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CONTROL ASPECTS OF VERY HIGH FLUX RESEARCH REACTORS

by

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LIST OF SYMBOLS

Symbol	* Subscript	Meaning	Units
	numeral	Indicates reactor region	
Φ^*		Neutron Flux:	$n/cm^2 \cdot sec$
	f	fast	
	s	slow	
σf		Microscopic fission cross section	cm^2
Σf		Macroscopic fission cross section	cm^{-1}
σ^*		Microscopic thermal absorption cross section:	cm^2
	letter	indicates the material	
P		Power	fis/sec
V		Volume	cm^3
VF		Volume fraction:	
	letter	indicates material	
A*		Core area:	cm^2
	i	internal lateral surface	
	o	external lateral surface	
S		Source of neutrons	
B_z		Axial geometrical buckling	cm^{-1}
τ		Fermi age	cm^2
D*		Diffusion coefficient:	cm
	f	fast group	
	s	slow group	
c		Conversion factor = 3.1×10^{10}	$fis/watt \cdot sec$
Σa^*		Macroscopic thermal absorption cross section:	cm^{-1}
	letter	indicates material	

LIST OF SYMBOLS (Cont'd.)

Symbol	* Subscript	Meaning	Units
ν		2.47 atoms/fission for U^{235}	
α		$\Sigma a_u / \Sigma f = 1.183$	
η		ν / α	
j^*	f s	Neutron current: fast slow	n/cm ² .sec
P _{NL}		Non-leakage probability	
N _a		Neutrons absorbed	n/sec
N _L		Neutrons leaking	n/sec
ϵ		Fast fission factor	
f		Thermal utilization factor	
k _∞		Multiplication factor for an infinite pile	
k _{eff}		Effective multiplication factor	
ρ		Reactivity	
L		Diffusion length	cm

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FOREWORD

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ABSTRACT

This work is directed toward the computation of reactor characteristics over a fuel cycle in a way suitable to the investigation of the control aspects of the reactor problem.

The high flux operation of the type of reactor under consideration has required the study of the dynamic problem including as independent variables the neutron energy, the space and time. An additional condition of versatility in the method used to obtain the solutions, is necessary to explore the control aspects of the problem.

These general premises have placed the problem beyond what is published in the literature and rule out the digital computers and hand numerical computations as suitable methods to obtain the solutions. The accuracy necessary for control study and the condition of versatility imposed have suggested its solution by analog methods. The problem as presented, however, is beyond the standard use of the analog machines. The normal possibilities of the analog computers were surveyed and a method was found to solve the reactor dynamic problem.

The classical approach of considering the neutron energy dependency in groups and the spatial dependency in reactor regions is used. Only cases of regular symmetry are considered, so that the reactor three dimensional configuration is reduced by analytical methods to the study involving one space coordinate. Time is considered as a continuous variable.

Full advantage is made of the peculiarities of the reactor operation. Since only long term solutions are considered, the short term variations, such as neutron kinetics and reactor servo control, act as the average effect of keeping the reactor critical during operation. This fact has suggested the idea of representing the flux distribution in a steady-state basis and superimposing onto this solution the

slow time varying parameters. This scheme can be handled in an analog machine provided the reactor critical condition during operation is assured and the spatial solution is not disturbed by the superposition of the time solution. To satisfy the first requirement a criticality calculator and control poison simulator are included. The second requirement is met by eliminating any sort of time delay in the flux simulator. This was accomplished by replacing the diffusion differential equations with a first order finite difference approximation. The power produced in each core region is added up to the total desired reactor power output. This sum is maintained constant automatically and sets the level of the flux distribution accordingly, as the uranium is depleted.

The Mighty Mouse reactor (A-Conf 15/P/423-1958) is simulated and the analog results compared against published data. With the spatial dependency represented by three core and three reflector regions, the fast and slow flux distributions are within 5% of the digital computer solution of the same problem. At the end of the fuel cycle, the flux distribution is essentially that of the digital solution with the values lowered 5 to 10%. Compared with hand numerical computations made considering the same number of core regions, the analog results show agreement within 1%.

Some fundamental aspects of the long term reactor dynamics are discussed, based on results from the simulator. Illustrative examples of power transients, shut-down, control by burnable poison and localized control rods are included and discussed.

The method of solution presented does not replace or overlap with existing methods. It is felt that considerable time can be saved in the reactor project by a combined use of the available methods, including this analog method. The procedure is not bounded by any empirical

or numerical limitation. Thus, its field of application is open for long term reactor problems in general, provided the equations representing the reactor operation are properly introduced into the analog machine. It can also be used as a pedagogical aid in demonstrating advanced reactor theory concepts.

PART I

INTRODUCTION

This work was motivated by the necessity of studying some control aspects of a very high flux research reactor such as the proposed Mighty Mouse Reactor Project,^(11,12) to provide more knowledge and confidence about its behavior in operation.

The Mighty Mouse project pioneers the next generation of research reactors. The proposed reactor is a 250 Megawatt thermal reactor with highly enriched uranium, heterogeneous annular cylinder core, cooled and moderated by heavy water. It has been developed exclusively for research purposes and is designed to produce a thermal neutron flux in the inner reflector region of approximately 5×10^{15} neutrons/cm²-sec. This flux is one order of magnitude higher than in any existing reactor.

The very high flux operation of a reactor has required to study the control problems including the spatial dependency. Versatility in the method used to obtain the solutions is necessary to explore the control aspects of the reactor problem. These two conditions have placed the problem beyond the analytical methods presented in the literature.^(5,14) The condition of versatility eliminates the digital computers and hand numerical computations as available methods to obtain the solutions. Hence, it was felt that a work in the direction of providing a method suitable for this type of application would be well justified by itself.

A new technique which will furnish more detailed information about a reactor with such a high flux will add to human knowledge and also permit a more basic understanding of lower flux reactor technology where several problems are small in magnitude and virtually neglected.

A. Historical Review

The nuclear technology is recent but well developed. The increasing demand for nuclear reactors associated with complex power systems has stressed the necessity of more detailed information about reactor control and also about the behavior of the reactor system as a whole. Many recent developments in the reactor control technology are consequences of this large power reactor program.

Meanwhile, the advance of technology and the accumulation of experience have allowed the engineers to aim for higher power densities from the reactors. As a result, the burn-up of fissionable material began to appear as one of the most important problems, since it has definite effects on the reactor dynamics and determines the practical life of a reactor core.

The state of reactor technology with respect to the burn-up problems is still far from adequate. As a general rule the methods of solving the long term reactor problem incorporate simplifying assumptions which do not guarantee a good prediction of the core life, unless a large number of man hours or expensive digital computer set-ups are used. These limitations complicate the studies in this area, making the progress slow.

Burn-up has some consequences in the control field, due mainly to its effect on the concentrations of xenon and samarium. When the specifications call for higher fluxes, as in the case of the present investigation, these problems become a source of uncertainty in the reactor project. Therefore, they must be adequately accounted for.

Reactor control studies usually separate the problems in function of the time basis which is associated with it.⁽¹⁸⁾ As a general rule, the phenomena occurring during a short time do not affect those happening during a much larger time. For example: neutron kinetics

and automatic rod control are associated with short time variations with solutions represented in seconds of actual time. Start-up of complete reactor systems, xenon and power transients are in general accounted for in an intermediate time basis; while burn-up problems and shut-down studies are long term problems usually expressed in hours of actual time.

Well established methods are available to study short time variations in a reactor.^(18,20,6) Recent work has been done to simulate the start-up problem.^(4,7) It should be noticed, that although the start up of nuclear reactors being one of intermediate time, there is virtually no burn-up associated with this problem.

The fundamental aspects of the long term reactor behavior have been studied extensively.^(5,6,18) No attempt, however, has been made to consider the spatial flux distribution in problems created by a high burn-up rate since the inherent difficulties are formidable.

The reactor problem usually is assumed to be a function of three independent variables: neutron energy, space and time.

The standard reference literature for this field^(5,14) uses the approach of solving the non-linear reactor problems by linearizing the equations taking one function at a time and assuming all others as constants. The independent variables are made separable, or one or two of them assumed constant, so that individual solutions are found by common applied mathematics techniques. This method produces good results provided the variances of the functions are small so that the interaction effects are not pronounced. This condition is satisfied for all practical purposes when the neutron flux is kept low, i.e., below 10^{13} n/cm²-sec.

When a more rigorous treatment of the subject is sought, however, the equations to be solved become more and more complex, reaching quickly a point beyond the scope of commonly used analytical

methods. Hand numerical methods using desk calculators are available to study concomitantly more than one function. These methods lead to reasonable draft design data and are used extensively. More accurate solutions in a shorter time are available using a digital computer and are required for reactors with flux levels of 10^{13} n/cm²-sec. or higher.

A generally accepted practice is to take the energy dependency in groups to divide the spatial distribution in reactor regions, all at a definite instant of time. This procedure is repeated for different instants of time with proper adjustment of the time varying parameters.

The solution of the reactor problem within this scheme varies in complexity as a function of the number of neutron energy groups and reactor regions considered. They can be found by analytical or hand numerical methods. Whenever possible, these methods are replaced by a digital computer solution operating on standard codes available for some specific aspects of the reactor problem.⁽¹⁷⁾

This treatment assumes the neutron energy as independent of time. This is well justified since the slowing down time is always very small ($4.6 \cdot 10^{-5}$ sec. in heavy water). So, for most of the reactor problems, the slowing down is assumed instantaneous.

For control studies, time is the most important variable and necessarily should be made continuous. However, the high burn-up rate associated with very high flux reactors has required to include the spatial dependency in the formulation of the reactor equations. This is necessary to show the interactions between the reactor regions as a function of time. The additional consideration of time as a continuous variable in this study shifts immediately the problem to a higher order of complexity.

To program a digital computer code for control studies in this level would be prohibitive in price and time. The digital computer

approach, besides being very expensive, is not really suitable for the early stages of the reactor design or to study the control aspects of the reactor problem. In either case the accuracy of the results is not the critical aspect. Rather, it is essential to have the possibility of changing the reactor parameters to obtain a quick appreciation of their individual or combined effects.

As a matter of simplification, it was suggested that the energy dependency be eliminated by considering only the thermal neutron energy group. The approximation was found unacceptable for a reactor of the Mighty Mouse type. The slowing down length in heavy water is 11 cm. This means a high percentage of the neutrons born in one region in the core will slow down to thermal energy in the reflector or in another core region. The flux distribution obtained with one neutron group would be so distorted from the reality that it would invalidate this type of solution.

Since the neutron energy is not considered as a function of time, the problem can be separated into two sets of equations: one set with energy and position as the independent variables and the other with position and time as the independent variables. The spatial dependency is the common interconnecting variable. The number of independent variables is therefore fixed at two, although two distinct sets of equations are required.

The problem is well characterized, but there is no known standard technique available which would provide the solutions within a desirable time.

The conditions of low accuracy and versatility required for the purposes of the present investigation would be easily met if the problem could be worked in an analog computer. The analog computer machines, however, are designed to handle only one independent variable.⁽¹⁰⁾ There is a tendency, therefore, to disregard the possibility

of analog computer application in problems involving more than one independent variable.⁽⁹⁾

Several analog techniques are available to determine the static distribution of flux in a reactor.^(1,14) These methods are used for criticality and perturbation studies, but do not consider the time dependency.

A complete survey of the possibilities of the analog machine would be necessary to determine its applicability for the purposes of this work.

B. Approach to the Problem

1. General Remarks

One of the main purposes of the present work is to study the new aspects of control problems created by the requirement of operating a nuclear research reactor at a very high flux. From experience in the field a thermal reactor with the following characteristics was chosen as a basis for study:

1. Highly enriched uranium core.
2. Heterogeneous core with symmetrical configuration.
3. Heavy water moderated and cooled.
4. Neutron flux in the range of 10^{15} to 10^{16} neutrons/cm²-sec.

The first two characteristics would include the CP-5 (Argonne Research Reactor), the MTR (Materials Testing Reactor) and the ETR (Engineering Testing Reactor). Only a research reactor of the Mighty Mouse type would represent all four characteristics.

The heat generation in a unit volume of a reactor core is directly proportional to the number of fissions occurring at that volume. The number of fissions per second is given by the product of flux Φ and

the macroscopic cross sections Σf . Σf is the number of fissionable nuclei per unit volume - uranium in this case - times the fission cross section per atom.⁽¹³⁾

The number of fissions per second can be converted to power in watts from the equivalence 1 watt = 3.1×10^{10} fissions/sec.⁽¹⁹⁾

Thus, the power per unit volume can be expressed in this form:

$$P = \frac{\Sigma f}{c} \cdot U \cdot \Phi$$

where U and Φ are functions of time and position inside the core.

The standard literature does not consider U and Φ as functions of two independent variables.^(5,14) It is common to assume an average uranium concentration and an average flux value throughout the core. Since this method neglects completely the uranium depletion necessary to account for the energy produced during the reactor operation, more rigorous methods, usually numerical, are required in a reactor project. During the reactor operation, as the uranium is depleted the flux must increase in the same proportion, if the power is to be maintained constant.

The initial uranium concentration is determined from critical studies and the projected core life for given operating conditions. Thus, bringing the flux to a very high level, implies raising the power accordingly. A reactor of the type here considered could not have a large core since the total power produced would be immense when operating at high fluxes.

Operation at a high power level in a small core encounters the following principal consequences:

1. A high rate of heat removal with its consequences on the reactor mode of operation.
2. A high rate of uranium burn-up with its consequences on reactor physics and control.

In low flux highly enriched uranium thermal reactors, the power is also very small so that the burn-up is negligible. In this case both flux and power are essentially constants and a generally accepted practice is to use flux detectors acting on instruments displaying an equivalent power output. The difference for a very high flux reactor can be illustrated with some characteristic data of the Mighty Mouse reactor. For that reactor, about 50% of the initial uranium loading is burned in five days of constant full power operation. The flux roughly doubles during the fuel cycle.

From an engineering point of view, a heat system operating at a constant power output is simpler and easier to control. Since xenon-135 production rate is proportional to power in the past, and its destruction rate is proportional to present power, a constant flux operation leads to an increasing inventory of Xe-135. Also the flux is not as high as it is with constant power operation so that kinetic chemistry experiments dependent on ϕ^2 or more are severely limited by constant flux operation. If a constant flux operation is desired, a continuous reduction of the total power output is necessary due to the uranium depletion. The constant power output operation is assumed throughout the present work.

