Phil
289
9/20

RECEIVED

1967

MASTER

RECEIVED BY DTIC SEP 22 1967

UNIVERSITY of CALIFORNIA
LIVERMORE

264

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

FIG. 3 2 m, MN .65

TID-4500, UC-33
Propulsion Systems
and Energy Conversion

Lawrence Radiation Laboratory
UNIVERSITY OF CALIFORNIA
LIVERMORE

UCRL-50294

HEAT PIPE RADIATOR FOR A
50-MW t SPACE POWER PLANT

Richard W. Werner
Gustav A. Carlson

June 30, 1967

Printed in USA. Available from the Clearinghouse for Federal
Scientific and Technical Information, National Bureau of Standards,
U. S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy \$3.00; Microfiche \$0.65.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Blank Page

CONTENTS

	<u>Page No.</u>
ABSTRACT	1
NOMENCLATURE	2
INTRODUCTION	5
CONVENTIONAL RADIATORS	6
VAPOR FIN RADIATORS	7
THE HEAT PIPE RADIATOR	10
HEAT PIPE OPERATION AND HEAT TRANSFER	12
HEAT PIPE WORKING FLUIDS	14
COMPUTER DESIGN OF THE RADIATOR	16
Input Data	16
Material Properties	16
Heat Pipe Design	17
Manifold Design	18
Radiator Planform	19
Feed and Return Line Design	20
Line and Manifold Wall Thicknesses	21
Meteoroid Protection of Lines and Manifolds	22
Heat Rejection from Manifolds and Feed Line	22
Mass Calculations	22
HPRAD4 DESIGNS FOR SPR-6 RADIATOR	25
Heat Pipe Input Flux and Axial Flux Safety Factor	25
Heat Pipe Diameter	26
Heat Pipe Survival Probability	27
Radiator Survival Probability	27
Pressure Drop of Working Fluid	28
Heat Pipe Fluid	28
PRESSURE DROP ANALYSIS	29
METEOROID CRITERIA	31
Flux Distribution	32
Penetration Equations	32
Application of Barrier Thickness Equation	33
COMPARISON OF HEAT PIPES AND SOLID CONDUCTORS	34
AN ADDITIONAL METEOROID HAZARD	36
SUBCOOLING	37
REFERENCES	38

Blank Page

HEAT PIPE RADIATOR FOR A
50-MWt SPACE POWER PLANT

ABSTRACT

The concept and design of a heat pipe radiator suitable for high power levels of approximately 50 MWt are discussed. Design considerations include meteoroid protection, heat transfer, stress, fluid flow, pressure drop, etc. A representative heat pipe design is shown to reject 46 MW at 1100°K using a total mass of 13,220 kg for a specific weight of 1.322 kg/kWe. The radiator is used as part of a Rankine-cycle nuclear reactor power plant suitable for a manned Mars mission of 10,000 hours.

NOMENCLATURE

A = vulnerable area, m^2 [ft^2 in equation (39)].

A_c = flow area, cm^2 .

b = groove height = $r_w - r_v$, cm .

b = thin plate adjustment constant = 1.5.

C_p = specific heat at constant pressure, $W\text{-sec}/g\text{-}^\circ K$.

C = sonic velocity of target material,

$$C = 12 \sqrt{\frac{E g_c}{\rho_t}} \text{ ft/sec,}$$

ρ_t inside sonic velocity term is in lbm/ft^3 .

d = projectile diameter, cm .

D = pipe diameter, cm .

E = elastic modulus, lb/ft^2 .

f = Darcy-Weisbach friction factor.

F = heat pipe view factor ($2/\pi$ for close-packed tubes).

g = acceleration of gravity, cm/sec^2 .

g_c = gravitational constant.

G = mass flow rate per unit area, $g/cm^2\text{-sec}$.

H = condensing heat transfer coefficient, $W/cm^2\text{-}^\circ K$.

$\bar{H} = -\ln P(0)$, average number of penetrations on area A in time τ .

K = thermal conductivity, $W/cm\text{-}^\circ K$.

ℓ = axial pipe coordinate, cm .

ℓ_c = length of heat pipe condenser section, cm .

ℓ_e = length of heat pipe evaporator section, cm .

L = latent heat of vaporization, $dyne\text{-}cm/g$.

m = meteoroid mass, g .

N = total number of heat pipes.

P = pressure, bars.

$P(0)$ = survival probability.

q, Q = heat flow, W or dyne-cm/sec depending on the units of the equation.

r = radius, cm.

r_c = capillary radius (groove half-width), cm.

r_v = radius of vapor conduit, cm.

r_w = internal radius of heat pipe including wicking, cm.

S = wetted perimeter, cm.

t = thickness, cm [ft in equation (39)].

T = temperature, °K.

v_g = vapor specific volume, cm^3/g .

v_l = liquid specific volume, cm^3/g .

$\bar{V}$ = mean meteoroid velocity, km/sec.

w = mass flow rate, g/sec.

X = fluid quality.

Z = vapor fin tube length, cm.

Z = heat pipe length, cm.

α = meteoroid flux constant, $\text{g}^\beta/\text{m}^2\text{-sec}$.

β = constant in meteoroid equation = 4/3.

γ = surface tension, dyne/cm.

γ = material proportionality constant in meteoroid equation, ranging from 1.5 to 3.0.

ϵ = emissivity.

η = viscosity, g/cm-sec.

θ = wetting angle.

θ = constant in meteoroid equation = 2/3.

μ = viscosity, g/cm-sec.

ρ = barrier material density, lbm/ft^3 .

ρ_l = liquid density, g/cm^3 .

ρ_p = projectile density, g/cm^3 .

ρ_t = target density, g/cm^3 , except lbm/ft^3 in sonic velocity term C .

ρ_v = vapor density, g/cm^3 .

σ = Stefan-Boltzmann constant = $5.67 \times 10^{-12} \text{ W}/\text{cm}^2\text{-}^\circ\text{K}^4$.

σ = stress, bars.

τ = mission time, sec [hr in equation (39)].

ϕ = angle with horizontal.

ϕ = constant in meteoroid equation = $1/2$.

INTRODUCTION

The manned exploration of space requires a method of rejecting waste heat from the spacecraft which is extremely reliable and at the same time of low specific weight. Many systems of heat rejection have been studied and proposed, such as moving belt radiators, rotating radiant disks, vortex tubes, and refrigeration cycles. The radiator studies conducted at LRL, after a systems review, started with the classical fin-and-tube approach. Then, after the most optimum of the fin-and-tube radiators was found to be extremely heavy due to the required meteoroid protection, a change was made to a heat-pipe/heat-exchanger model. The heat pipe approach, at the high power levels in which LRL was interested, proved to be a good choice, with a "near optimum" design specific weight of 1.2 kg/kWe.

The heat pipe approach as it is used in this report differs in design principle from the vapor fin approach, which is also a heat pipe technique, in that the heat pipes form the secondary leg of a liquid-metal heat exchanger with direct contact between heat pipe and fluid to be cooled. This direct contact increases to a marked degree the heat transfer effectiveness of the system and also allows free selection of ratios of heat absorption to heat rejection.

The general constraints in the design of this radiator were as follows

1. Mission time 10,000 hours.
2. Heat rejection 46 megawatts.
3. Survival probability 0.99.
4. Rejection temperature about 1100°K.
5. Liquid metal to be potassium.

A manned Mars landing is the mission considered, and the radiator is part of a Rankine-cycle, closed-loop space power plant which uses a nuclear reactor heat source and turboalternators for power generation (Fig. 1). This 10-MWe space power reactor is designated SPR-6.

For high power levels such as are considered for a manned Mars mission, the radiator must consist of large surface areas even with favorably high radiation temperatures. For instance, at a representative radiator temperature of 1100°K

Fig. 1. Closed-loop space powerplant.

and an emissivity of about 0.9 the maximum rejection rate to a zero degree sink is only 7.5 W/cm^2 , quite small considering that the quantity of waste energy may be as high as 50 million watts.

CONVENTIONAL RADIATORS

Conventional radiators generally employ thin, solid fins as extended surfaces between fluid-carrying tubes. The purpose of the fins is to reduce the surface occupied by flow passages and thus reduce the area vulnerable to critical penetration from meteoroids. The fins, as solid conductors, have limited effectiveness since they cannot be isothermal, and the radiator size and weight increases as a consequence. Furthermore, the fluid-carrying tubes must be heavily armored against meteoroids which adds further to the total weight and imposes a severe weight penalty on the radiator design. The radiator thus can become the largest and heaviest single component of the space power supply and substantially control the entire system analysis when low specific weight is the design criterion.

