TECHNICAL PROGRESS REPORT

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on

Potential Well Formation in Electrostatic Confinement Devices

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Division of
Controlled Thermonuclear Research

by

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ABSTRACT

The goal of this program is to evaluate the feasibility of using inertial electrostatic forces to confine energetic plasmas capable of sustaining fusion reactions. We have shown experimentally that electron injection into a spherical system under high vacuum conditions produces a deep negative potential well. Our recent theoretical analysis of this system gives excellent agreement with the experimental results. Of much more importance is our observation of a stable multiple potential well structure (ion rich region in the center) when electrons are injected into a low pressure deuterium gas background. The extension of the theoretical analysis to explain these results do give qualitative agreement with our experimental results.

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A. Theoretical Studies

Due to the sharply curtailed funding of this project, our most recent effects have concentrated on finding a theoretical basis for understanding our previous experimental observations. The extension of the Lavrentyev-Dolan analysis has allowed us to formulate a theory that successfully explains the potential well formation in spherical geometry when electrons are injected under high vacuum conditions. This analysis¹ gives excellent agreement with our previous experimental results². In particular, our analysis was performed for a Gaussian distribution in total and angular energies. This was one of the topics proposed in our contract renewal. A detailed summary of our results, both experimental and theoretical, are contained in the two papers attached as appendices.^{3,4} These papers were presented at the New York Academy of Sciences Conference on Electrostatic and Electromagnetic Confinement of Plasmas and the Phenomenology of Relativistic Electron Beams, March 4-7, 1974.

In continuing our theoretical analysis, we have extended our work to the consideration of the double potential structure formed when electrons are injected into a low pressure deuterium gas background. In our initial analysis of potential well formation when electrons and ions are present⁴ we assumed narrow spreads in total and angular energies and we obtained a multiple potential solution, although not a unique solution. Our latest studies using a narrow spread in electron energies and an angular spread in electron energies of about 5% (consistent with our experimental results) and using a narrow spread in ion angular energies but a broad spread in ion total energy (consistent

with a background ionization source of ions) we found unique double potential well solutions which seem to be reasonably close to those suggested by the experiment. Our present work involves more detailed study of these cases.

Experimental Work

The sharp curtailment of funds available for this project virtually eliminated any possibility of pursuing our planned experimental program. We had hoped to examine the potential wells formed when ion injection was used. Instead we pursued some small additional studies dealing with electron injection and which did not require additional funds for equipment as modification of our apparatus. We are presently undertaking minor refinements of the apparatus so that we can examine the dynamics of potential well formation and decay. This will give us some much needed insight into the ion production and loss mechanisms in our apparatus.

REFERENCES

1. D. A. Swanson, B. E. Cherrington, J. T. Verdeyen, "Theory of the Negative Electrostatic Well in Spherical Geometry" to be published in Physics of Fluids.
2. D. A. Swanson, B. E. Cherrington, J. T. Verdeyen, "Potential Well Structure in an Inertial Electrostatic Plasma Confinement Device", Physics of Fluids, 16, 1939 (1973).
3. J. T. Verdeyen, B. E. Cherrington, D. A. Swanson, and D. J. Meeker, "Recent Developments in Electrostatic Confinement-Experimental", presented at New York Academy of Sciences Conference on Electrostatic and Electromagnetic Confinement of Plasmas and the Phenomenology of Relativistic Electron Beams, March 4-7, 1974. To be published in the Annals of the New York Academy of Science.
4. B. E. Cherrington, J. T. Verdeyen, D. A. Swanson, "Recent Developments in Electrostatic Confinement-Theoretical". Presented at New York Academy of Sciences Conference on Electrostatic and Electromagnetic Confinement of Plasmas and the Phenomenology of Relativistic Electron Beams, March 4-7, 1974. To be published in the Annals of the New York Academy of Science.

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LIST OF PUBLICATIONS

<u>TITLE</u>	<u>AUTHORS</u>	<u>JOURNAL/STATUS</u>	<u>PERCENTAGE AEC SUPPORT</u>
Measurement of Electron Density in a Cylindrical Inertial Electrostatic Plasma Confinement Device	D. J. Meeker J. T. Verdeyen B. E. Cherrington	Journal of Applied Physics, <u>44</u> , 5347 (1973)	0
Multiple Potential Well Structure Created by Electron Injection in Spherical Geometry	D. A. Swanson B. E. Cherrington J. T. Verdeyen	Applied Physics Letter, <u>23</u> , 125 (1973)	0
Potential Well Structure in an Inertial Electrostatic Plasma Confinement Device	D. A. Swanson B. E. Cherrington J. T. Verdeyen	Physics Fluids, <u>16</u> , 1939 (1973)	50
Theory of the Negative Electrostatic Well in Spherical Geometry	D. A. Swanson B. E. Cherrington J. T. Verdeyen	To be published in Physics Fluids	100
Recent Developments in Electrostatic Confinement- Experimental	J. T. Verdeyen B. E. Cherrington D. A. Swanson D. J. Meeker	To be published in Annals of New York Academy of Science	100
Recent Developments in Electrostatic Confinement- Theoretical	B. E. Cherrington J. T. Verdeyen D. A. Swanson	To be published in Annals of New York Academy of Science	100

