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A Model for the Leading Light Cone Singularity
of the Current Commutator

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Abstract

We investigate a model for the current commutator near the light cone where the forward matrix element of the leading bilocal operator $v_\mu(x,p)$ satisfies two equations $\partial_\mu v_\mu(x,p) = O(x^2)$ and $\square v_\mu(x,p) = O(x^2)$. In this model the electromagnetic self-energy becomes finite and the scale-breaking term for $W_1(\nu, q^2)$ can be predicted in terms of the scaling function for $\nu W_2(\nu, q^2)$.

I. Introduction

The results of the deep inelastic e-p scattering experiments support the leading structure¹ of the current commutator near the light cone² as realized in the free quark model. One may now ask how far one can go with the free quark model predictions. In this paper we adopt the hypothesis that only the leading term can be abstracted from the free quark model. The non-leading terms are subject to constraints of Lorentz invariance, causality and current conservation, but otherwise arbitrary. In this general context we study a specific model. Our model for the current commutator is based on the assumption that the dynamics of the leading term can be extracted from a massless free quark model. For the forward matrix element of the leading bilocal operator $v_\mu(x,p)$ we thus propose two constraint equations which define the model:

$$\partial_\mu v_\mu(x,p) = 0(x^2) \quad (1.1)$$

$$\square v_\mu(x,p) = 0(x^2) \quad (1.2)$$

The motivations for this analysis are at least two-fold. First we know that for the massive free quark model the leading light cone singularity of the current commutator does not depend on the masses of the constituent quark-fields. Thus perhaps the massless free field phenomena might adequately represent the structure of the leading term. Then, a significant justification³ of our Eq(1.1) lies in the fact that it makes the logarithmic divergent term of the electromagnetic self-energy vanish identically, making all self masses finite. Now,

one cannot just propose Eq(1.1) only without making it an ad hoc conjecture on the dynamics of the leading singularity. So for the sake of exploring completely the basis of such an assumption, we shall consider both Eqs(1.1) and (1.2) together.

The Fritzsche-Gell-Mann structure of the leading light cone current commutator relates the inelastic e-N scattering scaling functions $F_2(\xi = -q^2/2\nu)$ to $F_1(\xi)$. In our model one can further predict the inelastic e-N scattering scaling function $H_1(\xi)$ in terms of the already measured $F_2(\xi)$. The model becomes more interesting due to the fact that $R = \sigma_S/\sigma_T$ is not identically zero and $H_1(\xi)$ is definitely needed for the cancellation of the log divergence in the expression for the self-energy.

For simplicity we consider here only the spin-averaged matrix element of the electromagnetic current commutator. In Section II we describe the model for the leading bilocal operator of the current commutator near the light cone and develop a set of four equations to be satisfied by its matrix elements. In Section III we discuss the implications of current conservation on the non-leading singularities of the current commutator and show that our model does not imply vanishing of the non-leading terms. We also comment here on the freedom of the operators inherent in the tensorial structures of the singularities. In Section IV we solve the model for the case of the deep inelastic e-p scattering and obtain an expression for the leading non-vanishing structure functions. We also discuss here the divergence of the electromagnetic self-energy and exhibit that the self-masses would be finite in this model. Finally Section V is devoted to a short discussion of

the prediction of this model from the experimental point of view. We present here numerical estimates of $H_1(\xi)$ from the fits of the data for $F_2(\xi)$ and suggest that a measurement of $H_1(\xi)$ can immediately test this model.

