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PHYSICS OF WATER MODERATED THORIUM REACTORS

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Abstract

Historically, the interest in thorium arose because the value of η for thermal neutron absorption in U-233 showed the possibility of obtaining a net breeding gain in thermal reactors. More recently, as a result of the increased availability of highly enriched uranium, the practicality of thorium converter reactors has become an important question.

The pressurized water reactor is one of the important reactor types now under development in which the use of thorium may prove to be particularly attractive. In these reactors thorium is superior to U-238 in two respects. It has a larger radiative capture cross section which reduces the neutron losses to water, structural materials and fission products, and it produces upon conversion a better fuel for thermal reactors. These factors combine to give a pressurized water thorium converter an advantage of as much as 0.2 in conversion ratio and a reactivity life as much as twice as long for a given excess multiplication of the reactor core.

The use of thorium in pressurized water reactors has been studied by The Babcock & Wilcox Company in connection with the Consolidated Edison Thorium Reactor (CETR) project. Engineering and economic considerations of fuel element design, heat removal and fuel cycle have led to the choice of a design in which the fuel is highly enriched uranium oxide mixed with thorium oxide contained in close-packed metal tubes. Performance specification and design requirements set metal-to-water ratios in the range 0.8 to 1.3. The economic requirement of long fuel element life leads to consideration of cores having fuel irradiations of 15,000 to 20,000 MWD/Ton of thorium.

As a consequence of the high metal-to-water ratio and high initial fuel loadings, 20% to 30% of the fissions are epi-cadmium. Design information for these cores has been obtained from critical experiments on thorium metal and thorium oxide water lattices performed at the Critical Experiment Laboratory of The Babcock & Wilcox Company.

Critical experiments on several water moderated enriched uranium-thorium assemblies are reported. Bucklings and critical masses have been measured for thorium metal and thorium oxide lattices at several thorium to uranium and thorium to water ratios. The epi-cadmium to thermal fission rates have been measured.

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Values of infinite medium multiplication constants and migration areas have been derived using the supplementary measurement of the rate of change of reactivity with water height as a function of water height. In the derivation of k_{∞} and M^2 from the water height measurements neglect of the fourth moment of the slowing down kernel leads to errors of several percent in k_{∞} and M^2 for the lattices with the larger multiplication constants ($k_{\infty} \approx 1.3$). The contribution of the fourth moment to the non-leakage probability has been calculated and used in the interpretation of the water height measurements.

The effective resonance integrals of the metal and ThO_2 in the water lattices have been derived from measurements of cadmium ratios of ~~thorium~~. The results are in good agreement with measurements of resonance integrals for individual lumps after the shadowing (Dancoff) correction is made.

The experimental results have been compared with theory and can be correlated for the purpose of design calculations with a modified two group diffusion model in which the effect of resonance fission is included.

This paper is divided into two parts. The first part gives a summary of experimental reactor physics data for thorium water reactors and the second part presents reactivity lifetime considerations including a comparison of the reactivity lifetime of U-238 and thorium reactors fueled with U-235.

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Historically, the interest in thorium arose because the value of η for thermal neutron absorption in U-233 showed the possibility of obtaining a net breeding gain in thermal reactors. More recently, as a result of the increased availability of highly enriched uranium, the practicality of thorium converter reactors has become an important question.

The pressurized water reactor is one of the important reactor types now under development in which the use of thorium may prove to be particularly attractive. Due to the superior resistance of oxide fuel elements to radiation damage, the more recent designs of these reactors have been almost universally based on the use of high density oxides of fuel and fertile material contained in either stainless steel or zirconium alloy tubing.

In the case of oxide fuels thorium shows a considerable nuclear advantage over U-238 which is not present for metallic fuel. For the oxide fuels the density of uranium and thorium are nearly the same and, hence, the larger radiative capture cross section of thorium results in significantly lower neutron losses to water, structural materials and fission product poisons. The fast fission effect in uranium oxide is not large enough to offset the neutron losses due to these thermal neutron absorption losses and as a consequence, thorium oxide converters have an advantage of as much as 0.2 in conversion ratio.

On the other hand, for metal-water lattices, the higher density of uranium metal as compared to thorium metal compensates to some extent for the smaller radiative capture cross section of U-238. Furthermore, since the fast fission effect in close packed uranium metal-water lattices is many times larger than in thorium, the conversion ratios for the uranium metal lattices are larger than for thorium.

* The Babcock & Wilcox Co., Atomic Energy Division, Lynchburg, Virginia, U.S.A. Including work by H.S. Barringer, M.L. Batch, J.C. Deddens, C.J. Heindl, M.M. Levine, R.H. Lewis, W.K. Luckow, G.K. Rhode, N.L. Snidow, The Babcock & Wilcox Co.

The reactivity life of a thorium converter can be as much as twice as long as the life of a U-238 converter for a given excess multiplication of the reactor core. The larger reactivity life for thorium is due to a combination of three factors; (1) higher conversion ratio, (2) larger value of η for U-233 than for Pu-239 and (3) a larger mass of fuel for a given reactivity.