RECENT DEVELOPMENTS IN ELECTROSTATIC CONFINEMENT - EXPERIMENTAL*

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I. Introduction

The motivation for this work on electrostatic plasma confinement was provided by the experimental results published by Hirsch.¹ In order to put our work into proper perspective, let us review some of the striking features of his work. The experimental system was based on the earlier ideas of Farnsworth² and consisted of six energetic ion beams, arranged symmetrically around a grounded permeable sphere and focussed to converge at the center. When the ion beams were accelerated through potentials of 50 to 150 kV, copious quantities of neutrons ($10^6 - 5 \times 10^9$ /sec) were emitted from the device and the neutron emission depended inversely on the pressure of the deuterium or D-T mixtures utilized (0.1 to 8 microns). Some of the more prosaic explanations for the neutron production can be dismissed on the basis of the experiments. The fact that the neutron production increased with decreasing pressure eliminates the beam-gas interaction theory. The decrease in performance when the beams were purposely misaligned runs counter to the suggestions that the origin of neutrons might be a beam-solid target effect. Various experiments were performed to ascertain the origin of these neutrons, in

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particular to see if they resulted from fusion in the center of the device and led to the formation of a simple model for the device's operation.

This model was based on the solution to Poisson's equation for the bipolar diode with ions being created at the grounded anode and accelerated through a permeable, electron emitting cathode held at a negative potential. Solutions to this equation for monoenergetic particles with perfect radial aiming, leads to the prediction of alternating layers of positive and negative space charge with large electrostatic pressure appearing in the center of the device. Such regions could confine the energetic ions for sufficient time for fusion reactions to take place as was observed experimentally.

In spite of the severity of the assumption of perfect radial focussing and monoenergetic particles, this model predicted many features which were consistent with the observation in these early experiments. For instance, multiple peaks of gamma emission were observed from the device suggestive of bremsstrahlung being emitted by electrons passing through the multiple virtual anodes.

Our experiments at the University of Illinois have been designed to examine the potential structure and charge densities within the interior of a similar device so as to ascertain whether such alternating layers of space charge can exist. In spite of the obvious limitation of the simple model proposed by Hirsch, it

appears to explain many of the features found in his experiments, and serves as a useful guide for ours. Deep potential wells (i.e., 20 to 80% of the applied potential) have been measured with single species (electron) injection, with multiple wells being formed in the presence of background gas. Furthermore, the background plasma density shows an oscillatory behavior in space. Both results are suggested by the simple theory and partially confirmed by the more exact (but complex) theories of Larentyev,³ Dolan,⁴ and Swanson,^{5,6} and Imel.⁷ The following sections will describe the methods we have used to analyze such plasmas in an electrostatic environment.

Although this version of electrostatic confinement was advanced with the goal of a fusion reactor,² - and it might achieve that aim - the results showing the existence of stable potential wells capable of trapping particles of opposite charge are applicable to other fusion schemes.

II. Electron-Beam Probing of Potential Wells

A. Electron injection into a hard Vacuum.

The apparatus used to study the potential wells that might be formed in an electrostatic containment device is shown in Fig. 1. Electrons are injected into the interior of the device through the slot in the grounded spherical shell by pulsing the hot cathode negative by 100-300 volts. These electrons are attracted by a highly

transparent (93% open) anode held at a positive potential. Even though the initial source of electrons is decidedly non-uniform, experimental evidence - to be discussed later - allows one to consider the injection to be spherically symmetric.

An electron beam from an oscilloscope gun is used to probe the interior of the device. Sinusoidal voltages are applied to the deflection plates leading to a circular Lissajous pattern on the screen on the far side. The amplitude of these voltages controls the apex angle of the cone swept out by the probing beam.

Obviously, the attractive positive potential of the anode alone will tend to contract the circle, whereas any negative charge on the interior of this cone will scatter or expand the circle. Finally, if there were to be both positive and negative space charge layers as suggested by Hirsch's early model, the change in trajectory will depend upon the net charge contained within the cone of the probing beam.

The energy of the probing beam and the deflection plate voltages are independent parameters of the experiment. By varying these quantities, one can test the consistency of an assumed potential variation on the interior of the device. Although it is always possible to pick a potential function to explain scattering data at one energy, a close approximation to the true function is necessary to match data for a 50:1 variation on the beam parameter.

This procedure is easier and in some cases more accurate than attempting an inversion technique similar to the Abel⁸ process.

We have, therefore, used the beam probing data to check the consistency of our assumed potentials.

The success of this approach is illustrated in Fig. 2 where the focussing properties of the anode alone are presented. The photographic insert indicates how the graphical data was taken. One compares the diameter of the Lissajous "circle" for various values of the ratio V_A/V_G leading to the graph shown. Incidentally, this comparison, $D(V_A/V_G)/D(0)$, is independent of deflection plate voltages (i.e., the initial diameter) and is a unique function of the ratio V_A/V_G . Consequently the probing beam can be treated as a "paraxial ray" obeying the linear differential equation:⁹

$$\frac{d^2 r}{dz^2} + \frac{V'_0(z)}{2V_0(z)} \frac{dr}{dz} + \frac{V''_0(z)}{4V_0(z)} r = 0 \quad (1)$$

where $V_0(z)$ is the potential variation along the axis of the probing beam with the primes indicating the derivative with respect to z , and r is the distance from the axis. Equation 1, of course, is exactly equivalent to the equations of motion in a cylindrically symmetric system provided $(dr/dz)^2$ is small compared to 1.

The fact that the equation is linear and of second order allows us to use the matrix formulation in our data interpretation that has proved so successful with optical beams.¹⁰

$$\begin{bmatrix} r_{\text{out}} \\ r'_{\text{out}} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_{\text{INITIAL}} \\ r'_{\text{initial}} \end{bmatrix} \quad (2)$$

with the initial position being zero and the initial slope given by $D(0)/2$ divided by the distance from the gun to screen.