One of the assumptions of the present study has been to restrict the investigation to heterogeneous reactors with regular symmetrical configuration. By regular symmetry is meant a figure which can be expressed in simple cases of rectangular, cylindrical or spherical coordinates. It includes almost all core configurations in existence but excludes some odd fuel arrangements permitted in the Argonaut and other low power research reactors. The reason behind this limitation is to permit a one space dimensional treatment of the diffusion of neutrons, and to avoid the use of leakage coefficients determined by empirical methods.

The condition of heterogeneity was included to characterize specifically the type of reactor. The problems in an heterogeneous reactor are essentially different from those in an homogeneous one. In the latter, the uranium concentration can be varied during the reactor operation, and, at the same time, the fission products can be removed, so that, no core life is associated with this type of reactor. In an heterogeneous reactor, the fissionable material is canned and mechanically assembled in fuel elements, which are arranged in a configuration forming the core. Actually, due to the small mesh formed by the fuel plates, the heterogeneous core is considered homogeneous in all theoretical treatments of diffusion and slowing down of neutrons.^(5,14) In some reactors where large uranium bars are used as fuel elements, there is a flux distortion in the neighborhood of each bar which may require a small correction from the results obtained assuming an homogeneous core. For the type of reactor considered in this study, the high rate of heat removal requires very thin fuel plates. So, the fuel elements are essentially transparent as far as the diffusion of neutrons is concerned, and the assumption of homogeneity is used throughout this work without restrictions.

2. Spatial Dependency.

The spatial distribution in a reactor is a three dimensional problem. Taking advantage of core symmetry, it can be reduced to a solution in one or two dimensions. For the accuracy required by the control aspects of the reactor problem, it was found sufficient to study the spatial dependency in only one dimension. There is no theoretical limitation in extending this representation to more dimensions. The only practical limitation is the amount of equipment available in the computer installation.

The condition of regular symmetry allows the elimination of at least one dimension. The sphere is a one dimension case. The slab or cylinder are two dimensional ones which can be reduced to one dimension by simple analytical techniques.

Cylindrical coordinates are used throughout this work. Two reasons have determined this choice: 1) It is the most complex case of regular symmetry, making easier the application of the method here proposed for a simpler configuration. 2) The necessity of comparing the results obtained in this work against reliable data. In this case the only available data refers to the Mighty Mouse Reactor which is a case of cylindrical symmetry.

The symmetry of a cylindrical shape requires the solution in two dimensions - radial and axial. To reduce it to a single dimension a cosine flux distribution in the axial direction was assumed, based on the standard method of separation of variables.⁽⁵⁾ The effects of this assumption in the radial equations are properly taken in account. The details are shown in Appendix I. The problem of flux spatial dependency was then simplified to the study of its behavior in the radial direction, but still a function of neutron energy and time.

3. Energy Dependency.

The interaction of neutrons with matter is a function of their energy.⁽⁵⁾ In the reactor diffusion equation, this is indicated by the variation of cross sections with neutron energy.

The energy dependency was divided in two groups: fast (or epithermal) and slow (or thermal). This method assumes average values of cross sections over the energy region each group includes. It produces results in good agreement with the experiments and the deviation from a treatment with many groups is small in the present case. Again, there is no theoretical limitation in increasing the

number of groups; practically it would only require more computing equipment. For reactor studies of the type considered here, the two group approach is adequate and well presented in the literature.^(5,14)

The first characteristic of the reactor chosen for this work specified a uranium 235 reactor. This condition is taken strictly, i.e., no uranium 238 is present. This same assumption was made in the Mighty Mouse study⁽¹¹⁾ although it is known the purest U-235 released by the U.S. Atomic Energy Commission contains about 7% of U-238.

The presence of U-235 in a reactor has several important consequences related with the fuel burn-up. Its effect in the proposed reactor study is summarized below:

- a) U-238 has high resonance peaks in the energy spectrum of the cross sections. This effect is minimized by the excellent moderating properties of heavy water and the low amount of U-238 present.
- b) The thermal neutron absorption cross section of U-238 is small, requiring a large irradiation to produce a significant amount of plutonium. For the type of reactor here considered the total irradiation in a fuel cycle is only about 10^{21} neutrons/cm² and this effect can be discarded.
- c) The ratio of thermal neutron capture cross section of U-235 relative to U-238 is very large, which combined with the small percentage of U-238 present, allows the assumption of a pure U-235 reactor.

The macroscopic epithermal capture cross section of U-235 is small so that the resonance escape probability, i.e., the fraction of fast neutrons which escape capture while being slowed down is practically equal to one.⁽⁵⁾ Also, since the epithermal fission cross section is small, there is essentially no contribution due to the fast fission process and $\epsilon = 1$.⁽⁵⁾ Thus, for a U-235 reactor,

$$k_{\infty} = \eta f$$

for all practical applications. These conclusions were automatically assumed when writing the diffusion equations in Part II.

4. Some Aspects of Time Behavior.

This investigation will be directed toward the problems associated with the high burn-up rate. These are essentially problems of long time variations. They may require an intermediate time scale to reveal details of some fast transients.

The Mighty Mouse Reactor Project has proposed a fuel cycle of 120 hours.⁽¹²⁾ Even if a later design call for a higher initial uranium load, the core life would be within the same order of magnitude. For higher fluxes, the core life would be even smaller.

The build-up of non-saturating fission products poison is a function of irradiation. For a U-235 specimen, the poison build-up is essentially a linear function of irradiation up to about 2×10^{21} neutrons/cm². The slope of this line is 44 barns per initial atom of U-235 per 10^{21} neutrons/cm² of irradiation.⁽¹⁶⁾ In the Mighty Mouse Project a poison build-up of 50 barns per fission was used in the computations. These two constants are equivalent for the total irradiation in consideration. The Mighty Mouse data was used in this work. No difficulty is associated with this choice since the irradiation or the fission rate are readily available from the method of solution here proposed.

It should be noted that the irradiation is the time integral of flux. So, it is different for every point in the core, depending on the spatial flux distribution. As a corollary, this same reasoning applies to the fission products.

Some of the fission products, xenon and samarium, have different behavior and a much larger effect on the reactor operation due to their large cross sections. While the remaining fission product poison increases as a linear function of irradiation, xenon and samarium are functions of the uranium concentration, time and flux. For this reason their effects are accounted for independently, and the mention of fission products in this study excludes xenon and samarium.

The short core life associated with a very high flux reactor allows one to neglect the samarium effect since it has not time to appear.⁽¹²⁾ In this work only the xenon effect is considered. The samarium equations and the justifications for this assumption are explained in Appendix II. The solution of the samarium problem is very similar to the xenon problem, and so, its appreciation, if required, would need only more computer equipment.

5. Analog Aspects of the Investigation.

The analog aspect of the problem is to find a method to solve by an analog computer a system of equations involving three independent variables. One set of equations relates flux with energy and position. The other relates flux with position and time. Fortunately the energy is not considered a function of time as was explained in Section A, so that the only common variable between these equations is the spatial dependency.

Actually, since the analog machines are designed to handle only one independent variable some new basic idea should be introduced to surmount the difficulty.

It is proposed that the two solutions be superimposed, one with respect to the space and the other with respect to time; both are represented by the same computer time variable, but with a time

scaling factor so different that one solution is seen by the other as if it were undertaking a quasi-steady state behavior.

The time variation for the type of reactor problems associated with the burn-up is measured in hours or days of core life. The spatial flux variations are very small within a few hours of reactor operation. This would suggest considering the space as the fast variable and the real time as the slow one. The complete solution is represented by a family of curves displayed in a plane normal to the time axis and spaced at regular intervals.

This concept requires a special relay amplifier arrangement to stop and reset the spatial solution at some distance from the origin, i.e., at a corresponding time interval, so that a new solution can be traced for the next interval. This process is repeated until the end of core life. The spatial reset mechanism is independent of the computer units associated with the time solution. This requires to isolate from the machine master control switch the computer units associated with one of the two solutions. Usually this is not practicable if a single machine console is available. For an installation with two computer consoles this possibility is feasible. The two solutions are worked simultaneously but in different machines, with the interacting signals connected through external trunk lines. The machines are started simultaneously. The time solution would be displayed continuously in one machine while the spatial solution would be repeated at definite intervals set by the repetition rate desired.

The applicability of this scheme to solve the problems associated with this work has been explored to some extent. Later considerations have suggested a different approach, still within the same basic idea. It was felt, however, the scheme mentioned above would be of help for different applications. For this reason it was presented in some details.

Some difficulties were experienced when trying to apply this particular scheme to the type of reactor problem outlined in the preceding sections. For example, the Mighty Mouse reactor incorporates two different media radially arranged: a reflector medium inside the annulus followed by a core medium and then another reflector of the same kind. This introduces two points of discontinuity in the diffusion equations, bringing up difficult boundary conditions to be satisfied and requiring separating the spatial equations in three different ranges along the radial axis, the solution in one medium being used as the input function to the next. This is further complicated when applied to a set of equations representing the two group approach. Besides, this approach does not incorporate the versatility which would be desired for the present work. For these reasons, the first scheme was abandoned and a new one presented itself within the same basic framework, taking advantage of some particular aspects of a nuclear reactor operation.

For a critical reactor, the flux at a given time has a definite spatial distribution which is only a function of the diffusion equations' coefficients. Therefore, the reactor power level has solely the effect of lowering or raising this flux distribution uniformly. Even for a reactor operating at a very high power density, the variations of the diffusion equations' coefficients with time are very small. These changes are slow varying functions and can be represented in an actual time scale of hours. An investigation of these long term effects, which is the main purpose of this work, would see the reactor kinetic behavior, i.e., the short time variation, as represented by the average effect of keeping the reactor critical during operation.

These facts have suggested the idea of representing the flux distribution on a steady-state basis and superimposing onto this solution the slow time varying coefficients. With this scheme, the spatial

distribution would be seen by the time varying functions as if it were assuming a quasi-steady-state behavior.

To design a simulator within this scheme, two conditions were considered absolutely necessary:

- 1) The reactor should be critical all the time
- 2) The solution displayed by the spatial flux simulator is to be computed rapidly and incorporate a high degree of stability.

The first condition implies the incorporation of a criticality calculator actuating a control rod simulator as in an actual reactor operation. The second condition rules out the use of integrators associated with the flux simulator. No time delay is tolerated in this simulation.

Several tentative proposals were made to simulate a reactor system within this scheme in a way suitable for the kind of investigation required in this work.

The possibility of a continuous solution for the radial dependency was suppressed by condition number one since the time delay associated with the integration of the diffusion equations would invalidate the method. This has shifted the idea to the finite difference approximation of the diffusion differential equations. It implies displaying the flux distribution in a stepwise basis. The reactor core is divided into a finite number of regions. The diffusion coefficients are calculated for a unit volume located at the center and representative of the medium enclosed by the region.

The finite difference flux simulation has assumed a critical reactor. This implies a steady-state distribution when the time varying functions are not introduced. At this situation, the flux distribution is only a function of the reactor physical parameters, i.e., fission and absorption cross sections, diffusion coefficients, Fermi age, leakage coefficients among the regions and external leakage.

The leakage coefficients are determined by the difference approximation. They are functions of the spacing of each region and the diffusion coefficients. The external boundary condition can be approximated in several different ways, the most common being to assume the flux zero at the extrapolated boundary.⁽⁵⁾

With some external boundary condition fixed, a simulator representing the difference equations should assume the critical steady-state condition at each representative point with the interactions from all the regions and the external boundary included.

The Mighty Mouse report⁽¹²⁾ has used three core regions to describe the burn-up effects taking in account the flux variations. To verify the method and be able to compare some basic results, three core regions are also used here.

C. Statement of the Problem

One of the purposes of the present work is to investigate the unusual control problems associated with research reactors operating at a very high flux level.

Some of these problems are:

- a) Determination of the control poison necessary to keep the reactor critical at any instant during the core life.
- b) Investigation of the location of control poison in any reactor region, with appreciation of its effect in the flux distribution and in the core life.
- c) Calculation of the spatial concentration of xenon as a function of time.
- d) Appreciation of xenon and local power stability.
- e) Transient studies for step changes in reactor power.
- f) Effects resulting from operating the reactor at different power levels.

- g) Calculation of reactivity as a function of time after a shut-down at any instant during the core life.

There are several problems associated with these mentioned above. The method used to obtain the solutions must incorporate versatility to permit ample application.

The high burn-up rate inherent to high flux reactor operation requires the appreciation of the long term reactor problem taking in consideration the spatial and time variation of all the reactor functions

This study has been made considering only one space coordinate. The reactor physical three dimensional configuration is reduced by analytical methods to a single space variable. The neutron diffusion equations are approximated by finite difference equations expressed in two groups of neutron energy. This implies to represent the spatial dependency by a finite number of points or regions.

Time is a continuous independent variable. The problem is restricted to long time variations of the reactor functions. The non-linear coefficients of the diffusion difference equations are continuously introduced as a function of time with each individual absorbing effect accounted independently.

A criticality calculator acting on a control poison simulator is required to keep the reactor critical when in operation. Neutron kinetics and the type of reactor servo control are problems involving short time variations and are not included in this work.

The power produced in each core region is added up to furnish the total desired reactor power output. This sum is to be maintained constant automatically. The level of the flux distribution is to be set by the total power output taking in consideration the uranium depletion.

The study of these non-linear reactor problems is to be made in an analog computer to incorporate the necessary versatility required by the problems. However, the solution of these problems with the degree of completeness summarized above is beyond current analog techniques, and a good part of this work is to be devoted to overcome this difficulty. Empirical or numerical limitations are avoided to keep the work general.

Since the solution by an analog machine will be a new method to handle this kind of problem, it requires a detailed comparison against available data calculated by different methods. This comparative study of the accuracy of the results should determine the degree of confidence and the limitations of the method.

PART II

THEORY

A. Power Equations

The high flux research reactor considered in this work operates at a constant power output. Since uranium burns as a function of the spatial flux distribution, the uranium depletion will vary spatially as a function of time, and as a consequence, the power produced per region will also vary with time.

