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EFFECT OF EXTERNAL PLASMA ON SIMPLE MIRROR REACTOR Q

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EFFECT OF EXTERNAL PLASMA ON SIMPLE MIRROR REACTOR Q

ABSTRACT

The plasma energy drain incurred by the presence of cold plasma outside the mirrors of a simple mirror reactor is calculated as a function of ϵ (ϵ is defined as the density of external cold plasma divided by the density of hot interior plasma). The resultant lowering of the reactor Q (Q is defined as fusion power divided by injected power) is shown to be significant ($Q < 1$) for $\epsilon > 5 \times 10^{-3}$, with a maximum reduction in Q by a factor of 7 as $\epsilon \rightarrow 1$.

INTRODUCTION

Recently, there has been renewed interest in simple magnetic mirror geometries stabilized by line-tying to cold external plasma. The question arises as to whether an economic energy balance determined by $Q \gtrsim 1$, where Q is defined as fusion power divided by injected power, can be maintained in the face of the inevitable energy drain on the hot plasma by the cold plasma. The purpose of this work is to calculate Q as a function of ϵ (ϵ is defined as cold external plasma density divided by hot central plasma density), using approximate analytical methods. A given set of initial reactor parameters will be assumed to illustrate the effect of external cold plasma on the reactor Q. No attempt will be made to vary the initial reactor parameters to optimize Q. However, the possibility of recovering $Q > 1$ at $\epsilon = 1$ in a two-component mode of operation will be discussed.

SIMPLE MIRROR REACTOR MODEL

Table 1 gives the parameters for a simple mirror reactor (no cold plasma injection) as provided by Gus Carlson,¹ along with appropriate plasma quantities obtained from Fokker-Planck calculations.² An effective area A_{eff} and length L_{eff} can be defined such that

$$\frac{n_0^2 A_{eff} L_{eff}}{(\pi r)} \equiv \frac{I_h}{q} \quad (1)$$

and

$$\frac{L_{eff}}{\sqrt{\frac{A_{eff}}{\pi}}} \equiv \frac{L}{r_p}, \quad (2)$$

where $n\tau$ is the average ion confinement parameter which depends on E_0 , R , and kT_e . With $R = 10$ and $E_0 = 200$ keV, a multi-species Fokker-Planck calculation by Futch,² et al. gives $n\tau = 7.24 \times 10^{13}$ for the above model with spatially uniform plasma (no external gas source). With the other parameters from Table 1, Eqs. (1) and (2) give $A_{\text{eff}} = 7 \text{ m}^2$ and $L_{\text{eff}} = 4.3 \text{ m}$. This effective area and length will be useful for calculating particle and energy losses from the plasma, and comparing results with spatially-independent Fokker-Planck calculations.

Table 1. Simple mirror reactor parameters
(no cold gas).

Injection energy $E_0 = 200$ keV
Fusion power $P_{\text{fus}} = 1000$ MW
$B_0, \text{ vac} = 50$ kG
$\beta = 0.9$
$R = R_{\text{vac}} / \sqrt{1-\beta} = 9.5$
Plasma length $L = 14.1$ m
Plasma radius $r_p = 3.21$ m
Peak ion density $n_0 = 2.34 \times 10^{14} \text{ cm}^{-3}$
Reactor $Q = 1.41$
Injection current $I_h = 3500$ amperes
Average ion energy $\bar{E}_i = 242$ keV (including α particles)
Electron temperature $kT_e = 21.5$ keV
Ambipolar potential $q\phi_m = 5 kT_e$

Figures 1a and 1b show qualitatively the field, density, and potential profiles of the reactor model with exterior expander regions where the field is decreased to very low values before reaching the walls. A large expansion ratio (≥ 100) is necessary to reduce the power flux density of escaping plasma to tolerable levels at the wall.

For the purposes of line tying, the average line density $\int n \cdot ds$ over the length of the flux tube between the mirror and wall is probably more relevant than the average exterior density by itself. For gas sources at the mirror, the line density is roughly constant along the flux expander for any expansion ratio, simplifying the treatment of such expanders in line-tying theories. Placing a gas source at the end

walls for $n_{\text{ext}}(z) = \text{const.}$ along the expander would result in a higher average line density, which may allow a lower value of n_{ext} for the same degree of line tying. However, the potential drop in the expander, if any, would have to be provided by a discrete sheath at the wall. Whether or not such a sheath could exist depends on surface effects, such as secondary emission by electrons and ions, which is difficult to predict with a high neutral density at the wall. Besides, a line-tied, low Q , simple mirror reactor may require direct converters which are not compatible with a gas source at the ends. For electron heat loss to be manageable, some retarding sheath potential is necessary. To ensure at least a distributed potential drop, the gas sources are located just outside the mirror throats, allowing the cold plasma density to fall to a low value at the walls.

