lower Plenum

$$\begin{aligned} -\int_{LPT}^{HSB} dP &= \int_{LPT}^{HSB} dP_{Head} + \int_{LPT}^{HSB} dP_{Friction} + \int_{LPT}^{HSB} dP_{acc} \\ &= \int_{LPT}^{HSB} \frac{g}{g_c} \rho_D dL + \int_{LPT}^{HSB} dP_{Friction} + \int_{LPT}^{HSB} \rho_D \frac{v dv}{g_c} \end{aligned}$$

Assuming $L_{LPT} = L_{HSB}$ and defining

$$\int_{LPT}^{HSB} dP_{Friction} = K_{CHSB} \frac{v_{HSB}^2}{2g_c} \rho_D ,$$

there is obtained

$$\begin{aligned} P_{LPT} - P_{HSB} &= \rho_D \left[\frac{v^2}{2g_c} \right]_{LPT}^{HSB} + K_{CHSB} \frac{v_{HSB}^2}{2g_c} \rho_D \\ &= \rho_D \left[\frac{v_{HSB}^2}{2g_c} - \frac{v_{LPT}^2}{2g_c} \right] + K_{CHSB} \frac{v_{HSB}^2}{2g_c} \rho_D . \end{aligned}$$

Substituting

$$v_{LPT}^2 = v_{HSB}^2 (A_{HS}/A_{LP})^2$$

and

$$v_{HSB}^2 = v_{sat}^2 (\rho_f/\rho_D)^2 ,$$

there is obtained

$$P_{LPT} - P_{HSB} = \frac{\rho_f^2 v_{sat}^2}{\rho_D 2g_c} \left[1 - \left(\frac{A_{HS}}{A_{LP}} \right)^2 + K_{CHSB} \right]$$

or

$$P_{LPT} - P_{HSB} = \frac{v_{sat}^2}{2g_c} [N_4] \quad (V.9)$$

$$\text{with } N_4 \equiv (\rho_f^2/\rho_D) \left[1 - (A_{HS}/A_{LP})^2 + K_{CHSB} \right] .$$

2. Nonboiling Region

Since a large fraction of the nonboiling region may be in local boiling, the question immediately arises concerning the length of the local boiling region and the uncertainty in the distribution and magnitude of the local boiling voids. For the purpose of this analysis it is tacitly assumed that the local boiling voids adhere to the wall and therefore do not decrease the density of the fluid. The acceleration pressure drop due to the pressure of local boiling voids will be neglected here and incorporated into a total void acceleration pressure drop calculation in the boiling section.

The inception of local boiling and hence the length of the local boiling region can be estimated in the following manner. The wall temperature in the local boiling region is calculated using the Jens-Lottes equation:

$$\Delta T_{\text{sat}} = 60 \left(\frac{q}{10^6} \right)^{0.25} / eP/900$$

and the value plotted on a temperature length graph as illustrated below.

The wall temperature in the nonboiling region is then calculated by

$$T_w - T_f = q''/h \quad ,$$

where h is given by the Colburn equation

$$h = 0.023 \frac{k}{D} (Re)^{0.8} (Pr)^{0.4} \quad .$$

The temperature in the nonboiling region is also plotted vs the channel length and the intersection of the two wall temperature profiles is designated as the inception of local boiling.

Having established the length of the local boiling region the total pressure drop across the nonboiling region can be computed in the following manner:

$$\begin{aligned}
-\int_{HSB}^{\text{sat}} dP &= \int_{HSB}^{\text{sat}} dP_{\text{Head}} + \int_{HSB}^{\text{sat}} dP_{\text{acc}} + \int_{HSB}^{\text{LB}} dP_{\text{Friction}} + \int_{\text{LB}}^{\text{sat}} dP_{\text{Friction}} \\
&= \int_{HSB}^{\text{sat}} \frac{g}{g_c} \rho dL + \int_{HSB}^{\text{sat}} \rho \frac{v dv}{g_c} + \int_{HSB}^{\text{LB}} f_{\text{HS}} \frac{v^2}{2g} \frac{\rho}{D_{\text{HS}}} dL \\
&\quad + \int_{\text{LB}}^{\text{sat}} \bar{R}_{\text{LB}} f_{\text{HS}} \frac{v^2}{2g} \frac{\rho}{D_{\text{HS}}} dL
\end{aligned}$$

where $\bar{R}_{\text{LB}}$ = ratio of the local boiling pressure drop at the total flow rate,

$$\Delta P_{\text{LB}} / \Delta P_{\text{SP}} \quad .$$

It should be noted that the density ρ , friction factor f , and velocity v (indirectly) are functions of the temperature and hence length. However, for all practical purposes an average density, velocity and isothermal friction factor based on a mean temperature of the fluid in the various segments can be used for calculating the pressure drop due to friction and head loss unless extreme conditions are encountered, such as near the critical point. The acceleration integral can be expanded by substituting

$$G = \rho v = \text{constant}$$

and

$$dv = - \frac{G d\rho}{\rho^2} \quad .$$

Therefore,

$$\begin{aligned}
-\int_{HSB}^{\text{sat}} dP &= \frac{g}{g_c} \bar{\rho}_{\text{NB}} \int_{HSB}^{\text{sat}} dL - \frac{G^2}{g_c} \int_{HSB}^{\text{sat}} \frac{d\rho}{\rho^2} + \bar{\rho}_{\text{NB}} \bar{f} \frac{v_{\text{NB}}^2}{2g_c D_{\text{HS}}} \int_{HSB}^{\text{LB}} dL \\
&\quad + \bar{\rho}_{\text{LB}} \bar{R}_{\text{LB}} \frac{\bar{f}_{\text{LB}} v_{\text{LB}}^2}{2g_c D_{\text{HS}}} \int_{\text{LB}}^{\text{sat}} dL \quad .
\end{aligned}$$

Simplifying

$$\begin{aligned}
P_{\text{HSB}} - P_{\text{sat}} &= \frac{g}{g_c} \bar{\rho}_{\text{NB}} (L_{\text{sat}} - L_{\text{HSB}}) + \frac{G^2}{g_c} \left[\frac{1}{\rho_f} - \frac{1}{\rho_{\text{HSB}}} \right] \\
&\quad + \frac{\bar{\rho}_f^2}{\rho_{\text{NB}}} \frac{v_{\text{sat}}^2 \bar{f}}{2g_c D_{\text{HS}}} (L_{\text{LB}} - L_{\text{HSB}}) \\
&\quad + \frac{\rho^2}{\text{LB}} \bar{R}_{\text{LB}} \bar{f}_{\text{LB}} \frac{v_{\text{sat}}^2}{2g_c D_{\text{HS}}} (L_{\text{LB}} - L_{\text{HSB}})
\end{aligned}$$

or

$$P_{\text{HSB}} - P_{\text{sat}} = \frac{g}{g_c} (L_{\text{sat}} - L_{\text{HSB}}) \bar{\rho}_{\text{NB}} + \frac{v_{\text{sat}}^2}{2g_c} [N_5 + N_6 + N_7] \quad (\text{V.10})$$

3. Boiling Zone

A simplified analysis of the two-phase flow regime can be made by writing a force balance on a differential element in the boiling zone where there is a transfer of mass between the two phases, as depicted below.

The assumptions made are:

- (1) The pressure is constant across any plane normal to the direction of flow.
- (2) The sum of the areas at a plane normal to the direction of flow occupied by the gas and liquid phase equals the total pipe area.

A force balance on the gas phase yields

$$\begin{aligned} PA_g - (P + dP)(A_g + dA_g) - dF_{fg} - (A_g dL \rho_g + dW_g)(g/g_c) \\ = (1/g_c)[(W_g + dW_g)(V_g + dV_g) - W_g V_g - dW_g V_g] \end{aligned}$$

F_{fg} is defined as the force expended by the gaseous phase in overcoming friction. Simplifying,

$$\begin{aligned} PA_g - PA_g - P dA_g - A_g dP - dP dA_g - dF_{fg} - A_g dL \rho_g (g/g_c) + dW_g (g/g_c) \\ = (1/g_c)[W_g V_g + W_g dV_g + V_g dW_g + dW_g dV_g - W_g V_g - V_g dW_g] \end{aligned}$$

or, neglecting terms of second order,

$$\begin{aligned} -P dA_g - A_g dP - dF_{fg} - A_g dL \rho_g (g/g_c) - dW_g (g/g_c) \\ = (1/g_c)[W_g dV_g + V_g dW_g - V_g dW_g] \end{aligned} \quad (V.11)$$

Also, a force balance on the liquid phase gives

$$\begin{aligned} PA_\ell - (P + dP)(A_\ell + dA_\ell) - dF_{f\ell} - [A_\ell dL \rho_\ell + dW_\ell](g/g_c) \\ = (1/g_c)[(W_\ell + dW_\ell)(V_\ell + dV_\ell) - W_\ell V_\ell - dW_\ell V_\ell] \end{aligned}$$

Again $F_{f\ell}$ is defined as the force expended by the liquid phase in overcoming friction. Simplification yields

$$\begin{aligned} PA_\ell - PA_\ell - P dA_\ell - A_\ell dP - dP dA_\ell - dF_{f\ell} - A_\ell dL \rho_\ell (g/g_c) - (g/g_c) dW_\ell \\ = [W_\ell V_\ell + W_\ell dV_\ell + V_\ell dW_\ell + dW_\ell dV_\ell - W_\ell V_\ell - V_\ell dW_\ell - W_\ell V_\ell](1/g_c) \end{aligned}$$

or

$$-P dA_\ell - A_\ell dP - dF_{f\ell} - A_\ell dL \rho_\ell (g/g_c) = [W_\ell dV_\ell](1/g_c) \quad (V.12)$$

Summing Eq. (V.11) and (V.12)

$$\begin{aligned} P(-dA_\ell - dA_g) - dP(A_g + A_\ell) - dF_{fg} - dF_{f\ell} - (g/g_c)dL(A_\ell \rho_\ell + A_g \rho_g) \\ + dW_\ell(g/g_c) + dW_g(g/g_c) = (1/g_c)[W_\ell dV_\ell + V_\ell dW_\ell + W_g dV_g + V_g dW_g], \end{aligned} \quad (V.13)$$

since

$$dA_\ell = -dA_g, \quad A_g + A_\ell = A_p, \quad \text{and} \quad dW_\ell = -dW_g$$

Further,

$$d(W_\ell V_\ell + W_g V_g) = [W_\ell dV_\ell + V_\ell dW_\ell + W_g dV_g + V_g dW_g](1/g_c) ,$$

and

$$(dF_{fg} + dF_{fl}) = dP_{TPF} A_p$$

by definition, where dP_{TPF} is the two-phase frictional pressure drop loss. Accordingly Eq. (V.13) becomes

$$-dP A_p - dP_{TPF} A_p - (g/g_c) dL (A_\ell \rho_\ell + A_g \rho_g) = (1/g_c) d(W_\ell V_\ell + W_g V_g) .$$

Dividing by A_p ,

$$-dP - dP_{TPF} - g/g_c (A_\ell/A_p \rho_\ell + A_g/A_p \rho_g) dL = \frac{d(W_\ell V_\ell + W_g V_g)}{g_c A_p} \quad (V.13a)$$

$$\text{Defining } \alpha = A_g/A_p ,$$

$$(1 - \alpha) = A_\ell/A_p$$

and substituting into (V.13a);

$$-dP - dP_{TPF} - g/g_c [(1 - \alpha)\rho_\ell + \alpha \rho_g] dL = \frac{d(W_\ell V_\ell + W_g V_g)}{g_c A_p} . \quad (V.14)$$

Applying Eq. (V.14) over the boiling length,

$$\begin{aligned} -\int_{\text{sat}}^{\text{HSE}} dP &= \int_{\text{sat}}^{\text{HSE}} \frac{d(W_\ell V_\ell + W_g V_g)}{g_c A_p} + \int_{\text{sat}}^{\text{HSE}} dP_{TPF} \\ &\quad + g/g_c \int_{\text{sat}}^{\text{HSE}} [(1 - \alpha)\rho_\ell + \alpha \rho_g] dL \end{aligned}$$

or

$$\begin{aligned} P_{\text{sat}} - P_{\text{HSE}} &= 1/g_c A_p \left[W_\ell V_\ell + W_g V_g \right]_{\text{sat}}^{\text{HSE}} + \left[\Delta P_{TPF} \right]_{\text{sat}}^{\text{HSE}} \\ &\quad + \bar{\rho}_B \left[L \right]_{\text{sat}}^{\text{HSE}} , \end{aligned}$$

where

$$\bar{\rho}_B = \frac{\int_{\text{sat}}^{\text{HSE}} \rho dL}{L_{\text{HSE}} - L_{\text{sat}}} .$$

Also defining

$$\bar{R} = \Delta P_{TPF} / \Delta P_{SPF}$$

as the ratio of the two-phase frictional pressure drop in the boiling zone to the single-phase pressure drop at the total flow rate,

$$\left[\Delta P_{TPF} \right]_{sat}^{HSE} = \bar{R} f_{HS} \rho_f \frac{v_{sat}^2}{2g_c} \frac{L_{HSE} - L_{sat}}{D_{HS}},$$

so that

$$\begin{aligned} P_{sat} - P_{HSE} &= \frac{1}{g_c A_p} \left[W_{l_{HSE}} V_{l_{HSE}} + W_{g_{HSE}} V_{g_{HSE}} - W_{l_{sat}} V_{sat} - W_{g_{sat}} V_{g_{sat}} \right] \\ &\quad + \bar{R} f_{HS} \rho_f \frac{v_{sat}^2}{2g_c D_{HS}} (L_{HSE} - L_{sat}) + \bar{\rho}_B (L_{HSE} - L_{sat}) \\ &= \frac{1}{g_c A_p} \left[\frac{W_T(1-x_e)(W_T)(1-x_e)}{\rho_f(1-\alpha_e) A_p} + \frac{x_e W_T(x_e W_T)}{\alpha_e A_p \rho_g} - W_T \frac{G}{\rho_f} \right] \\ &\quad + \bar{\rho}_B (L_{HSE} - L_{sat}) + \bar{R} f_{HS} \rho_f \frac{v_{sat}^2}{2g_c} (L_{HSE} - L_{sat}) \end{aligned}$$

Noting that $W_T/A_p = G$ and simplifying

$$\begin{aligned} P_{sat} - P_{HSE} &= \left[\frac{(1-x_e)^2}{(1-\alpha_e)} \frac{1}{\rho_f} + \frac{x_e^2}{\alpha_e} \frac{1}{\rho_g} - \frac{1}{\rho_f} \right] \frac{G^2}{g_c} + \bar{\rho}_B (L_{HSE} - L_{sat}) \\ &\quad + \bar{R} \frac{f_{HS}}{D_{HS}} \rho_f \frac{v_{sat}^2}{2g_c} (L_{HSE} - L_{sat}) \end{aligned}$$

The quantity $\left[\frac{(1-x_e)^2}{1-\alpha_e} \frac{1}{\rho_f} + \frac{x_e^2}{\alpha_e} \frac{1}{\rho_g} - \frac{1}{\rho_f} \right]$ is the acceleration multiplier, r , as defined by Martinelli and Nelson. Therefore,

$$P_{sat} - P_{HSE} = \frac{r G^2}{g_c} + \bar{R} \frac{f_{HS}}{D_{HS}} \frac{v_{sat}^2}{2g_c} \rho_f (L_{HSE} - L_{sat}) + \bar{\rho}_B (L_{HSE} - L_{sat})$$

or

$$P_{sat} - P_{HSE} = \frac{v^2}{2g_c} \left[N_8 + N_9 \right] + \bar{\rho}_B (L_{HSE} - L_{sat}) \quad (V.15)$$