The sum of the power produced per region must add up to the constant total power output. So, as time elapses, the local power will vary but its sum is to be maintained constant by some additional device. This statement is expressed by equation

$$\sum_{n=2}^4 P_n = P_0 = \text{Constant} \quad (1)^*$$

Uranium is depleted at a rate determined by its thermal neutron capture. From this capture, a fraction of U-235 is converted in U-236 which does not fission; the remainder undergoes the fission process releasing energy. The ratio of total (capture plus fission) and fission cross sections, defined as α , is a constant equal to 1.183. So, the uranium depletion is directly related to the number of fissions per second or the power being produced. It is expressed as

$$\frac{dU}{dt} = -\Sigma a_U \Phi = -\alpha \sigma_f U \Phi \quad (2)$$

The uranium concentration at any instant is the time integral of equation (2), for a given initial concentration determined by the fuel loading U_0 atoms/cm³. For the present problem it is necessary to generate the power per region and equation (2) is written in a general form as

*The symbols used in this work are summarized in the List of Symbols.

$$\frac{dU_n}{dt} = -\alpha \sigma f U_n \Phi_n \quad (3)$$

$n = 2, 3 \text{ and } 4.$

The initial fuel loading is uniformly distributed in the core, therefore, U_0 is the same in any region.

Expression (1) in the same units as equation (3) is written as

$$\sum_{n=2}^4 \sigma f U_n \Phi_n V_n - P_0 \frac{\text{fissions}}{\text{sec}} = 0 \quad (4)$$

$V_n = \text{volume of region } n.$

Since the flux distribution is independent of the power level at a given instant, the solution of equations (3) and (4) is used to determine the reference level of the spatial flux distribution, so that the resultant values of Φ_n will satisfy equation (3). This condition will guarantee the reactor operating at a constant power output and at the same time, by feeding back the resulting Φ_n into equation (3), the uranium burn-up rate is correctly established.

The flux reference level is determined by the operation of a simulated simplified high speed servo-motor control system. When the system assumes a power different from the reference power output P_0 equation (4) shows an error. The integral of this error signal is used to determine the flux reference level. The error becomes zero when equations (3) and (4) are satisfied, meaning the proper flux level is established.

B. Flux Representation

The flux distribution is approximated by dividing the reactor in six regions, equally spaced 10 centimeter apart. This choice corresponds to the three core regions used in the Mighty Mouse computations.⁽¹²⁾ It also furnishes an approximate value of the flux in the

reflector so that the neutron leakage can be calculated. Figure 1 shows the reactor arrangement and the location of the six regions.

The external reflector includes only two regions. This choice was made to save computer equipment when simulating the equations. To fix the boundary conditions in this case, the flux in the seventh region is assumed to maintain a constant ratio to the flux in the sixth region. The relative flux values were taken from the flux plots in Figure 12 for the core at its initial condition, i.e., at $t = 0$. The same data can be obtained entirely by the analog machine, simulating the flux for the condition of $t = 0$ with a great number of reactor regions, terminated at the condition of zero flux at the extrapolated boundary. This more accurate flux plot without time dependency would furnish the same fractional values above mentioned.

The steady state diffusion equation for neutrons in one energy group can be expressed in the form of equation (5),⁽¹⁾

$$\nabla \cdot D \nabla \Phi - \Sigma \Phi + \Sigma' \Phi' = 0 \quad (5)$$

where D is the diffusion coefficient of the energy group, Φ is the flux of the group, Φ' is the flux in the preceding group, Σ is the macroscopic cross section for loss by capture or by slowing down out of the group and Σ' is either the macroscopic slowing down cross section for neutrons in the primed group if $\Sigma' \Phi'$ is the slowing down source, or a macroscopic yield cross section for the primed group if $\Sigma' \Phi'$ is a fission source.

When applied to the particular case of this work where the energy spectrum is divided in two groups, the following set of equations are obtained for the two different media in the reactor:

In the core: $n = 2, 3, 4.$

Fig 1 - Location of regions in the Mighty Mouse Reactor.

$$\text{fast group} \quad \nabla \cdot D_{fn} \nabla \Phi_{fn} - \frac{D_{fn}}{\tau} \Phi_{fn} + \nu \Sigma_{fn} \Phi_{sn} = 0 \quad (6)$$

$$\text{slow group} \quad \nabla \cdot D_{sn} \nabla \Phi_{sn} - \Sigma_{an} \Phi_{sn} + \frac{D_{fn}}{\tau} \Phi_{fn} = 0 \quad (7)$$

In the reflector: $n = 1.5, 6, 7$.

$$\text{fast group} \quad \nabla \cdot D_{fn} \nabla \Phi_{fn} - \frac{D_{fn}}{\tau} \Phi_{fn} = 0 \quad (8)$$

$$\text{slow group} \quad \nabla \cdot D_{sn} \nabla \Phi_{sn} - \Sigma_{an} \Phi_{sn} + \frac{D_{fn}}{\tau} \Phi_{fn} = 0 \quad (9)$$

In these equations, the diffusion coefficient D and the Fermi age τ are assumed constant during the core life. The last term of equation (6) represents the source of fast neutrons produced by the thermal flux. This source is a function of the uranium concentration per region and is introduced from the simultaneous solution of (3), (4) and the diffusion equations. In the reflector, the total thermal absorption cross section Σ_a is constant. In the core regions, it is the sum of the absorbing effects of xenon, uranium, fission products, control rods, fuel cladding and moderator. From these, only the last two are constants. All the other are varying functions of time and as such are considered in the Σ_a calculator.

Actually, the right sides of equations (5) to (9) represent the rate of change of the total number of neutrons in that particular region or group.⁽⁴⁾ Making the right side zero means the total number of neutrons present in the reactor is constant, i.e., the reactor is critical. It implies a steady-state spatial flux distribution. A quasi-steady-state behavior is achieved when slowly varying time coefficients are introduced in those equations.

The first term of equations (6) to (9) represents the number of neutrons diffusing in an elemental volume per second. It is commonly referred as the leakage term. A finite difference approximation is used in place of the differential terms resulting from the vector operations indicated in the leakage terms.

In cylindrical coordinates with r dependency only, the leakage terms are expressed as

$$1/r \frac{\partial}{\partial r} D(r) \cdot 1/r \frac{\partial \Phi}{\partial r}$$

or

$$-D(r) \left[\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} \right] - \frac{\partial \Phi}{\partial r} \frac{\partial D}{\partial r} \quad (10)$$

The point representative of each region is taken at the center of the region and is numbered from the reactor center or axis where $r_n = 0$. Thus, $r_n = n\Delta r$ where Δr is the spacing among the regions. The finite difference approximation method is used to represent the differential terms⁽⁵⁾ of (10).

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial r^2} &= \frac{\Phi_{n+1} - 2\Phi_n + \Phi_{n-1}}{\Delta r^2} \\ \frac{\partial \Phi}{\partial r} &= \frac{\Phi_{n+1} - \Phi_{n-1}}{2\Delta r} \\ \frac{\partial D}{\partial r} &= \frac{D_{n+1} - D_{n-1}}{2\Delta r} \end{aligned}$$

Collecting the coefficients of the flux terms, expression (11) is obtained. It represents the net leakage from region n in function of the flux values in the neighborhood of that region.

$$\begin{aligned} & - \frac{1}{2\Delta r^2} \left\{ \left[\frac{(2n+1) D_n}{n} + \frac{\Delta D}{2} \right] \Phi_{n+1} - 4 D_n \Phi_n + \right. \\ & \left. \left[\frac{(2n-1) D_n}{n} - \frac{\Delta D}{2} \right] \Phi_{n-1} \right\} \quad (11) \end{aligned}$$

The coefficients attached to the flux values in (11) are called here leakage coefficients. For the fast group they are:

$$\begin{aligned} b_n' &= \frac{1}{2\Delta r^2} \left[\frac{(2n+1)}{n} D_{fn} + \frac{\Delta D_{fn}}{2} \right] \\ c_n' &= \frac{2D_{fn}}{\Delta r^2} \\ d_n' &= \frac{1}{2\Delta r^2} \left[\frac{(2n-1)}{n} D_{fn} - \frac{\Delta D_{fn}}{2} \right] \end{aligned}$$

and for the slow group:

$$b_n = \frac{1}{2 \Delta r^2} \left[\frac{(2n+1)}{n} D_{sn} + \frac{\Delta D_{sn}}{2} \right]$$

$$c_n = \frac{2 D_{sn}}{\Delta r^2}$$

$$d_n = \frac{1}{2 \Delta r^2} \left[\frac{(2n-1)}{n} D_{sn} - \frac{\Delta D_{sn}}{2} \right]$$

Replacing expression (11) for the leakage terms of equations (6) to (9) with the proper leakage coefficients, the following set of equations are obtained for the six reactor regions in consideration.

Fast group equations: (12)

$$\begin{aligned} \text{Region 1} \quad & b_1' \Phi_{f2} - \left(c_1' + \frac{Df_1}{\tau} \right) \Phi_{f1} + d_1 \Phi_{f0} = 0 \\ \text{Region 2} \quad & b_2' \Phi_{f3} - \left(c_2' + \frac{Df_2}{\tau} \right) \Phi_{f2} + d_2' \Phi_{f1} + \nu \Sigma_{f2} \Phi_{s2} = 0 \\ \text{Region 3} \quad & b_3' \Phi_{f4} - \left(c_3' + \frac{Df_3}{\tau} \right) \Phi_{f3} + d_3' \Phi_{f2} + \nu \Sigma_{f3} \Phi_{s3} = 0 \\ \text{Region 4} \quad & b_4' \Phi_{f5} - \left(c_4' + \frac{Df_4}{\tau} \right) \Phi_{f4} + d_4' \Phi_{f3} + \nu \Sigma_{f4} \Phi_{s4} = 0 \\ \text{Region 5} \quad & b_5' \Phi_{f6} - \left(c_5' + \frac{Df_5}{\tau} \right) \Phi_{f5} + d_5' \Phi_{f4} = 0 \\ \text{Region 6} \quad & - \left(c_6'^* + \frac{Df_6}{\tau} \right) \Phi_{f6} + d_6' \Phi_{f5} = 0 \end{aligned} \quad (12)$$

Note: $c_6'^*$ means corrected c_6' value, resulting from the assumption that

$$\Phi_{f7} = k_1 \Phi_{f6}.$$

Slow group equations: (13)

$$\begin{aligned} \text{Region 1} \quad & b_1 \Phi_{s2} (c_1'^* + \Sigma_{a1}) \Phi_{s1} + \frac{Df_1}{\tau} \Phi_{f1} = 0 \\ \text{Region 2} \quad & b_2 \Phi_{s3} - c_2 \Phi_{s2} + d_2 \Phi_{s1} - \Sigma_{a2} \Phi_{s2} + \frac{Df_2}{\tau} \Phi_{f2} = 0 \\ \text{Region 3} \quad & b_3 \Phi_{s4} - c_3 \Phi_{s3} + d_3 \Phi_{s2} - \Sigma_{a3} \Phi_{s3} + \frac{Df_3}{\tau} \Phi_{f3} = 0 \\ \text{Region 4} \quad & b_4 \Phi_{s5} - c_4 \Phi_{s4} + d_4 \Phi_{s3} - \Sigma_{a4} \Phi_{s4} + \frac{Df_4}{\tau} \Phi_{f4} = 0 \\ \text{Region 5} \quad & b_5 \Phi_{s6} - (c_5 + \Sigma_{a5}) \Phi_{s5} + d_5 \Phi_{s4} + \frac{Df_5}{\tau} \Phi_{f5} = 0 \end{aligned} \quad (13)$$

$$\text{Region 6} \quad - (c_6^* + \Sigma_{a6}) \Phi_{s6} + d_6 \Phi_{s5} + \frac{Df_6}{\tau} \Phi_{fb} = 0$$

Note: Φ_{s0} and Φ_{s7} are assumed a constant fraction of Φ_{s1} and Φ_{s6} respectively. The coefficients c_1^* and c_6^* include the correction. The asterisk means corrected value.

The set of equations (12) and (13) are solved for the flux values in each group and in each region. A very fast simulator is required to produce a solution which will not be disturbed by the changes in the time varying coefficients.

The source terms of equations (12) are non-linear. They are generated by multiplying the uranium concentration resulting from the simultaneous solution of (3) and (4) by the corresponding flux determined from equations (12) and (13). All slowing down cross sections represented by D/τ in (12) are corrected by adding $DB_{\frac{1}{2}}^2$ to account for the axial cosine flux distribution as shown in Appendix I. This same correction applies to all thermal absorption cross sections in (13).

A Σ_a calculator is required to sum up all absorbing effects in the core regions and multiply this result by the corresponding flux value in order to generate the proper non-linear absorbing terms in the thermal diffusion equations (13).

The remaining terms of equations (12) and (13) are constants. The leakage coefficients are determined exclusively by the reactor physical configuration, the diffusion constant and the width of the reactor regions.

C. Σ_a Calculator

The total thermal neutron absorption cross section in the core regions is determined by the sum of the absorbing effects of uranium, xenon, fission products, control elements, fuel cladding and moderator.

The uranium macroscopic thermal absorption cross section is determined by the integration of equations (2) for a given initial fuel loading, multiplied by the microscopic absorption cross section σ_a .

The absorbing effect of xenon 135 is determined by equation (22) after the simultaneous solution of (20) and (21). The non-linear coefficients of these equations require the introduction of variables determined by the simultaneous solution of power and flux.

The fission product macroscopic absorption cross section is determined by equation (23) using the simultaneous solution of power and flux.

The type of fuel element determines the constant effect. The volume fraction of moderator, cladding and uranium alloy element is determined for the particular fuel element. Multiplying the cross section of each material by its volume fraction, the individual contributions are determined. Its sum, corrected by the addition of DB_z^2 is the total constant absorbing effect. As in the Mighty Mouse type of fuel element, where the cladding and uranium matrix are made of the same material, the constant macroscopic cross section is calculated from expression (14).

$$\Sigma_{amc} n = \Sigma a_m \cdot V_m + \Sigma a_c \cdot VF_c + D_{sn} B_z^2 \quad (14)$$

The reactor critical studies are made incorporating a definite amount of reactivity which can be controlled by some external device. The reactor is first made critical with all this control poison inserted. As the uranium burns and the fission products build up, the control poison is removed by an amount just necessary to keep the reactor critical. This work is done by the reactor servo control system. Since the solution of equations (12) and (13) require a critical reactor, a criticality calculator is incorporated, associated with a servo control simulator. An actual type of servo control is not mentioned

in this work, only its function is simulated. It is assumed the control absorbing material is uniformly distributed throughout the core and operated simultaneously in all core regions by the output of a servo simulator. This output, $\Sigma a_{cn} \text{ cm}^{-1}$, is added to form the total Σa . Then,

$$\Sigma a_n = \Sigma a_{mcn} + \Sigma a_{un} + \Sigma a_{xn} + \Sigma a_{pn} + \Sigma a_{cn} \quad (15)$$

The total thermal absorption cross section is multiplied by Φ_{sn} to produce the non-linear absorbing terms of (13). The product $\Sigma a_n \cdot \Phi_{sn}$ is available from the Σa calculator.