The expanding field converts the electrons' perpendicular energy to parallel energy, which sets up the distributed potential sufficient to eventually slow the electrons to the accelerated ion velocities before reaching the wall. The electrons collected at the wall at the total ion loss rate then have negligible kinetic energy, and electrons

Fig. 1. Field, density, and potential profiles of a reactor model.

injected with the ions at the midplane are counted as lost, carrying away an energy of $q\phi_{\text{tot}}$. The resulting loss surface, with expanders external to the mirrors, are portions of a spherical surface intersected by the loss cones in velocity space, for electrons at the midplane (Fig. 1c). Electrons at the midplane occupying the double-hatched region of one loss cone between the energies $q\phi_m$ and $q\phi_{\text{tot}}$ occupy a hemisphere of radius $\sqrt{2\phi_{\text{ext}}/m_e}$ in velocity space at the mirror throat, with the other half-sphere there filled by electrons reflected by the exterior potential drop in the expander (Fig. 1d). Electrons escape from the system by collisional diffusion which increases their energy above the top of the potential barrier ϕ_{tot} out in the expander. The average incremental energy above ϕ_{tot} with which an electron escapes is small compared to ϕ_{tot} (and kT_e) because under reactor conditions the electron scattering mean-free-path, especially for electrons out in the Maxwellian tail, is much longer than the system length. This point will be discussed further in a following section on electron loss rates.

ION LOSS RATES WITH COLD PLASMA SOURCE

Gordon Hamilton³ has extended the Fokker-Planck calculations of Futch, et al.² to include the effect of adding cold electrons at the midplane. The resulting decrease in electron temperature for a given current of cold electrons injected at the midplane will, in general, be different for the same current of cold electrons injected at the mirror throat, since in the latter case the electrons will fall into the potential drop of the mirror, equivalent to injection of electrons of energy $q\phi_m$ at the midplane. Also, Hamilton assumed an absorbing boundary for particles at the mirror, whereas here the ambipolar potential profile will be altered with external expanders. However, as long as cold ions from the gas source do not appreciably interact with the hot mirror-confined ions (penetration limited by $q\phi_m$) and since $n\tau$ for the hot ions is not substantially affected by the mirror potential drop for large mirror ratios,¹ the effect of injecting electrons in either manner is mainly to change the drag on hot ions by lowering the electron temperature in different amounts. Therefore the results of Hamilton's calculations on reactor Q and $n\tau$ are plotted in Fig. 2 as a function of kT_e rather than the electron injection rate, and the purpose of this work is to calculate the equilibrium electron temperature for injection of cold electrons at the mirror, and then use Fig. 2 to obtain Q . Hamilton's calculations show that for a fixed hot ion injection energy E_0 , lowering kT_e by cold electron injection results in a lower average ion energy $\bar{E}_i$ as well as a lower $n\tau$ due to increased slowing down of hot ions relative to side scatter. In order for the effective mirror ratio to remain high as T_e decreases, the plasma beta $\sim n_0 \bar{E}_i$ must be kept constant, so n_0 must be increased as $1/\bar{E}_i$, starting with n_0 from Table 1. The result using Hamilton's results for $\bar{E}_i$ (kT_e) is shown in Fig. 2. The loss rate of hot mirror confined ions J_h is obtained from Eq. (1):

Fig. 2. Q , $n\tau$, and n_0 as a function of kT_e . $E_0 = 200$ keV, $R = 10$, $\beta = \text{constant}$.

$$J_h (kT_e) = \frac{I}{q} = A_{\text{eff}} L_{\text{eff}} \left(\frac{n_0^2}{nT} \right), \quad (3)$$

where (n_0^2/nT) is obtained from Fig. 2.

Since the hot ions that escape the mirror have a parallel energy $\approx \phi_m$ at the mirror, the density of hot ions n_h at the mirror can be estimated from

$$n_h(z_m) \sqrt{\frac{2\phi_m}{m_i}} \frac{A_{\text{eff}}}{R} \approx J_h. \quad (4)$$

Using Eq. (3) with Table 1, a minimum value of $n_h(z_m)/n_0 \approx 4 \times 10^{-5}$ is obtained.

For $z > z_m$, n_h falls roughly along with B for $\phi_{\text{ext}} \ll \phi_m$.

The injection of a density n_c of cold ions at a mirror raises the density not only at the source in the mirror throat, but also to some distance z_0 into the mirror-confined plasma, because the mirror drop ϕ_m is distributed over the hot ion density profile (see Fig. 1b). A local minimum in the total electron density and potential cannot exist if there is no local particle sink, so the total density and potential must decrease monotonically away from the center. A small gradient in ϕ between z_0 and z_m supports the cold ion pressure, so that the cold ions do not penetrate all the way between the mirrors unless the external density is raised to the order of n_0 , and $\phi_m \rightarrow 0$.

The cold ion confinement time (a sound transit time over a distance $z_m - z_0$) is sufficiently short compared to equilibration times with hot ions and electrons that kT_i (cold) $\ll kT_e$ and $q\phi_m$. Thus, the total density and potential is nearly flat between z_0 and z_m . The point z_0 at which $n_{\text{ext}}(z_m) = n_c(z_m) + n_h(z_m) \approx n_h(z_0)$ approaches the midplane as $n_{\text{ext}} \rightarrow n_0$.

