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ANALOG COMPUTER

- B-5 TESTS -

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SUMMARY

The response of a single degree of freedom system to excitation by a sinusoidal lateral displacement of its support structure was determined on the analog computer. The system was damped by coulomb friction and coupled by linear springs. Presented in this report is an explanation of the response characteristics, some typical curves of displacements, velocities, etc., and the computer circuit diagram. The constants used in the analog program were approximately equal in magnitude to those of the B-5 partial core.

DISCUSSION

Friction or coulomb damping P is a form of energy dissipation caused by the rubbing of two surfaces. It is a function of normal pressure and the condition of the sliding surfaces, but not of velocity as is viscous damping. The force required to "break" static friction is usually larger than the sliding friction force. In this problem the magnitude of friction is considered to be constant.

A model of the system is shown in Figure 1.

The equation of motion can be written as:

$$M\ddot{X}_2 + K(X_2 - X_1) \pm P = 0$$

A response curve plotted in dimensionless form is shown in Figure 2. The dotted line represents the boundary between non-stop or slip motion and motion with stops occurring. The parameter $\frac{P}{KX_1}$ is the ratio of the frictional damping force to the input force transmitted by the spring K , both acting on the mass M . ω_n is the undamped natural frequency, 10.3 cps for the constants used in this problem.

The response is typical of a system under the influence of coulomb damping. For low values of $\frac{P}{KX_1}$ the response become infinite. This occurrence can be explained using energy considerations. The input force is proportional to amplitude while the friction force is a constant. The energy dissipated per cycle by each is proportional to amplitude. Input energy to the system is therefore proportional to the square of the amplitude while that dissipated by frictional damping is proportional only to the first power of amplitude. A critical ratio exists, $\frac{\pi}{4}$ as first determined by Prof. Den Hartog, below which the frictional damping energy is not sufficient to prevent an infinite response.

As the input amplitude X_1 is increased, the break-away frequency decreases, break-away occurring when X_2 no longer follows X_1 . Slightly above the break-away frequency the sum of the inertia and spring forces are not always greater than the friction force and as a result stops occur in the motion for a portion of the cycle, i.e. $(X_2 - X_1) = \text{const.}$

During slip motion the friction force P is a maximum positive or negative value always acting in a direction opposite to the relative velocity.

During a stop in the relative motion the friction force counteracts the inertia force and the constant spring force. The mass is effectively connected rigidly to the input making the response acceleration $\ddot{X}_2$ equal to $\ddot{X}_1$. As the inertia force $M\ddot{X}_1$ increases P is limited at its maximum value and slip motion again occurs.

At frequencies lower than break-away, the mass follows the input exactly, the inertia force never becoming great enough to "break" friction.

Figures 3, 4, 5 and 6 are the response curves at various frequencies to an input amplitude of .1" $\frac{P}{KX_1} = 1.1$. These curves clearly indicate the sequence of events as they occur with increasing input frequency. Reading from top to bottom of the page these curves are of:

Input Displacement	X_1	
Relative Displacement	Y	$Y = X_2 - X_1$
Response Acceleration	$\ddot{X}_2$	
Response Velocity	$\dot{X}_2$	
Response Displacement	X_2	
Friction Force	P	

Figure 1 represents a point slightly above break-away frequency. Stops or freezing occurs for a large percentage of the period, as evidenced by the large flat portion on the relative response curve Y . During the stop $\ddot{X}_2$ equals $\ddot{X}_1$ (not shown) and the friction force assumes the same shape as the acceleration.

$$P = K(X_2 - X_1) + M\ddot{X}_1$$

As the input inertia force $M\ddot{X}_1$ increases friction is limited or "broken" and slip motion occurs.

Figure 4 represents a point near the dotted boundry shown on Figure 2. At this higher frequency the input inertial force has increased in magnitude ($\ddot{X}_1 = \omega^2 X_1$) and as a result the sum of the spring and input forces are enough to overcome frictional resistance to relative motion for all but a small portion of the cycle.

When the frequency is increased to 13 cps, Figure 5, the inertia and spring forces are always great enough to overcome friction and no stops occur in the motion.

At much higher frequencies damping becomes essentially ineffective, as in the case of viscous damping, and the response X_2 approaches zero.

Examination of the curves shown in Figure 7 reveal the effect on the response waveshapes produced by the discontinuous nature of coulomb friction. It can be seen that the curvature of the response displacement X_2 is non-symmetrical. Displacement

curvature is proportional to the second derivative $\ddot{X}_2$ - acceleration. The acceleration or inertia force in turn is the sum of all forces acting on the mass. The abrupt reversal in sign of the friction force thus produces an abrupt change in the acceleration and a corresponding change in curvature of the displacement. This change is reflected as a discontinuity on the velocity response curve $\dot{X}_2$ at the point of friction reversal.

The analog circuit is shown in Figure 8. The calculation of coulomb friction force P can be explained using this circuit diagram. The relative velocity is calculated and put into the high gain amplifier. The purpose of the diode bridge is to put a hard limit on the output voltage equal to $P_{\max}$. A change in the sign of the relative velocity changes the sign of P and during oscillation P switches between plus and minus $P_{\max}$. The operation of this circuit is such that even a small difference in velocities $\dot{X}_1$ and $\dot{X}_2$ causes voltage P to limit at $P_{\max}$ in the direction to reduce this velocity difference. Should this relative velocity go to zero, stop motion, voltage P will maintain this state until it limits at $P_{\max}$ and $\dot{X}_2$ can no longer follow $\dot{X}_1$.

CONCLUSIONS

A theoretical model of the B-5 partial core has been developed and analyzed. Comparing test results with these theoretical results have enabled us to:

1. Determine how closely the B-5 system resembles a simple analytical model.
2. Better understand the characteristics of the response data.
3. Better plan and execute future vibration tests.

FIGURE 2
"RESPONSE CURVE"

FIGURE 3
ANALOG CURVE

FIGURE 4
ANALOG CURVE

FIGURE 5
ANALOG CURVE

FIGURE 6
ANALOG CURVE

FIGURE 7
ANALOG CURVE

FIGURE 8
"ANALOG CIRCUIT"