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THE FAST EXPONENTIAL EXPERIMENT

Work Done by

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THE FAST EXPONENTIAL EXPERIMENT

Report Prepared by

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I. PREFACE

It is the hope of the authors to present in this report those experimental data obtained from the Fast Exponential Experiment which are publishable in a unified fashion as of the date of the report, together with such theoretical treatments as are pertinent. The material covered represents the original program of the experiment. In order to have a complete and comprehensive report, and at the same time to reduce cross reference to other reports which may not be readily available to all readers, the experimental facilities and procedures are outlined in detail. Procedures which are used in obtaining data in several sections, and which may be of more general interest, are included in the fourth chapter. "Single shot" experiments are described and reported together later in the chapter. Also included are the procedures followed to assure safe handling of the large amounts of fissionable material used in the experiment. It is believed that such experience is of broad enough interest to justify its inclusion. Wherever the error of a measurement is known, it is included with the data.

Most of this report was written by members of the Staff as memoranda for internal distribution at this Laboratory, and many sections have been directly incorporated into this document. The authors of these memoranda were: J. W. Armstrong, F. C. Beyer, R. H. Bryan, A. E. Engfer, H. H. Hummel, D. H. Lennox, F. H. Martens, W. A. Reardon, N. Rosenzweig, A. B. Smith, and B. I. Spinrad.

A precis of this report was also presented to the International Conference on Peaceful Uses of Atomic Energy as Paper No. 598.

II. DESCRIPTION OF THE FACILITY

In order to study, without the inherent and largely unknown danger of a critical assembly, the properties of a reactive system utilizing fast neutrons, the Fast Exponential Experiment was set up. The experiment consisted of two weakly interacting nuclear systems: a source pile of U^{235} , moderated with H_2O to provide neutrons to the exponential assembly; and the assembly itself. Between the two systems there was located a slab of uranium of such thickness that thermal and resonance neutrons from the source pile were absorbed before reaching the assembly, and such that the most reactive assembly did not greatly affect the reactivity of the source pile. By analogy with previous vertical exponential assemblies, this insulating uranium slab was referred to as the pedestal. Figures 1, 2, and 3, respectively, are plan, elevation and detail drawings of the Fast Exponential Experiment.

A. The Source Reactor

1. Fuel Elements

The core of the source reactor was made of MTR fuel plates in Borax-type assemblies.¹ Each fuel plate contained 15.7 grams of U^{235} and, with ten plates in each assembly, each fuel assembly contained 157 grams of U^{235} . Several assemblies which contained only four fuel plates, with provisions for inserting up to six additional plates, were furnished. These modified assemblies, plus adjustable stops on the control rods, were used to limit the amount of reactivity available to the operator. The core as finally set up contained 2.31 kg of U^{235} . Figure 4 is a drawing of a fuel assembly showing the pertinent dimensions.

2. Control Rods

Three control rods, made of 0.060" thick x 2-3/16" wide x 33-1/2" long cadmium sandwiched in aluminum, were used in the source core. The two outer rods were completely withdrawn from the core and used as safety rods; the center rod was used as a regulating rod and the maximum amount of withdrawal was limited by a mechanical stop.

¹Smith, C. D., Droston, F. W., and Kerze, F., Jr., "Production of Fuel Assemblies for the Materials Testing Reactor Mock-Up Critical Experiments," Report ORNL-951, Oak Ridge National Laboratory, Oak Ridge, Tennessee (April 9, 1951).

FIG. 1
FAST EXPONENTIAL EXPERIMENT

FIG. 2
FAST EXPONENTIAL EXPERIMENT - ELEVATION

FIG. 3
FAST EXPONENTIAL EXPERIMENT
PLAN DETAIL

FIGURE 4.
SOURCE CORE FUEL ELEMENT.

3. Control Rod Drive Units

The drive units which positioned these rods had been used for previous critical experiments.² They consisted of drums powered by gear motors through an electromagnetic clutch. The drums were mounted on ball bearings and were free to rotate on the shaft when the clutch was de-energized automatically at scram or power failure, or at will by the operator. The rods were connected to the drum through flexible steel cables which unwound as the rods dropped. When near the bottom of its travel, the rod was decelerated smoothly by shock absorbers and came to rest fully inserted. The drive motors were 1/20 hp and were operated by a 90-volt dc power supply. The gear drive allowed a maximum withdrawal rate of 60 cm per minute, corresponding to a reactivity increase of less than 22 in hours per second; interlocks allowed only one rod at a time to be withdrawn at this speed.

4. Core Shielding

Lead, used to provide gamma-ray shielding after shutdown, was placed around three sides, top, and bottom of the core. A formed lead brick, designed to prevent streaming, was stacked on three sides of the core to a depth of 6". Below the core, lead bricks were stacked to a thickness of 3-7/8". Above the core, 12 pieces of 1/2" thick lead sheet were stacked to give a thickness of 6". Holes were cut in the upper lead shield to allow for passage of the three control rods.

5. Tank

The source reactor tank, 4' square and 5' high, was constructed of 1/8" and 1/4" thick steel plates and was reinforced with steel angles and channel sections to prevent excessive deflections. An aluminum plate, 36" wide x 34-3/4" long x 3/4" thick, was set in the side of the tank between the pedestal and the source core. A 3/8" diameter neoprene gasket was used to seal this aluminum "window." The top of the tank was sealed with aluminum covers to prevent access to the core and the control rods, so that the available reactivity could not be easily increased.

6. Water System

The water used for moderating the source reactor was stored in a sump tank located in the pit (cf. Fig. 1). This water was pumped from the sump tank to the source tank through a 1-1/2" stainless steel pipe. In this line was a 1-1/2" solenoid-operated valve; a 1" valve was located in a by-pass line around the 1-1/2" valve. The 1-1/2" solenoid valve remained open until

²Brittan, R. O., et al., "Experiments on Zero Power Reactors," ANL-4684, p. 11 (August 24, 1951).

the water in the source tank reached a predetermined level, at which time the valve was closed. Water continued to flow into the tank through the 1" by-pass line. The 1" valve was set to permit a predetermined amount of water to continually flow to the tank to make up for leakage over the weir and around the flap valve. A 2" line from the source tank led to an adjustable weir which was also used to control the water height.

The main water dump from the source tank was through a 6" pipe connected to a header at the top of the sump tank. In this header was located the flap valve used to dump the water. Approximately 30 seconds were required to drain the water from the source tank whereas approximately 10 minutes were needed to fill the tank. An ion exchange column was connected to the system to permit purification of the water when necessary.

7. Pedestal

The pedestal is the normal uranium slab located between the source core and the exponential. The slab was constructed of uranium bricks, 2" x 3" x 6", and was 8" thick, 24" wide and 36" high. It had been experimentally determined that this thickness of uranium absorbed the thermal and resonance neutrons from the source pile before they reached the exponential assembly, and decoupled the multiplication of the most reactive assembly from that of the source pile to the desired degree.

8. Shield Tanks

Three shield tanks were placed around the sides and top of the pedestal. The tanks were originally designed to be filled with a borax or cadmium solution, with lead bricks in the sections of the tanks closest to the source core. However, it was determined experimentally that a more desirable shield material was a mixture, in alternate layers, of Lucite and cadmium. The Lucite was approximately 0.4" thick, and the cadmium 0.010". The shield tanks were made of 3/16" thick steel and had an outside depth of 6".

9. Source

A Po-Be source of about 5 curies, producing about 10^7 neutrons per second, was located below the source core near its southeast corner. It was driven up into an inverted source well to within a centimeter of the bottom or dropped to a depth of 100 cm into an oil-filled floor well. The source was mounted at the end of a rod set in an elevator which moved vertically on linear ball bearings along two parallel rods. The elevator was cable-operated by a gear motor.

The following table includes some of the pertinent facts about the source core with the 8" uranium pedestal in place.

Table I

SOURCE CORE DATA

Height	61.0 cm
Width	48.6 cm
Maximum Thickness	22.8 cm
Adjusted Thickness*	19.0 cm
Weight of U ²³⁵	2.355 kg
Volume Ratio (metal to water)	0.4:1
Density of U ²³⁵	0.0412 gm/cc
Total Volume of Core	67.8 liters
Adjusted Volume of Core*	56.8 liters

*The adjusted values are the values obtained by considering the height and width to be fixed and then calculating what thickness (and consequently volume) would give a uniform fuel density.

B. The Exponential Experiment

1. Fuel Plates

The fuel plates were fabricated from enriched and natural uranium pieces in such a way that the enriched sheet was snugly sealed into a natural uranium cladding at top, bottom, sides, and edges. This technique insured that the enriched uranium could never get above a concentration of 25% in any assembly, and that contact with air, sodium, or cleaning agents would not corrode the valuable enriched material. The completed "sandwich" was about 0.076" thick, 24" long, and 2.76" wide, and had an average weight of 1.560 kg, of which 376 g were U²³⁵.

A zirconium button was welded to each plate to limit the density of plates in an assembly.

2. Dilution Plates

The dilution plates were made of normal uranium and were nominally 0.076" thick x 24.00" long x 2.750" wide. These plates had a 1/8" wide x 1/8" deep notch in each end to aid in identifying the enriched from the normal plates. The average weight of a dilution plate was 1.502 kg.

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3. Fuel Cans

The fuel cans were made of 0.078" thick Armco iron and had external dimensions of 3" x 3" x 24-1/2" long. The can was open at the top and had a 1/8" thick bottom. On the bottom of the can was a boss, 7/64" high, which had a hole through it, tapped for a 1/8" pipe plug. A cover plate, which also contained a 1/8" pipe thread, could be used to seal the open end of the fuel can. The holes in the cover plate and the bottom of the fuel can were used to fill the cans with sodium. Figure 5 shows a fuel plate and a fuel can.

The fuel cans were loaded with specified numbers of fuel plates and dilution plates. With a 3:1 ratio of U^{238} to U^{235} , 15 fuel plates were used per fuel can. For the 7:1 ratio of U^{238} to U^{235} , 7 or 8 fuel plates were used per fuel can. 8 or 7 dilution plates were also used, respectively, so as to have a total of 15 plates per fuel can. Using the 7:1 ratio, the fuel cans were arranged in a checkerboard pattern with regard to the number of fuel plates in each can. The number of fuel plates in any two adjacent cans was then 15. With the 5:1 ratio, 10 fuel plates and 5 dilution plates were used per can. Figure 6 is a photograph of the exponential assembly where the 5:1 ratio was used.

4. Reflector

A bottom reflector of 12" thickness was always used beneath the enriched assembly. Lead, iron and normal uranium were procured in brick form, 2" x 3" x 6", so that the reflector could be easily stacked on the table to the exact width of the assembly. Slots were milled into the bricks so that flux traverses could be made through the reflector in the vertical direction. Later, aluminum bars, 1-1/2" x 3" x 24", were procured to be used as reflector for the 5:1 and 7:1 loadings. A series of special reflector studies was made with the 5:1 assembly, as reported in Chapter III. This material was improvised as the experiments required.

5. Table

The top of the table on which the fuel cans and the reflector were loaded was 33-1/2" long by 34-5/8" wide and 3/4" thick. This table top was supported by a welded I-beam frame which, in turn, was welded to a 3/4" steel plate. A 2" thickness of lead was poured in the bottom of the table. This lead was intended to act as a bottom gamma shadow shield after shut-down. Fastened to the bottom of the 3/4" steel plate were 4 steel casters, two of which were grooved and the other two flat. The grooved casters ran on an inverted V-shaped bar which acted as a guide for the table when driven up to or retracted from the pedestal. Two 12", 50-lb I beams were used as supporting rails for the table.

Fig. 5. Fuel Can & Fuel Plate

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FIG. 6 - EXPONENTIAL ASSEMBLY - 5:1 RATIO

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6. Drive Unit

The drive unit used for positioning the exponential table consisted of four main units: namely, a 3/4-hp, 220v dc motor; a gear reduction unit; a ball bearing lead screw and nut; and a selsyn indicating system. The speed of the drive motor could be controlled to move the exponential table at a rate of from 0 - 6 inches per minute. The position of the table was accurately shown by the selsyn indicating system.

7. Channels

Channels were provided to permit measuring devices to reach the interior of the assembly so that data might be obtained. There was, in general, one channel in each of the three directions. The convention decided upon was to let X be the direction perpendicular to the leakage face of the reactor - or the exponential direction. The Z direction was taken to be the vertical and the Y as the horizontal across the assembly.

In order to get space for the Y and Z channels, 5/16" aluminum sheets were placed between the center rows and center layers of cans to cause a separation sufficient to place a stainless steel channel guide. These thin-walled guides prevented binding and catching of detectors as they moved through the assembly.

In the X direction, the guide channel was inserted in a fuel can in which several of the plates were cut lengthwise to accommodate the channel.

C. Operational Procedure

To achieve maximum safety, and still to have a reasonable operating procedure for the Fast Exponential Experiment, certain rules and procedures were laid down, and instruments and controls were provided. The monitoring and control systems are shown diagrammatically in Figs. 7 and 8. In addition, Table II gives a more detailed description of the six monitoring circuits.

Having six such circuits, there was some overlapping of functions, so that the pile could still be run if some of the instruments should malfunction. The following conditions had to be met before starting the pile:

1. The period trip circuit (P-VII) must be operating.
2. Either P-I or P-II must be operating.
3. Two of the three level trips (P-II, P-IV, and P-VI) must be operating.

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FIG. 7
MONITOR AND SAFETY CIRCUITS

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Table II

MONITORING CIRCUITS

	<u>Circuit Detector</u>	<u>Amplifying System</u>	<u>Function</u>
M-I	B ¹⁰ -Lined Proportional Counter	Nuclear Measurements Corporation Scaling Unit	Actuates audible flux monitor
P-I	B ¹⁰ F ₃ -Filled Ion Chamber	Applied Physics Corp. Vibrating Reed Electrometer	Records neutron flux level
P-II	B ¹⁰ F ₃ -Filled Ion Chamber	Applied Physics Corp. Vibrating Reed Electrometer	Records neutron flux level, operates flux level trip
P-IV	Anthracene Crystal Photomultiplier Tube	DC Amplifier	Operates maximum level trip
P-VI	Argon-Filled Ion Chamber	DC Amplifier	Operates flux level trip
P-VII	NaI (Tl) Crystal Photomultiplier Tube	Logarithmic Amplifier and Period Meter	Records flux level and period, operates period trip

The control system was designed to prevent inadvertent addition of too much reactivity or too high a rate of increase of reactivity. There was six components which affected the reactivity or flux level of the pile: the three control rods, the primary neutron source, the exponential table, and the personnel shield. The drive selector and rod selector permitted the operator to move only one component at a time. The drive mechanisms were designed so that the components could not move faster than some safe maximum, but controls were provided to allow slower motion of the components. Selsyn indicators on the control panel indicated the position of the components. There were also lights to indicate the extremes of travel of the source, exponential table and shield.