The use of thorium in pressurized water reactors has been studied by The Babcock & Wilcox Company in connection with the Consolidated Edison Thorium Reactor (CETR) project. Engineering and economic considerations of fuel element design, heat removal and fuel cycle have led to the choice of a design in which the fuel is highly enriched uranium oxide mixed with thorium oxide contained in close-packed metal tubes. Performance specification and design requirements set metal-to-water ratios in the range 0.8 to 1.3. The economic requirement of long fuel element life leads to consideration of cores having fuel irradiations of 15,000 to 20,000 MWD/Ton of thorium.

As a consequence of the high metal-to-water ratio and high initial fuel loadings, 20% to 30% of the fissions are epi-cadmium. Design information for these cores has been obtained from critical experiments on thorium metal and thorium oxide water lattices performed at the Critical Experiment Laboratory of The Babcock & Wilcox Company.

This paper is divided into two parts. The first part gives a summary of experimental reactor physics data for thorium water reactors and the second part presents reactivity lifetime considerations, including a comparison of the reactivity lifetime of U-238 and thorium reactors fueled with U-235.

1.0 THORIUM METAL AND OXIDE WATER LATTICE EXPERIMENTS

Experiments have been performed on several enriched uranium-thorium-water critical assemblies. These experiments have included both plate type elements containing thorium metal and pin type elements containing thorium oxide. Infinite medium lattice parameters have been determined for both types of assemblies for several uranium-to-thorium ratios.

1.1 DESCRIPTION OF THORIUM METAL CORES

The thorium metal experiments were carried out on critical assemblies constructed from plate-type elements. The assembly was water-reflected and arranged in a water tank. The cross section of a typical fuel element is shown in Figure 1. The active length is six feet. Two sizes of fuel elements were used; the smaller elements were positioned adjacent to the control rods. The fuel loading per element was changed by varying the number of fuel tapes in each element.

The fuel tape was 47.3% by weight U_3O_8 highly enriched in U-235. The remainder of the tape was polyethylene. The fuel bearing portion of the tape was 0.044" thick with a $1\frac{1}{2}$ mil cladding of pure polyethylene on

each side to give a total thickness of 0.047". The average mass of U-235, per tape in the large element was 83.70 grams; that in the small element 74.46 grams. The thorium and fuel tapes were bound to the aluminum plates by a double-surfaced adhesive tape approximately 2 mils thick.

1.2 DESCRIPTION OF THORIUM OXIDE CORES

A total of four oxide cores have been investigated. These cores were constructed from pin-type elements containing pellets of thorium oxide-uranium oxide fuel. The pins were stainless steel clad and had an active length of 4.0 feet. The fuel pellets were 0.260" in diameter and the cladding thickness was 0.019". The overall pin diameter was 0.312". One-half the pins were of density 8.2 gms/cm³ and contained 15 atoms of thorium for each atom of U-235; the remaining pins were of density 8.6 gms/cm³ and contained 25.8 atoms of thorium per atom of U-235. Experiments were carried out on assemblies of two different metal-to-water ratios; 1.119 and 0.892 (total pin volume-to-water volume).

1.3 PHYSICAL CONSTANTS

Physical constants for each of the seven cores investigated are listed in Table I.

TABLE I

Physical Constants for Experimental Cores

Core	Type	$\frac{N_{25}^{(a)}}{N_{02}}$	$\frac{N_w^{(a)}}{N_{02}}$	$\frac{N_s^{(a)}}{N_{02}}$	$\frac{\Sigma_1^{(b)}}{\Sigma_{02}}$
I	5-Tape Plate	.0239	4.31	3.35	.036
II	5½-Tape Plate	.0268	4.34	3.40	.037
III	6-Tape Plate	.0296	4.47	3.42	.039
IV	Pins, M/W = 1.119	.0667	2.48	1.67	.012
V	Pins, M/W = 1.119	.0388	2.40	1.62	.008
VI	Pins, M/W = 0.892	.0667	3.10	1.67	.012
VII	Pins, M/W = 0.892	.0388	3.02	1.62	.008

(a) Atomic concentration symbols: N_{25} for U-235, N_{02} for Th, N_w for water molecule, N_s for structural material (Al for plate cores, stainless steel for pin cores).

(b) Ratio of macroscopic absorption cross section of miscellaneous impurities to that of Th.

1.4 MATERIAL BUCKLING

The clean critical masses of all the assemblies in the shape of right circular cylinders were determined. The critical radii and masses are given in Table II.

TABLE II

Critical Sizes of Clean Critical Assemblies

Core	Critical Radius (cm)	Critical Height (cm)	Critical Mass (Kg of U-235)
I	75.0(a)	182.8	126.5(a)
II	46.2	182.8	52.96
III	37.2	182.8	37.03
IV	24.3	121.9	38.0
V	47.5	121.9	88.1
VI	21.9	121.9	27.7
VII	38.8	121.9	55.1

(a) Computed for uniform 5-tape loading based on critical core with 12 6-tape elements in center (critical radius = 62.8 cm, critical mass = 90.36 Kg).