4. Expansion or Contraction to a Riser

a. Expansion. The expansion for the two-phase fluid is treated in the same manner as for a single-phase fluid. Since only the overall pressure

drop is desired, a momentum balance can be written across the expansion:*

$$M_g V_{gHSE} + M_f V_{fHSE} + P_{HSE} A_R = M_g V_{gRB} + M_f V_{fRB} + P_{RB} A_R ;$$

$$P_{HSE} - P_{RB} = \frac{M_g}{A_R} (V_{gRB} - V_{gHSE}) + \frac{M_f}{A_R} (V_{fRB} - V_{fHSE}) ;$$

$$V_{gRB} = \frac{W_g}{\rho_g A_R \alpha_{RB}} = \frac{x_{HSE} \rho_f A_{HS} V_{sat}}{\rho_g A_R \alpha_{RB}} ;$$

$$V_{gHSE} = \frac{W_g}{\rho_g A_{HS} \alpha_{HSE}} = \frac{x_{HSE} \rho_f A_{HS} V_{sat}}{\rho_g A_{HS} \alpha_{HSE}} = \frac{x_{HSE} \rho_f V_{sat}}{\rho_g \alpha_{HSE}} ;$$

$$V_{fRB} = \frac{W_{fRB}}{\rho_f A_R (1 - \alpha_{RB})} = \frac{(1 - x_{HSE}) \rho_f A_{HS} V_{sat}}{\rho_f A_R (1 - \alpha_{RB})} = \frac{(1 - x_{HSE}) A_{HS}}{(1 - \alpha_{RB}) A_R} V_{sat} ;$$

$$V_{fHSE} = \frac{W_{fHSE}}{\rho_f A_{HS} (1 - \alpha_{HSE})} = \frac{(1 - x_{HSE}) \rho_f A_{HS} V_{sat}}{\rho_f A_{HS} (1 - \alpha_{HSE})} = \frac{(1 - x_{HSE})}{(1 - \alpha_{HSE})} V_{sat} ;$$

$$M_g = \frac{x_{HSE} W_T}{g} = \frac{x_{HSE} \rho_f A_{HS} V_{sat}}{g} ;$$

$$M_f = \frac{(1 - x_{HSE}) W_T}{g} = \frac{(1 - x_{HSE}) \rho_f A_{HS} V_{sat}}{g} ;$$

and

$$\begin{aligned} P_{HSE} - P_{RB} = & \frac{x_{HSE} \rho_f A_{HS} V_{sat}}{\rho_g A_R} \left[\frac{x_{HSE} \rho_f A_{HS} V_{sat}}{\rho_g A_R \alpha_{RB}} - \frac{x_{HSE} \rho_f V_{sat}}{\rho_g \alpha_{HSE}} \right] \\ & + \frac{(1 - x_{HSE}) \rho_f A_{HS} V_{sat}}{\rho_g A_R} \\ & \left[\frac{(1 - x_{HSE}) A_{HS} V_{sat}}{A_R (1 - \alpha_{RB})} - \frac{(1 - x_{HSE}) V_{sat}}{(1 - \alpha_{HSE})} \right] . \end{aligned}$$

Substituting $A_{HS}/A_R = \sigma$ and simplifying,

$$\begin{aligned} P_{HSE} - P_{RB} = & \left(\frac{\rho_f V_{sat}}{2g} \right)^2 2\sigma \\ & \left[x_{HSE}^2 \frac{\rho_f}{\rho_g} \left(\frac{\sigma}{\alpha_{RB}} - \frac{1}{\alpha_{HSE}} \right) + (1 - x_{HSE})^2 \left(\frac{\sigma}{1 - \alpha_{RB}} - \frac{1}{1 - \alpha_{HSE}} \right) \right] . \end{aligned} \quad (V.16)$$

*Personal Communication from F. Romie, American Standard Co.

A general equation for an expansion between any two risers (unheated flow channels) of areas A_i and A_{i+1} is

$$P_i - P_{i+1} = \frac{v_{sat}^2}{2g_c} 2\rho_f \frac{A_{HS}}{A_{i+1}} \left[\frac{x_{HSE}^2 \rho_f}{\rho_g} \left(\frac{A_{HS}}{A_{i+1}} \frac{1}{\alpha_{i+1}} - \frac{A_{HS}}{A_i} \frac{1}{\alpha_i} \right) + (1 - x_{HSE})^2 \left(\frac{A_{HS}}{A_{i+1}(1 - \alpha_{i+1})} - \frac{A_{HS}}{A_i} \frac{1}{\alpha_i} \right) \right]. \quad (V.17)$$

For $A_i = A_{HS}$ and $A_{i+1} = A_{RB}$,

$$P_i - P_{i+1} = \frac{v_{sat}^2}{2g_c} [N_{10}] \quad (V.18)$$

b. Contraction. The contraction is treated in an analogous manner but is more subject to question. A more rigorous treatment similar to the one made for a single-phase fluid is difficult because of uncertainties in energy balance for two-phase fluids and numerous questions concerning the vena contracta. Therefore a simplified approach was taken and the following assumptions were made:

- (1) A vena contracta does not exist.
- (2) The pressure is constant across plane 2, as depicted in the diagram below.

Writing a momentum balance as before

$$M_g V_g + M_\ell V_{\ell_1} + P_1 A_1 = M_g V_{g_2} + M_\ell V_{\ell_2} + P_2 A_1$$

or

$$P_{HSE} - P_{RB} = \frac{M_g}{A_{HS}} (V_{gRB} - V_{gHSE}) + \frac{M_\ell}{A_{HS}} (V_{\ell RB} - V_{\ell HSE})$$

Substituting for the velocity, etc., and simplifying,

$$P_{HSE} - P_{RB} = \rho_f \frac{v_{sat}^2}{2g} (2)\sigma \left[x_{HSE}^2 \frac{\rho_f}{\rho_g} \left(\frac{A_{HS}}{A_R \alpha_{RB}} - \frac{1}{\alpha_{HSE}} \right) + (1 - x_{HSE})^2 \left(\frac{A_{HS}}{A_R (1 - \alpha_{RB})} - \frac{1}{1 - \alpha_{HSE}} \right) \right] \quad (V.19)$$

This is identical to Eq. (V.16). Therefore, Eq. (V.17) is a general equation for both an expansion or a contraction from the heated section. The general equation for any contraction is

$$P_i - P_{i+1} = \frac{v_{sat}^2}{2g_c} (2\rho_f) \left(\frac{A_{HS}}{A_i} \right) \left[x_{HSE}^2 \frac{\rho_f}{\rho_g} \left(\frac{A_{HS}}{A_{i+1} (\alpha_{i+1})} - \frac{A_{HS}}{A_i \alpha_i} \right) \right] \quad (V.19a)$$

As a result the pressure drop for a series of expansions and contractions can be written as

$$\sum \Delta P_i = \frac{v_{sat}^2}{2g_c} \sum N_{i0} \quad (V.20)$$

c. Riser

$$\int_{RB}^{RT} dP = \int_{RB}^{RT} dP_{Head} + \int_{RB}^{RT} dP_{acc} + \int_{RB}^{RT} dP_{Friction}$$

Since there is no change in voids in the adiabatic riser section,

$$\int_{RB}^{RT} dP_{acc} = 0$$

and

$$-\int_{RB}^{RT} dP = \int_{RB}^{RT} \rho_R dL + \int_{RB}^{RT} f_R \frac{v_R^2}{2g_c} \rho_f \frac{dL}{D_R}$$

where R is defined as $\Delta P_{TPF}/\Delta P_{SPF}$ in the riser. Then

$$\left[P \right]_{RT}^{RB} = \rho_R \left[L \right]_{RB}^{RT} + R f_R \frac{v_{sat}^2}{2g_c} \left(\frac{A_{HS}}{A_R} \right)^2 \frac{\rho_f}{D_R} \left[L \right]_{RB}^{RT}$$

and

$$P_{RB} - P_{RT} = \rho_R (L_{RT} - L_{RB}) + R f_R \frac{v_{sat}^2}{2g_c} \left(\frac{A_{HS}}{A_R} \right)^2 \frac{\rho_f}{D_R} (L_{RT} - L_{RB})$$

or

$$P_{RB} - P_{RT} = \rho_R (L_{RT} - L_{RB}) + \frac{v_{sat}^2}{2g_c} [N_{11}] \quad (V.21)$$

5. Expansion into Upper Plenum

The expansion into the upper plenum is given by Eq. (V.7).

Summing Eqs. (V.5), (V.6), (V.7), (V.8), (V.9), (V.10), (V.15), (V.17), and (V.21) to satisfy Eq. (V.1) and simplifying,

$$\begin{aligned} & \frac{v_{sat}^2}{2g} [N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8 + N_9 + \Sigma N_i + N_{11}] \\ & - \rho_D (L_{DT} - L_{DB}) + \rho_D [L_{LPT} - L_{LPB}] + \rho_{NB} (L_{sat} - L_{HSB}) \\ & + \bar{\rho}_B (L_{HSE} - L_{sat}) + \rho_R (L_{RT} - L_{RB}) = 0 \end{aligned} \quad (V.22)$$

Since

$$(L_{DT} - L_{DB}) = L_{RT} - L_{LPB} ,$$

then

$$L_{DT} - L_{DB} = (L_{LPT} - L_{LPB}) + (L_{sat} - L_{HSB}) + (L_{HSE} - L_{sat}) + (L_{RT} - L_{RB}) . \quad (V.23)$$

Substituting Eq. (V.23) into Eq. (V.22) and simplifying,

$$\begin{aligned} & \frac{v_{sat}^2}{2g} [N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8 + N_9 + \Sigma N_i + N_{11}] \\ & = (\rho_D - \bar{\rho}_{NB})(L_{sat} - L_{HSB}) + (\rho_D - \bar{\rho}_B)(L_{HSE} - L_{sat}) \\ & + (\rho_D - \rho_R)(L_{RT} - L_{RB}) \end{aligned} \quad (V.24)$$

Defining and substituting,

$$L_{\text{sat}} - L_{\text{HSB}} = L_{\text{NB}} = \text{nonboiling length}$$

$$L_{\text{HSE}} - L_{\text{sat}} = L_{\text{B}} = \text{boiling length}$$

$$L_{\text{RT}} - L_{\text{RB}} = L_{\text{R}} = \text{riser length}$$

$$L_{\text{DT}} - L_{\text{DB}} = L_{\text{D}} = \text{downcomer length}$$

$$\sum L_{\text{Ri}} (\rho_{\text{D}} - \rho_{\text{Ri}}) = L_{\text{R}} (\rho_{\text{D}} - \rho_{\text{R}})$$

and

$$\sum N_{\text{Ri}} = N_{11} \quad ,$$

Equation (V.24) becomes, for systems that may have a series of risers with varying diameters,

$$\frac{v_{\text{sat}}^2}{2g} = \frac{L_{\text{NB}}(\rho_{\text{D}} - \bar{\rho}_{\text{NB}}) + L_{\text{B}}(\rho_{\text{D}} - \bar{\rho}_{\text{B}}) + \sum L_{\text{Ri}}(\rho_{\text{D}} - \rho_{\text{Ri}})}{N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8 + N_9 + \sum N_i + \sum N_{11}} \quad (\text{V.25})$$

The individual N terms in Eq. (V.25) may be classified in the following manner:

$N_1 = \rho_f \left(\frac{A_{\text{HS}}}{A_{\text{D}}} \right)^2 \left[1 + K_{\text{cUP}} \right]$	= acceleration of the fluid from the upper plenum to downcomer and frictional resistance at contraction.
$N_2 = f_{\text{D}} \left(\frac{L_{\text{D}}}{D_{\text{D}}} \right) \left(\frac{\rho_f^2}{\rho_{\text{D}}} \right) \left(\frac{A_{\text{HS}}}{A_{\text{D}}} \right)^2$	= frictional resistance in downcomer.
$N_3 = \frac{\rho_f^2}{\rho_{\text{D}}} \left(\frac{A_{\text{HS}}}{A_{\text{D}}} \right)^2 \left[\left(\frac{A_{\text{D}}}{A_{\text{LP}}} \right)^2 - 1 + K_{\text{ELPB}} \right]$	= deceleration of fluid and frictional losses at the expansion from the downcomer to lower plenum.
$N_4 = \frac{\rho_f^2}{\rho_{\text{D}}} \left[1 - \left(\frac{A_{\text{HS}}}{A_{\text{LP}}} \right)^2 + K_{\text{CHSB}} \right]$	= acceleration of fluid and frictional losses at the contraction from lower plenum to the heated section.
$N_5 = 2\rho_f \left[\frac{\rho_{\text{HSB}} - \rho_f}{\rho_{\text{HSB}}} \right]$	= acceleration of fluid in nonboiling length due to density change with temperature.
$N_6 = \frac{\rho_f^2}{\rho_{\text{NB}}} \left(\frac{f_{\text{HS}}}{D_{\text{HS}}} \right) (L_{\text{NB}})$	= frictional resistance in the nonboiling segment of the heated section.

$$N_7 = \frac{\rho_f^2}{\rho_{LB}} (\bar{R}_{LB}) \left(\frac{f_{HS}}{D_{HS}} \right) (L_{LB}) = \text{frictional resistance in the local boiling segment of the heated section.}$$

$$N_8 = 2(r)(\rho_f)^2 = \text{acceleration of fluid due to formation of steam voids in the heated channel.}$$

$$N_9 = \rho_f (\bar{R}) \left(\frac{f_{HS}}{D_{HS}} \right) (L_B) = \text{frictional resistance in the boiling segment.}$$

$$N_{10} = 2\rho_f \left(\frac{A_{HS}}{A_{i+1}} \right) \left\{ x_{HSE}^2 \left(\frac{\rho_f}{\rho_g} \frac{1}{(\alpha_{i+1})} - \frac{A_{HS}}{A_i} \frac{1}{2_i} \right) + \left(1 - x_{HSE} \right)^2 \left[\left(\frac{A_{HS}}{A_{i+1}} \right) \frac{1}{(1 - \alpha_i)} - \frac{A_{HS}}{A_i \alpha_i} \right] \right\} \text{ total pressure drop due to an expansion or contraction.}$$

$$N_{11} = R f_R \left(\frac{A_{HS}}{A_R} \right)^2 \frac{\rho_f}{D_R} L_R = \text{frictional loss in the adiabatic risers.}$$

The various factors in Eq. (V.25), such as R , $\bar{R}_B$, $\bar{R}_{LB}$, ρ_B , V_g/V_f , etc., can be obtained from empirical correlations or by analytical means as discussed previously.

C. FORCED CIRCULATION SYSTEM ANALYSIS

The basic analysis presented for the natural circulation system applied equally to the forced circulation system with slight modification. The pump in the forced circulation system must overcome the frictional and acceleration losses in the loop plus the differential hydrostatic head between the downcomer and riser section. The differential head in the loop is a negative quantity since the downcomer density is greater than the riser density. Equation (V.25) can therefore be rewritten as

$$W_P = \frac{v^2}{2g_c \rho_D} \left[N_1 + \dots + N_1 + \dots - N_{pc} \right] - \left[\left(1 - \frac{\bar{\rho}_{NB}}{\rho_D} \right) L_{NB} + \left(1 - \frac{\bar{\rho}_R}{\rho_D} \right) L_B + \sum \left(1 - \frac{\rho_{R_i}}{\rho_D} \right) L_{R_i} \right],$$

Where W_P is the pump work requirement (based on the downcomer density), $N_1 + \dots - N_1 - \dots$ are defined by Eq. (V.25), and N_{pc} is the number of velocity heads lost in the pump circuit exterior to the reactor.

D. COMPARISON OF ANALYSES WITH LABORATORY DATA

The validity of the analyses and the assumptions made therein was checked by comparing the calculation procedure with performance test data obtained from laboratory loops. These tests were run on a basic

natural circulation loop in which the riser geometry was varied to introduce contractions and expansion into the flow circuit. A series of runs was made for each geometry at various pressures and powers. The power was increased for each set of runs until hydrodynamic instability ensued.

The laboratory loop used for these performance tests is schematically illustrated in Fig. III.2. The geometries studied were a one-inch diameter heated section, 4 feet in length, with risers $\frac{3}{4}$, 1 and 2 in. in diameter.

The performance of each of the geometries studied was precalculated, using the methods outlined in the previous section. The calculated recirculation velocity was plotted as a function of the exit steam volume fraction of the heated section, as shown in Figs. V.2-V.9.

It should be noted that there is some question concerning the absolute value of the measured steam volume fraction due to instrumentation difficulties which were discovered after completion of the testing program. This probable error applies only to the data for the 1-in. and $\frac{3}{4}$ -in. riser geometries. The measured velocities are correct and therefore the trend of the data is also essentially accurate. The only change that could result would be to shift the data to the right on the abscissa. In spite of the probable error in the data, some interesting observations can be made from each of the three geometries studied.

1. 2-in. Diameter Riser

In a boiling system the acceleration pressure drops due to the system geometry and the formation of steam voids in the boiling section represent a large fraction of the pressure drop due to friction and acceleration of the fluid. For the three geometries studied, the acceleration pressure drops accounted for a substantial percentage of the combined pressure drops. Also, the percentage of the pressure drop due to acceleration of the fluid generally increased as the void level increased. Therefore the amount of pressure recovery occurring at the expansions must be accounted for in the hydrodynamic analysis in order to accurately predict the performance. If zero recovery is assumed, the calculated recirculation velocity will be too low; if total recovery is assumed, the calculated velocity is too high. This was borne out by the data.

As can be seen in Figs. V.2-V.4, the laboratory data check the calculated performance very well for the three pressures studied. The use of the momentum balance for calculating the pressure recovery across an expansion therefore appears to be validated.