II. The Model

As explained in the introduction the model we shall discuss here is based on a speculation on the leading singularity of the current commutator near the light cone. We assume that for the deep inelastic phenomena nature is not only following free field theory predictions but for the leading phenomenon the masses of the underlying fields can be neglected. This we do in the spirit of extracting several constraint equations for the matrix element of the leading operator, which we then hypothesize to be true in the real world. For a free quark-model the electromagnetic current commutator near the light cone can be written¹ as

$$C_{\alpha\beta}(x) = \pi i \left[j_{\alpha}^{\text{e.m.}}(x/2), j_{\beta}^{\text{e.m.}}(-x/2) \right] \triangleq \partial_{\mu} \left(\epsilon(x^0) \delta(x^2) \right) S_{\alpha\beta\mu\nu} v_{\nu}(x) + \dots, \quad (2.1)$$

where we have omitted terms which vanish for the spin-averaged matrix element. $v_{\nu}(x)$ is a bilocal operator and can be expressed in terms of the quark-fields $\psi(x)$ by

$$v_{\mu}(x) = \bar{\psi}(x/2) \frac{i}{2} \gamma_{\mu} \psi(-x/2) - \bar{\psi}(-x/2) \frac{i}{2} \gamma_{\mu} \psi(x/2) \quad (2.2)$$

$$\text{and} \quad S_{\alpha\beta\mu\nu} = g_{\alpha\nu} g_{\beta\mu} + g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu} \quad (2.3)$$

We now assume that the underlying fields $\psi(x)$ satisfy the massless Dirac and Klein-Gordon equations:

$$\gamma_\mu \partial_\mu \psi = 0 \quad \text{and} \quad \partial_\mu \bar{\psi} \gamma_\mu = 0 \quad (2.4)$$

and

$$\square \psi = 0. \quad (2.5)$$

Define the matrix element of the bilocal operators by

$$v_\mu(x, p_2, p_1) = \langle p_2 | v_\mu(x) | p_1 \rangle \quad (2.6)$$

Eq(2.4) immediately tells us that $\partial_\mu v_\mu(x) = 0$. In terms of the matrix elements, this generates the following two conditions:

$$\partial_\mu v_\mu(x, p_2, p_1) = 0 \quad (2.7)$$

and

$$(p_2 - p_1)_\mu v_\mu(x, p_2, p_1) = 0 \quad (2.8)$$

Eq(2.5) produces two further constraint equations

$$\left[\square + \frac{1}{4}(p_2 - p_1)^2 \right] v_\mu(x, p_2, p_1) = 0 \quad (2.9)$$

and

$$(p_2 - p_1)_\alpha \langle p_2 | \partial_\alpha v_\mu(x) | p_1 \rangle = 0 \quad (2.10)$$

In deriving Eqs(2.7) to (2.10) we have used the fact that $v_\mu(x)$ is odd under $x \rightarrow -x$. We now throw away the model of massless quark-fields and assume that, in the strongest version of our model, these relations remain true for the leading operator $v_\mu(x)$. We do not impose any additional constraints on the non-leading terms except those arising from general principles of Lorentz covariance and causality. This will be discussed

in the next section. Minkowski⁴ has recently studied the im-

plications of Eq(2.9) for the case of non-forward scattering. He has, however, not used the other conditions of the present model.

For the deep inelastic e-p scattering, where only the forward matrix element is concerned we have $p_1 = p_2 = p$ and only two equations define the model. We shall now accept a weaker version of the above generalizations and demand that these relations be true only to order x^2 , as expressed in Eqs(1.1) and (1.2).

III. Light Cone Singularities and Current Conservation

In this section we shall study the non-leading singularities of the current commutator with particular reference to current conservation and the proposed equations for the model. For simplicity let us consider only canonical light cone singularities. The conclusions will essentially not change if non-canonical singularities are present. The electromagnetic current commutator near the light cone can then be written as in Eq(2.1)

$$C_{\alpha\beta}(x) = \partial_\mu \left(\epsilon(x^0) \delta(x^2) \right) S_{\alpha\beta\mu\nu} v_\nu(x) + \epsilon(x^0) \delta(x^2) R_{\alpha\beta}(x) + \epsilon(x^0) \theta(x^2) J_{\alpha\beta}(x) + \dots \quad (3.1)$$

The non-leading terms $R_{\alpha\beta}(x)$ and $J_{\alpha\beta}(x)$ are symmetric in α, β and Lorentz invariant but otherwise arbitrary.

