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BETA GAMMA DIRECTIONAL CORRELATION IN Re^{186}

by

T. B. Novey, M. S. Freedman, F. T. Porter,
and F. Wagner, Jr.

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ABSTRACT

The directional correlation between the non-unique first forbidden 934-kev beta group and the 137-kev E2 gamma transition in the decay of Re^{186} has been measured as a function of beta energy above 150 kev. Analysis of the experimental data included a correction which takes into account the effect of the backscattering of electrons from the crystal, for the finite energy resolution of the anthracene crystal scintillation spectrometer. The differential anisotropy increases with increasing beta energy to 0.158 ± 0.006 . The directional correlation $W(\theta) = 1 + 0.105 \pm 0.003 \cos^2\theta$ for beta energies of 257 - 934 kev, with no evidence for a $\cos^4\theta$ term, in agreement with theoretical expectation for a first forbidden beta transition.

Limited ranges for the ratios of the nuclear matrix elements governing the beta decay were obtained by comparison with theory, assuming the following: the spin and parity sequence (1-, 2+, 0+); the beta interaction mixture scalar plus tensor; point nuclear charge. Higher order terms in $2\rho/\alpha Z$ were included.

I. INTRODUCTION

Concurrent with the investigation¹ of the shape of the beta distribution of 89-hr Re^{186} , a study was made of the directional correlation of the 934-kev beta branch with the 137-kev gamma.² The primary purpose of this experiment was to secure an independent determination of the nuclear matrix element ratios governing this beta decay for purposes of comparison with those derived from beta spectroscopy. An added motive was to obtain a measurement of the correction to be applied to the magnetic

¹Porter, F. T., M. S. Freedman, T. B. Novey, and F. Wagner, Jr., ANL-5525 (February, 1956).

²Preliminary reports of these experiments have been given; Porter, Freedman, Novey, and Wagner, Jr., Phys. Rev. 98, 214 (1955); Phys. Rev. 99, 671 (A) (1955).

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spectrometer coincidence data to derive the "integrated over 4π solid angle" spectral shape (Appendix II of Ref. 1). We measured both the differential (with beta energy) and integral correlations.

In the only previously reported³ measurement of this beta-gamma directional correlation an upper limit of 0.07 was set for the anisotropy. An anisotropy would be expected to be associated with a non-allowed shape, and the work of Guss, Killion, and Porter⁴ on the composite beta spectrum gave strong indication of the non-allowed shape of the main inner beta spectrum ($E_0 = 934$ kev). Thus the original impetus to the work reported here and in Ref. 1 was to check the contradiction implied in these results.

The decay scheme of Re^{186} is given in Fig. 1 as based mainly on the work of Metzger and Hill⁵ and of Steffen;⁵ the quantitative values are those determined in the beta spectrum measurements.¹ The 137.2-kev gamma ray is known to be E2 in character from the L sub-shell relative conversion ratios,⁶ from the K-conversion coefficient,^{1,5} and from the 627-kev gamma-137-kev gamma angular correlation.³ The lifetime of the lower $2(+)$ state in Os^{186} is $0.8\text{--}1.8(10^{-9})$ sec.⁷ From the first forbidden character of the 934-kev beta transition and the pure E2 character of the 137-kev gamma ray, a correlation function $W(\theta)$ of the form

$$W(\theta) = 1 + A(E) \cos^2\theta \quad (1)$$

is anticipated, if contributions from the third and higher order forbidden terms are neglected.

II. EXPERIMENTAL

A. Apparatus

The sources were prepared from the identical supply of neutron-activated and aged KReO_4 as used for the spectrometer sources¹ and were similarly volatilized in vacuum onto $200\text{-}\mu\text{g}/\text{cm}^2$ Al foils. The average source thickness over the $1/4$ -in. diameter sources was $12\text{ }\mu\text{g}/\text{cm}^2$. The foils were mounted on a 2 in. diameter thin aluminum ring to minimize source mount scattering.

³J. P. Hurley and P. S. Jastram, Phys. Rev. 95, 627 (1954).

⁴Guss, Killion, and Porter, Phys. Rev. 95, 627 (1954).

⁵F. R. Metzger and R. D. Hill Phys. Rev. 82, 646 (1951); R. M. Steffen, Phys. Rev. 82, 827 (1951).

⁶J. B. Swan and R. D. Hill, Phys. Rev. 91, 424 (1953).

⁷F. K. McGowan, Phys. Rev. 81, 1066 (1951) - $0.8(10^{-9})$ sec - probably better value; C. C. McMullen and M. W. Johns, Phys. Rev. 91, 418 (1953) - $1.8(10^{-9})$ sec.

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Fig. 1. Decay scheme of Re^{186}

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The source holder was placed in a centering device in a 7 in. diameter by 7 in. high chamber⁸ with 1/8 in. thick Bakelite walls which was evacuated to ca 1 mm Hg. The plane of the source was oriented at 45° to the beta direction.

