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LASER-INDUCED INSTABILITIES AND ANOMALOUS ABSORPTION IN DENSE PLASMAS[†]

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INTRODUCTION

In the last three years or so, the subject of radiation-induced parametric instabilities in plasmas has become an exceedingly active field of research. These instabilities have been observed in many experiments. High power transmitting antennas can excite these instabilities in the F-layer of the ionosphere where the resulting enhanced wave fluctuations are observed by radar backscatter techniques. Numerous laboratory experiments using Q-machines, DP devices and discharges and employing radiation sources in the radio-frequency to microwave regime have studied these instabilities. Many detailed theoretical predictions have been verified by these experiments, but at this early point there also remain many unanswered questions. Perhaps the most exciting but least understood application of these effects is toward the heating of laser produced plasmas for laser-induced fusion.

This is such a rapidly advancing subject that it is impossible to do it justice in a forty-five minute talk. I will therefore limit myself to the simplest cases which have received the most study. These instabilities were first seriously studied in 1965. (References 1, 2, and 5a.) At that time, however, available laser intensities were four orders of magnitude below the threshold intensities needed to excite these instabilities. Today available power densities are several orders of magnitude above this threshold.

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The basic processes which interest us particularly for laser-fusion research are anomalous absorption and anomalous backscattering processes, all of which can be classed as parametric instabilities. These processes can be represented schematically as in Fig. 1. 1. The anomalous absorption processes are three wave decay

ABSORPTIVE DECAY INSTABILITIES

BACKSCATTER INSTABILITIES

$$\omega_0 = \omega_1 + \omega_2$$

$$\underline{k}_0 = \underline{k}_1 + \underline{k}_2$$

Figure 1. Schematic representation of three wave parametric instabilities. a) Absorptive decay instabilities b) Backscatter instability. (Decay of plasma waves is indicated by inverse Cerenkov effect into particle energy.)

processes in which an intense electromagnetic wave is transformed by nonlinear interaction into two predominantly longitudinal plasma waves which ultimately give their energy to random particle motion via Landau or collisional damping. Thus electromagnetic energy is converted into plasma energy. In the case of an isotropic plasma ω_1 may correspond to a high frequency electron-plasma (Langmuir) waves [$\omega_1^2 = (\omega_{pe}^2 + 3k_1^2 v_{Te}^2)$] and ω_2 to an ion-acoustic wave [$\omega_2^2 = k_2^2 (T_e/m_i)$]. We call this the electron-ion decay instability ($\omega_1 \sim \omega_{pe}$, $\omega_2 \sim \omega_{pi}$). If ω_1 and ω_2 both correspond to Langmuir waves, we call this the electron-electron decay instability ($\omega_1 \sim \omega_{pe}$, $\omega_2 \sim \omega_{pe}$). (3) The absorptive parametric instabilities appeared at first sight to be just what was needed for plasma heating of laser-fusion pellets. (4) However, computer simulations, experiments, and some preliminary theory shows that these instabilities often produce very high energy tails on the electron velocity distribution which have detrimental preheating effects on laser-fusion schemes which rely on extreme compression of fuel pellets.

The backscatter instabilities in an isotropic plasma have two names: Stimulated Raman scattering (SRS)⁵ if the excited plasma wave is a Langmuir wave, and stimulated Brillouin scattering (SBS)⁶ if ω_2 corresponds to an ion-acoustic wave. In the inhomogeneous plasma encountered in laser-plasma interaction, these instabilities mostly backscatter the incident laser light. They do relatively little plasma heating and may prevent the radiation from reaching the region of density where the absorptive instabilities can act and where appreciable linear absorption by inverse bremsstrahlung can occur.

SUMMARY OF LINEAR THEORY

Next I will give a brief sketch of a phenomenological linearized theory of these instabilities which can all be treated from a common point of view. In a homogeneous, but not necessarily isotropic plasma, we can write the space-time Fourier transform of Maxwell's equations in the following form

$$\Lambda_{ij}(\underline{k}, \omega) E_j(\underline{k}, \omega) = \frac{4\pi i}{\omega} J_i(\underline{k}, \omega) \quad (2.1)$$

$$\Lambda_{ij}(\underline{k}, \omega) = \frac{k^2 c^2}{\omega^2} [\delta_{ij} - \frac{k_i k_j}{|k|^2} - \epsilon_{ij}(\underline{k}, \omega)] \quad (2.2)$$

where $\epsilon_{ij}(\underline{k}, \omega)$ is the dielectric tensor which takes into account the linear induced currents in the plasma. The current $J_i^{NL}(\underline{k}, \omega)$ arises from nonlinear induced currents which result from the beating of the intense pump wave

$$E_j^0(\underline{r}, t) = \frac{E_0}{2} j e^{i(\underline{k}_0 \cdot \underline{r} - \omega_0 t)} + c.c. \quad (2.3)$$

with waves excited in the plasma. If the pump is sufficiently weak, we can write J^{NL} as a series and Eq. (2.1) becomes¹

$$\begin{aligned} \Lambda_{ij}(\underline{k}, \omega) E_j(\underline{k}, \omega) &= \chi_{ijk}^{(1)} E_j^0 E_k(\underline{k} - \underline{k}_0, \omega - \omega_0) \\ &\quad + \chi_{ijk}^{(1)} E_j^{0*} E_k(\underline{k} + \underline{k}_0, \omega + \omega_0) \\ &\quad + \text{terms of form } \chi^{(n)} (E^0)^n E_k(\underline{k} - n\underline{k}_0, \omega - n\omega_0) \end{aligned} \quad (2.4)$$

The coefficients such as $\chi_{ijk}^{(1)}$ (which depend on $\underline{k}$, $\underline{k}_0$, ω , ω_0) are called nonlinear susceptibility coefficients. This is the point where microscopic plasma physics comes in. Either from the kinetic equation (e.g., Vlasov eq.) or the fluid equations, we must compute the nonlinear response of the current to the electric fields in the plasma. I will not have time in this talk to discuss the details of these important but straightforward calculations.^(1,3,7) Many cases have been considered by various workers. The χ_{ijk} (and higher order) coefficients have important symmetry properties which again I will not discuss here.