The rate of cold ion injection J_c required to maintain a stagnant density $n_c(z_m)$ just behind the gas sources has been found by Lee and McNamara⁴ to be given by

$$J_c \approx 2 \left(\frac{1}{2} n_c(z_m) \right) \left(\sqrt{\frac{kT_e}{m_i}} \right) \left(\frac{A_{\text{eff}}}{R} \right), \quad (5)$$

where a flow velocity just outside the mirrors of $C_s = \sqrt{(kT_e + 5/3 kT_i)/m_i} \approx \sqrt{kT_e/m_i}$ is assumed to satisfy the Bohm sheath condition. A quantity that can be easily measured experimentally and related directly to theories of line tying is the ratio of external density at the mirror throat n_{ext} to the central density n_0 , defined as

$$\epsilon \equiv \frac{n_{\text{ext}}}{n_0} = \frac{n_c(z_m) + n_h(z_m)}{n_c(0) + n_h(0)}. \quad (6)$$

For $n_c = 0$, ϵ has a minimum of 4×10^{-5} , given by Eq. 4, due to the escaping hot ions. For $q\phi_m \gg kT_i$ (cold), the cold ions do not penetrate to the midplane and $n_c(0) \approx 0$. As ϵ approaches unity and $q\phi_m$ becomes sufficiently small compared to kT_i (cold), the cold plasma can dominate both inside and outside the mirror (the two-component limit). In this limit, cold ion drag on the hot ions is important, raising kT_i (cold) $\gtrsim kT_e$.⁵ However, over most of the range $4 \times 10^{-5} \ll \epsilon \leq 1$, $\epsilon \approx n_c(z_m)/n_h(0)$.

ELECTRON LOSS RATES (NO COLD PLASMA)

For the reactor parameters assumed in this study, the electron-electron scattering m.f.p. (for a test electron energy at the average electron energy) $\lambda_{ee} > 10^4$ meters, which is much longer than the length of the high density plasma region where scattering can occur (collisions are negligible in the expander). For electrons near the ambipolar potential energy $\gg kT_e$, the collision length and diffusion times in velocity space are even longer. Once an electron's energy exceeds the ambipolar potential, it can be lost in one bounce time $t_b \sim 10^{-8}$ s. Therefore, if classical collisions are the only available mechanism for populating the high energy tail of the electron distribution, the distribution of electrons above the ambipolar energy will be depleted from that of a Maxwellian, and the electron loss rate will be controlled by the rate at which they diffuse to the edge of the ambipolar cutoff. In other words, if we could assume the distribution to be completely Maxwellian, the electron loss rate would be calculated by integrating (vf) over the volume of velocity space occupied by the loss region for electrons (Fig. 1c). But if the loss region is depleted, the loss rate should be calculated by

$$J_e = -A_{\text{eff}} L_{\text{eff}} \int D_v \nabla_v f d S_v \quad (7)$$

"Loss surface in
velocity space",

where D_v is the velocity space diffusion coefficient, and f is an appropriate distribution which decreases smoothly to zero at the loss surface.

Larry Hall⁶ has suggested that electrons always Maxwellianize much faster than with classical collisions due to sheath oscillations. Early experimental evidence for this by Langmuir,⁷ and a later experiment by Gabor et al.,⁸ are quoted in support of this view. However, a more recent experiment by Harp⁹ indicates that quiescent sheaths can exist without rf fluctuations. The fact that sheath oscillations sometimes do occur may be due to disturbances elsewhere in the plasma propagating to the sheath, and not due to a fundamentally unstable nature of the sheath itself.

One result of a depleted electron distribution is a lower ambipolar potential to keep the electron loss rates in balance. Futch,² et al. find $q\phi = 5 kT_e$ with a Fokker-Planck treatment for our model reactor parameters, while an assumption of a Maxwellian distribution would give $q\phi = kT_e \ln(n_0/n_{\text{ext}}) \approx 10 kT_e$ for $n_{\text{ext}} = 4 \times 10^{-5} n_0$. With n_{ext} reduced by another factor of 10^3 in the expander, a total drop $q\phi \approx 17 kT_e$ would be calculated using a Boltzman factor. The resulting higher energy lost per electron using a Maxwellian is necessary to explain the energy loss from the 2XII experiment, according to Hall. However, he assumes one hot electron removed from the plasma per ion lost, which implies a non-emitting wall. Since plasma potentials were not directly measured in the 2XII experiment, the measured values of n_e , T_e , W_i , and τ can be as equally-well accounted for by assuming a smaller value of $q\phi/kT_e$, and some nonzero secondary emission coefficient which results in more than one hot electron lost per ion.