We first stress⁵ that the operators as given above are not defined unambiguously. We easily verify that Eq(3.1) remains invariant under the following substitutions:

$$\begin{aligned}
 v_\nu(x) &\rightarrow v'_\nu(x) = v_\nu(x) + x^2 u_\nu(x) + \chi^4 \bar{v}_\nu(x) \\
 R_{\alpha\beta}(x) &\rightarrow R'_{\alpha\beta}(x) = R_{\alpha\beta}(x) - 2g_{\alpha\beta} \chi_\nu u_\nu(x) \\
 &\quad + 2(x_\alpha u_\beta(x) + x_\beta u_\alpha(x)) + x^2 \bar{R}_{\alpha\beta}(x)
 \end{aligned}
 \tag{3.2}$$

and

$$J_{\alpha\beta}(x) \rightarrow J'_{\alpha\beta}(x) = J_{\alpha\beta}(x),$$

where $u_\nu(x)$, $\bar{v}_\nu(x)$ and $\bar{R}_{\alpha\beta}(x)$ are arbitrary operators. Thus there is a freedom inherent in the selection of the operators for the current commutator near the light cone. This freedom may be called a gauge freedom corresponding to the above gauge transformation. The free quark-model expression for $v_\nu(x)$ given in Eq(2.2) can be true only in one particular choice of the gauge and acceptance of Eq(2.2) means that the gauge is fixed by the free quark model. The non-leading term $R_{\alpha\beta}(x)$ cannot then be arbitrary up to order $O(x^2)$.

The current conservation conditions demand two equations to be satisfied by the commutator $C_{\alpha\beta}(x)$:

$$\partial_\alpha C_{\alpha\beta}(x) = 0 \quad (\text{first current conserved}) \tag{3.3}$$

and

$$\partial_\beta C_{\alpha\beta}(x) = 0 \quad (\text{second current conserved}) \tag{3.4}$$

Consider Eq(3.3) first, which implies that

$$\partial_\alpha J_{\alpha\beta}(x) = 0 \tag{3.5}$$

and

$$\partial_\mu (\epsilon(x^0) \delta(x^2)) [S_{\alpha\beta\mu\nu} \partial_\alpha v_\nu + R_{\mu\beta}] + \epsilon(x^0) \delta(x^2) [\partial_\alpha R_{\alpha\beta} + 2x_\alpha J_{\alpha\beta}] = 0 \quad (3.6)$$

Multiplying Eq(2.7) by x^2 , we see that the coefficient of $\partial_\mu (\epsilon(x^0) \delta(x^2))$ term must be of the form

$$S_{\alpha\beta\mu\nu} \partial_\alpha v_\nu + R_{\mu\beta} = x_\mu \Sigma_\beta + x_\lambda \Lambda_{\mu\lambda, \beta} + O(x^2) \quad (3.7)$$

where Σ_β and $\Lambda_{\mu\lambda, \beta} = -\Lambda_{\lambda\mu, \beta}$ are unknown operators. To determine the current conservation constraint on $R_{\alpha\beta}$, we write

$$\begin{aligned} \Lambda_{\mu\lambda, \beta} = & (g_{\beta\mu} \Gamma_\lambda - g_{\beta\lambda} \Gamma_\mu) + (g_{\lambda\beta} \chi_\mu - g_{\mu\beta} \chi_\lambda) \\ & + (x_\mu \theta_{\lambda\beta} - x_\lambda \theta_{\mu\beta}) + O(x^2), \end{aligned} \quad (3.8)$$

where Γ_λ , χ_μ and $\theta_{\lambda\beta}$ are unknown operators. Then the symmetric part of Eq(3.7) gives us

$$\begin{aligned} R_{\mu\beta} = & -g_{\beta\mu} \left\{ \partial_\alpha v_\alpha(x) - x_\alpha Z_\alpha \right\} + \frac{1}{2} x_\mu \left\{ \Sigma_\beta + x_\lambda \theta_{\lambda\beta} - Z_\beta \right\} \\ & + \frac{1}{2} x_\beta \left\{ \Sigma_\mu + x_\lambda \theta_{\lambda\mu} - Z_\mu \right\} + O(x^2) \end{aligned} \quad (3.9)$$

