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**PROGRAM REVIEW
OF THE WATER BOILER REACTOR
KINETIC EXPERIMENTS**

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ABSTRACT

A series of reactor kinetic experiments will be performed on low power homogeneous reactor cores at the North American Aviation Field Test Laboratory in the Santa Susana Mountains near Los Angeles, California. Initial tests will be concerned with a spherical geometry core similar to the Los Alamos "SUPO" type water boiler. Subsequent tests are tentatively planned to include both open and closed cylindrical geometries. The experimental facility, apparatus, and program are described. Theoretical studies made in preparation for the program include analogue computations of power vs time and total energy release for various step increases in reactivity. The calculations include the effects on reactivity of radiolytic production of gas and of inertial pressure increases during the transients. Analysis of reactor oscillations which permit the determination of heat transfer characteristics, prompt neutron lifetime and effective fraction of delayed neutrons is given.

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I. INTRODUCTION

The increasing demand for low power research and training reactors plus the desire to locate the reactors in highly populated regions has precipitated detailed consideration of their ultimate safety. Experimental tests conducted by the Argonne National Laboratory¹ during the past two years have provided data upon which the safety aspects of the pool type research reactor can be judged and, in addition, have firmly established the value of such tests. The Atomic Energy Commission has also deemed it necessary to determine experimentally the behavior of homogeneous reactors intended for applications similar to those of the pool type. Such a program was undertaken at the Commission's direction by North American Aviation, Inc., in September 1954. The prime objective of this program is to establish a firm basis for evaluation of the safety of homogeneous reactor designs with regard to accidental release of large quantities of reactivity. The intent is to investigate a sufficient number of the parameters during the study to permit optimization of core design. The optimum design may be defined as that which, in the event all potential reactivity is released, presents no hazard to the reactor operating personnel, the installation or the environs.

The first test reactor and its auxiliaries will be installed in an underground reinforced concrete building located at the Santa Susana Field Test Laboratory of North American Aviation, Inc. In addition to the underground building, a small instrument shelter, a remote control building, waste storage tanks and a disposal stack comprise the facility. The construction of this installation has been completed. The experimental phase of the program is scheduled to commence during the month of February, 1956.

II. PROGRAM REVIEW

There are a sufficient number of parameters which affect the magnitude of an accidental power excursion in low power homogeneous reactors to make a comprehensive investigation nearly interminable. To illustrate this, some of the considerations which should be included in such a study are listed below:

1. Core geometry
 - a. Sphere
 - (1) Orifice size
 - (2) Orifice geometry
 - b. Cylinder
 - (1) Open ended
 - (2) Flat ends and orificed
 - (3) Domed and orificed
2. Reflector
 - a. Material
 - b. Thickness
 - c. Density
3. Initial fuel solution temperature
4. Initial core atmosphere pressure
5. Amount of reactivity released
6. Reactivity addition rate
7. Initial power level
8. Configuration of internal components of core
9. Void volume above fuel
10. Overflow chamber effect
11. Gas explosions

In the interests of economy of both time and money it has been necessary to restrict the current program to the testing of the reactor model of prime interest and to a study of those parameters which bear directly on the transient characteristics of that reactor. It is planned to start the testing of other models after a complete evaluation of the first test model is obtained.

The initial experiments will be performed with a reactor core patterned after the Los Alamos SUPO reactor.² The only specific difference between the test core and the SUPO core which is likely to make their transient behavior somewhat dissimilar is the amount of free volume in their effluent lines - the test model being more favorable in this respect. The operating parameters which will affect the energy release in such a core under transient conditions (other than the amount of reactivity involved and the time scale on which it is

released) are, in the expected order of importance:

1. Void volume above solution
2. Initial solution temperature
3. Initial core pressure
4. Initial power level

In the cases of high initial power levels some latitude in the initial temperatures must be sacrificed in order that delayed neutrons achieve equilibrium before the transient is induced. A delay time - after initial power is achieved, before the transient occurs, and during which the delayed neutron concentration will become essentially constant - will be about two minutes. Operating the cooling system at capacity will help minimize temperature rise in the core during this waiting period, but progressively higher initial powers will require progressively higher initial temperatures.

The above variables, together with variations in the amounts of reactivity and rates of reactivity release, constitute the basis of the experimental program as presently conceived. The void volume above the fuel solution will be varied from 0 to about 20 per cent of the core volume. Solution temperatures will range from about 20° to 90° C. Core pressures will be varied over the practical working range of the gas recombination system, that is, 0 to -100 inches of water, gage. Initial power levels from zero to at least 50 kw will be investigated. Reactivities up to approximately 4 per cent will be introduced on time scales ranging from about 0.040 to 3 seconds.

The general program may be divided into seven distinct phases as follows:

1. Facility and equipment planning and design
2. Facility and equipment construction and procurement
3. Reactor and measurement equipment installation
4. Pre-operational testing and calibration of equipment
5. Operational testing and calibration of equipment
6. Transient tests
7. Interpretation of results.

Concurrent with these phases is a continuing theoretical study to predict and interpret the results of the tests themselves. This effort was undertaken immediately upon activation of the program, and some of the results obtained to date are reviewed in this report. At the present time the program effort is

well into the third phase, with completion of the experimental facility and equipment components scheduled for February 1. The following two phases of equipment installation and testing will take an additional six weeks, allowing transient testing to begin in the month of March.

The pre-transient phase of the experimental program will be concerned mainly with calibrations of the equipment and determinations of the nuclear constants of the test reactor. Mass and temperature coefficients of reactivity along with calibrations of control, safety, and poison rods will be determined during loading of excess reactivity. With an amount of reactivity representative of operating conditions, the prompt neutron lifetime and the effective fraction of delayed neutrons will be determined by a pile oscillator technique. Energy coefficient of gas production will be determined as a function of both power level and temperature, although it is not expected to vary significantly with these parameters. The total amount of mixed gaseous fission products present in the sweep gas system of the reactor at equilibrium rated power will also be measured. Over-all heat transfer coefficients will be evaluated in a series of oscillation experiments so that post-burst behavior in an appreciable number of the tests can be predicted reliably.

The amount of reactivity to be released in the first test model will be determined by either of two criteria, whichever proves limiting: (1) that amount of reactivity normally provided in a 50-kw reactor in order to overcome the negative temperature and power coefficients and provide adequate experimental flexibility at equilibrium 50-kw operation, or (2) that amount which is capable of destroying the integrity of the core container of the test model. If the first criterion proves to be limiting, the maximum reactivity released will be about 4 per cent. Half this amount is required to reach 80° C solution temperature at a 50-kw equilibrium power level, and the remainder is a realistic requirement for experimental flexibility, as for example, absorbing samples installed in or near the core, reflector removal for beam production, margin of control, etc. The second, or destructive criterion, cannot be estimated at the present with the degree of confidence desired. Therefore, the kinetic phase of the program will start with the introductions of small (e. g., 0.2 to 0.3 per cent) amounts of reactivity. The amounts will then be increased step-wise in small

enough increments that the results of each succeeding transient can be predicted from the preceding results with sufficient accuracy to guarantee that no rupture will occur in the system.

To prevent a potential rupture of the system from becoming the limiting criterion, a considerable ruggedization of the reactor core vessel has been effected. Present calculations indicate that, with the ruggedized core design, the tests involving reactivity releases of up to 4 per cent can be accomplished.

Destructive testing has not been included in the current program. The test cores will not be subjected to transient pressures greater than that to which the assembly has been statically tested. Incremental changes in parameter values will be limited in magnitude to assure reliable prediction of peak pressure before an excursion is made. If transient pressure is predicted to exceed the static test pressure, the experiment will not be performed. The peak pressures attained are of primary importance in evaluating the severity of the excursions. Therefore much emphasis will be placed on accurate records of these values throughout the reactor system.

In order that no limitations on the accuracy of the data be introduced from the instrumentation, all components will have response times suitable for accurate reproduction of a one-millisecond reactor period. Since the release of 4 per cent in reactivity results in a stable period of about 4 milliseconds, a sufficient margin of safety in instrument response will have been included in any tests involving that or lesser amounts of reactivity.

III. EXPERIMENTAL FACILITY

A. SITE

The experimental site is located at the Field Test Station of North American Aviation, Inc. This 5,000-acre reserve is situated at the west end of the San Fernando Valley, about 30 miles from central Los Angeles, as shown in Fig. 1. A small section of the reserve has been set aside for AEC activities - primarily the construction of the Sodium Reactor Experiment. The facility for the kinetic experiments is located about 500 feet from the main SRE building. The area layout is given in Fig. 2, while Fig. 3 is an aerial photograph of the site. A plot plan of the kinetic experiment facility is shown in Fig. 4.

B. REACTOR TEST BUILDING

The Reactor Test Building, a plan of which is shown in Fig. 5, is an underground concrete structure. This building will house the reactor test core, the control rod, gas handling and fuel storage systems, and the necessary detection instruments. The building is 15-feet by 26-feet by 10-feet high with outside walls and floor of reinforced concrete, 8-inches thick. The roof is a one-foot thick reinforced concrete slab. Six feet of earth fill covers the building for shielding purposes. The interior is divided into three gas-tight rooms: the reactor room (12 feet by 15 feet), the gas and fuel handling room (6-1/2 feet by 7-1/2 feet), and the valve gallery (6-1/2 feet by 7-1/2 feet). Concrete shielding walls two-feet thick separate the three rooms. A ventilation system is provided for emergency use and can be used to purge the atmosphere in any or all of the rooms. The exhausted air is discharged from a disposal stack.

Fig. 1. Location of North American Aviation Field Test Station

Fig. 2. Location of KEWB Experimental Facility on the SRE Site at Santa Susana

C. CONTROL AND ELECTRICAL BUILDINGS

A 12-foot by 20-foot prefabricated sheet steel building, located 200 feet from the reactor test building (See Fig. 3), will serve as the control station. All recording instruments necessary to operate the reactor in a normal fashion plus the transient instrumentation are located in this building. In order to reduce cable loss, the preamplifier circuits and some power supplies are located in a small shelter, called the electrical building, located within a few feet of the testing building. All cable-runs pass through this shelter, but only those lines feeding instruments located there are interrupted. The majority of the lines are unbroken between the test and control buildings.

Fig. 3. Aerial View of SRE Area Showing Location of the Experimental Facility

D. WASTE DISPOSAL

The waste disposal system for the facility consists of three underground tanks and a 60-foot exhaust stack with a 2000-cfm blower system. The stack and blower are the means by which gas withdrawn from the reactor system may be released to atmosphere after sufficient decay to insure tolerance level discharge. A 300-gallon tank, initially at vacuum, will be used to collect gas directly from the reactor system. This low-pressure source provides not only the necessary temporary storage capacity but also, in conjunction with an automatic pressure regulator located between it and the reactor, insures that the reactor atmosphere is at the desired negative pressure. The collection tank is evacuated when required, and the vacuum pump exhaust is fed to the stack blower intake. The system is so interlocked that discharge of excessive activity or blower motor failure will automatically interrupt the active gas flow.