Whether or not a cut-off electron distribution is more appropriate than a complete Maxwellian is yet to be verified by experiment, but for the purposes of this study, a cut-off electron distribution will be assumed.

Returning to Eq. (7), a reasonable model for the cut-off electron distribution function is needed. Berk¹⁰ has found an analytic solution to the Fokker-Planck equation for electrons at the midplane ($\phi = 0$), with a source at $v = 0$, and for the boundary condition $f \rightarrow 0$ as $v \rightarrow v_{\max}$:

$$f(v) = e^{-v^2/\bar{v}^2} \int_v^{v_{\max}} dv e^{+u^2/\bar{v}^2}, \quad (8)$$

where

$$\bar{v} \equiv \sqrt{\frac{2 kT_e}{m_e}}. \quad (9)$$

Equation (8) can be approximated to within 10% for $0 < v < v_{\max}$ by

$$f(v) \approx e^{-v^2/\bar{v}^2} - e^{-v_{\max}^2/\bar{v}^2}. \quad (10)$$

Normalizing this f so that $n_0 = \int_0^{v_{\max}} f(v) 4\pi v^2 dv$ gives:

$$f(v) = \frac{n_0}{4\pi \bar{v}^3 \left[\frac{\pi}{4} - \left(\frac{\alpha}{2} + \frac{\alpha^3}{3} \right) e^{-\alpha^2} \right]} \left\{ e^{-v^2/\bar{v}^2} - e^{-\alpha^2} \right\}, \quad (11)$$

where

$$\alpha^2 \equiv v_{\max}^2/\bar{v}^2 = q\phi_{\text{tot}}/kT_e. \quad (12)$$

This f approaches a Maxwellian for $\alpha^2 \gg 1$, and $v \ll v_{\max}$. It also is spherically symmetric, whereas the loss surface of Fig. 1c would tend to make the distribution anisotropic for energies near $q\phi_{\text{tot}}$. Consider the diffusion coefficients¹¹ that would obtain if f were Maxwellian, which should be a good approximation for the bulk of the electrons if $q\phi_{\text{tot}} \gg kT_e$:

$$\langle (\Delta v_{||})^2 \rangle \approx \frac{C}{v} G\left(\frac{v}{\bar{v}}\right), \quad (13)$$

$$\langle (\Delta v_{\perp})^2 \rangle \approx \frac{C}{v} \left\{ \Phi\left(\frac{v}{\bar{v}}\right) - G\left(\frac{v}{\bar{v}}\right) \right\}, \quad (14)$$

where the constant $C = 3 \times 10^{19} n_0$, n is in cm^{-3} , and v is in cm/sec . For $v \gg \bar{v}$ near the loss surface, the functions G and $\Phi - G$ can be approximated by

$$G \approx \frac{\bar{v}^2}{2v^2} \quad (15)$$

and

$$\Phi - G \approx 1, \quad (16)$$

so the diffusion coefficients become

$$\langle (\Delta v_{||})^2 \rangle \approx 3 \times 10^{19} \frac{n_0}{v} \left(\frac{\bar{v}^2}{2v^2} \right) \left(\frac{(\text{cm/sec})^2}{\text{sec}} \right), \quad (17)$$

$$\langle (\Delta v_{\perp})^2 \rangle \approx 3 \times 10^{19} \frac{n_0}{v}. \quad (18)$$

For $v \gg \bar{v}$, diffusion in angle is much faster than diffusion in radius in velocity space, which tends to keep the distribution below $q\phi_{\text{tot}}$ fairly isotropic. Above $q\phi_{\text{tot}}$ and at perpendicular velocities well outside the loss cone, Fokker-Planck calculations¹² show that side diffusion is not sufficient to keep the distribution cut off at $q\phi_{\text{tot}}$ at angles $\theta \gg \theta_c$ (see Fig. 1c). However, electrons diffusing beyond $q\phi_{\text{tot}}$ outside the loss cone are mostly balanced by electrons beyond $q\phi_{\text{tot}}$ slowing down by dynamical friction, as in a steady state Maxwellian. Because of the decrease in diffusion rates at higher velocities, the portion of the loss cone surfaces nearest the origin in velocity space will collect the bulk of the escaping electrons. If one neglects the loss of electrons across the lateral surfaces of the loss cones in Fig. 1c compared to the diffusion across the spherical surfaces intersected by the loss cones, Eq. (7) can be approximated by

$$J_e \approx - \left[\langle (\Delta v_{||})^2 \rangle \frac{df}{dv} \right]_{v=v_{\text{max}}} \cdot 4\pi v_{\text{max}}^2 \Psi A_{\text{eff}} L_{\text{eff}}, \quad (19)$$

where the diffusion coefficient is obtained from Eq. (7) for $v = v_{\text{max}}$, and Ψ is the fraction of the velocity space surface within the loss cones given by

$$\Psi = 1 - \cos \theta_c = 1 - \sqrt{1 - \frac{1}{R}}. \quad (20)$$

For $R = 10$ used in this model, Eq. (2) gives $\Psi = 0.054$. The distribution gradient at $v = v_{\text{max}}$ is computed from Eq. (11):

$$\frac{df}{dv} \approx -2 \left(\frac{n_0}{\pi^{3/2} \bar{v}^3} \right) \left(\frac{v}{\bar{v}^2} \right) e^{-v^2/\bar{v}^2}, \quad (21)$$

where we have made the approximation that $\alpha^2 \gg 1$ in the normalization constant. Thus

we have at $v = v_{\max}$:

$$\left. \frac{df}{dv} \right|_{v=v_{\max}} \approx - \left(\frac{2n_0 \alpha}{\pi^{3/2} v^4} \right) e^{-\alpha^2} \quad (22)$$