For Cores II and III radial buckling was determined by defining an effective core radius to give an area equivalent to the actual assembly and assuming a radial reflector savings of 7.0 cm. The axial buckling was determined by using a total reflector savings of 15 cm. These savings were computed with constants consistent with a direct measurement of vertical reflector worth on the five tape core. The material buckling of a 5-tape core was deduced from the third configuration by assuming the effective material buckling of the core to be given by a statistical weighting of the 5-tape and 6-tape bucklings. The bucklings thus found are tabulated in Table III.

TABLE III

Experimental Buckling Data

Core	Extrapolation Distance (cm)		Buckling (10^{-3} cm^{-2})		
	Radial	Vertical	Radial	Axial	Total
I	7.0(a)	15.0(a)	0.87 ± 0.11	0.25 ± 0.02	1.12 ± 0.11
II	7.0(a)	15.0(a)	2.04 ± 0.17	0.25 ± 0.02	2.29 ± 0.18
III	7.0(a)	15.0(a)	2.96 ± 0.24	0.25 ± 0.02	3.21 ± 0.25
IV	7.6	13.8	5.61 ± 0.05	0.54 ± 0.01	6.15 ± 0.06
V	8.0	12.6	2.13 ± 0.02	0.54 ± 0.01	2.67 ± 0.03
VI	8.3	12.6	6.15 ± 0.07	0.54 ± 0.01	6.69 ± 0.08
VII	8.7	13.4	2.37 ± 0.02	0.54 ± 0.01	2.91 ± 0.03

(a) Extrapolation distances for plate cores were not measured directly.

The material buckling for each pin core was obtained from flux traverses made with cadmium-covered indium foils. Early measurements showed the ratio of resonance to thermal flux to be constant over the cores.

Since the resonance flux proved to be less sensitive to minor variations in pin positioning than the thermal flux, the cadmium covered data was used in the analysis. The radial and axial bucklings and extrapolation distances are given in Table III.

1.5 WATER HEIGHT MEASUREMENTS

The determination of rate of change of reactivity with water height as a function of water height was made in order to obtain experimental data connecting the infinite medium multiplication constant and the non-leakage probability⁽¹⁾. Assuming that k_{eff} for a critical assembly is given by $k_{\infty} P(B^2)$, where k_{∞} is the infinite medium multiplication constant and P is the non-leakage probability for the assembly, it follows that

$$\frac{1}{k_{\infty}} \frac{\partial P^{-1}}{\partial B_z^2} = \frac{(h + \lambda)^3}{2\pi^2} \frac{d\rho}{dh} = C \quad (1)$$

In equation (1) the assumption has been made that B_z^2 can be written as $[\pi/(h+\lambda)]^2$, where h is the physical core height, λ is the vertical reflector savings and ρ is the reactivity. The quantity $(\partial P^{-1}/\partial B_z^2)$ will be constant for a given set of water height measurements; however, its exact form will depend upon the model chosen for the non-leakage probability. The constant C , has been determined for Cores III, IV, VI, and VII. In each case the reactor period was measured for several incremental changes in water height as a function of water height. The observed periods were converted to reactivities through the inhour equation. Since the age for delayed neutrons is approximately one-third to one-half that for fission neutrons, it was necessary to correct the delay fractions for their relative non-leakage probabilities. The resulting average delay fractions, $\bar{\beta}$, and the derived values of C are given in Table IV. The errors shown are root mean errors computed by standard least squares procedures on the basis of external consistency of the data.

TABLE IV

Core	$\bar{\beta}$	C (cm ²)	U-235 Cadmium Fraction(a)	Thorium Cadmium Fraction(a)	Conversion Ratio
I	.00657		.910 ± .005	.653(b)	.63
III	.00690	36.6 ± 3.5	.905(b)	.653 ± .004	.50
IV	.00726	44.0 ± 2.5	.775(b)	.321 ± .007	.37
V	.00683		.827(b)	.438 ± .006	.50
VI	.00740	35.8 ± 2.6	.819 ± .002	.402 ± .006	.31
VII	.00693		.864 ± .002		

(a) Counting errors only

(b) Estimated values

1.6 THERMAL FLUX RATIOS

In order to obtain the ratio of the average thermal fluxes in the different regions of the lattice cell, fine structure measurements were made on five-, and six-tape elements. Holes were punched in the aluminum plates for locating foils. Lucite spacers were used to support the foils in the water gaps. In some locations foils were taped to the thorium plates. Comparisons with multi-region P_3 calculations showed excellent agreement. The calculated curves, normalized to experiment, were used to determine the average flux ratios given in Table V.

Average flux ratios were also measured for the pin cores. For measurements of thermal fine structure and the cadmium fraction of thorium described in Section 1.8, a special test bundle of 25 fuel pins was inserted in the cores. The spacing between the pins was maintained to much closer than average tolerance by means of lucite spacer blocks bored to the same pitch as the core.