The 1/4 in. thick by 1-1/4 in. diameter anthracene beta detector crystal was sealed with silicone grease to a 3/4 in. long Lucite light pipe; this in turn was sealed to the inside surface of the 2 in. diameter by 2-mm thick Pyrex window on the vacuum chamber, also with silicone grease. The RCA 6342 photomultiplier and its 1/4 in. long light pipe were clamped against the window with an intervening layer of 10⁶-centistoke silicone oil. Resolution for the 624-kev Ba^{137m} conversion electron line was ca 18%. The beta detector and chamber were mounted on a turntable which rotated with respect to the gamma detector.

The gamma crystal, a right cylinder of NaI·Tl, 2-15/32-in. diameter by 2-9/16-in. high, was mounted on a DuMont 6363 3-in. cathode photomultiplier at ca 1.5% gamma geometry. Resolution for the 661-kev Ba^{137m} gamma ray was 8%.

The electronic system consisted of a fast-slow coincidence arrangement with single channel pulse height analysers for energy selection, similar to that used in the beta coincidence spectroscopy.¹ The negative pulses from each photoanode were applied to identical circuits. These consisted of a cascaded amplifier stage in parallel with a standard 6AK5 cathode follower which operated the relatively slow linear amplifier and pulse height analysers. The cascaded amplifiers, which have very low output impedance and a fast rise time, were connected via terminated 185-ohm cables to the distributed line amplifiers. The pulses from the cascaded amplifier stage showed a rise time of 11 mμsec on a Tektronix type 517 oscilloscope (minimum rise time of 7 mμsec). The gain of this stage was 0.95. The 6AK5 cathode follower would give a gain of only 0.5 with a 185-ohm load and a rise time of 10 mμsec.

Two distributed line amplifiers were used in both beta and gamma channels, and amplified pulses were fed into the pulse limiter of a Bell, Graham, and Petch⁹ diode coincidence circuit, modified by McGowan.¹⁰ This circuit uses a common delay line pulse shaper and a biased G7A diode detector of coincident pulses. The linear amplifiers were Atomic Instrument Company Model 218, operated with 1-μsec delay line shaping; these drove the Detectolab Model DZ15 single channel pulse analyzers which had

⁸T. B. Novey, Phys. Rev. 89, 672 (1953).

⁹Bell, Graham, and Petch, Can. J. Phys. 30, 35 (1952).

¹⁰F. K. McGowan, private communication.

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an average dead time of 1 μ sec. These units were built to Oak Ridge National Laboratory specifications. The outputs of the analysers were mixed with the fast coincidence circuit output in a slow triple coincidence circuit, of 4- μ sec resolving time.

The common delay line pulse shaping time used in these experiments was 20 m μ sec. With independent sources the resolving time, 2τ , was measured as 30-35 m μ sec instead of the anticipated 40 m μ sec. For this range of resolving times a coincidence efficiency of >90% was realized.

Angular positioning and recording of data were accomplished automatically, usually at 20-40 minute intervals. The motor driven turntable on which the vacuum chamber and beta crystal and the photomultiplier were mounted had provision for selecting any angle in 15° steps and for reversing to enable it to scan back and forth over a preselected range. The four-decade scalers from the beta (B) and gamma (G) analysers and the two-decade scaler from the triple coincidence circuit (T) drove Streeter-Amet traffic counters. A Microflex timer controlled the cycling. The counters were turned off during the repositioning. No observable, instrumentally introduced angular asymmetries were present, as based on previous experience with the apparatus.

B. Treatment of Data

1. Calculation of Real Coincidence Rate. Real coincidence rates (R) were derived from the formula

$$R = \frac{T - 2\tau GB}{GB} e^{-\lambda(t - t_0)} \quad (2)$$

The absolute value of the coincidence efficiency was not material for these experiments, insofar as the measurement of anisotropy was concerned, as in the comparison of real coincidences at different angles; the efficiency cancels out if it is constant during the measurement over angles.

In the measurement of 2τ with independent sources of Re^{186} the circuits saw the identical pulse height distributions and counting rates as those seen in the actual measurements. During the experiments the resolving time was measured daily. No dependence of 2τ on beta energy was observed.

Correction for the small variation in gamma solid angle with the beta-gamma detection angle, θ , (due to imperfect location of the source on the rotation axis) and for radioactive decay of the sample was made by referring observed coincidences to unit analysed gamma recorded. It was ascertained that the gamma background count was independent of relative angle between the gamma and beta detectors, so that no contribution to the β - γ angular correlation arose from sources other than the one under investigation. This background was negligible.

A further correction for drifts in the over-all gain of the beta channel was made by dividing by $Be^{\lambda(t-t_0)}$, where t_0 was the starting time of each angular sweep cycle. Such drifts in the analyzed beta (B) count were observed, to the extent of ca 1.5% deviation from the value calculated from the 88.9-hr half life. The measured anisotropy can be affected in two ways by such drifts, which are attributed to photomultiplier gain variation with temperature, and not to analyser window width fluctuations. Since the variation in the beta count is due to a change in the beta pulse amplitude (and hence beta energy) being examined, the real coincidence rate will be affected by: a) the change in the number of inner group betas in the window and b) the shift in the β - γ anisotropy therein. If the real coincidence rate is approximately proportional to the total (inner group plus ground state transitions) analysed beta count, i.e., if the anisotropy is a slowly varying function of beta energy and the two beta components are similar in shape, then the indicated division by $Be^{\lambda(t-t_0)}$ approximately compensates for the former effect. These conditions are closely met in Re^{186} . The maximum drifts in observed values of B corresponded to energy shifts of less than 25 kev and these were randomly positive and negative. Therefore, the second effect, the shift in the anisotropy being examined at any instant, was small and was included in the indicated energy uncertainty.