The first radiator design considered and evaluated at LRL was this classical tube-and-fin model in which vapor out of the turbines or energy converter is caused to flow through a multiplicity of tubes where heat is rejected by 1) direct radiation from the tubes to space, and 2) conduction through the fins, and then radiation to space. A typical fin-tube geometry applicable to SPR-6 is shown in Fig. 2.

Fig. 2. Typical fin-tube geometry.

Pertinent characteristics of this radiator obtained from a computer calculation (called TOAST) are listed below.

Heat rejected, MW	47
Inlet fluid temperature, °K	1100
Number of tubes	349
Tube length, m	8.7
Radiator width, m	18.1
Radiator length, m	27.6
Projected area, m ²	498 (one side)
Weights:	

	<u>kg</u>	<u>% of total</u>
Inlet duct	1,820	6
Outlet duct	1,190	4
Barrier	16,100	56
Fins	9,750	34
Tube (liners)	96	—
Fluid (potassium)	<u>10</u>	<u>—</u>
Total radiator	28,800	100

The total weight for meteoroid protection is not explicit in these data. In addition to the 56% direct barrier weight, 25% of the fin weight is used to carry the fin from the tube wall to the outside diameter of the barrier, giving a total of 64.5% of the radiator weight attributable to meteoroid protection. Also, the tube shrouded in the barrier becomes a very ineffective emitter.

VAPOR FIN RADIATORS

In an attempt to gain some weight relief from the fin-and-tube arrangement, a second approach to radiator design¹ was reviewed for possible use. This is one proposed by NASA in which the solid fin is replaced by what is termed a vapor fin. A representative geometry for a vapor-fin radiator is shown in Fig. 3.

Fig. 3. Typical geometry of vapor fin radiator.

Haller *et al.*¹ show that for low power systems (500-1000 kWe) designed for 500-day life, substantial radiator weight savings of as much as 50% may be realized with the vapor-chamber fin-tube radiator over that of the solid fin and tube. The weight saving is stated to be less for higher power systems.

There are two principal heat transfer limits inherent in the vapor fin approach which seriously limit its applicability:

1. The fluid-carrying tubes are always separated from the vapor chamber by a meteoroid barrier thickness which is substantial and which creates a significant temperature drop between the fluid in the tube and the vapor in the fin. Since heat rejection is a fourth-power function of the vapor chamber wall temperature, any temperature degradation between fluid and wall lowers effectiveness and creates a heavier system.
2. The area available for condensing heat transfer from the fluid to the tube wall is limited to one-half the tube circumference per vapor chamber. This seriously limits the total quantity of heat into the chamber and directly sets the maximum length of the vapor fin by the heat balance between condensing and radiant transfer.

These two points can be illustrated by an example. Let us assume that representative tube and armor dimensions can be obtained from the TOAST code previously cited and that the vapor fin geometry can be approximately as shown in Fig. 4.

Fig. 4. Schematic of vapor fin geometry.

By conduction through the barrier wall whose thickness is $r_2 - r_1$,

$$q_{in} = \frac{\pi K Z (T_p - T_w)}{\ln(r_2/r_1)} \text{ watts.} \quad (1)$$

It can be reasonably assumed that when the capillary-lined cavity is functioning as a vapor fin the evaporator wall temperature T_w is equal to the radiating surface temperature T_r . Then by radiation from the surface of the vapor fin

$$q_{out} = \sigma \epsilon 2 L Z T_w^4 \text{ watts.} \quad (2)$$

By heat balance between (1) and (2)

$$\frac{\pi K Z (T_p - T_w)}{\ln(r_2/r_1)} = \sigma Z \epsilon 2 L T_w^4 \quad (3)$$

or

$$T_p = T_w + \frac{(\sigma Z \epsilon 2 L T_w^4) \ln(r_2/r_1)}{\pi K Z}. \quad (4)$$

For condensing liquid metals a heat flux of 50 W/cm^2 is attainable² at the fluid surface r_1 . Using $r_1 = 0.8 \text{ cm}$ and $r_2 = 2.7 \text{ cm}$ from TOAST and a thermal conductivity for niobium of $60 \text{ W/cm}^2\text{-}^\circ\text{C-cm}$, the temperature drop due to conduction across the barrier wall is obtained as follows:

$$50 \text{ W/cm}^2 \times (\pi Z \times 0.8) \text{ cm}^2 = \frac{\pi Z \times 0.6 (T_p - T_w)}{\ln(2.7/0.8)},$$

thus

$$\Delta T = 81^\circ\text{C}.$$

This is a seriously large temperature depression.

For liquid metals a film heat transfer coefficient $h_c = 4.25 \text{ W/cm}^2\text{-}^\circ\text{C}$ is representative.³ For the 50-W/cm^2 heat flux the temperature drop across the film is therefore about 12°C .

For SPR-6, T^* is taken as the turbine exit temperature and is equal to approximately 1110°K . Therefore, we can calculate

$$T_p = 1098^\circ\text{K},$$

$$T_w = 1017^\circ\text{K}.$$

The vapor fin maximum length L can now be determined as a function of r_1 for this particular case. That is

$$L = \frac{q_{in}/Z}{2\sigma\epsilon T_w^4} = \frac{50 \times \pi \times r_1}{2 \times 5,668 \times 10^{-12} \times 0.9 \times 1.96 \times 10^{12}} = 6.4 \text{ cm.}$$

It is apparent that using the vapor fin in this arrangement is not particularly advantageous. The conduction temperature drop across the meteoroid barrier decreases the radiator effectiveness by:

$$- \left[1 - \left(\frac{1098}{1017} \right)^4 \right] \times 100 = 37.5\%.$$

Limiting the heat input area cancels one of the vapor fin's greatest virtues, that of being able to transport energy for long distances isothermally and thus provide large amplification of heat transfer area. The vapor chamber is a fin without significant temperature drop and should therefore be made as long as possible.

THE HEAT PIPE RADIATOR

How does one resolve this dilemma of providing necessary meteoroid protection which does not sacrifice heat transfer capability? The solution lies in placing heat pipes inside the fluid-carrying tubes rather than appended to them as the vapor fins were. In this case the pipes can be supplied with heat from a source which is well protected from meteoroids and at the same time has a low presented area and thus a low contribution to total weight. The fluid-carrying tube can be of such a diameter that its thickness for structural reasons can approach the thickness required for meteoroid protection.

In this concept the inlet and outlet ducts originally used for vapor delivery and fluid return in the other systems become by suitable adjustment the primary leg of a heat exchanger. One end of each of a large number of heat pipes is inserted into this ducting to a depth suitable for heat transfer (Fig. 5). These heat pipes operate in parallel, functionally independent one from the other, and form the separate, secondary leg of the heat exchanger. Because a large number of independent heat pipes are involved, they can be designed for a minimum weight for a specified probability that a certain fraction will remain unpunctured during the mission.⁴ The ducting would carry the same amount of meteoroid protection as would be required for any radiator piping design. This arrangement can be visualized in a first concept as a large U-tube out of which protrude the heat pipes in a plane array as shown in Fig. 6.

Since the thin-walled heat pipe is inserted directly into the meteoroid-shielded duct, the large temperature drop caused by conduction loss seen in the vapor fin is eliminated. Additionally, the area available for condensing heat transfer may now be set at whatever value is optimum.

The effect of the puncture of a heat pipe is only that it will cool and not contribute as a heat exchange member. Since a large number are involved and redundancy can be incorporated, the loss of a considerable number can be tolerated.

Fig. 5. Heat pipes and duct.

Fig. 6. Heat pipe radiator concept.

HEAT PIPE OPERATION AND HEAT TRANSFER

The advantages of introducing the heat pipe principle to produce space radiators of low mass and minimum planform area lies in its unique feature of being a lightweight, geometrically adaptable, isothermal device capable of transporting large quantities of energy.