A second storage tank of 1000-gallon capacity is buried beneath floor level of the test building and will retain all liquid waste from the facility. This tank is equipped with pump-out connections, so its service may be renewed if and when occasion demands.

The third underground tank is also of 1000-gallon capacity and will retain the reactor cooling water so that it may be checked for activity before release to ground drainage. The reactor is cooled by a "once through" tap-water system. Calculation of induced activity based on chemical analysis of well samples from the vicinity of the site indicate less than drinking tolerance activity levels will be present upon discharge from the core. However, the possibility of migration of fuel solution into the cooling water in the case of a breach of one or more of the cooling coils has required the holdup precaution. This tank, as well as the waste liquid storage tank, is equipped with level alarm to indicate impending overflow. The capacity of 1000 gallons allows equilibrium reactor operation at 50 kw for about one hour.

Figures 6-10 are progress photographs which were made during the period of construction and installation of the facility at the Field Test Station. Figure 6 shows the concrete slab floor of the underground test building. Figure 7 is a view of the forms for the walls and top slab of the test building. Figure 8 shows the completed underground building with the retaining walls, waste disposal tanks, and stack prior to the back-filling operation. The completed test building is

Fig. 4. Plot Plan of the Experimental Facility for the Reactor Kinetic Program

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Fig. 5. Plan View of the Reactor Test Building

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Fig. 6. Concrete Slab Floor of Test Building

Fig. 7. Forming for Walls and Slab Top of Building

Fig. 8. Completed Test Building Prior to Back-Filling Operation

Fig. 9. View of Completed Underground Test Building

shown in Fig. 9 with the electrical building at the right. Figure 10 is a view of the site showing the control and electrical buildings with the underground test building and the stack in the background.

IV. REACTOR DESCRIPTION

A. GENERAL

All existing low power homogeneous reactors,²⁻⁶ of the type frequently referred to as water boilers, have three common characteristics: (1) small core vessels of less than 20-liters capacity, (2) graphite reflector, and (3) a plumbing system external to the reflector for processing of the gases evolved from the core during operation. However, design details which affect transient characteristics vary considerably between both existing and proposed models. For this reason the first model to be tested for kinetic behavior will be a replica of a particular reactor design - one on which safety information is

Fig. 10. View of Site Showing Completed Test Building and Control and Electrical Buildings

required in connection with licensing for operation at a specific location. The rated power level of this first test model will represent what is generally considered as the current practical limit for such reactors, namely 50 kilowatts. Also representative is the core geometry, which will be spherical. This geometry, by merit of most favorable neutron economy, has proved to be the most popular configuration.

Fig. 11. Layout of First Reactor Core Vessel
to be Subjected to Transient Tests

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In all respects except biological shielding and experimental facilities the first test reactor will be a close duplicate of the proposed 50-kw unit. Control, cooling and gas recombination systems will be direct reproductions, while the core will be modified in order to assure a reasonable amount of latitude and safety during the investigations and to provide for special pressure and temperature measurements - without alteration of transient behavior.

B. CORE

Figure 11 shows the essential features of the core vessel. The sphere is 12.3 inches inside diameter and will have a minimum wall thickness of 0.220 inches. Ten lengths of stainless-steel tubing 1/4-inch OD by 0.028-inch wall, each 9-feet long, provide sufficient cooling capacity to maintain the fuel solution at 80° C when the reactor is operating at 50 kw. The inlet and outlet lines are manifolded along one side of the sphere, and the coils are essentially horizontal. A 2-1/4 inch OD tube is re-entrant in the sphere along a horizontal diameter. This tube will accommodate the poison rod to be used as the mechanism for introducing the reactivity increments. Four additional re-entrant tubes, equally spaced about the vertical diameter, provide passages for the control and safety rods. These rods are normal sintered B_4C rods contained in thimbles. They are positioned by conventional rack and pinion drives and fall freely when released by electromagnets attaching them to their individual racks. These rods control a total of about 8 per cent reactivity.

A vertical effluent line is attached to the top of the core. This tube, 2 inches in diameter and 18-inches long, leads to a deflection dome which facilitates gas-liquid separation in the event of expulsion of fuel solution from the core. The solution is temporarily retained in an annular reservoir while the hydrogen-oxygen gases are allowed to pass through a stainless-steel wire filter for entrainment elimination before being swept to the recombiner. Drainage from the holdup reservoir to the core is accomplished by gravity feed through a 3/16-inch hole at the bottom of the 2-liter reservoir. Above the entrainment filter the vessel is brought to a dome configuration and gas sweep lines entering at this location conduct the evolved hydrogen and oxygen from the core vessel to the recombiner and return the condensate to the core.

C. REFLECTOR

The reflector consists of AGOT-grade graphite bars stacked to form a 56-inch cube. The bars are nominally 4-inches square and unmachined except for those pieces adjacent to the core. Since the edges are somewhat rounded, the gross density is less than the normal 1.6 gm/cm^3 by a few per cent.

Two channels of graphite are readily removable from the stack. One channel will contain the start-up source, and the second will house neutron detectors used in the experiments. This latter channel will allow positioning of detectors close to the core for subcritical analysis purposes. All replaceable attachments to the core are located in one quadrant of the reflector, and the graphite in this quadrant is arranged for convenient unstacking. A cutaway perspective view of the reactor is shown in Fig. 12.

The reflector rests on a concrete pedestal and is completely enclosed by a gas-tight reinforced sheet aluminum container. This container is capable of withstanding an internal pressure of 2 psig and will serve to confine liquid and gas released by the core if rupture of that container should occur. As added precautions, a holdup trench outside the aluminum enclosure is intended to trap any escaping liquid, and the core-reflector unit is contained in a room made gas-tight by use of a specially designed gasketed door and by application of a sealant paint to the inner concrete surfaces. This room, and in fact all gas-tight enclosures in the facility, will be pressure tested prior to experimentation.

D. GAS AND FUEL HANDLING SYSTEMS

The plumbing associated with the test reactor is located in a metal lined gas-tight room adjacent to the reactor room but separated from it by two feet of concrete. The gas-recombination system is located in an alcove in this room, and all vessels and piping necessary for manipulations and storage of fuel and decontamination solutions are also located there. After reactor operation is undertaken, entry into this room will be made rarely, and then only for maintenance purposes. Operations involving equipment in the room, which include transporting fuel to and from the core, fuel concentration and dilution, decontamination of all lines and vessels, measurement of gas production from the core, etc., will be conducted remotely through a shielding wall of two feet of concrete.

Fig. 12. Isometric View of the Reactor

The gas handling system provides for circulation of 7 cfm of gas through the core vessel. The sweep-gas dilutes hydrogen and oxygen produced in the core and conducts the mixture through a platinized alumina catalyst bed, where recombination is effected, and returns the condensate to the core. An aspirator provides the driving pressure for sweep-gas flow, and phase separation takes place in a 20-liter reservoir which also serves as the housing for an entrainment filter and the catalyst bed. A tap-water heat exchanger at the inlet to the canned rotor aspirator pump removes the heat of recombination from the system. The working liquid for the aspirator is distilled water and is maintained at constant volume in the reservoir by action of an overflow weir. Fuel solution, received from the core assembly as a result of a violent excursion, is partially returned to the core as excess volume from the reservoir. Water vapor evolved subsequently from the core allows that vessel to act as a distillation unit, retaining the uranium it receives while continuously diluting any uranium solution remaining in the recombiner reservoir. Energy necessary to evaporate water from the core may be obtained by operation of the reactor with restricted cooling or by electrical heating.

The fuel handling system consists primarily of three vessels with inter-connecting piping. A 3-liter bowl is used to introduce the initial charges of uranium into the system and to contain fuel solution during concentration by vacuum distillation. The contents of this vessel drain by gravity into a 20-liter cylindrical reservoir, 6 inches in diameter, which serves as the main storage tank for fuel solution in the reactor system. A distillation condenser is mounted above the 3-liter vessel, and will be used to trap water vapor during concentration of the fuel solution contained in the 3-liter vessel.

The gas recombiner is paralleled in the reactor system by a ballast tank with a capacity of 20 liters. If the recombiner is blocked-off from the system and the evacuated ballast tank substituted for it, the reactor may operate for about 2.5 kwh before the system atmosphere achieves positive pressure. This arrangement will be used to measure the energy coefficient of gas production under different power, temperature and pressure conditions. The measurements of gas volume will be made by noting pressure changes resulting from operation of the reactor.

Pure gases for purging the system and for replacing liquid may be added to the gas and fuel handling systems by means of water traps. Also, the entire reactor system can be evacuated or flooded with wash-water or dilute acid solutions. All piping and vessels have been designed to accomplish complete gas displacement when the system is filled with rinsing solutions. It is hoped that sufficient decontamination can be effected by rinsing, gas displacement, and evacuation to permit the required maintenance activities without more vigorous measures.

To minimize the quantity of liquid held-up in the plumbing lines after drainage and to minimize pressure drop in sweep-gas lines, a rotating ball-type valve will be employed. These valves, procured from the Jamesbury Company, Worcester, Massachusetts, consist essentially of a hollow cylinder in which two ring-gaskets in compression fix the location of a stainless-steel ball having a hole coaxial with the cylindrical housing. A 90-degree rotation of the ball affords complete cut-off of the line since misalignment of the ball hole and housing passage results. The gasket material will be black neoprene since a valve with gaskets of this material has been found to withstand 5×10^7 R of fission product gamma radiation, hydrogen-oxygen internal explosions and repeated disassembly and reassembly without loss of helium leak-tightness. Quarter-turn operation also is valuable, since some forty valves are necessary to perform all the operations required in the experiments. An elevation view of the piping and vessels which constitute the plumbing for the reactor system is shown in Fig. 13.

E. REACTIVITY DEVICES

The reactivity will be added to the test reactor by withdrawal of poison from the core. Two mechanisms will be utilized in order that the time scale of withdrawal may be varied over a wide range, that is, about 0.040 second to 3.0 seconds. The shortest time scale is intended as an approximation to a step increase in reactivity, while the extended time scales will yield information related to ramp rates involved in more typical accidents. In neither case will reactivity be added as a linear function of time since both mechanisms are constant acceleration devices, and the position-effectiveness curve for the poison rod will be of the typical "S" shape.

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Fig. 13. Elevation View of the Plumbing for the Gas Handling and Fuel Handling Systems

Figure 14 shows the salient features of the fast-ejection poison rod apparatus. The rod itself is a four segment cylinder, 16-inches long, of normal sintered B_4C compound 1-9/16 inches OD. There is a small diameter void down the center of the rod necessitated by the original application of the units as sandblast nozzles. This hole will be utilized in making integral neutron distribution measurements during transients. The rod is expected to control a minimum of 4 per cent reactivity when centered in the 2-inch "glory hole" which is on a diameter of the test core. The rod segments are contained in an aluminum tube which is rigidly attached to the piston element of a piston-cylinder arrangement mounted adjacent to the reflector. On the opposite end of the poison-rod tube a second rigid member supports a slide-wire contactor. When moved slowly this contactor will report the position of the rod in terms of resistance in a potentiometer circuit; when the rod is ejected at high velocity the contactor is withdrawn from the immediate vicinity of the slide wire by an accelerating force component. The latter provision was made to protect both slide wire and contactor from frictional abuse. The position of the poison rod as a function of time during ejection, from which the reactivity-time function will be inferred, is obtained from an oscillograph record of a series of pulses equi-spaced at intervals of about an inch over the rod's travel. These pulses are generated in a number of individual coils wound around the cylinder by the passage of a small permanent magnet mounted in the edge of the piston mentioned above. The pulses are large enough to be fed directly to a galvanometer in the oscillograph without amplification, so that a simple series arrangement of all of the coils and the galvanometer comprises the circuitry.