A number of interlocks prevented the operator from performing some act in improper sequence if that act would cause an unsafe condition. These interlocks and their functions are listed as follows:

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1. Door 13 interlock - de-energized the magnetic clutches on the rods when the assembly room door was opened, preventing anyone from entering the room with the pile critical.
2. Flux level interlock - Prevented the rods from being withdrawn with a low flux in the reactor. After several hours shutdown, there were too few neutrons in the reactor core to indicate the true multiplication. Without this provision, the operator could conceivably have withdrawn the rods far beyond the critical point, believing there was still no multiplication.
3. Water level interlock - Prevented the rods from being withdrawn without a full tank of water. If the rods were withdrawn to critical position or farther without water up, the addition of water could have given an extremely rapid increase in reactivity.
4. Power Delay - There was a thirty-second delay between turning on the rod power switch and the time power was available to operate the rods or the source. During this time, alarm panels containing both lights and bells located at all danger points were actuated. If anyone were inadvertently left in the assembly room, he had time to leave. It was the responsibility of the operator to check his personnel.

Four overrides were provided to enable the operator to bypass some of the above interlocks when necessary. Three of these overrides were in the form of spring-loaded switches which had to be held down with considerable pressure while the operation was taking place.

1. Loading override - This was a key switch which permitted the rods to be raised when loading the pile. Personnel had to be in the assembly room during this procedure, so it was desirable to have both the door open and the rods cocked. However, this override opened a switch in the primary circuit of the dump valve magnetic clutch, making certain that the moderator was removed from the pile. It also shut off the alarms.
2. Water level override - This permitted the rods to be raised for loading, for checking trip circuits, or to cock safety rods when starting up.
3. Flux level override - This permitted the rods to be raised for loading or when checking trip circuits.

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4. Scram circuit override - Each instrument scram circuit was provided with an override button to allow the range of the instrument to be changed, or when checking trips, to override a low level trip circuit while testing a higher level circuit. The overrides also reset the scram circuit after scram had occurred.

A scram signal could originate in one of three ways - high period, high power, or a hand scram from one of four buttons available to the operator. When such a signal occurred, the safety and control rods were dropped. For normal shutdown, the rods were dropped by the operator and the water kept in the tank until the end of the day. This provided greater shielding for the personnel.

Two resistance thermometers were installed in the source pile tank. One indicated the temperature in the tank, and the other the temperature difference between the water in the core and that outside the lead brick surrounding the core.

In addition to the electrical and mechanical safeguards listed above, certain operational limitations were imposed. These conditions were.

1. No more than 0.5% k excess was available to operator.
2. At least 1.5% k was available for shutdown.
3. On opening the assembly room door, water was dumped whenever access to the control rods was possible.
4. Safety and control rod withdrawal rate was limited so as to add less than 0.05% Δk per second.

The two operators conducting the experiment had to ascertain that these conditions were met before operating the reactor. If insufficient data were available to make an accurate estimate, conservative values were chosen. If, in spite of these precautions, an unsafe condition prevailed, the experiment was stopped and additional safety precautions taken.

The operations of the Fast Exponential were divided roughly into two types. The first, and most dangerous, involved any change in the fuel loading, control rod positioning, or change in the reflector on the leakage face. During this time, all operation was closely scrutinized by two of the qualified operators. Any changes to be made were agreed upon by the two staff members and all other interested personnel.

The second phase of operation came when a certain loading and geometry were to be used for a long period of time. The reactor was then sealed to prevent access to the fuel and control rods. Excess reactivity was limited to less than 200 inhours. During this time the reactor always became critical with the same control rod settings. The pile could then be operated by a qualified technician under the supervision of the staff members.

III. PRELIMINARY STUDIES

A. Neutron Flux Distribution within the Source Core

The thermal neutron distribution within the core was obtained using Fe-Mn wires as detectors. The wires were arranged throughout the fuel as shown in Fig. 9. With these detectors, the neutron distributions in the respective directions were obtained (Figs. 10 - 12). In the transverse, or Y, direction, the distribution was a close approximation to a cosine function. The core edge was too near the lead shield to permit investigation of the very narrow water reflector. However, in the vertical, or Z, direction, the distribution clearly shows the thermal peaking in the upper reflector. Again, the Z distribution approximated a cosine function. In the X, or front to back, direction there is an extrapolated distance out into the pedestal and a peak in the reflector to the rear. The flux distributions shown are generally typical of cores of this type.

Some estimate of the energy distribution of the neutrons within the core is given by the cadmium ratio of Mn. Figure 10 shows the cadmium ratio to be about 10, rising slowly as one approaches the reflector.

B. The Pedestal as a Neutron Source

The neutron attenuation of the pedestal was studied, using indium foils as detectors. The result is shown in Fig. 13. It is noticed that the response of this detector fell by two orders of magnitude in traversing the pedestal. This should not be confused with the total neutron attenuation, since the attenuation of the neutron distribution within the pedestal is highly energy dependent. No small detector with a flat energy response being known, it is impossible to obtain a true representation of the total flux attenuation. One may only conclude that there is a rapid drop in neutron intensity at the indium resonance energies and below.

Since the Exponential Experiment rested against the face of the pedestal, the neutron distribution on this surface was of interest. As the work progressed, it became apparent that there was a sizable room background incident upon this face. For this reason the neutron distribution was examined with the surface exposed to the room and with a cover of some six inches of uranium. Figure 14 is an isoflux plot of the bare pedestal face using indium detectors. An examination shows that the emerging neutron distribution follows the general outline of the core. A flux peak in the Y direction and a suggestion of one in the Z direction are observable.

In comparison with the bare pedestal, Fig. 15 shows the Y-axis neutron distribution obtained with uranium detectors when the pedestal was covered with six inches of uranium to exclude the room effects. The general characteristics are similar to those of the fully exposed pedestal face.

FIGURE 9.
CORE FLUX MEASURING POINTS.

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FIGURE 14.
ISO-FLUX PLOT
OF PEDESTAL
IN DETECTORS.

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The sharp thermal peak just past the edge of the pedestal would exert a perturbing influence on the distribution within any exponential assembly placed against the pedestal. The cause of this peak was traced to the two tanks surrounding the pedestal which, as originally designed, contained lead. Since lead is not a good reflector or absorber of the neutrons emerging from the core, it was replaced by various combinations of borax, Lucite, and cadmium. After a careful study of the different effects with miniature fission chambers, described in the next chapter, it was decided that the peaking could best be removed by a cadmium-Lucite sandwich. A comparison of this effect, before and after the side tanks were loaded with the cadmium sandwich, is also seen in Fig. 15.

C. Power Calibration of the Source Reactor

It was desirable to have a measure of the intensity of the reaction within the fuel in terms of commonly utilized units. To this end the power level of the Exponential Source Reactor was calibrated in watts. Two methods were applied.

The first was basically a comparison with the Argonne Standard Pile. With the Exponential Reactor running at a convenient power, several indium foils were irradiated at various positions within the core. The absolute value of the flux at these measured points was determined by comparing the activities of these detectors as induced in the reactor and in the standard pile. Knowing the neutron distribution within the core, the average value of the flux was easily obtained. The power level in watts was determined simply as:

$$\text{Power} = C \int_{\text{core}} (\text{Fission flux density})(\text{cross section})(\text{fuel content}) dV$$

where C is a constant determined from the energy of fission. Care was taken to correct to fission flux density by comparing the respective cadmium ratios of indium and uranium within the core.

A second method was used as a rough check on the above determinations. At a relatively low power, the reactor was brought to as accurate a setting of criticality as possible. A Po-Be source of known strength was then rapidly inserted in the core. The resulting positive period of the reactor was carefully measured. Applying simple kinetic reactor theory, this period was related to the rate of the initial reaction as follows:

$$\text{Power} = \frac{8 \times 10^{-5} ST}{e \ell^*}$$

where

e = number of neutrons/fission
 ℓ^* = mean neutron life time

S = source strength
 T = doubling time

This second method yielded a result differing from the standard pile method by only $\sim 30\%$. This was remarkable in view of approximate values of ℓ^* and the effective source strength used in evaluating the above expression.

D. Reflector Studies

An experiment to determine the effects of various materials as reflectors on the leakage face of the source pile was performed. The unreflected core is shown in Fig. 16. The three cadmium control rods, placed as illustrated, were initially calibrated quite accurately, and their values were rechecked throughout the experiment. Large increases in reactivity due to the reflectors were equalized by removal of fuel, first from the fourth, or rear, row of elements and later from the third row. The four reflectors were Lucite, iron, lead, and natural uranium. In the case of Lucite, a flat sheet, 0.424" thick, was placed against one face of the core and the change in reactivity measured. Additional layers were added until a total of nine layers, approximately 3.7" thick, were on the front face. The reactivity differences were measured by the change in the position of the control rod and by the removal of fuel of known worth. The three metallic reflectors were composed of bricks, 2" x 3" x 6", allowing stacking of material in 2" layers. In the case of uranium, after the effect of the 8" reflector was determined, the last layer was removed and the bricks turned so they formed a final 3" layer, giving a 9" reflector. The data obtained for the Lucite reflectors are given in Table III; for the metal reflectors, in Table IV.

Table III

Lucite Thickness (in.)	$\Delta k/k$ for Lucite
0.431	1.8
0.862	3.15
1.293	4.25
1.730	4.60
2.155	4.95
3.020	5.04
3.880	5.20
∞^*	5.35
x_0	0.952

Table IV

Reflector Thickness (in.)	$\Delta k/k$ for		
	U	Pb	Fe
2	3.56	3.6	3.3
4	4.30	5.1	5.6
6	8.00	6.1	5.7
8	8.35	6.6	5.9
9	8.42		
∞^*	8.55	7.0	5.6
x_0	2.08	2.94	1.96

* Thickness Extrapolated

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The experimental measurements were adjusted to least squares fit of the empirical formula:

$$\Delta k/k = a(1 - e^{-x/x_0})$$

where x is the thickness of the reflector and x_0 the characteristic thickness of the material. It is seen (Fig. 17) that the curves fit quite well in all cases. The first point, however, lies slightly above the calculated curve. This may well be due to the change in neutron spectrum with reflector thickness and to the inaccuracy of the empirical curve itself.

Figure 17 is a composite graph of experimental results. The curves represent the empirical expression above with the constants adjusted to the indicated experimental points. There were certain inaccuracies in making the measurements, so it is felt that the points fit the curve to the degree of the experimental error.

E. Single Can Critical

For safe design of a system for cleaning sodium from the fuel cans, it was necessary to know the effect of controlled or accidental addition of moderator to a loaded fuel can. It was calculated that if the voids in a fuel element were filled with alcohol, the assembly would be subcritical by a factor of ten in mass. In order to substantiate this calculation, a multiplication experiment was performed. Existing instrumentation from previous critical work was utilized for monitoring purposes. To provide a "scram" system, a fast-acting moderator release valve was fitted to a fuel can, and a cadmium-covered rod was suspended so as to fall between one side of the fuel and reflector. Both safety circuits were actuated by any of three instruments responsive to a pre-set neutron or gamma flux level. One additional instrument was set to trip on a minimum period of 10 seconds.

The fuel can was enclosed on four sides with a paraffin reflector and the counting rate determined as a function of water height and fuel loading. No multiplication was detected within a 2% statistical uncertainty for any of the incremental loadings or for that corresponding to the maximum physical capacity of the fuel container.

For the most concentrated loading planned, the fuel can contained 5.63 kg U^{235} distributed uniformly among 15 fuel plates. Spacing between plates was maintained by a zirconium bottom welded to one side of each plate. However, by deliberately arranging some plates back-to-back, it was possible to force 19 fuel strips into the container. This combination was also investigated to see if inadvertent overloading of the can would increase the criticality hazard. The volume per cent composition of the fuel element when filled with water was:

FIGURE 16.
CORE CONFIGURATION FOR REFLECTOR STUDIES

FIGURE 17.
REACTIVITY EFFECT OF VARIOUS
REFLECTORS ON SOURCE CORE.

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Uranium	34.45
Iron	10.61
H ₂ O	54.94
U ²³⁸ /U ²³⁵ ratio	3.17:1

The assembly was made in a cell shielded by 3' concrete walls. Visual access to the experiment was through zinc bromide radiation windows in the wall. A wooden supporting framework for the fuel can, which was large enough to permit stacking of paraffin blocks to form a 6" thick reflector, was made. Water flow into the bottom of the fuel can was controlled by a "fail safe" solenoid valve actuated from the control room. A sight glass indicating water level was read through a radiation window with a cathetometer.

On a platform above the assembly, the magnetic clutch and drive unit for the single cadmium safety rod were mounted. De-energization of the magnetic clutch by a scram signal permitted the rod to fall freely into its sheath in the reflector. A synchro-transmitter geared to the rod drive unit indicated position on the receiver located in the control room. A second shutdown control was provided by a moderator release valve actuated by the level and period instrumentation. A 2" diameter hole in the bottom of the fuel can was sealed by a rubber stopper attached to the face plate of a magnetic clutch. A 20-pound lead weight was bolted to the clutch face plate so that a scram signal would permit the weight to fall and remove the stopper.

The results indicated negligible multiplication with any physically possible loading. The instruments used were an Applied Physics vibrating reed electrometer, a scintillation counter trip circuit, two dc amplifiers, a pile period meter, and two N.M.C. scaling units. The chambers feeding these instruments were placed around the assembly in the best position to "see" multiplication of neutrons from a Ra-Be source placed next to the fuel can between the can and the reflector.

To determine multiplication, counts were taken with 5, 10, 15 and 19 fuel plates in the can. For each loading the counts (c) were made as a function of H₂O height, with and without the control rod inserted. Plots of $1/c$ and $1/\Delta c$ gave horizontal straight lines within the statistics of counts.

F. Fuel Storage

The fuel was shipped from the fabricators, the General Metals and Controls Corporation of Attlehead, Massachusetts, in lots of six drums or less. These drums were 14" in diameter and 24" high, giving a volume of 2.2 cu ft per can. The drums had dome-shaped lids which were fitted with water-tight gaskets, so there was no danger of flooding the cans, and

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the drums would not be stacked one on top of the other. This prevented the drums from being stacked in a three-dimensional array. Each drum contained fifteen enriched plates, approximately 5.6 kg. During processing and fabrication both at General Metals and at the laboratory where the plates were welded, the drums were kept in groups of six or less. As they arrived at Building 316, where they were to be used, a test was made to check the multiplication of groups of more than six drums.

A radium-beryllium source was set in the center of an hexagonal array of six drums. A long counter was used to determine the counting rate. As more drums arrived, they were added to the array. Previous calculations indicated that the reactivity of such a system would be negligible, so no elaborate safety mechanisms were provided; instead, a rope was tied around one drum so that it could be removed from a distance. Since only one drum was added at a time, this was considered a sufficient disassembly mechanism. A long forked stick was used to push the drums forward. At no time was there a noticeable counting rate increase, so that the storage system was deemed safe.