For the "singles" rates employed, the corrections for analyser dead times and for slow chance coincidences were negligible, ca 0.2%; this also cancels in the comparison at different angles. In the experiments maximum analysed beta (B) and gamma (G) rates were each 200,000c/min. In order to assess any rate effects the beta spectrum of Tl^{204} was measured using strong and weak sources so as to give maximum beta integral counting rates of 2.5×10^6 c/min and 2×10^5 c/min; the spectrum shape was seen to be independent of the rate over this range and to have the expected form. Thus the linear amplifier and pulse height analyser were not subject to severe base line shifts or resolving time losses in the anisotropy measurements.

The error in the chance rate calculation resulting from the omission of the subtractions of R' [Eq. (1a), Ref. 1] was 0.2%; since the chance-to-real ratios were ca 5%, the error in the real rate was less than 0.01% and this itself cancels out in the comparison of real coincidences at different angles.

2. Geometrical Corrections. For the 1.5% geometrical efficiency used, the correction for angular resolution is important. Correction formulae for circular detectors and a line source are given by Walter *et al.*,¹¹ by Frankel,¹² and by Feingold and Frankel.¹³ Only a very small error is

¹¹Walter, Huber, and Zunti, *Helv. Phys. Acta* 23, 697 (1950).

¹²S. Frankel, *Phys. Rev.* 83, 673 (1951).

¹³A. M. Feingold and S. Frankel, *Phys. Rev.* 97, 1025 (1955).

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introduced by applying these formulae to the case of a 1/4 in. diameter circular source. The geometrical correction amounts to 10.0% of the anisotropy.

3. Correction for Finite Beta Energy Resolution. A major instrumental distortion of the true anisotropy arises from well-recognized effects which in scintillation spectroscopy distort the beta spectrum distribution. These are mainly the following: 1) the statistical response of the phosphor-photomultiplier combination; 2) the finite width of the pulse analyser "window" employed; and 3) the most important effect, backscattering of electrons from the phosphor. In principle all of these distortions can be taken into account in one procedure¹⁴ by applying a "finite energy resolution correction" based on a manifold of line shapes of monoenergetic conversion line sources, including the backscattering "tails." Using this procedure¹⁵ we have computed the true composite beta spectrum of Re¹⁸⁶ (Fig. 2) from scintillation spectrometry data; the corrected spectrum matched the total spectrum observed on the magnetic lens spectrometer¹ (corrected for its resolution width, of course) to <2% above 200 kev, whereas the uncorrected spectrum deviated above the true spectrum by 20% at 200 kev.

If complete and reliable coincidence spectra were available at 90° and 180° to the gamma direction, the true anisotropy as a function of energy could be computed from these spectra corrected as above. However, since data was accumulated in this experiment by scanning over angles at a single energy (selected with a single channel pulse analyser), the coincidence energy spectrum at each angle produced over a long period (weeks) is not as reliable as if completely scanned in energy at each angle in a relatively short time (as, for example, with a multichannel analyser). The choice of angle-sweeping over energy-sweeping is based on considerations of relative ease of achieving the necessary (short term) stability-over-angles of the coincidence efficiency, resolving time, and analyser calibrations.

¹⁴Correction for Phosphor Backscattering in Electron Scintillation Spectrometry, Freedman, Porter, Novey, and Wagner, Jr., (to be published).

¹⁵The electron lines studied were: $e_{K(+L)}$ (976 kev) of Bi²⁰⁷ in coincidence with γ (569 kev); e_K (624 kev) of Ba^{137m} in coincidence with K-X-rays; $e_{K(+L)}$ (481 kev) of Bi²⁰⁷ in coincidence with γ (1064 kev); $e_{K(+L)}$ (162 kev) of In^{114m}. These line shapes were observed with the single channel pulse analyser with the same window width (100 kev) as was used in measuring the anisotropy and the beta spectrum. In general, they are characterized by a Gaussian shape with a low, constant intensity backscattering tail extending to zero pulse amplitude, with a counting rate on the tail equal to 0.075 ± 0.010 of the peak counting rate, independent of initial electron energy.

Fig. 2. Demonstration of the finite energy resolution correction for the scintillation beta spectrometer. Curve (a) is the true total beta spectrum of Re^{186} obtained from magnetic spectrometer data. Curve (b) is the spectrum observed with the anthracene crystal in the angular correlation apparatus. Curve (c) (dashed) is curve (b) corrected for finite energy resolution. All curves are normalized at 450 keV.

To compute the energy resolution correction to our data we obtained the true spectrum distribution of the main inner group of Re^{186} from the magnetic coincidence spectrometer.¹ From this spectrum and the manifold of line shapes for the scintillation coincidence apparatus one can derive the true anisotropy $A(E)$ by a process of successive approximations. The validity of the procedure is attested to, at least insofar as the accuracy and applicability of the manifold of line shapes is concerned, by the excellence of the match of the corrected scintillation spectrum to the true spectrum. The details of the method will be given in Ref. 14. It should be remarked that the line shapes must be measured under identical conditions as the spectrum is taken. For the procedure we used, in which the beta spectrum was obtained from the magnetic spectrometer, the coincidence efficiency for the scintillation counter as a function of energy must be known, whereas if the anisotropy is computed from the corrected spectrum shapes at 90° and 180° , this is not necessary. We took the coincidence efficiency as independent of beta energy for the purposes of the correction.