The accelerating force applied to the poison-rod piston is achieved by release of compressed gas stored in a receiver, the outlet of which is obstructed by the piston. The initial pressure in the receiver may be varied conveniently to any desired level, which allows a wide range of ejection times. An initial pressure of approximately 100 psi will result in the minimum ejection time of about 0.040 second. The pressure source is a conventional 2500-psi nitrogen cylinder located in the control building. Release of the gas from the receiver is accomplished by means of a solenoid-operated plunger which opens a one-inch diameter orifice in the receiver. Partial deceleration of the rod results from the build-up of pressure ahead of the piston; this pressure is then released, and a neoprene bumper absorbs the remaining momentum. The driving pressure is

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Fig. 14. Diagram of Fast Reactivity Release Mechanism

maintained until the rod reaches static equilibrium. The rod is returned to the desired starting position by manipulation of gas pressure on either side of the piston.

Incorporation of all time scale requirements in one reactivity release mechanism has proved impractical. In order that slower poison withdrawal times might be realized, a second apparatus will be employed. It consists essentially of a counterweight-pulley-cable arrangement. A variety of withdrawal times, initially chosen as 1 to 3 seconds in 0.5-second intervals, is made possible by adding rotational inertia to the system in fixed quantities by means of magnetic clutches which engage small flywheels. The axis of the series of flywheels is common to that of a pinion gear driven by a rack, upon which the pulley-counterweight arrangement acts. The rack is a connecting link between the cable and the poison rod, which is identical in structure to the one previously described. The rack also operates a 360-degree continuous rotation Helipot which, in series with a battery and galvanometer, provides a record of displacement versus time for the rod after it is released. A reversible motor moving the rack through a pinion gear repositions the rod and, upon automatically energizing its magnetic clutch, the motor acts as the braking mechanism for the assembly. Figure 15 shows the details of the apparatus, which will be mounted on the reflector face opposite that to which the rapid ejection device is mounted.

V. INSTRUMENTATION

A. GENERAL

The instrumentation system is composed of two major portions. One portion comprises those instruments, controls, and actuators which provide working control of the reactor. These enable the operator to start-up and shut-down the reactor and operate it in a conventional manner. This group of instruments is designated as the "operational" instrumentation. The second portion comprises those instruments, controls, actuators, and recording devices which enable the operator to initiate a reactivity release and record the dynamic behavior of the reactor following the release. These devices, taken together, are designated as the "transient" instrumentation. Each major portion will be described in detail.

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Fig. 15. Diagram of Slow Reactivity Release Mechanism

B. OPERATIONAL INSTRUMENTATION

A block diagram of the nuclear instruments for normal reactor operation is shown in Fig. 16. Two pulse-counting channels are provided for pre-start-up and low power level indication. The detectors are BF_3 chambers placed on top of the gas-tight container which encloses the reflector. One channel is connected through a pulse amplifier to a linear-count rate meter with range switching for accurate determination of power level in the low range. The

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Fig. 16. Instrumentation for Normal Reactor Operation

other channel is similarly connected to a four-decade logarithmic-count rate meter for wide-range monitoring at low levels. Two ionization chamber channels are also provided. One furnishes input current to an electrometer capable of measuring and displaying currents from 10^{-10} amp to 10^{-4} amp. The electrometer output is also used as the input current to a combined servo-amplifier and level recorder. The servo-amplifier output is used to control the position of one of the control rods to hold the reactor power level constant at a desired value. The second ionization chamber provides a current to a logarithmic amplifier whose output is displayed on a level recording covering six decades without scale switching. Thus this channel provides wide-range monitoring of power level at high values.

Four automatic safety trips are provided during normal reactor operation. As shown in Fig. 16, the output of the linear-count rate meter is fed to a meter relay so that an excessive counting rate will actuate the scram circuit. Likewise, the current into the electrometer is passed through a meter relay to the scram circuit, and the amplified output from the electrometer is also fed to a second meter relay and then to the scram circuit for additional power level protection. A voltage proportional to the period of the reactor is taken from the output of the logarithmic amplifier and used to actuate the scram circuit through the period meter relay if the reactor period should become undesirably short.

C. TRANSIENT INSTRUMENTATION

A block diagram of the instrumentation for recording the dynamic behavior of the reactor following a reactivity release is shown in Fig. 17 and 18.

Power level information is furnished by three channels operating from ion chambers. Two of these chambers are connected to logarithmic amplifiers with a range of about six decades. The outputs of these amplifiers are linear with respect to the logarithm of the input current from the ion chambers, thus enabling six decades of neutron level to be recorded on a single galvanometer trace. The output current of the amplifier is in the range from 0 to 200 microamperes so that a current amplifier is required between the logarithmic amplifier and the fast galvanometer on which the data are recorded. The third chamber is connected to a current amplifier directly and is intended to measure the power

level on a linear scale during an excursion. The use of a linear channel for this purpose gives a more accurate recording of the power level near, and including, the peak value.

A group of about twenty-four strain gages will be mounted at various points on the outside surface of the reactor core, the overflow chamber, and the ballast tank in the gas-handling system. These gages will be calibrated in position prior to actual reactor operation by the application of hydrostatic pressure to the system, so that the relationship of gage output to pressure at the gage locations will be known. Each active strain gage will constitute one leg of a four-leg

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Fig. 17. Instrumentation for Transient Neutron Levels

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Fig. 18. Instrumentation for Supplementary Data

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Wheatstone bridge. The bridge is energized by a three-kilocycle carrier wave from a carrier amplifier. The output will be amplified and demodulated by the carrier amplifier, and then fed to a galvanometer in a recording oscillograph. A device termed a carrier amplifier-multiplier is used to enable each channel of a four channel carrier amplifier to select one strain gage from a group of six gages assigned to that channel for recording during an excursion and to read each other gage in the group after the excursion without the necessity of re-balancing the channel controls between readings. Thus both elastic and permanent deformation of the reactor and ballast chamber may be accurately measured.

The locations of the strain gages, pressure pickups, and thermocouples are shown with respect to the major parts of the reactor system in Fig. 19.

Pressure pickups are provided to measure the initial static pressure in the sphere, the reaction and impact pressures during expulsion of fuel solution, and pressure surges from hydrogen-oxygen explosions. All pickups are miniature, flush-mounted diaphragm types of stainless-steel construction and, with one exception, employ unbonded strain gages connected in a bridge arrangement to yield an electrical signal. The gages are wound on the arms of a star-spring which detects the motion of the diaphragm. The pickups intended to measure reaction pressure and explosion pressure are used in conjunction with a 20-kilocycle four-channel carrier amplifier. The outputs of the amplifier are recorded on galvanometers whose frequency response is essentially flat to 3000 cycles per second. The pickups intended to measure static pressure and also to monitor the pressure drop across the aspirator pump are used in conjunction with a 3-kilocycle carrier amplifier, and the output is displayed on meters located in the control room and in the reactor building. The one pressure pickup which does not employ a strain-gage bridge is of similar construction and appearance to the others, but the stainless-steel diaphragm acts as one plate of a small capacitor. The second capacitor plate is an insulated electrode behind the diaphragm. The small variation in capacitance resulting from deformation of the diaphragm by pressure is introduced into a sharply tuned oscillator circuit which provides a small direct current output proportional to the pressure at the diaphragm. This current is amplified and fed to a very fast galvanometer. The two types of diaphragm pickups are shown in Fig. 20.

Temperature measurements in the fuel solution are made by iron-constantan thermocouples in stainless-steel sheaths with compressed MgO insulation. The wires are 0.010 inch in diameter, and the tips are bare except for a thin layer of platinum sputtered on for chemical protection. The stainless-steel sheath is brought through the sphere wall by welding it to both ends of a hole through a stainless-steel cap screw which is screwed into a boss-type fitting in the sphere. The output is taken directly to a galvanometer of fairly high sensitivity. Tests have shown this to be a reliable method of obtaining fast response temperature information in the fuel solution.

- LEGEND:
- PRESSURE PICK-UPS:
- 1 - 0-25 PSIA
 - 2 - 0-100 PSIG
 - 3 - 0-100 PSIG
 - 4 - 0-1000 PSIG
 - 5 - 0-1000 PSIG
 - 6 - 0-25 PSIA
 - 7 - 0-25 PSIA
 - 8 - 0-1000 PSIG

- THERMOCOUPLES:
- 1 - CORE FUEL TEMPERATURE
 - 2 - CORE FUEL TEMPERATURE
 - 3 - CATALYST BED TEMPERATURE
 - 4 - BALLAST CHAMBER TEMPERATURE
- X STRAIN GAGES AT VARIOUS POINTS

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Fig. 19. Locations of Supplementary Pickups

Microphones are located on the core and recombiner assembly as an aid in detecting equipment malfunction and to give aural perception of the reactor and gas handling system behavior during an excursion. The microphone outputs are amplified and presented over a speaker in the control room and recorded on magnetic tape for further study. It is expected that comparative estimates of the violence of the excursions can be made from these recordings and that a useful qualitative picture of the hydrodynamic behavior of the fuel will be given by them.

D. PHYSICAL LOCATION OF EQUIPMENT

Figures 16, 17, and 18 have been drawn to show the location of equipment at the underground reactor building, at the electrical building above ground and adjacent to the reactor building, and at the control room. The dotted lines represent the underground cables and wires joining these locations. Approximately two-hundred wires are provided in direct-burial multi-conductor cables for 110-volt power and control circuits. The numerous coaxial cables, thermocouple

Fig. 20. Pressure Transducers to Measure Transient Pressure.
(The Left is the Variable Capacitance Instrument;
the Other Contains an Unbonded Strain Gage.)

extension wires, carrier cables, and microphone and speaker cables are placed in non-metallic ducts. The equipment in Fig. 16 is used for routine operation of the reactor and is located in a console in the control room. The recording oscillographs and carrier amplifiers are located on a table. Auxiliary racks between the console and recording table contain the control circuits and connectors which coordinate the conduct of an induced transient, as well as such miscellaneous equipment as the tape recorder, intercom amplifier, and carrier amplifier-multiplier.

VI. HYDROGEN-OXYGEN EXPLOSION HAZARD

Approximately four liters of hydrogen and oxygen gases are produced per megawatt-second of energy release in a conventional water-boiler core.^{4, 7} Under normal operation these highly flammable gases, present in stoichiometric amounts from water decomposition, are immediately diluted to non-inflammable concentrations by sweep-gas circulated through the core. However, when several megawatt-seconds of energy are released nearly instantaneously in the core, explosive and even detonatable conditions and concentrations will exist throughout the system until recombination is effected. A source of ignition is present in the form of the recombiner catalyst, since a surge of gas will raise the temperature of the bed - particularly at the inlet face - to the required temperature. Also, nuclear radiation of sufficient intensity may be capable of causing ignition, although no evidence of this phenomenon has been noted in the literature.