Three experiments were performed on the fuel in the drums in an attempt to measure the multiplication of the worst possible stacking geometry. To minimize the direct effect of the source, the source was thermalized with paraffin and four drums carefully arranged to present the largest possible cross section to thermal neutrons. The detector was a Cd - shielded BF₃ counter of the "long - counter type." The drums were stacked near the wall of the storage vault, so that the wall, as well as the floor, would provide reflection of neutrons. The array of drums is shown in Fig. 18. The observed counting rates were:

a)	background without source	~1 cpm
b)	bare thermal source	3617 (20 min)
c)	1st trial	4220 (20 min)
d)	2nd trial	4080 (20 min)
e)	3rd trial	3979

A second set of experiments, in which thermal and fission sources were used, yielded:

<u>Number of drums</u>	<u>Fission Source</u>	<u>Thermal Source</u>
10	6935 cpm	2255 cpm
12	7053 cpm	2293 cpm
15	7034 cpm	2432 cpm

The detector was buried in the array, approximately two drums away from the source.

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FIGURE 18.
STORAGE SAFETY EXPERIMENT PLAN

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A third set of measurements using a thermal source and detecting epi-Cd neutrons yielded:

<u>Number of drums</u>	<u>Trial I</u>	<u>Trial II</u>
10	3876 cpm	4015 cpm (two trials)
16	4118 cpm	

In this trial the detector could "see" the source at all times.

The small increase between bare source and the first trial was no doubt due to slow neutron fission in the fuel in the first six drums (i.e., first generation neutrons). The first six drums thus became the source for the final two arrays.

The net decrease in neutrons observed may have been due to shielding of the source by the drums, although this is a barely significant difference.

There was no observable multiplication in the system. A value of $m = 1.002$ was computed by the method given in LA-1273³ for the multiplication of the system, based on the assumptions that the material was uniformly distributed in spheres of 300 cm radius and that the source is a point at $r = 0$. Considering the approximation involved in spherizing the geometry, the computed figure, which would be expected to be an over-estimate, was in accord with the experimental result.

For actual storage, the drums were lined up in rows of four, perpendicular to the walls, with about four or five feet between rows. This provided access to all the cans and gave a neat appearance to the storage area.

G. Initial Loading of the Fast Exponential Assembly

The initial loading of the fuel forming the Fast Exponential Assembly was done in small steps, each of which was considered to be absolutely safe as far as criticality was concerned. Several methods of determining the multiplication were employed, although the multiplication remained so low that results were observed by only one system. This system employed a radium-beryllium neutron source which was placed near the assembly and several counters, both bare and paraffin covered, in and near the assembly. The increase in the counting rate of the detectors then gave a measure of the multiplication. The reciprocal of the counting rate was plotted against the mass of fuel loaded. An extrapolation

³Carlson, Bengt, "A Note on the Calculation of Neutron Multiplication," LA-1273, (July 16, 1951).

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of this curve would show the critical mass. A representative curve from one of the counters is shown in Fig. 19. Superimposed on this are two theoretical curves which show the plot: (1) if the assembly were to go critical when completely loaded ($k_{\infty} = 2$; $\kappa = 0.05$); or (2) if the assembly would never go critical ($k_{\infty} = 2$; $\kappa = 0.05i$). It can be seen that our assembly was even less reactive than this latter case.

The following alternate methods were tested to determine multiplication, but, in all cases, the multiplication was too low to be observed above background:

1. Using the spontaneous fission neutrons as a distributed source, the counting rate was taken as loading proceeded. The Ra-Be source was removed.
2. The critical position of the source pile control rod was noted after each loading. This gave an increase of 100 inhours when the first row of cans was loaded, but no further indication.
3. Counting rates and instrument readings were taken with the pile neutron source in place, and the safety rods cocked. This was done thinking the reactivity of the combined system might be great enough to measure.

In addition to determining whether the exponential itself was safe, a "fat man" experiment was performed. In this, a large wooden box, approximately 24" high, 15" wide, 10" thick, filled with paraffin was suspended by two cables so that it would be free to swing away from the exponential. It was pulled forward with a cable attached to a magnetic clutch connected to the reactor scram circuit so that it would be de-energized on scram. The fat man was caught on the backswing by a latch so that it could not swing forward again and increase the reactivity. This was repeated several times during the loading, and at no time did the multiplication show any significant increase. When the exponential was fully loaded, a second box was set on the opposite side, very close to the exponential. The fat man was again brought up as close as possible to the exponential; there was no significant increase in reactivity. This proved that two men could be on either side of the exponential without danger of criticality.

These tests, with the most reactive configuration, showed no danger of criticality. However, all major changes were made employing the same system of multiplication measurements. When a minor change in the assembly was made, a monitor was used to give the operators an aural indication of the flux.

FIGURE 19.
MULTIPLICATION OF EXPONENTIAL ASSEMBLY

H. Shielding

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It was quite evident in the early stages of operation that the fast neutron flux level in the control room and in the surrounding areas would be far above maximum permissible levels whenever the neutron flux in the exponential assembly was sufficient to permit irradiation of low-cross section foils, i.e., Np^{237} , U^{238} etc. A plan view of the cell is shown in Fig. 20. The exterior walls were of brick, 12" thick. The shadow shield wall between the assembly and control room was monolithic concrete, 3' thick up to a height of 14'. Above this section was a 10' high section of 12" brick which reached up to the ceiling of the assembly room. A wall made of pre-cast concrete blocks separated the two reactor cells. This wall was also 3' thick. Two doors were located on the south wall: an emergency personnel door and a freight entrance. The ceiling was of 8" concrete. There was a large door, 6' x 8', leading from the control room into the east storage room. There was also a 6' x 8' opening between the storage and assembly rooms.

Figure 20 shows typical paths of fast neutrons from the assembly which reach the control room by suffering only one collision in the walls or air. Figure 21 shows how neutrons could reach the control room by passing through the thinner ceilings and scattering from the air or upper section of the north wall. Three baffle walls in the east storage room cut the neutron flux which entered through the storage room to a point where the major component of the flux was received through the ceiling and side walls. With these baffle walls in place, it was found that the limiting factor would be the flux at the guard post at the northern entrance to the area.

The neutron attenuation length in brick and concrete was measured to be 3.8" and 3.1", respectively. These values were used to calculate the leakage from the different areas. The leakage from the east wall was found by assuming that the east storage wing did not exist and that the 6' x 8' opening was filled with 12" of brick. The following results were found, assuming a flux of 1 neutron per second per square foot incident on the walls.

	Area (Sq. ft)	Compo- sition	Atten- uation	% Trans- mission	Total Leakage	% of Total Leakage
Ceiling	728	8" conc.	13	7.5%	55	23
West	672	36" conc.	10^5	-	-	-
South	504	12" brick	23	4.3%	22	9
Doors	120	1/2" wood	1	100	120	51
East	672	12" brick	23	4.3%	29	12
North (lower)	364	36" conc.	10^5	-	-	-
North (upper)	260	12" brick	23	4.3%	11	5
TOTAL	3220			7.4%	237	100

FIGURE 20.
PLAN OF ASSEMBLY CELL

FIGURE 21.
TYPICAL PATHS OF NEUTRONS
INTO CONTROL ROOM.

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A fast neutron survey was made at several points around the building. The top number in the four sets of numbers shown in Fig. 22 is the original fast neutron dose in number/sq cm/sec at a power of 25 watts. After a water shield, 9" thick, was added to the ceiling, another survey was taken, giving the second set of numbers. After this survey was taken, shielding was placed in front of the freight door. This shielding consisted of 16" of pre-cast concrete block, and a baffle was constructed around the emergency personnel door so it could still be used for its original purpose and yet be shielded from the reactor. After the construction of these walls the third set of numbers was obtained by the survey.

The 9" of water should have reduced the leakage of neutrons through the ceiling by an additional factor of 7.5. This would reduce the total leakage to 80%. This figure is in good agreement with three of the values in Fig. 22. The 16" shields at the doors should have reduced the leakage through those areas to less than 1% of the original leakage. This would make the total leakage only 30% of the original value, which is also in agreement with the observed data. Because the leakage was of the same order of magnitude through all the surfaces of the room, a heavy shielding would have to be placed on all walls and the ceiling in order to reduce the leakage appreciably.

It was then decided to test the effect of shielding the exponential assembly itself. This was done by placing a 16" concrete block wall along both sides of the assembly. The effect of these walls on the leakage was negligible, and a considerable perturbation in the equilibrium flux inside the assembly was noted.

A decision was made to do all experiments that were possible with the flux reduced to a point where it did not constitute a hazard. This maximum level was 25 watts for four hours running time per day.

FIGURE 22.

NEUTRON DOSES AT 25 WATTS
OPERATING POWER.

IV. EXPERIMENTAL TECHNIQUE AND EQUIPMENT

A. Miniature Fission Chambers

The research program required a method of obtaining neutron flux distributions in the exponential assembly with detectors of various energy response, i.e., such as the two major isotopes of uranium, neptunium and plutonium. The miniature fission chambers described here solved many of the problems associated with these measurements:

1. They required a much lower flux than did foils to give the same statistical accuracy.
2. Experimental time was used more advantageously, since the radiation and the counting was done simultaneously.
3. Counting time could be adjusted immediately to give the proper statistics.
4. If an interesting effect in a flux plot occurred, it could be examined carefully immediately, as could any anomalous results.
5. There were no background or decay corrections to be made.
6. The chambers, in contrast with foils, required a very small amount of the very scarce isotopes.
7. It was not necessary to have an inventory sitting on the shelf cooling off from previous runs.

1. Construction

The shell of the chambers (see Fig. 23) was milled from cold rolled steel in two sections. When properly hard soldered together, the steel provided the necessary physical strength to contain the pressures used in the filling. The completed shell was dip-tinned in soft solder to provide a base for the installation of the end plug and insulating seal. Assembled, the shell had a material thickness of only 0.080". Experiment showed that this thickness of the combined materials produced no noticeable neutron flux depression in the epithermal energy region.

The central paddle-shaped electrode was supported by an hermetic* seal soldered to the chamber and extending through the milled hole. The exterior portion of the seal was seated within a recess as illustrated in the figure. This recess, provided with a set screw, formed a

*Hermetic Seals, Newark, New Jersey

FIGURE 23.
SHELL BLANK. 2 Required

FIGURE 24.
END PLUG.

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socket for the probe unit which established the physical and electrical contact between the chamber, preamplifier, and the motor drive unit. The open end of the counter shell was closed with a brass plug, Fig. 24, machined to shape, and soft soldered into place. This plug contained the 1/8" copper filling tube. The electrode was a 10-mil stainless steel sheet, hard soldered to a copper wire leading through the hermetic seal. An expanded view of the chamber is shown in Fig. 25.

The method of depositing the coating of fissionable material on the paddle is described by Rossi and Staub;⁴ briefly, the fissionable material is converted to a nitrate and then dissolved in a few drops of ethyl alcohol. This solution is mixed with about 5 cc of Zapon Lacquer, * 1:20 dilution. The fissionable material, carried in the lacquer binder, is sprayed or painted upon the paddle. A very thin coating is applied and then baked at 500° F. At this temperature, the lacquer burns off and the remaining fissionable material is converted to a hard, smooth, oxide coating. As many as fifty such applications may be necessary to build up a thickness of 100 $\mu\text{gm}/\text{cm}^2$. Beyond this thickness, absorption of the fission fragments in the layer becomes excessive. With this method, good coatings of plutonium, neptunium, uranium-235 and 238 were obtained.

The chamber was filled with argon to a pressure of 10 atm in order to give the necessary stopping power within the limited active volume of the chamber.

2. Electronics

A block diagram of the circuitry necessary for the utilization of these chambers is shown in Fig. 26. (For stability and simplicity, the high voltage was supplied by a 300-volt dry battery.) The polarity is arbitrary as it governs only the direction of the output pulse. In such a form, the high voltage supply may be incorporated within the preamplifier forming an exceptionally convenient unit.

The preamplifier is the only component demanding special construction and care. It is essentially an inverse feed-back amplifier with a gain of 150 and a response to 10 megacycles. Fig. 27 gives the preamplifier circuit that has proved satisfactory. The output of the preamplifier drives the cable to the remotely placed linear amplifier. This unit is a standard pulse amplifier** with negative feedback, variable gain, and

⁴B. B. Rossi and H. H. Staub, Ionization Chambers and Counters, National Nuclear Energy Series, Vol. V-2, McGraw Hill and Co. New York (1949).

*Zapon Lacquer, Hercules Powder Co.

**Linear Amplifier, Model 218, Atomic Instrument Co., Cambridge, Mass.

FIGURE 25.
CHAMBER.

FIGURE 26.
BLOCK DIAGRAM OF CIRCUITRY.

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FIGURE 27.
PRE-AMPLIFIER.

frequency response. (A gain of 3000 has proved to be sufficient.) The output of this linear amplifier goes to an integral discriminator set to bias out the alpha particle pulses due to the natural disintegrations of the fissionable loadings. The pulses from the discriminator are fed into a scaling circuit located nearby. The resolving time of the completed unit is restricted to 5μ sec by this scaler. As an aid in the general analysis of the chamber performance, a differential pulse height analyzer may be added to the above circuit.

3. Drive Unit

An aluminum bracket, mounted on rods, supported the drive units, which consisted of the dc drive motor, the driving gear, two idler pulleys, the selsyn transmitter, and a junction box. The dc drive motor was a 24-volt reversible motor (Bodine type V-10-R) with a built-in gear train which could turn at a maximum speed of 19 revolutions per minute. The driving gear and selsyn were directly coupled to the output shaft. The gear was of such a size that the rack moved precisely ten centimeters per revolution. The motor was controlled from the control room by a three-position switch. Its speed was varied by changing the output of a dc supply. For each drive unit a selsyn indicator was also mounted near the motor control.

The fission chambers themselves were mounted on the ends of probes, 3' long, made from three pieces of brass. To provide a race for the idler pulleys, a $1/8$ " square piece of brass was silver-soldered to a section of $1/4$ " brass tubing, and the gear rack was silver-soldered to the opposite side of the tubing. The probe could then be set into the idler pulley and gear of the driving system. The outer insulation and braided shield of a piece of RG 59 U coaxial cable was removed and the center conductor and the insulator threaded through the tube. This was arranged with a fitting to make the electrical connections when the chamber was attached.

This whole apparatus could be mounted on support bars outside of the exponential assembly so that the fission chamber was free to travel through a channel. The chamber position for a given selsyn reading could be measured prior to operating the reactor and, as long as the probe was not run beyond the limits of the rack, this position remained constant. Since the motor was reversible and its speed adjustable, a very close setting, about 1 millimeter, of the fission chamber was possible. Shown in Fig. 28 are three chambers, a probe, the drive unit and a preamp containing the high voltage battery.

4. Performance

Operating as pulse counters, these fission chambers have proven very dependable. A typical voltage plateau is shown in Fig. 29. It is well enough defined to provide good stability with a reasonably stable

FIG. 28 - FISSION CHAMBERS AND ASSOCIATED EQUIPMENT

FIGURE 29.
VOLTAGE PLATEAU.