4. Beta Energy Calibration. For the measurement of the differential correlation (as a function of beta energy), the energy calibration of the beta pulse analyser was based on the 624-keV conversion line of $\text{Ba}^{137\text{m}}$. For the anthracene scintillator a linear relationship between pulse amplitude and beta energy was assumed to hold above 100 keV, with extrapolated intercept at zero pulse amplitude of 22 keV based on the work of Hopkins¹⁶ and of Taylor, *et al.*¹⁷ Immediately after this calibration the energy spectrum of the composite Re^{186} spectrum was taken, and a linear extrapolation of the essentially linear portion of the energy spectrum between 500 keV and 700 keV was made towards the energy axis. The resulting intercept, which occurred at 910 keV, furnished an internal fiducial point for subsequent recalibrations. Another internal standard was available at low energy in the form of the L-conversion line of the 137.2-keV gamma, at 125 keV, which stands out clearly above the beta continuum. The linear calibration curve used in each measurement at a given energy was based on a redetermination of the 125-keV and 910-keV points at the beginning and end of each ca 8-12-hour run. These lines passed through the 22-keV intercept.

C. Extranuclear Field Effects

For the purpose of correcting the magnetic spectrometer coincidence data, the correlation results from the solid sources of volatilized KReO_4 are, in principle, directly applicable, even if the anisotropy were found to suffer attenuation due to crystalline field effects. The sources used were prepared identically, and the resolving time used in either experiment (35 m μ sec or 120 m μ sec) was very long in comparison to the lifetime of the intermediate

¹⁶J. I. Hopkins, Rev. Sci. Instr. 22, 29 (1951).

¹⁷Taylor, Jentschke, Remley, Eby, and Kruger, Phys. Rev. 84, 1034 (1951).

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state, 0.8-1.8 m μ sec.⁷ However, if a fitting to β - γ angular correlation theory is to be attempted, some consideration must be given to the magnitude of such attenuation effects.

Only small attenuations are expected for lifetimes of intermediate states of the order of 1 m μ sec in solid sources. Fraser and Milton¹⁸ have measured the attenuations as a function of delay time in In¹¹¹ and Am²⁴¹ solid sources, from which one can deduce a half-life for attenuation of these correlations of the order of 5 m μ sec. From the experimental lifetime of the intermediate state of Re¹⁸⁶, 0.8 m μ sec, one would estimate an attenuation of ca 5-10%, making the crude assumption that the combined influence of crystalline field gradients and nuclear electric quadrupole moments are similar in our case and theirs.

As a check on the possibility that large attenuations might have occurred in our volatilized solid sources, we examined the anisotropy for beta energies of 700-934 kev, using as a source an aqueous solution of KReO₄. One microliter of this solution was spread over 1 cm² as a thin film (1 mg/cm²) between 0.5-mg/cm² rubber hydrochloride film and a 1-mg/cm² mica square; the anisotropy was measured in an atmosphere of helium saturated with water vapor by passing through a bubbling column to prevent the source from drying out. The observed anisotropy was 0.17 ± 0.015 . A second experiment for beta energies of 500-934 kev gave an anisotropy of 0.092 ± 0.015 . From the A(E) curve obtained with solid sources (Fig. 5b, page 16) there was obtained, for the average anisotropy over these energy ranges, 0.147 and 0.130, respectively. The agreement is quite poor, but perhaps indicates that severe attenuation does not occur in the solid sources. This point requires more detailed investigation. We assume, in making the comparison with theory, that the results with the solid sources truly represent the unattenuated directional correlation.

D. Inner Bremsstrahlung Effect

In Ref. 1 it was argued that the influence of inner bremsstrahlung (I. B.) on the determination of beta spectral shapes, even for such a case as Re¹⁸⁶, is negligible because of the small I. B. intensity and the strong angular correlation of β - I. B. which discriminates against the angles used in the magnetic spectrometer. For the angular correlation experiments, however, the gamma discrimination is differential rather than integral above a lower limit, and for the case of the 137-kev photopeak of Re¹⁸⁶, only ca 1/3 of the I. B. spectrum above the 100 kev is accepted, giving a contribution of 0.0025 for the ratio of β - I. B. coincidences to β - γ coincidences over all angles. Fig. 12 of Ref. 8 demonstrates that ca 80% of this contribution (or 0.002) will be seen at 90° and none at 180°, leading to a 0.2% reduction of the β - γ anisotropy. In comparison to the ca 10% β - γ anisotropy, this effect was neglected.

¹⁸J. S. Fraser and J. C. D. Milton, Phys. Rev. 94, 795 (1954).

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E. Gamma-Gamma Coincidences

The contribution of γ - γ coincidences to the observed correlation, arising from the very weak 627-keV - 137-keV gamma cascade, was computed to be negligible.