The problem of determining the peak pressures which the core assembly must withstand in the event of gas explosions has been approached experimentally. Figure 21 is a photograph of a core mockup specifically designed for explosion tests. A series of tests have been made with this model to establish peak pressures delivered to the walls of the container and the extent of internal component damage as functions of ignition location, water level in the sphere, and initial pressure of the stoichiometric mixtures. The results of these experiments have been applied to the design specifications of the first nuclear test core to insure that the integrity of the vessel will be maintained under the most realistically pessimistic conditions.

Fig. 21. Reactor Core Mockup Used for the Hydrogen-Oxygen Explosion Tests

VII. GENERAL THEORETICAL CONSIDERATIONS

Concurrent with the planning of the facility and the experimental program, a series of theoretical studies of the transient behavior of water-boiler type reactors has been in progress. The main objectives have been the following:

1. To facilitate specification of adequate transient instrumentation.
2. To indicate the relative importance of the various parameters.
3. To predict the dependence of peak power, pressure, etc., on the more important parameters, especially in the neighborhood of potentially damaging situations.

Additional analysis has been concerned with the possibility of measuring the reactor constants during the static phase of the experimental program by observing both free and forced power oscillations of small amplitude.

The results of the transient studies indicate that for a given set of conditions (fixed initial power, temperature, free expansion volume, etc.) the range of introduced reactivity steps may be qualitatively divided into four regimes. These are characterized by the type of shutdown mechanism which predominates and by the manner in which the time scale for the energy burst influences the shutdown mechanism. This break-down is summarized in Table I.

The first range is approximately from zero to 0.4 per cent, corresponding to stable reactor periods greater than ten seconds. In this range the shutdown is caused primarily by temperature rise, because the transient is proceeding slowly enough that most of the radiolytic gas produced in the core solution escapes without contributing to the shutdown reactivity. Transient behavior in this range has previously been investigated,^{8,9} and the chief purpose of considering this range is to check the methods and procedures developed for the subsequent tests.

The second range is from 0.4 to 1.0 per cent, corresponding to reactor periods between 10 and 0.05 seconds. This covers the range in which the time scale of the energy burst is comparable to the average residence time of radiolytic gas bubbles in the solution, so that gas production will begin to play a significant role in the shutdown.

The third range is from 1.0 to 2.0 per cent, with periods from 50 to 10 milliseconds. Here the time scale for the transient is very short compared

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TABLE I. KEWB TRANSIENT EXPERIMENTS

Group	Reactivity Release	Reactor Period	Primary Shutdown Mechanism	Inertial Pressure	Time Required to Reach Peak Power	Peak Power	Energy Release	Remarks
I	0-0.4%	∞ -10 sec	Temp Coeff.	None	∞ -200 sec	0-20 kw	0-1.8 Mw sec	Transients which have been previously investigated serving as checkout of methods and procedures.
II	0.4-1.0%	10-0.05 sec	Temp Coeff. and Gas Coeff.	None	200-1 sec	20-80 kw	1.8-3.8 Mw sec	Transition range from predominance of temperature to predominance of gas production.
III	1.0-2.0%	0.05-0.01 sec	Gas Coeff.	0-0.5 atm	1-0.2 sec	0.08-20 Mw	3.8-1.2 Mw sec	The gas coefficient becomes the predominant shutdown mechanism for the first power surge. The inertial pressures are not large enough to change the gas coefficient appreciably.
IV	2.0-4.0%	10-4 milli-sec	Gas Coeff.	0.5-6.0 atm	0.2-0.1 sec	20-150 Mw 30-600 Mw*	1.2-3.4 Mw sec 1.8-14 Mw sec*	The gas coefficient is the dominant shutdown mechanism, but the inertial pressure causes a variation in the coeff. which increases the peak power and the total energy release.

*Corrected for transient pressure

to the bubble residence time, and most of the gas generated will contribute to shutting down the reactor. Since the energy coefficient of reactivity for gas production is an order of magnitude larger than that for temperature effects, the severity of these excursions is much less than would be anticipated from consideration of temperature alone. Subsequent to the burst which lasts for a small fraction of a second, the loss of negative reactivity by bubbling will cause a new power rise; however, this second surge is similar to the effect of introducing reactivity at a fairly slow "ramp" rate. In fact, the return of reactivity by loss of gas from the core may be compared with a "power demand" situation in which the rate of reactivity introduction decreases as the surge proceeds. The rate, which is initially of the order of one dollar per second, is sufficiently great that some overshoot will occur, but in contrast with the initial burst, the effect is a very mild transient for which the time scale is several seconds or more.

These events occur in a time interval that is still small compared with the time for significant heat loss from the core, which is of the order of a minute even with coolant flowing. The reactor will therefore behave during the first few seconds after the initial burst as though it were seeking an equilibrium characterized by an asymptotic approach to zero power at a temperature prescribed by the initial reactivity step. Actually, the reactor can undergo damped oscillations of very long periods (hours), approaching an equilibrium power determined by the rate of heat removal. The temperature will not be greatly different from that attained in the hypothetical adiabatic case above; in either case the net reactivity asymptotically approaches zero.

The role played by gas production is extremely important. Although the steady power coefficient of reactivity arising from gas production is less than that for temperature, the dominance of gas production in determining the transient energy coefficient of reactivity not only limits greatly the peak power in a burst but also slows up the subsequent release of energy that must eventually compensate for the loss of gas from the solution. Because of this change of time scale, the total energy release will be approximately halved by the gas production, and only a small fraction of this energy will be produced in an intense burst.

The fourth range listed in Table I is from 2.0 to 4.0 per cent, with periods from 10 to 4 milliseconds. This range is characterized by the appearance of inertial pressure, which arises because the mixture of core solution and radiolytic gas cannot expand rapidly enough to keep up with the rapidly rising rate of energy production. (The phenomenon is analagous to transient thermal stresses in a metal which is heated so rapidly that expansion cannot keep up with the temperature rise.)

The results for an open-ended cylindrical core indicate an inertial pressure rise of 6 atmospheres at the largest reactivity release now contemplated (4 per cent). It is important to note that this pressure is a direct consequence of gas production; if the large contribution of radiolytic gas to the compressibility were not present, the inertial effects would be negligibly small even for a reactivity release of 4 per cent.

Several other phenomena are important in determining the energy release. If the core vessel has a limited volume for expansion, with a small diameter relief tube, there will be a pressure surge beginning when the vessel is filled, whereupon subsequent expansion encounters a resistance to transient flow in the relief tube. This effect becomes important at different reactivities, depending on the amount of expansion volume initially available. In the spherical core to be tested first, all transients will first be observed with the greatest possible initial void volume. Because the peak pressure for a given reactivity step increases rapidly as the initial void volume is reduced, the theoretical calculations allowing a large margin for error will be carefully used as a guide when the effect of initial void volume is being investigated.

Subsequent tests to be performed on a cylindrical core having a large expansion space above the solution are expected to show that this configuration has much advantage in intrinsic safety over the sphere and the closed cylinder, particularly if provision is made for holding up overflow in a reservoir outside the region of maximum statistical weight. The spherical core has an additional disadvantage in that the reduction in reactivity caused by gas bubbles in an underfull sphere is partially offset by the reduced neutron leakage from the more nearly spherical geometry. This effect is not large, and is essentially independent of

solution height, because both the density effect on reactivity and the geometrical effect on leakage are approximately proportional to the total volume of gas bubbles.

The onset of boiling is another phenomenon of importance in determining the energy release. Here the important parameter is initial temperature, which may be varied either by initiating a transient at various equilibrium power levels or by passing hot water through the cooling coils when the reactor is shut down. If the boiling point is reached some time after the initial gas-limited burst (i. e., during the relatively slow second power surge), the effect of boiling will be a sudden shutdown by steam formation. On the other hand, if 100°C is reached during a pressure surge caused by inertial effects or by limited void space, the results indicate that no boiling can occur because the pressure-temperature curve during the surge will be rising more rapidly than the equilibrium vaporization curve. Boiling will then contribute only to minimizing the tail on the power-time curve after peak pressure has been reached.

Initial power level is an important parameter for small reactivity steps, as evidenced by the fact that the peak power in a small transient starting from "zero power" may be much less than the nominal operating power level of the reactor. For fast transients, the initial power level is important only as it affects the initial temperature and hence the time at which boiling begins. The peak power in a fast transient is essentially unaffected by the starting concentration of delayed neutron emitters, which is the only other parameter influenced by the initial power level. It should also be noted that the initial solution level will depend on the power because of the presence of an equilibrium gas-bubble concentration. This will become important at small values of the initial void volume.

The rate of input of reactivity is important in those cases where the shutdown mechanisms begin to act significantly before the full amount of available reactivity is inserted. This will occur for faster transients and for higher initial power levels. For a given amount of available reactivity, the step input yields the transient with the highest peak power, while at the other extreme there is a range of low rates of reactivity input for which there will be no power overshoot at all but rather a slow transition to a new equilibrium power. On the

other hand, the energy release varies only a factor of two from one extreme to the other. Since peak power is much more indicative of potentially hazardous situations, and since it will vary greatly with the reactivity rate, this rate is an important parameter for experimental investigation. Because the step input represents the limiting case for transients with a given total reactivity available, most of the computations to date are concerned with step inputs, and only these are reported here.

Other parameters of lesser importance may be noted. For example, the energy coefficient of reactivity for radiolytic gas will be inversely proportional to the initial absolute pressure over the solution. Also, the prompt neutron generation time, to which the peak power is approximately inversely proportional, will depend on the type of reflector, the fuel concentration, and the positions of safety and control rods. Finally, the details of the holdup and return of expelled solution will influence the power-time history.

The possibility of explosive recombination of hydrogen and oxygen has not been considered in the transient calculations. In the fast transients where the evolution of gas is significantly great, explosion is unlikely until after the gas has largely been separated from the core solution, at which time the effect of an explosion on the reactivity would not be great. The primary enclosure has been designed to withstand explosive recombination as described elsewhere in this report.

In the following, the basic kinetic equations for a water-boiler reactor are presented, together with a discussion of the choice of numerical coefficients and some of the limitations of the theoretical model. The results of electronic analogue computations are then presented, and the modifications required for consideration of inertial pressure, limited void space and boiling are discussed. Finally, the linearized kinetic equations are employed to indicate the usefulness of reactor oscillation experiments.