For the purposes of this talk, we will regard these coefficients as given phenomenological quantities. The advantage of this formulation, which essentially is that commonly used in nonlinear optics,

¹ repeated indices are summed

is that all the parametric instabilities in a plasma can be viewed from a common point of view. On the other hand, the approach using the particle kinetic equations or fluid equations which emphasizes the particle variables rather than the associated wave variables seems to be favored by plasma physicists. In this approach it is harder to see the common features of all these instabilities.

Equation (2.4) as written has been linearized by neglecting terms such as $(2\pi)^{-4} \int d^3k' d\omega' \chi_{ij}^{(1)} E_1(k', \omega') E_k(k-k', \omega-\omega')$ which are of second order or higher in the "weak" Fourier coefficients. Such terms must be retained to study the nonlinear theory of the saturation level of these instabilities. In the linearized theory, we also assume that the pump parameters are fixed (the origin of the term parametric). Then the expansion on the right-hand side of Eq. (2.4) may contain arbitrarily high powers of $|E_0|$. More detailed considerations^(3,7) show that only the leading terms of this expansion are required for a nearly collisionless plasma if

$$d_{oe} = \frac{eE_0}{m_e \omega_0^2} \ll \frac{\lambda}{2\pi} \quad (2.6)$$

That is, if the excursion amplitude of an electron in the pump field, d_{oe} , is much less than the wavelength of the plasma waves λ involved. In cases of large E_0 , the coefficients of the beat amplitudes are often Bessel functions of argument (d_{oe}/λ) .^(3,7)

Although it is not an essential restriction in carrying out the linearized theory, I will assume for simplicity that the directed velocity $v_0 = (eE_0/m\omega_0)$ of electrons in the oscillating laser field is much less than the thermal velocity $v_T = (k_B T/m)^{1/2}$. This guarantees that $|\underline{k} \cdot \underline{d}_{oe}| = (v_0/v_T)(k/k_{De})(\omega_0/\omega)^2 \ll 1$ provided $k \ll k_{De}$ -- the electron Debye wavenumber. For $k_{De} = 1 \text{ keV}$ and a laser intensity $I_0 = cE_0^2/4\pi$ of 10^{15} W/cm^2 , the ratio is $v_0^2/v_T^2 = 3 \times 10^{-2}$.

Even in the linearized theory we have an infinite set of coupled equations for the various Fourier beat amplitudes. This set of equations can often be truncated by noting that waves of a given polarization can only be resonantly excited if their linear dispersion relation is satisfied. We introduce a complete set¹ of polarization vectors $\hat{e}_{i\alpha}$ ($\alpha=1,2,3$) which diagonalize the Maxwell operator $\Lambda_{ij}(\underline{k}, \omega)$ in Eq. (2.1)

$$\Lambda_{ij}(\underline{k}, \omega) \hat{e}_{j\alpha}(\underline{k}, \omega) = \Lambda_{\alpha}(\underline{k}, \omega) \hat{e}_{i\alpha}(\underline{k}, \omega) \quad (2.7)$$

¹Actually, a biorthogonal set since Λ_{ij} is non-hermitian.

The (usually complex) solutions of $\Lambda[\underline{k}, \hat{\omega}(\underline{k})] = 0$ for $\hat{\omega}$ in terms of $\underline{k}$ give the linear dispersion relation for waves of polarization $\hat{e}_{i\alpha}$. In general these solutions are complex values $\hat{\omega} = \omega - i\lambda$ for fixed $\underline{k}$. In other words, $\Lambda^{-1}(\underline{k}, \omega)$ is the linear response function for the harmonic oscillator representing the mode with polarization $\hat{e}(\underline{k}, \omega)$ and wave vector $\underline{k}$. For the example of an homogeneous isotropic plasma, we have three mutually perpendicular polarization vectors $\hat{e}_i$ ($i=1,2,3$) where $\hat{e}_{i1} = \hat{e}_i$ with $\Lambda_1(\underline{k}, \omega) = -\epsilon_L(\underline{k}, \omega)$ where ϵ_L is the usual longitudinal dielectric function. For $i=2,3$, we have two mutually perpendicular transverse polarization vectors and $\Lambda_i(\underline{k}, \omega) = (k^2 c^2 / \omega^2) - \epsilon_T(\underline{k}, \omega)$ where ϵ_T is the transverse dielectric function. We represent the coefficients $E_i(\underline{k}, \omega)$ as a superposition of these polarization vectors with coefficients $\hat{E}_i(\underline{k}, \omega)$. Then, barring certain degeneracies, it is often the case that if the frequency and wavevector matching condition

$$\omega_0(\underline{k}_0) = \omega_1(\underline{k}_1) + \omega_2(\underline{k}_2) \quad (2.8)$$

$$\underline{k}_0 = \underline{k}_1 + \underline{k}_2$$

is satisfied for two modes, $\alpha=1$ and $\alpha=2$, that only these two modes can be simultaneously resonantly excited by the parametric interaction and the Eqs. (2.4) reduce to a 2×2 set.