III. RESULTS

A. Integral Anisotropy

The integral β - γ anisotropy in Re^{186} was observed, accepting beta particles of energy ≥ 257 keV. The 137.2-keV gamma peak was selected differentially so as to reject all X rays and to reduce the number of coincidences due to the 307-keV β with the 627-keV gamma. Less than 1% of the counts was due to X- β coincidences and $< 0.001\%$ due to the weak 307-keV β group. Data was taken every 15° from 90° to 270° ; the real coincidence rate (per analyzed gamma) exhibited symmetrical behavior about 180° . Chance rates averaged ca 1% of true coincidence rates.

In Fig. 3 the correlation function $W(\theta)$ is plotted vs $\cos^2\theta$. The straight line, a least squares fit to the data using only the $\cos^2\theta$ term, as expected for the spin sequence involved, $1(-) \rightarrow 2(+) \rightarrow 0(+)$, corresponds to a value for the integral anisotropy $A = 0.105 \pm 0.003$ for beta energies of 257-934 keV. A least squares fit of the data to a series including a $\cos^4\theta$ term gave the result, $W = 1 + (0.127 \pm 0.016) \cos^2\theta - (0.025 \pm 0.018) \cos^4\theta$. We do not believe that this represents a significant detection of a $\cos^4\theta$ term.

B. Differential Anisotropy

The differential angular correlation experiments required about 5 weeks of counting in the final run. At each energy the sequence 90° - 180° - 270° was scanned forward and backward many times, counting 20 min. at each angle. Each energy was examined a number of times throughout the experiment, so that a total of ca 200,000 coincidences was obtained at each energy. No asymmetry between 90° and 270° was observed.

The gamma analyser was adjusted as in the integral correlation experiment. The correlation was examined in 100-keV increments of beta energy. Several Re^{186} sources were used so as to maintain counting rates within preset limits. The ratio of true/chance coincidences exceeded 7 at all energies.

Figure 4, a plot of the anisotropy $A(E)$ vs the beta energy in keV, presents the experimental results. The vertical flags represent statistical errors only. The error contributed by the uncertainty in 2τ is negligible

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Fig. 3. Integral beta-gamma directional correlation in Re¹⁸⁶ vs $\cos^2 \theta$ for beta energies in the range 257-934 kev. The straight line is the least squares fit to the data assuming no $\cos^4 \theta$ term contribution. The slope of this line is 0.0955; when corrected for finite angular resolution this corresponds to an anisotropy of 0.105 ± 0.003 .

Fig. 4. The beta-gamma directional anisotropy in Re¹⁸⁶ as a function of beta particle energy. The vertical flags on the experimental points represent errors due only to counting statistics. The horizontal flags represent uncertainty in the energy. All corrections except that for the energy resolution of the β detector have been applied. Curve (a) is the "best fit" to the data. Curves (b) and (c) give our estimate of extreme limits of error taking into account the counting statistics and uncertainty in the energy.

in comparison. Horizontal flags represent uncertainty in energy calibrations. All corrections, except that for the energy resolution, have been applied to the data. The central curve (a) is drawn to represent the best fit to the data, and the upper (b) and lower (c) curves give our estimate of the extreme edges of an "error band" within which the true experimental curve is judged to lie, taking into account both the uncertainties in statistics and in energy.

Curve (a), Fig. 5, reproduces the central curve of Fig. 4; application of the correction for the finite energy resolution of the beta scintillation spectrometer transforms it into curve (b). A comparison of the curves shows that the correction is small ($< 4\%$) and positive above ca 550 kev and rises to about 37% (negative) at 150 kev. With a small additional contribution to the total error to account for the estimated inaccuracies in the resolution correction, the error band curves of Fig. 4 transform into those of Fig. 5, curves (c) and (d).

Curve (e) of Fig. 5 presents that function $A(E)$ that was used to compute the correction for the $\beta - \gamma$ anisotropy to be applied to the magnetic spectrometer coincidence spectrum.¹⁹ This curve is based on earlier measurements, with poorer statistics, and a more approximate energy resolution correction. The final "best" curve (b) lies within the error band (not shown) of curve (e).

Integration of the corrected curve (b) from 257-934 kev leads to a value for the anisotropy of 0.101, in comparison to 0.105 ± 0.003 obtained in the integral experiment.

IV. COMPARISON OF THEORY AND EXPERIMENT

A. Theoretical Formulation

In this section the observed $\beta - \gamma$ directional correlation as a function of β energy in Re^{186} is compared with the theoretical expression for this correlation. From this comparison one obtains the ratios of nuclear matrix elements effective in the β transition.

The treatment of $\beta - \gamma$ directional correlation as first given by Falkoff and Uhlenbeck²⁰ has been extended to give formulations not restricted by the approximation $\alpha Z \ll 1$ and to include the cross terms

¹⁹See Ref. 1. footnote 73a.

²⁰D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. 79, 323, 334 (1950).