Of course the V_0 in Eq. (1) is for zero space charge. When the electrons are injected into the device, this condition is not true. However, if one assumes that the potential function in the presence of space charge can be expressed as a two-term Taylor series, viz.

$$V(r,z) = V_0(z) + \frac{r^2}{2} V_2(z) \quad (3)$$

then Eq. (1) becomes for a paraxial electron;

$$\frac{\partial^2 r}{\partial z^2} + \frac{V_0'(z)}{2V_0(z)} \frac{\partial r}{\partial z} - \frac{V_2(z)}{2a^2 V_0} r = 0 \quad (4)$$

Again, we have a linear second order differential equation permitting the matrix formulation of Eq. (2).

The linearity implied by Eq. (4) is observed with electron injection into the central region. One intuitively expects, and one

observes the probing beam being scattered by the accumulated charge. But, again, the ratio $D(V_A/V_G)/D(0)$ is independent of $D(0)$ and is a function of the ratio of V_A/V_G and the injected current.

Experimental data illustrating the scattering by the accumulated space charge are shown in Fig. 3. Again, we plot the diameter of the screen pattern normalized to the initial diameter (i.e., with $V_a = I_a = 0$) as a function of the focussing parameter V_A/V_G for a constant current of 60 mA. Figure 3 also shows the theoretical ratio if one uses the Langmuir model¹¹ of injected electrons with perfect radial focussing with zero angular momentum. Obviously, the experimental data deviates considerably from Langmuir's prediction.

The fact that the normalized deflection is independent of the initial trajectory of the probing beam - over the range of experimental parameters available to us - shows that the probing ray obeys a linear differential equation such as Eq. (4). If one assumes spherical symmetry (and Fig. 5 verifies this), then it is reasonable to use a parabolic potential well as a model for the analysis in the interior of the anode. The depth of the parabolic well is characterized by the parameter, f , as being a fraction of anode voltage (i.e., the well is $f \cdot V_a$ deep).

The theoretical predictions based on this model agree reasonably well with experimental data as shown by the dashed curves of Fig. 3. The more exact calculations, to be discussed in the following paper, justify this assumption and show that it is a consequence of the angular momentum of the injected electrons.

From data such as this, one can estimate the functional dependence of the well depth on anode voltage and injected current. For instance, the fraction $f = 0.64$ agrees reasonably well with the experimental points for $V_a = 500$ volts. Thus, the absolute depth is 0.64×500 or 320 volts. Similarly, the case of $V_a = 1000$ volts indicates $f = 0.36$ or a depth of 360 volts. The close coincidence of these two values suggests that the well depth is only a function of the injected current and is rather insensitive to anode voltage.

This is shown in Fig. 4 which was constructed from graphs of data such as Fig. 3. The variation of well depth with injected current is also indicated here. The more exact theoretical model justifies the nearly linear dependence of well depth on current - for low values of f .

B. Bipolar injection

The ideal test of the simple model of electrostatic confinement would utilize ion and electron injection. Unfortunately, this is not easily achieved in an experiment, and we have relied on background ionization of gas atoms by the injected electrons to provide ions in the system. Whereas the system was continuously pumped for the experiments in IIA, ($p \approx 2 \times 10^{-8}$ TORR), here a small amount of deuterium is admitted to the chamber of Fig. 1. The pressure was not allowed to rise to values at which it affected the optics of probing beams, however.

Figure 5 shows photographs of the effect that one observes as the pressure is increased. Here, we show 8 arcs of a circle corresponding to the times at which the probing gun is unblanked during one revolution (counter-clockwise) of the circular Lissajou pattern (16.6 msec). During each time the gun is unblanked, electrons are injected for 300 usec. Thus, the initial and final portions of the arcs represent the beam position with no injected current, and the central portion is that with injected current. Other experiments have demonstrated that this effect is a steady state phenomena, the use of pulsed operation is merely to prevent electrode heating and destruction.

Notice that in the hard vacuum case, Fig. 5(a), the probing beam is scattered in the manner discussed previously. As the pressure is increased as in Fig. 5(b), the beam appears to "sag" back toward its initial trajectory - almost as if the ions are neutralizing the negative space charge. If this were true uniformly within the interior of the anode, the beam should return to its initial position with $I = 0$.

But, Fig. 5(c) shows that the beam is attracted toward the axis for the higher pressure case (i.e., higher ion generation rate). This experiment strongly suggests a region of positive space charge within the negative well created by the electrons.

There are two possible space charge distributions which would explain Fig. 5; a weak positively charged interior (of the anode)

corresponding to a high loss rate of the electrons from plasma interior, or a shell structure (like an onion) with the net positive charged inner region trapped by the surrounding negative space charge.

Figure 6 demonstrates that the latter viewpoint is the correct one. Here, the cone angle of the probing beam is varied. When the beam trajectory passes close to the center, Fig. 6(a), it is attracted towards the axis; when it misses the central region, Fig. 6(c), it sees only a net negative space charge. This physical interpretation is strikingly similar to the predictions of Hirsch's model.

Although the existence of the double well is clearly established, the analysis of this case is not nearly as advanced as it is on the negative well of Section IIA. This is due to the fact that experimental uncertainties about the shape of the negative well compound the difficulty, making data reduction extremely tedious and prone to error. Nevertheless, one can make reasonable assumptions, coupled with experimental data to obtain approximate values of the positive charge density.