Several special pins containing pellets hand selected for uniformity and mechanical integrity were fitted with removable end plugs. Flux measurements inside the fuel were obtained by removing half the pellets from one of the special tubes, inserting an aluminum wrapped dysprosium-aluminum foil, replacing the fuel pellets, and exposing the assembly in the test insert. Simultaneously, a sector foil was exposed in the test bundle exterior to the pins. The foils were then counted to obtain an average level of activation in the fuel and in the water. The flux ratio for the water moderator can be determined as the ratio of the two average activations. The experimental results are given in Table V.

TABLE V

Experimental Flux Ratios			
Core	$\bar{\phi}_{02}/\bar{\phi}_{25}$	$\bar{\phi}_s/\bar{\phi}_{25}$	$\bar{\phi}_w/\bar{\phi}_{25}$
I	1.035	1.032	1.067
III	1.023	1.019	1.061
VI	1.000	1.14	1.17
VII	1.000		1.14

1.7 CADMIUM FRACTION FOR U-235

The cadmium fraction of fission product activity was determined for Core I in the 5-tape material. The cadmium fraction is defined as (bare foil activity - Cd covered foil activity)/bare foil activity. Bare and cadmium covered foils punched from the polyethylene fuel tape were exposed at various positions in Core I.

Similarly, the fraction for thin U-235 foils (plated onto aluminum) was determined for two of the pin cores. The foils were exposed in the test insert in the manner given above for the flux ratio measurements.

Experimental results are given in Table IV.

1.8 CADMIUM FRACTION FOR THORIUM

The cadmium fraction for thorium was measured for Core III (6-tape). Thorium foils 0.250 inches in diameter and 0.056 inch thick were punched from a thorium plate and irradiated with aluminum covers and with cadmium covers. In order to eliminate the uncertainties introduced by flux depressions within the foil, following exposure the foils were dissolved in HCl; standard amounts were transferred to gold foils and evaporated to dryness. A piece of "Scotch" tape was placed over the residue to stop the alpha activity and to prevent moisture absorption by the chloride film. The residues were then beta counted. Measurements of the cadmium fraction were made at four different positions in the bundle. The average value of the cadmium fraction, corrected for thermal fine structure, is given in Table IV.

The cadmium fraction for thin wafers of thorium oxide was also measured for Cores IV, V, VI, and VII. A sandwich of three oxide wafers, each 20 mils thick, was exposed in the test insert in the same manner as the dysprosium foils described in Section 1.6. Each foil was exposed twice, once with aluminum spacer foils and once with the cadmium sleeve and foils as shown in Figure 2. The experimental results are presented in Table IV.

1.9 DERIVATION OF k_{∞} AND M^2 FROM BUCKLING AND WATER HEIGHT MEASUREMENTS

As was pointed out in Section 1.5, the water height measurements determine a constant, C. For large critical assemblies with small leakage, P^{-1} can be approximated by $1 + M^2 B^2$, where M^2 is the effective migration area. For k_{∞} greater than about 1.15, however, the higher moments of the neutron slowing down kernel become important in calculating the neutron leakage. The correction term for the fourth moment is $\frac{\gamma \tau^2 B^4}{4}$ where $\gamma = 1 - \frac{r^4}{(120\tau^2)}$. τ is the neutron age to thermal and r^4 is the fourth moment of the neutron slowing down kernel for a point source of fission neutrons.

The effective multiplication constant can be written as

$$k_{\text{eff}} = k_{\infty} \left[1 + M^2 B^2 + \tau(L^2 + \gamma\tau)B^4 \right]^{-1} \quad (2)$$

and the constant, C, as

$$C = k_{\infty}^{-1} \left[M^2 + 2\tau(L^2 + \gamma\tau)B^2 \right] \quad (3)$$

Simultaneous solution of these equations for $k_{\text{eff}} = 1$ gives

$$k_{\infty} = \frac{1 - \tau(L^2 + \gamma\tau)B^4}{1 - B^2 C} \quad (4)$$

and

$$M^2 = Ck_{\infty} - 2\tau(L^2 + \gamma\tau)B^2 \quad (5)$$

The fourth moment correction for water and aluminum-water mixtures can be estimated from measurements of the slowing down density of fission neutrons from a point source by Roberts, Hill, and Fitch⁽²⁾. For water $\gamma = -0.07$ and for an equal volume ratio of water to aluminum $\gamma = 0.25$. Since $\gamma = 0$ for a Yukawa kernel, the one group diffusion model for calculating the neutron leakage from water moderated reactors should be good through terms of order r^4 in the moment expansion of the neutron slowing down kernel. It is interesting to note that for an equal volume of aluminum and water the value of γ lies halfway between the value for a Yukawa and a Gaussian kernel ($\gamma = 0.50$).

The value of γ for the thorium oxide water lattices has been calculated using a Selengut-Goertzel kernel for scattering with hydrogen. Inelastic scattering in thorium was included. These calculations give $\gamma = -0.1$ for water and $\gamma = 0.2$ for the thorium oxide - water lattices.