D. Xenon and Samarium Problem

Because of their peculiar behavior in the thermal reactor operation and their large thermal absorption cross section, the study of the nuclides xenon 135 and samarium 149 is of special interest.

Assuming only thermal neutrons present and proper average values of cross section, the differential equations for the rate of change of concentration of specific elements can be written for a given flux and uranium concentration.

In general, take fission product A with a direct fission yield γ_A . Then

$$\frac{dA}{dt} = \left[\begin{array}{c} \text{Production rate from} \\ \text{fission} \end{array} \right] - \left[\begin{array}{c} \text{Destruction + Decay rates} \end{array} \right] \quad (16)$$

Production rate = $\gamma_A \Sigma f \Phi \text{ nuclei/cm}^3\text{-second}$.

where $\Sigma f \Phi$ is the number of fissions per cubic centimeter-second.

Let λ_A be the radioactive decay of isotope A. Then, at any instant, the natural rate of radioactive decay of A will be $\lambda_A A \text{ nuclei per cubic centimeter-sec}$.

In addition, A will be removed by neutron capture at a rate equal to $\sigma_A \Phi A \text{ nuclei/cm}^3\text{-sec}$. where σ_A is the thermal absorption cross section of A.

To make equation (16) more general, it is assumed that some precursor isotope B will decay into isotope A with a natural radioactive decay λ_B . So, a production term equal to $\lambda_B \cdot B$ is added to (16). Then,

$$\frac{dA}{dt} = \underbrace{\text{Production rate from fission}}_{\text{term (a)}} + \underbrace{\text{Decay from precursor}}_{\text{term (b)}} - \underbrace{\text{Destruction rate}}_{\text{term (c)}} - \underbrace{\text{Decay rate}}_{\text{term (d)}} \quad (17)$$

Applied to fission product A, equation (17) is expressed in this form:

$$\frac{dA}{dt} = \gamma_A \Sigma_f \Phi + \lambda_B B - \sigma_A A \Phi - \lambda_A A \quad .$$

In the standard literature^(5,14,18) the study of equation (17) assumes a reactor with constant power density operation. Also, an average flux value is calculated and used in term (c). For a reactor of the type studied in this work the assumptions just mentioned are untenable. Even for the assumption of a constant power density, the average flux will vary as a function of the uranium depletion. Actually, for a constant reactor power output, the power density per region will vary as function of the burn-up effects. The same thing occurs with the spatial flux distribution. In the present study, these variations are approximated by the continuous generation of flux and power in regions equally spaced inside the reactor. These variables are used in terms (a) and (c) of equation (17).

In the next section, equation (17) is applied to xenon. In Appendix II, equation (17) is applied to samarium to show how the order of magnitude of its effect can be determined.

The macroscopic cross-section of samarium is zero at $t = 0$ and the calculated value at the end of the core life is still small. One can say samarium has not time to appear. Its appreciation together with the remaining reactor absorbing terms is therefore neglected.

E. Xenon Effect

Xenon 135 is an unstable isotope with a very high thermal absorption cross section. It is formed directly from fission with a fractional yield $\gamma_X = .003$ atoms per fission. It is also formed from the following isotope decay chain, starting with a fractional yield of tellurium $\gamma_I = .064$.

The half life of tellurium 135 is very short and its cross section negligible. Thus, iodine 135 is assumed as a direct fission product with the same yield. By thermal neutron capture, xenon 135 changes to xenon 136 with negligible cross section. Cesium cross section is also negligible.

Equation (17) is used to describe the xenon production. The following constants are used:

γ_X = fractional yield of Xe-135 as a direct fission product.

γ_I = fractional yield of I-135 as a direct fission product.

σ_X = microscopic absorption cross section of Xe-135.

σ_I = microscopic absorption cross section of I-135.

λ_X = decay constant of xenon 135

λ_I = decay constant of iodine 135.

$$\frac{dI}{dt} = \gamma_I \Sigma f \Phi + 0 - \lambda_I I - \sigma_I I \Phi \quad (18)$$

$$\frac{dX}{dt} = \gamma_X \Sigma f \Phi + \lambda_I I - \lambda_X X - \sigma_X X \Phi \quad (19)$$

where I and X are the number of iodine and xenon atoms per cm^3 .

Notice the last two terms of equation (18). Even for a flux in the range considered in this work, λ_I is about one thousand times $\sigma_I \Phi$. Actually the iodine cross section is small and its absorbing effect negligible, even for very high fluxes. So, the last term of (18) is discarded. To solve the equations in function of time it is necessary to

introduce continuously the power density $\Sigma f \Phi$ and the flux Φ . These are time variables. They depend also on the spatial power and flux distribution. To account for the spatial dependency, the equations are written for each core region. The simultaneous solution of equations (3) and (4) for the power density and flux in each region are used in the corresponding xenon equations. Equations (18) and (19) are then reduced and written in a general form as (20) and (21).

$$\frac{dI_n}{dt} = \gamma_I \Sigma f_n \Phi_{sn} - \lambda_I I_n \quad (20)$$

$$\frac{dX_n}{dt} = \gamma_X \Sigma f_n \Phi_{sn} + \lambda_I I_n - \lambda_X - \sigma_X X_n \Phi_{sn} \quad (21)$$

This set of equations is solved by analog techniques in the xenon simulator. Its output appears multiplied by σ_X and is expressed as

$$\Sigma a_{xn} = \sigma_X X_n \quad (22)$$

$n = 2, 3$ and 4 representing the three core regions.

F. Fission Products Effect

Non-saturating fission products accumulate at a rate of about 50 barns per fission. To generate the corresponding macroscopic absorption cross section in function of time, this value must be multiplied by the number of fissions per second and integrated over the core life.

The number of fissions per second $\sigma_f U \Phi$ varies with time and is furnished by the power simulator. To account for the spatial dependency, the fission products macroscopic cross section is generated for each core region. The general expression is

$$\Sigma a_{pn} = \sigma_p \sigma_f \int_0^t U_n \Phi_n dt \quad (23)$$

$n = 2, 3$ and 4 representing the three core regions.

G. The Reactivity Calculator

There are several ways to express the critical condition in a reactor. Some will be summarized here to furnish a pictorial view of the problem.

In the standard literature the following expression is used to describe the critical reactor:

$$k_{\text{eff}} = \frac{k_{\infty} \cdot e^{-B^2 \tau}}{1 + L^2 B^2} = 1 \quad (24)$$

where B^2 is the geometrical buckling, L the diffusion length and τ the Fermi-age. As established in Part I-B-3, $k_{\infty} = \eta f$ where the thermal utilization factor f is defined as

$$f = \frac{\int_{\text{core}} \Sigma a_u \Phi_s dV}{\int_{\text{core}} \Sigma a \Phi_s dV} \approx \frac{\sum_{n=2}^4 \Sigma a_{un} \Phi_{sn} V_n}{\sum_{n=2}^4 \Sigma a_n \Phi_{sn} V_n} \quad (25)$$

and can be determined from the reactor simulator. The term $e^{-B^2 \tau}$ is the probability of fast leakage and $[1 + L^2 B^2]^{-1}$ is the probability of thermal leakage. $L^2 = D/\Sigma a$ can also be calculated from the simulator. However, B^2 varies considerably during the core life (since the reflector savings are functions of the Σa in the core) making it impractical to solve equation (24).

Another approach is to define the critical condition as

$$k_{\infty} \cdot \text{probability of non-leakage} = 1 \quad (26)$$

Calling N_a the total number of neutrons absorbed in the core per second and N_L the net total number of neutrons leaking from the core per second, the non-leakage probability is defined as

$$P_{NL} = \frac{N_a}{N_a + N_L} \quad (27)$$

where

$$N_a = \int_{\text{core}} \Sigma a \Phi_s dV - \sum_{n=2}^4 \Sigma a_n \Phi_{sn} V_n \text{ neutrons/sec.} \quad (28)$$

By net leakage from the core it is meant the difference between the number of neutrons leaving the core and those returning due to the reflector effect. However, the axial leakage is already added to the Σa values and included in N_a . This introduces a permanent error in the non-leakage probability unless a different simulator is used.

Another way to define the critical condition is by the neutron steady state equation

$$\text{Leakage} + \text{Absorption} - \text{Production} = 0 \quad (29)$$

There are different procedures to determine the total leakage plus total absorption. Equation (29) may be defined for any particular section of the reactor or to the reactor as a whole. In each case the core is included in the region of definition of equation (29), the production term is given by

$$\text{Production} = \int_{\text{core}} \nu \Sigma f \Phi_s dV \approx \sum_{n=2}^4 \nu \sigma f U_n \Phi_{sn} V_n \quad (30)$$

Defining equation (29) for the core only, the net number of neutrons per second leaving the core in the radial direction can be calculated from expression (31).

$$N_L = -j_{fi} \cdot A_i - j_{si} \cdot A_i + j_{f0} \cdot A_0 + j_{s0} \cdot A_0 \quad (31)$$

where j is the net neutron current vector normal to the boundary surface area, A_i is the core inner surface $= 2\pi R_i h$, A_0 is the core outer surface $= 2\pi R_0 h$, f and s stand for fast and slow neutrons, and h is the core height. By definition

$$j = -D \frac{\delta \Phi}{\delta r}$$

which in terms of the finite difference approximation used in this work is expressed as

$$j_{n,n+1} = -D_{n,n+1} \frac{\Phi_{n+1} - \Phi_n}{\Delta r} \quad (32)$$

where $D_{n,n+1} = \frac{D_n + D_{n+1}}{2}$

Replacing expression (32) into (31), the radial leakage from the core is obtained in function of the flux values as indicated for the particular arrangement of core regions chosen in this work.

$$N_L = \frac{A_i}{\Delta r} \left[D_{f12}(\Phi_{f2} - \Phi_{f1}) + D_{s12}(\Phi_{s2} - \Phi_{s1}) \right] - \frac{A_0}{\Delta r} \left[D_{f45}(\Phi_{f5} - \Phi_{f4}) + D_{s45}(\Phi_{s5} - \Phi_{s4}) \right] \quad (33)$$

This expression can be represented in terms of the flux values resulting from the solution of equations (12) and (13). The critical condition (29) can thus be established within the same degree of approximation determined by the number of regions used in the reactor simulator.

For some applications the Albedo concept⁽⁴⁾ can be used to determine the outer leakage. However, a sufficient number of outer reflector regions is required to allow the assumption that the fast flux is zero at the boundary where this concept is applied. For a reactor with a D₂O reflector, a minimum of 30 centimeters beyond the core outer surface would be required to permit its use.

Several different ways can be used to simulate the servo control, even through the simulation of an actual physical system. The driving control signal can be generated, for instance, by the error resulting from equation (29) when the system deviates from critical. In this work, since the reactivity display as a function of time after a shut-down simulation is a desirable function to be determined, equation (29) is expressed in terms of reactivity ρ . By definition, the reactor is critical when $k_{eff} = 1$ or

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = 0 \quad (34)$$

To determine ρ as a function of known expressions, equation (29) is written in the following form

$$\frac{\text{Production}}{\text{Absorption} + \text{Leakage}} = 1 \text{ and is replaced for } k_{eff} \text{ in (34).}$$

Simplifying, it reduces to (35)

$$\rho = \frac{\text{Production} - (\text{Absorption} + \text{Leakage})}{\text{Production}} = 0 \quad (35)$$

Condition (35) is calculated continuously during the core life. Any deviation from criticality will generate a reactivity signal which is integrated, simulating a servo-motor. The output of this integrator is the amount of $\Sigma a_{\text{control}}$ introduced in the Σa calculator. By adding or subtracting poison in the reactor, this servo control will maintain the reactivity, zero, or the reactor, critical during operation.

Whenever possible, the coefficients of the reactor equations are kept in a per unit volume basis in order to simplify the physical evaluation of the different effects. Several times these coefficients are multiplied by the corresponding region volume. This is an approximation of the same order of the finite-difference and only can be minimized by increasing the number of reactor regions. Experience in this field has indicated that three core regions is sufficient to furnish the engineering information required by some areas of the reactor design project. The implications resulting from an increased number of regions are discussed in the remaining parts of this work.

PART III

LONG TERM ANALOG REACTOR SIMULATION

A. Selection of a Particular Reactor

The Mighty Mouse reactor is simulated in this work. Essentially there was no choice to be made about the type of reactor to be simulated since the Mighty Mouse reports(11,12) were the only source of available data about a reactor with flux in the $10^{15} - 10^{16}$ neutron/cm²-sec range, at the time of writing. The application of the method proposed in this work to an hypothetical reactor would lead to results with no past experience available to evaluate their physical significance and justify some degree of confidence. However, if by applying it to the Mighty Mouse, it is possible to reproduce some basic results of that project within the same degree of accuracy, this would be enough proof of the possibilities of such a scheme and would guarantee a definite confidence in the remainder simulation information, even if applied to different reactors.

The possibilities of application and the amount of information available from a reactor simulator as the one proposed are ample. Surely the exploration and inclusion in this work of some of the applications would be more impressive than the meticulous and detailed effort chosen to make from the method here presented a powerful engineering tool for future reactor design. Thus, emphasis is placed in simulating a reactor already designed, and discuss the basic results in comparison with available computation data, rather than developing some particular simulator information.

No additional assumption or particular setup is necessary to simulate the Mighty Mouse reactor. Only basic reactor parameters and constants are used, making this simulation typical for reactor studies involving very high fluxes. The basic Mighty Mouse Reactor

Data is presented in Table I.

TABLE I
MIGHTY MOUSE REACTOR DATA

Volume:

Core - V	508,900 cm ³
Inner reflector - V _i	63,600 cm ³
Region 1 - V ₁	56,500 cm ³
Region 2 - V ₂	113,100 cm ³
Region 3 - V ₃	169,600 cm ³
Region 4 - V ₄	226, 200 cm ³
Region 5 - V ₅	282,700 cm ³
Region 6 - V ₆	339,300 cm ³

Area:

A _i - inner core surface area	8,480 cm ²
A _o - outer core surface area	25,400 cm ²

Total power - P₀ 250 mw

Uranium loading - U₀ 8.050 x 10²⁴ atoms/reactor
 - U₀ 1.582 x 10¹⁹ atoms/cm³

Reactor parameters and constants:

B _Z ² - Transverse buckling	0.00055 cm ⁻²
D ₄₅ fast - Diffusion coefficient	1.282 cm
D ₄₅ slow - Diffusion coefficient	0.808 cm
τ - Two group age	200 cm ²

Core:

Inner radius	15 cm
Outer radius	45 cm
Height	90 cm
VF-D ₂ O Volume fraction of D ₂ O	0.8
VF-Mg Volume fraction of Mg	0.2
ν - Number of neutrons/fission	2.47
D _f - Diffusion coefficient	1.315
D _s - Diffusion coefficient	0.862

Reflector:

D _f - Diffusion coefficient	1.25
D _s - Diffusion coefficient	0.754

The long time simulator presented in this work is very complex due to the interdependence of almost all reactor variables. Figure 2 is a block diagram of the reactor functions with lines showing the flow of information required to generate them. The complexity arises because all secondary functions feed back correcting signals to the main functions, sometimes through an intricate course.