The negative well formed in the hard vacuum case, shown in Fig. 5, corresponded to a 30% depression in the potential within the anode with 20 mA injected current. Since the anode voltage was 2500 volts, the absolute well depth is 750 V and the naked electron density is $1.6 \times 10^8 \text{ cm}^{-3}$. The positive ion density in the high pressure case must exceed this value over a limited range of the central region.

If we assume that the net positive charge is also uniform over a limited region, say $r < R_1$, one can attempt to match the theoretical prediction of the deflection for various values of R_1 to the experiment. This procedure allowed us to estimate R_1 to be ~ 1.0 cm (compared to 3.2 cm radius for the anode). Thus the local ion density would be $3.2 \times 10^8 \text{ cm}^{-3}$ for $r < 1$ cm with a potential rise from the edge of the well of 49.6 volts or a rise of 125 volts from the bottom of the well.

At first glance, there is an inconsistency in this calculation - the center is some 625 volts below the anode potential and the ions could never escape over the potential barrier. If one assumes that the dominant ion is D_2^+ and uses a resonant charge transfer cross-section of 10^{-14} to 10^{-15} cm^2 , and picks an average energy of the trapped ions to be 13 volts (consistent with the inferred potential distribution), then the ion lifetime is 14 to 140 $\mu\text{sec.}$ for the pressure used here (8×10^{-5} TORR). Considering the approximations involved, this is amazingly close to the time measured by Swanson¹² for the formulation of equilibrium situation. Hence we conclude that the picture of a stable, positively charged, interior is consistent with known rate processes.

III. Measurement of the Charge Density Distribution

Although the electron beam measurements have established that stable, multiple potential wells can exist, one can only infer

the difference between the ion and electron densities from the potential variation. In order to obtain these desired quantities directly, a cylindrical system was utilized in conjunction with a microwave Fabry-Perot^{13,14} cavity (at 35 and 70 GHz) or a laser heterodyne system¹⁵ operating at 3.39μ . The cylindrical system is convenient for these types of measurements since both depend upon the integrated density along the axis of measurement. If end effects are ignored, the cylindrical system affords the most straightforward interpretation of data.

An end view of the 16 cm long system used is shown in Fig. 7. This system can be used with electron or ion injection into the central region; only the latter is discussed here. The electrons emitted by the hot cathode are attracted by the accelerating grid and repelled by the center electrode held at a constant negative potential. The ions formed in the space between the grids are then attracted towards the central region through the highly transparent (92% open) electrode.

Figure 8 shows the electron density as measured with a microwave Fabry-Perot cavity in the central region as a function of the injected ion current with the center electrode voltage held constant. These measurements are a spatial average with the local density weighted by the square of the microwave field.¹⁶ In other words, no attempt is made at this point to infer spatial structure to the density.

It is important to recognize that the electron density is the measured quantity even though ions are the injected particles.

Consequently the non-linear increase of equilibrium electron density with injected ion current shown in Fig. 7 is a strong indication of trapping of the electrons formed in the central region by the ionic space charge, similar to the reverse situation discussed earlier in Section IIB.

Spatially resolved measurements of the electron density taken with the laser heterodyne system are shown in Fig. 9. Due to the very small size of the laser beam (~ 1 mm diameter), the averaging process is only applicable for the axial coordinate. Even though there is a small amount of dissymmetry to the measurements, most of the spatial variations are far in excess of the errors involved. Consequently, this definitely shows a spatially oscillatory form of the plasma - again similar to that suggested by Hirsch's simple model.

It should be clear, however, that these densities are associated with an inhomogeneous plasma and are not the "naked" charge densities. A simple application of Gauss' Law indicates that the uncovered charge density can be at most $5 \times 10^8 \text{ cm}^{-3}$ for the potentials used here. Since the densities are on the order of 10^{11} to 10^{12} cm^{-3} , this must represent the nearly compensated charge density of the background plasma.

IV. Summary and Conclusions

There is not a complete one-to-one correspondence between this work and the earlier experiments. Hirsch used guns for the ion

source and depended upon electrons emitted by ion bombardment of the center electrode for the negative charge. This is an excellent (but difficult) approximation to the simple model since the charged particles originate at a well defined location.

In the experiments described here, the use of gas ionization to generate the ions makes the identification of the energy and angular momentum of the particle difficult. Of course, the potentials used here are orders of magnitude lower than those used by Hirsch. Nevertheless, many of the ideas suggested in this early work are observed in these experiments.

Although electrostatic confinement was initiated as alternative to magnetic confinement for fusion purposes, it should be clear that any system permitting the escape (or injection) of a single type of charged species will have the space charge fields and resulting influence on the plasma as have been observed here.

It is not possible to estimate the viability of this approach to fusion without some assumption of a model. Hirsch did this using the general ideas of the simple monenergetic model and concluded that one could conceive of small power reactor if the internal circulating current between virtual electrodes was much larger ($\sim 10^4$) than the external current. Elmore, Tuck, and Watson¹⁷ examined the case of a central core of ions confined by the injection of an electron current. They concluded that it would require excessive external power input and would probably be unstable. They did not, however, allow for the multiple well structure as has been observed here.

Although r-f oscillations (in the 10 to 500 MHz range) appear on the anode in our experiments with electron injection, the presence (or absence) does not affect the gross behavior of the central region. Indeed, it appears as if those oscillations are transit time^{18,19} phenomena of electrons interacting with the anode circuit.

To extrapolate these experimental results to regions of fusion interest requires considerable faith in the slope of data taken at 1-3 kV remaining constant to 300 kV! Obviously, more data is needed.