A heat pipe, a nearly empty cavity, is a self-contained thermal conductance device without moving parts which transfers heat as latent energy by evaporating a working fluid in a heating zone and condensing the vapor thus produced in a cooling zone. The heat pipe transports heat at substantially isothermal conditions. As shown in Fig. 7, the pipe has only three components (1) a container, (2) a capillary wick, and (3) a heat transfer

Fig. 7. Schematic of heat pipe.

fluid. It has two principal regions (a) the evaporator, where heat is absorbed in the form of latent heat of vaporization, and (b) the condenser, where heat is rejected by condensation. Fluid return is via the wick and capillary pumping.

The necessary condition for heat pipe operation is that the capillary pumping force be equal to or greater than all the losses in the cycle. In general form:

$$\boxed{\begin{array}{c} \text{Pressure} \\ \text{rise due to} \\ \text{capillary} \\ \text{forces} \end{array}} \cong \boxed{\begin{array}{c} \text{Pressure} \\ \text{drop in} \\ \text{the liquid} \end{array}} + \boxed{\begin{array}{c} \text{Pressure} \\ \text{head in the} \\ \text{liquid due} \\ \text{to gravity} \end{array}} + \boxed{\begin{array}{c} \text{Pressure} \\ \text{drop in} \\ \text{the vapor} \end{array}}$$

For pipes in which the capillary structures is a series of axial grooves of rectangular cross section covered with a single layer of mesh, the applicable equation is:

$$\boxed{\frac{2\gamma \cos \theta}{r_c}} \cong \boxed{\frac{3\eta QZ}{4b\pi L \rho_1 r_v r_c^2}} + \boxed{p_1 gZ \sin \theta} + \boxed{\frac{(1 - 4/\pi^2)Q^2}{8\rho_v r_v^4 L^2}} \quad (5)$$

Equation (5) assumes that the intervening wall between grooves is negligibly thin at the inner radius (the wall is of triangular rather than trapezoidal cross section).

Equation (5) can be expressed as

$$AQ^2 + D + \frac{BQ}{r_c^2} = \frac{C}{r_c} \quad (5A)$$

where

$$A = \frac{(1 - \frac{4}{\pi^2})}{8\rho_v r_v^4 L^2}, \quad D = \rho_1 gZ \sin \theta, \quad B = \frac{3\eta Z}{4b\pi L \rho_1 r_v}, \quad \text{and } C = 2\gamma \cos \theta.$$

Solving for Q,

$$Q = \frac{-B \pm \sqrt{B^2 - 4Ar_c^2 (Dr_c^2 - Cr_c)}}{2Ar_c^2}. \quad (6)$$

Differentiating (5A) with respect to r_c and setting dQ/dr_c equal to zero yields

$$\frac{B}{Ar_c^3} \left(B^2 - 4ADr_c^4 + 4ACr_c^3 \right)^{1/2} \pm \frac{B}{Ar_c^3} \left(B + \frac{ACr_c^3}{B} \right) = 0, \quad (7)$$

which may be rewritten

$$\left(r_c^3 \right) + \left(\frac{4DB^2}{AC^2} r_c \right) - \left(\frac{2B^2}{AC} \right) = 0. \quad (7A)$$

For the case where the gravity term is zero (i. e., in space or for $\phi = 0$), the optimum value for r_c is

$$r_{c,opt} = \left(\frac{2B^2}{AC} \right)^{1/3} \quad (8)$$

and the axial power is

$$Q = \frac{-B + \sqrt{B^2 + 4Ar_c^3 C}}{2Ar_c^2}. \quad (9)$$

For the special case of optimum r_c and maximum power,

$$Q_{max} = \frac{B}{A(r_{c,opt})^2}. \quad (10)$$

HEAT PIPE WORKING FLUIDS

The desirable properties of heat pipe fluids are⁵:

1. High latent heat.
2. High thermal conductivity.
3. Low viscosity.
4. High surface tension.
5. High wetting ability.
6. Suitable boiling point.

Furthermore, it can be shown⁶ that the relative worth of any two working fluids can be compared by the parameter λ , the axial heat flux, which can be expressed as

$$\lambda \propto L \left(\frac{\rho_\ell \rho_v \gamma^2}{\eta_\ell Z} \right)^{1/3}. \quad (11)$$

The choice of fluids starts tentatively with a boiling point selected to mesh with the known operating temperature. At the operating temperature the vapor pressure should be in the region of the pressure/temperature curve where pressure changes produce small temperature changes (see Fig. 8). This generally will be at a level of at least 300-400 torr. At the same time a reasonable pressure limit should be imposed (perhaps less than 10 atm) so that containment is not a particular problem and thin-walled tubes can be used for good heat transfer and light weight.

At the temperature of interest for SPR-6, 1100°K (at the turbine exit), there are four working fluids that could be considered: cesium, potassium, sodium, and rubidium.

Table 1 lists properties of these four fluids that determine their relative worths and gives a figure of merit for each.

Thus, from a heat transfer standpoint for an individual heat pipe, potassium and sodium are substantially superior to either cesium or rubidium. Potassium and sodium are the two fluids which are currently used for radiator analysis. It is important that this figure of merit, λ , not be misconstrued. For a single pipe it is certainly valid. For use in a radiator system with many pipes it must be weighed against the many other variables of the system. For instance, if the number of heat pipes required when using potassium has been calculated for a specific system

Fig. 8. Operating region for working fluid.

Table 1. Properties and figures of merit for four potential working fluids.

Working fluid	Latent heat of vaporization L	Liquid density, ρ_l	Vapor density, ρ_v	Surface tension, γ	γ_2	Liquid viscosity, η_l	$\left(\frac{\rho_l \rho_v \gamma^2}{r_l Z}\right)^{1/3}$	Merit, λ
	$\frac{\text{dyne-cm}}{\text{g}}$	$\frac{\text{g}}{\text{cm}^3}$	$\frac{\text{g}}{\text{cm}^3}$	$\frac{\text{dynes}}{\text{cm}}$		$\frac{\text{g}}{\text{sec-cm}}$	for unit Z	
Cesium	6.14×10^9	1.3	5×10^{-3}	33	1.1×10^3	1.1×10^{-3}	18.6	1.14×10^{11}
Potassium	1.89×10^{10}	0.55	1×10^{-3}	60	3.6×10^3	1.1×10^{-3}	12.2	2.31×10^{11}
Sodium	4.02×10^{10}	0.68	2×10^{-4}	120	1.44×10^4	2.7×10^{-3}	9.0	3.6×10^{11}
Rubidium	8.9×10^9	1.2	3×10^{-3}	55	3×10^3	1.4×10^{-3}	13.7	1.22×10^{11}

heat load, it does not follow that (a) the number of pipes can be decreased or (b) that the length of individual pipe can be shortened simply by going to sodium. This is because at a given temperature and heat rejection the condensing (radiating) area of the heat pipe assembly must remain a constant, i. e., the ability to lose heat to space is the governing factor. The potential of the figure of merit as it relates to the total system lies more probably in the ability to run off-optimum with the best fluid. It may be possible to use less capillary wick (and hence have less weight) as an off-optimum case. The comparison of sodium and potassium for a system which uses an optimum heat pipe is discussed on p. 28.

COMPUTER DESIGN OF THE RADIATOR

The computer subroutine HPRAD4 is a fast-running code which designs a heat pipe radiator for a given set of input parameters. Very little optimization is carried out internally, but optimum designs can be quickly found by calling the subroutine with different input data sets. This discussion follows the order of calculations carried out by the code.

Input Data

The input data includes the mass flow rate of the working fluid to be condensed as well as its inlet temperature and quality and maximum allowable fractional pressure drop. The input data also includes the mission time, the design survival probability, an overall condensing heat transfer coefficient for the working fluid, and the maximum allowable wall stress for the piping. The input data set is completed by specifying three heat pipe parameters: the inside diameter (including wicking structure), the heat pipe input flux, and the axial flux safety factor. These last two variables will be explained in the heat pipe design discussion.