In order to obtain an estimate of the uncertainty in the values of k_{∞} and M^2 due to the fourth moment correction to P^{-1} , calculations were made for both $\gamma = 0$ and $\gamma = 0.2$. The results are given in Table VI. For the cores having $k_{\infty} \approx 1.3$, there is a difference of about 15% in the value of M^2 and a difference of 2% in k_{∞} .

The same difficulty arises in the interpretation of the boron poison method⁽³⁾ of measuring M^2 . Additional experimental work is needed; either with critical assemblies having a larger range in fuel enrichment for a fixed lattice or direct measurement of the first three even moments of the slowing down density distribution of fission neutrons from a point source in these lattices.

TABLE VI

Core	k_{∞}		M^2	
	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0$	$\gamma = 0.2$
I	$1.05 \pm .01$	$1.05 \pm .01$	$41 \pm 5(a)$	$39 \pm 5(a)$
II	$1.09 \pm .01$	$1.09 \pm .01$	$41 \pm 5(a)$	$39 \pm 5(a)$
III	$1.13 \pm .01$	$1.13 \pm .01$	41 ± 5	39 ± 5
IV	$1.37 \pm .02$	$1.34 \pm .02$	60 ± 4	52 ± 4
V	$1.16 \pm .02$	$1.14 \pm .02$	$60 \pm 4(b)$	$52 \pm 4(b)$
VI	$1.31 \pm .02$	$1.30 \pm .02$	46 ± 4	42 ± 4
VII	$1.13 \pm .02$	$1.12 \pm .02$	$46 \pm 4(c)$	$42 \pm 4(c)$

(a) Assumed the same as for Core III

(b) τ assumed the same as for Core IV

(c) τ assumed the same as for Core VI

1.10 THEORETICAL MODEL FOR CALCULATION OF k_{∞}

In the lattices described above 10 to 25% of the fissions are epi-cadmium. The experimental data can be interpreted reasonably well by introducing an epi-thermal or fast multiplication constant, k_1 . The infinite medium multiplication constant is then the sum of the thermal and fast multiplication constant, i. e., $k_{\infty} = k_1 + k_2$.

The fast multiplication constant is defined by setting $k_1 \Sigma_1 \phi_1$ equal to the number of fission neutrons produced per unit volume in unit time by the absorption of fast neutrons and is given by

$$k_1 = \frac{\int_{E_0}^{\infty} \nu \Sigma_f(E) \phi(E) dE}{\Sigma_1 \phi_1} \quad (6)$$

$\Sigma_1 \phi_1$ is the slowing down density into the thermal neutron group in the absence of resonance absorption. The energy, E_0 , which divides the fast and thermal neutron groups is taken to be cadmium cutoff for convenience in the interpretation of the experiments.

The neutron spectrum $\phi(E)$ and k_1 have been calculated taking into account the details of the resonance structure of the uranium and thorium and the geometry of the lattice. However, a simple approximation to k_1 can be obtained by noting that almost all of the resonance absorption in thorium occurs at higher neutron energies than does most of the resonance absorption in U-235, i. e., the thorium screens the uranium. This approximation gives

$$k_1 = \eta_r (1 - P_{25}) P_{02} \epsilon \quad (7)$$

where η_r is the average value of η for U-235 for epi-cadmium absorption, ϵ is the fast fission factor for thorium, and P_{02} and P_{25} are the resonance escape probabilities respectively for Th and U-235.

The resonance escape probabilities are given by the usual formula

$$P = \exp (-NR_{\text{eff}}/\xi \Sigma_s) \quad (8)$$

where the effective resonance integral, R_{eff} , for each absorber is defined to account for shadowing effects as well as the effects of lumping. The effective resonance integrals include all the absorption above cadmium cutoff. The practice of subtracting the $1/v$ part of the absorption leads to an error of about 0.07 in the average value of η for a lattice in which 25% of the fissions are epi-cadmium.

A comparison of the screening approximation with the more detailed spectral calculation of k_1 for the experimental lattices is given in Table VII. The spectral calculations were normalized to give the same effective resonance integrals for each absorber as were used in the screening approxi-

mations. The effective resonance integrals for thorium were obtained from measurements described in Section 1.13 and are given in Table VII. The values $\eta_r = 1.62$ and $R_{25} = 411$ b. were used for all the calculations.

The thermal multiplication constants listed in Table VII were computed from

$$k_2 = \eta_{th} f P_{02} P_{25} P_x \epsilon \quad (9)$$

where η_{th} is the number of fission neutrons per thermal absorption in U-235, f is the thermal utilization for U-235, and P_x is the resonance escape probability for the structural materials (mostly epi-cadmium $1/v$ absorption). The hardening of the thermal neutron spectrum, estimated by averaging the cross sections with respect to a Wigner-Wilkins spectrum, reduces the thermal multiplication constants by about 1%.