FIGURE 30.
PULSE HEIGHT ANALYSIS.

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discriminator. In the present work, the operating potential was 300 volts. Although this was not the ideal setting, it was a good compromise between performance and electronic expediency. Because interest was in the use of the chamber as a detector, the energy resolution of the fission particles was not of importance. However, the proportionality between fragment energy and pulse height was a good indication of the proper functioning of the instrument. With electrode coatings of $100 \mu\text{gm}/\text{cm}^2$ maximum thickness, an investigation of the chamber output with a differential pulse height analyzer clearly showed the fission fragment energy distribution⁵ (See Fig. 30). The two peaks corresponding to particle energies of 60 and 90 Mev were clearly evident.

In making experimental measurements using these chambers as detectors, the bias level was set slightly above the alpha particle energy. Fortunately at this setting the maximum sensitivity was obtained, and also the greatest stability to discriminator drift. During the recording of data, any long time drift in the sensitivity was avoided by the repeated check of several arbitrary reference points. Three units could be used within the assembly, so that neutron distribution measurements in the three spatial axes could be simultaneously determined.

B. Flux Monitor

Located near the linear amplifiers used with the miniature fission chambers were four counting circuits, utilizing Berkeley decimal scaler components. One of these scaling components was driven by a flux monitor, a boron-10-lined proportional counter which measured the leakage flux from the reactor. The total number of counts from this monitor gave a number proportional to the flux integrated over the time interval of counting. This monitor count could be used to normalize successive runs to an arbitrary total flux.

The detector was located so as to give a counting rate of approximately 100,000 counts/min. This was about ten times greater than the rate of the experimental detectors, so no appreciable error was due to the monitor statistics.

A timing circuit using an X-ray time switch was available so that counting with all the units could be started and/or stopped simultaneously. This also permitted the experimenter to accurately reproduce counting times from one run to the next.

⁵Brunton, D. C. and Hanna, G. C., "Energy Distribution of Fission Fragments from U-235 and U-233," Can. Jour. Res. 28, 190 (1950).
Brunton, D. C. and Thompson, W. B., "Energy Distribution of Fission Fragments from Pu-239," Can. Jour. Res., 28, 498 (1950).

C. Fission Ratios Using Counters

Some indication of the neutron energy distribution was obtained by measuring the ratio of U^{235}/U^{238} fission per atom in an assembly. This measurement could be made with the miniature fission chambers.

Two miniature chambers were used, one filled with uranium enriched in the 235 isotope and the other with natural uranium. These were arranged with the usual circuitry and monitor system. The individual measurements could thus be normalized to a common level of radiation. For any given measurement of the ratio the bias was arbitrarily set for each chamber. This level, together with the fissionable material and individual geometric properties, determined the absolute sensitivity. This sensitivity could be calibrated by thermal irradiation in the following manner.

With a given bias, a U^{235} chamber and a natural uranium chamber were exposed to the thermal flux obtained in the center of a large paraffin block placed in the leakage flux from the source core. To assure true thermal conditions the response was measured by taking Cd differences; thus the integrals were from zero energy to Cd cut off. The response in counts/monitor count N was given by:

$$N_{25}^{th} = e_{25} A \int_0^{Cd} \sigma_{f25} \phi_{th}(E) dE \quad (4.1)$$

$$N_{nat}^{th} = e_n \rho \int_0^{Cd} \sigma_{f25} \phi_{th}(E) dE \quad (4.2)$$

where $\rho \cong 0.007115$ (atomic fraction of U^{235} in natural uranium), σ_f is the fission cross section, ϕ the flux, and A the fraction of U^{235} in the enriched coating. e_n and e_{25} are the relative efficiencies determined by geometry, loading and bias. In this calibration the counters were assumed to depress the flux to an equal extent, a reasonable assumption as the materials and construction are identical.

The same two chambers were now placed in the unknown fast flux of the assembly, $\phi(E)$. When normalized to the same radiation level the counting rates are:

$$N_{25}^f = e_{25} \left[A \int_0^\infty \sigma_{f25} \phi(E) dE + (1 - A) \int_0^\infty \sigma_{f28} \phi(E) dE \right] \quad (4.3)$$

$$N_{nat}^f = e_n \left[\rho \int_0^\infty \sigma_{f25} \phi(E) dE + (1 - \rho) \int_0^\infty \sigma_{f28} \phi(E) dE \right] \quad (4.4)$$

Here the effect of the threshold fission of U^{238} is introduced and the integration is over all energies. From equations (4.1) and (4.2)

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$$\frac{e_{25}}{e_n} = \frac{N_{25}^{th}}{N_n^{th}} \frac{\rho}{A} \quad (4.5)$$

From equations (4.3) and (4.4) it follows that

$$\frac{N_{25}^f}{N_n^f} = \frac{e_{25}}{e_n} \frac{AR + (1 - A)}{\rho R + (1 - \rho)} \quad (4.6)$$

where

$$R = \frac{\int_0^\infty \sigma_{f_{25}} \phi(E) dE}{\int_0^\infty \sigma_{f_{28}} \phi(E) dE} \quad (4.7)$$

and is the fission ratio to be found. For simplicity, the $(1 - A)$ term in the numerator of equation (4.6) is eliminated as it causes a negligible error in even the hardest (fission) spectrum. Then eliminating the quantity e_{25}/e_{28} from equation (4.6) and solving for R , we have

$$R = \frac{1/\rho - 1}{\left(\frac{N_{25}}{N_n}\right)_{th} \left(\frac{N_n}{N_{25}}\right)_f - 1} \quad (4.8)$$

D. Fission Ratios Using Catcher Foils

If a thin aluminum foil is placed in contact with a polished uranium plate and this system exposed to a neutron flux, some of the fission fragments from atoms near the surface of the uranium will be ejected into the aluminum foil and stopped. These fission fragments will then decay according to their own characteristic lifetime. This foil may then be counted in an ordinary beta counter and a number proportional to the number of fissions obtained. If the plate is made from a pure isotope of uranium, a number proportional to the fission rate of that isotope is obtained. By using the two different isotopes the fission ratio can be quickly determined. This represents an advance over the use of a uranium foil directly in the beta counter, because it removes the background of the uranium and also does away with the effects of radiation histories of the foils.

A holder was made from two pieces of aluminum, $1-1/8" \times 1-5/8" \times 1/16"$. Four recesses, exactly 1 cm square and 0.010" deep, were milled in one piece to receive the uranium plates. The aluminum foils were placed directly over the plates and the cover piece fastened with four small screws. A fifth screw held the whole assembly to a long thin iron spear with which the holder was inserted into a channel in the exponential assembly. The holder was always placed as near as possible to the center of the cosine curves in the Y and Z directions, so that the flux variation over its area would be minimized.

Three uranium plates were used. One was made of enriched U^{235} , one of natural uranium, and the third of highly depleted uranium. The fourth division contained a bare aluminum foil so that any induced activity in the aluminum might be observed. The aluminum foils were accurately cut from very pure aluminum sheet (Reynold's Wrap) whose thickness was 0.0006". The standard irradiation time of two hours was made in all cases. Twenty minutes after the pile was scrammed, the counting was started on the aluminum foils. The counts lasted five minutes, with a two-minute interval between to change the foils. Each foil was counted in each of four counters. A previous determination had shown that both foils, the natural and the enriched, decayed during the counting interval according to the law $N = N_0 t^{-0.675}$. Using this relation, the second and third and fourth counting rates were corrected back to the time of the first counting rate. Then the total counts from all four counters for each of the foils were found. Since there was some U^{238} in the enriched foil and some U^{235} in the natural foil a correction was made.

The enriched uranium plate contains a fraction, A , of U^{235} and $(1 - A)$ of U^{238} . Its fission rate, f , is, then,

$$f_e = A f_{25} + (1 - A) f_{28} \quad (4.9)$$

Similarly, for a natural plate containing 0.7115% U^{235} ,

$$f_n = 0.007115 f_{25} + 0.992885 f_{28} \quad (4.10)$$

For the depleted plate we assume that

$$f_d = f_{28} \quad (4.11)$$

This is an error of less than 1% for any assembly where the U^{235}/U^{238} fission ratio is less than 1000. The counting rate N of each aluminum foil is proportional to the fission rate of its own uranium plate, or

$$N_e = K f_e \quad (4.12)$$

$$N_n = K f_n \quad (4.13)$$

$$N_d = K f_d \quad (4.14)$$

K is a constant of proportionality. We can immediately write

$$N_e = K [A f_{25} + (1 - A) f_{28}] \quad (4.15)$$

$$N_n = K (0.007115 f_{25} + 0.992885 f_{28}) \quad (4.16)$$

$$N_d = K (f_{28}) \quad (4.17)$$

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This will then give us two independent solutions for the U^{235}/U^{238} fission ratio: one from the ratio of the enriched to the natural, and one from the enriched to the depleted. Dividing (4.15) by (4.16), rearranging, and letting $\frac{N_e}{N_n} = R_{en}$, we have

$$\frac{f_{25}}{f_{28}} = \frac{0.992885 R_{en} - (1 - A)}{A - 0.007115 R_{en}} \quad (4.18)$$

Since R_{en} is usually much greater than 1, the $(1 - A)$ term in the numerator may be dropped, and the expression further simplified to

$$\frac{f_{25}}{f_{28}} = \frac{0.992885}{\frac{A}{R_{en}} - 0.007115} \quad (4.19)$$

A similar, but even simpler, operation on (4.15) and (4.17) gives the second ratio

$$\frac{f_{25}}{f_{28}} = \frac{N_e}{A N_d} = \frac{R_{ed}}{A} \quad (4.20)$$

If the foils are not identically the same size, there will be an error directly proportional to the difference in the areas of the foils. To eliminate this the uranium plates were cut on the same foil cutter. A piece of the aluminum foil was folded twice so that the four foils could be cut simultaneously.

The polish of the uranium plates is also a factor governing the accuracy. All plates were polished approximately the same length of time on new crocus cloth and were then thoroughly cleaned in acetone before the exposure.

The only assumption made in this experiment is that the fission particles from both isotopes of uranium decay at the same rate. A comparison of the results obtained by using the catcher technique and by using the miniature fission chambers showed the two methods agreed well within the statistical errors of the measurements. A second indication was obtained by placing the enriched and the depleted foils in a special fission chamber which held the two foils back to back in identical compartments. The two compartments had the same gas filling, so the efficiencies should have been the same. This fission chamber was placed in a flux where the ratio had previously been measured by the catcher technique, and the ratio was again measured. The two results were in excellent agreement.

The statistical errors in the measurement were quite large. The background was a fairly high percentage of the total counts, especially with that from the depleted foils. The total number of counts obtainable from the depleted foil was quite small. The statistical error due only to the counting of the foils was of the order of 10% when the fission rate was low (~ 20) and increased to about 25% when the ratio was the order of 100.

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V. EXPONENTIAL EXPERIMENTS

A. Theory of Buckling

The square root of the buckling, κ , is determined for a uniform homogeneous region where N groups of neutrons are being considered as the fundamental eigenvalue of the coupled set of equations

$$n_j = \frac{\nu_j + \sum_{i=1}^{j-1} n_i (\sigma_{in})_i x(i \rightarrow j) + q_{j-1}}{(\sigma_{in})_j + (\sigma_f)_j + (\sigma_c)_j + \kappa f\left(\frac{\kappa}{\sigma_j}\right) + q_j} \quad j = 1, \dots, N \quad (5.1)$$

The spatially independent numbers n_j are group fluxes for a fission rate of one neutron/cc/sec and are the components of the eigenvector corresponding to κ . Here $(\sigma_{in})_j$, $(\sigma_f)_j$, and $(\sigma_c)_j$ are, respectively, average macroscopic cross sections in cm^{-1} for inelastic scattering, fission, and radiative capture for group j . Their sum will be denoted as the reaction cross section, $(\sigma_r)_j$. The transport cross section σ_j is defined as the sum of the elastic transport cross section $[\sigma_s(1 - \bar{\mu})]$ and the reaction cross section. The ν_j are fission neutrons born in group j per fission. The term q_j is properly the slowing down density at the lower energy limit of group j but was usually taken as simply $n_j \left(\frac{\sigma_s(1 - \bar{\mu})}{\Delta u} \right)_j$, where the term in parenthesis is an effective elastic removal cross section based on the elastic scattering portion of the group average transport cross section. $x(i \rightarrow j)$ represents fractional inelastic scattering distribution from group i to group j . $f\left(\frac{\kappa}{\sigma_j}\right)$, for the asymptotic solution of the Boltzmann equation in the transport approximation, is given by $\frac{1}{\tan^{-1} \frac{\kappa}{\sigma_j}} - \frac{1}{\left(\frac{\kappa}{\sigma_j}\right)}$. For small $\frac{\kappa}{\sigma_j}$ this reduces to the diffusion theory expression, $\frac{\kappa^2}{3\sigma_j}$. The asymptotic solution was used in the calculations, but the correction to diffusion theory was found to be negligible for the Fast Exponential Experiment.

Considering that there is no contribution from inelastic scattering to group 1, and assuming that there is no contribution from elastic scattering to group 2, $n(1)$ and $n(2)$ may be expressed as

$$n(1) = \frac{\nu(1)}{\sigma_r(1) + \kappa f(x_1)} \quad (5.2)$$

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and

$$n(2) = \frac{\nu(2) + \sigma_{in}(1)n(1)}{\sigma_r(2) + \kappa f(x_2) + \left(\frac{3\sigma_s}{\Delta u}\right)_2} \quad (5.3)$$

κ is conveniently determined by source iteration: a trial value is selected, determining n_j for the highest energy group $j = 1$, the other n 's then being found in turn. The fission rate $\sum_j n_j \sigma_{fj}$ is then computed and the process repeated with a different value of κ until convergence is achieved.

The criterion for determining κ is that all the $n_j > 0$ and, because of the normalization of the n_j , that $\sum_{j=1}^N n_j \sigma_{fj} = 1$.

The calculated values of κ for the three different ratios of U^{238} to U^{235} are listed in Tables V, VI, and VII as different parameters are varied.

The cross sections used in making these calculations are given in Appendix A, along with a discussion of their reliability and the source of information. Since the available information is not as complete and as accurate as is desired, the effect of varying certain cross sections was calculated. Three different expressions were used for the inelastic spectrum of uranium. Three values of the quantity $(1 - \bar{\mu})$ for uranium (assumed to be the same for U^{235} and U^{238}) are listed. These changes naturally cause the average cross sections for various materials to change. The average cross sections for several interesting materials are also listed in the three tables. Tables VIII, IX and X contain the group fluxes normalized to 10,000 for each condition listed in Tables V, VI, and VII.