Material Properties

Material properties are required for the structural materials and for the working fluid and heat pipe fluid. The structural material properties are assumed to be essentially constant over the temperature range of the radiator. The properties required in the code are the density and modulus of elasticity for the heat pipe material, the manifold and supply line material, and the meteoroid barrier material. The thermal conductivity of the barrier material is also specified. The fluid properties are obtained from two fluid property subroutines, KVAP and TRANP. After specifying the working fluid, these subroutines provide the following property data: inlet pressure and enthalpy; outlet pressure, temperature, and enthalpy; mean values, at $T_F = (T_{in} + T_{out})/2$, for the viscosity and specific volume of the saturated liquid, for the specific volume of the saturated vapor, and for the rate of change of the specific volume of the saturated vapor with respect to the pressure. Each heat pipe is considered to be exposed to the working fluid at its mean temperature T_F . The heat pipe temperature T_P (assumed to be essentially constant) is calculated as

$$T_P = T_F - \frac{\text{Flux}}{H} \quad (12)$$

where Flux is the heat pipe input flux and H is the condensing heat transfer coefficient. (The heat pipe input flux must be maintained below some value at which boiling in the wick structure becomes a danger. Heat pipes have been successfully run at input fluxes in excess of 100 watts/cm².⁷ The input parameter Flux is generally chosen to be less than this number.) After specifying the heat pipe fluid, the fluid property subroutines provide the following data: surface tension, heat of vaporization, pressure, viscosity and density of the saturated liquid, and viscosity and density of the saturated vapor.

Heat Pipe Design

The heat pipes are assumed to be right circular cylinders. The wick structure consists of axially directed rectangular grooves cut on the inside walls of the heat pipes. The heat pipe equations used imply adjacent grooves (the groove separation is zero at the vapor duct radius) and a single layer of screen covering the grooves.

The ratio of the length of the heat pipe condenser section to the length of the evaporator section is calculated as

$$\frac{\ell_c}{\ell_e} \times \frac{H(T_F - T_P)}{\sigma \epsilon F T_P^4} . \quad (13)$$

The vapor duct radius of the heat pipe, r_v , is chosen to be some fraction of the inside radius of the pipe, r_w . It can be shown that an optimum heat pipe (in terms of maximum axial heat flow) is obtained for

$$r_v = \frac{5r_w}{6} . \quad (14)$$

Because an off-optimum heat pipe may sometimes result in a system mass reduction, the r_v , r_w relation given by equation (14) should not be considered inviolable.

An iterative procedure is carried out to determine the appropriate length for the heat pipe. A length Z is guessed and the heat pipe equations (8) and (10) are used to calculate the optimum groove half-width, r_c , and the maximum axial heat transfer, $Q_{\max}$. (The only variable in these equations which has not yet been mentioned is the wetting angle θ . In the absence of any exact information θ has so far been considered to be zero.) $Q_{\max}$ is then divided by the axial flux safety factor SF.

$$Q = \frac{Q_{\max}}{SF} . \quad (15)$$

Q is the design heat transfer for the heat pipe. SF has been varied between 1 and 5. An SF higher than unity serves two purposes. It results in a shorter heat pipe which reduces

the meteoroid vulnerability, and it safeguards the heat pipe by selecting an operation level below the calculated maximum. The heat into the heat pipe is now calculated as

$$Q_{in} = 2\pi r_w \ell_e H (T_F - T_P). \quad (16)$$

The evaporator length ℓ_e is obtained by combining equation (13) with the geometric relation

$$\ell_e + \ell_c = Z. \quad (17)$$

Q_{in} is now compared with the design heat transfer for the heat pipe Q . The guessed value for Z is revised until the two heats are identical. At this point the individual heat pipe is designed. The three heat transfer terms – convection to the pipe, axial transfer in the pipe, and radiation away from the pipe – are all equal.

The number of heat pipes necessary to condense all of the working fluid is calculated by dividing the total heat of condensation by the design heat transfer for the individual heat pipe.

The final heat pipe calculations are those concerned with meteoroid protection. As fully explained in the section on meteoroid criteria, the survival of the necessary number of heat pipes is assured through an appropriate tradeoff between individual pipe protection (by virtue of its structural wall) and the addition of redundant pipes.

Manifold Design

Having calculated the heat pipe dimensions as well as the total number of pipes N (including the redundant pipes), the code proceeds to design the manifold, in which the working fluid flows past the evaporator ends of the heat pipes. The manifold is considered to be a right circular cylinder pierced by heat pipes as shown in Fig. 9.

Fig. 9. Schematic of manifold.

There is one heat pipe per axial plane in the manifold. The condenser ends of the heat pipes (outside the manifold) form a plane. The evaporator ends of the pipes may be bent so as to present a staggered view to the working fluid flowing through the manifold. Given this arrangement, the manifold inside diameter and total length is straightforwardly calculated from the heat pipe diameter, heat pipe evaporator length, and total number of pipes.

Next, the manifold is divided into parallel segments to reduce the pressure

drop of the condensing working fluid. In order to ensure symmetric radiator planforms only even numbers of segments are considered. The pressure drop in a manifold segment is calculated as described in the section on pressure drop using the working fluid properties, the dimensions of the manifold segment, and the mass flow rate per segment. The wetted perimeter S and the flow area A_c are calculated as

$$S = (\pi + 2)D_m \quad (18)$$

and

$$A_c = \frac{\pi}{4} D_m^2 - D_p D_m \quad (19)$$

where D_m is the inside diameter of the manifold and D_p is the outside diameter of the heat pipe. Manifold segmentation is continued until the calculated pressure drop is less than the maximum allowable value. (The total allowable pressure drop is assumed to be equally divided between the manifolds, feed line, and return line.)

Radiator Planform

The manifold segments are arranged as shown in Fig. 10 and connected by feed and return lines. Figure 10 shows the radiator in an axial feed line orientation. Figure 11 shows the same planform in a lateral feed line orientation. The computer code chooses

Fig. 10. Orientation 1 axial feed line.

Fig. 11. Orientation 2 lateral feed line.

whichever orientation results in a radiator whose axial dimension is greater than its lateral dimension. This restriction can be important to the total system because the shield size increases with radiator width. The simplicity of this orientation selection does have some drawbacks. In the almost square case the lateral feed line orientation will result in a heavier system because of the extra feed line leg. The lateral feed line orientation also results in the shadowing of some of the heat pipes by the feed line. This complication is ignored in the code. However, these drawbacks are academic for the 45-50 MW radiator because the optimum radiator designs have been found to be in the axial feed line orientation. If the code is to be used to design much smaller heat pipe radiators (with fewer manifold segments), it may be necessary to change the planform and orientation restrictions.

Feed and Return Line Design

The feed and return line lengths are easily calculated knowing the manifold and heat pipe dimensions and the orientation of the radiator. Then the lines are sized using the pressure drop criterion.

1. The Feed Line

The feed line is assumed to carry the working fluid at a constant quality equal to its inlet quality. The pressure drop is calculated for a constant diameter pipe which carries the total flow rate of the working fluid. The pipe length used in the calculation depends on the radiator orientation. In the axial feed line case the radiator length is used; in the lateral feed line case half the length plus half the width is used. The diameter D_0 is increased until the pressure drop is less than the maximum allowable value.

After D_0 has been determined the feed line may be tapered since the mass flow rate decreases linearly as a function of position along its distribution length. For a constant pressure gradient an approximate solution to the pressure drop relation for a linearly decreasing flow rate yields

$$D_{fl} = D_0 \left(1 - \frac{x}{X}\right)^{0.375} \quad (20)$$

where

D_{fl} is the feed line diameter,

D_0 (as previously calculated) is the feed line diameter at the beginning of the distribution length,

x is the position along the distribution length,

X is the total distribution length.

2. The Return Line

The return line is assumed to carry the working fluid in an all-liquid state. The pressure drop is calculated for a constant-diameter pipe. In both radiator orientations half the flow rate is assumed to travel the length of the radiator plus half its width. The diameter of the line is increased until the pressure drop is sufficiently low. The return lines are not tapered because the resulting mass reduction would not be significant.

Line and Manifold Wall Thicknesses

The line and manifold wall thicknesses t_i are calculated from the simple hoop stress relation using the inlet pressure of the working fluid P_{in} , the allowable stress σ as specified in the input, and the appropriate diameter D_i .

$$t_i = \frac{P_{in}}{\sigma} \frac{D_i}{2} . \quad (21)$$

In some cases this calculation may result in an unrealistically low wall thickness. In such cases the thickness (and pipe mass) may be scaled up as desired.

Meteoroid Protection of Lines and Manifolds

The lines and manifolds are now provided with a layer of armor of sufficient thickness to provide protection from meteoroids. This application of the barrier thickness equation is discussed in the section on meteoroid criteria. In general, the meteoroid armor is considered to be of different material than the ducting it protects. However, the case of thick-walled, "self-protecting" ducting may be calculated by specifying equal material properties for the ducts and armor.

