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RESONANCE SELF-SHIELDING FOR HYDROGENOUS REACTOR

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Contract No. DA-44-009 Eng. 2868

Issued December 4, 1956

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Resonance Self-Shielding for Hydrogenous Reactor

Raymond L. Murray

A correction factor to apply to the macroscopic absorption cross section of U^{238} in the resonance region for a heterogeneous hydrogen-moderated reactor is sought. In the age-diffusion multigroup theory, with Goertzel-Selengut treatment of slowing, the value of $\Sigma_a^{238}(u)$ appears as a part of Σ_a in

$$D \nabla^2 \phi - \phi (\Sigma_a + \Sigma_{SH}) + \int_0^u \phi \Sigma_s e^{-(u-u')} du' + S = \frac{\partial q}{\partial u} \quad (1)$$

where ϕ depends on lethargy u and position r . The third term on the left is associated with scattering of neutrons with hydrogen, S is the fission source strength, and q is the slowing down density. Because of self-shielding of the fuel elements and the geometric arrangement of the fuel and moderator, the absorption cross-section must be modified to an effective value $(\Sigma_a^{238})_e$.

Solution of Boltzmann equation for infinite hydrogen moderator.

The generalized multigroup equation (1) would yield the correct energy distribution of flux if the fuel and moderator were homogeneously mixed. We first solve for the flux or slowing down density in a limiting case in which an analytic result is possible:

- (a) moderation only by hydrogen
- (b) infinite medium
- (c) resonance absorption by U^{238} only.

- (d) homogeneous mixture of fuel and moderator
- (e) fission occurs at a single energy, far above the resonance region.

The actual multigroup machine calculation should reproduce the solution of this problem, since no cognizance can be taken directly of heterogeneity. The simplified Boltzmann equation is

$$\phi(u) \left(\Sigma_a(u) + \Sigma_s(u) \right) = \int_0^u \phi(u') \Sigma_s(u') e^{-(u-u')} du' + \delta(u) \quad (2)$$

where the fission source has been normalized to one neutron per unit volume per second at lethargy zero (fission energy). Let the collision density be

$$\psi(u) = \phi(u) \left[\Sigma_a(u) + \Sigma_s(u) \right] \quad (3)$$

and the probability of scattering relative to total scattering and absorption be

$$P_S(u) = \frac{\Sigma_s(u)}{\Sigma_a(u) + \Sigma_s(u)} \quad (4)$$

Then equation (2) becomes

$$\psi(u) = \int_0^u \psi(u') P_S(u') e^{-(u-u')} du' + \delta(u) \quad (5)$$

Differentiate both sides of this equation with respect to u to obtain

$$\dot{\psi}(u) = -e^{-u} \int_0^u \psi P_S e^{u'} du' + e^{-u} [\psi P_S e^u] \quad (6)$$

Add equations (5) and (6) to obtain

$$\psi + \psi(1 - P_S) = \delta(u)$$

The solution, using the initial condition $\psi(0) = 1$ from equation

$$(5), \text{ is } \psi(u) = e^{-\int_0^u [1 - P_S(u')] du'} = e^{-\int_0^u \frac{\Sigma_a}{\Sigma_a + \Sigma_s} du'} \quad (7)$$

or the slowing down density is

$$q(u) = (1)p(u) \quad (8)$$

Effects of self-shielding.

The resonance escape factor is now generalized to account for volume fractions, flux depression and geometric factors.

Write the integrand in equation (7) as the capture probability,

$$P_C = \frac{V_0 \Sigma_0 \bar{\sigma}_0}{C V_0 \Sigma_0 \bar{\sigma}_0 + V_1 \Sigma_1 \bar{\sigma}_1} \quad (9)$$

where cross sections refer to absorption in fuel (0) and scattering in moderator (1) as separate media, V's are volumes or volume fractions, and C takes account of the multiplicity of fuel rods or plates. The effective cross section will appear in

$$P_C = \frac{V_0 (\Sigma_0)_e}{V_0 (\Sigma_0)_e + V_1 \Sigma_1} \quad (10)$$

Equating (9) and (10) and letting the self-shielding factor be

$f = \bar{\sigma}_0 / \bar{\sigma}_1$, we find

$$\frac{(\Sigma_0)_e}{\Sigma_0} = \frac{1}{\frac{1}{f} + (C - 1) \frac{V_0 \Sigma_0}{V_1 \Sigma_1}} \quad (11)$$

so that when C and f are known, the effective cross section is also. The values of f are quoted in KAPL-1262 with the following approximate formulas:

$$\begin{array}{ll} \text{cylinder} & f \approx 1 - \frac{8}{3} \frac{V_0 \Sigma_0}{S} \\ \text{sphere} & f \approx 1 - \frac{9}{4} \frac{V_0 \Sigma_0}{S} \\ \text{plates} & f \approx 1 - \frac{V_0 \Sigma_0}{S} \left(0.9228 + \ln \frac{S}{2V_0 \Sigma_0} \right) \end{array}$$

Where V_0/S is the appropriate volume to surface ratio of the fuel bearing element, $R/3$ for sphere, $R/2$ for cylinder, w for slab (of half-width w).