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Table V

RESULTS OF BUCKLING CALCULATIONS FOR ENRICHED EXPONENTIAL CORE

3.13:1 $U^{238}:U^{235}$ Atom Ratio

Run	$\frac{U^{238}}{U^{235}}$ atom ratio	Vol% Al	Vol% Fe	Vol% U	U 1- μ	U Incl. Spect.	Average Cross-Sections, Barns														
							$\sigma_c^{U^{238}}$	$\sigma_f^{U^{238}}$	κ cm ⁻¹	$\bar{\Sigma}_f$ cm ⁻¹	$\bar{\Sigma}_a$ cm ⁻¹	$\frac{(\nu-1)\bar{\Sigma}_f-\bar{\Sigma}_a}{\kappa^2}$	$\sigma_f^{U^{235}}$ I	$\sigma_f^{U^{235}}$ II	$\sigma_c^{U^{235}}$	$\sigma_f^{U^{238}}$	$\sigma_c^{U^{238}}$	σ_{Au} σ_c	σ_{np}^{237}	$\sigma_f^{Pu^{239}}$	
1	3.13	14.76	10.36	35.11	I	dE	I	I	0.0604	0.00723	0.00363	2.00	1.502	1.437	0.350	0.0865	0.1675	0.335	0.566	1.819	
2						EdE			0.0590	0.00684	0.00313	2.07	1.413	1.367	0.276	0.0843	0.1508	0.270	0.622	1.809	
3						E ^{1/2} _e -EdE			0.0600	0.00709	0.00346	2.01	1.471	1.410	0.326	0.0850	0.1620	0.309	0.562	1.809	
10	0	10.36	35.11		I	E ^{1/2} _e -EdE	I	I	0.0548	0.00705	0.00333	2.45	1.442	1.389	0.299	0.0903	0.1555	0.289	0.602	1.812	
15	0	0	35.11		I	E ^{1/2} _e -EdE	I	I	0.0504	0.00709	0.00305	3.01	1.421	1.374	0.280	0.1007	0.1495	0.272	0.647	1.817	
39	14.76	10.36	35.11		I	dE	II	I	0.0624	0.00730	0.00328	1.98	1.518	1.449	0.255	0.0866	0.1704	0.346	0.562	1.821	
38						EdE			0.0601	0.00688	0.00296	2.06	1.425	1.376	0.232	0.0835	0.1531	0.278	0.615	1.809	
37						E ^{1/2} _e -EdE			0.0617	0.00714	0.00318	2.00	1.484	1.419	0.247	0.0855	0.1642	0.318	0.559	1.811	
48	14.76	10.36	35.11	U^{235} I, U^{238} II		E ^{1/2} _e -EdE	I	I	0.0594	0.00708	0.00345	2.06									
32	14.76	10.36	35.11	III		E ^{1/2} _e -EdE	I	I	0.0621	0.00712	0.00350	1.88									
52	14.76	10.36	35.11	I		E ^{1/2} _e -EdE	I	II	0.0583	0.00684	0.00353	2.00	1.482	1.417	0.336	0.0822	0.1646	0.317	0.548	1.808	
24	29.51	10.36	35.11	I		E ^{1/2} _e -EdE	I	I	0.0647	0.00715	0.00368	1.70	1.500	1.431	0.351	0.0810	0.1683	0.329	0.530	1.808	
19	54.5 (Na)	10.36	35.11	I		E ^{1/2} _e -EdE	I	I	0.0637	0.00716	0.00361	1.78	1.497	1.431	0.348	0.0828	0.1680	0.329	0.525	1.809	

N.B.: σ_{in} for U^{235} and U^{238} Curve I (0.2-Mev threshold), Appendix A, for Tables V, VI, VII.

Table VI

RESULTS OF BUCKLING CALCULATIONS FOR ENRICHED EXPONENTIAL CORE

5.0-1 U²³⁵/U²³⁸ Atom Ratio

Run	U ²³⁵ /U ²³⁸ atom ratio	Vol% Al	Vol% Fe	Vol% U	U 1-1/2	U Inel. Spect.	σ _c ^{U²³⁵}	κ ⁻¹ cm ⁻¹	Σ _{f,1} cm ⁻¹	Σ _a cm ⁻¹	$\frac{(\nu-1)\sum f - \sum a}{\kappa^2}$	Average Cross-Sections, Barns							
												σ _f ^{U²³⁵} I	σ _f ^{U²³⁵} II	σ _c ^{U²³⁵}	σ _f ^{U²³⁵}	σ _c ^{U²³⁸}	σ _f ^{U²³⁸}	σ _c ^{Au}	σ _f ^{Nb²³⁷}
7	5.00	14.76	10.36	34.59	I	dE	I	I	0.00526	0.00370	1.97	1.563	1.482	0.404	0.0683	0.1823	0.381	0.479	1.812
8						EdE		0.0456	0.00494	0.00325	2.04	1.464	1.404	0.324	0.0655	0.1639	0.307	0.527	1.798
9						E ^{1/2} _e -dE		0.0462	0.00514	0.00354	1.98	1.529	1.452	0.379	0.0665	0.1762	0.351	0.473	1.801
11	0		10.36	34.59	I	dE	I	I	0.00521	0.00349	2.42	1.527	1.456	0.372	0.0723	0.1746	0.355	0.516	1.813
12						EdE		0.0416	0.00487	0.00297	2.53	1.418	1.371	0.283	0.0697	0.1544	0.277	0.582	1.800
13						E ^{1/2} _e -dE		0.0423	0.00508	0.00333	2.44	1.487	1.422	0.343	0.0706	0.1676	0.322	0.513	1.801

Table VII

RESULTS OF BUCKLING CALCULATIONS FOR ENRICHED EXPONENTIAL CORE

6.82-1 U²³⁵/U²³⁸ Atom Ratio

Run	U ²³⁵ /U ²³⁸ atom ratio	Vol% Al	Vol% Fe	Vol% U	U 1-1/2	U Inel. Spect.	$\sigma_c^{U^{235}}$	$\sigma_f^{U^{235}}$	κ^{-1}	$\bar{\Sigma}_{f,1}$ cm ⁻¹	$\bar{\Sigma}_{a,1}$ cm ⁻¹	$\frac{(\nu-1)\bar{\Sigma}_f - \bar{\Sigma}_a}{\kappa^2}$	Average Cross-Sections, Barns								
													$\sigma_f^{I,235}$	$\sigma_f^{II,235}$	$\sigma_c^{U^{238}}$	$\sigma_f^{U^{238}}$	σ_c^{Au}	$\sigma_f^{Nb^{237}}$	$\sigma_f^{Pu^{249}}$		
4	6.82	14.76	10.36	34.33	I	dE	-	I	-I	0.0364	-0.00420	0.00378	1.94	1.611	1.518	0.447	0.0573	0.1933	0.416	0.420	1.810
5						EdE				0.0363	0.00395	0.00332	2.01	1.507	1.434	0.362	0.0546	0.1738	0.337	0.465	1.793
6						E ^{1/2} _e -EdE				0.0364	0.00411	0.00362	1.95	1.575	1.484	0.419	0.0559	0.1865	0.384	0.416	1.798
14	0	0	10.36	34.33	I	E ^{1/2} _e -EdE	I	I	I	0.0337	0.00404	0.00337	2.41	1.523	1.448	0.374	0.0587	0.1764	0.348	0.452	1.795
16	0	0	0	34.33	I	E ^{1/2} _e -EdE	I	I	I	0.0316	0.00409	0.00315	3.00	1.490	1.423	0.344	0.0676	0.1684	0.323	0.500	1.798
40	14.76	10.36	10.36	34.33	I	dE	II	I	I	0.0388	0.00424	0.00350	1.92	1.628	1.531	0.292	0.0572	0.1965	0.429	0.416	1.813
41						EdE				0.0378	0.00397	0.00315	2.00	1.520	1.444	0.265	0.0544	0.1764	0.347	0.460	1.795
42	6.82	14.76	10.36	34.33	I	E ^{1/2} _e -EdE				0.0385	0.00414	0.00338	1.93	1.590	1.496	0.283	0.0557	0.1894	0.395	0.411	1.801
54	14.76	10.36	10.36	34.33	I	E ^{1/2} _e -EdE	I	II	I	0.0342	0.00391	0.00369	1.94	1.589	1.494	0.432	0.0537	0.1895	0.393	0.403	1.798
25	29.51	10.36	10.36	34.33	I	E ^{1/2} _e -EdE	I	I	I	0.0385	0.00418	0.00388	1.64	1.625	1.523	0.462	0.0533	0.1966	0.421	0.386	1.803
23	54.5(Nu)	10.36	10.36	34.33	I	E ^{1/2} _e -EdE	I	I	I	0.0382	0.00419	0.00361	1.72	1.619	1.522	0.455	0.0552	0.1955	0.419	0.384	1.804

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Table VIII

GROUP FLUXES FROM BUCKLING CALCULATIONS3.13:1 U²³⁸:U²³⁵

Run	1	2	3	10	15	39	38
Group/ κ *	0.0600	0.0600	0.0600	0.0550	0.0501	0.0625	0.0600
1	1545	1489	1521	1612	1800	1549	1476
2	774	921	737	835	910	770	911
3	311	380	303	321	344	308	375
4	325	397	322	374	402	321	392
5	452	547	457	468	493	445	539
6	557	663	573	607	578	544	652
7	306	363	319	334	337	298	357
8	330	389	348	366	368	321	382
9	355	414	378	391	401	345	405
10	373	427	400	428	444	362	417
11	490	547	528	508	490	473	534
12	575	622	621	606	545	552	605
13	626	648	675	589	555	602	635
14	537	521	573	616	591	523	517
15	634	563	663	674	622	635	574
16	752	580	748	594	528	785	622
17	625	373	554	444	398	680	421
18	252	107	188	153	130	284	127
19	118	33	68	55	44	136	40
20	51	7	16	16	12	57	9
Run	37	48	32	52	24	19	
Group/ κ *	0.0625			0.0575	0.0650	0.0625	
1	1530			1471	1453	1488	
2	736			717	667	662	
3	301			296	286	229	
4	319			315	290	284	
5	451			448	437	319	
6	562			563	543	518	
7	312			314	304	359	
8	339			343	331	466	
9	368			373	362	431	
10	389			395	379	417	
11	510			523	525	481	
12	596			618	620	619	
13	650			677	726	635	
14	559			579	559	554	
15	664			679	652	819	
16	782			787	850	764	
17	607			599	669	588	
18	215			204	234	233	
19	80			73	88	95	
20	19			15	17	27	

*These are values used in flux calculations, not critical values.

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Table IX

GROUP FLUXES FROM BUCKLING CALCULATIONS5.00:1 $U^{238}:U^{235}$

Run	7	8	9	11	12	13
<u>Group/κ</u>	<u>0.0365</u>	<u>0.0460</u>	<u>0.0460</u>	<u>0.0425</u>	<u>0.0425</u>	<u>0.0425</u>
1	1216	1151	1187	1282	1218	1255
2	646	763	610	742	894	699
3	268	326	259	286	363	278
4	281	344	277	332	423	329
5	402	489	404	416	524	422
6	510	614	522	549	682	569
7	284	343	295	302	376	319
8	311	373	326	332	411	355
9	339	403	359	356	433	384
10	359	421	384	392	466	426
11	486	559	523	477	549	521
12	594	665	642	595	660	652
13	680	734	736	597	628	652
14	589	601	634	653	636	704
15	723	682	765	780	683	819
16	934	774	946	766	555	752
17	814	530	743	662	351	567
18	333	159	260	277	101	196
19	159	50	97	131	31	70
20	63	9	21	63	8	19

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Table X

GROUP FLUXES FROM BUCKLING CALCULATIONS6.82:1 $U^{238}:U^{235}$

Run	4	5	6	14	16	40	41
Group/ κ	0.0365	0.0365	0.0365	0.0335	0.0325	0.0390	0.0375
1	1018	957	994	1040	1199	1018	953
2	561	661	529	606	688	557	655
3	237	288	228	246	276	234	285
4	250	307	246	295	333	247	303
5	364	444	365	384	427	358	438
6	470	569	480	533	519	461	560
7	265	322	274	303	314	259	316
8	292	354	306	341	355	285	347
9	321	386	340	371	397	313	378
10	342	407	365	415	457	333	397
11	473	552	508	518	526	458	538
12	593	676	641	671	617	572	657
13	703	775	760	686	655	676	754
14	615	646	663	760	747	596	632
15	776	758	825	927	871	766	756
16	1067	923	1088	883	764	1085	953
17	972	673	901	672	579	1028	729
18	404	211	325	232	188	444	241
19	196	69	126	83	63	221	84
20	72	12	25	22	16	80	14

Run	42	54	25	23
Group/ κ	0.0385	0.0340	0.0380	0.0383
1	993	956	953	989
2	525	511	473	477
3	226	221	211	167
4	243	238	216	211
5	359	355	339	239
6	471	469	440	403
7	269	268	252	291
8	299	299	280	401
9	331	333	313	384
10	356	358	332	379
11	493	499	482	451
12	619	632	607	617
13	734	755	783	675
14	645	662	616	605
15	816	833	763	1013
16	1112	1127	1177	1086
17	960	960	1095	937
18	362	352	435	413
19	146	136	193	193
20	29	25	33	57

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B. Experimental Measurement of Buckling

For the boundary conditions of the experiment, the spatial distribution of the fundamental mode is

$$\phi = X(x) Y(y) Z(z), \quad (5.4)$$

where

$$\left. \begin{aligned} X(x) &= A_1 (e^{-\kappa_x x} + \gamma e^{\kappa_x x}) \\ Y(y) &= A_2 \cos \kappa_y (y - y_0) \\ Z(z) &= A_3 \cos \kappa_z (z - z_0) \end{aligned} \right\} \quad (5.5)$$

and

$$B = \sqrt{\kappa_y^2 + \kappa_z^2 - \kappa_x^2} \quad (5.6)$$

By accurately measuring the flux distribution in the assembly, experimental values for all the parameters, particularly the κ 's, in these equations may be determined.

For each different combination of enrichment, reflector and coolant, a complete set of flux traverses were made. Each set consisted of at least one traverse in every direction with each of the different types of fission chambers.

Figure 31 shows the geometrical arrangement for the three different uranium mixtures. Figure 6 (p. 16) is a photograph of the completed assembly with the 5:1 uranium mixture and natural uranium reflector. The three fission chambers and drives are in place. These arrangements remained essentially the same for all the different reflectors and coolant combinations.

A minimum of twelve flux traverses were made for each of the 18 different assemblies. Of all these (more than two hundred), three are shown in Figs. 32, 33, and 34 as examples of X, Y, and Z traverses. The fitted curve as determined by the AVIDAC is shown in a solid line. The experimental points shown have been normalized to an arbitrary monitor count.

The normalization to the monitor count was made for every traverse. The ratios between the normalized counting rates for corresponding points taken with U^{235} and U^{238} chambers were then found. In the central region of the core, this ratio is flat within 1%, as shown in Fig. 35, indicating energy equilibrium.

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3:1 URANIUM MIXTURE.

5:1 URANIUM MIXTURE.

7:1 URANIUM MIXTURE.