Heat Rejection from Manifolds and Feed Line

The heat radiated directly to space from the armored manifolds and feed line is calculated. The surface temperature of the armor used in this calculation is determined by considering three temperature drops: the working-fluid-to-pipe-wall temperature drop (using the condensing heat transfer coefficient), the pipe-wall-to-armor temperature drop (using a contact heat transfer coefficient), and the conduction temperature drop across the armor.

The number of heat pipes may now be reduced because some of the heat (approximately 10%) is rejected by the ducting. The code returns to the manifold design section and designs a radiator with an appropriately lower number of heat pipes. Revision of the number of pipes followed by a redesign of the manifolds and lines is continued until the total heat rejection matches the total heat load.

Mass Calculations

The total radiator mass is calculated as the sum of the masses of six components: the heat pipes, the heat pipe fluid, the manifolds, the feed line, the return lines, and the meteoroid armor. The working fluid inventory in the radiator is also calculated, but this mass is not included in the total radiator mass.

The following is a copy of a portion of the HPRAD4 code output which illustrates the design of a specific radiator for SPR-6.

1

HPRAD4, HEAT PIPE RADIATOR

```
INPLT DATA
RADIATOR INLET TEMP, DEG K= 1.100E+03
FLCW RATE, G/SEC= 2.850E+04
INLET QUALITY= 8.500E-01
MAX FRACTICNAL P DROP CF PRIMARY= 5.000E-02
```

LIFETIME,HR= 1.000E+04
SURVIVAL PRCB= 5.900E-01
HEAT COEFF,W/CM/CM/DEG K= 4.250E+00
MAX INPUT FLUX,W/CM/CM= 5.000E+01
AXIAL FLUX SAFETY FACTOR= 2.000E+00
HEAT PIPE ID INCLUDING WICKING,CM= 1.500E+00

PRIMARY PIPING STRESS,BAR= 3.500E+02
HPP,AVERAGE HITS PER HEAT PIPE= 1.000E-01

END OF INPLT DATA

MATERIAL DENSITIES,G/CM**3
HEAT PIPE= 8.560E+00 (Niobium)
MANIFOLD AND LINE= 8.560E+00 (Niobium)
BARRIER= 1.860E+00 (Beryllium)

MATERIAL MODULUS OF ELASTICITY,DYNE/CM**2
HEAT PIPE= 1.100E+12
BARRIER= 2.100E+12

BARRIER CONDUCTIVITY,W/CM/K= 6.000E-01
MANIFOLD-BARRIER INTERFACE HEAT CCEFF,W/CM**2/K= 5.670E+00

AVERAGE PRIMARY FLUID PROPERTIES (Potassium)
LIQ VIS,G/SEC/CM= 1.085E-03
LIQ DENS,G/CM**3= 5.487E-01
VAP DENS,G/CM**3= 8.227E-04
DVGDP,CM**5/G/DYNE=-5.782E-04
ENTHALPY CHANGE,DYNE CM/G= 1.602E+10

HEAT PIPE FLUID PROPERTIES (Potassium)
SURF TENS,DYNE/CM= 6.126E+01
LIQ VIS,G/SEC/CM= 1.100E-03
VAP VIS,G/SEC/CM= 2.005E-04
LIQ DENS,G/CM**3= 5.543E-01
VAP DENS,G/CM**3= 7.541E-04
HEAT OF VAP,DYNE CM/G= 1.888E+10

HEAT REJECTION REQUIRED,KW= 4.566E+04

HEAT PIPE TEMP,DEG K= 1.085E+03
BARRIER SURFACE TEMP= 1.081E+03

CALCULATED HEAT REJECTION,KW= 4.567E+04
HEAT REJECTION BREAKDOWN,PERCENTS
HEAT PIPES= 89.59 MANIFOLDS= 7.96 FEED LINE= 2.45

PRESSURE IN BARS
HEAT PIPE PRIMARY IN PRIMARY OUT
1.648E+00 1.863E+00 1.770E+00

D VAPOR	C WICK	TH PIPE	L EVAP	L CCND
1.25CE+00	1.500E+00	6.746E-02	7.872E+00	8.740E+01

DI MAN	L MAN	R CAP	GROOVES	PIPES
8.64CE+00	3.677E+04	2.043E-02	96	22489

MASS FLOW IN HEAT PIPE,GM/SEC= 1.071E+00
 RACIAL REYNOLDS NO= 1.080E+02 (ANALYSIS VALID ONLY IF MUCH GREATER

THAN ONE)

MAX HEAT PIPE WALL STRESSES,BARS
 COMPRESSION OF PUNCTURED PIPE= 2.258E+01
 TENSION OF WHOLE PIPE= 1.832E+01

PARALLEL MANIFOLD SEGMENTATION

SEGMENTS	REYNOLDS	FR P DROP	SEGMENT LENGTH
44	1.098E+05	1.211E-02	8.356E+02

ORIENTATION,AXIAL FEED LINE,LATERAL MANIFOLDS

FEED LINE ID,CM= 5.836E+01 FR P DROP= 1.554E-02
 TOTAL LENGTH,CM= 2.096E+03
 RETURN LINE ID,CM= 1.300E+01 FR P DROP= 1.452E-02
 TOTAL LENGTH,CM= 5.863E+03

NOTE,FEED LINE ID IS AT ENTRANCE,DISTRIBUTION LENGTH IS TAPERED
 AS $(1-X/L)**0.375$

PIPE WALL THICKNESSES,CM		
MANIFOLD	FEED LINE	RETURN LINE
2.14E-02	1.554E-01	3.461E-02

METEOROID BARRIER THICKNESS,CM= 1.087E+00

RADIATOR MASSES IN KG		
HEAT PIPES	H PIPE LIQ	MANIFOLDS
7.001E+03	5.824E+02	1.706E+02
FEED LINE	RETURN LINES	BARRIER
3.731E+02	7.112E+01	3.560E+03
TOTAL MASS= 1.176E+04		

MASS PERCENTS					
H PIPE	H PIPE LIQ	MAN	FEED	RETURN	BARRIER
59.54	4.95	1.45	3.17	0.60	30.28

RAD LENGTH,CM= 2.191E+03 RAD WIDTH,CM= 1.756E+03

PRIMARY FLUID MASS,KG= 5.205E+02
 DISTRIBUTION OF PRIMARY FLUID,MASS PERCENTS

FEED	MAN	RETURN
0.60	17.35	82.05

It is meaningless to compare individual radiator components for the fin-and-tube case as generated by TOAST with the heat-pipe-radiator case as generated by HPRAD4. The designs are different. The codes are different. Inlet ducts, outlet ducts, barriers, fluid, etc., are defined differently for each code.

The total radiators can be compared without confusion as follows:

	<u>Fin and tube</u>	<u>Heat pipe</u>
Heat rejected, MW	47	45.66
Inlet fluid temperature, °K	1,100	1,100
Total mass, kg	28,800	11,760
Specific weight, kg/kWe	2.88	1.176
Relative weight	1.0	0.41

HPRAD4 DESIGNS FOR SPR-6 RADIATOR

The SPR-6 power system requires a radiator capable of rejecting approximately 45 MW of heat. This heat rejection is accomplished through the condensation of a 28.5-kg/sec flow of potassium, which enters the radiator at a quality of 85%. The radiator is to be designed for a 10,000-hour mission with a 99% probability of survival. The computer code HPRAD4 has been used to investigate the effect of various parameters on heat pipe radiator mass. While the parameter variation has not been exhaustive, enough has been done to specify a "near-optimum" radiator design.

Heat Pipe Input Flux and Axial Flux Safety Factor

The heat pipe input flux and axial flux safety factor jointly determine the length of the individual heat pipe as well as its separation into evaporator and condenser sections. For a heat pipe of given length, higher input fluxes mean shorter evaporator sections and longer condenser sections. This can be advantageous because heat rejection goes up with heat pipe condenser area. Also, size and mass of the heat pipe manifold decrease as the heat pipe evaporator section becomes shorter. However, these advantages of a high input flux are eventually overtaken by the disadvantages: longer heat pipe condenser sections require thicker walls to provide the same meteoroid protection, and small manifolds require much segmentation which increases the amount of feed line piping. Thus there exists an optimum value for the heat pipe input flux. It can also be argued that there should be an optimum value for the axial flux safety factor. Given a ratio of condenser section length to evaporator section length, higher safety factors mean shorter heat pipes,

Fig. 12. Total mass vs axial flux safety factor for four input fluxes.

but more of them. The shorter pipes may be constructed with thinner walls because of their reduced vulnerability to meteoroids. However, this mass reduction tends to be offset by the amount of manifolding required to accommodate the increased number of heat pipes. Thus there exists an optimum value for the axial flux safety factor.