VIII. THEORETICAL MODEL

A. BASIC EQUATIONS

The time-dependent neutron density is assumed to satisfy two coupled first-order differential equations:

$$\dot{n} = \frac{\rho - \beta}{\tau} n + \lambda c, \quad \dots (1)$$

$$\dot{c} = \frac{\beta}{\tau} n - \lambda c, \quad \dots (2)$$

where n is the density of thermal neutrons, c is the density of fission fragments which are delayed neutron precursors, and ρ is the reactivity defined by

$$\rho = \frac{k - 1}{k}. \quad \dots (3)$$

Here k represents the effective multiplication constant, which is unity for criticality; hence a steady state is characterized by

$$\rho = 0; \quad c_0 = \beta n_0 / \lambda \tau. \quad \dots (4)$$

The constants β and λ represent respectively the total fraction of all neutrons produced which are delayed and the effective value of the mean decay constant for delayed neutron emitters, which is taken to be

$$\frac{1}{\lambda} = \frac{1}{\beta} \sum_i \frac{\beta_i}{\lambda_i} \quad \dots (5)$$

where the subscripts refer to the properties of individual species of delayed neutron emitters. The quantity τ is the prompt neutron generation time. It is assumed that Eq. (1) and (2) are rendered applicable to a reflected reactor simply by choosing an appropriate value for τ .

The densities c and n may both be expressed in units of reactor power. For the present purpose, n is taken to be the volume average power in kilowatts.

In general, Eq. (1) is non-linear because the physical effects of energy production in a transient state cause changes in the neutron multiplication. In the water boiler the two major effects of energy release are temperature rise and radiolytic gas production. It is postulated that the temperature obeys

the linear differential equation

$$T = Kn - \gamma (T - T_s) \quad \dots(6)$$

where K is the reciprocal of the heat capacity of the reactor core, γ is the reciprocal of the relaxation time for heat transfer, and T_s is the temperature of the coolant (or of the surroundings if no coolant is flowing). Steady state values of n and T are related by

$$Kn_o = \gamma (T_o - T_s). \quad \dots(7)$$

Combining Eq. (6) and (7) yields

$$T = K(n - n_o) - \gamma T, \quad \dots(8)$$

where for convenience the temperature is measured on a scale such that $T_o = 0$.

The volume of radiolytic gas present in the solution is assumed to satisfy an analogous equation

$$\dot{V} = G (n - n_o) - \sigma V \quad \dots(9)$$

where G is the rate of gas production and σ is the reciprocal of the average residence time of gas bubbles in the solution. Here V is to be interpreted as the gas volume in excess of the steady state amount at a power level n_o .

The reactivity is then assumed to satisfy a linear expression

$$\rho = \rho_o + \alpha T + \phi V \quad \dots(10)$$

where ρ_o represents the externally introduced reactivity (an arbitrary function of time), and α and ϕ are respectively the temperature and gas void volume coefficients of reactivity. Both coefficients are negative for the water boiler.

The kinetic behavior is then assumed to be characterized by Eq. (1), (2), (8), (9) and (10). Note that the energy coefficients of reactivity for temperature and gas production are αK and ϕG respectively, while the steady-state power coefficients of reactivity are $\alpha K/\gamma$ and $\phi G/\sigma$ respectively. Numerical values of the various constants employed in the

computations are discussed below, and the modifications made to account for other phenomena are described later.

B. NUMERICAL VALUES

Numerical values employed in the kinetic calculations are as follows:

β = delayed neutron fraction	0.0075
λ = mean decay constant for delayed neutron precursors	0.08 sec ⁻¹
τ = prompt neutron generation time	120 μ sec
K = reciprocal heat capacity of core	0.01735 °C/kw-sec
γ = reciprocal relaxation time for heat flow	0.018 sec ⁻¹
G = radiolytic gas production rate (STP)	4.16 cm ³ /kw-sec
σ = reciprocal residence time for gas bubbles	0.5 sec ⁻¹
α = temperature coefficient of reactivity	-2.4 x 10 ⁻⁴ /°C
ϕ = gas coefficient of reactivity	-7 x 10 ⁻⁶ /cm ³

Values for the delayed neutron fraction β and the mean decay constant λ are from Glasstone and Edlund.¹⁰ It is known that the effective value of β will be influenced by the neutron spectrum, because delayed neutrons originate at lower energies than prompt neutrons and hence are subjected to lower probability of fast leakage. The effective value of λ may be defined in several ways, depending upon the rate at which the power is changing. Means for improving the numerical values of these coefficients will be further investigated when experimental data are available.

The value for the prompt neutron generation time τ is considered typical of a graphite-reflected water boiler. An important part of the static tests will be to determine fairly precise experimental values for τ , but it should be noted that the assumption of a constant value for τ is an idealization even for a given reactor. The prompt neutron generation time will vary somewhat with the fuel loading and the position of control rods, and during a transient it will be influenced slightly by the mechanisms which are causing changes in reactivity.

The heat capacity is calculated for 13 liters of solution and 750 cm³ of coolant. The value of the heat transfer constant γ is rather arbitrarily

chosen to match the observation that the average temperature difference between the solution and the coolant is about one degree centigrade for each kilowatt in steady operation. Actually, for the duration of the analogue computer runs, the influence of heat transfer variations is negligible.

The gas production rate (energy coefficient of gas production) corresponds to a measured value of $15 \text{ cm}^3 / \text{watt-hour}$ obtained from the Water Boiler Neutron Source at North American Aviation, Inc.⁴ The value for σ corresponds to an arbitrary choice of 2 seconds for the average residence time of gas bubbles in the core. Since the average bubble size may vary with the power, and since statistical fluctuations are not taken into account, this model must be regarded as an idealization. In particular, the implications of a shorter average residence time are that the gas coefficient of reactivity would not become effective until a somewhat larger reactivity step is reached, and that the rate of reactivity return after the first burst would be greater.

The value listed for the temperature coefficient of reactivity is typical of measured values of this quantity for a water-boiler reactor near room temperature.^{3, 8} Since the absolute magnitude of this coefficient increases with increasing temperature so that the resultant peak power is overestimated, no attempt has been made to include the effect of temperature rise on the temperature coefficient.

The gas volume coefficient of reactivity may be estimated from simple density considerations which yield

$$\frac{d\rho}{d\delta} = -\frac{4}{3} \frac{k_{\infty}^{-1}}{k_{\infty}}, \quad \dots (11)$$

where δ is the volume dilatation $\Delta V/V$. For $k_{\infty} = 1.6$, this gives $d\rho/d\delta = -0.5$ and $\phi = -3.8 \times 10^{-5}$ per cm^3 .

An alternate value may be obtained from a very bold application of SUPO data,¹¹ which yields an estimate of $-0.17 \text{ gm U-235 reactivity effect per kw}$ at steady operation. Using the conversion that 1 gm is equivalent to 0.034 per cent reactivity, this yields an equilibrium power coefficient of reactivity

$$\phi G/\sigma = -5.8 \times 10^{-5} \text{ per kw}, \quad \dots (12)$$

whence $\phi = -7 \times 10^{-6}$ per cm^3 . Because of the large variation (more than a factor of five) between these two estimates for ϕ , the smaller value has been used in the calculations. Even so, the energy coefficient of reactivity for gas, which becomes $\phi G = -29.1 \times 10^{-6}$ per kw-sec, is seven times larger than the energy coefficient of reactivity for temperature ($\alpha K = -4.16 \times 10^{-6}$ per kw-sec). Note that the larger value for ϕ would yield an energy coefficient thirty-eight times larger for gas than for temperature.

In contrast, the steady state power coefficient for temperature is

$$\alpha K/\gamma = -2.31 \times 10^{-4} \text{ per kw} \quad \dots (13)$$

Comparison with Eq. (12) shows that the steady power coefficient for temperature is four times as large as that for gas production.

IX. TRANSIENT STUDIES

A. TRANSIENTS WITH CONSTANT COEFFICIENTS

Before considering the effects of inertial pressure, limited void space, etc., a series of analogue computations was performed in which these various modifying effects were neglected. The pressure was taken to be constant at one atmosphere, the expansion volume was not limited, and boiling was not considered in order that the implications of the kinetic equations with constant reactivity coefficients could be explored.

The results of a sample analogue computation are shown in Fig. 22. The reactivity step is 2 or 1.5 per cent, and the transient is typical of the third range listed in Table I. Gas production is the major limiting factor in the 8-megawatt first burst, for which the time is taken to be zero at a power of 0.01 watt.

Events subsequent to the first burst are shown in the same figure by use of an expanded vertical scale and a compressed time scale. The slow return of reactivity as radiolytic gas escapes from the solution gives rise to a second peak in power, this time only 150 kw with a much slower time scale.

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Fig. 22. Analogue Computation for \$2 Step ($\rho_0 = 0.015$) Increase in Reactivity

Because the energy coefficient for gas production is seven times larger than that for temperature, the energy and peak power in the first burst might be expected to be one-eighth as large as anticipated on the basis of temperature alone. Using the numerical values of the previous section, the energy required to halt a fast burst by changing the net reactivity from ρ_0 to $-\rho_0$ for temperature alone is $480 \rho_0$, or 7.2 megawatt seconds for a \$2 step ($\rho_0 = 0.015$). For combined temperature and gas, it is $60 \rho_0$ or 0.9 megawatt second.

Since the temperature must ultimately rise far enough to stop the reaction, the energy production which was halted by the gas effect on reactivity must resume. However, the effect of spreading out the time scale for this remaining energy production is to reduce the total energy, because for relatively slow reactivity introduction it is sufficient merely to reduce the net reactivity to zero. The end effect is that the total energy is roughly one half of what would be generated in a fast burst limited by temperature alone. This results in a release of $240 \rho_0$ or 3.6 megawatt seconds for a \$2 step.

The analogue computation of Fig. 22 shows an energy production of only 0.6 megawatt second for the first burst. That this is less than the 0.9 megawatt second predicted by the simple model is consistent with the fact that the first burst in Fig. 22 is over before the reactivity reaches $-\rho_0$.

Not shown is the steadily decreasing rate of energy production after 10 seconds, during which the power is seeking a final equilibrium value given by

$$n = - \rho_0 \left[(\alpha K / \gamma) + (\phi G / \sigma) \right]^{-1}, \quad \dots \quad (14)$$

which in this case is 52 kw. The energy production eventually exceeds the 3.6 megawatt seconds of the simple adiabatic model, and it is somewhat arbitrary to assign a unique total energy to this particular transient because of the manner in which the transient merges into steady operation.

For higher reactivity steps the time scale is shortened, and the assignment of a total energy production for a transient is more clearcut. This is because the transient events of interest are essentially over before transient heat transfer becomes significant, so that the energy vs time curves show a nearly asymptotic approach to the $240 \rho_0$ of the simple model long before transient heat transfer is significant.

This behavior is illustrated in Fig. 23, where the energies for a series of analogue computations are plotted as a function of the reactivity step ρ_0 . It is seen that for small reactivities the total energy varies as $480 \rho_0$, as predicted, and that for large reactivities the energy in the first burst and the final energy are respectively asymptotic to the straight lines $60 \rho_0$ and $240 \rho_0$. As pointed out previously, the assignment of a total energy in the transition range around $\rho_0 = 0.015$ is somewhat arbitrary because of the lack of a clearcut separation between transient and steady-state operation.