$$\Lambda_1(\underline{k}, \omega) \hat{E}_1(\underline{k}, \omega) = \chi_{102}^{(1)} \hat{E}_0 \hat{E}_2(\underline{k}-\underline{k}_0, \omega-\omega_0) \quad (2.9)$$

$$\Lambda_2(\underline{k}-\underline{k}_0, \omega-\omega_0) \hat{E}_2(\underline{k}-\underline{k}_0, \omega-\omega_0) = \chi_{201}^{(1)} \hat{E}_0^* \hat{E}_1(\underline{k}, \omega)$$

where $\chi_{102}^{(1)}$ is the projection of $\chi_{ijk}^{(1)}$ on the polarizations of the three waves involved

$$\chi_{102}^{(1)} = \hat{e}_{i1} \chi_{ijk}^{(1)} \hat{e}_{j0} \hat{e}_{k2} \quad (2.10)$$

where $\hat{e}_{j0}$ is the polarization vector of the pump. We see here that the beating of one Fourier component with the pump acts as a source or driver for the other Fourier component. This "bootstrapping" or regenerative effect is the source of the instability. The effect is strong only if the two driven harmonic oscillators are simultaneously near resonance, a condition which leads to Eq. (2.8).

The dispersion relation for the coupled wave problem is obtained by setting the determinant of Eqs. (2.9) to zero. If we solve this for complex ω for fixed values of $\underline{k}$, we find that there are temporally growing solutions corresponding to instability provided

the pump amplitude is greater than a threshold value. If we define a quantity Γ of dimensions sec^{-1}

$$\Gamma^2(\underline{k}) = \frac{x_{102}x_{201}|\mathcal{E}_0|^2}{\left.\frac{\partial \text{Re}\Lambda_1}{\partial \omega}\right|_{\omega=\omega_1} \left.\frac{\partial \text{Re}\Lambda_2}{\partial \omega}\right|_{\omega_0-\omega=\omega_2}} \quad (2.11)$$

the threshold condition can be written as ^(1,7)

$$\Gamma^2(\underline{k}) > \Gamma_m^2 = \gamma_1\gamma_2 \quad (2.12)$$

the threshold is determined by the uncoupled wave damping decrements. These relations hold for all three-wave decay-type parametric instabilities including those in a magneto-plasma provided only that the uncoupled waves are sharply defined as modes, i.e., $\gamma_1/\omega_1 \ll 1$, $\gamma_2/\omega_2 \ll 1$. The quantity Γ is the growth rate of the coupled modes for perfect frequency matching, $\Delta\omega = \omega_0 - \omega_1 - \omega_2 = 0$, in the absence of damping ($\gamma_1 - \gamma_2 = 0$).

Near threshold, the growth rate depends critically on $\gamma_1\gamma_2$ and $\Delta\omega$.

$$\gamma_g(\epsilon_0, \underline{k}) = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \left\{ \frac{\Gamma^2(\underline{k})}{\gamma_1\gamma_2} \frac{1}{1 + \left(\frac{\Delta\omega}{\gamma_1+\gamma_2}\right)^2} - 1 \right\} \quad (2.13)$$

Notice that for $\Delta\omega=0$, $\gamma_g=0$ for $\Gamma^2=\Gamma_m^2$. For $\Delta\omega \neq 0$, a higher value of Γ^2 is required to produce instability ($\gamma_g > 0$).

In this near-threshold limit, γ_g behaves linearly with $|\mathcal{E}_0|^2$. In the electron-ion decay instability and in SBS when the pump is sufficiently high above threshold so that the growth rates becomes comparable in magnitude to the ion acoustic frequency, the dispersion relation predicts ⁽⁹⁾ that the growth rate then is proportional to $|\mathcal{E}_0|^{2/3}$.

If the solution of the dispersion relation is used to relate ω to $\underline{k}$ in Eqs. (2.9), we can solve for the amplitude ratio $\mathcal{E}_1(\underline{k}, \omega)/\mathcal{E}_2(\underline{k}-\underline{k}_0, \omega-\omega_0)$. The result can be summarized in the so called Manley-Rowe Relations which simply express the fact that for a three wave decay process for every pump photon absorbed, a plasmon appears in the modes 1 and 2. Letting n_0 , n_1 , and n_2 represent the quantum occupation numbers (or more strictly, the change in these quantities), we have

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$$n_0 = n_1 = n_2 \quad (2.14)$$

or since the energy in the mode α is related to n_α by $W_\alpha = n_\alpha h\omega_\alpha$, we have

$$\frac{W_0}{\omega_0} = \frac{W_1}{\omega_2} = \frac{W_2}{\omega_3} \quad (2.15)$$

Thus the energies in the coupled modes are, in the ratios of their frequencies, the greatest energy in the highest frequency mode. If the SBS and SRS instabilities are strongly excited, this means that most of the absorbed pump energy will appear in the scattered light wave and relatively little will be absorbed in the lower frequency plasma waves. In Table I, the properties are listed of the four decay-type parametric instabilities which can be excited in a non-magnetized plasma by an intense electromagnetic wave pump. The geometry of the $\mathbf{k}$ vectors, which satisfy the matching condition, relative to the pump polarization is shown, the 3 frequency dispersion relations involved, the form of Γ^2 for each instability. Numerical values of the pump intensity $I_0 = E_0^2 c / 8\pi$ in Watts/cm² at threshold for the case of an Nd-glass laser with $n = 10^{21}$ cm⁻³ at the critical density ($\omega_0 = \omega_c$) and $T = 1$ keV and for the case of a CO₂ laser with $n_c = 10^{19}$ cm⁻³. For the two absorptive decay