FIGURE V.2

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A
NATURAL CIRCULATION SYSTEM 2-in. RISER; 600 psig

FIGURE V.3

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A
NATURAL CIRCULATION SYSTEM. 2-in. RISER; 1000 psig

FIGURE V.4

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A
NATURAL CIRCULATION SYSTEM. 2-in. RISER; 1500 psig

FIGURE V.5

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A
NATURAL CIRCULATION SYSTEM. 3/4-in. RISER; 600 psig

FIGURE V.6

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A NATURAL CIRCULATION SYSTEM. 3/4 in. RISER; 1000 psig

FIGURE V.7

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A NATURAL CIRCULATION SYSTEM. 3/4 in. RISER; 1500 psig

FIGURE V.8

COMPARISON OF THE PREDICTED AND CALCULATED PERFORMANCE OF A NATURAL CIRCULATION SYSTEM. 1 in. RISER; 600 psig

FIGURE V.9

COMPARISON OF THE PREDICTED AND MEASURED PERFORMANCE OF A NATURAL CIRCULATION SYSTEM. 1 in. RISER; 1000 psig

2. $\frac{3}{4}$ -in. Riser Geometry

In addition to the acceleration pressure drop due to the contraction from the lower plenum and formation of steam voids, an additional acceleration pressure drop occurs at the contraction from the 1-in. heated section to the $\frac{3}{4}$ -in. riser. The system is rather unique, however, because for all practical purposes the acceleration pressure drops must be regarded as irrecoverable; it is assumed that there is no pressure recovery. This can be attributed to the fact that the expansion into the upper plenum involves a very large change of area. Thus, as for a single-phase fluid, the entire fluid velocity head can be lost; that is, ΔP across the expansion approaches zero. This was actually borne out by measurement. Two static pressure taps were placed at the expansion into the upper plenum, one in the riser and the other just above the riser exit in the upper plenum. In all instances the measured pressure drop was zero or very slightly positive.

Although the data check the calculated performance very well, as shown in Figs. V.5 to V.7, the validity of using a momentum balance across the contraction from the 1-in. pipe to the $\frac{3}{4}$ -in. riser cannot be completely established. The frictional pressure drops are very high, especially for the $\frac{3}{4}$ -in. riser, and as a result even though the acceleration pressure drop is substantial, it is not the dominant loss. Therefore any error made in calculating the frictional losses would be reflected in the magnitude of the pressure drop across the contraction. If we assume that the absolute frictional pressure drops are essentially correct (as they are thought to be, however), a momentum balance across a contraction appears to yield satisfactory estimates of the pressure drop.

3. 1-in. Riser

For this geometry the two-phase frictional pressure drop is the dominant loss in the system. Therefore the comparison of the data with the calculated performance tends to demonstrate the validity of the method used for calculating the two-phase friction factor. As discussed previously, the acceleration pressure drop due to void formation and the contraction from the lower plenum can be treated as irrecoverable.

As can be seen from Figs. V.8 and V.9, there is good agreement between data and calculations. The data could not be obtained over a wider range of voids because of the onset of instability.

From the comparisons given above, the validity of the calculation procedure presented appears to be established. Perhaps the major uncertainties remaining concern the contraction of a two-phase mixture and the two-phase friction factors over wide mass velocity ranges and pressures. These problems are under study currently.

SECTION VI
THERMAL-HYDRAULIC DESIGN PROCEDURES
FOR BOILING WATER REACTORS

M. Petrick

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A. NOMENCLATURE

A_{coolant}	Total cross-sectional area of coolant
B^2	Buckling
D_C	Core diameter
D_e	Mean equivalent diameter of flow channel
D_r	Diameter of fuel rod
f	Thermal utilization; Fanning friction factor
f_1, f_2	Arbitrary functions
h_B	Boiling heat transfer coefficient
h_c	Contact resistance between cladding and fuel
h_f	Saturation enthalpy
h_{fg}	Latent heat of vaporization
h_g	Enthalpy of saturated vapor
h_i	Enthalpy of water at inlet to the core
h_{in}	Enthalpy of inlet water
h_M, h_m	Enthalpy of makeup water
k_c	Thermal conductivity of cladding
k_{eff}	Effective multiplication factor
$\bar{k}_R$	Estimated mean heat conductivity of fuel rod
k_1, k_2	Arbitrary constants
L^*	Diffusion length
L_B	Boiling length
L_C	Core height or length
L_{NB}	Nonboiling length
L_T	Total heated length
M/F	Volumetric ratio of moderator to fuel
N	Slope of plot of slip ratio vs water velocity
n	Number of fuel rods in core
ΔP	Pressure differential
ΔP_I	Total pressure differential through risers and downcomers
ΔP_{II}	Pressure differential in fuel elements

p	Resonance escape probability
P_R	Total reactor power
$P.D. \cdot kw/L$	Power density in kw/liter
Q	Power
Q_{avg}	Average power per fuel element
Q_B	Heat input to boiling section
Q_{NB}	Heat input to nonboiling section
Q_r	Power of fuel element at radius r
Q_t	Total heat input
q	Power
q''	Heat flux
q''_c	Average heat flux in core
$R, \bar{R}$	Two-phase friction factor multipliers
$R_{max/avg}$	Ratio of the maximum to average heat flux in core
r	Acceleration pressure drop multiplier; Radius
S/F	Volumetric ratio of structure (fuel cladding, control rods, superstructure, etc.) to fuel
$T_{\mathcal{L} \max}$	Maximum fuel rod centerline temperature in core
T_{sat}	Saturation temperature
t_c	Cladding thickness
V	Recirculation velocity
$\bar{V}$	Average velocity
V_g/V_l	Slip ratio
$\bar{V}_o$	Average velocity of elements feeding a given external riser
V_r	Axial flow velocity at radius r
W	Fluid flow rate
W_A	Annular fluid flow
W_c	Central fluid flow
W_m	Flow rate of makeup water
W_S	Total steam flow
W_{sA}	Steam flow rate to outer annular region
W_{sC}	Total steam flow to control nonhomogeneous riser

W_T	Total recirculation flow rate
X	Mixture quality
x_A	Quality in outer riser
x_C	Quality in central riser
x_e	Exit mixture quality
$\bar{x}_e$	Average exit mixture quality
α	Steam volume fraction; void fraction
α_e	Exit steam volume fraction
ϵ	Fast-fission factor
ρ	Density
$\bar{\rho}_B$	Average boiling density
ρ_D	Density in downcomer
ρ_f	Density of saturated liquid
ρ_g	Density of gas
ρ_l	Density of saturated liquid
ρ_{NB}	Average density in nonboiling section
ρ_{R_i}	Two-phase mixture density; local riser density
τ	Fermi age

B. INTRODUCTION

In order to realize economic nuclear power the desirability of incorporating certain design objectives becomes readily apparent, if the desired goal is to be realized. A few of the objectives that the boiling water reactor designer might seek are:

1. derive the highest specific power and power density from the core;
2. obtain the longest fuel life possible;
3. obtain the highest power density per unit void;
4. obtain a high fuel conversion ratio;
5. employ natural circulation of the core coolant;
6. use internal gravity vapor-liquid separation;

7. use low enrichment fuel;
8. produce superheated steam;
9. use ceramic (UO_2), rather than metallic, fuel;
10. obtain the highest neutron economy possible by elimination of unnecessary core, structural material, etc.

Unfortunately, a substantial degree of incompatibility exists between the various objectives. Basically, the incompatibility stems from conflicting physics and engineering requirements and, as a result, the reactor design which evolves generally represents a compromise between the physics, thermodynamics and hydraulics design objectives. However, in arriving at the compromise design a number of design parameters are usually taxed to their limits. The result is that in many instances either one or several of these parameters or design criterion actually establishes the limits of reactor performance. Therefore, before a calculation procedure for reactor core analysis is presented, a brief discussion of a few of the major design parameters or criterion of boiling water reactors is given. An attempt is made to point out how these criterion affect reactor design and ranges and conditions under which they may tend to become limiting. The calculation procedure is then illustrated by a sample core analysis.

C. BOILING WATER REACTOR DESIGN CRITERIA

1. Moderator to Fuel Ratio

The moderator to fuel ratio is perhaps the most important reactor design parameter because of its immediate impact on core physics and because it essentially represents the bridge between physics and engineering. Establishing the moderator to fuel ratio of a reactor, as one might suspect, is the area of greatest conflict between the engineer and physicist because of the incompatibility of the most desirable objectives of each.

From an engineering viewpoint it is desirable to have a large moderator to fuel ratio to obtain a large equivalent diameter of the core coolant flow channel. This is especially true for the natural-circulation boiling water reactor, since the major hydrodynamic resistance exists within the core. As the moderator to fuel ratio decreases, the recirculation velocity also decreases due to the increased frictional resistance within the core. As a result, the reactor power, and hence performance, drops, since the reactor power is essentially proportional to the recirculation velocity for a fixed amount of reactivity tied up in steam voids.

Also, as the moderator to fuel ratio is decreased, the coolant power density increases. As a result, the core recirculation flow rate must be increased if the same mean core steam volume fraction is to be maintained. The moderator to fuel ratio could eventually be lowered to a point where it would be impossible to obtain the desired coolant flow rate by natural convection and, as a result, forced circulation of the coolant would have to be employed. On the other hand, if the moderator to fuel ratio is increased too far, the maximum heat flux becomes very large and either the fuel centerline temperature becomes excessive or the critical maximum heat flux is reached.

From a physics viewpoint, the moderator to fuel ratio strongly influence a number of important core design criteria such as fuel cycle costs, reactor stability, etc. The effect of the moderator to fuel ratio on the reactor physics can be seen by a superficial examination of the factors of the criticality equation for a bare, enriched, water-moderated reactor:

$$k_{\text{eff}} = \frac{\eta f \epsilon p}{1 + (L^2 + \tau) B^2} \quad .$$

Of particular interest are the terms f and p , the thermal utilization and the resonance escape probability, which are related to the moderator to fuel ratio as follows:

$$f \approx \frac{1}{k_1 + k_2(M/F)}$$

and

$$p \approx \frac{1}{f_1(M/F)/f_2(M/F)} \quad ,$$

where $f_2(M/F)$ changes more rapidly than $f_1(M/F)$.

As can be seen, increasing the moderator to fuel ratio lowers the thermal utilization and increases the resonance escape probability. If this trend is carried too far, the thermal utilization decreases faster than the increase of the resonance escape probability and the enrichment of the core would have to be increased. Decreasing the moderator to fuel ratio will eventually lead to a required increase in enrichment due to the reverse effect. As a result there is an optimum moderator to fuel ratio for a minimum fuel enrichment.

The enrichment, however, is but one of several factors which must be considered in evaluating the fuel cycle costs. The number of fuel rods in the core increases as the moderator to fuel ratio decreases, since the fuel rod diameter is usually fixed independently by the limiting centerline temperature. Increasing the number of fuel pins increases the fuel

fabrication costs. However, as the moderator to fuel ratio is decreased, the resonance escape probability decreases and hence the conversion ratio increases. The production of additional fissionable material enhances the fuel burnup (MWD/ton) considerably. The fuel burnup can also be increased by adding enrichment. This, however, must be weighed against the increased cost and the lowering of the conversion ratio.

The fuel lifetime and fuel inventory are also strongly affected by the moderator to fuel ratio. The characteristics of a reactor system having a high moderator to fuel ratio are a low fuel inventory, low fuel burnup, high average heat flux, low fuel lifetime (if no excess enrichment is added), high specific power, and low coolant power density system. The low moderator to fuel core on the other hand will have a longer fuel lifetime, higher fuel burnup, higher coolant power density, lower average heat flux, high fuel inventory and a low specific power. All of these factors must be studied and weighed.

The effect of the moderator to fuel ratio on the reactor stability and safety must also be evaluated. The boiling water reactor must be designed with a negative power coefficient of reactivity due to formation of steam voids within the core; that is, as the water moderator is expelled from the core, the reactivity must decrease. It should be noted, however, that a very large negative void coefficient is not desirable either. If the negative void coefficient is too large, reactor instability can result due to the self-regulating feedback control characteristic of the system.

The magnitude of the negative void coefficient will depend upon the moderator to fuel ratio and the core buckling. There is a range of moderator to fuel ratio where the effects of the variation of the resonance escape probability (p) and the thermal utilization (f) on the core reactivity will essentially cancel each other. As a result, the core buckling can then be set so that the void coefficient will be negative due to void formation and the resultant neutron exchange. In general, the lower the water to metal ratio, the more negative the void coefficient becomes:

To establish this moderator to fuel ratio, all the various factors affecting both the engineering and physics requirements of the core must be thoroughly investigated. Only then can a compromise on the moderator to fuel ratio be reached.

2. Critical Maximum Heat Flux (Burnout)

A major uncertainty in boiling reactor design is the prediction of the critical heat flux. The critical heat flux is defined as the transition from nucleate to film boiling and is marked by a sharply defined increase in the temperature of the heat transfer surface. If the heat flux is sufficiently great, the resulting thermal excursion results in physical destruction of the

heat transfer surface. "Burnout" is a term usually used interchangeably with critical heat flux, although it is sometimes used to denote the point of actual failure. Since the first usage is a misnomer and the second subject to such extraneous variables as material properties, pressure and/or thermal stresses, and corrosion rates and effects, "burnout" is an ambiguous and ill-defined term.

The critical heat flux and the determining conditions thereof have been of considerable interest for at least a decade, and numerous experimental investigations have been made. However, no correlations of net boiling "burnout" have been developed to describe adequately its relationships with all various system parameters. The burnout heat flux is a function of the mass velocity, mixture enthalpy, system pressure, and possibly such additional factors as surface condition, geometry, etc. Much of the work that has been done to date on the critical heat flux problem has been summed up and reviewed in WAPD-188. One of the most important conclusions listed in this report is that for nonuniform axial flux distribution the critical flux may not occur at either the point of maximum heat flux or maximum mixture enthalpy (exit condition). Thus in the boiling water reactor where highly skewed axial power distributions have been measured, the core exit condition cannot be used for establishing the burnout limits. The entire channel must be analyzed and the local heat fluxes and fluid conditions compared with existing burnout data and correlations to establish the burnout point. As the critical pressure is approached, the critical maximum flux decreases and the net boiling length in the core approaches zero. At very high pressures, the power peak will therefore occur in a region of local or subcooled boiling. It is also possible that, even though the critical maximum flux is exceeded, a true burnout need not necessarily occur, providing the film boiling coefficient is large enough. Recent film boiling data have shown that the temperature rise accompanying the departure from nucleate boiling is not excessive for heat fluxes of $q < 200,000$ Btu. Thus in a boiling water reactor which uses the oxide as fuel and stainless steel as cladding it may be possible to operate in the film boiling region under certain conditions. However, more such data are needed at heat fluxes of the order of magnitude existing in reactors (150,000 to 450,000 Btu/hr ft²) and at comparable mass velocities.

In evaluating the probable burnout point in a boiling water reactor operating with natural circulation of the coolant, consideration must also be given to the stability of the system. When a system is hydrodynamically unstable, this departure from nucleate boiling can occur at lower value of heat flux than the value obtained under forced circulation for identical conditions. This is demonstrated in Fig. VI.1, which shows a trace of the recirculation velocity and the point of burnout for a natural-circulation system which became hydrodynamically unstable. The estimated burnout heat flux under forced-circulation conditions is $\sim 700,000$ Btu/hr ft², whereas the burnout occurred at a flux of $\sim 560,000$ Btu/hr ft². However, the fact that the

system is hydrodynamically unstable does not preclude a burnout at any heat flux. Needless to say, there is practically no information available on natural circulation burnout or the effect of instability on burnout.

FIGURE VI.1
VELOCITY AS A FUNCTION OF TIME FOR A NATURAL CIRCULATION BURNOUT

3. Vapor-Liquid Separation

In a reactor, such as EBWR, where steam is withdrawn directly from the reactor vessel a reactor power is ultimately reached where the vapor-liquid separation problem becomes so acute that excessive moisture carryover and steam carryunder occur, adversely affecting reactor operation. The present trend of increasing the power and operating pressure of boiling water reactors has caused vapor-liquid separation to become one of the major problems in boiling water reactor plant engineering.

Knowledge concerning the mechanism and various factors influencing the separation of steam from steam-water mixtures in reactor vessels or in separate steam drums is quite incomplete. As a result of lack of information, sizing of the reactor vessel and internals with any degree of confidence becomes difficult at best.

The little information that has been reported has appeared in the English and Russian literature. However, the experimental data obtained by different techniques are in some cases inconsistent, and the given equations do not predict correctly the fraction of moisture carried by the steam for certain ranges of steam loads and pressures.

In a natural-circulation boiling water reactor of the EBWR type the vapor-liquid separation problem can be divided into the following parts; (1) liquid carryover in the effluent steam; (2) steam carryunder in the downcomer; (3) effectiveness of liquid makeup water injection and the rapidity of steam quenching thereafter; (4) transport of steam through stagnant steam-water mixtures (vapor holdup above core).