References

1. Hirsch, R.L., J. Appl. Phys. 38, 4522, (1967). See also Phys. Fluids 11, 2486 (1969).
2. Farnsworth, P.T. , U. S. Patent No. 3,258,402 issued June 28, 1966.
3. Lavrentyev, O.A., Magnitnye Lovushki Vypusk 3 (Nankova Dumka, Kiev, U.S.S.R. 1968) p. 77-147 [AEC report No. AEC-tr-7002, 1970].
4. Dolan, T.J., J.T. Verdeyen, B.E. Cherrington, and D.J. Meeker, J. Appl. Phys. 43, 1950 (1972).
5. Swanson, D.A., B.E. Cherrington, J.T. Verdeyen, Phys. Fluids. (To be published).
6. Swanson, D.A., B.E. Cherrington, J.T. Verdeyen, Appl. Phys. Let. 23, 301, (1973).
7. Imel, G., M.S. Thesis, Pennsylvania State University, (1973).
8. Bockasten, K., J. Opt. Society 51, 943, (1961). (See Also W.M. Black, Ph.D. thesis, Department of Electrical Engineering, Pennsylvania State University, (1971).
9. Spangenberg, K., "Vacuum Tubes," McGraw-Hill, p. 350. New York, New York, (1948).
10. Kogelnik, H., and T. Li, Appl. Optics 5, 1550, (1966).
11. Langmuir, I., and K.R. Blodgett, Phys. Rev. 24, 49, (1924).
12. Swanson, D.A., B.E. Cherrington, J.T. Verdeyen, Phys. Fluids 16, 1939, (1973).

Reference - 2 -

13. Meeker, D.J., J.T. Verdeyen, B.E. Cherrington, J. Appl. Phys. 44, 5347, (1973).
14. Chaffin, R.J. and J.B. Beyer, IEEE Trans. Microwave Theory Tech. MTT-16, 37, (1968).
15. Liu, C.S., J.T. Verdeyen, B.E. Cherrington, J. Appl. Phys. 40, 201, (1969).
16. Brown, S.C., "Basic Data of Plasma Physics." (Wiley, New York, 1959), Chaps. 1 and 4.
17. Elmore, W.C., J.L. Tuck, and K.M. Watson, Phys. Fluids 2, 239, (1959).
18. Hirsch, R.L., Phys. Fluids 11, 2486, (1969).
19. Klevans, E., (Private Communication).

Figure Captions

- Fig. 1. Schematic diagram of the spherical device used for electron beam probing. The radius of the inner transparent anode is 3.18 cm.; the radius of the grounded sphere is 10 cm.
- Fig. 2. Electrostatic focussing properties of the anode. The photographic insert illustrates the method of taking the data present in the graph. The ellipticity shown here is due to the unequal distances from the horizontal and vertical deflection plates to the anode.
- Fig. 3. Normalized experimental and theoretical deflections vs. normalized anode voltage for a constant injected current.
- Fig. 4. Absolute well depths ($f \times V_A$) under the conditions of constant current and constant voltage.
- Fig. 5. A photograph of the deflection patterns seen at various background pressures.
- Fig. 6. Change in the deflection pattern as the initial slope of the probing beam is changed. The arc marked B corresponds to the deflection when the current is injected for 300 μ sec.
- Fig. 7. End view of the cylindrical device, the length of the outer cylinder is 20 cm, with a 16 cm inner electrode.
- Fig. 8. Electron density in the cylindrical device as measured with the microwave Fabry-Perot cavity.
- Fig. 9. Spatially resolved electron density on the interior of the cylindrical device as determined with a laser heterodyne system.

FIG. 5. PERDEYEN, CARRAGHAN

(a) $p = 2 \times 10^{-7}$

(b) $p = 3 \times 10^{-6}$

(c) $p = 1.5 \times 10^{-5}$

HYPOTHETICAL ELECTRON BEAM TRAJECTORIES

ELECTRON BEAM PATTERNS

FIG. 7. VERDEYEN, CHEERINGTON
SUMMIT, MICHIGAN

FIG. 9. VERDEYEN, CHERINGTON

RECENT DEVELOPMENTS IN ELECTROSTATIC CONFINEMENT - THEORETICAL *

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Introduction

The early interest in inertial-electrostatic plasma confinement as a possible technique for confining energetic ions for fusion purposes arose, in part, from the theoretical prediction, based on Poisson's equation, of the existence of alternating potential wells inside a spherical electrode into which were injected positive and negative charge species.¹ This suggested that sufficiently intense electrostatic fields could be generated near the center of such devices so that extremely energetic ions could be contained, and fusion could possibly result. Not only is that original electrostatic confinement concept still of interest, the idea that injected charged beams can give rise to strong electrostatic fields capable of trapping charged particles is also of broad interest in magnetic or electromagnetic confinement systems, particularly those employing neutral or ion beam injection.