To arrive at an estimate of C, we consider the limiting problem of completely black fuel regions. For this case, all neutrons incident on the surface of the fuel ($\bar{\sigma}_1/4$)S are absorbed within it at a rate $V_0 \bar{\sigma}_0 \Sigma_0$ and hence

$$\frac{\bar{\sigma}_0}{\bar{\sigma}_1} = \frac{S}{4 \Sigma_0 V_0} \quad (12)$$

If we let $P_{C\infty}$ be the capture probability for black fuel, then equation (9) gives

$$C = \frac{1}{P_{C\infty}} - \frac{V_1 \Sigma_1 \bar{\sigma}_1}{V_0 \Sigma_0 \bar{\sigma}_0} = \frac{1}{P_{C\infty}} - \frac{4 \Sigma_1 V_1}{S} \quad (13)$$

It will be noted that the function C is dependent only on moderator cross sections and hence will be slowly varying with neutron energy.

Evaluation of P_{Co} for plane geometry.

The probability that a neutron arising at a point P at a distance b from an effectively infinite plane (see Figure 1) will strike the plane

Figure 1. Geometry for deriving P_{Co} for a plane.

is given by

$$j = \int_{\rho=d}^{\infty} \frac{e^{-\sum_s r}}{4\pi r^2} \cos \theta \, 2\pi \rho \, d\rho \quad (14)$$

Since $\cos \theta = b/r$, $r^2 = b^2 + \rho^2$, and $\rho \, d\rho = r \, dr$, this reduces to

$$j = \frac{b}{2} \int_{r=b}^{\infty} \frac{e^{-\sum_s r}}{r^2} \, dr \quad (15)$$

By the definition of the exponential integral function*

* Tables of Functions and Zeros of Functions. Bureau of Standards Applied Mathematics Series 37. Page 57 ff.

$$E_n(x) = x^{n-1} \int_x^\infty \frac{e^{-y} dy}{y^n} \quad (16)$$

This may be written

$$j = \frac{E_2(\sum_S b)}{2} \quad (17)$$

The average probability over the space between parallel plates separated by a distance d is

$$P_c = \frac{2 \int_0^d j db}{d} = \frac{1}{d} \int_0^d E_2(\sum_S b) db \quad (18)$$

where the factor of 2 is due to the presence of two plates.

From the property of the functions $E_n(x)$,

$$E_{n+1}(x) = \int_x^\infty E_n(y) dy \quad (19)$$

this becomes

$$P_{c\infty} = \frac{1}{d \sum_S} \left[E_3(0) - E_3(\sum_S d) \right] \quad (20)$$

which is the probability of absorption in a black plane system.

Summarizing, the working formulas for the ratio of effective cross section for a resonance absorber to that given in BNL-325 are

$$\frac{\sum_{0e}}{\sum_0} = \frac{1}{\frac{1}{f} + (C-1) \frac{V_0 \sum_0}{V_1 \sum_1}}$$

where

$$C = \frac{1}{P_{\infty}} - \frac{4V_1 \Sigma_1}{S}$$

$$P_{\infty} = \frac{1}{d \Sigma_1} \left[E_3(0) - E_3(d \Sigma_1) \right]$$

$$f \approx 1 - \frac{V_0 \Sigma_0}{S} \left(0.9228 + \ln \frac{S}{2V_0 \Sigma_0} \right)$$

Numerical Example.

Assume natural U plates, 0.060 inch thick, water gap 0.100 inch, at an energy such that $\sigma_a^{238} = 50$ barns, $\sigma_s^H = 20$ barns. With $N_{238} = 0.048$, $N_H = 0.067$, we find $\Sigma_0 = 2.40 \text{ cm}^{-1}$, $\Sigma_1 = 1.34 \text{ cm}^{-1}$. Also $d = 0.254 \text{ cm}$, and for a half cell $V_0 = (0.030)(2.54) = 0.0762 \text{ cm}^3$, $V_1 = (0.050)(2.54) = 0.127 \text{ cm}^3$. The surface area S is simply 1 cm^2 for a unit area reference column.

Substituting,

$$f = 1 - \frac{(0.0762)(2.40)}{1} \left[0.9228 + \ln \frac{1}{(2)(0.0762)(2.40)} \right]$$

or

$$f = 0.647 \text{ and } \frac{1}{f} = 1.546.$$

Next $d \Sigma_1 = 0.340$, $E_3(0) = 0.5$, $E_3(0.34) = 0.282$ and $P_{\infty} = 0.641$.

$$\text{Thus } C = \frac{1}{0.641} - \frac{4(0.127)(1.34)}{1} = 0.879.$$

Finally, the correction factor to the absorption is

$$\frac{\Sigma_{0e}}{\Sigma_0} = \frac{1}{1.546 - 0.121 \left[\frac{(0.0762)(2.40)}{(0.127)(1.34)} \right]} = 0.706$$

This would imply that the proper σ_a^{238} to use would be 35.3 barns rather than 50 barns. The average over a lethargy group may be

performed graphically.

The problem of self-shielding effects in cylindrical geometry is somewhat more complex and will be derived in a later report.