TABLE I

GEOMETRY	FREQUENCIES $\omega_0 = \omega_1 + \omega_2$	Γ^2	THRESHOLDS W/cm ² Nd CO ₂
<p>e-i Decay</p>	$\omega_0 = \omega_{en}(k_0)$ $\omega_1 = \omega_L(k_1)$ $\omega_2 = \omega_A(k_2)$	$\frac{\Lambda_0^2}{16} \frac{(\hat{k} \cdot \hat{e}_0)^2}{E_0^2} \omega_1 \omega_2$ $(\Lambda_0 = \frac{E_0}{4\pi n T_e})$	Nd 10 ¹³ 10 ¹⁰ 10 ¹¹ ($T_e \gg T_i$)
<p>e-e Decay</p>	$\omega_0 = \omega_{en}(k_0)$ $\omega_1 = \omega_L(k_1)$ $\omega_2 = \omega_L(k_2)$	$\frac{\Lambda_0^2}{64} \frac{k_0^2}{k_1^2 k_2^2} (\hat{k} \cdot \hat{e}_0)^2 (\hat{k} \cdot \hat{e}_0) \omega_1 \omega_2$	~10 ¹³ 10 ⁹
<p>SRS</p>	$\omega_0 = \omega_{en}(k_0)$ $\omega_1 = \omega_{en}(k_1)$ $\omega_2 = \omega_A(k_2)$	$\frac{\Lambda_0^2}{16} (\hat{e}_1 \cdot \hat{e}_0)^2 \frac{\omega_{pe}^4}{\omega_1^2} \omega_1 \omega_2$	~10 ¹³ 10 ¹⁰
<p>SRS</p>	$\omega_0 = \omega_{en}(k_0)$ $\omega_1 = \omega_{en}(k_1)$ $\omega_2 = \omega_L(k_2)$	$\frac{\Lambda_0^2}{16} \frac{k_0^2}{k_1^2 k_2^2} (\hat{e}_1 \cdot \hat{e}_0)^2 \frac{\omega_{pe}^2}{\omega_1^2 \omega_2^2} \omega_1 \omega_2$	~10 ¹³ 10 ⁹

Electromagnetic $\omega_{en}(k) = (\omega_{pe}^2 + k^2 c^2)^{1/2}$; Langmuir $\omega_L(k) = (\omega_{pe}^2 + 3k^2 v_e^2)^{1/2}$; Ion Acoustic $\omega_A = k(\gamma T_e / m_i)^{1/2}$

6-1/2 x 9-7/8 in. trim
3/16" type metal
10% reduction
3" x 52" piece working size
30 x 40" piece final size

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instabilities, only the cases $k_0 \ll k_1, k_2$ are shown and for SRS and SBS only the geometry for backscatter² is shown.

There are several cases of parametric instabilities in which one mode is actually not well defined or is a quasi-mode. This is the case for the e-i decay instability⁽¹⁾ or SBS for $T_e = T_i$ for which the ion acoustic mode is not well-defined. In this case, the process we are discussing may be more correctly described as induced scattering (or conversion) from ions of transverse waves into Langmuir waves or transverse waves (see Fig. 2).

In the case in which mode 2 is a longitudinal wave, this growth rate is given by⁽¹⁾

$$\gamma_g(E_0, k) = \frac{x_{102} x_{201} |\epsilon_0|^2}{\frac{\partial \text{Re} \omega_1}{\partial \omega}} \left| \text{Im} \frac{1}{\epsilon(k - k_0, \omega_1 - \omega_0)} \right| - \gamma_1 \quad (2.16)$$

(This assumes that the real parts coupled mode frequencies are not shifted far from their uncoupled values.) The condition for maximum growth, i.e., maximum $\text{Im} \epsilon^{-1}(k - k_0, \omega_1 - \omega_0)$ provides a matching condition $\omega_1(k) - \omega_0 = -1.7 (k/k_{De}) \sqrt{m_e/m_i} \omega_p$ which can be obtained from tables of the plasma dispersion function.

The simple two-mode coupling scheme fails in several important cases. In these cases, the "anti-stokes" mode with Fourier component $\omega + \omega_0, \underline{k} + \underline{k}_0$ can also be resonantly excited in addition to the Fourier components at $\omega, \underline{k}$ and $\omega - \omega_0, \underline{k} - \underline{k}_0$ already considered. In this problem, we have 3 coupled equations to solve and thus a somewhat more complicated dispersion relation.^(8,9) Two important cases arise:

1) All three modes resonantly coupled: This can occur in the electron-ion decay instability if $2\omega_0 \ll \gamma_1$ and for forward Brillouin and Raman scattering. The instability threshold⁽⁸⁾ is generally higher if the third mode can be resonantly coupled.