Figure 12 shows the effect of the heat pipe input flux and axial flux safety factor on the total radiator mass. All of the results are for radiators using 1.5-cm-diam potassium-filled heat pipes. The piping is sized for a 5% pressure drop of the working fluid. A "near optimum" radiator is seen to result for an input flux of 50 W/cm² and an axial flux safety factor of 2. (The 75-W/cm², safety-factor-of-1 case is not chosen because it is considered undesirable to operate the heat pipes at maximum capacity.)

Heat Pipe Diameter

The total radiator mass is a strong function of the heat pipe diameter. As heat pipe diameter decreases the required wall thickness decreases. This results in a re-

Fig. 13. Total mass vs heat pipe diameter.

duction in heat pipe mass, even though the number of heat pipes increases. At some point, however, this mass decrease is negated by the increased amount of piping required to accommodate the large number of small heat pipes. Figure 13 shows the effect of heat pipe diameter on total radiator mass. The results shown are for potassium-filled heat pipes with an input flux of 50 W/cm² and an axial flux safety factor of 2. The optimum heat pipe diameter is seen to be between 1.0 and 1.5 cm.

Heat Pipe Survival Probability

As stated previously there is a tradeoff between individual heat pipe protection from meteoroids and the amount of redundancy included in the design. $\bar{H}$ has been defined as $-\ln P(0)$ where $P(0)$ is the survival probability of the individual heat pipe. $\bar{H}$ is the average number of damaging hits per heat pipe. A low value of $\bar{H}$ means a high degree of protection for the individual heat pipe and little redundancy. It is shown theoretically that $\bar{H} = 0.20$ results in the minimum total heat pipe mass. Because the heat pipes do not make up the total radiator, however, it is to be expected that the minimum total radiator mass will occur for a somewhat lower $\bar{H}$. In Fig. 14 it is seen that the optimum value of $\bar{H}$ is 0.10. The results shown are for 1.5-cm-diam potassium-filled heat pipes with an input flux of 50 W/cm^2 and an axial flux safety factor of 2.

Fig. 14. Mass vs $\bar{H}$.

Radiator Survival Probability

It has been specified that the radiator piping carry enough meteoroid armor to ensure a 99% probability of survival. Figure 15 shows the penalty which must be paid for survival probabilities in excess of 99%. The higher total mass at higher survival probabilities is due entirely to the increase in armor thickness. The armor thickness increases from 1.09 cm at $P(0) = 0.99$ to 3.77 cm at $P(0) = 0.9999$. Note that "adding a 9" to the survival probability approximately doubles the thickness and mass of the armor.

Fig. 15. Mass vs $P(0)$.

Pressure Drop of Working Fluid

Higher allowable pressure drops result in lighter radiators because manifold segmentation decreases and feed and return lines decrease both in length and diameter.

Fig. 16. Total mass vs pressure drop.

Figure 16 shows the effect of varying the allowable pressure drop from 5 to 20%. The mass reduction resulting from the increased allowable pressure drop is seen to be rather small. In a system study this effect must be balanced with the increase in pump mass due to higher pressure drops. The results shown are for 1.5-cm-diam potassium-filled heat pipes with an input flux of 50 W/cm^2 and an axial flux safety factor of 3.

Heat Pipe Fluid

It can be argued that sodium is a better heat pipe fluid than potassium. However, the use of sodium-filled heat pipes for the SPR-6 radiator does not necessarily result in

Fig. 17. Total mass vs axial flux safety factor.

a lighter radiator. Figure 17 presents the same parameter variation as Fig. 12 except that the heat pipes are filled with sodium rather than potassium. The minimum mass radiator of Fig. 17 (at an input flux of 50 W/cm^2 and a safety factor of 3) is actually 150 kg heavier than the minimum mass radiator of Fig. 12. In one sense, however, the sodium heat pipe radiator is "better" than the potassium radiator. At an input flux of 50 W/cm^2 , the sodium design reaches its minimum mass at a higher axial flux safety factor than the potassium design. Thus, the sodium pipes operate at a lower fraction of their capacity than the potassium pipes. Unfortunately, this "advantage" does not appear as a reduction in radiator mass.

PRESSURE DROP ANALYSIS

An influencing factor in the radiator planform is the allowable pressure drop in the duct which contains the heat pipes and in the supply and return headers. For this analysis it is assumed that there is two-phase flow, that the vapor quality is equal to that at the turbine exhaust, and that the return fluid is all liquid with subcooling as a consequence of carrying excess heat pipes for redundancy.

The allowable pressure drop is taken as 5% of the inlet pressure, equally divided between supply and return headers and duct. This is a somewhat arbitrary constraint based on the logic that the use of Owens' equation (see below) for two-phase flow has greater validity the less the pressure drop.

The pressure drop of the condensing working fluid in the radiator is obtained by integrating the two-phase pressure gradient derived by the method of W. L. Owens.⁸ Owens' pressure gradient is derived for a two-phase mixture flowing in a constant-area circular pipe with the assumption of equal velocities for the two phases (the fog flow model). The result is

$$\frac{dP}{d\ell} = \frac{-\frac{fG^2 v_\ell}{2Dg_c} \left[1 + X \left(\frac{v_g}{v_\ell} - 1 \right) \right] + \frac{G^2 v_\ell}{g_c} \left(\frac{v_g}{v_\ell} - 1 \right) \frac{dx}{d\ell} + \frac{\sin \phi}{v_\ell \left[1 + X \left(\frac{v_g}{v_\ell} - 1 \right) \right]}}{1 + X \frac{dv_g}{dP} \frac{G^2}{g_c}} \quad (22)$$

Following Owens, the pressure gradient for two-phase flow in a constant-area noncircular duct (or a circular duct with internal frictional surfaces) may be derived as

$$\frac{dP}{d\ell} = \frac{-\frac{fG^2 v_\ell S}{8g_c A_c} \left[1 + X \left(\frac{v_g}{v_\ell} - 1 \right) \right] + \frac{G^2 v_\ell}{g_c} \left(\frac{v_g}{v_\ell} - 1 \right) \frac{dx}{d\ell} + \frac{\sin \phi}{v_\ell \left[1 + X \left(\frac{v_g}{v_\ell} - 1 \right) \right]}}{1 + X \frac{dv_g}{dP} \frac{G^2}{g_c}} \quad (23)$$

where

S = wetter perimeter,

A_c = flow area.

Neglecting the gravity term and substituting for G in terms of the mass flow rate w and the flow area A_c yields

$$\frac{dP}{d\ell} = \frac{-\frac{w^2}{g_c A_c^2} \left\{ (v_g - v_\ell) \frac{dx}{d\ell} + \frac{fS}{8A_c} \left[(1 - X) v_\ell + X v_g \right] \right\}}{1 + \frac{w^2}{A_c^2 g_c} \frac{dv_g}{dP} X} \quad (24)$$

Now assume that

1. v_g , v_ℓ , and dv_g/dP are essentially constant over the range of integration.
2. Quality varies linearly with position, $X = X_1 + (X_2 - X_1) \frac{\ell}{L}$, where L = length of duct.