FIGURE 31
EXPONENTIAL GEOMETRIES

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FIG. 35
FISSION RATIOS IN THE X DIRECTION
 $\circ = U^{235}/U^{238}$ $+ = U^{235}/Np$

The ratios are not a very sensitive test of energy equilibrium, but are the best indicators available at the present time. Further investigations are being conducted to see if equilibrium is achieved; and, if not, what is the effect on the results.

Appendix B contains the κ 's as determined for each detector and direction as a function of enrichment, reflector and coolant. These κ 's were determined from the experimental data by least-squares curve fitting as described in Appendix C.

The experimental values of κ_x^2 , κ_y^2 , κ_z^2 and B, where $B = \sqrt{\kappa_y^2 + \kappa_z^2 - \kappa_x^2}$, are given in Tables XI, XII and XIII with the applicable theoretical values, so a comparison may be made. The experimental values given here represent the weighted average of all the determinations of κ for each mixture-coolant combination. When possible, averaged κ 's for each direction are given, and the mean buckling computed from these averages.

Reflector savings, discussed in Section C of this chapter, are also listed in Tables XI, XII, and XIII.

The variation in B with different enrichments (i.e., mixtures) of uranium is shown graphically in Fig. 36. The effect on the buckling of varying the aluminum concentration with the 5:1 uranium mixture is shown in Fig. 37. The point for the aluminum equivalent of sodium was found by calculating the per cent of aluminum necessary to give the same macroscopic cross section as the 55% sodium. The U^{235}/U^{238} fission ratios are also given. These are discussed in Section D of this chapter.

C. Reflector Savings

Reflector savings calculations were made using only nine energy groups, obtained by collapsing the twenty groups given in Appendix A. The cross sections for Fe and Al were weighted according to $\left(\frac{1}{\sigma_s}\right)^2$, which assumes constant slowing down density in the reflector.

Because of the separability of the dependence of the flux on the x, y, and z coordinates in the equilibrium spectrum, the effect of the reflector on the enriched core can be obtained by a one-dimensional calculation in the z direction. This calculation was performed on the NYU UNIVAC by multigroup diffusion theory, using a code prepared by members of the ANL staff. The coupled equations to be solved contained the same terms as those in Eq. (5.1), Section V A. The flux n_j is now a function of z, and the cross sections are, of course, different in core and reflector. The leakage term $\kappa f \left(\frac{\kappa}{\sigma_j}\right)$ becomes $\frac{\nabla^2 n_j(z)}{3 \sigma_j n_j(z)}$, which, because of the separability of

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Table XI

RESULTS OF MEASUREMENTSU²³⁵/U²³⁸ Atom Ratio = 1:3.13 Al = 14%

Reflector	κ_y^2 (cm ⁻²)	κ_x^2 (cm ⁻²)	κ_z^2 (cm ⁻²)	$\kappa_z^2 - \kappa_x^2$ (cm ⁻²)	B (cm ⁻¹)	From κ_z , Zo Reflector Savings (cm)	From $\bar{B}$, $\bar{\kappa}_y$, κ_x Reflector Savings (cm)	Reflector Savings (Theoretical) (cm)
U	0.00436 ± 0.00007	0.00272 ± 0.00009	0.00132 ± 0.00005	-0.00140 ± 0.00010	0.0545 ± 0.0011	22.25	27.59	18.4
Pb	0.00436 ± 0.00007	0.00317 ± 0.00007	0.00135 ± 0.00003	-0.00182 ± 0.00005	0.0504 ± 0.0009	19.77	19.41	18.4
Fe	0.00436 ± 0.00007	0.00306 ± 0.00007	0.00136 ± 0.00002	-0.00170 ± 0.00007	0.0516 ± 0.0009	29.79	28.85	19.3

$$\bar{\kappa}_y^2 = 0.00436 \pm 0.00007 \text{ cm}^{-2}$$

$$\overline{\kappa_z^2 - \kappa_x^2} = -0.00173 \pm 0.00008 \text{ cm}^{-2}$$

$$\bar{B}^2 = 0.00263 \pm 0.00011 \text{ cm}^{-2}$$

$$\bar{B} = 0.0512 \pm 0.0011 \text{ cm}^{-1} \text{ (Experiment)}$$

$$= 0.006005 \text{ (Theory)}$$

*Average of all Y traverse made with 1:3 mix.

Table XII a

RESULTS OF MEASUREMENTS

 U^{235}/U^{238} Atom Ratio = 1/5

Al = 14%

Reflector	κ_y^2 (cm ⁻²)	κ_x^2 (cm ⁻²)	κ_z^2 (cm ⁻²)	$\kappa_z^2 - \kappa_x^2$ (cm ⁻²)	B (cm ⁻¹)	Reflector Savings from κ_z , Z_0 (cm)	Reflector Savings from B, κ_y , κ_x (cm)	Reflector Savings (Theoretical) (cm)
U	0.00279 ±0.00005	0.00216 ±0.00003	0.001016 ±0.000007	-0.00114 ±0.00003	0.0406 ±0.0007	27.72 ± 2.0	30.52 ± 2.0	18.8
Pb	0.00270 ±0.00003	0.00208 ±0.00001	0.00099 ±0.00001	-0.00109 ±0.00001	0.0401 ±0.0007	34.18 ± 3.0	33.57 ± 3.0	22.8
Al	0.00266 ±0.00002	0.00221 ±0.00003	0.001043 ±0.000003	-0.00117 ±0.00003	0.0386 ±0.0005	30.40 ± 2.0	27.53 ± 2.0	20.1
Fe	0.00268 ±0.00004	0.00208 ±0.00004	0.00103 ±0.00001	-0.00105 ±0.00004	0.0404 ±0.0007	29.91 ± 3.0	30.74 ± 3.0	22.6
*Hetro #1	0.00276 ±0.00001	0.00208 ±0.00005	0.00101 ±0.00003	-0.00107 ±0.00006	0.0411 ±0.0007	28.27 ± 4.0	29.35 ± 4.0	-
*Hetro #2	0.00260 ±0.00003	0.00195 ±0.00005	0.00098 ±0.00001	-0.00097 ±0.00005	0.0404 ±0.0007	33.72 ± 2.0	33.88 ± 2.0	-

$$\overline{\kappa_y^2} = 0.00273 \pm 0.00002 \text{ cm}^{-2}$$

$$(\overline{\kappa_z^2} - \overline{\kappa_x^2}) = -0.00110 \pm 0.00001 \text{ cm}^{-2}$$

$$B^2 = 0.00163 \pm 0.00002 \text{ cm}^{-2}$$

$$B = 0.0404 \pm 0.0002 \text{ cm}^{-1} \text{ (Experimental)}$$

$$= 0.0462 \text{ (Theory)}$$

*See Section C of this Chapter.

Table XII b

RESULTS OF MEASUREMENTS

 U^{235}/U^{238} Atom Ratio = 1/5

Reflector Fe

Coolant	κ_y^2 (cm ⁻²)	κ_x^2 (cm ⁻²)	κ_z^2 (cm ⁻²)	$\kappa_z^2 - \kappa_x^2$ (cm ⁻²)	B (cm ⁻¹)	Reflector Savings from κ_z , Z_0 (cm)
0% Al	0.00255 ±0.00004	0.00190 ±0.00001	0.00082 ±0.00003	-0.00108 ±0.00003	0.0383 ±0.0007	42.97 ± 4.0
14% Al	0.00268 ±0.00004	0.00208 ±0.00004	0.00103 ±0.00001	-0.00105 ±0.00004	0.0404 ±0.0007	29.91 ± 3.0
28% Al	0.00298 ±0.00004	0.00190 ±0.00005	0.000987 ±0.00004	-0.00091 ±0.00005	0.0455 ±0.0006	31.40 ± 4.0
55% Na	0.00284 ±0.00003	0.00213 ±0.00003	0.00112 ±0.00002	-0.00101 ±0.00004	0.0428 ±0.0006	27.32 ± 3.0

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Table XIII

RESULTS OF MEASUREMENTS U^{235}/U^{238} Atom Ratio 1/6.82 Al = 14%

Reflector	κ_y^2 (cm ⁻²)	κ_x^2 (cm ⁻²)	κ_z^2 (cm ⁻²)	$\kappa_z^2 - \kappa_x^2$ (cm ⁻²)	B (cm ⁻¹)	Reflector Savings from κ_z , Zo (cm)	Reflector Savings from $\bar{B}$, $\bar{\kappa}_y$, κ_x (cm)	Reflector Savings (Theoretical) (cm)
Fe	0.00180 ±0.00002	0.00186 ±0.00010	0.00104 ±0.00001	-0.00082 ±0.00010	0.0313 ±0.0016	30.26 ± 1.0 cm	28.58 ± 1.0 cm	24.5
Pb	0.00181 ±0.00002	0.00174 ±0.00004	0.00096 ±0.00002	-0.00078 ±0.00004	0.0321 ±0.0007	32.73 ± 1.0 cm	32.23 ± 1.0 cm	26.1
Al	0.00171 ±0.00003	0.00177 ±0.00002	0.00102 ±0.00002	-0.00075 ±0.00003	0.0310 ±0.0007	29.04 ± 1.0 cm	29.40 ± 1.0 cm	22.3
U	0.00179 ±0.00002	0.00189 ±0.00002	0.00113 ±0.00002	-0.00076 ±0.00003	0.0321 ±0.0006	28.11 ± 1.0 cm	27.54 ± 1.0 cm	18.3

$$\bar{\kappa}_y^2 = 0.00178 \pm 0.00002 \text{ cm}^{-2}$$

$$\kappa_z^2 - \kappa_x^2 = -0.00076 \pm 0.00001 \text{ cm}^{-2}$$

$$\bar{B} = 0.0320 \pm 0.0003 \text{ cm}^{-1} \text{ (Experimental)}$$

$$= 0.0364 \text{ (Theory)}$$

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FIG. 36

BUCKLINGS AND U^{235}/U^{238} FISSION RATIOS VS ENRICHMENT

FIG. 37

BUCKLINGS AND U^{235}/U^{238} FISSION RATIOS VS Al CONCENTRATES

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solutions, can be written as $\frac{-(\kappa_x^2 + \kappa_y^2)}{3\sigma_j} + \frac{\nabla_z^2 n(Z)}{3\sigma_j n(Z)}$. The quantity $(\kappa_x^2 + \kappa_y^2)$

is constant for all groups and both regions, and so can be determined as an eigenvalue in the Z calculation. The reflector savings is defined as

$\left(\frac{\pi}{2\kappa_z}\right) - L$, where L is the distance from the cosine maximum in the core to the core-reflector interface.

Fitting the experimental curves gives Z_0 , the point of the maximum of the cosine of the Z-traverse and κ_z , and, consequently, the reflector savings as just defined. An alternate way of obtaining reflector savings may also be used, based on the assumption that the extrapolation distance of the bare upper surface of the experiment is identical with the extrapolation distance in the Y direction. Then, knowing κ_x , κ_y and B, κ_z may be determined, and Z_0 located at $\frac{\pi}{2\kappa_z}$ from the extrapolated end point. While this method is in general less precise than the direct determination, it serves as a useful check.

Tables XI, XII, and XIII, which also give the buckling data, list the theoretical and the two different experimental values of the savings. A graphical comparison is made in Figs. 38 and 39. The theoretical values must be interpreted in light of the reliability of the cross sections used, as given in Appendix A.

The elemental reflectors (U, Fe, Pb and Al) were composed of machined bricks or bars which fitted closely to form a solid reflector. Slots were milled into certain pieces so that the Z channel could penetrate the reflector. The two heterogeneous reflectors were formed by loading spare fuel cans with the appropriate number of plates of iron, aluminum and natural uranium. Their compositions were:

	<u>Fe</u>	<u>U</u>	<u>Al</u>	<u>Void</u>
Hetro # 1	10.6%	50%	10%	29.4%
Hetro # 2	55%	23%	5.4%	16.6%

As is seen in Table XII A, the reflector savings increased as the void concentration decreased. Presumably, then, an array of U rods, Na-cooled in an iron block, would make a better reflector for a fast reactor than a pure U-Na array of the same U:Na composition.

D. Fission Ratios

The U^{235}/U^{238} fission ratios were measured, using the fission chamber method explained in Chapter IV, for every mixture-coolant combination. After the development of the catcher foil technique, this method

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FIG. 38
REFLECTOR SAVINGS
U AND Pb

FIG. 39
REFLECTOR SAVINGS
Al AND Fe

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was also used. The fission ratios, R , are listed in Table XIV; three values are given: the theoretical and two experimental values. The theoretical values were obtained by using the 20-group cross sections listed in Appendix A and the group fluxes in Section A of this chapter, as

$$R = \frac{\sum_{i=1}^{20} n_i (\sigma_{f_{25}})_i}{\sum_{i=1}^{20} n_i (\sigma_{f_{238}})_i} \quad (5.7)$$

Table XIV

U²³⁵/U²³⁸ FISSION RATIOS

Mixture	Coolant	Ratios as determined by		
		Theory	Fission Chambers	Catcher Foils
3:1	14% Al	17.3	20.3	-
5:1	0% Al	21.0	25	25.5
	14% Al	23.0	26	-
	28% Al	24.4	28	-
	55% Na	24.5	30	27.9
7:1	14% Al	28.0	34	34.2

The U²³⁵/U²³⁸ fission ratio in a fission spectrum was measured to examine the discrepancy between the experimental and theoretical values of the fission ratios. A fission plate was set up in a thermal column of CP-5 to produce a fission spectrum. The U²³⁵ and U²³⁸ chambers were properly shielded from direct thermal neutrons by cadmium covers. Both chambers could be exposed simultaneously at adjacent points, where the flux should have been identical. The chambers were interchanged, nevertheless, to make sure this was true. Thermal calibration was obtained by exposure to the direct thermal flux. Because of the proximity of massive objects, like the floor and the pile shielding, some scattered neutrons were present which would produce an apparently higher ratio. The value measured for R in a fission spectrum was 5.0 ± 0.35 .

Using the standard values of σ_f^{25} ,⁶ σ_f^{28} ,⁶ and $\phi(E)$,⁷ one can integrate to obtain the fission ratio. Such a calculation gave a value of 4.2. This ratio has been measured directly by Leachman⁸ to be 4.0 ± 0.4 .

If one accepts the fission spectrum measurement as a calibration, then the fission chamber results of Table XIV ought to be diminished by 16%, bringing them closely into line with theory. However, because of the scattered neutron background in the fission spectrum measurement, this treatment is not believed to be realistic.

E. Control Experiments

A pile oscillator was built to test various materials in the spectrum of the Fast Exponential assembly for use as control rods. The oscillator consisted of a motor and supports which would drive a sample to the center of the assembly in one second, hold it there for nine seconds, withdraw it to the edge in one second and hold it withdrawn for nine seconds. This cycle would then repeat. When the sample was inserted, it formed a sandwich with a miniature fission chamber. The sample shielded approximately 3/4 of the total solid angle around the counter. A counting rate meter was connected to a recorder to indicate the variation in counting rate. Two samples, Lucite and tungsten, which were believed to give the greatest variation in intensity, were used. The samples were in two pieces, each 1" x 3" x 1/4". No detectable difference in counting rate was observed with either sample, using either a U²³⁵ or a U²³⁸ chamber. It can be concluded that either the samples used were too small or that the effects of the materials in the high energy flux were negligible.