2) Purely growing mode.⁽⁹⁾ In this case, the stokes and anti-stokes modes $\omega - \omega_0$ and $\omega + \omega_0$ are resonantly coupled but the mode ω is shifted far from a resonance. For the case $k = 0$, i.e., a homogeneous pump field, this leads to a root of the dispersion relation which is unstable but has a zero real frequency, i.e., a purely growing instability with wavevector $\underline{k}$. The Fourier components at $\omega \pm \omega_0$ then oscillate at exactly the pump frequency ω_0 and grow in time with the growth rate $\text{Im} \omega$. When $k \neq 0$, this mode with wavevector $\underline{k}$ retains a small real frequency proportional to k_0 . The

(a) e-i Decay for $T_e \approx T_i$

Induced Scattering from Ions

(b) Purely Growing or Oscillating Two Stream (OTS) Instability

Figure 2. a) Decay instability for $T_e = T_i$ which is best represented as induced scattering or i -conversion of E.M. waves into Langmuir waves from ions. b) Schematic representation of mode coupling for the purely growing instability which involves an off resonant ion wave $(\omega, \underline{k})$.

threshold and growth rate for this instability is nearly equal to that of the electron-ion decay instability if the electron (T_e) and ion temperatures (T_i) are nearly equal. It has a higher threshold if $T_e \gg T_i$. This instability excites electrostatic waves with

$k < k_c$. It appears that the quasi-linear decay of shorter wavelength waves excited by the purely-growing instability are responsible for the high energy electron tails observed in the computer simulations.⁽¹⁰⁾ In the forward scattering geometry of SRS and SBS, i.e., when the scattered light wave is in the same direction as the laser wave, the "anti-stokes" mode $\underline{k} + \underline{k}_0, \omega + \omega_0$ can likewise be resonantly excited. The resulting zero frequency or low frequency instability goes by the name of the self-focusing instability^(10a) or the modulational instability^(10b) depending on the relative orientation of $\underline{k}$ and $\underline{k}_0$. A schematic diagram of the wave coupling involved in these instabilities is shown in Fig. 2.

PARAMETRIC INSTABILITIES IN INHOMOGENEOUS PLASMAS

All actual experiments are performed in bounded or inhomogeneous plasmas. In these situations, it is necessary to understand the spatial behavior of the parametric instabilities. Consider first a plane-layered plasma whose properties vary only in the z direction. We also restrict attention to the case in which the pump electromagnetic wave propagates with $\underline{k}_0$ in the z direction, polarized in the transverse direction. (Thus, we avoid the important problems of tunneling and longitudinal wave resonance associated with oblique incidence.)

Consider for a moment a homogeneous plasma. The dispersion relation arising from Eqs. (2.9) can also be solved for complex $\underline{k}$ for fixed real values of ω . Then for given values of k_x, k_y , we can find complex solutions for k_z corresponding to spatially growing (or convective) modes provided

$$\gamma_1 \gamma_2 = \Gamma_m^2 \ll \Gamma^2(k) \ll \frac{|v_1 v_2|}{4} \left(\frac{\gamma_1}{v_1} - \frac{\gamma_2}{v_2} \right)^2 \quad (3.1)$$

where $\underline{k} = (k_x, k_y, k_z)$ and v_1, v_2 are the group velocity components of the uncoupled waves along the z (or pump) direction. These quantities generally depend on (k_x, k_y, k_z) . In this regime, a wave packet of unstable modes will grow as it moves in space but will not grow in time at a fixed point in space.

However, one can show by several methods of analysis^(11,12) that for the backward wave case $v_1 v_2 < 0$, the dispersion relation leads to absolute temporal instability (growth in time at a fixed spatial point) provided

$$\Gamma^2 > \Gamma_T^2 = \frac{|v_1 v_2|}{4} \left(\frac{\gamma_1}{v_1} - \frac{\gamma_2}{v_2} \right)^2. \quad (3.2)$$

This threshold is generally higher than that (Γ) for spatial growth except for the $\omega_0 = 2\omega$ decay instability for which $\gamma_1/v_1 = -\gamma_2/v_2$ so that $\Gamma_T = \Gamma_0$. For ^{pe}SRS and SBS there is an intermediate pump region of spatial growth where $\Gamma < \Gamma < \Gamma_T$. In the absolute instability regime for SBS and SRS, there is spatial growth as well as temporal growth; the peak of the back scattered pulse moves with a velocity $[v_1 + v_2]/2$ and grows. This spatial growth may be more significant than the temporal growth for short laser pulses.⁽¹³⁾ In this case, the absolute regime may not differ significantly from the convective regime.

In an inhomogeneous plasma, the k matching condition $k_0 = k_1 + k_2$ can only be satisfied locally since k is not a constant over the ray path of the wave. The region over which matching can be approximately satisfied is called the interaction region. For the convective instabilities, the waves will grow exponentially in space only in this limited region and thus the gradients limit the size to which the excited waves can grow. Of course, nonlinear effects may still be a limiting factor if this spatial amplification is sufficiently large. We impose the rather arbitrary criterion for threshold in this case that the wave (power) grows by a factor e^A where, say, A is 5 in the interaction region. The stronger the gradient the smaller the interaction region and so the pump amplitude must be stronger to achieve the same total spatial amplification over the interaction region. This type of reasoning leads to the criterion^(14,12)

$$A < \frac{2\pi\Gamma^2}{k' v_1 v_2} \quad (3.3)$$

with $k' = d(\Delta k)/dz$ where $\Delta k(z) = k_{z0}(z) - k_{z1}(z) - k_{z2}(z)$ is the local mismatch of the z components of the wavevectors calculated in the WKB approximation for the three waves; k' is inversely proportional to the scale height, H , of the density gradient. In Table II are shown the threshold conditions dependent on the density gradient for our 4 instabilities. The factor in parenthesis multiplying $\gamma_1 \gamma_2$ in these formulas is the enhancement factor of the threshold due to gradient effects. The rough order of magnitude of this factor is given for the same laser-plasma parameters as used in Table I.