B. Simulation of Power and Uranium Concentration

The power equations (3) and (4) are solved by the analog computer simultaneously with the flux equations (12) and (13).

Figure 3 shows a simplified diagram for the power simulation associated with a simulated servo system necessary to enforce equation (4).

To reduce the difference between the reference power output and the total power produced in the reactor, i.e., the error signal resulting from equation (14), an integrator with a 0.1 second time constant is used in the servo simulator. Actually fluxuations in power not larger than 0.2% are obtained with this set up.

Analog computer integrators have a practical limit of integrating time associated with the equipment accuracy. For standard commercial units, the range of operation is limited to three decades and it is a normal practice to limit the integrating time to three minutes. A reactor of the type here considered will have a life shorter than 180 hours; these facts have suggested using the convenient time scaling of one hour of reactor time being represented by one second of computer time.

The power equations (3) and (4) are converted to machine equations by the transformations indicated on Table II.

Fig 2 - Reactor System Block Diagram

$$\sum_{n=2}^4 \sigma f U_n \bar{\Phi}_n V_n - P_o = 0 \quad (4)$$

$$\frac{dU_n}{dt} = -\alpha \sigma f U_n \bar{\Phi}_n \quad (3)$$

Fig 3 - Power simulation simplified diagram

TABLE II
COMPUTER SCALING FACTORS

Computer quantity	= Scaling factor	x Reactor quantity	Problem 1 factor value
t'	a_1	t	$1/3600$
U'	a_2	U	0.632×10^{19}
Φ'	a_3	Φ	10^{-16}
P'	a_4	P	10^{-19}
coeff. Φ'	a_5	coeff. Φ	10
I'	a_6	I	10^{-17}
Xe'	a_7	Xe	0.27959×10^{-17}
Na', N_L', N_P'	a_8	Na, N_L, N_P	10^{-4}

TABLE III
REACTOR PHYSICS CONSTANTS

γI - Fractional yield of Te-135	0.064
γx - Fractional yield of Xe-135	0.003
λI - Iodine decay constant	0.1033/hour
λx - Xenon decay constant	0.07524/hour
λ_{sm} - Samarium decay constant	0.0144/hour
γ_{nd} - Fractional yield of Nd-149	0.014

Thermal group cross sections:

Material	$\Sigma_{\text{capture}} \text{ cm}^{-1}$	$\Sigma_{\text{transport}} \text{ cm}^{-1}$	σ_{abs} barns	σ_{fission} barns
D ₂ O	0.000057	0.44203		
Mg	0.00292	0.16704		
U-235			597.25	504.86
Xe-135			2.7959×10^6	
Sm-149			5.3×10^4	
I			5.94	

They are:

$$\frac{dU_n'}{dt'} = \frac{\alpha \sigma f}{a_1 a_3} U_n' \Phi_{sn}' \quad (36)$$

where

$$\frac{\alpha \sigma f}{a_1 a_3} = 0.0215 \text{ and } U_0' = a_2 U_0 = 1$$

and

$$\frac{a_4 \sigma f}{a_2 a_3} \left[V_2 U_2' \Phi_{s2}' + V_3 U_3' \Phi_{s3}' + V_4 U_4' \Phi_{s4}' \right] - a_4 P_0 = 0 \quad (37)$$

or

$$0.9034 U_2' \Phi_{s2}' + 1.354 U_3' \Phi_{s3}' + 1.8068 U_4' \Phi_{s4}' - 0.7925 = 0$$

The uranium scaling factor is set to produce one machine unit for its initial condition or initial loading. The flux scaling factor allows one to read directly from the machine the flux values in fractions of 10^{-16} neutrons/cm²-sec. This choice permits a quick appreciation of the machine results.

Electronic multipliers are used to perform the product of U_n' and Φ_{sn}' . The requirements call for very fast resolution from this part of the system. For this particular case it can be obtained also with servo-multipliers. The uranium concentration is depleted at a small and slow varying rate during the core life and is only slightly affected by some reactor power transient. The flux is a sensitive function of the reactor parameters and variables and can undertake extremely fast changes during a power transient. The product $U_n' \cdot \Phi_{sn}'$ can thus be accomplished by a servo unit, connecting the fast input to the multiplying potentiometer and the slow varying input to the servo-motor. This connection guarantees a very fast resolution in the flux path.

The power simulator generates U_n' and $U_n' \cdot \Phi_{sn}'$ continuously during the reactor life. Multiplied by proper coefficients, these functions provide the interacting effects of power with the other sections of the reactor simulator.

C. Two Group, Six Region Flux Simulator

The requirements for the flux simulator call for a very fast resolution associated with a good degree of stability. These two specifications are of fundamental importance if the reactor simulator is to operate properly allowing step changes in the reactor variables without spoiling the solutions.

Equations (12) and (13) are formed by the algebraic sum of a number of terms. With exception of the source terms in (12) and the absorption terms in the core regions of (13), all others are flux values multiplied by constant coefficients. Representing the flux as electric voltage and the coefficients as resistances, no time delay is associated with these terms. Also, the preceding section has shown that no time delay is associated with the source terms of (12) and, as will be shown in Section D, this condition is also true for the absorbing terms.

Using analog computer techniques,⁽¹⁰⁾ each equation can be simulated by a single high gain amplifier, each term being represented by an input voltage. Since the system operates in a closed loop, feedback is inherently associated with each high gain amplifier, enforcing the condition of a null total input voltage.

Figure 4 shows a simplified diagram of the analog flux simulator for the fast and slow equations in one region, including the leakage contribution from the neighbor regions. The blocks represent the generation of the time varying coefficients.

The radial flux distribution produced by the arrangement of Figure 4 depends entirely on the coefficients of the diffusion equations. Thus, it is not a function of the power operating level as in the case of an actual critical reactor. However, the level of this flux distribution is directly related to the reactor power and this condition must be satisfied by the flux simulator.

Fig 4 - Simplified Flux Simulator Per Region

The steady-state diffusion equations form a closed loop system with no signal associated with it. To compute some solution, a driving voltage must be introduced at some point of the loop. For convenience it was chosen to make $d' \Phi_0'$ the reference flux level voltage calculated from the power simulator as shown in Figure 3. It serves the two purposes of closing the feedback loop between power and flux and furnishing a voltage necessary to drive the computing units.

Some further details are associated with the flux and power simulator. From the analog computer construction standards, the input signals to the integrators are applied only when the master control switch is in OPERATE position. Since the flux level servo control is simulated by an integrator in series with the feedback path between power and flux, it opens the loop in RESET and HOLD positions. This condition is to be avoided because it makes it difficult to check statically the solution at $t = 0$, and also makes it impossible to hold the problem at some time t without spoiling the solution. It is eliminated by converting an operational amplifier into an integrator by means of an external feedback capacitor. The unit operates independent of the master control switch. This is the type of set-up used in the flux level controller of Figure 3.

The flux simulator is closed in the outer regions by introducing boundary conditions as indicated in Part II-B. Thus, from the Mighty Mouse flux plots presented in Figure 12,

$$\Phi_{f7} = k_1 \Phi_{f6}, k_1 = 0.42$$

$$\Phi_{s0} = k_2 \Phi_{s1}, k_2 = 1.10$$

$$\Phi_{s7} = k_3 \Phi_{s6}, k_3 = 0.85$$

The constant coefficients of equations (12) and (13) are calculated using the data presented in Table I. The width of the reactor regions Δr is 10 centimeters. The thermal diffusion coefficient in the

core is not reported(8,9) and is calculated from the expression

$$D_{sc} = 1/3 \Sigma tr = 0.862$$

where

$$\Sigma tr = VFD_2O \Sigma tr_{D_2O} + VFMg \Sigma tr_{Mg}$$

The axial buckling is calculated from the expression $B_z^2 = \frac{\pi^2}{(h + \Delta h)^2}$ where $h + \Delta h$ is the extrapolated height, which for the Mighty Mouse reactor is 134 cm. The coefficients of (12) and (13) are summarized in Table IV, including the variables coefficients calculated for the condition of $t = 0$. At this condition the coefficient of the fission terms is easily obtained since $U_{on}' = 1$ and it reduces to a constant, $\frac{\nu\sigma f}{a_2} = 0.01971$. The values presented for the core absorbing coefficients at $t = 0$ are results from the Σa calculator.

TABLE IV
COEFFICIENTS OF THE DIFFUSION EQUATIONS

n	b'	c'	d'	$\frac{D_f}{\tau} + D_f B_z^2$	$\frac{\nu\sigma f}{a_2}$
1	.01891	.02500	.00609	.00694	.01971(t = 0) .01971(t = 0) .01971(t = 0)
2	.01660	.02630	.00970	.00730	
3	.01534	.02630	.01096	.00730	
4	.01463	.02630	.01167	.00730	
5	.01359	.02500	.01141	.00694	
6	.01354	.02500 $c_6^*.01931$	.01146	.00694	
n	b	c	d	$\frac{D_f}{\tau}$	$\Sigma a + D_s B_z^2$
1	.01158	.01508	.00350	.00625	.00047
2	.01104	.01724	.00619	.00657	.01455(t = 0)
3	.01006	.01724	.00718	.00657	.01455(t = 0)
4	.00943	.01724	.00781	.00657	.01455(t = 0)
5	.00802	.01508	.00705	.00625	.00047
6	.00817	.01508 $c_1^*.01123$	.00691	.00625 $c_6^*.00814$	.00047

The coefficients of all the terms in the flux simulator are multiplied by a scaling factor a_5 in order to increase the accuracy of the potentiometer setting.

D. Σa Simulation

Regions 1, 5 and 6 are reflector regions, where the absorption cross section is that of plain heavy water, corrected for the axial leakage:

$$a_5 \Sigma a_n = a_5 (\Sigma a_{D_2O} + D_s B_z^2) = 0.00047 \text{ cm}^{-1} \quad n = 1, 5, 6$$

For the core regions, Σa is a function of time determined by equation (15). Figure 5 shows a simplified diagram of how the absorbing effects are accounted in the reactor simulator. The individual terms of equation (15) are calculated using the Mighty Mouse constants and converted into machine units as follows.

1. Absorption in Moderator and Cladding.

The constant absorption cross section term is determined by expression (14) and is represented in machine units as

$$a_5 \Sigma a_{mc} = 0.0110$$

2. Uranium Capture.

By definition,

$$a_5 \Sigma a_{un} = a_5 \sigma_a U_n = \frac{a_5 \sigma_a}{a_2} U_n' = 0.0945 U_n'$$

The uranium concentration is generated continuously as a function of time in the power simulator. It is multiplied by a constant to generate the macroscopic cross section. At $t = 0$, $U_n' = 1$ and the constant above is used as a checking value.

Fig 5 - $\sum a$ Simulator Block Diagram

3. Xenon Cross Section.

Xenon is simulated by solving in the analog machine the non-linear equations (20) and (21). It requires the simulation of the reactor as a whole in order to generate the non-linear terms of those equations. For convenience in reducing the number of potentiometers, a scaling factor is used to produce $a_5 \Sigma a_{xN}$ at the output of the xenon simulator.

Using the transformations indicated in Table II, equations (20) and (21) are expressed in terms of computer variables X' and I' as:

$$\frac{dI'}{dt'} = \frac{a_6 \gamma I \sigma f}{a_1 a_2 a_3} U' \cdot \Phi'_S - \frac{\lambda_1}{a_1} I' \quad (38)$$

$$\frac{dX'}{dt'} = \frac{a_6 \gamma x \sigma f}{a_1 a_2 a_3} U' \cdot \Phi'_S + \frac{a_7 \lambda I}{a_1 a_6} I' - \frac{\lambda x}{a_1} X' - \frac{\sigma x}{a_1 a_3} X' \cdot \Phi'_S \quad (39)$$

$$a_5 \Sigma_x = a_5 \sigma_x X = \frac{a_5 \sigma x}{a_7} X' = a_5 X'$$

The variable $a_5 X'$ is obtained from the solution of (38) and (39) with all terms independent of X' in (39) multiplied by $a_5 = 10$. Replacing the constants these two equations are:

$$\frac{dI'}{dt'} = 0.1840 U' \cdot \Phi'_S - 0.1033 I'$$

$$\frac{d(10X')}{dt'} = 0.0241 U' \Phi'_S + 0.2888 I' - 0.0752 (10X') - 100.65 (10X') \Phi'_S$$

These equations apply to all three core regions, but have different input variables. At $t = 0$ both xenon and iodine are absent and the initial conditions are zero.

4. Absorption in Non-saturating Fission Products.

The non-saturating fission product macroscopic cross section is generated by solving equation (23).

$$\Sigma a_p = \sigma_p \sigma f U \cdot \Phi_S \text{ cm}^{-1} \text{ sec}^{-1}$$

Converting into machine units it becomes:

$$a_5 \Sigma_{ap} = \frac{a_5 \sigma_p \sigma_f}{a_1 a_2 a_3} U' \cdot \Phi'_s$$

With the constants replaced, the total effect at any time t is given by:

$$a_5 \Sigma_{ap} = \int_0^t 0.000143 U' \cdot \Phi'_s dt$$

The constant which multiplies U' , Φ'_s is too small to be set by a potentiometer. It must be multiplied by a factor and after integration reduced accordingly. Since this absorbing term is a product of fission, it starts with zero initial condition.

5. Control Poison.

The control absorption cross section of expression (15) is the exact amount of control poison required to make the reactor critical at any instant. Since the simulation of transient operating conditions are in the objectives, this cross section must be controlled by a fast servo simulator in computer time units or by a conventional reactor servo in actual time basis.

For the Mighty Mouse reactor, the estimated initial Σa_c necessary to reach criticality is equal to 0.004 cm^{-1} . This value multiplied by a_5 is used as the initial condition for the integrator which simulates the servo system. Physically this means the control poison is initially introduced in an amount just enough to make the reactor critical. When the computer master control switch is turned to OPERATE, the servo will add or subtract poison in order to maintain criticality.