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Fig. 5. The beta-gamma directional anisotropy in Re^{186} with correction for finite energy resolution of the beta scintillation spectrometer. Curve (a) repeats the central curve of Fig. 4 and transforms into curve (b) on application of the energy resolution correction. Curves (c) and (d) define the error band about (b), including a small contribution from uncertainty in the energy resolution correction. Curve (e) is that function $A(E)$ which was used in correcting the magnetic spectrometer coincidence data; it is based on preliminary measurements and is shown here for completeness. See footnote 73a in Ref. 1.

associated with a combination of the five beta decay interactions.^{21,22,23} Several attempts have also been made to take into account the effects of finite nuclear size.²⁴ In the present work we assume point nuclear charge.

The theory for the $\beta - \gamma$ angular correlation involving first forbidden beta decay is given by Yamada and Morita²⁵ and by I. Hauser.²³ In the present section we have followed the notation and formulation of Hauser as his work contains a complete formulation of the problem of interest, and in particular explicit expressions for the calculation of the quantities L_{10} and N_{10} which enter into the cross term coefficients. Specifically we consider the case of a first forbidden β transition followed by an E2 gamma transition in which the sequence of nuclear spins and parities is $[1(-), 2(+), 0(+)]$.

The $\beta - \gamma$ correlation is expressed as

$$W(\theta, E) = \sum_k A_k(E) B_k P_k(\cos \theta) \quad k = 0, 2 \quad (3)$$

where W is the relative probability of emission of beta rays with energy E at angle θ to the gamma direction, and the P_k are Legendre polynomials. The coefficients $A_k(E)$ depend only on those variables which describe the β decay, and the B_k ²⁶ depend only on the gamma transition. Including the presence only of the first forbidden matrix elements, one expects a correlation function of the form (B_0 is unity)

$$W(\theta, E) = A_0(E) + A_2(E) B_2 P_2(\cos \theta)$$

²¹A review of work up to 1954 is contained in the article by H. Frauenfelder, Beta and Gamma-Ray Spectroscopy, edited by K. Siegbahn, (Interscience Publishers, 1955), Chapter XIX.

²²H. Takebe, Prog. Th. Phys. Japan 12, 561 (1954); 12, 574 (1954); 12, 747 (1954).

²³I. Hauser, unpublished doctoral thesis. State University of Iowa.

²⁴M. Yamada, Prog. Th. Phys. Japan 10, 241 (1953); M. E. Rose and D. K. Holmes, Phys. Rev. 83, 190 (1951).

²⁵M. Yamada and M. Morita, Prog. Th. Phys. Japan 8, 431 (1952); M. Morita, Prog. Th. Phys. Japan 10, 363 (1953).

²⁶ B_k is equivalent to the symbol $A_k^{(2)}$ of Ferentz and Rosenzweig, whose tables cover the cases of pure or mixed multipolarity transitions. [M. Ferentz and N. Rosenzweig, Table of F Coefficients, ANL-5324.] For our case of a pure E2 transition B_k reduces to $F_k(202) = -0.5976$ as given by L. C. Biedenharn and M. E. Rose., Rev. Mod. Phys. 25, 729 (1953).

or

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$$W'(\theta, E) = 1 + \frac{A_2(E)B_2}{A_0(E)} P_2(\cos \theta) \quad (4)$$

Experimentally one measures the anisotropy $A(E)$, where $A(E)$ is defined by the equation

$$W''(\theta, E) = 1 + A(E)\cos^2 \theta \quad (5)$$

or

$$A(E) = [R(\pi, E)/R(\pi/2, E)] - 1 \quad (6)$$

$$W'' \propto R \quad (\text{see Eq. 2})$$

It then follows from the definition of $P_2(\cos \theta)$ that

$$A_2(E)/A_0(E) = 2A(E)/\{B_2[3 + A(E)]\} \quad (7)$$

Finally the quantities $A_2(E)$ and $A_0(E)$ must be specified. $A_0(E)/D_0(E)$ is just the familiar angular-independent beta spectrum shape "correction factor," while $A_2(E)/D(E)$ is the beta spectrum "shape factor" of the angular-dependent term in Eq. (3). Here $D(E)$ is the allowed beta energy distribution. [$A_0(E)$ might be obtained from an experiment in which an integration over all directions of emission of the beta particles is involved, as in either a beta spectrometer "singles" measurement, in a coincidence measurement with no angular discrimination, or in a beta spectrometer coincidence experiment by correcting with the measured anisotropy $A(E)$.¹] In measurement of the directional correlation between beta and gamma rays only the ratio $B_2A_2(E)/A_0(E)$ is obtained.

For the "a la mode" beta decay interaction combination scalar plus tensor, as was used in fitting the spectrum shape,¹ and for a first forbidden transition from $1(-)$ to $2(+)$,^{23,25}

$$\begin{aligned} D^{-1}(E)A_0(E) = & x^2(M_0 + K^2L_0/3 + 2KN_0/3 + 2L_1) + v^2L_0 \\ & + u^2(K^2L_0/8 + 9L_1/8) + 2xv(N_0 + KL_0/3) \\ & + 2x(M_0 - L_1) + 2v(N_0 - KL_0/3) \\ & + (M_0 + K^2L_0/6 - 2KN_0/3 + L_1/2) \quad , \end{aligned} \quad (8)$$

$$\begin{aligned} D^{-1}(E)A_2(E) = & 4(3/2)^{1/2} w_2(11)[x - 1/2][x(N_{10} - KL_{10}/3 + L_1/2) \\ & + (N_{10} + KL_{10}/3 - L_1/4) - vL_{10}] + 3(3/2)^{1/2} w_2(21) u \\ & \cdot [x(N_{10} - KL_{10}/3 - L_1) + (N_{10} + KL_{10}/3 + L_1/2) - vL_{10}] \\ & - 9/8(7/2)^{1/2} w_2(22) u^2L_1 \end{aligned} \quad (9)$$

and

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$$D(E) = pEK^2 F_0(Z, E) .$$