Note that the electron-ion decay instability threshold is the least strongly effected by the gradient since in the geometry chosen, the excited electrostatic waves propagate normal to the density gradient and thus "stay" in the interaction region except for diffraction spreading of the waves along the z direction.⁽¹⁵⁾ SRS and SRS have their thresholds raised more strongly by the gradient because the backscattered waves rapidly propagate out of the interaction region. The enhancement factor for SRS is dominated by the temperature gradient if $k^2 c^2 / \omega_p^2 \gg 1$ since the acoustic mode frequency depends only on temperature.^p In the plane-layered geometry considered here, the sidescattering SRS or SBS process in which the scattered light wave propagates normal to the gradient also has a lower inhomogeneity threshold.^(15a,b) The plasma wave involved propagates at 45° to the gradient direction. Side-scattering which has a higher homogeneous threshold and lower growth rate may dominate backscatter for sufficiently strong gradients provided the laser beam width is large enough to permit appreciable growth in the transverse direction.^(15b) The electron-electron decay

instability usually behaves as an absolute instability since $k_1 = -k_2$ if $k_1 \gg k_0$ so that $\gamma_1/v_1 = -\gamma_2/v_2$. The propagation velocity of this coupled mode ($v_0 = (v_1 + v_2)/2$) is zero and we find only temporal growth in the interval Δz surrounding the interaction region. (12) In this interval, the wave growth may saturate for large times due to inhomogeneity effects (13) or may be limited by nonlinear effects.

Recent work indicates an important sensitivity of the backward wave, inhomogeneous problem to boundary conditions. If a linear mismatch $k(z) = k'(z)$ is assumed and also that the right and left going waves decay at $z = \pm \infty$, respectively, it can be shown (13) that no absolute instability exists in an inhomogeneous plasma, i.e., a pulse will begin to grow at the absolute instability growth rate but will ultimately saturate and die away as the pulse spreads out of the interaction region. However, if the homogeneous plasma is bounded (as it always is), an absolute instability which grows exponentially in the asymptotic time limit can be excited even when the amplitudes of the oppositely traveling waves are taken to be zero at the entrance boundaries. (12)

TABLE II

STABILITY	CONNECTIVE GROWTH THRESHOLD	ENHANCEMENT FACTOR
$\omega < 1$ DECAY	$\Gamma^2 > \gamma_1 \gamma_2 (1 + \frac{A}{2} \frac{v_e}{v_c} \frac{1}{H k_1})$ (H is scale height of density gradient)	$H = .01 - .1 \text{ cm}$ $1 - 10$
SRS	$\Gamma^2 > \gamma_1 \gamma_2 (1 + \frac{A}{4n} \frac{\omega_0^2}{\omega_1 \omega_2} \frac{k_1^2}{v_c^2 H})$	$10^4 - 10^5$
$\omega > 1$ DECAY	NONE	
SRS	$\Gamma^2 > \gamma_1 \gamma_2 (1 + \frac{A}{2n} [\frac{\omega_2}{v_2} \frac{v_0}{v_c} \frac{1}{k_0 H} + \frac{k_0^2 c^2}{\omega_p^2} \frac{H}{H_T}])$ (H_T is scale height of temperature gradient)	$10 - 10^2$
PURELY GROWING (O/S)	NONE	

NONLINEAR THEORY

The nonlinear theory of the parametric instabilities is, of course, the most poorly understood area of interest. In this regard, computer simulations have been very helpful in indicating important nonlinear effects, especially in cases of very strong pumping. I will concentrate on some recent results on the saturation of the electron-ion decay instability which is the case that has received the most attention from the "analytical theorists."

The question of most interest is "how much of the electromagnetic energy of the pump can be converted into heating of electrons and ions?" In this process, we have electromagnetic energy of the pump transformed into energy associated with coherent electrostatic plasma waves which is ultimately converted into random particle energy. Since the rate of energy loss by the pump by the parametric process is exactly equal to the energy gained by the plasma waves, we can define a nonlinear conductivity (or sometimes misleadingly identified with an effective collision frequency, ν_{eff}).⁽¹⁶⁾

$$\sigma_{\text{NL}}^{\text{NL}}(E_0) E_0^2 \equiv \frac{\nu_{\text{eff}}}{4\pi} E_0^2 = \int \frac{d^3k}{(2\pi)^3} \left[\gamma_g(E_0, k) - \gamma_g(0, k) \right] \frac{\langle |E_k|^2 \rangle}{4\pi} \quad (4.1)$$

Here the left hand side is the effective rate of pump energy depletion and the right hand side is the rate of growth of the energy of the excited electrostatic waves. (Note -- we must subtract the growth rate for $E_0 = 0$, a negative contribution corresponding to damping, since we want only that part of the growth rate due to the pump.) Alternatively, the nonlinear conductivity can be viewed as arising from the interaction of the pump wave with the enhanced density fluctuation levels arising from the instability.⁽⁴⁾ The well-known Dawson-Oberman formula^(16a) predicts an increased σ_{NL} arising from the enhanced ion density fluctuation level. Not so well-known is the increased σ_{NL} due to enhanced electron-density fluctuations which is usually a much weaker effect^(16b) but may dominate in a case to be discussed below. It is clear that to compute the wave intensity $I(\underline{k}) = \langle |E_k|^2 \rangle$, we need a saturated or nonlinear theory of the instability since E_k grows without bound as $\exp[\gamma_g(E_0, k)t]$ in the linear theory, and no finite steady state wave energy would result.