Fig. 23. Energy vs Reactivity Step from Analogue Computations

Included in Fig. 23 are the results of small reactivity transients conducted with the Livermore water-boiler reactor.⁸ These lie on a line of slope 50 per cent greater than the analogue results for small reactivity releases. In estimating potential hazards, the existence of this discrepancy (even though the two reactors are not identical) has been recognized by arbitrarily multiplying calculated energies by 1.5. Before any actual transient experiments are performed, more analytical work will be performed with the aid of a better representation for delayed neutrons and better values for the various coefficients. It must also be recalled that in Fig. 23 the effects of inertial pressure, limited void volume, etc., are not yet included.

Figure 24 shows peak power and final equilibrium power vs reactivity step, ρ_0 . The peak-power curves are taken from the analogue computer traces, and the equilibrium power is calculated from Eq. (14).

Also shown in Fig. 24 are observed power peaks for the Livermore reactor transients.⁸ These points appear to fall on a curve of similar shape which lies above the computed curve. This is to be expected since the use of only one group of delayed neutrons with the constants employed here predicts a significantly longer reactor period for small reactivities. Figure 25 shows the one-group inhour curve corresponding to the analogue computations, compared with the five-group inhour curve which will be used to determine the reactivity associated with the observed reactor period of each transient. This comparison in Fig. 25 shows also that the one-group formula (primarily because of the smaller value for prompt neutron generation time) yields a significant underestimate of the reactor period for large reactivities, so that the analogue computations tend to overestimate, rather than underestimate, the peak power in the range, except that inertial effects are not yet included.

Earlier computations (not reported here) were based on a model which uses a simple formula for peak power derived on the basis of no delayed neutrons:

$$n_{\max} = \rho_0^2 / 2\alpha K \tau. \quad \dots (15)$$

This is modified to include the effect of delayed neutrons by using an effective prompt generation time τ_e ; the inhour equation for no delayed neutrons

$$\tau_e = \tau / \rho_0$$

is used to rewrite Eq. (15) as

$$n_{\max} = \rho_0 / 2 \alpha K \tau_e. \quad \dots (16)$$

The value used for τ_e was then taken from the five-group inhour curve of Fig. 25.

Peak power vs ρ_0 obtained in this manner is found to be in excellent agreement with the Livermore data, whereas the replacement of αK by

Fig. 24. Power vs Reactivity Step from Analogue Computations

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Fig. 25. Water Boiler Inhour Curves

$\alpha K + \phi G$ in Eq. (16) yields results which lie below the analogue values for the larger reactivity transients where gas production contributes to shutting down the reactor.

In any case, as mentioned in connection with Fig. 24, the results of static measurements of the reactor constants will be used for more sophisticated calculations, so that any remaining inconsistencies with existing data may be removed.

B. EFFECT OF INERTIAL PRESSURE

This section summarizes the results of theoretical studies to date on the effect of inertial pressure. The presentation consists of: (1) derivation of the equation for transient pressure in a partially enclosed container of water, (2) development of the equation of state for a mixture of water and radiolytic gas, and (3) calculation of peak power as a function of reactor period. The results have served as a rough indication of the magnitude of inertial effects and as a guide for planning the experimental program, but it is recognized that the large uncertainties in this type of calculation necessitate careful exploration of the range of reactivities below the point at which inertial effects are predicted to become important.

The only previous investigation of this phenomenon appears to be some work by Hansen.¹² It is necessary to solve, subject to the proper boundary conditions, the equation of state

$$dP = \left(\frac{\partial P}{\partial T} \right)_V dT + \left(\frac{\partial P}{\partial V} \right)_T dV \quad \dots (17)$$

and the equation of motion

$$\rho \frac{\partial^2 \vec{S}}{\partial t^2} = -\nabla P, \quad \dots (18)$$

where pressure, temperature, volume change, and displacement $\vec{S}$ are functions of position and time. (Here the symbol ρ stands for mass density.) Using the definition of bulk modulus

$$B = -V \left(\frac{\partial P}{\partial V} \right)_T,$$

and assuming a linear equation of state, Eq. (17) becomes

$$P = T \left(\frac{\partial P}{\partial T} \right)_v - B \nabla \cdot \vec{S}, \quad \dots (19)$$

where

$$\nabla \cdot \vec{S} = \Delta V / V = - \Delta \rho / \rho. \quad \dots (20)$$

Neglecting the energy taken by gas production, heat transfer, and elastic deformation, the temperature is given by $dT/dt = Kn$, where the symbols have the same meaning as in Eq. (6). The simplest geometrical model which retains the essential characteristic of a partially free surface is that of a rigid cylinder with ends open to atmospheric pressure. It is not expected that the elasticity of the metal shell can contribute significantly to pressure relief.

Assuming exponential time dependence with reciprocal period ω , Eq. (18) becomes

$$\omega^2 \rho \vec{S} = - \nabla P.$$

Substituting for P from Eq. (19), and writing S_r and S_z for the radial and axial components of the displacement vector, yields in the absence of rotation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2} - \frac{\omega^2 \rho}{B} \right) S_r = \frac{1}{B} \left(\frac{\partial P}{\partial T} \right)_v \frac{\partial T}{\partial r} \quad \dots (21)$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{\omega^2 \rho}{B} \right) S_z = \frac{1}{B} \left(\frac{\partial P}{\partial T} \right)_v \frac{\partial T}{\partial z}$$

where the displacements and temperature are to be interpreted as coefficients multiplying the factor $\exp(\omega t)$.

Consider the temperature distribution

$$T = T_0 J_0(k_1 r) \cos k_2 z. \quad \dots (22)$$

The differential equations obtained by substituting Eq. (22) into Eq. (21) are satisfied by

$$\begin{aligned} S_r &= A \left[J_1(k_1 r) - \frac{J_1(k_1 a)}{I_1(\mu a)} I_1(\mu r) \right] \cos k_2 z \\ S_z &= A \left[\frac{k_2}{k_1} J_0(k_1 r) + \frac{k_2 J_1(k_1 a)}{\mu I_1(\mu a)} I_0(\mu r) \right] \sin k_2 z \end{aligned} \quad \dots (23)$$

where the radial displacement has been chosen to vanish at the cylindrical surface $r = a$, and where

$$A = \frac{k_1 T_o}{B} \left(\frac{\partial P}{\partial T} \right)_v / (k_1^2 + k_2^2 + \omega^2 \rho / B), \quad \dots (24)$$

$$\mu^2 = k_2^2 + \omega^2 \rho / B. \quad \dots (25)$$

The free surfaces are represented by the boundary condition $P(r, \pm b) = 0$, where b is the half-height of the cylinder. It is readily shown that the solution given by Eq. (23) cannot satisfy this condition unless T also vanishes on the free surface, which requires

$$k_2 = \pi / 2b. \quad \dots (26)$$

For any other value of k_2 the displacements must be infinite series whose coefficients are determined by expanding $T(r, \pm b)$ in a series of radial functions. The use of Eq. (23) and (26) is therefore a simplification whose effect is difficult to estimate, but it does seem reasonable that a non-vanishing driving temperature at the free surface would tend to relieve internal pressure more rapidly, leading to a less severe inertial effect.

The pressure is then calculated by substituting Eq. (23) and (24) into Eq. (19). The result is

$$P = P_o \left[J_0(k_1 r) + \frac{k_1 J_1(a)}{\mu I_1(\mu a)} I_0(\mu r) \right] \cos k_2 z, \quad \dots (27)$$

where

$$P_o = \frac{T_o (\partial P / \partial T)_v \omega^2 \rho / B}{k_1^2 + k_2^2 + \omega^2 \rho / B},$$

with k_2 and μ given by Eq. (25) and (26). In Eq. (27) the factor $\exp(\omega t)$ is now to be considered as contained in T_o .

It is interesting to obtain an estimate of the inertial effect in water alone. Calculating separately the two terms on the right-hand side of Eq. (19) and identifying them as a driving term and a pressure-relief term respectively, the result is that both terms rise exponentially in time with the driving term leading the relief term by a time lag

$$\Delta t = \left(\frac{\omega \rho}{B} \right) / \left(\frac{1}{k_1^2 + k_2^2} + \frac{k_1^2 / \mu^2}{k_1^2 + k_2^2 + \omega^2 \rho / B} \right) \dots (28)$$

For numerical calculations for water, $\rho = 1$ and $B = 20,000$ atm.

Corresponding to a two-liter void in a 30-cm diameter sphere, the radius $r = a$ of the free surface is 9.6 cm. Equating the volume of half the cylinder to the volume of water in the core vessel, $b = 48.6$ cm. Letting the radial power distribution extrapolate to zero at twice the cylinder radius yields $k_1 = 0.125 \text{ cm}^{-1}$, and, by equation (26), $k_2 = 0.0323 \text{ cm}^{-1}$. The resulting time lag is very nearly proportional to ω , ranging from 5 microseconds at a reactor period of 10 milliseconds to 50 microseconds at a period of 1 millisecond. The extreme smallness of this time lag indicates that inertial effects would be negligible for water itself, but the effect of gas production is to reduce greatly the bulk modulus so that the time lag may increase and allow an appreciable inertial pressure.

Since most of the fission energy goes into thermal energy, it is possible to derive an equation of state for the mixture of solution and gas bubbles in which a given temperature rise corresponds to the production of a certain amount of gas. For the liquid

$$dP = \left(\frac{\partial P}{\partial T} \right)_{V_w} dT + \left(\frac{\partial P}{\partial V_w} \right)_T dV_w, \dots (29)$$

where V_w is the liquid volume. If V_g is the gas volume, the total volume V_t is $V_w + V_g$ and

$$dV_w = dV_t - dV_g.$$

Hence

$$dP = \left(\frac{\partial P}{\partial T}\right)_{V_w} dT + \left(\frac{\partial P}{\partial V_w}\right)_T dV_t - \left(\frac{\partial P}{\partial V_w}\right)_T dV_g \dots (30)$$

Using the gas law,

$$V_g = NRT/P,$$

where the number of moles is

$$N = \Delta T / K\phi_g,$$

and where ϕ_g is the total fission energy corresponding to the production of one mole of radiolytic gas. Hence

$$V_g = RT\Delta T / K\phi_g P, \quad \dots (31)$$

from which dV_g may be determined and substituted into Eq. (30). The resulting equation may be solved for dP and the result compared directly with Eq. (29) to yield the effective values for the thermodynamic derivatives for the mixture of solution and gas:

$$\left(\frac{\partial P}{\partial T}\right)_{V_t} = \left[\left(\frac{\partial P}{\partial T}\right)_{V_w} + \frac{BR(T+\Delta T)}{KV_w\phi_g P} \right] / \left(1 + \frac{BRT\Delta T}{KV_w\phi_g P^2} \right), \quad \dots (32)$$

$$\left(\frac{\partial P}{\partial V_t}\right)_T = -B / \left[V_w \left(1 + \frac{BRT\Delta T}{KV_w\phi_g P^2} \right) \right], \quad \dots (33)$$

where $(\partial P / \partial T)_{V_w}$ and B are respectively the pressure coefficient and bulk modulus for the liquid alone.