Items (1) to (5) above are added together to produce the total thermal absorption cross section, $a_5 \Sigma a_n$. The core absorbing terms are non-linear and require independent multipliers to generate them. The same restrictions about the resolution time explained in Part III-B.

apply to these multiplications. Servo-multipliers are used in this case. The absorption cross sections are slow varying functions and are used to drive the servo of the multiplier; the flux is connected to the multiplying potentiometer. The output of these multipliers are the thermal absorbing terms of the diffusion equations in the core regions. They also determine the total number of neutrons absorbed per region when multiplied by the respective volume, information which is used in the criticality calculator.

E. Reactivity Calculator and Reactor Control Simulator

The critical condition (29) was defined in Part II-G as enclosing only the core. To save computer equipment, advantage is taken of the fact that there is no radial leakage at the reactor geometrical center. Thus, for this simulation, condition (29) is defined as including the core and internal reflector. As a consequence, the terms multiplied by A_i in (33) are canceled and the absorption plus axial leakage in the inner reflector added to (28).

The reflector thermal absorption cross section including axial leakage is the same as for region I. The average flux in the inner reflector can be assumed as being that of region I. There is no epithermal absorption cross section but only fast axial leakage. The term added to (28), can then be expressed as

$$Na_i = \Sigma a_1 \Phi_{s1} V_i + D_{f1} B_z^2 V_i$$

Transforming expression (28) into machine units it becomes

$$\begin{aligned} \text{Abs} = & \frac{a_8}{a_5} V_2 \Sigma a_2 \cdot \Phi'_{s2} + \frac{a_8}{a_5} V_3 \Sigma a_3 \cdot \Phi'_{s3} + \frac{a_8}{a_5} V_4 \Sigma a_4 \cdot \Phi'_{s4} \\ & + a_8 V_i \Sigma a_1 \Phi'_{s1} + a_8 D_{f1} B_z^2 V_i \Phi'_{f1} \end{aligned}$$

or, replacing the constants

$$\begin{aligned} \text{Abs} = & 1.13 \Sigma a_2 \Phi'_{s2} + 1.696 \Sigma a_3 \Phi'_{s3} + 2.262 \Sigma a_4 \Phi'_{s4} + 0.0030 \Phi'_{s1} \\ & + 0.0044 \Phi'_{f1} \end{aligned}$$

The values of $\Sigma a_n \cdot \Phi'_{sn}$ are furnished from the Σa calculator. The flux values are taken from the flux simulator.

The net radial leakage from the region under consideration is determined from expression (33), simplified from the elimination of the internal radial leakage. The scaling factor a_8 is used to convert to machine units.

$$N_L = - \frac{A_0 a_8}{\Delta r} [D_{f45} (\Phi'_{f5} - \Phi'_{f4}) + D_{s45} (\Phi'_{s5} - \Phi'_{s4})]$$

or, replacing the constants,

$$N_L = - 0.3262 \Phi'_{f5} + 0.3262 \Phi'_{f4} - 0.2056 \Phi'_{s5} + 0.2056 \Phi'_{s4}$$

The production of neutrons is determined from expression (30), multiplying the values of $U_n \cdot \Phi'_{sn}$ generated in the power simulator by constant potentiometers. In machine units equation (30) reads

$$\text{Prod.} = \frac{a_8 \nu \sigma f V_2}{a_2} U'_2 \cdot \Phi'_{s2} + \frac{a_8 \nu \sigma f V_3}{a_2} U'_3 \cdot \Phi'_{s3} + \frac{a_8 \nu \sigma f V_4}{a_2} U'_4 \cdot \Phi'_{s4}$$

or, replacing the constants,

$$\text{Prod.} = 0.2221 U'_2 \cdot \Phi'_2 + 0.3332 U'_3 \cdot \Phi'_3 + 0.4444 U'_4 \cdot \Phi'_4$$

Figure 6 shows a simplified diagram of the reactivity calculator associated with the servo control simulator. This section of the simulator represents the standard reactor operation where the reactivity is maintained around zero by adjusting the absorption through control poison.

F. Reactor Simulator

The proposed reactor simulator is obtained interconnecting the separate computer diagrams shown in Figures 7 through 11. The potentiometer setting associated with this simulator is presented in Table V. Current analog computer techniques are used in operating the simulator. A few special remarks will be mentioned here.

Fig 6 - Reactivity Calculator and Reactor Control Simulator

fig. 7

fig. 8

fig. 9

fig. 10

**CRITICALITY CALCULATOR
AND
CONTROL SIMULATOR**

fig. 11

ANALOG COMPUTER
POTENTIOMETER SETTINGS

TABLE V

PROBLEM NO. _____
DRAWING NO. _____
DATE _____

POTENTIOMETER NO.	MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE					
1		P_0	7925			
2		$a_4 \sigma_F V_2 / a_2 a_3$	9034			
3		$a_4 \sigma_F V_3 / a_2 a_3$	2708			
4		$\alpha \sigma_F / a_1 a_3$	0215			
5		$\alpha \sigma_F / a_1 a_3$	0215			
6		$a_4 \sigma_F V_4 / a_2 a_3$	3608			
7		$\alpha \sigma_F / a_1 a_3$	0215			
8		$11 a_5 \sigma_F \sigma_F / a_1 a_2 a_3$	0016			
9		$11 a_5 \sigma_F \sigma_F / a_1 a_2 a_3$	0016			
10		$11 a_5 \sigma_F \sigma_F / a_1 a_2 a_3$	0016			
11				Adj.		
12		λ_x / a_1	0752			
13		$a_6 \sigma_F \delta_x / a_1 a_2 a_3$	0241			
14				Adj.		
16		$\sigma_x / 10 a_1 a_3 - 10$	0652			
17		λ_x / a_1	0752			
18		$a_7 \lambda_1 / a_1 a_6$	2888			
19		$a_8 A_0 D_{45} / 5 \Delta r$	0652			
20		$a_5 \Sigma a_{60} / 5$	0080			
21		$a_5 D_{41} / \tau$	0625			
22		$a_5 (c_1 + \Sigma a_1)$	1170			
23		$a_5 b_1$	1158			
24		$a_5 d_2$	0619			
25		$a_5 D_{42} / \tau$	0657			

ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____
DRAWING NO. _____
DATE _____

POTENTIOMETER NO.	MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE					
26		$a_5 c_2 / 5$	0345			
27		$a_5 d_3$	0718			
28		$a_5 D_{46} / \tau$	0625			
29		$a_5 D_{43} / \tau$	0657			
30		$a_5 c_3 / 5$	0345			
31		$a_5 b_2 / 5$	0221			
32		$a_5 d_4$	0781			
33		$a_8 A_0 D_{45} / \Delta r$	2056			
34		$a_5 D_{44} / \tau$	0657			
35		$a_5 c_4 / 5$	0345			
36		$a_5 b_3 / 5$	0201			
37		$a_5 d_5$	0705			
38		$a_8 A_0 D_{45} / \Delta r$	3262			
39		$a_5 D_{45} / \tau$	0625			
40		$a_5 (c_5 + \Sigma a_5) / 5$	0311			
41		$a_5 (c_1 + \frac{D_{41}}{\tau} + D_{41} B_2^2)$	3191			
42		$a_5 d_2$	0970			
43		$a_5 \gamma \sigma_F / a_2$	1971			
44		$a_5 (c_2 + \frac{D_{42}}{\tau} + D_{42} B_2^2) / 5$	0672			
45		$a_5 b_1$	1891	open		
46		$a_5 d_3 / 5$	0219			
47		$a_5 \gamma \sigma_F / a_2$	1971			
48		$a_5 (c_3 + \frac{D_{43}}{\tau} + D_{43} B_2^2)$	3360			
49		$a_5 b_2$	1660			

ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____
DRAWING NO. _____
DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
50		$a_5 d_4$	1167				
51		$a_5 \sqrt{D_4} / a_2$	1971				
52		$a_5 (C_4 + \frac{D_4}{5} + D_4 B_2^2) / 5$	0672				
53		$a_5 b_3$	1534				
54		$a_5 d_5$	1141				
55		$a_5 (C_5 + \frac{D_5}{5} + D_5 B_2^2)$	3194				
56		$a_5 b_4$	1163				
57		$a_5 d_6$	1146				
58		$a_5 b_5$	1359				
59		$a_5 (C_6 + \frac{D_6}{5} + D_6 B_2^2)$	2625				
60		$a_5 b_4 / 5$	0189				
61		$a_5 d_6$	0691				
62		$a_5 (C_6 + \sum a_6)$	0861				
63		$a_5 b_5$	0802				
64		$a_8 A_0 D_{545} / \Delta r$	2056				
65		$a_8 V_i D_1 B_2^2$	0044				
66		$a_6 \gamma_i \sigma_f / a_1 a_2 a_3$	1840				
67		$a_5 \sigma_v / 5 a_2$	0189				
68		$a_5 \sigma_v / 5 a_2$	0189				
69		$a_5 \sigma_v / 5 a_2$	0189				
70		$a_5 \sum a_{mc2}$	0110				
71		$a_5 \sum a_{mc3}$	0110				
72		$a_5 \sum a_{mc4}$	0110				
73		$a_8 V_i \sum a_1^*$	0030				

AMD-2C (8-57)

ANALOG COMPUTER
POTENTIOMETER SETTINGS

PROBLEM NO. _____
DRAWING NO. _____
DATE _____

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
74		λ_1 / a_1	1033				
75		$a_8 \sqrt{2} / 5 a_5$	2260				
76		$a_8 \sqrt{3} / 10 a_5$	1696				
77		$a_8 \sqrt{4} / 10 a_5$	2262				
78		$a_8 \sigma_f \sqrt{2} / a_2$	2271				
79		$a_8 \sigma_f \sqrt{3} / a_2$	3332				
80		$a_8 \sigma_f \sqrt{4} / a_2$	4444				
81		$a_6 \gamma_i \sigma_f / a_1 a_2 a_3$	1810				
82		$a_6 \gamma_x \sigma_f / a_1 a_2 a_3$	0211				
83		$a_7 \lambda_1 / a_1 a_6$	2888				
84		λ_1 / a_1	1033				
85		λ_x / a_1	0752				
86		$\sigma_x / 10 a_1 a_3 - 10$	0652				
87		$a_6 \gamma_x \sigma_f / a_1 a_2 a_3$	0211				
88		$a_6 \gamma_i \sigma_f / a_1 a_2 a_3$	1810				
89		$a_7 \lambda_1 / a_1 a_6$	2888				
90		λ_1 / a_1	1033				
91		$\sigma_x / 10 a_1 a_3 - 10$	0652				
92		$\sigma_b / a_1 a_3$	1279				
93		$\sigma_b / a_1 a_3$	1279				
94		$\sigma_b / a_1 a_3$	1279				
95		$a_5 \sum a_{B0}$	0236				
96		$a_5 \sum a_{B0}$	0236				
97		$a_5 \sum a_{B0}$	0236				

AMD-2C (8-57)

The diagrams are prepared to be used in the Argonne National Laboratory PACE⁽¹⁵⁾ analog computer installation, consisting of two computer consoles with 56 amplifiers, 80 potentiometers, 10 servo-multipliers and 5 electronic multipliers in each one. About one console was used in the present simulation.

The simulator converges almost instantaneously to an equilibrium solution when the computer master control switch is turned to RESET. This allows the problem to be checked for its initial condition.

Special attention is to be given to the criticality calculator. It incorporated the division of two variables. With the electronic divider used, the denominator shall never be allowed to drop to zero, since this condition overloads the divider. Thus, the simulation of a complete shutdown to zero power is not possible. However, simulation of a shutdown to a small fraction of total power is easily accomplished.

The initial control absorption cross section necessary to bring the reactor critical is to be determined in connection with the critical studies. The method here presented can be used to approximate its correct value. In this case, the time dependency would be eliminated from the simulator and a large number of regions used. The results from this modified simulator would determine the value for $\Sigma_{a_{CO}}$ with a good degree of approximation. This result would then be used as the initial condition for the servo control integrator. The number of regions would be reduced again to permit the inclusion of the time dependency with the same amount of machine components. Certainly, the reactivity calculated by a simulator with a small number of regions will not match the initial Σ_{a_c} obtained by a more refined method, and some deviation from zero is apt to appear at the divider output with the computer in RESET position. Potentiometer 14

of Figure 11 is used to cancel this initial reactivity deviation from zero. In the present work, Σa_{c0} is taken from the Mighty Mouse report.(12)

Experience with the simulator has shown that a gain of 5 in the control poison feedback loop produces the best results. If the gain is too large, the servo-multipliers will overload caused by the reactivity fluxuations about zero. For a small gain, the control simulator gets sluggish. In this case almost no effect is observed in the remaining parts of the simulator; only the Σa_c curve shows small erratic deviations from the correct average value.

The potentiometer 11 of Figure 8 has no connection with the reactor simulator, and must be set in OFF position for normal operation. It is included to permit a static check of the flux simulator with the power feedback open. The integrator shown in Figure 8 is the reference flux level control, which must be wired independently of the computer master control switch in order to keep the power feedback loop closed, even with the switch in REST or HOLD position.

A simulator like the one presented here is able to furnish the following information directly from the machine:

- a) uranium concentration per region as a function of time
- b) fission rate per region as a function of time
- c) fast and slow neutron flux per region as a function of time
- d) the total thermal absorption cross section per region as a function of time
- e) the individual contribution of each absorbing effect as a function of time
- f) reactivity as a function of time if the control feedback loop is open

It is also possible to operate the reactor at any power level in the megawatt region by single adjustment of the reference power potentiometer (Pot 1).

The problem can be hold at any time during the core life and the parameters readjusted before continuing with the solution. Thus, simulation of step changes in power or variations of cross sections for step changes in temperature, can be investigated.

The control poison is assumed evenly distributed throughout the core, but this situation can be changed to confine the control rods where it seems more convenient. Localized burnable poison can also be simulated with some additional equipment.

The discussion and results obtained with this simulator are presented in the next two parts of this thesis.

PART IV

COMPARATIVE DISCUSSION OF THE BASIC RESULTS

This work has proposed to solve the reactor problem in a way suitable to the investigation of the long term control aspects of the solutions.

The high flux operation of the type of reactor under consideration required the study of problems including spatial dependency. To explore the control aspects of the solutions, an additional condition of versatility is necessary.

These general premises have placed the problem beyond what has been published in standard literature^(5,13,18) about the long term control aspects of reactor problems. The condition of versatility eliminates the digital computers and the hand numerical computations as methods available to obtain solutions.

