COO-2407-1

GENERALIZATION OF INTERNAL CENTRIFUGAL ZONE GROWTH OF METAL-CERAMIC COMPOSITES

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December 1, 1973 - August 31, 1974

Prepared for the U.S. Atomic Energy Commission
Under Contract No. AT(11-1)-2407

ABSTRACT

Our objectives are (1) to develop a realistic model of Internal Centrifugal Zone Growth (ICZG) and (2) to utilize the predictive capacities of this model to improve present ICZG systems and to extend the use of the ICZG concept in general. So far, we have developed mathematical models of hypothetical (infinitely long) samples heated by long RF induction coils; these models are believed to display several essential features of the ICZG process. Study of these models for constant material properties shows that a large surface-to-center temperature difference, needed for ICZG, is favored by small values of the parameter k/Lh where k is the thermal conductivity of the sample, h is the surface heat transfer coefficient, and L is the smaller of the sample radius or the RF skin depth. Moreover, computer solutions for the temperature and electromagnetic fields in cases where electrical conductivity is initially low but a strongly increasing function of temperature have displayed an interesting and perhaps technologically important heating instability phenomenon. For low values of an impressed RF field, there is very little heating of such a sample; whereas, after a critical value of the RF field has been exceeded, the slightly heated sample interacts with the RF field in an unstable manner, resulting in much higher steady state temperatures. Apparatus to test these predictions is now being assembled.

PROGRESS REPORT

A. Technical Progress

Briefly stated, the broad technical objectives of this contract are (1) to develop a realistic model of Internal Centrifugal Zone Growth (ICZG), and (2) to utilize the predictive capacities of this model to improve present ICZG systems and to extend the use of the ICZG concept in general. Both theoretical (mostly computer-aided modeling) and experimental work are to be carried out at Carnegie-Mellon University; whereas, some experimental work is to be carried out in cooperation with Dr. G. W. Clark at Oak Ridge.

During the first nine months of this contract, considerable progress has been made toward meeting these objectives. In particular:

1. We have developed mathematical models of simplified systems that are believed to display several of the essential features of ICZG process.

2. We have communicated the main predictions of our models to Dr. Clark and his group at Oak Ridge (meeting of June 3, 1974). These are:

   a) A large surface-to-center temperature difference, needed to internally melt a sample by induction heating while keeping its surface solid, is contingent upon obtaining a small value of the parameter k/Lh where k is the thermal conductivity of the sample, h is the surface heat transfer coefficient, and L is the lesser of the sample radius R or the electromagnetic skin depth δ.

b) Significant induction heating of materials (such as oxides) whose initially very low electrical conductivity is a strongly increasing function of temperature is characterized by an instability phenomenon which is possibly of great technological importance. For low values of an impressed RF field, there is very little heating of such a sample; whereas, after a critical value of the RF field (dependent upon material and process parameters) has been exceeded, the slightly heated sample interacts in an unstable way with the RF field, resulting ultimately in much higher steady state temperatures.

3. We have begun to purchase and assemble experimental equipment to test the above theoretical predictions.

A complete accounting of theoretical work to date is contained in a detailed technical report which accompanies.[1] A brief summary follows.

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1 R. A. Hartzell and R. F. Sekerka, "Generalization of Internal Centrifugal Zone Growth of Metal-Ceramic Composites," Technical Report No. COO-2407-2.

i) Mathematical Models

The mathematical models which we have developed to date pertain to cylindrical samples which are very long compared to their radii and which are also heated by very long induction coils. Temperatures and electromagnetic fields within such samples are therefore functions of only the radial coordinate, r. Such one-dimensional problems are tractable mathematically and yet can lead to considerable insight, especially in relation to the interactions among surface heat losses, temperature-dependent electrical conductivity, and electromagnetic heating.

Conditions faborable to the establishment of a large surface-to-center temperature difference can be estimated approximately from a model wherein all material properties are independent of temperature. Analytical solutions to this model can be obtained easily for a surface heat loss flux of the form $h(T_s - T_a)$ where h is a heat transfer coefficient (actualy dependent on temperature if the heat loss mechanism is radiation), T_s is the temperature at the surface and T_a is the temperature of the ambient. Details are given in reference 1 where it is shown that the temperature at radial coordinate r within the sample is given by the expressions

$$T(r) - T_a \propto \frac{4k}{Rh} + 1 - \frac{r^4}{R^4} \quad ; \quad \delta >> R$$