TABLE VII

Core		I	IV	V	VI	VII
R_{eff}	Th(barns)	23	18	18	20	20
k_1	Calc. A(a)	.069	.319	.204	.267	.169
	Calc. B(b)	.072	.322	.208	.270	.173
k_∞	Calc. A(a)	1.065	1.298	1.110	1.315	1.141
	Calc. B(b)	1.059	1.307	1.112	1.319	1.140
k_2	Calc. A(a)	.935	.754	.816	.797	.852
	Calc. B(b)	.932	.754	.813	.795	.849
$\overline{k}_\infty$	Measured	$.910 \pm .005$		$.819 \pm .002$		

(a) Calc. A is the screening approximation. Error in k_1 as given by Equation (7) due to neglect of $1/v$ absorption in water and structure is less than .02.

(b) Calc. B is a forty-group spectral calculation of k_1 .

1.11 RATIO OF k_2/k_∞

From the cadmium fission fractions given in Table IV it is possible to obtain an experimental determination of k_2/k_∞ . If the modified two-group model is used to interpret these measurements, the relation

$$(C.F.)_{235} = \frac{k_2}{\overline{k}_\infty} \quad (10)$$

is obtained, where $k_1(1+L^2B^2)$ has been replaced by k_1 in the denominator. This ratio can be combined with the k_∞ to yield k_1 and k_2 separately. Comparisons of computed and experimental values of k_2/k_∞ are presented in Table VII. Experimental values of k_∞ are given in Table VI.

1.12 CONVERSION RATIO

The cadmium fraction measurements for thorium and uranium can be combined to yield the conversion ratio which is, for this purpose, defined as the ratio of absorptions in thorium to absorptions in U-235. This ratio can be written as

$$\frac{\Sigma_{02}}{\Sigma_{25}} \frac{\bar{\phi}_{02}}{\bar{\phi}_{25}} \frac{(CF)_{25}}{(CF)_{02}} \frac{\eta^r}{\eta_{th}} \left[1 - (CF)_{25} \left(1 - \frac{\eta^r}{\eta_{th}} \right) \right]^{-1} \quad (11)$$

Conversion ratios deduced from this expression are tabulated in Table IV. Cadmium fractions for both U-235 and for thorium were not measured in all cores. Estimates of some values were obtained by comparing the available experimental data with the calculated values given in Table VII.

1.13 EFFECTIVE RESONANCE INTEGRALS FOR THORIUM AND THORIUM OXIDE

Resonance integrals measured for individual rods and plates of thorium metal show agreement with values of integrals derived from cadmium fraction measurements made in critical assemblies (Section 1.8). In the case of thorium oxide, however, considerable discrepancy between different measurements still remains. Additional measurements are being made.

Dayton and Pettus⁽⁴⁾, using a danger coefficient technique and absolute calibration against boron in a water moderated research reactor obtained the following results for individual lumps of thorium:

$$\text{Metal: } R_{\text{eff}} = 2.9 + 24.1 \sqrt{S/M} \text{ barns} \quad 0.1 \leq S/M \leq 1.3 \quad (12)$$

$$\text{Oxide: } R_{\text{eff}} = 35.2 \sqrt{S/M} \text{ barns} \quad 0.2 \leq S/M \leq 1.3 \quad (13)$$

$$S/M = \text{surface mass of thorium (cm}^2/\text{gm)}$$

The epi-cadmium $1/v$ contribution is included in these results.

In order to compare the integrals derived from cadmium fraction measurements in the critical assemblies, the effect of shadowing by neighboring pins and the volume term of the resonance integral must be determined.

The volume term can be found fitting the Dayton-Pettus data to

$$R_{\text{eff}} = a + b \left(C \frac{S}{M} \right) \quad (14)$$

where a and b ($C S/M$) are respectively the volume and surface terms and C is the probability that a resonance neutron incident on the lump have a collision in the lump^(5, 6, 7). The results for R_{eff} in barns are:

Metal	a = 6.8	b = 42.7
Oxide	a = 7.6	b = 47.2

The Dancoff and Ginsberg⁽⁸⁾ calculations for the reduction of the surface term due to shadowing by neighbors in a lattice have been extended to allow for the presence of cladding and cadmium foil used in the cadmium fraction measurements. The replacement of moderator by metal reduces the source of resonance neutrons adjacent to the absorber and increases the shadowing effect.

The fractional reduction F in the surface term for the oxide lattice (Core IV) due to shadowing by each neighbor is given as a function of pin separation a, pin radius R, and moderator macroscopic cross section Σ_m in Figure 3. The effect of cladding can be seen by comparing the dashed line for the case of no clad, $\Sigma_m R = 0.633$, with the corresponding solid line for the case of clad and the same pin radius.