Defining effective bulk modulus of the mixture as

$$B_{\text{eff}} = -V_t \left(\frac{\partial P}{\partial V_t} \right)_T,$$

and using Eq. (31), Eq. (33) becomes

$$B_{\text{eff}} = B \left(1 + \frac{RT\Delta T}{KV_w \phi_g P} \right) / \left(1 + \frac{BRT\Delta T}{KV_w \phi_g P^2} \right) \quad \dots (34)$$

Substituting numerical values into Eq. (32) and (34) yields

$$\left(\frac{\partial P}{\partial T} \right)_{V_t} = \left[\left(\frac{\partial P}{\partial T} \right)_{V_w} + \frac{340 P_s (T + \Delta T)}{T_s P} \right] / \left(1 + \frac{340 P_s T \Delta T}{T_s P^2} \right) \quad \dots (35)$$

$$B_{\text{eff}} = \left(B + \frac{340 P_s T \Delta T}{T_s P} \right) / \left(1 + \frac{340 P_s T \Delta T}{T_s P^2} \right), \quad \dots (36)$$

where the subscript *s* refers to standard conditions. The volume expansion coefficient is

$$\frac{1}{B_{\text{eff}}} \left(\frac{\partial P}{\partial T} \right)_{V_t} = \left[\left(\frac{\partial P}{\partial T} \right)_{V_w} + \frac{340 P_s (T + \Delta T)}{T_s P} \right] / \left(B + \frac{340 P_s T \Delta T}{T_s P} \right) \quad \dots (37)$$

Since the pressure coefficient for the liquid alone is 20 atm/°C, the effect of gas is to increase the pressure coefficient and volume expansion coefficient (at standard temperature and pressure) to eighteen times their value for water alone. Hence for small inertial pressures, the gas production is extremely important as a shutdown mechanism. Note that in the analogue computations the factor of eight for the total energy coefficient of reactivity relative to the effect of temperature rise is decidedly more pessimistic.

In principle, the evaluation of inertial pressure requires the solution of a partial differential equation containing pressure-dependent coefficients as given by Eq. (35) and (36). The procedure followed here is a stepwise calculation in time, using Eq. (27) to find the central pressure, with the

pressure coefficient and bulk modulus in each time interval determined by the time-average pressure and temperature within the interval.

This procedure allows construction of a family of curves of pressure vs power for various reactor periods $1/\omega$. These curves may then be used to modify the earlier analogue computations of peak power. For this purpose, it is assumed that the effect of inertial pressure is to reduce the reactivity coefficient ϕ for gas; hence for each period the peak power of the analogue computation is used to find the inertial pressure from the pressure vs power curve. Assuming that ϕ is proportional to the actual gas volume, and that the peak power is inversely proportional to the value of ϕ at peak power, this inertial pressure rise is used to calculate a new peak power. This procedure is iterated through the number of steps necessary to obtain convergence.

The resulting inertial pressure rise at peak power is plotted in Fig. 26 as a function of the period. It is seen that the pressure rise varies slightly more rapidly than the square of the reciprocal period.

Curves of peak power computed with and without consideration of inertial pressures are shown in Fig. 27. The relative increase in peak power $(n - n_0) / n_0$, where n and n_0 are peak powers with and without inertial pressures, are shown in Fig. 28. Note that in the important part of the range (periods from 5 to 10 milliseconds) the relative difference varies approximately as the reciprocal period squared.

Figure 29 shows the effect of inertial pressure on the energy production in the transients. The reference curves labelled "temperature only" and "constant pressure" are those of Fig. 23, but with a factor $3/2$ included. The effect of inertial pressure on the energy of the first burst is then obtained by assuming that the energy is augmented by the same factor as the peak power in Fig. 27. The augmentation of total energy is arbitrarily obtained by assuming that after the first burst the remaining energy production will be half of the difference between the energy of the first burst and the energy associated with a temperature-only burst.

An important consequence of the increased compressibility caused by gas production is a corresponding decrease in the speed of sound. In the course

Fig. 26. Pressure Rise at Peak Power
vs Reactor Period

Fig. 27. Effect of Inertial Pressure
on Peak Power

Fig. 28. Relative Increase in Peak Power Caused by Inertial Pressure

Fig. 29. Effect of Inertial Pressure on Energy Production

of the step-wise computations, the minimum sound speed in each transient may be noted and the time required for sound to traverse the core diameter calculated from it. Comparison of this time with the reactor period in each case shows that the time for sound traverse is consistently somewhat less than the period, but the two times are not significantly different in any of the cases considered.

Therefore, in order to estimate the likelihood of shock waves in the core solution, another approach is necessary. Forming S_z from Eq. (23), and using it to calculate the maximum rate of fluid displacement for the 4-milli-second transient, it is found that the maximum fluid displacement rate is ten times less than the minimum speed of sound in this transient. Hence the computation indicates that shock waves probably will not be encountered, but the range in which inertial pressure is significant must be explored very cautiously. In the extreme case cited, the calculated minimum sound speed is sixteen times less than that for water alone.

These computations are still restricted to the case of unlimited free volume above the solution. However, for the model used, the inertial pressure is overestimated because the power will drop below an exponential curve before peak power is attained.

C. EFFECT OF LIMITED EXPANSION VOLUME

If the available expansion volume is V_o , the pressure will begin to rise when the amount of gas produced equals V_o . Keeping the symbol V to mean gas volume at standard conditions as in Eq. (9), the pressure after $V = V_o$ will satisfy

$$1 + P = \frac{V}{V_o} \left(1 - \frac{F}{V_c} \right), \quad (38)$$

where P is gage pressure in atmospheres, F is the volume of mixed solution and gas that has been expelled via the relief tube, and V_c is the total internal volume of the water-boiler sphere.

The model chosen for use with the analogue computer is a linear differential equation for the flow rate $\dot{F}$ or $\dot{Q}$:

$$\dot{Q} = NP - \epsilon Q, \quad (39)$$

where ϵ is taken to be the reciprocal of the characteristic time for build-up from zero to steady flow in a two-inch diameter pipe two feet in length. This time is

$$\frac{1}{\epsilon} = \frac{L Q}{g h A} ,$$

where L = length, g = acceleration of gravity, h = pressure head, and A = cross-sectional area.

For a head of 2 atmospheres, this relation yields $\epsilon = 16.6 \text{ sec}^{-1}$. Since the equation is to be applied to a system which does not approach a steady flow but in which transient pressures are expected to exceed 2 atmospheres, this choice for ϵ underestimates pressure relief.

The acceleration term in Eq. (39) will be $AP/\rho L$, where ρ is the density, so that

$$N = A/\rho L.$$

The numerical value obtained is $N = 337 \text{ liter/atm-sec}^2$.

The effect on reactivity will be two-fold. In the first place, the reactivity coefficient ϕ is replaced by $\phi / (1 + P)$ to account for compression of gas bubbles. On the other hand, the expulsion of fuel via the relief tube contributes a new negative term in the expression for reactivity. Estimating a reactivity effect of 3 per cent per liter ejected, the reactivity equation which replaces Eq. (10) is

$$\rho = \rho_o + \alpha T + \frac{\phi V}{1 + P} - 0.03 F. \quad \dots (40)$$

The system is now characterized by Eq. (1), (2), (8), (9), (38), (39) and (40).

Figure 30 shows the manner in which the response to a \$2 step is modified by a limited void volume $V_o = 0.5$ liter. The peak power is 16 megawatts instead of 8 megawatts as in Fig. 22, and the peak pressure is 6.5 atmospheres.

Fig. 30. Analogue Computation for \$2 Step Increase in Reactivity ($\rho_o = 0.015$)

with 0.5 Liter Void Volume

Not shown is the reactivity curve, which tends to level off at the moment the sphere is filled, and later drops off rapidly toward more negative values as the flow rate increases.

For larger reactivity steps, where inertial pressures appear, an attempt was made to modify the original analogue computations before introducing the limited void volume. This was done by reducing the gas coefficient ϕ to correct for the calculated inertial pressure as shown in Fig. 26. The resulting analogue computations for unlimited void volume yielded slightly lower peak powers than those shown in Fig. 27.

The results of calculations to date for the effect of limited void volume are summarized in Table II. Inertial pressure is negligible for the \$2 step, and the appropriate inertial pressures (0.9 atmosphere for \$3 and 2.5 atmospheres for \$4) are added into the peak pressures tabulated. Not yet considered is the proper means of treatment for the manner in which the effect of limited void volume interacts with the inertial pressure by restricting pressure relief. This effect arises from the restriction of free surface as the solution level rises in a spherical container.

D. TRANSIENT BOILING

For a slow transient in which no superheating occurs, the effect of reaching the boiling point will be a sudden shutdown by steam formation. Analogue computations have been performed in which the temperature is held constant after the boiling point is reached, while the gas volume in Eq. (10) is replaced by the total volume of radiolytic gas and steam. The amount of steam produced by a given energy production is more than one hundred times the corresponding amount of radiolytic gas, and the effect is a shutdown almost as sudden as if a negative step change of reactivity were introduced. Subsequent heat transfer will eventually cause the power to rise, and if the initial temperature is near enough to the boiling point (depending upon the initial amount of reactivity introduced), the final equilibrium state can be a condition of steady boiling.

On the other hand, the presence of pressure increases and time lags in bubble formation prevent boiling contributing to shutdown during a fast burst, even if a temperature of 100° C is exceeded. The computations for inertial

TABLE II.

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EFFECT OF LIMITED VOID VOLUME V_o .

Reactivity Step ρ_o	V_o (liters)	Peak Power (Megawatts)	Peak Pressure (Atm.)
\$2 ($\rho_o = 0.015$)	0.5	16	6.5
	1.0	8.5	1.7
	1.5	8	0.35
	∞	8	0
\$3 ($\rho_o = 0.0225$)	1	80	66
	2	58	35
	3	40	21
	∞	38	0.9
\$4 ($\rho_o = 0.03$)	1	—	78
	2	390	18.5
	3	275	11.0
	∞	145	2.5

effects and limited void space show that while power is rising the pressure-temperature curve lies above the equilibrium-vaporization curve. Once the first rapid burst is over, boiling can effect those phenomena which occur on a longer time scale. Note that the curve of final equilibrium power plotted in Fig. 24 will not apply if the final temperature, $-\rho_o/\alpha$, which is calculated for no boiling, exceeds the boiling point. The value of ρ_o at which this occurs depends on the initial temperature.

X. OSCILLATIONS

A. DAMPED OSCILLATIONS

For small variations of power about an equilibrium power level n_o , Eq. (1) may be linearized by writing n_o for n in the non-linear term $\rho n/\tau$. The results may be combined with Eq. (2), (8), (9) and (10) to yield a linear fourth-order differential equation which is homogeneous in the variable $n-n_o$. Note that for free oscillations, $\rho_o = 0$ for $n=n_o$ to represent an equilibrium.