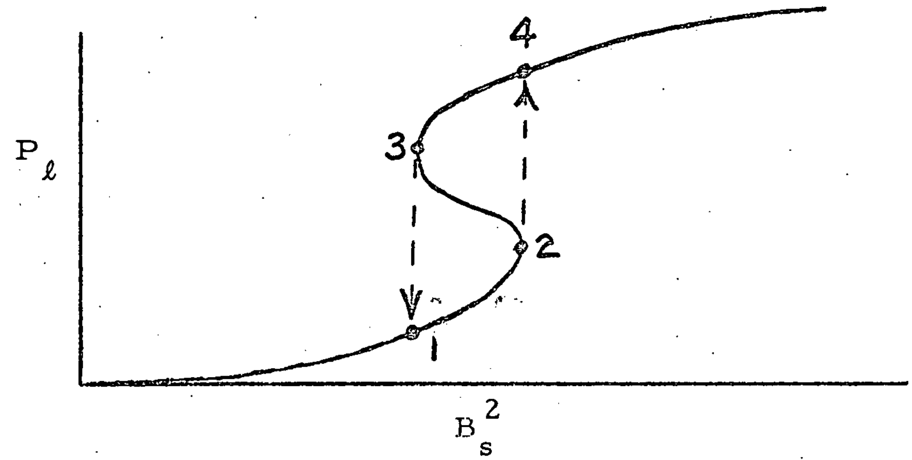
$$T(r) - T_a \propto \frac{2k}{\delta h} + 1 - e^{2(r-R)/\delta} \quad ; \quad \delta << R.$$

Here, k is the thermal conductivity of the sample, R is the radius of the sample, and δ is the R.F. skin depth. Hence we arrive at the criterion that significant dependence of T or r, and hence a significant surface-to-center temperature difference, will necessitate small values of k/Lh where L is the smaller of R or δ. This condition will not be exact but should still be approximately true even when the material parameters depend on temperature.

Analysis of the instability phenomenon which characterizes the heating of oxide-like materials necessitates computer-aided solution, via numerical methods, of coupled nonlinear differential equations for the electromagnetic and temperature fields. For an activated electrical conductivity of the form $\sigma = \sigma_o \exp(-Q/kT)$, the electromagnetic and temperature fields are strongly coupled because T determines σ, which influences the penetration of RF field, and σ influences T via the Joule heating. Details of the relevant equations, numerical techniques, and results may be found in reference 1. The more important results are summarized below.

Figure 1 shows a typical plot of power-per-unit-length, P_ℓ, induced in the sample versus the square of the amplitude of the impressed

RF field, B_s^2, at the surface of the sample. Note the typical "S" shape.



As B_s^2 is increased, P_ℓ increases slowly (e.g., point 1) and hence the temperature of the sample rises only slightly. However, as B_s^2 approaches a critical value corresponding to point 2 on the curve, the lower branch of the curve becomes unstable and, following a transient period, a value of P_ℓ corresponding to point 4 is obtained at steady state. Values of P_ℓ at point 4 should correspond to much larger steady state temperatures than at point 1; via the instability phenomenon, the otherwise poorly conducting sample has coupled strongly to the RF field.

We emphasize that this instability phenomenon, characterized by the steady state "S" curve, has been verified theoretically by computer solutions of linearized transient equations for the temperature field, valid in the vicinity of the lower leg of the "S" curve. For points in the "S" curve below point 2, these transient solutions decay with time; whereas, at point 2, an eigenvalue corresponding to solutions which increase with time begins to appear. It has not been proven, but is conjectured, that a similar instability phenomenon occurs with decreasing power from points 4 to 3 to 1.

ii) Experimental

Apparatus is now being purchased and assembled to test the above theoretical predictions, especially with respect to the theoretically predicted instability phenomenon in the heating of oxide-like materials. A 10 KW 450 killocycle RF generator was donated to this project by Westinghouse Electric Corporation. Various methods of measuring RF currents and magnetic fields are now being established. Details of contemplated experiments is contained in the proposed technical program for renewal, August 1974.

B. Expended and Contemplated Effort

This research is being conducted by the principal investigator, Professor Robert F. Sekerka, and one full-time graduate student, Mr. Robert A. Hartzell. Percentages of the expended and anticipated effort are given in tabular below:

	Expended									Anticipated		
	1973	1974								1974		
	D	J	F	M	A	M	J	J	A	S	O	N
Prof. Sekerka (Principal Investigator)	15%	<------------>					15%	50%	50%	15%	15%	15%
Mr. Hartzell (Graduate Student)	<------------------- 100% --------------------->											

In the remaining three months of the current contract period, effort will be devoted toward the evaluation of our theoretical results for material parameters corresponding to actual materials that will be candidates for experimental tests. Some experimental tests of the predicted unstable heating phenomenon will be attempted along with preliminary work toward development of a theoretical model for samples of finite length, as detailed in our proposed technical program for renewal.