The problem of carryunder of entrainment of steam in the liquid in a natural vapor-separation system is a major one in either a natural or forced convection system. The entrainment of the steam in the liquid in the downcomer is in general a function of the downcomer velocity and system pressure. In a natural-circulation system any entrainment that may occur in the downcomer adversely affects reactor operation, since the recirculation is reduced and this in turn lowers the reactor power. There is practically no information available that could be used for estimating the amount of carryunder that could be expected under normal reactor conditions.

The carryunder problem in a forced-circulation system could be just as severe. In general, the velocities are somewhat greater in such a system, both in the core and the downcomer. The entrainment of the steam into the suction lines of the pump could decrease the NPSH to a point where pump cavitation problems could become serious, depending on the condition of the makeup water and point of injection.

The seriousness of the steam carryunder problem will be determined by the rapidity with which the entrained steam bubbles are quenched and, of course, by the amount of steam entrained. It is conceivable that the entrained steam bubbles are not collapsed immediately because of insufficient mixing in the downcomer; as a result the steam would be carried a considerable distance before condensing. This will, to a large degree, be dependent on the mode of makeup water injection. The makeup water injection ring must be designed so that the makeup water is distributed evenly across and around the entire downcomer to insure fast, thorough mixing. Should substantial quantities of steam carryunder occur, the net driving head for natural circulation would be reduced. A reduction in net driving head would in turn reduce the recirculation flow rate and increase the core steam volume fraction.

In a closed reactor vessel the mixture-vapor interface of a two-phase fluid depends upon the initial water level and upon the steam void content (power level in a reactor system). The voids that are formed in the reactor core and riser and entrained in the downcomer displace an equal volume of water, which causes an increase in the mixture height. The water level above the riser is, in turn, further expanded by the vapor flowing through it, which creates a stagnant two-phase mixture or "bed." The final height of the two-phase mixture is a function of the superficial steam velocity (based on vessel diameter) and water content above the riser. Thus, for a given initial water level at saturation (with no voids present) the expansion

of the two-phase "bed" increases with increasing power because of the higher superficial steam velocity and the increased water content which was expelled from the region below the riser.

The height of the two-phase mixture cannot be obtained by purely analytical means. Recourse must be made to experimental data on vapor holdup through stagnant beds of water. Unfortunately, no data exist for flow of steam through stagnant water at 600 psi in large vessels. However, such data is available on air-water systems at atmospheric pressure and in vessels up to 36 inches in diameter.¹ Additional data have been reported by Behringer² for steam flowing through water in a 2-in. pipe at various pressures up to 600 psi. This data tended to check the work of Zmola¹ and serve as a basis of extrapolation of the air-water data to higher pressures for the large diameter vessels.

Calculations of the height of the vapor-mixture interface have been made for EBWR at 40 Mw and tend to explain the increase in carryover actually measured at that power level, since they show that the interface would have been very close to the steam ring at these powers.

Additional data on the transport of vapors through low-velocity water at the higher pressures must be obtained before optimum vessel sizes can be chosen.

The problem of entrainment of liquid with the steam will also influence strongly the sizing of the reactor vessel. Data in the Russian literature indicate that the entrainment is not a function of the superficial velocity alone but also of dome height; that is the liquid entrainment is a function of the steam dome volume. The superficial steam velocity is known to affect primary vapor separation through its effect on the steam "disengaging height" at the interface. The steam disengaging height is defined as the length required for the transition from the steam-water mixture to steam of constant quality. The existence of this zone, which probably results from the expulsion of liquid droplets by the disengaging steam at the vapor-liquid interface, has been demonstrated in EBWR (see Fig. VI.2). As can be seen, the disengaging height increases with increasing superficial steam velocity and hence reactor power. Therefore the minimum steam dome height for effective primary natural vapor-liquid separation must exceed the disengaging height.

¹ Transport of Gases through Liquid-Gas Mixture, P. C. Zmola, R. V. Bailey, F. M. Taylor and R. J. Planchet, Tulane University and Oak Ridge National Lab, unpublished.

² Behringer, P., Velocity in Rise of Steam Bubbles in Boiler Tubes, Verein deutscher Ingenieure Forschungsheft, 4 (1934).

FIGURE VII 2
DISENGAGING HEIGHT (INTERFACE THICKNESS) AS A FUNCTION OF POWER IN EBWP

It can be seen that sizing of the pressure vessel and, in fact, the actual reactor design depend to a very large degree on the vapor-liquid separation problem. If the primary vapor-liquid separation cannot be achieved within the reactor vessel, recourse must be made to external separation means such as steam drums, etc.

4. Reactor Geometry; Natural vs Forced Circulation

Recirculation of the coolant through the reactor core can be achieved by either natural or forced convection. Proper selection

of the mode of operation can be made only after a careful evaluation of the limitations imposed by reactor design parameters, such as reactor geometry, minimum flow-rate requirements, vapor separation, reactor stability, etc.

In a natural-convection system the recirculation flow rates that can be obtained are determined primarily by the system geometry. More specifically, the hydraulic diameters, relative flow areas and heights of the core, riser and downcomer are the controlling factors. Employment of natural circulation, therefore, tends to impose limitations upon the geometry that can be used. In a forced-circulation system the reactor geometry is not usually a major factor, providing the necessary pump head and capacity requirements do not exceed feasible limits.

Natural-circulation flow rates can be as high or higher than flows normally associated with a forced-circulation system. Laboratory tests on a natural-circulation loop showed that recirculation velocities up to 10 ft/sec could readily be obtained. Basically the system consisted of a heated section, a 4 ft-long pipe of 1-in. diameter, and a 2-in. diameter riser, 8 ft long. The flow rates usually associated with forced circulation are in the range of 8-12 ft/sec.

Natural-circulation velocities can be increased either by enlarging the core coolant channel hydraulic diameter and relative flow areas of the riser and downcomer or by adding riser height. A natural-circulation system featuring a riserless core or heated section yields recirculation velocities which are generally quite low, about 2-4 ft/sec.

However, by adding riser height of large hydraulic diameters, the net natural-circulation driving head is increased without introducing additional flow resistance. As a result the recirculation velocity can readily be increased.

An important reactor design parameter which can influence the selection of either natural or forced circulation is the vapor-separation problem. As mentioned previously, in a closed reactor vessel a power level is ultimately reached where the separation problem becomes acute and excessive moisture carryover and carryunder occurs which adversely affect reactor operation. When this point is reached, it is usually necessary to resort to external separation methods, such as the use of steam drums. Such a system has an excellent natural-circulation potential, since the basic design readily lends itself to incorporation of adequate riser height to obtain the desired performance. However, the effect of high risers on reactor stability is unknown at present.

Reactor stability considerations will also have a direct bearing on the selection of the type of circulation. Laboratory tests have shown that natural-circulation stability is a function of system geometry, pressure, and exit steam volume fraction of the heated channel. These tests have shown that there is apparently a threshold value of the exit steam volume fraction beyond which hydrodynamic instability occurs. This relationship is also a function of pressure; as the system pressure increases, the system will become unstable at a lower value of the exit steam volume fraction, but at a higher power density.

Hydrodynamic stability is strongly affected by the system geometry. Placement of restrictions in the downcomer tends to increase the stability of the system; that is, the threshold value of the exit steam volume fraction increases at the inception of instability. A series of tests which were run to study the effect of riser geometry showed that, for a system where riser flow area was one-half that of the test section, hydrodynamic instability was observed at a power density of 120 kw/l at 600 psi; the inlet section velocity was approximately 4 ft/sec. By comparison, stable operation was obtained at a power level of 350 kw/l for the riser geometry of twice the flow area of the test section. The recirculation velocity in this instance was 8 ft/sec.

It should be noted that, in general, natural-circulation systems are hydrodynamically stable over wide ranges of conditions, and from information gained recently it appears that the stability of such systems can be improved considerably by proper design. More information is needed, however, especially on high riser systems of large hydraulic diameter and of varying relative flow areas to the core.

Forced-convection systems with multiple flow channels are also subject to hydrodynamic instabilities which result from parallel channel effects. These effects are due to possible variations in the resistance of each channel while they are being maintained at a fixed pressure differential common to all. As a result, orificing of the individual channels is required to eliminate the possibility of flow fluctuations occurring. This is a much discussed subject, but little laboratory information is available.

In the final analysis, the merits of a natural-circulation or forced-circulation system must be subject to economic comparison. The initial capital investment and maintenance of the external pump loops of the forced-circulation system must be weighed against the cost of the much more simplified but possibly more limited natural-circulation system.

D. DESIGN PROCEDURE FOR BOILING WATER REACTORS

A design procedure which may be used in a boiling water reactor design study is outlined below. It should be recognized that the method of approach suggested is but one of a number that can be followed. A design procedure will, in fact, evolve during the design study and will be governed by the objectives and type of reactor under study. The sole purpose of this outline is to illustrate the various calculation procedures generally used in boiling water reactor design analysis.

(1) Establish the design objective. The design objectives usually are many and varied, such as (a) a specified total power, (b) the maximum power that can be obtained from a given size of pressure vessel, (c) a certain core power density, (d) achieving vapor-liquid separation within the reactor vessel, (e) natural or forced circulation, etc.

(2) Conduct a general parametric study of the various design parameters such as coolant channel hydraulic diameter, riser length, core diameter and height, mean core steam volume fraction, moderator to fuel ratio, structure to fuel ratio, etc. The purpose of this study would be to establish feasible limits on the various design parameters which would satisfy the design objectives. The thermal hydraulic portion of the study is tied to the core physics and fuel element design by the following equations:

$$nD_R^2 = \frac{D_c^2}{(M/F) + (S/F) + 1} \quad (VI.1)$$

$$D_e = \left[D_c^2 - nD_R^2 \left(1 + \frac{S}{F} \right) \right] / nD_R \quad (\text{VI.2})$$

$$P_R = n\pi D_R L_c q_c'' / 3.413 \times 10^6 \quad (\text{VI.3})$$

$$T_{L \max} = T_{\text{sat}} + R_{\text{max/avg}} q_c'' \left(\frac{D_R}{4k_R} + \frac{1}{h_c} + \frac{t_c}{k_c} + \frac{1}{h_B} \right) \quad (\text{VI.4})$$

These equations can be plotted in graphical form, as shown in Fig. VI.3. Such graphs can readily be used in a parametric study. Preliminary estimates of the core recirculation flow rate, steam volume fraction, power density, etc., can be obtained from generalized "average channel" calculations, which can be made rather quickly. The inherent assumptions in such an analysis are (1) a constant axial heat input, and, (2) the overall performance characteristics of the core can be estimated by an analysis of the average channel (based on power density). Although these assumptions are subject to much question, the calculated performance by the average channel method will not vary very greatly from a detailed analysis unless radical departures from ordinary reactor geometries are encountered.

The following procedure and equations can be used for the thermal hydraulic calculations:

$$P.D._{kw/L} = P_R(1000) / L_c A_{\text{coolant}} (28.3) \quad (\text{VI.5})$$

The cross-sectional flow area is related to the moderator to fuel ratio as follows:

$$A_{\text{coolant}} = \frac{\pi D_c^2}{4} - \frac{n\pi D_R^2}{4} \left[1 + \frac{S}{F} \right] \quad (\text{VI.6})$$

Therefore,

$$P.D._{kw/L} = \frac{P_R(1000)}{L_c (28.3) \left(\frac{\pi D_c^2}{4} - \frac{n\pi D_R^2}{4} \left[1 + \frac{S}{F} \right] \right)} \quad (\text{VI.7})$$

The average coolant power density is also related to the hydraulics of the system. From a heat balance on the boiling section of the average channel with a uniform heat input,

$$Q = W_{T^x} h_{fg} = V A_{\text{coolant}} \rho_l^x h_{fg} \quad (\text{VI.8})$$

FIG. VI.3

RELATIONSHIPS BETWEEN VARIOUS REACTOR DESIGN PARAMETERS

Converting to power density in kw/L,

$$P.D. \text{ kw/L} = \frac{Q(3600)}{3413 A_{\text{coolant}} L_B (28.3)} = \frac{V \rho_l x_e h_{fg}}{(28.3) \frac{L_B}{L_c} (3413)}$$

and

$$P.D. \text{ avg kw/L} = \frac{V x_e \rho_l h_{fg}}{\frac{L_B}{L_c} (L_c)} (0.0374) \quad (VI.9)$$

Equating Eq. VI.5 and VI.9,

$$V = 948 P_R L_B / L_c A_{\text{coolant}} x_e \rho_l h_{fg} \quad (VI.10)$$

Equation VI.10 merely defines the relationship between the recirculation velocity, core geometry and power. The recirculation velocity, V , is also, however, a function of the hydraulic characteristic of the system. The relationship between the velocity and the system geometry is given by Eq. V.25:

$$\frac{V^2}{2g} = \frac{L_{nB}(\rho_D - \bar{\rho}_{NB}) + L_B(\rho_D - \bar{\rho}_B) + \sum R_i(\rho_D - \rho_{Ri})}{N_1 + \dots + N_g + \sum N_i + \sum N_{Ri}} \quad (VI.11)$$

where

$$N_1 = \rho_f \left(\frac{A_{HS}}{A_D} \right)^2 \left[1 + K_{cUP} \right] \quad = \text{acceleration of the fluid from the upper plenum to downcomer and frictional resistance at contraction.}$$

$$N_2 = f_D \left(\frac{L_D}{D_D} \right) \left(\frac{\rho_f}{\rho_D} \right) \left(\frac{A_{HS}}{A_D} \right)^2 \quad = \text{frictional resistance in downcomer.}$$

$$N_3 = \frac{\rho_f^2}{\rho_D} \left(\frac{A_{HS}}{A_D} \right)^2 \left(\frac{\rho_f}{\rho_D} \right)^2 \left[\left(\frac{A_D}{A_{LP}} \right)^2 - 1 + K_{E_{LPB}} \right] \quad = \text{deceleration of fluid and frictional losses at the expansion from the downcomer to lower plenum}$$

$$N_4 = \frac{\rho_f^2}{\rho_D} \left[1 - \left(\frac{A_{HS}}{A_{LP}} \right)^2 + K_{cHSB} \right] \quad = \text{acceleration of fluid and frictional losses at the contraction from lower plenum to the heated section.}$$

$$N_5 = 2\rho_f \left[\frac{\rho_{HSB} - \rho_f}{\rho_{HSB}} \right]$$

= acceleration of fluid in nonboiling length due to density change with temperature.

$$N_6 = \frac{\rho_f^2}{\rho_{LB}} \left(\frac{f_{HS}}{D_{HS}} \right) (L_{NB})$$

= frictional resistance in the nonboiling segment of the heated section.

$$N_7 = \frac{\rho_f^2}{\rho_{LB}} (\bar{R}_{LB}) \left(\frac{f_{HS}}{D_{HS}} \right) (L_{LB})$$

= frictional resistance in the local boiling segment of the heated section.

$$N_8 = 2(r)(\rho_f)^2$$

= acceleration of fluid due to formation of steam voids in the heated channel.

$$N_9 = \rho_f (\bar{R}) \left(\frac{f_{HS}}{D_{HS}} \right) (L_B)$$

= frictional resistance in the boiling segment.

$$N_{10} = 2\rho_f \left(\frac{A_{HS}}{A_i + 1} \right) \left[x_{HSE}^2 \left\{ \frac{\rho_f}{\rho_g} \frac{1}{(\alpha_i + 1)} - \frac{A_{HS}}{A_i} \frac{1}{\alpha_i} \right\} + (1 - x_{HSE})^2 \left\{ \frac{A_{HS}}{A_i + 1} \frac{1}{(\alpha_i + 1)} - \frac{A_{HS}}{A_i \alpha_i} \right\} \right]$$

= total pressure drop due to an expansion or contraction.

$$N_{11} = R f_R \left(\frac{A_{HS}}{A_R} \right)^2 \frac{\rho_f}{D_R} L_R$$

= frictional loss in the adiabatic risers.

Thus the thermodynamic and hydraulic equations, VI.10 and VI.11, which describe the behavior of the average channel, must be solved simultaneously. The unknowns in the equation can be obtained as follows:

The coolant cross sectional flow area, $A_{coolant}$, can be estimated from eq. VI.6 if a fuel element design and core layout has not been decided upon.

The two-phase friction factor multipliers $\bar{R}$ and R can be obtained from the literature or by the methods suggested in Section V.