The non-linear nature of the reactor problem as it is presented in this work, has suggested its solution by analog methods. The solution of a complete long term reactor problem by analog methods was not found in the literature, however, and is beyond the current use of the machines. Thus, a sequence of actions was mandatory. First, the possibilities of the analog machines should be surveyed and adapted to the solution of this problem. Second, once the solutions obtained, the method should be tested against some absolute standard to determine its accuracy. Third, the solutions should be compared with solutions obtained by other methods in order to evaluate their practical value. Only at this stage could the control aspects of the solutions be interpreted within the limitations determined by the comparative study referred above.

This tedious procedure has been dictated by the fact that few data are available about control aspects of long term reactor problems in this flux range, and, unless supported by positive practical

confirmation of the validity of the method used, with the accuracy of the solutions tested, the presentation of results would be untenable to a scientific objection.

This part contains a comparative discussion of the results.

A. Comparative Results Against a Standard

The absolute standard solution is considered that obtained by a digital computer with the reactor at its initial condition. The basic reactor function, the neutron flux, is used for comparison.

The flux calculations for the Mighty Mouse reactor were previously made in a digital computer using three groups of neutron energy and sixty spatial points, one centimeter apart, in the portion of the reactor treated in this work. The digital machine used the finite difference method of representing the differential equations but with a high degree of approximation since it takes in account the effects of many regions in the neighborhood of the representative point. Thus, it is the more accurate man made solution and is considered here as the absolute standard for this type of problem. The standard is for a reactor problem using a definite set of parameters. The solutions of the same problem by other methods can thus be compared in an absolute basis.

The digital solution is calculated for fixed values of time. The parameters are then transported from one solution to the next problem at a different time. Thus, it would represent a standard for a given set of parameters only at $t = 0$. At any other situation it would be impossible to match the parameters generated in the continuous solution of the analog method.

The same reactor problem solved by digital methods was simulated in an analog machine via a two group treatment and dividing the spatial dependency in six regions with a first order difference approximation.

Figure 12 shows, drawn in black lines, the flux distribution calculated by a digital code⁽¹²⁾ for the same problem introduced in the analog machine. The flux values obtained by the analog simulator are plotted in this same figure to show the deviations. The actual results from the analog machine for the thermal and fast flux are presented in Figures 13 and 14 for the first five regions as a function of time. The plotting of the sixth region would interfere with a clear tracing of the others and was avoided.

The fast and slow flux deviations are within 5% of the standard digital values. This result shows that the spatial solution of the analog method is valid for this type of problem. The discrepancies were expected and are due to the use of a coarse mesh in calculating the flux. Actually the control aspects of the reactor problem would not require so much accuracy. However, it is intrinsically available, suggesting a much broader application than was originally sought.

The results have shown that a very small number of reflector regions is sufficient to represent the flux beyond the core boundaries. Also, the assumption that the fluxes in the seventh region would maintain a constant ratio to the fluxes in the sixth region (assumption made in Part II-B) is a good one, saving considerably in equipment used. This assumption has also shown consistency when applied to the internal reflector, maintaining in all cases the accuracy within the deviations of the remaining points.

Considering the large spatial rate of change of the flux values at the core boundaries where the diffusion medium changes, the results obtained with a few number of regions are surprisingly good. Only three core regions is sufficient in this case to represent the spatial solutions within 5% error. Since the amount of computer equipment required is essentially determined by the number of core regions, this fact will permit the application of the method here presented in

FIG. 12
 COMPARATIVE RESULTS OF FLUX DISTRIBUTION

FIG. 13
ANALOG DISPLAY OF THERMAL FLUX PER REGION AS A FUNCTION OF TIME

FIG. 14
ANALOG DISPLAY OF FAST FLUX PER REGION

all medium size analog computer installations within the same error margin. For reactors with no internal core boundaries, the results would certainly be more accurate for the same equipment used.

B. Comparative Results With Different Methods

1. Comparison with a Digital Solution

The digital computer results for the problem considered in this work is presented in Figure 12. As was mentioned in the preceding section, the digital solution for $t = 120$ hours cannot be used as a standard. However, it is a reliable result and is compared here with the analog solution at the end of the fuel cycle.

The comparison shows that after 120 hours of reactor operation, the flux shape generated by the analog machine is essentially that of the digital solution, with all values consistently lowered about 5 to 10%. These differences are common and expected from this type of problem. They may be caused by many different reasons like over or under estimation of reactor parameters in the digital solution; cumulative error due to the coarse mesh used for time in the digital solution and for space in the analog one; inaccuracy of the analog components; etc.

The results have demonstrated that the essence of the method, i.e., the superposition of the slow varying time functions over the very sensible steady-state flux equations is a good practical way of handling a problem with two independent variables in an analog machine.

2. Comparison with Numerical Computations

A set of numerical calculations has been performed about the same problem under discussion and are presented in the Geneva Conference proceedings.⁽¹²⁾ The same reactor parameters were used. The spatial dependency was included by considering three core regions.

Three aspects of the solutions were calculated for 24 hour intervals over the entire fuel cycle: 1) The uranium concentration; 2) The ratio of xenon cross section to the fission cross section; 3) The uniform poison cross section for criticality.

The number of core regions considered in this case is the same as in the analog simulator. The difference is that the analog machine has a continuous solution as a function of time, while the numerical calculations were performed at definite time intervals.

Figure 15 shows the continuous display by the analog machine of the uranium concentration per region as a function of time. Over this solution are plotted the points obtained from the numerical calculations.⁽¹²⁾ The results are within 1% of the machine values.

This performance shows that the hand computations for this type of problem can be eliminated with a substantial saving of time, money, and skilled personnel.

The calculations 2 and 3 are related to the control problems. The uranium fission cross section is available from Figure 15, multiplying the uranium concentration by a constant.

$$\Sigma f_n = \sigma f a_2 U_n^1 = 0.007987 U_n^1 \text{ cm}^{-1}$$

The xenon cross section is read directly from the analog machine plots. The ratio $\Sigma a_x(t)/\Sigma f_u(t)$ is then calculated at the end of each 24 hour interval, and compared with the published data. The results are presented in Table VI. Figure 16 shows the analog machine display of the xenon cross section in region III, with the points obtained by numerical calculations marked at the end of each 24 hour interval.

The comparison shows a practically constant deviation in all three core regions after the first 24 hours. Certainly the approximations made to handle the non-linear xenon differential equations and the large time interval between calculations have been the cause of

FIG. 15
URANIUM CONCENTRATION PER REGION AS A FUNCTION OF TIME

FIG. 16
 $\Sigma_{Xe}(t)$ IN REGION III

TABLE VI
COMPARISON OF RESULTS FOR $\frac{\Sigma a_x(t)}{\Sigma f_u(t)}$

Time (hrs)		Results from the Mighty Mouse report	Results from the analog computer
REGION II	24	0.06165	0.06133
	48	0.06059	0.06700
	72	0.06043	0.06700
	96	0.05934	0.06700
	120	0.05807	0.06700
REGION III	24	0.06241	0.06087
	48	0.06134	0.06687
	72	0.06115	0.06654
	96	0.06039	0.06646
	120	0.05917	0.06646
REGION IV	24	0.06217	0.06087
	48	0.06144	0.06615
	72	0.06110	0.06619
	96	0.06052	0.06605
	120	0.05948	0.06595

error in the numerical method. A very simple calculation will show that the analog solution is essentially correct:

$$\begin{aligned} \text{let } \sigma x &= 2.5 \times 10^{-18} \text{ cm}^2 & \text{and } \Phi &= 2 \times 10^{-15} \text{ n/cm}^2\text{-sec} , \\ \text{then } \sigma x &= 5 \times 10^{-3} / \text{sec.} \end{aligned}$$

The mean life of a xenon atom is then approximately 200 seconds which is much smaller than the half life of xenon by natural decay (9.2 hours). Therefore, almost as soon as a xenon atom appears, it is burnt. The rate of loss of xenon is then essentially $\Sigma a_x \cdot \Phi$ and after iodine has reached an equilibrium solution, the rate of creation of xenon is

$$(\gamma_I + \gamma_x) \Sigma f \cdot \Phi.$$

So,

$$\frac{\Sigma a_x}{\Sigma_f} \simeq \gamma_I + \gamma_x = 0.067 \quad (40)$$

which is essentially the value obtained by the analog machine after the initial transients are over.

This illustration has explained why the difference in xenon macroscopic cross section for the Mighty Mouse problem was negligible when solved in the analog machine for two different values of σ_x : 2.477×10^6 barns and 2.7959×10^6 barns.

This fact has suggested a simplification in the xenon simulator, valid within 1% accuracy for reactors operating at fluxes of 2×10^{15} n/cm²-sec or higher. It consists in replacing all the feedback components of the xenon integrator by a single wire connecting its output to the input. This procedure enforces condition (40) without interfering with the iodine solution. It has the advantage of saving two potentiometers, one multiplier and one amplifier per region.

Figure 17 shows the analog results obtained from the Σa calculator for region III. The following functions are displayed as a function of time:

- a) Non-saturating fission products macroscopic cross section.
- b) Xenon macroscopic cross section.
- c) Removed control poison cross section for criticality.
- d) Total absorption macroscopic cross section.

The uniformly distributed poison cross section for criticality calculated by numerical methods⁽¹²⁾ at the end of each 24 hour interval was subtracted from the initial $\Sigma a_{CO} = 0.004$ and marked in Figure 17.

Again the analog results are consistent. Xenon was underestimated in the numerical calculations; therefore, in the analog solution more control poison is removed to keep the reactor critical. The Σa_c curve shows that at the end of the fuel cycle it is still available to

FIG. 17
ANALOG RESULTS FROM THE Σa CALCULATOR

remove 0.00035 cm^{-1} . The numerical calculations have predicted for the same situation, 0.00068. The analog result shows that a choice in the reduction of the allowance of poison for experiments or of the fuel cycle is to be decided to account for the extra xenon present.

C. Conclusions

The comparative discussion of results has shown that the method proposed in this work to study the long term control aspects of the reactor problem is valid and incorporates a degree of accuracy which would suggest its application far beyond the control field.

The main practical limitation of the method is the amount of equipment required to simulate the long term reactor problem as desired. A good deal of judgment must guide the decisions that will establish the number of neutron energy groups and the number of regions compatible with the accuracy required and the number of computer components available. However, the possibility of representing the spatial solutions by a small number of regions with results within 5% accuracy, allows the application of this method in all medium size analog computer installations.

The accuracy of the method would suggest its application in the early design stage of a reactor project for a quick estimation of the optimum core size, configuration and loading, for given reactor requirements. Due to the low price of the analog computer operation, considerable amount of money and time would certainly be saved using the method here proposed.

The comprehensive analog simulation of the reactor problem suggests the application of this method as a powerful instruction aid, saving considerable time in instructing students in advanced reactor physics concepts. The analytical methods used currently are tedious and time consuming. Frequently they are omitted from the programs.

It is felt that this method can be used as a tool to solve problems given by the professors and whose solutions would be interpreted by the students. This certainly would contribute to bringing the nuclear engineers to a closer understanding of the reactor problem without getting too much involved in detailed aspects of reactor physics.

PART V

REACTOR CONTROL STUDIES USING THE SIMULATOR

A. Power Transients Simulation

As the simulator was constructed, the reactor reference operating power is automatically controlled at a fixed level determined by the setting of potentiometer 1. The scaling factor a_4 used allows, by single adjustment of Pot 1, to operate the reactor from 20% above to 90% below the full power level.

The possibility of simulating the accident case of an instantaneous power increase or decrease at any time during the fuel cycle is of special interest. The procedure consists in holding the problem by the computer master control switch and readjusting the power level to the new value as desired. The machine continues to solve the problem for the new condition as the switch is returned to OPERATE.

If necessary to obtain some details of the transients when a power change is made, the time scaling of the plotter can be readjusted while the machine is in the HOLD position. If this method is still not sufficiently accurate for the requirements, the whole time scaling of the computer solution can be changed by readjusting all potentiometers affected by the scaling factor a_1 .

An illustrative example of the computer results for a simulation as mentioned above is shown in Figure 18. It shows the xenon cross section and the control poison cross section for criticality. After 48 hours of normal operation, the reactor power is reduced by 50%. The time axis is driven at a rate of 0.1 in./sec (representing 0.1 in./hr of reactor time) and after the power reduction at a rate of 1 in./sec.

The results are only illustrative because a servo-driven vary-plotter was used and no study was made to determine its performance

FIG. 18
SIMULATION OF 50% POWER REDUCTION AFTER 48 HOURS

for this particular type of transient. The result as it is shows a total reactivity change of approximately 3.6%, corresponding to a Σa_c change of 0.00046 cm^{-1} . The control poison cross section plot shows that an average of about 0.127 cm^2 of poison must be removed from the reactor per second for criticality. This is equivalent to about $0.002\% \text{ k/sec}$ which must be removed during 30 minutes to compensate for the effects of the xenon transient.

The method here presented is no good for 100% shut-down simulation. When the variables represented as voltages in the analog machine become too small, the machine loses accuracy. It is true that proper scaling, taking advantage of the maximum working voltages of the units, would minimize this problem. For the present simulation, for example, this situation could be improved by increasing the scaling factor of all the coefficients of the flux equations (a_5). Also, the operating power scaling factor (a_4) could be set such that Pot 1 would operate near one machine unit. With the simulator as it is, it was observed that the criticality calculator shows a constant zero reactivity for a power level change with the problem held, only within some limits. The critical condition is independent of the power operating level. The machine shows perfect agreement with this statement, from full power to 10% of full power. Below this value the reactivity starts changing. As was mentioned above, one reason may be the inaccuracy inherent to the machine for being operated at very low voltages. Also, the few number of regions considered in the problem has required an arbitrary zeroing of the reactivity at $t = 0$. This correction may be non-linear for very low flux values and requires readjustment.

This fact is more the usual consequence of operating an instrument over a too wide range in a single scale than a limitation of the method itself.

The partial increase and decrease of power during the fuel cycle was simulated for 10% and 20% variations. It was observed that for a power change of $\pm 10\%$ of full operating power, the xenon cross section has changed about 6-8% in the first half of the fuel cycle and about 4-6% in the second half. The reactivity changes associated with these transient conditions are very small. For power transients of 20% of full power, the xenon cross section has changed about 14% in a short time. The total control poison change observed was about 40 cm², which is still within the possibilities of a rod type of control. For the 50% reduction reported with some details, the xenon cross section increases by 80% requiring special consideration in the reactor project.