A second study, in which slabs of various materials were used to split the exponential core in the Y - Z plane, was performed. A traverse along the X direction should show a lower flux level beyond the absorbing plates if sufficient absorber is present to give a detectable difference. The following slabs were used:

0.75" Boral sheet
0.562" B₄C in plastic

0.875" Lucite
1.125" Air gap

⁶Hughes, D. and Harvey, J., "Neutron Cross Sections," Report BNL-325 Brookhaven National Laboratory, Upton, New York, (July 1, 1955).

⁷Nereson, N., "Fission Neutron Spectrum of U²³⁵," Phys. Rev. 85, 600, (1952).

⁸Leachman, R. B. 25/28 Fission Ratio, Private communication, Los Alamos, 1954.

As a measure of the worth of control materials, the ratios of the flux at two positions were plotted for both the U^{235} and U^{238} chambers. These positions were chosen as 16 and 40 cm back from the front face of the exponential assembly. The results are summarized in Table XV.

Table XV

CONTROL SLAB EXPERIMENTAL RESULTS

Slab	U^{235} ratio (16cm/40cm)	U^{238} ratio (16cm/40cm)
0.75" Boral	3.50	3.38
B ₄ C & Plastic	3.48	3.23
Lucite	2.08	1.58
Air gap	3.18	3.12
No slab	3.51	3.23

It may be seen from Table XV that no ratio is appreciably higher than that obtained from a core with no material inserted. In fact, the air gap and Lucite tend to decrease the ratio: the Lucite, because it adds to the reactivity of the assembly, as would be expected; and the air gap, because it replaces alternating material.

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VI. EQUILIBRIUM NEUTRONS IN NATURAL URANIUM

The fast assembly and pedestal were replaced by a cube of natural uranium, 24" on a side, in order to study the equilibrium distribution of neutrons in this material. Figure 40 is a photograph of the "Snell block," as it is called. By the same methods as previously described, the diffusion length L , defined as $\frac{1}{iB}$ in the notation of the previous chapter, was found to be 10.0 ± 0.2 cm. This compares with the earlier value of 9.6 cm found by Feld, Slotin et al.⁹ If the diffusion length be calculated, using cross sections consistent with those in Appendix A except for the indicated changes in inelastic scattering threshold and inelastic spectrum of U^{238} , the following values are found:

Table XVI

EQUILIBRIUM RELAXATION LENGTH AND FISSION RATIO IN NATURAL U

<u>Inelastic Scattering Threshold, Mev</u>	<u>Inelastic Spectrum</u>	<u>L, cm.</u>	<u>U^{235}/U^{238} Fission Ratio</u>
0.2	$E^{1/2} e^{-E} dE$	8.80	225
0.2	dE	8.60	216
0.2	$E dE$	9.30	221
0.4	$E^{1/2} e^{-E} dE$	9.76	242
Experimental		10.0	363

It should be borne in mind that uncertainties in the capture and transport cross sections of U^{238} make inferences about inelastic scattering properties based on the diffusion length alone rather unreliable. A more complete list of theoretical results is given in Table XVII where the effect of variations of the inelastic, $(1 - \bar{\mu})$, and $\sigma_f U^{235}$ are shown. The average cross sections for various materials are also given.

⁹Slotin, L., Brolley, J. E., Feld, B. T., Scalettar, R., Stewart, R. B., "Neutron Multiplication in a Mass of Uranium Metal," CF-1627, (July 20, 1944).

FIG. 40 - NATURAL URANIUM EXPERIMENT ("SNELL BLOCK")

Table XVII

RESULTS OF BUCKLING CALCULATIONS FOR SNELL EXPERIMENT

Run	$1 - \bar{\mu}$ of U	U Incl. Spect.	$\sigma_{in}^{U^{238}}$	$\sigma_f^{U^{235}}$	$L = \frac{1}{K}$, cm	$\bar{\Sigma}_f$, cm	$\bar{\Sigma}_a$, cm	$D = \frac{(\nu - 1) \bar{\Sigma}_f - \bar{\Sigma}_a}{K^2}$	$\frac{\sigma_f^{U^{235}}}{\sigma_f}$, I	$\frac{\sigma_f^{U^{235}}}{\sigma_f}$, II	$\frac{\sigma_c^{U^{235}}}{\sigma_c}$	$\frac{\sigma_f^{U^{238}}}{\sigma_f}$	$\frac{\sigma_c^{U^{238}}}{\sigma_c}$	$\frac{\sigma_c^{Au}}{\sigma_c}$	$\frac{\sigma_f^{Np^{237}}}{\sigma_f}$	$\frac{\sigma_f^{Pu^{239}}}{\sigma_f}$
30	I	dE	I	I	8.60	0.001027	0.01238	0.799	1.915	1.719	0.731	0.00793	0.2558	0.609	0.0991	1.792
29		EdE			9.30	0.000954	0.01121	0.845	1.787	1.619	0.625	0.00731	0.2317	0.510	0.1238	1.766
31		E $\frac{1}{2}$ e ⁻ E _d E			8.80	0.000991	0.01193	0.809	1.869	1.677	0.698	0.00747	0.2468	0.566	0.0949	1.777
46	I	E $\frac{1}{2}$ e ⁻ E _d E	II	I	9.76	0.000885	0.01051	0.861	1.704	1.570	0.537	0.00647	0.2177	0.455	0.1001	1.748
47	II	E $\frac{1}{2}$ e ⁻ E _d E	I	I	9.13	0.000978	0.01183	0.863								
36	III	E $\frac{1}{2}$ e ⁻ E _d E	I	I	8.12	0.001026	0.01212	0.699								
55	I	E $\frac{1}{2}$ e ⁻ E _d E	II	II	9.63	0.000820	0.01060	0.870	1.714	1.577	0.547	0.00596	0.2195	0.461	0.0941	1.748

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Calculated values for the ratio of U^{235}/U^{238} fissions per atom are also given in Table XV, based on the revised U^{235} fission cross sections. The cross sections listed in Appendix A give ratios about 10% higher than those of Table XV. The tabulated values of this ratio are in excellent agreement with a value of 220 obtained by H. Paxton, et al.,¹⁰ of LASL in a cylinder 22" in diameter and about 36" high. Radiochemical techniques were used to obtain this value. The value measured by the authors is 363 ± 40 , using both the miniature fission chambers and the catcher foils.

The difference in the two results may be explained if equilibrium is not obtained in the smaller assembly. To test this, the fission ratio was measured for blocks whose height and length remained 24", while the thickness varied from 10" to 24". From the graph of the results, Fig. 41, it may be seen that equilibrium is not achieved in any assembly less than a 24" cube, if indeed it is achieved there. A few samples, not shown on Fig. 41, were analyzed radiochemically by R. Buchanan of the ANL Chemistry Division. While the errors were large, these results tended to agree with the Paxton ratio of 220.

An inelastic removal cross section for U^{238} from the 1.4 - Mev group of about 3 barns would be needed to agree with the higher ratio values, both in the natural uranium and in the enriched assemblies.

¹⁰ Paxton, H. C., et al., "Fission Ratios in Natural Uranium," Private Communication.

FIGURE 41
EFFECT OF URANIUM BLOCK SIZE ON FISSION RATIO

APPENDIX A

A Set of 20-Group Cross Sections

A set of 20-group cross sections for U^{235} , U^{238} , Fe, Na and Al* has been constructed. These cross sections have been used in calculations of the material buckling and the energy dependence of the equilibrium flux for comparison with and analysis of the results of the Argonne Exponential Experiments.

In the course of adopting these cross sections, the pertinent experimental and theoretical cross section data were examined. However, the provisional character of many of the values listed is to be emphasized. In the cases where direct experimental information is not available inferences from integral experiments and/or theoretical arguments were used; in some cases, sheer guesses enabled completion of the list.

Where there is considerable uncertainty, alternative sets of cross sections (labelled I, II, etc.) have been tried.

We regard as primary the following quantities: σ_T total, σ_f fission, σ_{in} total inelastic, $\chi(i \rightarrow j)$ inelastic transfer from group i to group j , σ_c capture cross section; and also $1 - \bar{\mu}$, where $\bar{\mu}$ is the average value of the cosine of the angle for elastic scattering. From these the following are derived:

$$\text{Reaction cross section} \quad \sigma_R = \sigma_f + \sigma_{in} + \sigma_c \quad (A.1)$$

$$\text{Elastic scattering} \quad \sigma_s = \sigma_T - \sigma_R \quad (A.2)$$

$$\text{Transport} \quad \sigma_{tr} = \sigma_s (1 - \bar{\mu}) + \sigma_R \quad (A.3)$$

The quantities which actually enter the multigroup calculations are σ_f , σ_{in} , $\chi(i \rightarrow j)$, σ_R , $\sigma_s (1 - \bar{\mu})$ and σ_{tr} .

We have listed the derived quantities for the light elements (Na, Fe and Al) for which we give only one set of primary cross sections. For the U isotopes there are so many possible combinations of the assumed cross sections that we do not list the derived cross sections. (They are computed from formulae (A.1), (A.2), (A.3) when a particular combination has been specified.)

*We also list fission cross sections for Pu^{249} and Np^{237} and n- γ cross sections for Au.

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Table A.I

FISSION SPECTRUM AND $(1 - \mu)$ FOR URANIUM

Group Number	Energy Interval (Mev)		Fission Spectrum $\nu(j)$	$1 - \bar{\mu}^*$ for U^{235} and U^{238}		
				I	II	III
1	1.4	- ∞	1.4056	.450	.450	.450
2	1.0	- 1.4	.3289	.660	.660	.660
3	.9	- 1.0	.0878	.680	.671	.700
4	.8	- .9	.0886	.685	.672	.740
5	.7	- .8	.0893	.695	.673	.780
6	.6	- .7	.0890	.710	.673	.820
7	.55	- .6	.0440	.720	.673	.830
8	.50	- .55	.0436	.725	.674	.860
9	.45	- .50	.0429	.735	.675	.890
10	.40	- .45	.0419	.745	.679	.910
11	.35	- .40	.0407	.755	.683	.930
12	.30	- .35	.0392	.770	.690	.950
13	.25	- .30	.0377	.785	.700	.970
14	.20	- .25	.0350	.795	.720	.980
15	.15	- .20	.0322	.820	.745	.990
16	.10	- .15	.0286	.845	.800	1.000
17	.050	- .10	.0227	.890	.865	1.000
18	.025	- .050	.0082	.940	.940	1.000
19	.010	- .025	.0031	.970	.970	1.000
20	.000	- .010	.0010	1.000	1.000	1.000

$\nu(j)$ is obtained by integration of $\sinh \sqrt{2E} e^{-E(7)^{**}}$ normalized to 2.51.

* $1 - \bar{\mu}$ was assumed to be the same for U^{235} and U^{238} . The data available at the time were measurements for the U^{238} fission group (effective energy about 2.8 Mev)¹¹ and the value of Walt and Barschall¹² of 0.67 at 1 Mev. Lower energy values of $1 - \bar{\mu}$ were varied for test purposes. Information that has recently become available^{12,13} indicates that the correct behavior is approximately midway between that of Curve I and of Curve III.

**See p. V. 30

¹¹Jurney, E. and Zabel, C. Unpublished measurements made at Los Alamos using U^{238} fission detector. Neutrons inelastically scattered to energies above the U^{238} fission threshold are included in elastic scattering.

¹²Walt, M., "Angular Distributions and Non-Elastic Neutrons Scattering," International Conference on the Peaceful Uses of Atomic Energy, Paper 588, August, 1955.

¹³Langsdorf, A., and Lane, R. O., "Angular Distribution of Moderate Energy Neutrons," Bull. Am. Phys. Soc. 30, No. 3, Paper F3 (1955), and private communication.

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Table A.II

CROSS SECTIONS FOR U^{238}

Group Number	Energy Interval (Mev)		$\sigma_T(6)$	$\sigma_f(6)$	σ_{in}^*		$\sigma_c(6)**$
					I	II	
1	1.4	- ∞	7.00	0.532	2.10	2.10	.050
2	1.0	- 1.4	6.83	0.046	1.50	1.50	.090
3	.9	- 1.0	6.92	0.012	1.27	1.27	.105
4	.8	- .9	7.08	0.006	1.11	1.11	.113
5	.7	- .8	7.25	0.002	.920	0.820	.123
6	.6	- .7	7.58	<div><div></div><div>Zero</div><div></div></div>	.820	0.670	.137
7	.55	- .6	7.75		.688	0.460	.140
8	.50	- .55	7.92		.611	0.320	.145
9	.45	- .50	8.25		.538	0.200	.150
10	.40	- .45	8.50		.465	0.100	.157
11	.35	- .40	8.83		.380	<div><div></div><div>Zero</div><div></div></div>	.165
12	.30	- .35	9.17		.296		.172
13	.25	- .30	9.58		.223		.185
14	.20	- .25	10.08		.150		.200
15	.15	- .20	10.75		<div><div></div><div>Zero</div><div></div></div>		.220
16	.10	- .15	11.50				.250
17	.050	- .10	12.42				.300
18	.025	- .050	13.50				.420
19	.010	- .025	14.67				.530
20	.000	- .010	15.50				.700

*Values were assigned to be consistent with Carter and Beyster¹⁴ and with the values of Engle and Hansen¹⁵ at the high energy end. Set II was assigned a higher threshold than I for test purposes.

**See p. 81.

¹⁴Bethe, H. A., Beyster, J. R., and Carter, R. E., "Inelastic Collision Cross Section Measurements for Fission Spectrum Neutrons," (unpublished).

¹⁵Engle, L., and Hansen, G. E., Los Alamos memorandum W-2-577, (unpublished).

Table A. III

CROSS SECTIONS FOR U²³⁵

Group Number	Energy Interval		$\sigma_T(6)$	$\sigma_f(6)$		σ_{in}^{**}		σ_c^{***}	
				I*	II	I	II	I	II
1	1.4	- ∞	7.00	1.25	1.25	1.40	1.40	.070	.070
2	1.0	- 1.4	6.50	1.30	1.30	1.10	1.10	.120	.147
3	.9	- 1.0	6.50	1.22	1.22	1.00	1.00	.125	.168
4	.8	- .9	6.58	1.15	1.15	0.86	0.86	.130	.178
5	.7	- .8	6.75	1.15	1.15	0.76	0.64	.140	.190
6	.6	- .7	7.08	1.17	1.17	0.64	0.47	.150	.205
7	.55	- .6	7.33	1.20	1.20	0.53	0.36	.155	.215
8	.50	- .55	7.50	1.23	1.23	0.48	0.25	.160	.223
9	.45	- .50	7.83	1.25	1.25	0.42	0.16	.165	.233
10	.40	- .45	8.00	1.30	1.30	0.36	0.08	.170	.243
11	.35	- .40	8.33	1.35	1.35	0.29	↑ Zero	.185	.255
12	.30	- .35	8.75	1.40	1.40	0.23		.200	.270
13	.25	- .30	9.17	1.50	1.47	0.17		.300	.285
14	.20	- .25	9.75	1.60	1.51	0.12		.400	.305
15	.15	- .20	10.50	1.75	1.59	↑ Zero	↓ Zero	.600	.330
16	.10	- .15	11.42	1.91	1.68			.800	.370
17	.050	- .10	12.75	2.20	1.87			1.000	.440
18	.025	- .050	14.17	2.60	2.25			1.300	.550
19	.010	- .025	14.67	3.30	3.00	↓ Zero	↓ Zero	1.650	.760
20	.000	- .010	16.00	4.00	4.50			2.000	1.000

*This is an older set, obtained from BNL-170, the earlier edition of the reference cited.