The acceleration pressure drop multiplier, r , is

$$r = \left[\frac{(1 - x_e)^2}{1 - \alpha_e} + \frac{x_e^2}{\alpha_e} \frac{\rho_f}{\rho_g} - 1 \right] \frac{1}{\rho_f} \quad (\text{VI.12})$$

The mean density in the boiling segment may be found as a function of pressure, slip ratio, and exit steam volume fraction. For the preliminary analysis of this type, the assumption is made that the slip ratio is constant along the heated length and that there is a uniform heat input.

$$\bar{\rho}_B = 1 - \frac{1 - (\rho_g/\rho_f)}{1 - \psi} \left\{ 1 - \left[\frac{1}{\alpha_e(1 - \psi)} - 1 \right] \ln \left[\frac{1}{1 - \alpha_e(1 - \psi)} \right] \right\}, \quad (\text{VI.13})$$

where

$$\psi \equiv \frac{V_g}{V} \frac{\rho_g}{\rho_l}$$

The two-phase mixture density in an adiabatic riser section is simply

$$\rho_{R_i} = (1 - \alpha_{R_i}) \rho_f + \alpha_{R_i} \rho_g \quad (\text{VI.14})$$

The change in voids due to a geometry change may be estimated from the following semi-empirical equation:

$$\alpha_2 = \frac{1}{\frac{(1/\alpha_1 - 1)}{(A_1/A_2)^N} + 1} \quad (\text{VI.15})$$

where N is the slope of the slip ratio vs water velocity plot as discussed in Section V.

The mixture quality, X , is related to the steam volume fraction, α , by the slip ratio V_g/V_l through the continuity equation. At any plane normal to the fluid flow in a channel,

$$W_g/W_f = xW_T/(1 - x)W_T = \rho_g A_g V_g / \rho_l A_l V_l$$

Introducing

$$A_g = \alpha A_T$$

and

$$A = (1 - \alpha) A_T$$

and simplifying,

$$x = \frac{1}{1 + \rho_l/\rho_g \frac{(1/\alpha - 1)}{V_g/V_l}} \quad , \quad (\text{VI.16})$$

The boiling length ratio may be obtained from an overall heat balance on a boiling channel

$$Q_T = W_T [h_f - h_i + x_e h_{fg}] = W_g [h_f - h_M + h_{fg}] \quad .$$

Inspection of the above equation will reveal that

$$Q_B/Q_T = h_{fg}/(h_f - h_M + h_{fg}) \quad . \quad (\text{VI.17})$$

This ratio is therefore dependent only on pressure and feedwater injection temperature.

For a uniform heat input,

$$Q_B/Q_T = L_B/L_T = h_{fg}/(h_g - h_M) \quad . \quad (\text{VI.18})$$

A direct solution of eq. VI.10 and VI.11 is not possible, since eq. VI.11, describing the hydraulic characteristics of the system, is implicit in V . The procedure generally used for solving these equations is as follows. The solution is found by an iterative process.

1. Assume a value of exit steam volume fraction, α_e .
2. Estimate a value of the slip ratio (using the correlations presented in the previous section) and calculate the quality, using eq. VI.16.
3. Calculate the boiling length from eq. VI.18.
4. Solve the flow equation (VI.11).
5. Re-evaluate the slip ratio and recalculate the quality if necessary. Repeat process until solution converges.
6. Calculate the velocity, using eq. VI.10 and the quality calculated in step 2.
7. If the velocity calculated in step 6 equals the velocity calculated in step 4, the desired solution has been obtained. If the two velocities are not equal, a new value of the exit steam volume fraction must be assumed and the entire procedure repeated until the velocities become equal.

Such an analysis is carried out for a series of differing reactor geometries of varying core flow equivalent diameters, core height, riser height, etc., and a series of curves of the type shown in Fig. VI.4 is obtained. Such curves are very useful in that they yield quick estimates of reactor performance for a great number of possible reactor designs. Preliminary sizing of the pressure vessel can also be carried out at this point since the total recirculation flow rates, core voids, etc., can be obtained.

FIG. VI. 4

EFFECT OF RISER HEIGHT AND EXIT STEAM VOLUME FRACTION
ON PERFORMANCE OF NATURAL CIRCULATION BOILING REACTORS;
 $D_e = 0.5 \text{ in.}$; $D_R = 1 \text{ in.}$; $P = 1000 \text{ lb/in.}^2$

The total recirculation flow rate is

$$W_T = A_{\text{coolant}} V \rho_l$$

The downcomer is generally sized so that it will have a negligible pressure drop due to friction and low enough velocity so that excessive carryunder of steam is not obtained. A thermal shield analysis is then made to establish the thickness. Combining these factors, the diameter of the vessel can be set. The height of the vessel is established by allowing sufficient length on the lower plenum, core, riser, and primary internal vapor separation, if desired. The required vessel height above the riser is a function of the reactor power, initial saturated zero-power water level, pressure, etc., as discussed previously in the section on vapor-separation problems.

(3) Select several of the most probable final designs and then make detailed physics, engineering and thermal-hydraulic calculations to arrive at the final design. The calculation procedure for a detailed core analysis is much more complex, due to the additional complicating factors of constant inlet subcooling across the core and the axial and radial power distribution within the core.

In such an analysis, the following unknowns are to be determined: (1) the core inlet subcooling, (2) total recirculation flow rate, (3) velocity profile across the core, (4) mean core steam volume fraction, (5) axial and radial steam volume fraction profiles, (6) axial and radial mixture quality profile, (7) boiling length profile across the core. These factors are a function of the axial and radial core power distributions. The power distribution, however, in turn is a function of the system hydraulics, since it is a function of the steam volume fraction distribution within the core. The core physics and hydrodynamics are coupled together by this interrelationship.

A procedure that can be followed for a core analysis is as follows:

1. Assume a radial and an axial power distribution and calculate the power density distribution.
2. Assume a velocity profile across the core.
3. Calculate the total recirculation flow rate, W_T :

$$W_T = \rho_f \int_0^R V_r dA; \quad = \rho_f \int_0^R V_r 2\pi r dr; \quad = 2\pi \rho_f \int_0^R V_r r dr$$

4. Calculate the core inlet subcooling by a heat balance at the top of the downcomer where the make-up water is injected:

$$W_m h_m + (1 - \bar{x}_e) W_T h_f = W_T h_i$$

Since $W_m = W_S = x W_T$,

$$\bar{x}_e W_T h_m + (1 - \bar{x}_e) W_T h_f = W_T h_i$$

or

$$h_i = \bar{x}_e h_m + (1 - \bar{x}_e) h_f$$

5. Select a number of points (channels) at appropriate core radii for analysis and determine the power release from step 1. More points should be selected near the outer region of the core because (1) the power distribution usually varies more markedly near the core periphery and (2) the velocity profile must be established more accurately since the greatest percentage of the total coolant flow occurs at the outer region of the core.
6. Calculate the exit mixture quality for each of the points (or channel) selected for study by a heat balance using the assumed velocity profile from step 2.

$$P.D._{kw}/L = 0.0374 V_r \rho_f (h_f - h_i - x_e h_{fg}) / L_c$$

7. Calculate the exit steam volume fraction profile using the mixture qualities in step 6. Also estimate the steam volume fraction in the various risers. The steam volume fraction is calculated by the following equation:

$$\frac{V_g}{V_l} = \left(\frac{x}{1-x} \right) \left(\frac{1-\alpha}{\alpha} \right) \frac{\rho_l}{\rho_g}$$

The slip ratio may be obtained from the slip ratio correlations presented in the previous section.

8. Calculate the boiling length for the various channels under study by the following relationships:

$$\frac{Q_{NB}}{Q_T} = \frac{(h_f - h_i)}{(h_f - h_i) + x_l h_{fg}}$$

and

$$Q_B/Q_T = 1 - Q_{NB}/Q_T$$

A plot of Q/Q_T vs the core length, L_c , can be obtained by integrating the power distribution. The point at which boiling begins is where

$$Q/Q_T = Q_{NB}/Q_T \quad .$$

9. Calculate the recirculation velocity by the flow equation (VI.11) at the selected points on the core radius.
10. Compare the recirculation velocity profile obtained from step 9 with the one initially assumed in step 2. If the profiles are the same, the performance of the core has been established. If the profiles are not the same, steps 2-10 must be repeated until agreement is reached.
11. After the calculation has been completed, the assumed power distributions must be recalculated using the new steam volume fraction profile. If the new power distribution does not check the one in step 1, the entire procedure must be repeated.

Alternate methods of solution can be used. For instance, a subcooling could have initially been assumed and then the velocity profile calculated. The solution would iterate on the subcooling instead.

To illustrate the procedure for analyzing a natural-circulation boiling water reactor, consider the proposed operation of EBWR (Experimental Boiling Water Reactor) at 100 Mw. Figure VI.5 lists the dimensions to be used in the calculation and indicates the relationship of the various riser sections.

There are two types of fuel elements in the core. In the central region (i.e., the region in which the elements feed the central portion of the non-homogeneous riser) there are 32 plate-type elements. Grouped around these elements are 28 rod-type elements with a higher enrichment, called spike elements. The spike elements feed into the outer (also called annular) portion of the non-homogeneous riser. Around the spike elements are grouped 87 more plate-type elements, completing the core.

Element	Number	Area per Element	Total Area
Plate - central	32	0.0580 ft ²	1.856 ft ²
Spike	28	0.0591 ft ²	1.655 ft ²
Plate - outer	87	0.0580 ft ²	5.046 ft ²
			8.557 ft ²

The average power per element is

$$Q_{avg} = \frac{100 \text{ Mw}}{147 \text{ elements}} \times 3.413 \times 10^6 \frac{\text{Btu}}{\text{hr Mw}} = 2.322 \times 10^6 \frac{\text{Btu}}{\text{hr}} \quad .$$

FIGURE VI.5

EBWR

	Area	D_e
Plate elements	0.0580 ft^2	0.0573 ft
Spike elements	0.0591 ft^2	0.0417 ft

FIGURE VI.6

EBWR 100-Mw RADIAL POWER PROFILE

FIGURE VI.7

EBWR 100-Mw AXIAL POWER PROFILE

FIGURE VI.8

EBWR 100-Mw RADIAL VELOCITY PROFILE

A graphical integration of the radial power distribution indicates that 26.24% of the power is generated in the central region. The operating pressure is 600 psia, and makeup water is fed to the reactor at 100°F. Thus, the total steam flow is

$$W_S = \frac{\text{Power}}{h_g - h_{in}} = \frac{3.413 \times 10^8}{1203.2 - 68.0} = 3.007 \times 10^5 \text{ lb/hr} \quad .$$

The steam flow to the central nonhomogeneous riser is

$$W_{s_c} = 0.2624 \times 3.007 \times 10^5 = 0.7890 \times 10^5 \text{ lb/hr} \quad .$$

The steam flow to the outer annular region is

$$W_{s_A} = (3.007 - 0.789) \times 10^5 = 2.218 \times 10^5 \text{ lb/hr} \quad .$$

A velocity profile for the entire core is assumed as a basis for calculation. Assume the velocity in the spike elements is 5.4 ft/sec and the velocity in the plate-type elements is everywhere 7 ft/sec.

The flow rates of the regions are then calculated.

$$\begin{aligned} W_C &= \text{central fluid flow} = \rho A V \\ &= 49.75 \text{ lb/ft}^3 \times 1.856 \text{ ft}^2 \times 7 \text{ ft/sec} \times 3.6 \times 10^3 \text{ sec/hr} \\ &= 2.327 \times 10^6 \text{ lb/hr} \quad . \end{aligned}$$

$$\begin{aligned} W_A &= \text{annular fluid flow} = \rho A_{\text{spike}} V_{\text{spike}} + \rho A_{\text{plate}} V_{\text{plate}} \\ &= 49.75 \times 1.655 \times 5.4 \times 3.6 \times 10^3 + 49.75 \times 5.046 \times 7 \times 3.6 \times 10^3 \\ &= 7.927 \times 10^6 \text{ lb/hr} \quad . \end{aligned}$$

The total core flow rate is thus

$$W_T = 1.0254 \times 10^7 \text{ lb/hr} \quad .$$

The net quality in the common riser is

$$x = W_S / W_T = 0.0293 \quad .$$

The quality in the central riser is

$$x_c = W_{s_c} / W_C = 0.0339 \quad .$$

The quality in the outer riser is

$$x_A = W_{sA} / W_S = 0.0280 \quad .$$

The subcooling is calculated by

$$\begin{aligned} h_{in} &= (1 - x)h_f + xh_m \\ &= 0.9707 \times 471.6 + 0.0293 \times 68 = 459.8 \text{ Btu/lb} \quad . \end{aligned}$$

Thus,

$$\Delta h_{\text{sub}} = 471.6 - 459.8 = 11.8 \text{ Btu/lb} \quad .$$

This value of subcooling is assumed to remain constant across the entire core.

The void fraction in the various regions is calculated by

$$\alpha = \left[1 + \frac{\rho_g}{\rho_f} \frac{V_g}{V_f} \frac{(1 - x)}{x} \right]^{-1} \quad .$$

Thus, the void fractions in the various regions are:

Central riser

$$\alpha_{R_2} = \left[1 + \frac{1.89}{38.3} \times \frac{0.9661}{0.0339} \right]^{-1} = 0.416$$

Outer riser

$$\alpha_{R_2} = \left[1 + \frac{1.87}{38.3} \times \frac{0.9720}{0.0280} \right]^{-1} = 0.371$$

Tapered region
of common riser

$$\alpha_R = \left[1 + \frac{1.82}{38.3} \frac{0.9707}{0.0293} \right]^{-1} = 0.388$$

Common riser

$$\alpha_{R_3} = \left[1 + \frac{1.59}{38.3} \frac{0.9707}{0.0293} \right]^{-1} = 0.536$$

Vessel above
common riser

$$\alpha_V = \left[1 + \frac{2.10}{38.3} \frac{0.9707}{0.0293} \right]^{-1} = 0.355$$

The previously discussed equation (VI.11) for computing natural-circulation velocity as applied to this system is

$$\frac{V_0^2}{2g} = \frac{a + b + c_1 + c_2 + c_3}{N_{11} + N_{12} + N_2 + N_3 + N_{41} + N_{42} + N_{43} + N_6 + N_7 + N_8 + N_{91} + N_{93} + N_{94}}.$$

The significances of the terms of the equation as applied to this problem will be dealt with explicitly below and the values will be calculated. The analysis of a system as complicated as this one can be simplified by treating the terms related to external risers and downcomers separately from the terms related to the elements. This is because the pressure differential between the entrance to an element at the lower plenum and the exit from an element at the end of the element portion of riser into the nonhomogeneous riser must be the same for all elements of the same length feeding the same nonhomogeneous riser.

Thus, by rearranging terms so that

$$\begin{aligned} \frac{V_0^2}{2g} \left[N_{11} + N_{12} + N_{42} + N_{43} + N_{93} + N_{94} \right] - c_2 - c_3 &= a + b + c_1 \\ - \frac{V_0^2}{2g} \left[N_2 + N_3 + N_{41} + N_6 + N_7 + N_8 + N_{91} \right] &, \end{aligned}$$

the left side of the equation contains terms related to the external risers and downcomers, whereas the right side of the equation contains terms related to the interior of an element.

The pressure differentials through various elements are compared to the pressure differentials through the external risers and downcomers. The velocities in the elements are adjusted until the pressure differentials agree.

The pressure differentials in the external risers and downcomers are

$$\Delta P_I = \frac{\bar{V}_0^2}{2g} \left[N_{11} + N_{12} + N_{42} + N_{43} + N_{93} + N_{94} \right] - c_2 - c_3$$

where $\bar{V}_0$ is the average velocity of the elements feeding a given external riser, for the velocity distribution originally assumed.