TABLE VIII

Material	Thickness or Diameter (inches)	S/M	Pitch (inches)	Single Lump R_{eff} (barns)	Calculated Lattice R_{eff} (barns)	Measured Lattice R_{eff} (barns)
Metal Plates Core III	.0558	1.23	.290	29.6	23.0	23.1 ± .9
Oxide Pins						
Core IV	.260	.900	.381(a)	33.3	21.3	18.8 ± 1.7
Core V	.260	.875	.381(a)	32.8	21.1	18.1 ± 1.7
Core VI	.260	.900	.403(a)	33.3	23.4	17.8 ± 1.7
Metal Pins ⁽⁹⁾						
	.387	.361	.567(b)	17.2	12.2	10.4 ± .8
	.387	.361	.624(b)	17.2	13.8	13.4 ± .9
	.387	.361	.677(b)	17.2	14.7	13.8 ± 1.4
	.387	.361	.770(b)	17.2	15.8	15.1 ± 1.0

(a) Square pitch

(b) Triangular pitch

The resonance escape probability for thorium in the lattices is given by

$$P_{02} = \frac{1}{1 + \frac{1 - (C.F.)_{02}}{(C.F.)_{02}} \frac{\Sigma_{02}}{\Sigma_a} P_{25} P_x} \quad (15)$$

Σ_{02} and Σ_a are the thorium and the total macroscopic absorption cross sections weighted by the disadvantage factors for the cell. The effective resonance integral is then defined by $R_{eff} = -(\xi \Sigma_s / N_{02}) \ln P_{02}$.

The effective resonance integrals for lattices computed by applying a shadowing correction to the Dayton-Pettus data are compared to the integrals derived from cadmium fractions measured in water lattices in Table VIII.

2.0 REACTIVITY LIFE CONSIDERATIONS

Recently pressurized water reactor designs for central station power plants have evolved which are almost universally based on the use of high density oxides of fuel and fertile material clad in either stainless steel or zirconium alloy tubing. The superior resistance of the oxide fuel elements to radiation damage (experimental data indicate the feasibility of irradiations in excess of 20,000 megawatt days of heat per metric ton of oxide (MWD/T)) gives a strong incentive to design reactor cores with long reactivity life to take advantage of the potentially long physical life of oxide fuel elements.

In view of the economic importance of achieving long core life, a series of calculations has been performed indicating the effects of the various core parameters on the reactivity life of a practical pressurized water core. The core, a right circular cylinder, has a radius of 100 cm and height of 250 cm. The fuel is high density oxide contained in 0.8 cm diameter tubing. Approximately 200 fuel tubes are assembled into a square bundle enclosed along its length by a zirconium alloy can. The power is assumed to be 500 MW and the average water temperature is 260°C. The volume fractions of the various materials are: oxide 0.27, water 0.47, fuel pin clad (Zr or stainless steel) 0.11, Zr fuel cans 0.13 and control rods 0.02.

The fuel pin, containing thorium oxide and clad with stainless steel, for which lattice measurements have been made, was compared with U-238 oxide as fertile material and alternately with zirconium as the clad material. A two-group calculation was used to determine excess reactivity, and this was repeated at a number of later times to take into account the buildup of the higher isotopes and fission products, burnout of U-235, and changes in the neutron spectrum. The effects of non-uniformities caused by control rods, and the non-uniform burnup of the fuel were not taken into account. Because of this, the actual core lives would be appreciably shorter than those shown, although relative comparisons should be valid.

The results of the calculations indicate that long reactivity lives are possible for cores using ThO₂ as fertile material than for those with UO₂, because it is possible to load more U-235 into the core, using the same reactivity control mechanism. In addition, the conversion ratios of the ThO₂ cores are higher than the UO₂ cores, also adding to the life. The computed core lives have large uncertainties, derived from experimental uncertainties in resonance absorption, in the average η of the fissionable material in the resonance region, and in the thermal and resonance cross sections of the gross fission products.

2.1 EXCESS REACTIVITY AND CONTROL

Almost unlimited reactivity life could be obtained if it were possible to add enough U-235 to the core. However the amount of U-235 that can be added in excess of that needed for criticality is limited by the available method of reactivity control. Mechanical feasibility limits the amount of black control rod surface that can be used; the possibility of a positive moderator temperature coefficient and corrosion may limit the amount of soluble poison that can be used; and the relative uncertainties of the cross sections (added to metallurgical uncertainties) may limit the addition of burnable absorber to the fuel. The amount of excess reactivity that can be handled by a given control system does not vary markedly in the cores studied, although somewhat more reactivity can be controlled in the cores with less U-235.

The effective multiplication constant, k_{eff} , as a function of U-235 loading is given in Figure 4. Since the critical masses of the thorium cores are larger than those of the U-238 cores, the excess mass of U-235 is greater in the thorium cores for a given reactivity.

2.2 REACTIVITY LIFE

In order to obtain the long reactivity life required for economic power generation in pressurized water reactors, relatively large amounts of fissionable material must be placed in the core.

The computed reactivity lives are shown in Figure 5. If the same initial reactivity can be controlled in all four cores studied, significantly longer core lives can be obtained for the thorium reactors than for comparable U-238 reactors. For the same core life, a much larger reactivity must be controlled for U-238 than for thorium. As an example, a zirconium clad thorium core which runs for 20,000 MWD/T and has an enrichment of 3.3% requires the control of 15% k_{ex} , while the U-238 core (2.7% enriched) of the same life requires control of 36% k_{ex} . Partial reloading of the core will of course reduce the reactivity requirements. In this case partial reloading of one-fifth of the core at a time reduces the maximum k_{ex} requirements to about 6% for thorium and 18% for U-238.