Substituting into Eq. (19), we get

$$J_e = \left(\frac{3 \times 10^{19} n_0 v^2}{2 v_{\max}^3} \right) \left(\frac{2n_0 \alpha e^{-\alpha^2}}{\pi^{3/2} v^4} \right) (4\pi v_{\max}^2) \Psi A_{\text{eff}} L_{\text{eff}},$$

or

$$J_e = \frac{1.2 \times 10^{20} n_0^2 e^{-\alpha^2} \Psi A_{\text{eff}} L_{\text{eff}}}{\sqrt{\pi} v^3} \quad (23)$$

Equating the electron loss rate to the hot ion loss rate gives an equation for α (no cold gas) in terms of kT_e and $n\tau$:

$$J_e = J_h \quad (24)$$

or

$$e^{-\alpha^2} = \frac{\sqrt{\pi} v^3}{1.2 \times 10^{20} n\tau \Psi} \quad (25)$$

where we have used Eq. (23) for J_e and Eq. (3) for J_h . Taking the value $kT_e = 21.5$ keV and $n\tau = 7.36 \times 10^{13} \text{ cm}^{-3} \text{ s}$ from Fig. 2 corresponding to no cold electron injection, Eq. (25) yields $\alpha^2 = 5.96$, a value somewhat greater than the value $\alpha^2 = 5.0$ obtained by the Fokker-Planck calculation.² This is to be expected since the electron loss surface area at $q\phi_{\text{tot}}$ is larger with an expander than without one. We shall now show that this larger value of α with an expander is also consistent with the assumed value of kT_e . The heat input rate to the electrons from the slowing down of ions is given by

$$P_{i \rightarrow e} = \frac{A_{\text{eff}} L_{\text{eff}} n_0 \bar{E}_i}{\tau_{ie}} \quad (26)$$

where τ_{ie} is the time for slowing down of ions on electron,¹¹ which can be written

$$\tau_{ie} = 1.4 \times 10^{12} \frac{kT_e^{3/2}}{n_0} \quad (27)$$

where kT_e is in keV and n_0 is in cm^{-3} . Since each electron lost carries away no more energy than $q\phi_{\text{tot}}$ with an expander, the power balance becomes, using Eqs. (3), (24), (26), and (27):

$$\frac{A_{\text{eff}} L_{\text{eff}} n_0^2 \bar{E}_i}{1.4 \times 10^{12} kT_e^{3/2}} = (J_e)(q\phi_{\text{tot}}) = \frac{A_{\text{eff}} L_{\text{eff}} n_0^2 q\phi_{\text{tot}}}{n\tau} \quad (28)$$

or

$$kT_e^{5/2} = \frac{n\tau}{1.4 \times 10^{12}} \frac{\bar{E}_i}{\alpha^2} \quad (29)$$

where kT_e is in keV.

Putting in $n\tau$ and $\bar{E}_i$ from Table 1, and $\alpha^2 = 5.96$, Eq. (29) gives $kT_e = 22$ keV. To understand why this model with an expander should give the same temperature with a higher value of $q\phi_{\text{tot}}$ than obtained in the Fokker-Planck calculation,² consider a power balance in the form of Eq. (28) for the Fokker-Planck model,² in which one would replace the energy lost per electron $q\phi_{\text{tot}}$ with $(q\phi_{\text{FP}} + \bar{E}_{e\perp})$, where $\bar{E}_{e\perp}$ is the mean perpendicular energy of escaping electrons. A reasonable estimate for $\bar{E}_{e\perp}$ would be $\bar{E}_{e\perp} \approx kT_e$, hence $q\phi_{\text{FP}} + \bar{E}_{e\perp} \approx 6 kT_e$, in agreement with the energy lost per electron with an expander.