The significances of the various terms are listed, and the calculation is illustrated for the central riser section. In the calculations of hydrostatic

head the common riser is assumed to be 1.5 feet shorter than its actual length, to account for steam carryover and allow for possible shortening of the riser.

friction loss in upper
portion of downcomer

$$\begin{aligned}
 N_{11} &= f_{D1} \left(\frac{L_{D1}}{D_{D1}} \right) \left(\frac{\rho_f^3}{\rho_D} \right) \left(\frac{A_C}{A_{D1}} \right)^2 \\
 &= (0.00905) \left(\frac{5}{2.75} \right) \left(\frac{49.75^2}{50.25} \right) \left(\frac{8.557}{22.14} \right)^2 \\
 &= 0.12
 \end{aligned}$$

friction loss in lower
portion of downcomer

$$\begin{aligned}
 N_{12} &= f_{D2} \left(\frac{L_{D2}}{D_{D2}} \right) \left(\frac{\rho_f^2}{\rho_D} \right) \left(\frac{A_C}{A_{D2}} \right)^2 \\
 &= (0.0093) \left(\frac{8.5}{1.412} \right) \left(\frac{49.75^2}{50.25} \right) \left(\frac{8.557}{13.305} \right)^2 \\
 &= 1.14
 \end{aligned}$$

friction loss in non-
homogeneous riser

$$\begin{aligned}
 N_{42} &= f_{R2} R_{R2} \left(\frac{L_{R2}}{D_{R2}} \right) \left(\frac{A_C}{A_{R2}} \right)^2 \rho_f \\
 &= (0.009) (6.00) \left(\frac{3.00}{2.254} \right) \left(\frac{1.856}{4.00} \right)^2 (49.75) \\
 &= 0.77
 \end{aligned}$$

friction loss in
common riser

$$\begin{aligned}
 N_{43} &= f_{R3} R_{R3} \left(\frac{L_{R3}}{D_{R3}} \right) \left(\frac{A_C}{A_{R3}} \right)^2 \rho_f \\
 &= (0.0078) (4.00) \left(\frac{4.5}{3.75} \right) \left(\frac{8.557}{11.04} \right)^2 (49.75) \\
 &= 1.12
 \end{aligned}$$

expansion from common
riser into vessel above

$$\begin{aligned}
 N_{9_3} &= 2\rho_f \left(\frac{A_C}{A_V} \right) \left[x^2 \frac{\rho_f}{\rho_g} \left(\frac{A_C}{A_V \alpha_V} - \frac{A_C}{A_{R_3} \alpha_{R_3}} \right) \right. \\
 &\quad \left. + (1-x)^2 \left(\frac{A_C}{A_V(1-\alpha_V)} - \frac{A_C}{A_{R_3}(1-\alpha_{R_3})} \right) \right] \\
 &= 2 (49.75) \left(\frac{8.557}{32.28} \right) \left[(0.0293)^2 (38.3) \left(\frac{8.557}{33.28 \times 0.355} \right) \right. \\
 &\quad \left. - \frac{8.557}{11.04 \times 0.536} \right] + (0.9707)^2 \left(\frac{8.557}{33.28 \times 0.645} \right. \\
 &\quad \left. - \frac{8.557}{11.04 \times 0.464} \right) \Bigg] = -31.24
 \end{aligned}$$

contraction loss from
nonhomogeneous riser
into common riser

$$\begin{aligned}
 N_{9_4} &= 2\rho_f \left(\frac{A_C}{A_R} \right) \left[x^2 \frac{\rho_f}{\rho_g} \left(\frac{A_C}{A_{R_3} \alpha_{R_3}} - \frac{A_C}{A_R \alpha_R} \right) \right. \\
 &\quad \left. + (1-x)^2 \left(\frac{A_C}{A_{R_3}(1-\alpha_{R_3})} - \frac{A_C}{A_R(1-\alpha_R)} \right) \right] \\
 &= 2 (49.75) \left(\frac{8.557}{16.44} \right) \left[(0.0293)^2 (38.3) \left(\frac{8.557}{11.04 \times 0.536} \right) \right. \\
 &\quad \left. - \frac{8.557}{16.44 \times 0.388} \right] + (0.9707)^2 \left(\frac{8.557}{11.04 \times 0.464} - \frac{8.557}{16.44 \times 0.612} \right) \Bigg] \\
 &= 40.15
 \end{aligned}$$

hydrostatic head in
nonhomogeneous riser

$$\begin{aligned}
 C_2 &= (\bar{\rho}_D - \bar{\rho}_{R_2}) L_{R_2} \\
 &= (50.25 - 29.59) 3 = 61.98 \text{ ft}
 \end{aligned}$$

hydrostatic head in
common riser

$$C_3 = (\rho_D - \rho_{R_3}) L_{R_3} \\ = (50.25 - 30.95) 3 = 57.90 \text{ ft} .$$

The summation of the n terms is 12.06, so that

$$(\bar{V}_0^2/2g) \sum N = (49.00/64.34)(12.06) = 9.18 \text{ ft} .$$

The total pressure differential through the risers and downcomers is
thus

$$\Delta P_I = -110.70 \text{ ft} .$$

The same type of analysis is used to find ΔP_I for the spike elements
and the outer plate-type elements. The results are:

spike

$$\Delta P_I = -91.63 \text{ ft}$$

plate, outer

$$\Delta P_I = -105.43 \text{ ft} .$$

The pressure differential through individual elements is now calculated. Six points in the core were selected to obtain the velocity profile. The points were radii of 10, 25, 40, 50, 60, and 70 cm. The radial power distribution shown in Fig. VI.6 was used to obtain the power at each point. Consider a point at radius 10 cm, to illustrate the calculation. The power of an element at radius 10 cm is

$$Q_r = \left(\frac{Q_r}{Q_{\text{avg}}} \right) Q_{\text{avg}} = (1.085)(2.322 \times 10^6) = 2.519 \times 10^6 \text{ Btu/hr} .$$

Several iterations on velocity will be necessary in order to find the correct velocity for an element. As a first assumption, $V_0 = 7 \text{ ft/sec}$ will be selected. Thus, the fluid flow rate for an element at radius 10 cm is

$$W = \rho AV = (49.75)(0.0580)(7.0)(3.6 \times 10^3) = 72,716 \text{ lb/hr} .$$

The power required to bring the subcooled liquid to saturation is

$$Q_{NB} = W\Delta h_{\text{sub}} = (72,716)(11.8) = 0.858 \times 10^6 \text{ Btu/hr} .$$

The nonboiling and boiling lengths of the element are now found by using Fig. VI.7, a plot of the fraction of total power along the axial length of the element.

$$Q_{NB}/Q_r = 0.858 \times 10^6 / 2.519 \times 10^6 = 0.341 \quad .$$

Thus, the nonboiling length is

$$L_{NB} = 30.4 \text{ cm} = 0.997 \text{ ft}$$

and the boiling length is

$$L_B = 3.003 \text{ ft} \quad .$$

The average density in the boiling region is found by selecting a number of equally spaced points along the boiling length, determining by Fig. VI.7 the fraction of total power generated up to each point, subtracting from that the fraction of the power used in the nonboiling length to reach saturation, thereby determining the quality at the point, by which the slip can be obtained so as to calculate the void fraction at the point, and thus obtaining the density at each point. The average of these densities along the boiling length is the average boiling density. This procedure must be used because the axial power distribution is not of a simple shape.

This calculation will be illustrated for the last point in the element. Here, all the power has been generated so

$$Q/Q_r = 1.000 \quad .$$

Thus,

$$(Q - Q_{NB})/Q_r = 1.000 - 0.341 = 0.659 \quad .$$

The quality is

$$\begin{aligned} x &= \frac{Q_B}{Wh_{fg}} = \frac{Q - Q_{NB}}{Wh_{fg}} = \frac{Q_r}{Wh_{fg}} \left(\frac{Q - Q_{NB}}{Q_r} \right) \\ &= \frac{2.519 \times 10^6}{(72,716)(731.6)} (0.659) = 0.0312 \quad . \end{aligned}$$

Thus, the void fraction in the element riser is

$$\alpha = \left[1 + \frac{1.50}{38.3} \times \frac{0.9688}{0.0312} \right]^{-1} = 0.451 \quad .$$

The density at the last point and in the element riser is

$$\begin{aligned}\rho &= (1 - \alpha)\rho_f + \alpha\rho_g = 0.549 \times 49.75 + 0.451 \times 1.3 \\ &= 27.90\end{aligned}$$

The same type of calculation for other points along the boiling length yields the average boiling density

$$\bar{\rho}_B = 33.39$$

The pressure differential in the elements are

$$\Delta P_{II} = a + b + c_1 - (V_0^2/2g) \left[N_2 + N_3 + N_{4_1} + N_6 + N_7 + N_8 + N_{9_1} \right],$$

where V_0 is the velocity assumed for the iteration. The significances of the various terms are listed, and the calculation is illustrated, using the 7 ft/sec velocity at radius 10 cm for which the previous terms were calculated.

friction loss in
nonboiling section
of element

$$\begin{aligned}N_2 &= f_C (L_{NB}/D_C) (\rho_f^2/\rho_{NB}) \\ &= (0.0146) \left(\frac{1 + 0.997}{0.0573} \right) \left(\frac{49.75^2}{50.00} \right) \\ &= 25.20\end{aligned}$$

(Note: 1 foot of unheated element added to L_{NB}).

friction loss in
boiling section
of element

$$\begin{aligned}N_3 &= f_C \bar{R} (L_B/D_C) \rho_f \\ &= (0.0146)(2.04) \left(\frac{0.25 + 3.003}{0.0573} \right) (49.75) \\ &= 84.11\end{aligned}$$

(Note: 0.25 ft of unheated element added to L_B).

friction loss in element
portion of riser

$$\begin{aligned} N_{4_1} &= f_{R_1} R_{R_1} (L_{R_1}/D_C) \rho_f \\ &= (0.0146)(3.43)(0.75/0.0573)(49.75) \\ &= 32.61 \end{aligned}$$

acceleration from
top of lower plenum
into heated element

$$N_6 = \rho_f^2 / \rho_D = 49.72^2 / 50.25 = 49.25$$

acceleration in
nonboiling region

$$\begin{aligned} N_7 &= 2\rho_f(\bar{\rho}_D - \rho_f)/\bar{\rho}_D \\ &= 2(49.75)(50.25 - 49.75)/50.25 = 0.99 \end{aligned}$$

acceleration due to
void formation

$$\begin{aligned} N_8 &= 2\rho_f \left[\frac{(1-x)^2}{(1-\alpha)} + \frac{x^2}{\alpha} \frac{\rho_f}{\rho_g} - 1 \right] \\ &= 2(49.75) \left[\frac{(0.9688)^2}{0.549} + \frac{(0.0312)^2}{0.451} (38.3) - 1 \right] \\ &= 78.80 \end{aligned}$$

expansion from
element riser into
nonhomogeneous riser

$$\begin{aligned} N_{9_1} &= 2\rho_f \left(\frac{A_C}{A_{R_2}} \right) \left[x^2 \frac{\rho_f}{\rho_g} \left(\frac{A_C}{A_{R_2} \alpha_{R_2}} - \frac{A_C}{A_{R_1} \alpha_{R_1}} \right) \right. \\ &\quad \left. + (1-x)^2 \left(\frac{A_C}{A_{R_2}(1-\alpha_{R_2})} - \frac{A_C}{A_{R_1}(1-\alpha_{R_1})} \right) \right] \\ &= 2(49.75) \left(\frac{1.856}{4.00} \right) \left[(0.0312)^2 (38.3) \left(\frac{1.856}{4.00 \times 0.416} - \frac{1.856}{1.856 \times 0.451} \right) \right. \\ &\quad \left. + (0.9688)^2 \left(\frac{1.856}{4.00 \times 0.584} - \frac{1.856}{1.856 \times 0.549} \right) \right] = -48.89 \end{aligned}$$

hydrostatic head in
nonboiling section

$$\begin{aligned} a &= (\bar{\rho}_D - \bar{\rho}_{NB}) L_{NB} \\ &= (50.25 - 50.00)(1 + 0.997) = 0.50 \text{ ft} \end{aligned}$$

hydrostatic head
in boiling section

$$\begin{aligned} b &= (\bar{\rho}_D - \bar{\rho}_B) L_B \\ &= (50.25 - 33.39)(0.25 + 3.003) = 54.85 \text{ ft} \end{aligned}$$

hydrostatic head
in element riser

$$\begin{aligned} c_1 &= (\bar{\rho}_D - \rho_{R_1}) L_{R_1} \\ &= (50.25 - 27.90)(0.75) = 16.76 \text{ ft} \end{aligned}$$

The summation of the n terms is 222.07, so that

$$(V_0^2/2g) \sum N = (49.00/64.34)(222.07) = 169.12 \text{ ft}$$

The total pressure differential through the element is

$$\Delta P_{II} = -97.01 \text{ ft}$$

Thus,

$$\Delta P_I - \Delta P_{II} = -110.70 + 97.01 = -13.69 \text{ ft}$$

The assumed velocity was, therefore, slightly low. Assuming $V_0 = 7.5$ ft/sec would yield $\Delta P_I - \Delta P_{II} = 0.98$ ft. Plotting $\Delta P_I - \Delta P_{II}$ against V_0 for several iterations yields $V_0 = 7.47$ ft/sec at radius 10 cm.

This procedure is applied to the other core radii selected to obtain the following velocity profile:

Radius, cm	V_0 , ft/sec
10	7.47
25	7.43
40	3.85
50	7.45
60	7.46
70	7.51

This velocity profile is plotted in Fig. VI.8.

A more complicated velocity profile would require a graphical integration of the velocity over the core to obtain the average velocities of the regions. In this case, however, it is sufficient to estimate them from the plot. In the central region $\bar{V} = 7.44$ ft/sec; in the spike region $\bar{V} = 3.85$ ft/sec; and in the outer plate region $\bar{V} = 7.48$ ft/sec. Thus, the flow rates for the core regions can be calculated as was done for the originally assumed velocity profile:

$$W_C = 2.473 \times 10^6 \text{ lb/hr}$$

$$W_A = 7.901 \times 10^6 \text{ lb/hr}$$

$$W_T = 1.0374 \times 10^6 \text{ lb/hr}$$

The quality for this flow is

$$x = (3.007 \times 10^5) / (1.0374 \times 10^7) = 0.0290$$

The subcooling is

$$h_{\text{sub}} = 459.9$$

or

$$\Delta h_{\text{sub}} = 11.7 \text{ Btu/lb}$$

Since the originally assumed velocity profile yielded a subcooling of $\Delta h = 11.8$ Btu/lb, the calculated velocity profile is quite accurate. Another velocity profile would need to be assumed and the calculation procedure repeated if the subcoolings did not agree.

APPENDIX

PHYSICAL AND THERMODYNAMIC PROPERTIES OF
LIGHT AND HEAVY WATER

Reprinted from
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Light and Heavy Water Cooled Systems (ANL)

CHAPTER 1.3. PHYSICAL AND THERMODYNAMIC PROPERTIES
OF LIGHT AND HEAVY WATER

by

P. A. Lottes

The properties of light and heavy liquid water and light water vapor frequently used in heat transfer calculations are presented in tabular and/or graphic form. Heavy water properties are available for relatively low temperatures only and are generally similar to those of light water. For most engineering calculations concerning heavy water at higher temperatures the corresponding property of light water, with the exception of density, has been used since data are not available above approximately 100° F.

The data are presented by individual property. Critical properties, constant pressure specific heat, thermal conductivity, dynamic viscosity, Prandtl number, density and latent heat values are included when available; exception is made for specific heat of light water vapor, and latent heat of light water because data presented in Keenan and Keyes* are the best available. In general, a tabular compilation of the physical and thermodynamic properties for light and heavy liquid water are presented. Whenever possible, a graphical comparison of these properties is included. Graphs of frequently used properties of light water vapor are included. Thermal conductivity data for heavy water are not available; consequently, thermal conductivity and the Prandtl number calculation, which includes the value of thermal conductivity, have been omitted.

A recent study by Wellman** presents a review of available data on the thermodynamic and physical properties of light liquid water in the form of tables of values; the study includes an evaluation of the accuracy of the

*J. H. Keenan and F. G. Keyes, "Thermodynamic Properties of Steam," 1st ed, John Wiley and Sons, Inc., New York, 1936

**E. J. Wellman, "A Survey of the Thermodynamic and Physical Properties of Water," M. S. Thesis, Purdue University, January, 1950.

values reported. These data were used in the preparation of this chapter with the exception that graphical data for the density and vapor pressure of light liquid water were taken from Keenan and Keyes.*

The data of Kirshenbaum** for heavy water have been recalculated into units used predominately in engineering heat transfer calculations.

Values for the properties of light water vapor were provided by Sibbitt† from an evaluation currently being made by Purdue University.

Table 1.3-1

CRITICAL PROPERTIES OF LIGHT WATER

Critical temperature	705.40	° F
	374.2	° C
Critical pressure	3206	psia
	218.2	atm
Critical density	19.9	lb/cu ft
	0.320	gm/ml

Table 1.3-2

CRITICAL PROPERTIES OF HEAVY WATER

Critical temperature	700.7	° F
	371.5	° C
Critical pressure	3212	psia
	218.6	atm
Critical density	22.7	lb/cu ft
	0.363	gm/ml

*J. H. Keenan and F. G. Keyes, "Thermodynamic Properties of Steam," 1st ed, John Wiley and Sons, Inc., New York, 1936.

**I. Kirshenbaum, "Physical Properties and Analysis of Heavy Water," 1st ed, McGraw Hill Book Company, Inc., New York, 1951.

†Communication from W. L. Sibbitt, School of Mechanical Engineering, Purdue University.