In order to trace the development of the theory, let us summarize the approach. Poisson's equation along with the equations of conservation of energy and continuity of current are simultaneously solved for a spherical system where the electrode voltages and

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currents are considered to be known. The solution for injection of a single charged species dates back to the studies of Langmuir.² In his study, it was assumed that the electrons started at a grounded cathode with no initial velocity and suffered no collisions in the system, therefore their radial velocity v_e can be expressed in terms of the potential at r , $V(r)$ as

$$v_e = \sqrt{\frac{2e V(r)}{m}} \quad (1)$$

Since the system is spherically symmetric, the current can be expressed as

$$I_e = 4\pi r^2 \rho_e v_e \quad (2)$$

where ρ_e is the electronic charge density and Poisson's equation can then be expressed as

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = \left[\frac{I}{4\pi \epsilon_0 \sqrt{2e/m}} \right] \frac{1}{r^2 \sqrt{V(r)}} \quad (3)$$

By normalizing the radius to the electrode (anode) radius, r_a , and the potential to the anode potential V_a ,

$$X = r/r_a \text{ and } Y = V(r)/V_a \quad (4)$$

and by defining

$$K_e = \frac{I_e}{V_a^{3/2}} \cdot \left(\frac{m}{2e}\right)^{1/2} \frac{1}{4\pi \epsilon_0} \quad (5)$$

which is proportional to the perveance. Poisson's equation can be expressed simply as

$$\frac{1}{x^2} \frac{\partial}{\partial x} x^2 \frac{\partial Y}{\partial x} = \frac{K_e}{x^2} Y^{-1/2} \quad (6)$$

Langmuir solved this equation under the condition of a space charge limited current and found a potential distribution as shown in Fig. 1. Obviously electron injection results in the formation of a virtual cathode under Langmuir's assumption. Unfortunately, Langmuir's theory predicts the existence of a virtual cathode no matter how small the current. This is at variance with our experimental results³ as well as those of Hirsch.⁴ Before we return to the modification of the theory, let's consider the extension of Langmuir's theory to two charge species, as done by Hirsch.¹

In the previous case, we considered that the electrons originated at zero velocity from a grounded cathode outside the anode and were accelerated through the permeable anode forming a virtual cathode within. Hirsch reversed this step, and assumed that ions were produced at a grounded anode and accelerated through a permeable cathode held at a negative high voltage $-V_0$. The results of this analysis would be the same, except that a virtual anode would be formed instead of a virtual cathode. Hirsch's addition was to allow the

cathode to emit electrons (at zero initial velocity) towards the center of the device.

In this case

$$\frac{1}{2} M v_1^2 = -e V(r) \quad (7)$$

and

$$\frac{1}{2} m v_e^2 = e [V(r) + V_0] \quad (8)$$

where $-V_0$ is the cathode potential.

Equation 6 then becomes

$$\frac{1}{x^2} \frac{\partial}{\partial x} x^2 \frac{\partial Y}{\partial x} = \frac{K_1}{x^2} [Y^{-1/2} - \lambda_1 (1 - Y)^{-1/2}] \quad (9)$$

where

$$\lambda_1 = \frac{I_e}{I_1} \left(\frac{M}{m}\right)^{1/2} \quad (10)$$

$$K_1 = \frac{I_1}{V_0^{3/2}} \left(\frac{M}{2e}\right)^{1/2} \frac{1}{4\pi \epsilon_0} \quad (11)$$

$$Y = -V(r)/V_0$$

and $X = r/r_a$ where r_a is the radius of the virtual anode (zero potential) formed within the real cathode. The solution of equation 9, an example of which is shown in Fig. 2, shows the existence of alternating virtual anodes and cathodes within the real cathode and the existence of very large electrostatic fields near the center of the device.

Now, obviously, the solutions to these single particle as well as two particle injection equations can be expected to give highly idealized results. Our experiments have shown that Langmuir's solution is not realistic for electron injection only. We have also shown experimentally that electrons and ions can form a multiple potential structure. Our most recent theoretical work, therefore, has been to attempt more realistic solutions to Poisson's equations for both single particle and dual particle injection to see if a reasonable correlation to our experimental data can be found and to see if a firmer theoretical foundation for electrostatic confinement can be laid.

Single Species Injection with Spreads in Energy and Momentum

In order to develop a more realistic model for electron injection only, we have included spreads in energy of the injected particles as well as spreads in their angular momentum. The technique used is the same as that used by Dolan⁵ based on Lavrentyev's⁶

analysis of Poisson's equation with spreads in energy and angular momenta.

Once again, we assume that electrons are injected into the interior of a permeable spherical anode held at a positive potential V_a . We no longer assume that the electrons have zero total energy. We now express the total electron energy, $-e W_e$ as

$$-e W_e = \frac{1}{2} m v_r^2 + \frac{1}{2} m v_\phi^2 - e V(r) \quad (12)$$

In this case, the electrons at the anode can have any chosen distributions of total energy of from 0 to $-e V_a$ corresponding to electron velocities at the anode of $\frac{2e V_a}{m}$ to zero respectively. We neglect collisions, so for any given particle, the total energy is conserved.

We also define an angular energy, $e S_e$, for each particle, which can be related to the angular momentum at the anode

$$e S_e = \frac{L^2}{2 m r_a^2} = \frac{1}{2} m v_\phi^2 \frac{r_a^2}{r^2} \quad (13)$$

Once again, collisions are neglected so the angular energy is conserved for any given particle and desired values of the angular energy distribution can be incorporated into the solution.

Referring to Eq. (12), the radial velocity

$$v_r = \sqrt{\frac{2e}{m}} [V(r) - W_e - S_e \frac{r_a^2}{r^2}]^{1/2} \quad (14)$$

as compared to Eq. (1).