ELECTRON ENERGY LOSS WITH COLD PLASMA

If we now consider the injection of cold plasma at the mirrors, we have the problem that the solution for f given by Eq. (8) can no longer be used since it develops from a source of electrons at zero energy at the midplane, whereas electrons injected at the mirrors have an energy $q\phi_m$ at the midplane. This will likely not change f much at low velocities $v \ll v_{\text{max}}$, since f is Maxwellian there anyway. At energies $\phi_m \sim \phi_{\text{tot}} \gg kT_e$, however, electron injection at the mirrors will peak up the density in velocity space at ϕ_m , giving rise to a steeper distribution gradient at $q\phi_{\text{tot}}$. Thus the electron loss rate at $q\phi_{\text{tot}}$ will increase to accommodate the additional injected electrons, partly because of a steeper df/dv at v_{max} , and partly because $q\phi_{\text{tot}}$ will be lower. However, for cold plasma currents J_c small compared to J_h , we may make the approximation that $q\phi_{\text{tot}}$ is constant, and for J_c large compared to J_h we will show that kT_e is relatively insensitive to $q\phi_{\text{tot}}$, and kT_e is what we're after.

Consider now the limit $\epsilon \rightarrow 1$, corresponding to $q\phi_m \rightarrow 0$, where electrons injected from both J_c and J_h have zero energy at the midplane, and Berk's distribution function Eq. (11) is again valid. Neglecting J_h compared to J_c in this limit, we can find the minimum value of $q\phi_{\text{tot}}$ setting J_e from Eq. (23) equal to J_c from Eq. (5) with $n_c = n_0$:

$$\text{Particle balance } J_e = J_c,$$

or

$$\frac{1.2 \times 10^{20} n_0^2 e^{-\alpha^2} \Psi A_{\text{eff}} L_{\text{eff}}}{\pi v^3} = n_0 \sqrt{\frac{kT_e}{m_i}} \frac{A_{\text{eff}}}{R} \quad ,$$

or

$$e^{-\alpha^2} = 9.66 \times 10^{12} \frac{kT_e}{n_0}, \quad (30)$$

where values of constants other than n_0 and kT_e are taken from Table 1 and kT_e is in keV. Also, we must have energy balance:

$$P_{i \rightarrow e} = (J_c)(q\phi_{tot}) = J_c \alpha^2 kT_e,$$

or

$$\frac{A_{eff} L_{eff} (n_0 \bar{E}_1) n_0}{1.4 \times 10^{12} kT_e^{3/2}} = n_0 \sqrt{\frac{kT_e}{m_i}} \frac{A_{eff}}{R} \alpha^2 kT_e, \quad (31)$$

or

$$kT_e = \left(\frac{7.945}{\alpha^2} \right)^{1/3}, \quad kT_e \text{ in keV}, \quad (32)$$

where we have put in the value of $n_0 \bar{E}_1 = \beta B_0^2 / 2\mu_0 = \text{const.}$ for the parameters in Table 1. Equations (30) and (32) give two equations for the two unknowns α^2 and kT_e [n_0 (kT_e) is given in Fig. 2]. The solutions by reiterative substitution are $\alpha^2 = 3.9$ and $kT_e = 1.26$ keV. The decrease in α^2 , or decrease in $q\phi_{tot}$ relative to kT_e , is due to the relatively poor cold ion confinement. The value of α^2 can be expected to decrease monotonically from 5.96 at $J_c = 0$ to 3.9 at $\epsilon \rightarrow 1$, due to steadily decreasing average total ion confinement time (hot + cold). This slow decrease in α^2 also results in Hamilton's calculations² with decreasing nT due to cold electron injection at the midplane. Since kT_e varies slowly with α^2 according to Eq. (32), only small errors in kT_e result from taking α^2 to be constant. Taking $\alpha^2 = 5.96$ in Eq. (32), for example, one obtains $kT_e = 1.1$ keV, a 15% error in kT_e . For smaller ϵ , the error is even less. For the present purposes this amount of error is acceptable. Taking $\alpha^2 = 5.96 = \text{const.}$ eliminates the need to calculate J_e explicitly in terms of α , since conservation of charge requires $J_e = J_c + J_h$, and we have the ion loss rates from Eqs. (3) and (5) to use in the energy balance equation for arbitrary ratios of J_c/J_h :

$$P_{i \rightarrow e} = (J_c)(q\phi_{ext}) + (J_h)(q\phi_{tot}). \quad (33)$$

Now that we have fixed $q\phi_{tot}$ in terms of kT_e , we need to determine the variation of $q\phi_{ext}$ with ϵ . Since the electron distribution departs from a Maxwellian for energies approaching $q\phi_{tot}$, the Boltzman factor cannot be used to calculate $q\phi_m$ for small values of ϵ at the mirror where $q\phi_m \lesssim q\phi_{tot}$. We must resort to the cut-off distribution, Eq. (11). The limitation of f to the case $J_c = 0$ does not restrict us for potentials

below $q\phi_m$, assuming that the presence of cold plasma for $J_c \neq 0$ does not significantly alter the hot ion density distribution. The density and therefore potential profile between the points $\pm z_0$ where $n_h(z_0) = n_{ext}(z_m)$ (see Fig. 1b), is nearly independent of J_c ; only the point z_0 changes with n_{ext} . Assuming the potential at $z = \pm z_0$ is nearly the same as at $z = \pm z_m$, ($kT_i \text{ cold} \approx 0$), we need only calculate

$$n_e(\phi_m) = n_h(z_0) \approx n_{ext}(z_m). \quad (34)$$

Hamilton and Moir¹³ find that magnetic effects on the electron density can be ignored; that is, $n_e(\phi, B) = n_e(\phi)$. This can be seen as a consequence of Liouville's theorem on the constancy of density in phase space. Considering electrons in the loss cones (Figs. 1c and 1d) one sees that the volume of velocity space within the loss cones expands as one moves from the midplane toward the mirror due to the increase in θ_c , but the volume of an element in configuration space correspondingly decreases along a flux tube. Only the potential gradient changes the electron density because the electric field changes the electron's energy.