where $Z_\mu = \Gamma_\mu - \chi_\mu$. Similar equations can be derived for Eq(2.5) and both the expressions for $R_{\mu\beta}$ must coincide at $x^2=0$. We see from these equations that the vanishing of the divergence of the leading bilocal operator does not imply vanishing of the next non-leading term, and vice versa. Even if we assume that the gauge, in which $v_\mu(x)$ is given by the free quark model expression, is also the gauge which makes $Z_\beta = \Sigma_\beta + x_\lambda \theta_{\lambda\beta}$, i.e., $R_{\mu\beta} = g_{\mu\beta} S$, then also the vanishing of $\partial_\alpha v_\alpha(x)$ does not make $R_{\mu\beta}$ zero. We also see that the current conservation do not

imply that $\partial_\mu (\epsilon(x^0) \delta(x^2)) S_{\alpha\beta\mu\nu} \partial_\alpha v_\nu(x)$ be zero as it was assumed by Manhanthappa and Yao⁶ (MY). Eq(3.7) clearly shows that the leading term alone cannot in general satisfy the current conservation relation. This can be true only if the non-leading terms vanish. Thus the M-Y model takes the massless free quark-model for the non-leading terms also, which we do not assume for our model.

IV. Predictions for the Deep Inelastic e-p Scattering and Finite Electromagnetic Self-Energy.

We discuss here the applications of the two constraint Eqs(1.1) and (1.2) for the deep inelastic e-p scattering phenomena. We assume canonical light cone singularities for the current commutator and then the scaling behavior of the structure functions* $v_1(\nu, q^2)$ and $v_2(\nu, q^2)$ in the Bjorken limit are given by

$$v_1(\nu, q^2) \xrightarrow{Bj} \frac{F_1(\xi)}{4\nu\xi^2} + \frac{H_1(\xi)}{2\nu^2\xi} + \dots \quad (4.1)$$

and

*The structure functions are defined by the spin-averaged forward matrix element of the current commutator as follows:

$$(2\pi^2 i)^{-1} \int d^4x e^{iq \cdot x} \langle p | C_{\alpha\beta}(x) | p \rangle = \{q_\mu q_\nu - g_{\mu\nu} q^2\} v_1 + \\ \{ (p_\mu q_\nu + p_\nu q_\mu) \nu - p_\mu p_\nu q^2 - g_{\mu\nu} \nu^2 \} v_2$$

where q^2 is the photon momentum squared and $\nu = p \cdot q / m_N$ its energy in the laboratory frame.

$$V_2(v, q^2) \xrightarrow{Bj} \frac{F_2(\xi)}{2v^2\xi} + \dots \quad (4.2)$$

where $\xi = 1/w = -q^2/2v$ and we have set $m_N = 1$. In terms of the structure functions $W_1(v, q^2)$ and $W_2(v, q^2)$ these are given by $W_1 \rightarrow F_1(\xi) - H_1(\xi)/v + \dots$, $vW_2 \rightarrow F_2(\xi) + \dots$ and $F_L(\xi) = F_2(\xi) - 2\xi F_1(\xi)$. The leading light cone singularity structure given by Eq(2.1) implies that $F_L(\xi) = 0$ and therefore the leading terms in V_1 and V_2 are $H_1(\xi)$ and $F_2(\xi)$ respectively. We shall now show that our model predicts a relation between these two structure functions.