The real parameters $\underline{x}$, $\underline{v}$, and $\underline{u}$, which involve the ratios of nuclear matrix elements, are to be determined from the comparison with the experimental data. They are defined as

$$\begin{aligned} \underline{x} &\equiv (i G_S \int \beta_{\underline{r}}) / (G_T \int \beta_{\underline{\sigma}} \underline{x} \underline{r}) \\ \underline{v} &\equiv (\int \beta_{\underline{\alpha}}) / (\int \beta_{\underline{\sigma}} \underline{x} \underline{r}) \\ \underline{u} &\equiv (i\beta B_{ij}) / (\int \beta_{\underline{\sigma}} \underline{x} \underline{r}) . \end{aligned} \quad (10)$$

The electron momentum is $\underline{p}$; the total electron energy is $\underline{E}$ (mc^2 units) and $K = E_0 - E$ is the neutrino energy. The atomic number of the daughter nucleus is $\underline{Z}$. The quantities L_λ , M_λ , N_λ , P_λ , Q_λ are combinations of electronic radial wave functions as given by Konopinski and Uhlenbeck²⁷ and by Pursey.²⁸ The quantities L_λ , M_λ , etc., were evaluated from the tables of Rose, Perry, and Dismuke²⁹ extended to five significant figures³⁰ and hence the point nuclear charge approximation is implicit in the results. Further, these tables at $Z = 76$ are computed for mass number 189 rather than 186 (our case).

The notation $w_k(JJ')$ is defined as

$$w_k(JJ') \equiv (-1)^{J_b - J_a} (2J_b + 1)^{1/2} W(J_b J_b JJ'; k J_a) , \quad (11)$$

where $\underline{W}$ is a Racah coefficient and J_a and J_b are the initial and final nuclear spins, respectively, in the beta transition.

The quantities L_{10} and N_{10} are given by Hauser³¹ in a form convenient for computation.

²⁷E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 69, 308 (1941).

²⁸D. Pursey, Phil. Mag. 42, 1193 (1951).

²⁹M. E. Rose and C. L. Perry, Phys. Rev. 90, 479 (1953); M. E. Rose, C. L. Perry, and N. M. Dismuke, Oak Ridge National Laboratory Report ORNL-1459 (unpublished).

³⁰These extended tables were derived from the raw data of the four-figure tables, ORNL-1459 by Rose, Perry, and Dismuke. The data were kindly supplied us by Dr. M. E. Rose. The necessity for the five-figure accuracy in computing A_0 is discussed in Ref. 1.

³¹The L_{10} of Hauser (op. cit.) is defined with the sign opposite to that of the L_{12} of Yamada. They are otherwise identical.

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$$L_{10} = 1/2 \sum_{\kappa=\pm 1} (L_1 - \kappa P_1)^{1/2} (L_0 + \kappa P_0)^{1/2} \cos \left[\delta_1(-\kappa E) + \delta_0(-\kappa E) + (\pi/2)(\gamma_1 - \gamma_0 + 1) + \sum_{n=1}^{\infty} S_n(g) \right] \quad (12)$$

$$N_{10} = 1/2 \sum_{\kappa=\pm 1} (L_1 - \kappa P_1)^{1/2} (M_0 - \kappa Q_0)^{1/2} \cos \left[\delta_1(-\kappa E) + \delta_0(-\kappa E) + (\pi/2)(\gamma_1 - \gamma_0 + 1) + \sum_{n=1}^{\infty} S_n(g) \right] ; \quad (13)$$

for positron emission, reverse the sign of N_{10} .

$$\begin{aligned} \delta_\lambda(\kappa E) &= \arctan \left\{ [g/(\gamma_\lambda + \lambda + 1)][1 - (\kappa/E)] \right\} , \\ g &= \alpha Z E / p , \quad \alpha = e^2 / \hbar c \\ \gamma_\lambda &= [(\lambda + 1)^2 - (\alpha Z)^2]^{1/2} \end{aligned} \quad (14)$$

and

$$S_n(g) = \arctan \left\{ [g(\gamma_{\lambda+1} - \gamma_\lambda - 1)] / [(\gamma_\lambda + n)(\gamma_{\lambda+1} - 1 + n) + g^2] \right\} .$$

For the calculation of $\sum_{n=1}^{\infty} S_n(g)$, the sum of the remainder of the terms past the N th is given by

$$\sum_{n=N+1}^{\infty} S_n(g) \simeq (1/2) S_{N+1}(g) + (\gamma_{\lambda+1} - \gamma_\lambda - 1) \arctan \left[\frac{g}{(1/2)(\gamma_{\lambda+1} + \gamma_\lambda + 1) + N} \right] \quad (15)$$

within an error which is less than $(1/2) S_{N+1}(g)$.