The distribution given by Eq. (11) applies only to electrons at the midplane. By making the transformation

$$\frac{1}{2}mv^2 = q\phi + \frac{1}{2}mu^2, \quad (35)$$

or

$$u^2 = v^2 - \frac{2q\phi}{m}, \quad (36)$$

and

$$du = \frac{v dv}{\sqrt{v^2 - \frac{2q\phi}{m}}}, \quad (37)$$

where u is the velocity at potential ϕ , we have:

$$n_e(\phi = \phi_m) = n_{ext} = \int_{u=0}^{u=u_{max}} f(u) 4\pi u^2 du$$

or

$$n_{ext} = \int_{\sqrt{\frac{2q\phi_m}{m}}}^{\sqrt{\frac{2q\phi_{tot}}{m}}} f(v) 4\pi \left(v^2 - \frac{2q\phi_m}{m} \right) \left(\frac{v dv}{\sqrt{v^2 - \frac{2q\phi_m}{m}}} \right). \quad (38)$$

Substituting for $f(v)$ from Eq. (11), we get

$$n_{\text{ext}} = \frac{4\pi n_0}{\pi^{3/2} \bar{v}^3} \int_{v_{\min}}^{v_{\max}} \left[e^{-v^2/\bar{v}^2} - e^{-\alpha^2} \right] \left(\sqrt{v^2 - v_{\min}^2} \right) v \, dv. \quad (39)$$

Defining

$$y = \frac{v}{\bar{v}}, \quad \beta = \frac{v_{\min}}{\bar{v}}, \quad v_{\min} = \sqrt{\frac{2q\phi_m}{m}},$$

Equation (39) can be written

$$\epsilon = \frac{4}{\sqrt{\pi}} I(\alpha, \beta), \quad (40)$$

where

$$I(\alpha, \beta) = \int_{\beta}^{\alpha} \left(e^{-y^2} - e^{-\alpha^2} \right) \left(\sqrt{y^2 - \beta^2} \right) y \, dy. \quad (41)$$

The integral I was numerically evaluated for $\alpha^2 = 5.96, 5, 4$, and 2.25 , and for several values of β corresponding to values of ϕ_m/ϕ_{tot} between zero and one. The results are shown in Fig. 3. Note that $\phi_m/\phi_{\text{tot}} \rightarrow 1$ as $\epsilon \rightarrow 0$, and $\phi_m/\phi_{\text{tot}} \rightarrow 0$ as $\epsilon \rightarrow 1$. The values of ϕ_m/ϕ_{tot} for a given ϵ are rather insensitive to α^2 .

The variation of Q with ϵ can now be determined in the following manner: one starts by choosing a value of ϵ between 4×10^{-5} and 1, and uses Fig. 3 to obtain ϕ_m/ϕ_{tot} . ϕ_{ext} is then obtained from

$$q\phi_{\text{ext}} = q\phi_{\text{tot}} \left(1 - \frac{\phi_m}{\phi_{\text{tot}}} \right),$$

or

$$q\phi_{\text{ext}} = \alpha^2 kT_e \left(1 - \frac{\phi_m}{\phi_{\text{tot}}} \right). \quad (42)$$

Taking $\alpha^2 = 5.96$, this $q\phi_{\text{ext}}$ is substituted into the energy balance equation, Eq. (33):

$$\frac{A_{\text{eff}} L_{\text{eff}} (n_0 \bar{E}_i) n_0}{1.4 \times 10^{12} kT_e^{3/2}} = n_0 (\epsilon - 4 \times 10^{-5}) \left(\sqrt{\frac{kT_e}{m_i}} \right) \left(\frac{A_{\text{eff}}}{R} \right) \alpha^2 kT_e \left(1 - \frac{\phi_m}{\phi_{\text{tot}}} \right) + \frac{A_{\text{eff}} L_{\text{eff}} n_0^2}{n\tau} \left(\alpha^2 kT_e \right). \quad (43)$$

Fig. 3. Ratio of mirror drop (ϕ_m) to total drop (ϕ_{tot}) as a function of ϵ .

Equation (43) is then solved for kT_e by reiterative substitution, with values for n_0 and $n\tau$ taken from Fig. 2 for each value of kT_e . The value of Q is then given in Fig. 2 for the value of kT_e that satisfies Eq. (43). The results for kT_e and Q are plotted in Fig. 4 as a function of ϵ .