Due to the presence of $\partial_\mu (\epsilon(x^0)\delta(x^2))$ term, the most general form for the forward matrix element of the leading bilocal operator can be written as

$$v_\mu(x, p) = x_\mu [f_1(x \cdot p) + x^2 g_1(x \cdot p)] + p_\mu [f_2(x \cdot p) + x^2 g_2(x \cdot p)] + O(x^4) \quad (4.3)$$

In terms of the scaling functions, the leading terms can be expressed as

$$f_1(p \cdot x) = -\frac{1}{4i} \int_{-1}^1 \frac{d\xi}{\xi p \cdot x} e^{i\xi p \cdot x} H_1(\xi) \quad (4.4)$$

and

$$f_2(p \cdot x) = -\frac{1}{2i} \int_{-1}^1 \frac{d\xi}{\xi} e^{i\xi p \cdot x} F_2(\xi) \quad (4.5)$$

The constraints of the model now demand that

$$f_2'(\alpha) + 4f_1(\alpha) + \alpha f_1'(\alpha) + 2\alpha g_2(\alpha) = 0, \quad (4.6)$$

$$f_2''(\alpha) + 2f_1'(\alpha) + 8g_2(\alpha) + 4\alpha g_2'(\alpha) = 0 \quad (4.7)$$

and

$$f_1''(\alpha) + 4\alpha g_1'(\alpha) + 12g_1(\alpha) = 0, \quad (4.8)$$

where we have written α for $p.x$. We have four functions f_1 , f_2 , g_1 and g_2 and three relations among them given by Eqs(4.6) to (4.8). We can therefore solve these differential equations to express the four functions in terms of one function only. Before solving these equations let us deviate a little to the problem of electromagnetic self-energy and study the implications of Eq(4.6), which was derived from Eq(1.1) alone. Putting $\alpha = 0$, we obtain

$$f_2'(0) + 4f_1(0) = 0 \quad (4.9)$$

In terms of the scaling functions, this gives

$$\int_0^1 d\xi [F_2(\xi) + 2H_1(\xi)] = 0, \quad (4.10)$$

which is exactly the condition necessary for the absence of logarithmic divergence in electromagnetic self-energy.² This implication of the vanishing $\partial_\mu v_\mu(x)$ has also been noted in Ref 3. We further note that since $F_2(\xi) = 0$ in this model, there is no quadratic divergence and the electromagnetic self-energy would be finite.

To solve the three differential equations we first use Eqs(4.6) and (4.7) to derive an equation for $f_1(\alpha)$ in terms of $f_2(\alpha)$:

$$2\alpha^3 f_1''(\alpha) + 10\alpha^2 f_1'(\alpha) + 8\alpha f_1(\alpha) = - \frac{d}{d\alpha} \{ \alpha^2 f_2'(\alpha) \} \quad (4.11)$$

This is now easily solved with the substitution $u_1 = 2\alpha^2 f_1(\alpha)$ obtaining

$$\frac{d}{d\alpha} (\alpha u_1') = - \frac{d}{d\alpha} (\alpha^2 f_2'(\alpha)) \quad (4.12)$$

Now, since u_1 is regular at $\alpha = 0$ due to analyticity in x -space we obtain

$$u_1' = -\alpha f_2'(\alpha) \quad (4.13)$$

Using again the same arguments of smooth behavior for $u_1(\alpha)$ at $\alpha = 0$, we finally derive

$$f_1(p.x) = - \frac{1}{2(p.x)^2} \int_0^{p.x} du \, u \, f_2'(u) \quad (4.14)$$

$g_2(\alpha)$ can now be obtained from Eq(4.6) and simplified with the help of Eq(4.13):

$$g_2(p.x) = - \frac{1}{4(p.x)} [f_2'(p.x) + 4f_1(p.x)] \quad (4.15)$$

Finally Eq(4.8) gives $g_1(\alpha)$ in terms of $f_1(\alpha)$.

