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a Partially-Filled Right Circular Cylinder inclined
to the Horizontal

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CALCULATION OF THE VOLUME OF LIQUID CONTAINED
IN A PARTIALLY-FILLED RIGHT CIRCULAR
CYLINDER INCLINED TO THE HORIZONTAL

Estimation of the volume contained in an inclined partially-filled cylindrical vessel (such as the HRT dump tanks) may be considerably simplified by use of the curve shown in Fig. 2. Consider the profile view of a circular cylinder shown in Fig. 1.

Fig. 1 - Partially-Filled Cylinder

The volume of a solid such as is represented by the shaded area is given by the formula:

$$V = \frac{R^3}{\tan \theta} \left[\left(\phi - \frac{\pi}{2} \right) \sin \phi + \cos \phi - \frac{1}{3} \cos^3 \phi \right] \quad (1)$$

$$\text{where } \phi = \sin^{-1} \left(\frac{R - d \sec \theta}{R} \right) \text{ and } -\pi/2 \leq \phi \leq \frac{\pi}{2} \quad (2)$$

This is the volume having one base which is cut off by a plane which passes through an edge of one base of a finite right circular cylinder and intersects the opposite face. (Note that if the plane passes through the cylinder "diagonally," i.e., $\sec \theta = 2R/d$, the formula gives

$$V = \frac{R^3}{2R/H} \left[-\frac{\pi}{2} \times -1 + 0 - \frac{\pi}{2} \times -1 - 0 \right] = \frac{\pi R^2 H}{2}$$

The function in the brackets, $F(\phi) = \phi \sin \phi + \cos \phi - \frac{\pi}{2} \sin \phi - \frac{1}{3} \cos^3 \phi$ is plotted in Fig. 2 against $\sin \phi$. After calculating $\sin \phi$ from the angle θ , the

radius, and the depth of liquid, the function $F(\phi)$ is simply read from the curve, multiplied by R^3 and divided by $\tan \theta$ to give the volume:

$$V = \frac{R^3}{\tan \theta} F(\phi) \quad (3)$$

$$\text{where } \sin \phi = 1 - \frac{d}{R} \sec \theta$$

The value of the volume so obtained is correct only if

$$d \leq 2R \cos \theta$$

$$\text{and } d \leq H_T \sin \theta$$

where H_T is the length of the entire cylinder.

In the several alternative cases, the volume contained is obtained as follows:

$$(a) \text{ If } d \geq 2R \cos \theta$$

$$\text{and } d \leq H_T \sin \theta$$

$$\text{then } V = \left(\frac{d \sec \theta}{R} - 1 \right) \frac{\pi R^3}{\tan \theta}$$

$$(b) \text{ If } d \leq 2R \cos \theta$$

$$\text{and } d \geq H_T \sin \theta$$

$$\text{then } V = \frac{R^3}{\tan \theta} [F(\phi_1) - F(\phi_2)]$$

$$\text{where } \sin \phi_1 = 1 - \frac{d}{R} \sec \theta$$

$$\text{and } \sin \phi_2 = 1 - \frac{d}{R} \sec \theta + \frac{H_T}{R} \tan \theta$$

As $\tan \theta$ approaches zero, it can be shown that this result approaches the formula for the partially-filled horizontal cylinder.

$$(c) \text{ If } d \geq 2R \cos \theta$$

$$\text{and } d \geq H_T \sin \theta$$

$$\text{then } V = \pi R^2 H_T - \frac{R^3}{\tan \theta} F(\phi)$$

where $\sin \phi = \frac{d \sec \theta - H_T \tan \theta}{R} - 1.$

Note that given the volume V_d corresponding to a depth d , the volume corresponding to a depth equal to $2R \cos \theta + H_T \sin \theta - d$ is simply $\pi R^2 H_T - V_d$. Therefore, if volumes have been calculated up to a depth = $\frac{1}{2} (2R \cos \theta + H_T \sin \theta)$, the volumes for depths greater than this are immediately available by subtracting the known volumes from the full volume $\pi R^2 H_T$.

The special case of $d = R \cos \theta = H_T \sin \theta$ is of interest because of the surprisingly simple result. This is the case of a plane intersecting a right circular cylinder through a diameter on one face and an edge on the other:

$$\sin \phi = 1 - \frac{d}{R} \sec \theta = 0$$

$$V = \frac{R^3}{R/H_T} \left[0 + 1 - 0 - \frac{1}{3} \right] = \frac{2}{3} R^2 H_T$$

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