For short irradiations there is a range where UO_2 is better than ThO_2 . This is the region of operation where Pa-233 has not reached equilibrium in the thorium systems. Because of the small fuel burnup and the frequent reloading, this region of operation is not considered economic for pressurized water reactors.

For exposures greater than 4,000 MWD/T core life is proportional to the excess fuel loading and increases more rapidly with fuel loading for thorium cores than for U-238 cores. This results in better fuel utilization for thorium cores. For example a zirconium clad thorium core reaches a life of 20,000 MWD/T starting with 54% excess mass, while the U-238 core with the same excess mass has a life of only 7400 MWD/T. The corresponding fractions of fuel consumed are 66% for thorium and 57% for U-238 (Figure 6).

The use of stainless steel clad increases core life, but at the expense of fuel utilization. The stainless steel clad thorium core with 54% excess mass has a life of 23,000 MWD/T but only consumes 52% of the fuel.

2.3 CONVERSION RATIO

The average conversion ratio is defined here by

$$\overline{CR} = \frac{\text{Mass of fissionable material produced during core life}}{\text{Mass of fissionable material consumed during core life}}$$

Values for the four types of cores are shown in Figure 7. The decrease in conversion ratio with increasing U-235 loading reflects the loss of neutrons to the control system as the reactivity increases. The larger values for thorium than for U-238 result from the larger thermal cross section of thorium, which competes more effectively for neutrons with the structural material, and also from the larger average value of η for U-233 than for Pu-239. The difference in conversion ratio between stainless steel and zirconium clad is greater for thorium than for U-238 because the stainless steel absorbs mostly thermal neutrons. Most of the U-238 conversion is from resonance absorptions, whereas 40% of the thorium conversion is by thermal absorptions. The larger value of conversion ratio for the thorium cores indicates the more efficient utilization of the original fuel, and is a basic factor in obtaining longer reactivity lives. The conversion ratio increases with time (Figure 8) because the excess reactivity decreases and fewer neutrons are lost to the control system.

2.4 TIME VARIATION OF k

The multiplication of the core varies with time as the U-235 is burned and as the new fissionable isotopes build up. Figure 8 shows this variation for the case of zirconium clad thorium core, 3.2% enriched. The thermal multiplication decreases with time as the net amount of fissionable material decreases and also as the fission products build up. The epithermal multiplication increases in spite of the fissionable material decrease because the product of the average η in the resonance region and the resonance integral is larger for U-233 than for U-235. These comments also apply to the other cores studied.

Initially k_{eff} decreases rapidly with time as equilibrium Xe and Sm build up, after which it decreases almost linearly as the fuel is burned out. The initial decrease is more rapid for the thorium cores because little U-233 is produced immediately. However, after Pa-233 reaches equilibrium, k_{eff} decreases less rapidly for the thorium cores because of the larger conversion ratio.

Due to the small rate of decrease of k_{eff} with time near the end of core life, an uncertainty in k_{eff} of 1% can mean an uncertainty in core life of 10-15%. The uncertainties in the resonance integrals, in the average η in the resonance region for all fissionable nuclei, and in the thermal cross section and resonance integral of the gross fission products contribute fairly large uncertainties to computed core lives.

2.5 RECYCLING OF FUEL

Recycling of the fuel in the thorium converters after removal of fission products will give an average conversion ratio of about 0.8 for 18,000 MWD/T exposure after three recycles. The U-235 make up per core loading will correspond to 70 kg or 0.23 gms per MWD of heat. The material balance for the first recycle of an 18,000 MWD/T core reload cycle using a zirconium clad thorium core is shown in Table IX. In the first recycle, the characteristics of the original core are very closely matched by the addition of 95 kg of U-235 to the used fuel. The U-235 make up is reduced by further recycling as the conversion ratio increases. In actual practice only part of the core would be reloaded, and the cycle time for this would be shorter. The non uniform loadings that this cycle implies would reduce the excess reactivity to be controlled.

TABLE IX

18,000 MWD/T Reload Cycle

Isotope	Original Core $\overline{CR} = 0.68$		First Recycle Core $\overline{CR} = 0.73$	
	Initial Mass (Kg)	Final Mass (Kg)	Initial Mass (Kg)	Final Mass (Kg)
U-233	0	152	170	211
U-234	0	12	12	31
U-235	476	177	272	104
U-236	0	56	56	83
Pa-233	0	18	0	18

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- -ALUMINUM PLATE (.203 CM)
- - FUEL TAPE (.124 CM)
- -THORIUM PLATE (.142 CM)
- WATER SPACES (.414 OR .533 CM)

FIGURE 1 CROSS SECTION OF PLATE
FUEL ELEMENT

FIGURE 2 SPECIAL TUBE FOR CADMIUM
FRACTION MEASUREMENT

FIGURE 3 DANC OFF FACTOR FOR OXIDE PIN LATTICES

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8