In the linearized theory discussed above for fixed plasma parameters, only a finite region of $\underline{k}$ space is unstable, i.e., corresponds to $\gamma_g(E_0, k) > 0$. This is the lense shaped region shown in Fig. 3a which is centered at $\underline{k}_m$ for which the frequency matching

condition $\Delta\omega(k) = \omega_0 - \omega_e(k) - \omega_i(k) = 0$ is exactly satisfied. The unstable region around k_m increases in size as ϵ_0 increases. For the cases $k_m \lesssim 2k_{De}$, the primary saturation mechanism of the electron-ion decay instability for equal electron and ion temperatures has been shown (7, 16, 17) to be the process of induced scattering of Langmuir waves from ions. This process is diagrammed schematically in Fig. 3b. Note that this is a ladder of successive induced scattering processes beginning with the primary interaction

(b) Induced Scattering of Langmuir Waves
from Ions

Parametric Ladder Scatters Langmuir
Waves Out of Unstable Region

Figure 3. a) Linearly unstable regions of k space for e-i decay instability are the lens shaped regions at $k \approx k_m$ centered about the direction of the pump $\vec{E}_0$. b) Schematic representation of the saturation process of induced scattering of Langmuir waves from ions.

which is the induced conversion of a transverse e.m. wave off ions into a Langmuir wave which in turn acts as a pump to scatter into still another Langmuir wave, etc. The system is stabilized by scattering waves out of the unstable region of k space into a stable region in which they are collisionally or Landau damped. Recent detailed numerical solutions⁽¹⁸⁾ of the nonlinear equations describing this process have produced a saturated wave spectrum concentrated in umbrella shaped regions as shown schematically in Fig. 4a. These regions spread toward values of k lower than k_m as shown schematically in Fig. 4b which shows the angle averaged intensity $\bar{I}_k = \frac{1}{2} \int_{-1}^1 d\mu I(k, \mu)$ where μ is the cosine of the angle between k and k_0 . Early analytical work^(16,17) which neglected the fine structure of the spectrum resulted in a curve such as the dotted curves in Fig. 4b. The spectrum gets higher and spreads to lower k as E_0 increases resulting in a total enhanced electrostatic wave energy proportional to E_0^4 , a result also found in the detailed numerical results. The peaks in the fine structure of the spectrum are separated by multiples of roughly $\Delta k = 2\omega(k)/V(k) = (2/3)k \sqrt{m_e/m_i}$ where $\omega \approx kc$ is an ion acoustic frequency, and $V(k)$ is the group velocity of the Langmuir waves. This saturated wave spectrum can be used in Eq. (4.1) to compute a nonlinear conductivity (or effective collision frequency)⁽¹⁹⁾

$$\sigma^{NL}(E_0) = \eta \frac{\omega_{pe} E_0^2}{16\pi n T_e} \quad (4.2)$$

where η varies between $1/3$ and $4/3$ depending on the details of the nonlinear theory used.⁽¹⁹⁾ As E_0 increases, the spectrum described above spreads to $k = 0$, and for higher values of E_0 , the total energy goes as E_0^2 (instead of E_0^4) with the result that σ^{NL} asymptotically approaches a constant value independent of E_0 . Application of the Manely-Rowe relations to the electron-ion decay instability shows that the energy in the low frequency mode (ion acoustic waves) is much less than the energy in Langmuir waves. Thus it can be shown⁽¹⁶⁾ that this enhanced conductivity arises from enhanced electron-density fluctuations rather than ion fluctuations.

Because of the flow of energy from pump into electrostatic waves, the amplitude of the pump inside the plasma is less than that predicted by linear theory for the same boundary conditions. To compute the steady state pump amplitude self-consistently inside an inhomogeneous plasma, it is necessary to solve the wave equation

$$\nabla^2 E_0(r) + \frac{\omega_0^2}{c^2} \left(1 - \frac{\omega_p^2(r)}{\omega_0^2} \left[1 - \frac{iv_{ei}}{\omega_0} - \frac{i\sigma^{NL}(E_0, r)}{\omega_0} \right] \right) E_0(r) = 0. \quad (4.3)$$

Figure 4. a) Regions of maximum intensity of the saturated e-i decay instability spectrum are shown to the right. The linear unstable region is shown to the left for comparison. b) Angle averaged spectrum of Langmuir waves. Solid lines represent structure found in detailed numerical solutions. Dashed lines show results of approximate analytical theory. Spectrum in purely growing region is purely schematic.

Here the term proportional $\sigma^{NL}(E_0, r)$ represents (the time derivative of) the currents associated with the excited electrostatic waves which react back on the pump field and the term involving ν_{ei} is the usual collisional absorption. In an inhomogeneous plasma, $\sigma^{NL}(E_0, r)$ can be obtained from a homogeneous calculation by replacing the plasma parameters by their local values if these do not vary appreciably over the distance of an electron mean free path.

In Fig. 5, typical numerical results are shown for a plane-layered plasma whose density varies linearly in the z direction. (19) The nonlinear conductivity discussed above was used in these calculations, with some additional modifications to take into account Landau damping which limits the instability to a narrow region in z near the reflection point $\omega_o = \omega_{pe}$ in which the matched k values of the excited Langmuir waves are much smaller than $k_{De}(z)$. In Fig. 5a, the ratio of local wave intensity squared to incident laser intensity is plotted versus distance from the reflection point $z = 0$ for which $\omega_o(0) = \omega_o$ for parameters typical of Nd-glass laser interactions. Here z is the density scale height H in units of free space wavelengths c/ω_o . The upper curve includes only linear collisional absorption while the lower curve includes the nonlinear absorption for an incident intensity of 10^{14} W/cm^2 . Note that the standing wave pattern and swelling behavior persist for rather strong nonlinear absorptions. In Fig. 5b, the net Poynting flux of the pump wave versus distance is plotted for the same two cases; the regions of largest slope indicating the regions of maximum absorption of energy. In Fig. 5c, the total reflection coefficient is plotted as a function of incident intensity for typical Nd-glass laser parameters.