By Fourier transforming Eq(4.14) and using the scaling functions we obtain

$$H_1(\xi) = -F_2(\xi) + \xi \int_{\xi}^1 dx \, F_2(x)/x^2, \quad (4.16)$$

which can also be written as

$$H_1(\xi) = \xi \int_{\xi}^1 dx \, F_2'(x)/x. \quad (4.17)$$

In deriving Eq(4.16) the following integral was used

$$\int d\lambda \, \frac{e^{-i\xi\lambda}}{\lambda} (e^{i\xi\lambda} - 1) = 2\pi i \, \theta(\xi - \xi) \text{ for } \xi > 0 \quad (4.18)$$

We now point out that Eq(4.16) should also hold for the M-Y model, which is a special case of our model with the stronger assumption that the massless free quark-model results can be abstracted for all the terms of the current commutator near the light cone.

V. Discussion and Numerical Estimates

From Eq(4.17) we see that $H_1(\xi)$ vanishes both at $\xi = 0$ and $\xi = 1$. The positivity of $F_2(\xi)$ and the vanishing of the log divergence in self energy as expressed in Eq(4.10), now tell us that $H_1(\xi)$ must be negative for some part of the physical interval, i.e. $0 \leq \xi \leq 1$. We note that

$$H_1(\xi) \approx -F_2(\xi) \quad (5.1)$$

near $\xi=1$. Thus from the known threshold behavior of $F_2(\xi)$ we expect that $H_1(\xi) \sim -(1-\xi)^3$ near $\xi=1$. The behavior of $H_1(\xi)$ near $\xi=0$, however, depends crucially on the slope of $F_2(\xi)$ near $\xi=0$. We have

$$\frac{d}{d\xi} \{H_1(\xi)/\xi\} = -F_2'(\xi)/\xi \quad (5.2)$$

If $F_2'(\xi)$ vanishes for some $\xi < \xi_0$, $H_1(\xi)$ will go to zero linearly as $\xi \rightarrow 0$. We do not expect such a non-analytic behavior for $F_2(\xi)$ and so let us assume that $F_2(\xi) \sim A + B\xi^\alpha$ ($\alpha < 1$) as $\xi \rightarrow 0$. Then $H_1(\xi) \sim B\alpha\xi^\alpha/(1-\alpha)$. Thus if $F_2(\xi)$ goes to a constant from above i.e. $B > 0$, $H_1(\xi)$ will go to zero from the positive side; and if $F_2(\xi)$ goes to a constant from below i.e. $B < 0$, $H_1(\xi)$ will go to zero from the negative side. In the previous case $H_1(\xi)$ must cross zero at least once. Finally, if $F_2(\xi) \sim C(1-\xi)^n$ then $H_1(\xi) \sim nc\xi \ln \xi$ as $\xi \rightarrow 0$.

Considering a fit of the data for $F_2(\xi)$ given by the SLAC-MIT group⁷, namely

$$F_2(\xi) = 1.274(1-\xi)^3 + 0.5989(1-\xi)^4 - 1.675(1-\xi)^5 \quad (5.3)$$

we have plotted in Figure 1 the values of $H_1(\xi)$ expected from this model. In this case we obtain

$$H_1(\xi) = -\xi [3.236 + 2.157(\ln \xi - 2\xi + \xi^2/2) + 1.993(1-\xi)^3 + 2.094(1-\xi)^4] \quad (5.4)$$

In Figure 1 we have also plotted, for comparison, the values of $F_2(\xi)$. In this case we note that $H_1(\xi)$ crosses to the positive side once, and has very small values in that region. However, this behavior of $H_1(\xi)$ for small ξ (large ω) cannot be taken seriously, because the behavior of $F_2(\xi)$ for small ξ is itself not clear at present.

Finally we emphasize that this model can be tested immediately as soon as we have some data for $H_1(\xi)$. Further applications of this model will be reported elsewhere.

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Figure Caption

Figure 1: A fit for $F_2(\omega)$ and the corresponding prediction for $H_1(\omega)$ of the deep inelastic e-p scattering are plotted together as a function of ω . The fit for $F_2(\omega)$ is taken from Ref. 7 and is given in Eq(5.3). In this fit $F_2(\infty) \simeq 0.2$. Note that $H_1(\omega) \simeq -F_2(\omega)$ near $\omega \sim 1$ and $H_1(\infty) = 0$.

Fig. 1