Table 1.3-3

CONSTANT PRESSURE SPECIFIC HEAT (c_p) OF LIQUID LIGHT WATER

Conversion Factor: From Btu/(lb)(°F) to cal/(gm)(°C) Multiply by 1

Temperature, °F	c_p , Btu/(lb)(°F)						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
32	1.0083	1.0032	1.0004	0.9979	0.9948	0.992	0.989
40	1.0048	1.0014	0.9986	0.9959	0.9926	0.991	0.988
50	1.0004	0.9991	0.9963	0.9933	0.9899	0.987	0.984
60	0.9990	0.9968	0.9939	0.9907	0.9872	0.984	0.981
70	0.9980	0.9949	0.9920	0.9888	0.9848	0.982	0.979
80	0.9975	0.9943	0.9912	0.9878	0.9840	0.981	0.978
90	0.9974	0.9938	0.9904	0.9871	0.9832	0.980	0.977
100	0.9976	0.9932	0.9897	0.9864	0.9824	0.978	0.975
110	0.9976	0.9932	0.9895	0.9860	0.9819	0.978	0.974
120	0.9977	0.9934	0.9895	0.9858	0.9817	0.978	0.974
130	0.9981	0.9937	0.9896	0.9856	0.9814	0.977	0.973
140	0.9988	0.9940	0.9897	0.9854	0.9811	0.977	0.973
150	0.9996	0.9950	0.9905	0.9861	0.9817	0.978	0.974
160	1.0004	0.9959	0.9913	0.9868	0.9823	0.978	0.974
170	1.0012	0.9969	0.9921	0.9876	0.9829	0.978	0.974
180	1.0022	0.9980	0.9931	0.9885	0.9836	0.979	0.975
190	1.0034	0.9984	0.9945	0.9896	0.9847	0.980	0.975
200	1.0047	1.0008	0.9958	0.9907	0.9857	0.981	0.976
210	1.0061	1.0021	0.9971	0.9919	0.9868	0.982	0.977
212	1.0064	1.0024	0.9974	0.9921	0.9870	0.982	0.977
220	1.0079	1.0039	0.9988	0.9934	0.9881	0.983	0.978
230	1.0097	1.0057	1.0005	0.9951	0.9896	0.985	0.980
240	1.0119	1.0075	1.0023	0.9967	0.9910	0.986	0.981
250	1.0142	1.0095	1.0041	0.9984	0.9925	0.988	0.983
260	1.0165	1.0117	1.0061	1.0003	0.9943	0.989	0.984
270	1.0191	1.0140	1.0092	1.0023	0.9961	0.991	0.986
280	1.0222	1.0163	1.0102	1.0042	0.9979	0.993	0.988
290	1.0255	1.0194	1.0131	1.0069	1.0005	0.995	0.989
300	1.0289	1.0232	1.0166	1.0101	1.0037	0.998	0.992
310	1.0322	1.0270	1.0200	1.0132	1.0068	1.001	0.995
320	1.0354	1.0307	1.0235	1.0164	1.010	1.004	0.998
330	1.0404	1.0353	1.0278	1.0205	1.014	1.008	1.002
340	1.0455	1.0399	1.0322	1.0246	1.017	1.011	1.005
350	1.0511	1.0445	1.0365	1.0287	1.021	1.014	1.008
360	1.0564	1.0496	1.0411	1.0331	1.025	1.018	1.012
370	1.0614	1.0553	1.0461	1.0377	1.029	1.021	1.014
380	1.0669	1.0611	1.0510	1.0424	1.034	1.026	1.019
390	1.0728	1.0668	1.0560	1.0471	1.038	1.030	1.022

Table 1.3-3 (Cont'd.)

Temperature, °F	c_p , Btu/(lb)(°F)						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
400	1.0794	1.074	1.062	1.053	1.043	1.035	1.027
410	1.0861	1.080	1.068	1.059	1.048	1.040	1.032
420	1.0941	1.087	1.075	1.065	1.053	1.045	1.037
430	1.1023	1.095	1.082	1.072	1.059	1.051	1.043
440	1.1114	1.105	1.091	1.079	1.066	1.057	1.048
450	1.1212	1.114	1.100	1.087	1.074	1.065	1.056
460	1.1319	1.124	1.109	1.095	1.081	1.071	1.061
470	1.1431	1.136	1.120	1.104	1.090	1.079	1.068
480	1.1545	1.149	1.131	1.114	1.099	1.087	1.075
490	1.1698	1.163	1.143	1.125	1.109	1.095	1.082
500	1.1861	1.176	1.154	1.135	1.118	1.102	1.088
510	1.21	1.19	1.171	1.151	1.132	1.115	1.100
520	1.23	1.21	1.188	1.166	1.146	1.127	1.110
530	1.25	1.23	1.205	1.182	1.160	1.139	1.121
540	1.28		1.225	1.199	1.175	1.152	1.132
550	1.31		1.252	1.218	1.192	1.168	1.147
560	1.34		1.278	1.238	1.209	1.182	1.158
570	1.37		1.304	1.257	1.227	1.198	1.172
580	1.41		1.341	1.283	1.246	1.216	1.189
590	1.46		1.382	1.310	1.265	1.230	1.200
600	1.51		1.448	1.346	1.290	1.25	1.21
610	1.58		1.52	1.385	1.317	1.27	1.23
620	1.65		1.62	1.441	1.352	1.29	1.25
630	1.71			1.509	1.392	1.32	1.27
640	1.88			1.597	1.442	1.36	1.31
650	2.09			1.701	1.505	1.40	1.34
660	2.34			1.817	1.576	1.43	1.36
670	2.84			2.14			
680	3.5			2.56			
690	5.5						
Accuracy for indicated temperature range, per cent							
32-100	0.3	0.3	0.5	0.8	1.1	1.5	1.5
100-200	0.2	0.2	0.4	0.8	0.6	1.5	1.5
200-300	0.2	0.2	0.4	0.8	0.6	1.5	1.5
300-400	0.3	0.3	0.5	0.9	0.9	1.5	1.5
400-500	0.3	0.3	0.5	0.9	0.9	1.5	1.5
500-530	1.0	1.0	0.7	1.0	0.5	1.5	1.5
530-600	1.5		0.7	1.0	0.5	1.5	1.5
600-620	1.5		1.5	1.0	0.5	1.5	1.5
620-660	3.5			3.5	0.5	1.5	1.5
660-680	4.0			4.0			
690	5.0						

Table 1.3-4

SPECIFIC HEAT (c_p) OF LIQUID HEAVY WATER AT 1 ATM^a

Conversion Factor: From Btu/(lb)(°F) to cal/(gm)(°C) Multiply by 1

Temperature		c_p Btu/(lb)(°F)	Temperature		c_p Btu/(lb)(°F)
°C	°F		°C	°F	
10	50	1.009	35	95	1.004
15	59	1.008	40	104	1.004
20	68	1.006	45	113	1.004
25	77	1.005	50	122	1.004
30	86	1.005			

^a Accuracy: $\pm 3.5\%$

FIG. 1 3-1
SPECIFIC HEAT DATA FOR LIGHT AND HEAVY LIQUID WATER
SUBMITTED FROM VALUES OF WELLMAN AND KIRSHENBAUM

Table 1.3-5

THERMAL CONDUCTIVITY (k) OF LIQUID LIGHT WATER

Conversion Factor: From Btu/(hr)(ft)(°F) to cal/(sec)(cm)(°C)
Multiply by 0.00413379

Temperature, °F	Saturated liquid	k, Btu/(hr)(ft)(°F)					
		Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
32	0.3185	0.3198	0.3211	0.3227	0.3246	0.3266	0.3286
40	0.3245	0.3260	0.3275	0.3294	0.3314	0.3335	0.3358
50	0.3319	0.3337	0.3356	0.3377	0.3398	0.3422	0.3447
60	0.3397	0.3414	0.3433	0.3454	0.3476	0.3500	0.3528
70	0.3472	0.3487	0.3508	0.3530	0.3551	0.3576	0.3604
80	0.3532	0.3537	0.3570	0.3590	0.3616	0.3640	0.3665
90	0.3589	0.3608	0.3629	0.3651	0.3676	0.3700	0.3723
100	0.3641	0.3659	0.3680	0.3703	0.3727	0.3751	0.3777
110	0.3689	0.3707	0.3727	0.3760	0.3773	0.3797	0.3822
120	0.3733	0.3751	0.3771	0.3792	0.3815	0.3839	0.3862
130	0.3772	0.3790	0.3810	0.3831	0.3853	0.3877	0.3900
140	0.3810	0.3828	0.3847	0.3868	0.3891	0.3914	0.3938
150	0.3836	0.3855	0.3875	0.3897	0.3920	0.3944	0.3968
160	0.3861	0.3880	0.3902	0.3924	0.3948	0.3973	0.3997
170	0.3884	0.3903	0.3925	0.3947	0.3971	0.3996	0.4020
180	0.3905	0.3924	0.3945	0.3968	0.3992	0.4017	0.4041
190	0.3923	0.3942	0.3964	0.3986	0.4010	0.4034	0.4059
200	0.3935	0.3957	0.3980	0.4003	0.4027	0.4051	0.4074
210	0.3943	0.3970	0.3995	0.4017	0.4044	0.4067	0.4088
212	0.3944	0.3972	0.3998	0.4020	0.4047	0.4070	0.4091
220	0.3950	0.3977	0.4003	0.4028	0.4055	0.4079	0.4103
230	0.3957	0.3983	0.4010	0.4037	0.4064	0.4091	0.4118
240	0.3961	0.3988	0.4016	0.4043	0.4071	0.4099	0.4127
250	0.3964	0.3992	0.4021	0.4048	0.4077	0.4106	0.4135
260	0.3964	0.3992	0.4021	0.4048	0.4077	0.4106	0.4135
270	0.3962	0.3991	0.4020	0.4048	0.4078	0.4107	0.4136
280	0.3959	0.3987	0.4018	0.4048	0.4079	0.4109	0.4140
290	0.3955	0.3984	0.4015	0.4047	0.4078	0.4110	0.4141
300	0.3952	0.3981	0.4013	0.4045	0.4076	0.4109	0.4141
310	0.3948	0.3975	0.4006	0.4038	0.4069	0.4102	0.4137
320	0.3944	0.3969	0.3998	0.4029	0.4060	0.4094	0.4131
330	0.3933	0.3958	0.3988	0.4019	0.4050	0.4084	0.4122
340	0.3921	0.3947	0.3977	0.4009	0.4040	0.4075	0.4114
350	0.3906	0.3933	0.3965	0.3997	0.4031	0.4069	0.4108
360	0.3891	0.3919	0.3951	0.3984	0.4020	0.4059	0.4101
370	0.3876	0.3904	0.3936	0.3970	0.4006	0.4046	0.4089
380	0.3857	0.3885	0.3919	0.3954	0.3991	0.4031	0.4077
390	0.3834	0.3863	0.3900	0.3937	0.3975	0.4017	0.4063

Table 1.3-5 (Cont'd.)

Temperature, °F	k, Btu/(hr)(ft)(°F)						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
400	0.3809	0.3840	0.3880	0.3920	0.3960	0.4000	0.4043
410	0.3783	0.3817	0.3860	0.3903	0.3944	0.3983	0.4021
420	0.3753	0.3787	0.3833	0.3877	0.3920	0.3960	0.3999
430	0.3724	0.3757	0.3806	0.3850	0.3894	0.3936	0.3976
440	0.3693	0.3728	0.3776	0.3821	0.3865	0.3907	0.3947
450	0.3670	0.3697	0.3745	0.3791	0.3836	0.3880	0.3922
460	0.3640	0.3664	0.3713	0.3761	0.3809	0.3857	0.3904
470	0.3608	0.3630	0.3678	0.3727	0.3775	0.3823	0.3872
480	0.3575	0.3595	0.3642	0.3689	0.3738	0.3784	0.3832
490	0.3535	0.3553	0.3603	0.3650	0.3698	0.3744	0.3791
500	0.3494	0.3510	0.3562	0.3611	0.3658	0.3705	0.3750
510	0.3446	0.3461	0.3519	0.3574	0.3626	0.3677	0.3725
520	0.3397	0.3410	0.3475	0.3535	0.3591	0.3645	0.3697
530	0.3348	0.3356	0.3424	0.3486	0.3545	0.3602	0.3657
540	0.3298		0.3371	0.3437	0.3499	0.3559	0.3617
550	0.3246		0.3318	0.3386	0.3452	0.3515	0.3577
560	0.3189		0.3256	0.3328	0.3398	0.3465	0.3530
570	0.3130		0.3190	0.3265	0.3339	0.3410	0.3478
580	0.3064		0.3118	0.3198	0.3277	0.3354	0.3429
590	0.2997		0.3045	0.3130	0.3215	0.3298	0.3380
600	0.2919		0.2962	0.3056	0.3146	0.3232	0.3318
610	0.2839		0.2875	0.2979	0.3074	0.3167	0.3257
620	0.2753		0.2778	0.2892	0.2997	0.3100	0.3199
630	0.2662			0.2800	0.2916	0.3030	0.3137
640	0.2565			0.2700	0.2831	0.2953	0.3068
650	0.2454			0.2588	0.2736	0.2869	0.2993
660	0.2335			0.2469	0.2634	0.2779	0.2914
670	0.2198			0.2319	0.2516	0.2679	0.2830
680	0.2056			0.2161	0.2395	0.2577	0.2744
690	0.1854				0.2242	0.2463	0.2661
Accuracy for indicated temperature range, per cent							
32-100	0.7	1.0	1.0	1.0	1.0	1.0	1.2
100-200	0.7	1.0	1.0	1.0	1.0	1.0	1.2
200-300	0.6	1.0	1.0	1.0	1.0	1.0	1.2
300-400	0.7	1.0	1.0	1.0	1.0	1.0	1.2
400-500	1.7	2.0	2.0	2.0	2.0	2.0	2.5
500-530	4.5	5.0	5.0	5.0	5.0	5.0	5.0
530-600	4.5		5.0	5.0	5.0	5.0	5.0
600-680	5.0		5.0	5.0	5.0	5.0	5.0
690	5.0				5.0	5.0	5.0

FIG. I.3-2
THERMAL CONDUCTIVITY VALUES FOR LIGHT LIQUID WATER.
SUBMITTED FROM VALUES OF WELLMAN

FIG I 3-3
THERMAL CONDUCTIVITY DATA FOR LIGHT WATER VAPOR
SUBMITTED FROM DATA OF KEYES AND SANDELL AND FROM TIMROTH AND
VARGAFTIG BY W. L. SIBBITT, PURDUE UNIVERSITY, JUNE 23, 1952

Table 1.3-6

DYNAMIC VISCOSITY OF (μ) LIQUID LIGHT WATER

Conversion Factor: From lb/(hr)(ft) to (Poises) Multiply by 0.00413379

Temperature, °F	μ , lb/(hr)(ft)						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
32	4.340	4.309	4.279	4.248	4.218	4.187	4.157
40	3.742	3.721	3.699	3.677	3.656	3.643	3.613
50	3.169	3.154	3.140	3.126	3.111	3.097	3.082
60	2.731	2.722	2.714	2.705	2.696	2.688	2.679
70	2.372	2.369	2.365	2.361	2.357	2.354	2.350
80	2.084	2.084	2.083	2.083	2.082	2.082	2.081
90	1.847	1.848	1.850	1.851	1.853	1.854	1.855
100	1.650	1.654	1.658	1.663	1.667	1.671	1.675
110	1.487	1.492	1.498	1.503	1.509	1.514	1.519
120	1.353	1.360	1.366	1.373	1.380	1.387	1.394
130	1.236	1.244	1.252	1.259	1.267	1.275	1.283
140	1.137	1.145	1.154	1.162	1.170	1.179	1.187
150	1.050	1.059	1.068	1.077	1.086	1.095	1.103
160	0.970	0.979	0.988	0.997	1.006	1.015	1.024
170	0.902	0.912	0.921	0.930	0.939	0.949	0.958
180	0.839	0.849	0.858	0.868	0.877	0.887	0.896
190	0.786	0.796	0.806	0.815	0.825	0.835	0.845
200	0.738	0.748	0.757	0.767	0.777	0.787	0.796
210	0.702	0.711	0.721	0.731	0.740	0.750	0.760
212	0.687	0.697	0.706	0.716	0.726	0.735	0.745
220	0.660	0.670	0.680	0.689	0.699	0.709	0.718
230	0.624	0.633	0.643	0.652	0.661	0.671	0.680
240	0.595	0.604	0.614	0.623	0.632	0.641	0.650
250	0.568	0.578	0.587	0.596	0.605	0.614	0.624
260	0.542	0.551	0.560	0.569	0.578	0.587	0.596
270	0.515	0.524	0.533	0.541	0.550	0.559	0.567
280	0.494	0.502	0.511	0.519	0.528	0.536	0.545
290	0.472	0.480	0.488	0.496	0.505	0.513	0.521
300	0.452	0.460	0.468	0.476	0.484	0.492	0.500
310	0.436	0.442	0.450	0.459	0.467	0.475	0.484
320	0.420	0.426	0.433	0.441	0.449	0.457	0.464
330	0.405	0.411	0.418	0.426	0.433	0.440	0.448
340	0.391	0.396	0.404	0.411	0.418	0.425	0.432
350	0.378	0.384	0.390	0.396	0.403	0.407	0.416
360	0.366	0.372	0.378	0.383	0.390	0.396	0.401
370	0.355	0.361	0.367	0.372	0.378	0.384	0.389
380	0.346	0.351	0.356	0.361	0.366	0.372	0.377
390	0.336	0.340	0.345	0.350	0.355	0.360	0.365

Table 1.3-6 (Cont'd.)