The solution of the linearized equation is a linear combination of four terms of the type $\exp(\omega t)$, the four values of ω being the four roots of the equation

$$\begin{aligned} \omega^4 + (\sigma + \gamma + \lambda + \beta/\tau)\omega^3 + [\sigma\gamma + (\sigma + \gamma)(\lambda + \beta/\tau) - (\alpha K + \phi G)n_o/\tau]\omega^2 \\ + [\sigma\gamma(\lambda + \beta/\tau) - (\sigma + \lambda)\alpha K n_o/\tau - (\gamma + \lambda)\phi G n_o/\tau]\omega \\ - \lambda(\alpha\sigma K + \phi\gamma G)n_o/\tau = 0. \end{aligned}$$

.... (41)

The magnitudes of the coefficients are such that the roots are well separated, and approximate solutions are readily obtained. Designating the four roots by $a+bi$, c , and d , the approximate solutions are:

$$\begin{aligned}
 -a &\simeq (1/2) \left[\gamma - (\alpha K n_o / \beta) - (\lambda \phi G n_o / \sigma \beta) \right], \\
 b &\simeq (-\lambda \alpha K n_o / \beta)^{1/2}, \quad \dots (42)
 \end{aligned}$$

$$c \simeq -\sigma,$$

$$d \simeq -\beta / \tau.$$

Here it has been assumed that n_o lies in the proper range for the existence of complex roots. It may be shown that for n_o either too large or small the system is overdamped.

A damped oscillation experiment was performed on the Livermore water boiler.⁸ With no coolant flowing, the reactor power was held constant at 500 watts by the steady withdrawal of an automatic control rod. When the rod reached its limiting position, the power decreased, undergoing a damped oscillation, and finally approached an equilibrium power of 65 watts. It was observed that when the oscillation was sufficiently damped to be described by a linear system, the damping factor was 1.5×10^{-4} per second and the angular frequency was 1.75×10^{-3} radians per second. These quantities are to be identified with $-a$ and b , respectively, of Eq. (42).

From the experiment, it is theoretically possible to determine simultaneously γ and T_s of Eq. (6) by observing (1) the cooling curve during the first power minimum at 5 watts and (2) the final equilibrium temperature. Unfortunately in this case the simultaneous solution failed because of the appearance of small differences between two large numbers, and it was necessary to assume that T_s was essentially room temperature, whence $\gamma \simeq 10^{-4}$.

Using this value for γ , regarding ϕ , G , and σ as unknown, and taking the other constants as given previously (except that α is changed to -2.6×10^{-4} per °C and K to $0.01428^\circ\text{C}/\text{kw-sec}$) yields the following results: First, calculating the angular frequency b in Eq. (42) gives the result $b = 1.60 \times 10^{-3}$ radians per second, in good agreement with the observed value. Second, taking the observed damping factor and solving the first equation of Eq. (42) for the power coefficient

of reactivity for radiolytic gas yields

$$\phi G / \sigma = -2.3 \times 10^{-4} \text{ per kw} \quad \dots (43)$$

This result is four times larger than that given in Eq. (12), and for the values of G and σ used previously, it corresponds to a gas coefficient of reactivity ϕ which is four times larger than that used in the transient calculations. Such a large coefficient is nevertheless consistent with Eq. (11).

If the power coefficient for gas is actually significantly smaller than that given by Eq. (43), as SUPO operating experience indicates it is, then the observed damping factor requires that α be several times 10^{-4} sec^{-1} . This in turn requires the effective temperature of the heat sink T_s to be about 84°F instead of room temperature.

It is apparent that an experimental determination of $\phi G / \sigma$ is critically dependent on the heat transfer, and several free-oscillation experiments will be performed with the KEWB reactor wherein an attempt will be made to establish the validity of the theoretical model used for representing heat transfer.

In the Livermore experiment discussed here, there seems to be no possibility of observing the reciprocal residence time σ for gas bubbles directly. The term in the expression for the reactor power which varies as $\exp(-\sigma t)$ is completely masked by the large non-linear oscillations at the start of the run. However, it seems likely that this term might be separated from the damped oscillations in an experiment where an oscillation is induced by an extremely small step input of reactivity with the reactor initially operating in the neighborhood of 100 watts, providing the reactivity could be introduced rapidly enough. Such an experiment can also yield directly information on the equilibrium power coefficients of reactivity by comparison of the initial and final equilibrium power levels.

B. FORCED OSCILLATIONS

If the reactivity is varied by external means such as a moving control rod, Eq. (10) is replaced by

$$\rho = \rho_1(t) + \alpha T + \phi V, \quad \dots (44)$$

where $\rho_1(t)$ is the externally controlled reactivity. The inhomogeneous term in the fourth-order linear differential equation in $n - n_0$ which has Eq. (41) as its characteristic equation is

$$(n_0/\tau)(D + \lambda)(D + \sigma)(D + \gamma)\rho_1(t), \quad \dots (45)$$

where D is the differential operator d/dt .

A more general scheme, valid for m groups of delayed neutrons, has been developed.¹³ The linearized differential equation, of order $m + 3$, may be written

$$P(D)(n - n_0) = (n_0/\tau) \left[\prod_{i=1}^{m+2} (D + \lambda_i) \right] \rho_1(t), \quad \dots (46)$$

where the polynomial $P(D)$ is given by

$$P(D) = \left[\prod_{i=1}^{m+2} (D + \lambda_i) \right] (D + \sum_{i=1}^m \beta_i / \tau) - \sum_{j=1}^{m+2} \left[\prod_{\substack{i=1 \\ i \neq j}}^{m+2} (D + \lambda_i) \right] e_j \lambda_j \dots (47)$$

Here advantage has been taken of the similarity in form among the equations for temperature, gas, and delayed neutrons by defining

$$e_i = \beta_i / \tau, \quad i = 1, \dots, m, \quad \dots (48)$$

$$e_{m+1} = n_0 \phi G / \sigma \tau, \quad \dots (49)$$

$$e_{m+2} = n_0 \alpha K / \gamma \tau. \quad \dots (50)$$

For $i = 1, \dots, m$, the λ_i are the usual delayed neutron decay constants, while

$$\lambda_{m+1} = \sigma,$$

$$\lambda_{m+2} = \gamma.$$

If the impressed reactivity is

$$\rho_1(t) = \rho_1 \exp(i\omega t), \quad \dots (51)$$

the deviation of power from equilibrium may be written as

$$n - n_0 = A \exp(i\omega t), \quad \dots (52)$$

where A is the complex amplitude, given by

$$A = n_0 \rho_1 / \tau (E_1 - iE_2) \quad \dots (53)$$

with

$$E_1 = \sum_{j=1}^m \frac{\beta_j}{\tau} - \sum_{j=1}^{m+2} \frac{e_j \lambda_j^2}{\omega^2 + \lambda_j^2}, \quad \dots (54)$$

$$E_2 = -\omega \left(1 + \sum_{j=1}^{m+2} \frac{e_j \lambda_j}{\omega^2 + \lambda_j^2} \right). \quad \dots (55)$$

Representing the complex amplitude as

$$A = |A| \exp(i\delta_1), \quad \dots (56)$$

the solution may be written as

$$n - n_0 = |A| \exp \left[i(\omega t + \delta_1) \right],$$

where

$$|A| = n_0 \rho_1 / \tau (E_1^2 + E_2^2)^{1/2}, \quad \dots (57)$$

$$\tan \delta_1 = E_2 / E_1. \quad \dots (58)$$

Solutions for temperature and radiolytic gas volume may also be obtained. These are

$$T = k(\omega^2 + \gamma^2)^{-1/2} |A| \exp \left[i(\omega t + \delta_2) \right],$$

$$\tan(\delta_2 - \delta_1) = -\omega / \gamma,$$

and

$$V = G(\omega^2 + \sigma^2)^{-1/2} |A| \exp \left[i(\omega t + \delta_3) \right],$$

$$\tan(\delta_3 - \delta_1) = \omega / \sigma.$$

Unfortunately, the attainable frequencies may not be sufficiently low to permit observation of the important range in phase shift between power and temperature, and the gas volume as a function of time is not observable.

Numerical computations have been performed for the power amplitude and phase shift as functions of the driving frequency ω in the case of one group of delayed neutrons. For these computations, $\alpha = -2.6 \times 10^{-4} / ^\circ\text{C}$, $K = 0.01428^\circ\text{C/kw-sec}$ and $\gamma = 10^{-2}$ (representative of coolant flow). Two values of the gas coefficient of reactivity ϕ were used in order to illustrate the effect of the possible extreme values for this coefficient. These values are $\phi = -7 \times 10^{-6}$ per cm^3 and $\phi = -3.5 \times 10^{-5}$ per cm^3 . The other constants are as used previously.

Figures 31 and 32 show respectively the frequency dependence of $|A|/n_0\rho_1$ and the phase shift δ_1 for various equilibrium powers with $\phi = -7 \times 10^{-6}$ per cm^3 , while Fig. 33 and 34 are the same except that $\phi = -3.5 \times 10^{-5}$. These curves show features of coupled linear networks with the coupling increasing with increasing equilibrium power as shown by the double peaks in the phase shift for higher powers. It is also evident that the larger gas coefficient of reactivity emphasizes the second peak in the phase-shift curve, while for the smaller gas coefficient the first peak is dominant.

For high frequency the amplitude curve is asymptotic to a line of slope minus one which extrapolates to $\omega = 1/\tau$ at $\log |A| = 0$, yielding a determination of the prompt neutron generation time. The plateau observed for angular frequencies in the range 1 to 10 sec^{-1} has a height $1/\beta$, while in the middle range of frequencies (for low power) another line of slope minus one is seen which extrapolates to $\omega = \lambda/\beta$ at $\log |A| = 0$. For the case of several groups of delayed neutrons, there will be several breaks in the curve, each associated with a straight line extrapolating to $\omega = \lambda_i/\beta_i$. The plateaus for extremely low frequency are given by the relation

$$|A| = -\rho_1 \left[\alpha k/\gamma + \phi G/\sigma \right]^{-1},$$

which has essentially the same significance as Eq. (14).

Phase shift curves are conventionally plotted as phase angle vs $\log \omega$, as in Fig.32 and 34. However, a log-log plot of $-\tan \delta_1$ reveals some additional regularities, notably a straight line of slope unity for large frequency which has intercept $\omega = \beta/\tau$ at $\log(-\tan \delta_1) = 0$, and a straight line of slope minus one in the middle range of frequencies (for low power) which has intercept $\omega = \lambda$. These points are noted for $\delta_1 = -45$ degrees in Fig.32 and 34.

Figures 31 through 34 cover a range of frequencies which is unattainable for practical reasons. For very low frequencies, statistical fluctuations and

Fig. 31. Power Amplitude Response vs Driving Frequency for $\phi = -7 \times 10^{-6}$ per cm^3 .

Fig. 32. Power Phase Shift vs Driving Frequency for $\phi = -7 \times 10^{-6}$ per cm^3 .

Fig. 33. Power Amplitude Response vs Driving Frequency for $\phi = -3.5 \times 10^{-5}$ per cm^3 .

Fig. 34. Power Phase Shift vs Driving Frequency for $\phi = -3.5 \times 10^{-5}$ per cm^3 .

prohibitively large periods (1.7 hours for 10^{-3} radians per second) are limiting. For high frequencies, the limits are inertia of the mechanical system and the finite time for neutron diffusion from the core to the detector. With the system now under consideration, the upper limit appears to be about 10^3 radians per second, which, in view of a prompt neutron generation time of the order of 10^{-4} second, may be in the range where the neutron diffusion time begins to be significant.

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