DISCUSSION OF RESULTS

Figure 4 represents the effect on Q of external plasma with a distributed sheath, with the plasma pressure (βB_0^2), geometry (A_{eff} , L_{eff}) and injection energy (E_0) held fixed. For cold plasma currents $J_c \lesssim J_h$, the temperature and Q are not greatly changed, since the additional electrons from J_c are lost with an energy $q\phi_{ext}$ which is a small fraction of $q\phi_{tot}$. Unfortunately, ϵ is small ($\sim 2 \times 10^{-4}$) for this magnitude of J_c due to the short containment time of the cold plasma; probably too small to provide much line-tying effect. For $J_c > 10 J_h$, the temperature is sufficiently lowered that $n\tau$ is dominated by electron drag, and $Q \sim n\tau \sim kT_e^{3/2} \sim 1/\sqrt{\epsilon}$. At this point ($\epsilon \sim 10^{-2}$), Q is sufficiently degraded below unity that the reactor must have an efficient direct converter to produce net power, and then the recirculating power is large. Unfortunately, a line-tying theory by Lieberman¹⁴ suggests that ϵ would have to be as high as unity to stabilize the lowest order flute mode with a plasma sheath. At $\epsilon = 1$, Q is reduced to a value ($Q = 0.2$) too low for net power unless a power-amplifying uranium blanket is used in a fission-fusion hybrid mode. A Q of 0.2 is at least acceptable for FERF, a neutron-producing test reactor, provided the parameters $E_0 = 200$ keV, $R = 10$, and $\beta = 0.9$ are attainable for FERF. A final point must be made that the low Q values and electron temperatures obtained at $\epsilon = 1$ are not invariably obtained; the values in Fig. 4 depend on the choice of parameters we made in Table 1. Inspection of the power balance in the two-component limit ($\epsilon \rightarrow 1$) by Eq. (31) shows that the kT_e increases as $(L_{eff}\beta B_0^2)^{1/3}$, so that by making the plasma longer and focusing the neutral beams into a higher density plasma (higher βB_0^2), one can force kT_e back up to high values (≈ 10 keV). Then, because only one species need be injected, an even higher Q can be obtained in the two-component mode,⁵ as indicated in Fig. 4. Thus, the two-component mode may be best for line tying ($\epsilon = 1$) for either a reactor or FERF.¹⁵

Fig. 4. kT_e and Q as a function of ϵ for a simple mirror reactor. $E_0 = 200$ keV, $R = 10$, β_e constant.

REFERENCES

1. G. Carlson, Lawrence Livermore Laboratory, private communication (Sept., 1974).
2. A. H. Futch, Jr., J. P. Holdrin, J. Killeen, and A. A. Marin, Plasma Physics 14, 211 (1972).
3. G. W. Hamilton, Lawrence Livermore Laboratory, private communication (Sept., 1974).
4. E. P. Lee and B. McNamara, Dynamics of a Plasma Discharge in a Mirror Field, Lawrence Livermore Laboratory, Rept. UCID-16445 (Jan., 1974).
5. W. Heckrotte, J. R. Hiskes, and R. P. Freis, Parametric Survey for the Linear Two-Component System, Lawrence Livermore Laboratory, Rept. UCRL-75953 (Sept., 1974).
6. L. S. Hall, Inadequacy of the Electron Dynamics Model in Current LLL Fokker-Planck Codes, Lawrence Livermore Laboratory, Internal Document UCIR-836 (Oct., 1974).
Readers outside the Laboratory who desire further information on LLL internal documents should address their inquiries to the Technical Information Department, Lawrence Livermore Laboratory, Livermore, California 94550.
7. I. Langmuir, Phys. Rev. 26, 585 (1925).
8. D. Gabor, E. A. Ash, D. Dracott, Nature 176, 916 (1955).
9. R. S. Harp, Stanford University Microwave Laboratory, Stanford, California, Rept. 1175 (June, 1964).
10. H. L. Berk, Boundary Conditions for Connective Wave Instabilities in Mirror Machines, Lawrence Livermore Laboratory, Rept. UCID-15591 (Oct., 1969).
11. L. Spitzer, Jr., Physics of Fully Ionized Gases, Second edition (Interscience Publishers, New York, 1967).
12. A. H. Futch and G. W. Hamilton, Lawrence Livermore Laboratory, private communication (Sept., 1974).
13. G. W. Hamilton and R. W. Moir, Conditions for Landau Damping and Wave Reflections in Open-Ended Magnetic Well Plasmas, Lawrence Livermore Laboratory, Rept. UCRL-50900 (June, 1970).
14. M. A. Lieberman, External Plasma Stabilization of Flute Modes in a Simple Mirror Reactor, Lawrence Livermore Laboratory, to be published.
15. T. K. Fowler, Two-Component FERF, Lawrence Livermore Laboratory, Rept. UCID-16466 (March, 1974).

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