The current consists of the motion of electrons with a distribution in W_e and S_e

$$I_e(S_e, W_e) = 4\pi r^2 \rho_e(r, S_e, W_e) v_r(S_e, W_e) \quad (15)$$

So that the charge density involves a double integral over S_e and W_e .

$$\rho_e(r) = \int_0^{V(r)} dW_e \int_0^{r_a} dS_e \frac{I_e(S_e, W_e)}{4\pi r^2 \sqrt{\frac{2e}{m} [V(r) - W_e - S_e \frac{r_a^2}{r^2}]^{1/2}}} \quad (16)$$

Consider the limits of integration on S_e . A particle with a given total and angular energy at the anode will proceed towards the center of the system only to the point where the radial velocity is zero

$$0 = V(r) - W_e - S_e \frac{r_a^2}{r^2} \quad (17)$$

So, for particles of a given total energy, the electrons that contribute to $\rho(r)$ have angular energies in the range

$$0 < S_e < \frac{e r_a^2}{2} [V(r) - W_e] \quad (18)$$

Also, the total current is

$$I = \int_0^{V_a} dW_e \int_0^{V_a - W_e} I_e(S_e, W_e) dS_e \quad (19)$$

Using the same normalization as before

$$\begin{aligned} X &= r/r_a \\ Y &= V(r)/V_a \\ Y_e &= W_e/V_a \\ Y_\phi &= S_e/V_a \end{aligned} \quad (20)$$

We obtain

$$\frac{1}{X^2} \frac{d}{dX} X^2 \frac{dY}{dX} = \frac{V_a}{4\pi \epsilon_0 \sqrt{\frac{2e V_a}{m}}} \int_0^Y dY_e \int_0^{X^2[Y-Y_e]} dY_\phi \frac{I_e(Y_\phi, Y_e)}{X^2[Y-Y_e-Y_\phi/X^2]^{1/2}} \quad (21)$$

$$\text{and } I = V_a^2 \int_0^1 dY_e \int_0^{1-Y_e} dY_\phi I_e(Y_e, Y_\phi) \quad (22)$$

Equations (21) and (22) were evaluated for square and Boltzmann (Gaussian) distributions as shown in Fig. 3. The results are quite similar, so we will discuss only the Gaussian distribution

$$I_e(Y_e, Y_\phi) = A_e e^{-Y_e/\sigma_e} e^{-Y_\phi/\sigma_\phi}. \quad (23)$$

In order to keep the volumes of the elements shown on Fig. 3 (circulating current) independent of the anode voltages and distribution widths σ_ϕ and σ_e , the value of A is taken to be

$$A = \frac{I}{V_a^2 \sigma_\phi \sigma_e} \left[1 - e^{-1/\sigma_e} - \left(\frac{\sigma_\phi}{\sigma_\phi - \sigma_e} \right) (e^{-1/\sigma_\phi} - e^{-1/\sigma_e}) \right]^{-1} \quad (24)$$

We also know that the circulating current can be written in terms of the anode current

$$I = I_a \left[\frac{2\eta}{1 - \eta^2} \right] \quad (25)$$

where η is the openness of the anode.

Equation (21) can therefore be rewritten as

$$\frac{d}{dX} X^2 \frac{dY}{dX} = K_e \int_0^Y dY_e \int_0^{X^2} dY_\phi \frac{e^{-Y_e/\sigma_e} e^{-Y_\phi/\sigma_\phi}}{[Y - Y_e - Y_\phi/X^2]^{1/2}} \quad (26)$$

where K_e , which is related to the perveance, is

$$K_e = \left[\frac{2\eta}{1 - \eta^2} \right] \left[\frac{1}{4\pi \epsilon_0 \sigma_e \sigma_\phi \sqrt{\frac{2e}{m}}} \right] \left[1 - e^{-1/\sigma_e} - \left(\frac{\sigma_\phi}{\sigma_\phi - \sigma_e} \right) (e^{-1/\sigma_\phi} - e^{-1/\sigma_e}) \right]^{-1} \frac{I_a}{V_a^{3/2}} \quad (27)$$

Equation (26) has been solved for the potential and charge distributions within the anode.

We have chosen $\eta = .9$ and we have used anode voltages and anode currents typical of those used in our experiments with an anode of radius 3.175 cm.

Figure 4 shows the potential formed within the anode for $V_a = 1000$ volts, $I_a = 100$ mA under the assumption of a narrow spread in total energies ($\sigma_e = 0.05$) and a spread of angular energies from very wide ($\sigma_\phi = 0.8$) to narrow ($\sigma_\phi = 0.05$). The results are very different from the Langmuir solution (Fig. 1). This figure shows that for narrow spreads in energy and angular momentum we approach the Langmuir solution, as expected, but that as the angular spread increases the depth of the well decreases considerably. We also see that relatively small amounts of angular momentum, which could be easily generated by the wire structure of the anode, would result in a potential well with an approximately parabolic shape. This parabolic shape is in excellent agreement with our experimental results. We have seen the sensitivity of the depth of the well on angular momentum. Figure 5 shows the dependence of the well shape on total energy spread for a fixed narrow spread (0.05) in angular momentum. As can be seen, spreads in total energy have relatively little effect on the well shape or depth. Angular momentum appears to be the important factor.

In order to facilitate comparison to our experimental results, Fig. 6 shows the potential variation for a fixed spread in total energy and angular energy $\sigma_e = \sigma_\phi = 0.05$ and for a fixed anode voltage, $V_a = 1000$ volts, but with a variable anode current. Each curve is labeled with four numbers. The first is the fractional well depth f , the second is the anode current (mA), the third is the anode voltage (volts) and the fourth is K_e which is related to the perveance. As can be seen, the well depth increases with increasing current for a given anode voltage. Figure 7 shows a plot of f , the fractional well depth, vs anode current for a fixed anode voltage of 2000 volts. Also shown are the experimental data points that we have obtained for the same conditions. The agreement is quite good and shows an essentially linear increase in well depth with increasing current.