B. $B_{ij} = 0$

From Eq. (8) and (9) it is seen that Eq. (7) is a quadratic in $\underline{x}$, $\underline{v}$, and $\underline{u}$. We consider first the case $u = 0$ ($B_{ij} = 0$). Then, for any energy $\underline{E}$ and the corresponding experimental value, $A(E)$, a conic section in the $\underline{x}, \underline{v}$ plane can be plotted. This graphical method of determining values for the

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nuclear parameters $\underline{x}$ and $\underline{y}$ is illustrated in Fig. 6. The ordinate in this figure is the same $\underline{y}$ which was introduced in Ref. 1. The necessary transformation is as follows:

$$y = [(1 + S)/2] (2\rho/\alpha Z) v - 1 - x \quad (16)$$

where $S = (1 - \alpha^2 Z^2)^{1/2}$, and with Rose et al.²⁹ we take $\rho = (1/2)\alpha A^{1/3}$, $A = 189$. Since the experimental values $A(E)$ have some uncertainty, two conics are generated at each energy corresponding to the upper and lower experimental error limits (Fig. 5c,d). In Fig. 6 the bands whose boundaries are these conics are shown for energies 368 and 818 keV ($p = 1.4$ and 2.4).

It is seen that the conic bands overlap in two regions within which (x, y) values lie which may give good fits to the angular correlation. Representative points from these regions are chosen to compute the anisotropy; dashed curves I, and II, Fig. 7 correspond to the points $(0.695, 0.035)$ and $(-1.0, 0.58)$, respectively. The solid curves reproduce the experimental error limits curves (Fig. 5c,d). It is seen that we are able to obtain an excellent comparison of experiment to theory without B_{ij} contributions.

The spectrum shape fitting for the inner group beta transition also selects (x, y) parameter values. A discussion of attempts to fit both shape and angular correlation simultaneously is given in Section VII of Ref. 1. It can be shown that only points in the neighborhood of $(-1.0, 0.58)$ even approximately fit the spectrum shape.

C. $B_{ij} \neq 0$

An attempt was made to investigate the sensitivity of the position of the overlap regions of the conics to small admixtures of B_{ij} . Values of $u = \pm 0.1$ were assumed. The resulting conic sections for $p = 0.8$ and $p = 2.4$ are shown in Fig. 8. It is apparent that the regions of overlap are very sensitive to small B_{ij} contributions. For $u = -0.1$, the overlap region in the neighborhood of $x = -1.0$, $y = 0.58$ is moved perhaps to give better agreement with the beta spectrum results, while for $u = +0.1$, the overlap moves to regions of the x, y plane not giving agreement with the beta shape (see Fig. 27a, Ref. 1). It does not follow from this, however, that large positive values of u will not give suitable overlap regions for both experiments. Values of u of 0.1 correspond to a ratio of $|i\beta B_{ij} / \int \beta \underline{\alpha}|$ of about 0.01 , whereas from ft values for decays, where $\int \beta \underline{\alpha}$ and B_{ij} are the largest contributors, one would expect values of ca. $1/3$ for this ratio. Thus one may reasonably expect even larger contribution from B_{ij} than were tried here. For $|u| < 1$, the directional correlation is more affected by B_{ij} contribution than is the spectrum shape, as u appears to the first power in $A_2(E)$ but only as u^2 in $A_0(E)$.

Fig. 6. Conic sections obtained in the graphical solution for the parameters $\underline{x}$ and $\underline{y}$ (Eq. 7). Because of experimental uncertainty a band (hatched) is generated rather than a single conic. Band A is for $p = 2.4$; band B is for $p = 1.4$. Points in the cross hatched region may produce good fits to the experimental β - γ anisotropy. The theoretical anisotropies for the points indicated are shown in Fig. 7.

Fig. 7. Comparison of the experimental and theoretical β - γ anisotropy in Re^{186} . The solid lines are the error limits on the experimental data. The dashed curve I is for the (x, y) point $(0.695, 0.035)$; dashed curve II is for the point $(-1.0, 0.582)$. The parameter u is zero ($B_{ij} = 0$). The parameters x , y , u are defined in Sections IV and V.

Fig. 8. Conic sections obtained in graphical solution of Eq. (7) for the Re^{186} beta-gamma correlation. Because of experimental uncertainty, bands, rather than single conics, are generated. These show the effects of including small contributions from the matrix element, βB_{ij} , i.e., the parameter $u = \pm 0.1$. The parameters x and y are defined in Sections IV and V. Vertically hatched bands are for $p = 0.8$; diagonally hatched bands are for $p = 2.4$.

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A complete analysis of the results to give optimum values of $\underline{x}$, $\underline{y}$, and $\underline{u}$ for both beta spectrum and directional correlation experiments is very time consuming and the use of an electronic computer is indicated. This work is in progress.

Rose and Holmes²⁴ have shown how extremely sensitive the important quantities L_λ , M_λ , etc, are to the nature of the nuclear charge distribution. The most obvious theoretical refinement seems to require the evaluation of these functions to the required 5-figure accuracy for various reasonable nuclear charge distributions. Were these available one might pursue, with more confidence as to their physical significance, the determination of the nuclear parameters.

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