Temperature, °F	μ , lb/(hr)(ft)						
	Saturated liquid	Pressure, psia, compressed					
		1000	2000	3000	4000	5000	6000
400	0.327	0.330	0.335	0.340	0.345	0.350	0.355
410	0.318	0.321	0.326	0.331	0.336	0.340	0.346
420	0.310	0.312	0.317	0.322	0.327	0.331	0.336
430	0.302	0.304	0.309	0.313	0.318	0.322	0.327
440	0.294	0.296	0.301	0.305	0.310	0.314	0.319
450	0.287	0.289	0.293	0.298	0.302	0.306	0.310
460	0.280	0.282	0.286	0.290	0.294	0.298	0.302
470	0.274	0.276	0.280	0.284	0.287	0.291	0.295
480	0.267	0.270	0.273	0.277	0.280	0.284	0.288
490	0.262	0.263	0.267	0.270	0.274	0.278	0.281
500	0.256	0.257	0.260	0.264	0.267	0.271	0.274
510	0.251	0.251	0.255	0.258	0.262	0.266	0.269
520	0.246	0.246	0.249	0.253	0.256	0.260	0.264
530	0.240	0.241	0.244	0.248	0.251	0.255	0.259
540	0.235		0.239	0.243	0.247	0.250	0.254
550	0.230		0.235	0.239	0.243	0.247	0.250
560	0.225		0.231	0.234	0.238	0.243	0.246
570	0.221		0.226	0.229	0.234	0.238	0.242
580	0.217		0.222	0.225	0.230	0.234	0.239
590	0.214		0.217	0.221	0.226	0.230	0.235
600	0.210		0.212	0.217	0.222	0.227	0.233
610	0.205		0.207	0.212	0.218	0.224	0.230
620	0.200		0.202	0.208	0.214	0.220	0.226
630	0.195			0.202	0.209	0.216	0.222
640	0.190			0.196	0.203	0.211	0.218
650	0.183			0.189	0.197	0.206	0.215
660	0.177			0.182	0.192	0.202	0.212
670	0.169			0.174	0.186	0.198	0.210
680	0.161			0.166	0.180	0.194	0.208
690	0.148				0.172	0.188	0.203
Accuracy for indicated temperature range, per cent							
32-100	0.5	0.6	0.6	0.6	0.6	1.0	1.0
100-200	0.7	0.8	0.8	0.8	0.8	1.0	1.0
200-300	1.5	2.0	2.0	2.0	2.0	2.5	2.5
300-400	1.5	2.0	2.0	2.0	2.0	2.5	2.5
400-500	3.0	3.5	3.5	3.5	3.5	4.0	4.0
500-530	4.0	4.5	4.5	4.5	4.5	5.0	5.0
530-600	4.0		4.5	4.5	4.5	5.0	5.0
600-620	7.0		7.0	7.0	7.0	8.0	8.0
620-680	7.0			7.0	7.0	8.0	8.0
690	7.0				7.0	8.0	8.0

Table 1 3-7

DYNAMIC VISCOSITY (μ) OF LIQUID HEAVY WATER AT 1 ATM^a

Temperature		μ		Temperature		μ	
°C	°F	Centipoise	lb/(hr)(ft)	°C	°F	Centipoise	lb/(hr)(ft)
5	41	1 988	4 809	25	77	1 102	2 667
10	50	1.684	1 074	30	86	0 972	2 352
15	59	1 450	3 508	35	95	0 863	2 089
20	68	1 260	3 048				

^aAccuracy: $\pm 0.5\%$ FIG I 3-4
DYNAMIC VISCOSITY DATA FOR LIGHT AND HEAVY LIQUID WATER

SUBMITTED FROM DATA OF WELLMAN AND KIRSHENBALM

FIG I 3-5
DYNAMIC VISCOSITY VALUES FOR LIGHT WATER VAPOR
SUBMITTED FROM DATA OF TIMROTH BY W L SIRBITT
PURDUE UNIVERSITY JUNE 23 1952

Table 1.3-8

PRANDTL NUMBER, $c_p \mu / k$, FOR LIGHT LIQUID WATER

Temperature, °F	$c_p \mu / k$						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
32	13.74	13.52	13.33	13.14	12.93	12.72	12.51
40	11.59	11.43	11.28	11.12	10.95	10.80	10.63
50	9.55	9.44	9.32	9.19	9.06	8.93	8.80
60	8.03	7.95	7.86	7.76	7.66	7.56	7.47
70	6.82	6.76	6.69	6.61	6.54	6.46	6.38
80	5.89	5.86	5.78	5.73	5.67	5.61	5.55
90	5.13	5.09	5.05	5.01	4.96	4.91	4.87
100	4.52	4.49	4.46	4.43	4.39	4.36	4.32
110	4.02	4.00	3.98	3.94	3.93	3.90	3.87
120	3.61	3.60	3.59	3.57	3.55	3.53	3.52
130	3.27	3.26	3.25	3.24	3.23	3.21	3.20
140	2.98	2.97	2.97	2.96	2.95	2.94	2.93
150	2.74	2.73	2.73	2.72	2.72	2.71	2.71
160	2.51	2.51	2.51	2.51	2.50	2.50	2.49
170	2.33	2.33	2.33	2.33	2.33	2.32	2.32
180	2.15	2.16	2.16	2.16	2.16	2.16	2.16
190	2.01	2.02	2.02	2.02	2.03	2.03	2.03
200	1.88	1.89	1.89	1.90	1.90	1.90	1.91
210	1.79	1.80	1.80	1.80	1.81	1.81	1.82
212	1.75	1.76	1.76	1.77	1.77	1.77	1.78
220	1.69	1.69	1.70	1.70	1.70	1.71	1.71
230	1.59	1.60	1.60	1.61	1.61	1.61	1.62
240	1.52	1.53	1.53	1.54	1.54	1.54	1.55
250	1.45	1.46	1.47	1.47	1.47	1.48	1.48
260	1.39	1.40	1.40	1.41	1.41	1.41	1.42
270	1.33	1.33	1.34	1.34	1.34	1.35	1.35
280	1.27	1.28	1.28	1.29	1.29	1.30	1.30
290	1.22	1.23	1.23	1.24	1.24	1.24	1.24
300	1.18	1.18	1.19	1.19	1.19	1.19	1.20
310	1.14	1.14	1.15	1.15	1.16	1.16	1.16
320	1.10	1.11	1.11	1.11	1.12	1.12	1.12
330	1.07	1.07	1.08	1.08	1.08	1.09	1.09
340	1.04	1.04	1.05	1.05	1.05	1.05	1.05
350	1.02	1.02	1.02	1.02	1.02	1.01	1.01
360	0.993	0.997	0.996	0.993	0.994	0.99	0.99
370	0.973	0.977	0.975	0.972	0.971	0.97	0.97
380	0.956	0.958	0.954	0.952	0.949	0.948	0.948
390	0.941	0.939	0.934	0.930	0.926	0.922	0.917

Table 1.3-8 (Cont'd.)

PRANDTL NUMBER, $c_p\mu/k$, FOR LIGHT LIQUID WATER

Temperature, °F	$c_p\mu/k$						
	Saturated liquid	Pressure, psia, compressed liquid					
		1000	2000	3000	4000	5000	6000
400	0.927	0.924	0.918	0.914	0.908	0.905	0.902
410	0.913	0.909	0.903	0.898	0.892	0.889	0.887
420	0.903	0.897	0.890	0.884	0.877	0.874	0.872
430	0.894	0.886	0.877	0.872	0.864	0.861	0.858
440	0.886	0.878	0.869	0.862	0.854	0.850	0.846
450	0.876	0.871	0.861	0.854	0.845	0.841	0.836
460	0.871	0.866	0.855	0.845	0.835	0.829	0.822
470	0.867	0.863	0.852	0.840	0.829	0.822	0.813
480	0.863	0.861	0.849	0.836	0.824	0.816	0.807
490	0.865	0.862	0.847	0.833	0.821	0.812	0.802
500	0.870	0.861	0.844	0.830	0.816	0.806	0.796
510	0.88	0.86	0.848	0.832	0.818	0.805	0.794
520	0.89	0.87	0.852	0.834	0.819	0.804	0.792
530	0.90	0.88	0.859	0.841	0.822	0.806	0.793
540	0.90		0.870	0.848	0.826	0.811	0.796
550	0.93		0.886	0.859	0.837	0.820	0.803
560	0.95		0.905	0.871	0.849	0.828	0.809
570	0.97		0.924	0.882	0.860	0.837	0.817
580	1.00		0.953	0.903	0.874	0.849	0.827
590	1.04		0.985	0.925	0.889	0.859	0.835
600	1.09		1.015	0.954	0.908	0.879	0.849
610	1.14		1.09	1.003	0.934	0.899	0.869
620	1.20		1.18	1.034	0.964	0.916	0.884
630	1.25			1.089	0.997	0.941	0.901
640	1.39			1.158	1.034	0.970	0.933
650	1.56			1.241	1.086	1.006	0.962
660	1.77			1.337	1.147	1.039	0.990
670	2.19			1.61			
680	2.75			1.97			
690	4.40						
Maximum error for indicated temperature range, per cent							
32-100	1.5	1.9	2.1	2.4	2.7	3.5	3.7
100-200	1.6	2.0	2.2	2.6	2.4	3.5	3.7
200-300	2.3	3.2	3.4	3.8	3.6	5.0	5.2
300-400	2.5	3.3	3.5	3.9	3.9	5.0	5.2
400-500	5.0	5.8	6.0	6.4	6.4	7.5	8.0
500-530	9.5	10.5	10.2	10.5	10.0	11.5	11.5
530-600	10.0		10.2	10.5	10.0	11.5	11.5
600-620	13.5		13.5	13.0	12.5	14.5	14.5
620-660	15.5			15.5	12.5	14.5	14.5
660-680	16.0			16.0			
690	17.0

FIG. I.3-6
PRANDTL NUMBER
VALUES FOR LIQ-
UID LIGHT WATER

FIG. I.3-7
GRAPHICAL PRESENTATION OF PRANDTL NUMBER
VALUE FOR LIGHT WATER VAPOR.

SUBMITTED FROM CALCULATIONS OF W. L. SIBBITT,
PURDUE UNIVERSITY, JUNE 23, 1952

FIG. I.3-8
DENSITY VALUES FOR LIGHT WATER (40-440 F).
SUBMITTED FROM VALUES OF KEENAN AND KEYES

FIG. I.3-9
DENSITY FOR LIGHT WATER (400-760 F).
SUBMITTED FROM VALUES OF KEENAN AND KEYES

Table 1.3-9

ABSOLUTE DENSITY (ρ) OF LIQUID HEAVY WATER AT 1 ATM^a

Temperature		ρ		Temperature		ρ	
°F	°C	lb/cu ft	Gm/cc	°F	°C	lb/cu ft	Gm/cc
41	5	69.0176	1.10555	68	20	69.0063	1.10537
50	10	69.0444	1.10598	77	25	68.9508	1.10448
51.8	11	69.0450	1.10599	86	30	68.8746	1.10326
53.6	12	69.0450	1.10599	95	35	68.7828	1.10179
55.4	13	69.0438	1.10597	104	40	68.6723	1.10002
59	15	69.0382	1.10588				

^aAccuracy: 0.003%

FIG. I.3-10

VAPOR PRESSURE VALUES FOR LIGHT WATER
 SUBMITTED FROM DATA OF KEENAN AND KEYES

Table 1.3-10

VAPOR PRESSURE OF LIQUID HEAVY WATER

Temperature		Pressure	
°C	°F	Atm (absolute)	Psia
0	32	0.00480	0.0705
10	50	.01025	.1506
20	68	.0200	.2939
30	86	.0368	.5408
40	104	.0647	.9508
50	122	.1100	1.616
60	140	.1797	2.641
70	158	.2843	4.178
80	176	.4363	6.412
90	194	.6520	9.582
100	212	.9503	13.96
110	230	1.353	19.88
120	248	1.888	27.75
130	266	2.584	37.97
140	284	3.474	51.05
150	302	4.596	67.54
160	320	5.993	88.07
170	338	7.707	113.3
180	356	9.788	143.8
190	374	12.29	180.6
200	392	15.26	224.3
210	410	18.77	275.8
220	428	22.87	336.1
230	446	27.63	406.0
240	464	33.17	487.5
371.5	700.7	218.6	3212

Table 1.3-11

LATENT HEAT OF VAPORIZATION OF HEAVY WATER^a

Temperature		ΔH	
°C	°F	Cal/gm	Btu/lb
3.82	38.9	554.66	998.39
10	50	549.47	989.05
25	77	541.53	974.76
40	104	531.50	956.69
60	140	519.86	935.75
80	176	507.98	914.36
100	212	495.65	892.16
120	248	482.76	868.98
140	284	469.08	844.35
160	320	454.70	818.47
180	356	439.28	790.69
200	392	422.70	760.86
220	428	404.63	728.33

^aAccuracy: $\pm 0.3\%$

Table 1.3-12

ENTHALPY OF SATURATED LIQUID VERSUS TEMPERATURE

Tempera- ture, t °F	Enthalpy of Saturated Liquid, h_f B/lb	Tempera- ture, t °F	Enthalpy of Saturated Liquid, h_f B/lb	Tempera- ture, t °F	Enthalpy of Saturated Liquid, h_f B/lb
90	57.99	125	92.91	160	127.89
91	58.99	126	93.91	161	128.89
92	59.99	127	94.91	162	129.89
93	60.98	128	95.91	163	130.89
94	61.98	129	96.90	164	131.89
95	62.98	130	97.90	165	132.89
96	63.98	131	98.90	166	133.89
97	64.97	132	99.90	167	134.89
98	65.97	133	100.90	168	135.90
99	66.97	134	101.90	169	136.90
100	67.97	135	102.90	170	137.90
101	68.96	136	103.90	171	138.90
102	69.96	137	104.89	172	139.90
103	70.96	138	105.89	173	140.90
104	71.96	139	106.89	174	141.90
105	72.95	140	107.89	175	142.91
106	73.95	141	108.89	176	143.91
107	74.95	142	109.89	177	144.91
108	75.95	143	110.89	178	145.91
109	76.94	144	111.89	179	146.92
110	77.94	145	112.89	180	147.92
111	78.94	146	113.89	181	148.92
112	79.94	147	114.89	182	149.92
113	80.94	148	115.89	183	150.93
114	81.93	148	116.89	184	151.93
115	82.93	150	117.89	185	152.93
116	82.93	151	118.89	186	153.94
117	84.93	152	119.89	187	154.94
118	85.92	153	120.89	188	155.94
119	86.92	154	121.89	189	156.95
120	87.92	155	122.89	190	157.95
121	88.92	156	123.89	191	158.95
122	89.92	157	124.89	192	159.96
123	90.91	158	125.89	193	160.96
124	91.91	159	126.89	194	161.97

Table 1.3-12 (Cont'd.)

Tempera- ture, t °F	Enthalpy of Saturated Liquid, h _f B/lb	Tempera- ture, t °F	Enthalpy of Saturated Liquid, h _f B/lb	Tempera- ture, t °F	Enthalpy of Saturated Liquid, h _f B/lb
195	162.97	250	218.48	310	279.92
196	163.97	252	220.51	312	281.99
197	164.98	254	222.54	314	284.06
198	165.98	256	224.58	316	286.13
199	166.99	258	226.61	318	288.20
200	167.99	260	228.64	320	290.28
202	170.00	262	230.68	322	292.36
204	172.02	264	232.72	324	294.43
206	174.03	266	234.76	326	296.52
208	176.04	268	236.80	328	298.60
210	178.05	270	238.84	330	300.68
212	180.07	272	240.88	332	302.77
214	182.08	274	242.92	334	304.86
216	184.10	276	244.96	336	306.95
218	186.11	278	247.01	338	309.04
220	188.13	280	249.06	340	311.13
222	190.15	282	251.10	342	313.23
224	192.17	284	253.15	344	315.33
226	194.18	286	255.20	346	317.43
228	196.20	288	257.26	348	319.53
230	198.23	290	259.31	350	321.63
232	200.25	292	261.36	352	323.74
234	202.27	294	263.42	354	325.85
236	204.29	296	265.48	356	327.96
238	206.32	298	267.53		
240	208.34	300	269.59		
242	210.37	302	271.66		
244	212.39	304	273.72		
246	214.42	306	275.78		
248	216.45	308	277.85		