Figure 8 shows a plot which is similar to Fig. 6 except that the anode current is held constant at 100 mA and the anode voltage is varied. Figure 9 shows a plot of the absolute well depth, $f V_a$, vs anode voltage for an anode current of 60 mA along with the experimental data points for the same conditions. Again, the agreement is quite good and shows that the absolute well depth is relatively independent of the anode voltage over quite a large range as long as the anode voltage is held fixed.

The excellent agreement between the theory and experiment for electron injection only indicates that this theoretical approach is valid. The theory also shows that the spread in angular energy

is the critical factor in determining the depth of the electrostatic wells formed. If square distribution functions are used, essentially similar results are obtained as we found in our previous work and which have also been found by Imel,⁷ so the form of the angular energy distributions are not critical, only the fact that they exist.

Two Species Injection with Spreads in Energy and Angular Momentum

The logical extension of the single species study is to two species injection to see if the multiple potential wells are still predicted. Certainly our experiments suggest that they do exist. A theoretical confirmation would be most useful in helping us to determine under what conditions the multiple potential well exists. Unfortunately, our experimental conditions cannot be easily represented by such a simple model. Collisions are obviously important in our experiment as a source of positive ions. Collisions were neglected in the theory. Possibly, however, modifications to the total energy and angular energy spreads could compensate.

Initial studies in this area have already been performed by Dolan. The approach is identical to that for single particle injection, only both electrons and ions are considered and both are considered to have spreads in total energy and angular energy. Dolan solved for the potential within the anode for an approximately Gaussian shaped energy distribution (rather than step function) of ions and electrons. Unfortunately, he found no multiple potential

well structure for the conditions of fairly wide total energy and angular energy distributions. The range of variables he considered was small, however, so we have continued to solve the equations with two charged species for a wide range of variables.

As a model of our experimental system, we have kept the electron spreads in total and angular energies relatively small and close to those values that seem to match our experimental electron injection only conditions. We expect that the ions are produced by ionizing collisions with the background gas within the anode, so we would expect the electrons to have a wide spread in total energies (corresponding to being born anywhere within the anode).

For our initial analysis, however, we have kept the spreads in total energy and angular energy relatively small. Under these conditions, we have found potential solutions which show a multiple potential structure. This is shown in Fig. 10. In fact, such solutions do appear to exist for a range of variables, although at the moment a strictly limited range. It should be pointed out, however, that there is still some question as to whether or not these solutions are unique. In some cases several solution, including those which are similar to Dolan's and show no multiple potential structure, are possible for the same conditions for which a solution showing a multiple potential well structure has been found.

Our present work is concerned with further examination of the conditions for and the uniqueness of the multiple potential

well solutions for two species injection in spherical geometry. Our initial results, although far from definitive, do indicate that there is some promise of using the present theoretical model to understand and predict multiple potential well formation in electrostatic confinement devices.

Conclusions

The solution of Poisson's equation in spherical coordinates for electron injection with spreads in total energy and in angular energy provides a realistic theoretical model for the potential within a spherical device for single species injection. The agreement between this theory and experimental results on similar devices is excellent. The essential factor in obtaining good agreement is the inclusion of the angular motion of the injected particles. Preliminary theoretical results for two species injection indicate that this theoretical approach may also be able to be used to provide a model of the multiple potential well structure observed in experimental devices for electron injection into a low pressure gas background.

References

1. Hirsch, R. L. J. Appl. Phys. 38: 4522 (1967).
2. Langmuir, I., K. R. Blodgett. Phys. Rev. 24: 49 (1924).
3. Swanson, D. A., B. E. Cherrington, J. T. Verdeyen. Phys. Fluids 16: 1939 (1973).
4. Hirsch, R. L. Phys. Fluids 11: 2486 (1969).
5. Dolan, T. J., J. T. Verdeyen, B. E. Cherrington, D. J. Meeker. J. Appl. Phys. 43: 1950 (1972).
6. Lavrentyev, O. A. Magnitnye Lovushki AEC report No. AEC-tr-7002, 1970.
7. Imel, G. M. S. Thesis, Pennsylvania State University (1973).

Figure Captions

- Fig. 1. Normalized potential vs. normalized radius within the anode for monoenergetic electron injection with no angular motion.
- Fig. 2. Normalized potential vs. radius for monoenergetic electron and ion injection with no angular momentum.
- Fig. 3. Assumed step function and Gaussian distributions of the electron current.
- Fig. 4. Normalized potential vs. normalized radius within the anode. The total energy spread is held constant and the angular energy spread is varied.
- Fig. 5. Normalized potential vs. normalized radius within the anode. The angular energy spread is held constant and the total energy is varied.
- Fig. 6. Normalized potential vs. normalized radius within the anode for a constant voltage and varying current:
- Fig. 7. Fractional well depth f , vs. anode current for $V_a = 2000$ Volts. Experimental and theoretical values.
- Fig. 8. Normalized potential vs. normalized radius within the anode for a constant current and varying voltage.
- Fig. 9. Absolute well depth, $f V_a$, vs. anode voltage for $I_a = 60$ mA. Experimental and theoretical values.
- Fig. 10. Multiple potential well structure for both electron and ion injection. A square distribution of energies has been used. (See Fig. 3).

Fig 1. (Continued)

Fig 2 Cherrin et al

Fig 3 Chasing the circle

Fig. 10 continued

Fig. 5 a. - cont'd

Fig. 6 Characteristic

Fig. 7 Cherington et al

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Fig. 9 Choung et al

Fig 10 Chw. ... et al