Substituting for X and integrating yields

$$\frac{P_2 - P_1}{P_1} = \frac{w^2 v_g}{g_c A_c^2 P_1} \int_0^L \frac{a\ell + b}{c\ell + d} d\ell \quad (25)$$

where

$$\begin{aligned} a &= \frac{fS}{8A_c L} (X_1 - X_2) \left(1 - \frac{v_\ell}{v_g} \right), \\ b &= \frac{1}{L} (X_1 - X_2) \left(1 - \frac{v_\ell}{v_g} \right) - \frac{fS}{8A_c} \left[(1 - X_1) \frac{v_\ell}{v_g} + X_1 \right], \\ c &= \frac{w^2}{g_c A_c^2 L} \frac{dv_g}{dP} (X_2 - X_1), \\ d &= 1 + \frac{w^2}{g_c A_c^2} \frac{dv_g}{dP} X_1. \end{aligned}$$

The integral may be evaluated exactly to yield

$$\frac{P_2 - P_1}{P_1} = \frac{w^2 v_g}{g_c A_c^2 P_1} \left(\frac{aL}{c} + \frac{bc - ad}{c^2} \ln \left| \frac{cL}{d} + 1 \right| \right). \quad (26)$$

If, as is often the case, $\left| \frac{cL}{d} \right| \ll 1$, then the result reduces to

$$\frac{P_2 - P_1}{P_1} = \frac{w^2 v_g}{g_c A_c^2 P_1} \left(\frac{aL^2}{2d} + \frac{bL}{d} \right). \quad (27)$$

The above solution reduces to the circular pipe case with the substitution

$$S = \pi D$$

and

$$A_c = \frac{\pi D^2}{4}.$$

It has been demonstrated by Owens that the two-phase friction factor should be chosen to be the same as that for single-phase liquid flow. For turbulent flow in a smooth-walled circular pipe, the Darcy-Weisbach friction factor is often given as

$$f = 0.184 \text{ Re}^{-0.2} \quad (28)$$

where Re is the Reynolds number. Writing the Reynolds number in terms of the flow area,

$$f = 0.184 \left(\frac{2w}{\mu \sqrt{\pi A_c}} \right)^{-0.2}. \quad (29)$$

Following Owens, f is calculated using the liquid viscosity for μ . For the circular pipe case the above relation for f reduces to the more familiar

$$f = 0.184 \left(\frac{4w}{\pi D \mu} \right)^{-0.2}.$$

METEOROID CRITERIA

The total mass of the radiator is the sum of the masses of the heat transport structure itself plus the not inconsiderable amount of meteoroid armor. The design of the space radiator is strongly influenced by the protection required against penetrating meteoroids. The data which are presently available on meteoroids are recognized as being imprecise, and the scaling laws which are used to determine penetration of structures by hypervelocity particles are estimates only and may be incorrect by a factor of 2 or 3.

Flux Distribution

The latest meteoroid criteria for flux distribution are taken from a NASA report⁹ and two personal communications between NASA and LRL.¹⁰ The meteoroid model used is a hybrid one consisting of the Whipple 1963A flux rate (which implicitly includes a meteoroid average density of 0.44 g/cc) with modified values of meteoroid velocity and density. The latest NASA constants are

$$F_{>} = \alpha m^{-\beta} \quad (30)$$

where $\beta = 1.34$ and $\alpha = 5.30 \times 10^{-11}$ for $F_{>}$ in hits per square foot per day and m in grams, or $\alpha = 6.60 \times 10^{-15}$ for $F_{>}$ in hits per square meter per second and m in grams, $\bar{V} = 20$ km/sec, $\rho_p = 0.2$ g/cc.

Penetration Equations

The general equation for expressing depth of penetration of hypervelocity particles into a plate of finite thickness¹¹ can be expressed as

$$t = b \gamma d \left(\frac{\rho_p}{\rho_t} \right)^\phi \left(\frac{\bar{V}}{C} \right)^\theta \quad (31)$$

An expression for thickness in terms of material properties is

$$t \propto E^{-1/3} \rho_t^{-1/6} \quad (32)$$

Depth of penetration or required thickness can be expressed in terms of meteoroid mass and flux. Assuming a spherical meteoroid particle and substituting

$$d = \left(\frac{6}{\pi} \right)^{1/3} m^{1/3} \rho_p^{-1/3} \text{ cm} \quad (33)$$

into (3) gives

$$t = 1.5 \gamma \left(\frac{6}{\pi} \right)^{1/3} \rho_p^{-1/3} \left(\frac{\rho_p}{\rho_t} \right)^\phi m^{1/3} \left(\frac{\bar{V}}{C} \right)^\theta \quad (34)$$

or

$$t = K m^{1/3} \left(\frac{\bar{V}}{C} \right)^\theta \quad (35)$$

where

$$K = 1.5\gamma \left(\frac{6}{\pi}\right)^{1/3} \rho_p^{-1/3} \left(\frac{\rho_p}{\rho_t}\right)^\phi. \quad (36)$$

The average number of penetrating impacts, $\bar{H}$, which will occur on a vulnerable area A in mission time τ by the assumed flux distribution is:

$$H = F_{>} A \tau = \ln P(0). \quad (37)$$

Combining equations (37), (35), and (30) results in an expression for calculating the required thickness for vulnerable areas,

$$t = 1.5\gamma \left(\frac{6}{\pi}\right)^{1/3} \rho_p^{-1/3} \left(\frac{\rho_p}{\rho_t}\right)^\phi \left(\frac{\bar{V}}{C}\right)^\theta \left(\frac{\alpha A \tau}{\bar{H}}\right)^{1/4}. \quad (38)$$

Equation (28) as it is used in our computer code has been modified by substitution of the values of V , ρ_p , etc., which are known, and by change of units to produce

$$t = 6.15 \frac{1}{E^{1/3} \rho^{1/6} \bar{H}^{1/4}} \left(\frac{\tau}{10,000}\right)^{1/4} A^{1/4}. \quad (39)$$

Application of Barrier Thickness Equation

1. Heat Pipe Manifolds and Supply Lines

The heat pipe manifolds and the supply and return lines are protected from meteoroids by the addition of a layer of beryllium armor. The required armor thickness is calculated from equation (39) using as vulnerable area the total surface area of the manifolds and the supply and return lines. The barrier calculation is made without regard to the protection ability of the pipe walls themselves. The design survival probability $P(0)$ is set at 0.99.

2. Heat Pipes

It is undesirable to add a separate layer of meteoroid armor to the heat pipes because of the detrimental heat transfer effect. Thus, the heat pipes must depend on their wall structure to provide protection. Also, redundancy may be introduced as a protection mechanism because each heat pipe is a separate entity. For a single heat pipe the average number of penetrating hits is given from equation (39) as

$$\bar{H} = \frac{C}{t^4} \quad (40)$$

where C is a constant and t is the wall thickness. If $\bar{H}$ is less than unity and the total number of heat pipes N is very large, then $\bar{H}N$ is the number of punctured pipes. If N_s is the number of heat pipes required for the completion of the mission it follows that

$$N = \frac{N_s}{1 - \bar{H}}. \quad (41)$$

If M_w is the total mass of the heat pipe walls then

$$M_w \propto Nt = \frac{N_s}{1 - \frac{C}{t^4}} t = \frac{N_s t^5}{t^4 - C}. \quad (42)$$

Minimizing M_w yields $\bar{H}_{opt} = 0.20$ and $N = 1.25 N_s$. (Because M_w may or may not be the dominant radiator mass term, $\bar{H}_{opt}$ is carried as a variable in our computer code.) The heat pipe wall thickness as calculated from equation (39) using $\bar{H} = \bar{H}_{opt}$ has been found to be more than adequate from the fluid pressure standpoint.

Equation (41) assumes that the number of heat pipes is large enough that the probability S of survival of N_s or more out of N pipes is sufficiently high. This probability may be calculated from the relation⁴

$$S = \sum_{n=N_s}^N \frac{N!}{n!(N-n)!} [1 - P(0)^{N-n} P(0)^n] \quad (43)$$

where $-\ln P(0) = \bar{H}$. For $\bar{H} = 0.20$ and $N_s = 0.8 N$ the probability S is greater than 0.99 for $N > 2500$ and greater than 0.999 for $N > 4000$. The number of heat pipes in a typical 45-MW radiator is considerably greater than 4000; therefore equation (41) is a valid redundancy relation.

COMPARISON OF HEAT PIPES AND SOLID CONDUCTORS

It is of interest to compare the heat transport qualities of a heat pipe and that of a solid rod of the same material.