The power transients simulated were of the step function type. In many practical applications the power follows a definite pattern or rate of variation. All these cases are within the versatility of the analog machine itself and do not need to be mentioned here.

B. Reactivity Display

For a normal reactor operating condition, the reactivity is maintained zero by the reactor control simulator. However, for some special conditions, it may be interesting to observe the machine display of reactivity as a function of time. This can be accomplished by opening the reactor control feedback loop. In the simulator here presented, the procedure is illustrated for a power reduction to 5% of its initial operating value at some time during the fuel cycle.

The procedure is as follows:

- a) Run the simulator for normal operating conditions and HOLD the problem at the time where the transient must be introduced.

- b) With the problem held, disconnect the summing junction of the integrator of the control poison simulator from its grid and connect it to the grid of another amplifier, here called the reactivity amplifier.
- c) Introduce the transient, in this case adjusting the power potentiometer (Pot 1) to 5% of its previous value. As was mentioned in Section A, this may require a rebalance to zero of the reactivity amplifier by means of Pot 14. The reactivity signal is then ready to be plotted with its sign reversed by the reactivity amplifier.
- d) Return the computer master switch to OPERATE.

Figure 19 shows a reactivity display from time zero to 120 hours with a shut-down simulation to 5% of full power after 22 hours of operation. The result shows a 32% negative reactivity peak reached only three hours after the shutdown. It decays reaching almost an equilibrium value of about 1%, 40 hours after the shutdown.

This same procedure was repeated with the transient introduced after 60 hours of reactor operation. The result has shown that the negative reactivity peak was raised to 37.5% within the same time, but the reactivity behavior is the same reaching about the same equilibrium value 40 hours after the shutdown.

These results are similar to the standard reactivity behavior for complete shutdown, presented in the literature. ⁽¹⁸⁾

C. Study of Control by Burnable Poison

In this section an illustrative example is given of how to study reactor control by means of burnable poison.

A good deal of work has been done on controlling the Mighty Mouse reactor by means of burnable poison. The method consists in

FIG. 19
REACTIVITY DISPLAY AS A FUNCTION OF TIME AFTER A SHUT-DOWN

loading the core with an initial amount of one or more isotopes with absorbing characteristics which will cause them to be destroyed by the existing neutron flux at a rate which should compensate to some extent the xenon and non-saturating fission product build-up. With this method, the amount of reactivity change necessary to be included in the control rods would be substantially reduced, with several beneficial consequences. ⁽²⁾ Such a work would have been drastically simplified, besides improving considerably the accuracy of the results if a simulator as proposed here were available at the time it was done. The illustrative problem can be expressed in this form: "Investigate the control effects of including 1200 cm² of boron 10 uniformly distributed in a fresh core. The effective thermal absorption cross section of boron 10 is $\sigma_B = 3554$ barns."

Boron will act as an absorber. Thus, its cross section must be accounted for in the Σa calculator. The problem consists in generating the boron cross section as a function of time and introduce it into the Σa calculator.

The rate of destruction of boron 10 is given by $\frac{dB_n}{dt} = -\sigma_B \cdot B_n \cdot \Phi_{sn}$ where B_n is the number of boron atoms per cubic centimeter per region. The equivalent machine equation is:

$$\frac{dB'_n}{dt'} = - \frac{\sigma_B}{a_1 a_3} B'_n \cdot \Phi'_{sn}$$

where $B' = a_9 B$

or, with the constants replaced,

$$\frac{dB'_n}{dt'} = - 0.1279 B'_n \cdot \Phi'_{sn} \quad (41)$$

For an initial $\Sigma a_B V = 1200$ cm², $\Sigma a_B = 0.002362$ cm⁻¹

or, at $t = 0$, $B_0 = 0.6646 \times 10^{18}$ atoms/cm³.

Let the initial B'_0 be one machine unit, then $a_9 = B_0^{-1} = 1.504 \times 10^{-18}$

The criticality calculator must receive at $t = 0$

$$a_5 \Sigma a_{B0} = 0.02362$$

Equation (41) is simulated per region with the following arrangement of computing units:

Fig. 20 - Boron burnable poison simulation

The results from the introduction of the given amount of evenly distributed boron 10 are plotted in Figure 21. The boron cross section curve is for region III. What is important in this case is the plot of control poison cross section for criticality. The reference level of the two curves has been dislocated to permit an easy interpretation of the results.

As was discussed in Part IV-B-2, the total Σa_c for criticality at $t = 0$ is 0.004 cm^{-1} . From this total, 0.00054 cm^{-1} is estimated as introduced by experiments. The specified amount of boron accounts for $\Sigma a_B = 0.00236$. There remains 0.0011 cm^{-1} to be taken care by control poison or rods. A close inspection of Figure 21 shows that Σa_B starts at 0.00236 and Σa_c starts at $0.00236 + 0.00110$. The difference to 0.004 is the constant allowed for experiments. Thus, when Σa_c crosses the 0.00236 line, no more control is available and the reactor shuts down by itself.

The results from Part IV-B-2 have shown that the proper amount of xenon (which was under-estimated by reference 9) would shorten the

FIG. 21
BORON BURNABLE POISON EFFECTS

reactor life by 6 hours if the same allowance was given to the experiments. With the introduction of boron, the reactor life was shortened by 10 hours, which means that 4 hours was due to the amount of boron not burned. However, a good regulation of the control rods curve was obtained, making it possible to control the reactor with half of the assigned control rod worth for $2/3$ of the core life.

The simulator determines concomitantly what is gained by the addition of burnable poison and the price paid for the improvement. These are the two basic facts on which all the design decisions are made.

This type of result has the additional advantage of suggesting the exact operating time where some additional black rods⁽²⁾ should be introduced to keep small the total reactivity control in the rods. Besides, the simulation of these black rods can also be included, permitting one to optimize not only the magnitude and time of the introduction but also allowing one to take advantage of the spatial distribution.

D. Odd Experiment with the Simulator

To verify the extreme versatility of the analog simulator, the same boron burnable poison as studied in the preceding section was included, but all control rod worth was removed from the core and placed in region I, i. e., in the internal reflector.

As a natural consequence the flux in region I dropped to about $1/3$ of its previous value. The spatial flux distribution was distorted, becoming higher in the outer regions to compensate for the depression in the internal regions.

With such a flux reduction in region I, the control rod worth was drastically reduced. This fact was practically verified, because during the reactor operation the reactivity could be maintained zero

by the control simulator for only about 10 hours. To improve this situation, the amount of Σa_c was increased. It happened, however, that the increasing of Σa_c quickly had no additional effect on the reactivity. Actually it was verified that the product $\Sigma a_c \cdot \Phi$ reached a definite limit.

Obviously this indicates the reactor could not be controlled in such a way. It suggests, however, the application of this method for the determination of the optimum load of material to be irradiated at the internal thimble of a reactor like the Mighty Mouse. One of the objectives of a very high flux reactor is the production of californium, obtained through the irradiation of curium. Curium has a fission cross section which will add some contribution to the neutron flux. It also acts as an absorber, leading to californium. These equations could be simulated and the problem optimized for maximum californium production.

E. Suggestions for Further Development

The possible applications of the method proposed in this work are ample and in each case require specific study to determine the limitations involved. A few areas will be mentioned here to illustrate the possibilities.

- a) Development of the work about flux instability of large power reactors ⁽²¹⁾ along these main lines:
 1. Experimental verification in the analog simulator of the theoretical results published.
 2. By applying the frequency response method to the simulator, determine the flux degree of stability, i. e., the transient behavior of the flux in a stable reactor.
 3. By correlation studies, to establish rules which would allow the estimation of transient behavior as a function of known reactor parameters.

- b) Application of this method to large power reactors, introducing the plutonium equations; also to reactors where the fuel loading varies spatially.
- c) By combined studies with reactor physics to determine the accuracy with which the temperature coefficients could be measured from the simulator. From these results establish some guide lines about the number of regions, groups, etc., compatible with these measurements. This item refers in general to perturbation studies.
- d) Attach a thermodynamics simulator to the reactor simulator here presented in order to study some spatial temperature effects.

F. Conclusions

The preceding sections are illustrative of the extreme versatility of the analog simulator. The field of application is open and in many cases it will depend on the ingenuity of the computer programmer.

The simulator represents the operation of a reactor on a long term basis. Proper programming of the situations and evaluation of their consequences are of fundamental importance in determining the quality of the information released by the machine. It is believed that a mutual overlapping of knowledge must exist between the nuclear physicist or engineer and the computer programmer. The speed, accuracy and consistency of the results will certainly be a function of how close they work together.

This work has followed its original objective of studying some control aspects of very high flux research reactors. The method here presented fulfills exactly the requirements for which it was primarily conceived. No attempt is made to determine the field of application of the method here presented. However, the method depends

exclusively on the basic reactor equations, and these are the ones which finally will determine its applicability, bounded by the basic limitations, i. e. , the inherent accuracy of the analog computer equipment and the time scaling between the space and actual time solutions.

The limitation of accuracy determined by the finite difference approximation involved in this method, or by the number of energy groups used, are of practical nature, determined by the amount of equipment available.

It is felt this work will fill a gap in the existing methods of solving the long term reactor problem allowing a more profitable use of all of them.

PART VI
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PART VII
APPENDIX I

The core configuration is studied in cylindrical coordinates with the axial dependency eliminated by an analytical method using the assumption of an axial cosine flux distribution. The reactor core has different radially spaced diffusion media, therefore, the diffusion coefficient D is only a radial function. So, to apply the analytical method, the reactor diffusion equation can be expressed in a simplified form as

$$-D \nabla^2 \phi + \Sigma a \phi = \text{Source } S \quad (42)$$

In cylindrical coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial Z^2} + \frac{1}{r} \frac{\partial}{\partial r} \cdot r \frac{\partial}{\partial r} \quad (43)$$

The axial cosine flux distribution is expressed as

$$\phi = \Phi(r) \cos B_z z \quad (44)$$

where B_z is the axial geometrical buckling.⁽⁴⁾

$$S = \nu \Sigma f \phi = \nu \Sigma f \Phi(r) \cos B_z z.$$

Let

$$S(r) = \nu \Sigma f \Phi(r)$$

then

$$S = S(r) \cos B_z z \quad (45)$$

and

$$\frac{\partial^2 \phi}{\partial Z^2} = -\Phi(r) B_z^2 \cos B_z z \quad (46)$$

Replacing (43), (44), (45) and (46) in (42) one obtains

$$\left[D B_z^2 - D \left(\frac{1}{r} \frac{\partial}{\partial r} \right) r \frac{\partial}{\partial r} \right] \Phi(r) + \Sigma a \Phi(r) = S(r)$$

$$D \nabla_r^2 \Phi + (\Sigma a + D B_z^2) \Phi = S(r)$$

Thus, the assumption of a cosine axial flux distribution has reduced the two dimensional problem to a single radial dependency, provided the absorption term in each neutron group is augmented by $D B_z^2$.

PART VIII
APPENDIX II

Samarium Effect

Samarium 149 is a stable isotope with an absorption cross-section for thermal neutrons of 5.3×10^4 barns. It is the end product of the decay chain:

The fractional yield of Nd 149 is $\gamma_{\text{Nd}} = 0.014$ of U-235 fissions by thermal neutrons. The cross sections of niodymium 149 and promethium 149 are essentially zero. When samarium 149 captures a thermal neutron it becomes samarium 150, stable, with no significant absorption cross section.

Applying equation (17) to the chain which produces samarium, the following equations are obtained:

Nd 149

$$\frac{d \text{ Nd}}{dt} = \gamma_{\text{Nd}} \Sigma f \Phi - \lambda_{\text{Nd}} \text{ Nd} + 0$$

Pm 149

$$\frac{d \text{ Pm}}{dt} = 0 + \lambda_{\text{Nd}} \text{ Nd} - \lambda_{\text{Pm}} \text{ Pm} - 0$$

Sm 149

$$\frac{d \text{ Sm}}{dt} = 0 + \lambda_{\text{Pm}} \text{ Pm} - 0 - \sigma_{\text{Sm}} \text{ Sm} \Phi$$

Since there is no burn-up in the intermediate elements Nd and Pm, the fission yield of Nd can be assumed as being that of promethium with a half life equal to the sum of the half life of the two elements. The above equations are thus reduced to:

$$\frac{d \text{ Pm}}{dt} = \gamma_{\text{Nd}} \Sigma f \Phi - \lambda'_{\text{Pm}} \text{ Pm} \quad (47)$$

and

$$\frac{d S_m}{dt} = \lambda'_{Pm} P_m - \sigma_{Sm} S_m \Phi \quad (48)$$

As was mentioned in the introduction, for a reactor of the type here considered, the samarium effect will not have time to appear. To appreciate only the order of magnitude of the samarium effect, a constant power density and a flux averaged over the core life are assumed. Let the power density be 500 watt/cm³ and $\bar{\Phi}$ equal to 3×10^{15} neutrons/cm³-sec.

Let $\lambda'_{Pm} = a$; $\sigma_{Sm} = b$ and $\gamma_{Nd} \Sigma f \Phi = k$. Then, applying Laplace's Transformation to (47) and (48):

$$sP_m + aP_m = \frac{k}{s} \quad \text{and} \quad sS_m + bS_m = aP_m$$

$$S_m = \frac{ak}{s(s+a)(s+b)} = \frac{k}{bs} + \frac{k}{(a-b)(s+a)} + \frac{ak}{b(b-a)(s+b)}$$

Taking the inverse transformation,

$$S_m = k \left[\frac{1}{b} + \frac{1}{b-a} e^{-at} + \frac{a}{b(b-a)} e^{-bt} \right]$$

and replacing the constants per unit volume:

$$k = .014 \times 500 \times 3.1 \times 10^{10} = 21.7 \times 10^{10} \text{ per sec.}$$

$$a = 4 \times 10^{-6} \text{ per second taking the combined half life of Nd and Pm as being 48.7 hours.}$$

$$b = 5.3 \times 10^{-20} \times 3 \times 10^{15} = 159 \times 10^{-6} \text{ per second}$$

$$S_m = 21.7 \times 10^{10} (6289 - 6451 e^{-4 \times 10^{-6}t} + 162 e^{-159 \times 10^{-6}t}) \frac{\text{atoms}}{\text{cm}^3}$$

The absorption macroscopic cross section of samarium

$\Sigma a_{Sm} = \sigma_{Sm} \cdot S_m$, calculated for t equal to 5 days, is

$$\Sigma a_{Sm} = .000059 \text{ cm}^{-1}$$