This is, of course, a very idealized theory since it neglects the effects of strong gradients which themselves may saturate the instability and effect of the backscatter instabilities which may reduce the electromagnetic energy which reaches the absorption region near $\omega_o = \omega_{pe}$. In this regard, one should consider E_{inc} to be the net intensity incident on the absorption region, subtracting away the backscattered intensity. This calculation is included here

Figure 5. a) Radiation amplitude squared (in units of incident amplitude) versus distance (in units of c/ω). The incident intensity is 10^{14} W/cm^2 . The plasma parameters correspond to a Nd-glass laser ($\lambda = 1.06 \mu\text{m}$) incident on a deuterium plasma ($\alpha = \sqrt{m_e/m_i} = 0.0165$). b) Negative Poynting Flux (in units of incident flux) versus distance for same parameters as above. c) Total reflection coefficient versus incident laser intensity from an overdense plasma for parameters above. Upper and lower curves correspond to values $1/3$ and $4/3$ respectively for the scaling parameter η . Vertical lines labelled by values of $P_c = E^2/E_{th}^2$, the ratio of local pump at the largest maxima in Fig. 5a to threshold power.

only as an example of a self-consistent theory. The problem of oblique incidence with E in the plane of incidence is of great interest. Here the incident electromagnetic wave can directly resonantly excite a longitudinal wave via the density gradient. This wave in turn can drive a parametric instability. It can be shown⁽¹⁹⁾ that a maximum absorption of 50% can arise from this effect. In a sufficiently weak gradient, the normal incidence case can lead to more absorption but in stronger gradients, oblique incidence absorption will dominate.

The nonlinear theory on which the calculations above are based has a number of important limitations. First, if $T_e = T_i$, as assumed, the purely growing instability discussed above has a threshold nearly equal to that of the decay instability. As mentioned above, this instability excites a zero frequency mode and a high frequency electrostatic wave with frequency exactly equal to ω_o and $k > k_c = \sqrt{(\omega_o^2 - \omega_p^2)/3V^2}$. This high frequency mode can decay by the induced scattering process into an electrostatic wave in the decay instability region $k < k_c$. This decay provides another parametric ladder process which tends to stabilize the purely growing instability. The spectrum for $k < k_c$ looks like Fig. 5b except that the peaks are displaced to lie between those in Fig. 5b. When $\omega_o > \omega_{pe}$, this process can increase the energy in the decay region ($k < k_c$) and thereby increase the nonlinear conductivity. This effect can be incorporated into the factor η in Eq. (4.2). In addition, computer simulations and numerical solution of coupled fluid equations show that the wave energy remaining in the region $k > k_c$ spreads toward larger k as P increases.⁽¹⁰⁾ These high k waves also contribute to σ_{NL} and provide low phase velocity waves to accelerate electrons to produce the high energy tails observed in the simulations and in experiments.^(10,20)

If $T_e \gg T_i$, the ion waves become weakly damped and the threshold for the decay instability is lower than that for the purely growing instability by a factor of $\sqrt{m_e/m_i}$. In this case, it is possible to excite only the decay instability. The nonlinear theory is complicated by the necessity of including ion wave nonlinearities. Some attempts^(21,22) have been made to do this. There are indications⁽²²⁾ that the usual weak-turbulence theories for the ion waves are not adequate to describe the saturation of the ion waves and that an orbit-perturbation approach may be necessary. However, under most conditions it is found that the spectrum, and more certainly, the total wave energy at saturation is unchanged from the result found for $T_e = T_i$, i.e., the inclusion of ion wave nonlinearities has little effect. Even in the case of Langmuir waves, the orbit-perturbation approach may provide the strongest nonlinearities for wavenumbers $k \gtrsim .2k_D$ where Landau damping effects begin to become important.⁽²³⁾ In this theory, the parametrically excited waves are locally absorbed in the unstable region of k space by a sort of resonance broadened Landau damping in which the resonance

width increases with the wave energy. The saturated spectrum is therefore roughly confined to the unstable region of the linear theory. The total wave energy predicted by this theory⁽²³⁾ is in qualitative accord with computer simulations⁽²⁴⁾ but the wave spectrum is not broadened in k as in the simulations.⁽¹⁰⁾ The induced scattering saturation mechanism discussed above also does not lead to results completely in accord with simulations.⁽²⁵⁾ This may be due to the inability of the simulation codes to treat close range particle-particle collisions accurately, or to a failure of the random phase approximation made in weak turbulence theory or the inapplicability of this approximation to the simulations.

A recent investigation of the weak turbulence saturation of the electron-electron decay instability ($\omega_o \approx 2\omega_{pe}$) has shown that induced scattering from ions is again the strongest nonlinear effect. Preliminary results⁽²⁶⁾ indicate that the total saturation wave energy in this case is proportional to $(k_o/k_m)^4 (m_i/m_e) E^4$ where k_o is the pump wave number and k_m the matched wavenumber ($\omega_o = 2\sqrt{\omega_{pe}^2 + 3k_m^2 v_e^2}$). This saturation level is generally much lower than that obtained^p for the electron-ion decay instability. Similar considerations can be applied to saturation of SRS.

Computer simulations^(5e, 10, 24) have been extremely important in demonstrating the dominant nonlinear effects for cases of strong pumping $v_o/v_e \gtrsim .1$. Here such highly nonlinear phenomena as particle trapping and wave breaking become important. In the back-scatter instabilities, pump depletion is seen to be an important saturation mechanism.

Among the important remaining theoretical problems is the need to study regimes in which the analytical weak turbulence theory and the computer simulations both apply so that we can better understand the limitations of both techniques.

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