The temperature gradient along the solid rod can be expressed as

$$\frac{dT}{dX} = \left(\frac{8\sigma\epsilon}{5KD} \right)^{1/2} T^{5/2}. \quad (44)$$

The heat into the rod from the Fourier conduction equation and (44) is:

$$\begin{aligned} q &= KA \left(\frac{8\sigma\epsilon}{5KD} \right)^{1/2} T_0^{5/2} \\ &= \left(\frac{\pi^2 KD^3 \sigma\epsilon}{10} \right)^{1/2} T_0^{5/2}. \end{aligned} \quad (45)$$

By integrating (44) the temperature distribution along the rod is obtained:

$$T = \left[T_0^{-3/2} + \frac{3}{2} \left(\frac{8\sigma\epsilon}{5KD} \right)^{1/2} X \right]^{-2/3}. \quad (46)$$

This can be more conveniently expressed as a ratio:

$$\frac{T}{T_0} = \left[\frac{1}{1 + \frac{3}{2} \left(\frac{8\sigma\epsilon T_0^3}{5KD} \right)^{1/2} X} \right]^{2/3}. \quad (46A)$$

This ratio is plotted in Fig. 18 for a reference temperature $T_0 = 1100^\circ\text{K}$. The heat into the heat pipe can be determined from the axial flux equation (10) multiplied by the cross-sectional area of the pipe.

$$Q_{ax} = \frac{10\eta\rho_v r_v^3 Z L r_w^2}{b\rho_\ell r_c^2}. \quad (47)$$

Using the HPRAD4 values previously cited for SPR-6,

$$\begin{aligned} T_0 &= 1100^\circ\text{K}, \\ D &= 1.5 \text{ cm}, \\ X &= 87 \text{ cm (condenser length)}, \\ K &= 0.6 \text{ W/cm}^2\text{-}^\circ\text{K cm} \\ \epsilon &= 0.9, \\ \sigma &= 5.67 \times 10^{-12} \text{ W/cm}^2\text{-}^\circ\text{K}^4, \end{aligned}$$

the heat into the solid rod from equation (45) is

$$q_{\text{solid rod}} = 128 \text{ watts.}$$

The heat into the heat pipe from equation (47) is

$$q_{\text{heat pipe}} = 4040 \text{ watts.}$$

Fig. 18. Temperature ratio T/T_0 as a function of rod length.

From the HPRAD3 output values, which did not utilize the heat pipes to their full capacity,

$$q_{\text{HPRAD3}} = 4040 / (\text{safety factor}) = 2020 \text{ watts.}$$

Each active heat pipe, including its fluid, was found to weigh 337 grams. The solid conductor pipe of the same length would weigh 1320 grams. A comparison of the heat transferred per unit weight is shown below.

	<u>Heat transferred, W</u>	<u>Weight, g</u>	<u>Watts/gram</u>
Solid rod	128	1320	0.097
Heat pipe (SPR-6)	2020	337	6.0

It can be seen that the system heat pipe on a weight basis is 60 times more effective than the solid rod.

AN ADDITIONAL METEOROID HAZARD

Leakage from a manifold via a doubly punctured heat pipe (one puncture outside the manifold, the other inside) is a meteoroid hazard that has not been quantitatively considered. It is felt, however, that this hazard is not serious.

The double wall thickness of heat pipe provides considerable protection. As shown by equation (39), the average number of penetrations per heat pipe is inversely proportional to the fourth power of the wall thickness. Thus the average number of double penetrations per heat pipe is a factor of 16 lower than the number of single penetrations. The probability of a double penetration is further decreased by the bumper screen effect.¹²

The probability of a double penetration that provides a manifold leakage path is much lower than the overall double-penetration probability because only meteoroids with particular flight directions can effect such a "critical" double penetration. For the case of a 1.5-cm-diam heat pipe piercing a manifold covered with a 1.0-cm-thick armor, only those meteoroids which pierce the heat pipe at an angle greater than 34° from the normal can possibly exit inside the manifold. This "critical" angle increases as meteoroid hits further from the manifold are considered. In addition to the reduced meteoroid flux which need be considered for "critical" double penetrations, the inclination of the "critical" meteoroid paths reduces the penetration because (a) the normal component of meteoroid velocity is proportional to the cosine of the angle from the normal, and (b) as indicated by oblique hypervelocity impact data, for angles greater than 50° penetration is less than that predicted by the normal component of velocity.¹²

The "critical" inclination angle may be further increased by the addition of a short collar around the heat pipe at the manifold barrier surface. For the case previously mentioned the addition of a 1.0-cm-long collar would increase the "critical" angle to 53° from the normal. The mass of such collars is already implicitly included in the computer design code because the barrier mass is calculated for a cylindrical shell with no correction made for the heat pipe entry holes.

SUBCOOLING

It is necessary to subcool the working fluid in the radiator so that the unavoidable entropy increase in the pumps does not force the fluid under the vapor dome. In previous LRL radiator analyses the amount of subcooling has been rather arbitrarily set at 100°R (55.6°K).

While not explicitly considered in HPRAD4, subcooling will occur due to the presence of the redundant heat pipes. An upper limit on the amount of subcooling, ΔT , is given by

$$\Delta T = \frac{q_{\text{sub}}}{wC_p}. \quad (48)$$

For 10% heat pipe redundancy it is clear that q_{sub} will be less than 10% of the heat of condensation. Thus, for SPR-6,

$$\begin{aligned} q_{\text{sub}} &< 4.57 \text{ MW}, \\ w &= 28.5 \text{ kg/sec}, \\ C_p &\approx 0.8 \text{ W-sec/g-}^\circ\text{K}. \end{aligned}$$

So, $\Delta T < 203^\circ\text{K}$. ΔT will be less than the limiting value because the temperature of the heat pipes in the subcooling section will be lowered considerably as a result of (a) the liquid-metal heat transfer coefficient which is less than the condensing heat transfer coefficient, and (b) the lowered source temperature itself.

For a constant mass flow rate the amount of subcooling will decrease with time as a result of heat pipe punctures. Control of the amount of subcooling has not been investigated.

REFERENCES

¹H. Haller, S. Lieblein, and B. Lindow, Analysis and Evaluation of a Vapor Chamber Fin-Tube Radiator for High Power Rankine Cycles, NASA-TN-D-2836 (May 1965).

²F. E. Tippets, J. R. Peterson, and S. G. Sawochka, Potassium Multiple Tube Test Condenser Design Study, NASA-CR-72137 (1966).

³D. Kroger, Heat Transfer During Film Condensation of Potassium Vapor, Massachusetts Institute of Technology, Cambridge, Mass., DSR-75239-42 (August 1966).

⁴R. English and D. Guentert, "Segmenting of Radiators for Meteoroid Protection," Am. Rocket Soc. J. 31, 1162 (1961).

⁵J. Deverall and J. Kemme, High Thermal Conductance Devices Utilizing the Boiling of Lithium or Silver, Los Alamos Scientific Laboratory, Los Alamos, N. Mex., LA-3211 (October 1964).

⁶J. Watts, Comparison of Ba and Ag as Heat Pipe Working Fluids, California Univ., Livermore Lawrence Radiation Lab., Research Projects Engineering Division, SPN-111 (unpublished document).

⁷J. Kemme, Heat Pipe Capability Experiments, Los Alamos Scientific Lab., Los Alamos, N. Mex., LASL-3585-MS (October 1966).

⁸W. Owens, "Two-Phase Pressure Gradient," International Developments in Heat Transfer, Pt. II (American Society of Mechanical Engineers, New York, 1963), pp. 363-368.

⁹N. Clough and S. Lieblein, Significance of Photographic Meteor Data in the Design of Meteoroid Protection for Large Space Vehicles, NASA-TN-D-2958 (August 1965).

¹⁰Letters from S. Lieblein, NASA, to C. Walter, LRL, Ref. 2720, Dec. 10, 1965, and Jan. 17, 1966.

¹¹F. L. Whipple, "On Meteoroids and Penetration," Interplanetary Missions Conference 9th AAS Annual Meeting, Los Angeles, California, Jan. 15, 1963.

¹²I. Loeffler, S. Lieblein, and N. Clough, "Meteoroid Protection for Space Radiators," Paper No. 2543-62, September 1962.

DISTRIBUTION

LRL Internal Distribution

Michael M May

J. Hadley

M. Heusinkveld

O. Kolar

T. Stubbs

J. Howard

W. Loewe

B. Myers

W. N. Ross

C. Walter

R. Batzel

J. Kane

A. Rothman

H. Teifeld

J. Freeman

W. Decker

R. Werner

12

G. Carlson

12

R. K. Wakerling, Berkeley

TID File

30

External Distribution

TID -4500, UC-33 Propulsion Systems and Energy Conversion

264

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights, or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission or employee of such contractor prepares, disseminates, or provides access to any information pursuant to his employment or contract with the Commission or his employment with such contractor.

WC/jlc