**Sets I and II were assigned different thresholds for the purpose of studying the effect of the threshold. High energy values consistent with Engle-Hansen.⁽¹⁵⁾

*** σ_c : Both I and II give an α of $\sim .14$ when integrated over the Godiva spectrum. II is consistent with Engle-Hansen.¹⁵ I, as suggested by D. Okrent, was chosen to agree with EBR data.¹⁶

¹⁶Lichtenberger, H. V., et al., "Operating Experience and Experimental Results Obtained from a Na-K Cooled Fast Reactor," Paper 813, International Conference on the Peaceful Uses of Atomic Energy, Geneva, Switzerland, (August, 1955).

Inelastic Transfer Coefficients for the U Isotopes

We assume that the elastic transfer coefficients $\chi(i \rightarrow j)$ have the form

$$\chi(i \rightarrow j) = \frac{\int_{E_j}^{E_j - 1} g(E) dE}{\int_0^{E_i} g(E) dE} \quad (\text{A.4})$$

$\chi(i \rightarrow j)$ is evidently properly normalized, viz:

$$\sum_{j=i+1}^N \chi(i \rightarrow j) = 1 \quad (\text{A.5})$$

We have considered three spectra as follows:

$$g(E) = \begin{array}{ll} 1 & \text{"flat"} \\ E & \text{"skew"} \\ E^{1/2} e^{-E} & \text{"Bethe"} \end{array}$$

The quantities which are needed for the multigroup calculation are the coefficients

$$g(j) = \int_{E_j}^{E_j - 1} g(E) dE \quad (\text{A.6})$$

They are listed for the three spectra.

Table A. IV

Group	<u>g(j)</u>		
	Flat	Skew	Bethe
1	-	-	-
2	.40	.96	.1320
3	.10	.19	.0378
4	.10	.17	.0392
5	.10	.15	.0410
6	.10	.13	.0420
7	.05	.0575	.0214
8	.05	.0525	.0215
9	.05	.0475	.0215
10	.05	.0425	.0214
11	.05	.0375	.0211
12	.05	.0325	.0207
13	.05	.0275	.0200
14	.05	.0225	.0191
15	.05	.0175	.0178
16	.05	.0125	.0158
17	.05	.0075	.0130
18	.025	.00187	.0049
19	.015	.000525	.0021
20	.010	.000100	.0007

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Table A.V.

OTHER DETECTOR CROSS-SECTIONS

Group Number	$\sigma_f(\text{Np}^{237})(6)$	$\sigma_f(\text{Pu}^{239})(6)$	$\sigma_{n-\gamma}(\text{Au})(6)$
1	1.486	2.00	.0664
2	1.330	1.89	.117
3	1.210	1.83	.140
4	1.130	1.82	.153
5	1.000	1.81	.168
6	.800	1.80	.185
7	.620	1.77	.200
8	.480	1.76	.213
9	.320	1.75	.230
10	.215	1.73	.245
11	.133	1.73	.269
12	.086	1.72	.295
13	.053	1.70	.330
14	<div style="display: flex; align-items: center; justify-content: center;"> <div style="writing-mode: vertical-rl; transform: rotate(180deg);">Zero</div> <div style="margin: 0 10px;">↑ ↓</div> </div>	1.70	.375
15		1.72	.440
16		1.74	.550
17		1.80	.760
18		2.00	1.200
19		2.22	2.000
20		2.70	3.000

Table A. VI

CROSS SECTIONS OF Al^{27}

Group Number	Energy Interval	Primary				Derived		
		$\sigma_T^{(6)}$	$\sigma_{in}^{(17,20)*}$	$\sigma_c^{(6)}$	$1 - \bar{\mu}^{(12,18)}$	σ_s	$\sigma_s(1 - \bar{\mu})$	σ_{tr}
1	1.4 - ∞	2.30	.40	.00035	.65	1.90	1.24	1.64
2	1.0 - 1.4	3.30	.30	.00038	.73	3.00	2.19	2.49
3	.9 - 1.0	2.80	.00	.00035	.81	2.80	2.27	2.27
4	.8 - .9	4.00	.00	.00060	.84	4.00	3.36	3.36
5	.7 - .8	3.25	.00	.00054	.87	3.25	2.83	2.83
6	.6 - .7	3.75	.00	.00065	.90	3.75	3.38	3.38
7	.55 - .6	4.00	.00	.00081	.92	4.00	3.68	3.68
8	.50 - .55	4.15	.00	.00067	.93	4.15	3.86	3.86
9	.45 - .50	4.05	.00	.00078	.95	4.05	3.85	3.85
10	.40 - .45	4.70	.00	.00135	.96	4.70	4.51	4.51
11	.35 - .40	3.50	.00	.00200	.98	3.50	3.43	3.43
12	.30 - .35	3.60	.00	.00090	.99	3.60	3.56	3.56
13	.25 - .30	2.25	.00	.00175	1.00	2.25	2.25	2.25
14	.20 - .25	4.80	.00	.00140	1.00	4.80	4.80	4.80
15	.15 - .20	6.00	.00	.00150	1.00	6.00	6.00	6.00
16	.10 - .15	3.50	.00	.00460	1.00	3.50	3.50	3.50
17	.050 - .10	3.40	.00	.00230	1.00	3.40	3.40	3.40
18	.025 - .050	4.00	.00	.00780	1.00	4.00	4.00	4.00
19	.010 - .025	1.15	.00	.00510	1.00	1.15	1.15	1.15
20	.000 - .010	1.40	.00	.00954	1.00	1.40	1.40	1.40

*First three levels of Al^{27} are at .843, 1.01 and 2.23 Mev.¹⁹

¹⁷Carter, R. E., and Beyster, J. R., "Inelastic Collisions Cross Sections for Fission Spectrum Neutrons," Phys. Rev. 90, 389 (A) (1953).

¹⁸Whitehead, W. D. and Snowden, S. C., "Angular Distribution of Neutrons Scattered from Aluminum, Iron, and Lead," Phys. Rev. 92, 114 (1953).

¹⁹Endt, P. M., and Kluyver, J. C., "Energy Levels of Light Nuclei (Z = 11 to Z = 20)," Rev. Mod. Phys., 26, 95 (1954).

²⁰Kielm, R. M., and Goodman, C., "Neutron Inelastic Scattering," Phys. Rev. 95, 989 (1954).

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Table A. VII

CROSS SECTIONS OF Na²³

Group Number	Energy Interval		Primary				Derived		
			$\sigma_T^{(6)}$	σ_{in}^*	$\sigma_c^{(6)}$	$1 - \bar{\mu}^{**}$	σ_s	$\sigma_s(1 - \bar{\mu})$	σ_{tr}
1	1.4	- ∞	2.70	.350	.00029	.65	2.35	1.53	1.88
2	1.0	- 1.4	2.85	.350	.00029	.73	2.50	1.83	2.18
3	.9	- 1.0	4.28	.350	.00029	.81	3.93	3.18	3.53
4	.8	- .9	4.38	.235	.00029	.84	4.15	3.49	3.73
5	.7	- .8	6.55	.225	.00029	.87	6.33	5.51	5.74
6	.6	- .7	5.52	.060	.00029	.90	5.46	4.91	4.97
7	.55	- .6	4.15	.140	.00029	.92	4.01	3.69	3.83
8	.50	- .55	2.70	.180	.00029	.93	2.52	2.34	2.52
9	.45	- .50	3.00	.020	.00029	.95	2.98	2.83	2.85
10	.40	- .45	3.75	Zero	.00029	.96	3.75	3.60	3.60
11	.35	- .40	3.80		.00029	.98	3.80	3.72	3.72
12	.30	- .35	3.40		.00029	.99	3.40	3.37	3.37
13	.25	- .30	3.00		.00029	1.00	3.00	3.00	3.00
14	.20	- .25	5.20		.00029	1.00	5.20	5.20	5.20
15	.15	- .20	3.50		.00029	1.00	3.50	3.50	3.50
16	.10	- .15	3.40		.00029	1.00	3.40	3.40	3.40
17	.050	- .10	4.50		.00029	1.00	4.50	4.50	4.50
18	.025	- .050	5.50		.00029	1.00	5.50	5.50	5.50
19	.010	- .025	5.20		.00029	1.00	5.20	5.20	5.20
20	.000	- .010	10.00		.00029	1.00	10.00	10.00	10.00

*The first two levels of Na²³ are at .439 and 2.07 Mev.¹⁹ σ_{in} below 1 Mev was inferred from a theoretical analysis by J. Monahan of ANL, Physics Division. High energy values are guessed.

** $1 - \bar{\mu}$ was assigned to be similar to Al.

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Table A. VIII

CROSS SECTIONS OF Fe

Group Number	Energy Interval		$\sigma_T(6)$	Primary			Derived		
				$\sigma_{in}(12,19)^*$	σ_c^{**}	$1 - \bar{\mu}(12,18,21)$	σ_s	$\sigma_s(1 - \bar{\mu})$	σ_{tr}
1	1.4	- ∞	2.80	.73	.006	.60	2.08	1.25	1.99
2	1.0	- 1.4	2.50	.50	.006	.70	1.99	1.39	1.90
3	.9	- 1.0	2.32	.25	.006	.82	2.06	1.69	1.95
4	.8	- .9	2.65	.10	.006	.85	2.56	2.25	2.36
5	.7	- .8	3.55	.00	.006	.87	3.55	3.09	3.10
6	.6	- .7	2.30	↑ Zero ↓	.006	.90	2.30	2.07	2.08
7	.55	- .6	2.60		.006	.92	2.60	2.39	2.40
8	.50	- .55	2.85		.006	.93	2.85	2.65	2.66
9	.45	- .50	3.40		.006	.93	3.40	3.16	3.17
10	.40	- .45	4.30		.006	.94	4.30	4.04	4.05
11	.35	- .40	4.00		.006	.95	4.00	3.80	3.81
12	.30	- .35	3.00		.006	.96	3.00	2.88	2.89
13	.25	- .30	2.90		.006	.97	2.90	2.81	2.82
14	.20	- .25	3.60		.006	.97	3.60	3.49	3.50
15	.15	- .20	3.70		.006	.98	3.70	3.63	3.64
16	.10	- .15	3.10		.006	.98	3.10	3.04	3.05
17	.050	- .10	3.90		.006	.99	3.90	3.86	3.87
18	.025	- .050	5.00		.006	1.00	5.00	5.00	5.00
19	.010	- .025	3.50		.006	1.00	3.50	3.50	3.50
20	.000	- .010	7.00		.007	1.00	7.00	7.00	7.00

*First excited level of Fe^{56} at .82 Mev. ²⁰** σ_c was taken to be $\frac{1}{v}$ from thermal until 6 mb is reached; suggested by H. A. Bethe.²¹Walt, M., and Barschall, H. H., "Scattering of 1-Mev Neutrons by Heavy Elements," Phys. Rev. 93, 1062 (1954).

Table A. IX

INELASTIC TRANSFER CROSS SECTIONS FOR SOME LIGHT ELEMENTS

Groups i	Na ²³				Al ²⁷		Fe			
	$\sigma(1 \rightarrow i)$	$\sigma(2 \rightarrow i)$	$\sigma(3 \rightarrow i)$	$\sigma(4 \rightarrow i)$	$\sigma(1 \rightarrow i)$	$\sigma(2 \rightarrow i)$	$\sigma(1 \rightarrow i)$	$\sigma(2 \rightarrow i)$	$\sigma(3 \rightarrow i)$	$\sigma(4 \rightarrow i)$
1	-	-	-	-	-	-	-	-	-	-
2	.100	-	-	-	.16624	-	.25958	-	-	-
3	.025	.0875	-	-	.04156	-	.06489	-	-	-
4	.025	.0875	-	-	.04156	-	.06489	-	-	-
5	.025	.0875	-	-	.04156	-	.06489	-	-	-
6	.025	.0875	-	-	.04156	-	.06489	-	-	-
7	.0125	-	.175	-	.02078	-	.03244	-	-	-
8	.0125	-	.175	-	.01558	.01429	.01622	.0625	-	-
9	.0125	-	-	.1175	.01558	.01429	.01622	.0625	-	-
10	.0125	-	-	.1175	.01558	.01429	.01622	.0625	-	-
11	.0125	-	-	-	-	.04285	.01622	.0625	-	-
12	.0125	-	-	-	-	.04285	.01622	.0625	-	-
13	.0125	-	-	-	-	.04286	.01622	.0625	-	-
14	.0125	-	-	-	-	.04286	.01622	.0625	-	-
15	.0125	-	-	-	-	.02857	.01622	.0625	-	-
16	.0125	-	-	-	-	.02857	.01622	-	.125	-
17	.0125	-	-	-	-	.02857	.01622	-	.125	-
18	.00625	-	-	-	-	-	.00811	-	-	.050
19	.00375	-	-	-	-	-	.00487	-	-	.030
20	.00250	-	-	-	-	-	.00324	-	-	.020

Note: In the IBM calculations, because of machine limitations, inelastic scattering for Na²³ from groups below 4 was set equal to zero.

APPENDIX B

This appendix contains tabulated values of κ 's as determined from AVIDAC least squares fits on experimental runs, along with appropriate averages. The major table headings (Table B.I, etc.) refer to core composition; the minor headings (B.Ia, etc.) refer to changes in dimension (X, Y, or Z) and/or reflector.

Table B.I

3.13:1 U²³⁸:U²³⁵ Atom Ratio, 14 Vol. % Al

a) U Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁸	0.053946	0.68	$\bar{\kappa}_x = 0.05214 \pm 0.00087 \text{ cm}^{-1}$
Pu	0.053352	0.57	
Np	0.051053	0.25	$\bar{\kappa}_x^2 = 0.00272 \pm 0.00009 \text{ cm}^{-2}$
Np	0.050015	0.77	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁸	0.037161	0.32	$\bar{\kappa}_z = 0.03638 \pm 0.00063 \text{ cm}^{-1}$
Np	0.037216	0.67	
U ²³⁵	0.035252	3.16	$\bar{\kappa}_z^2 = 0.00132 \pm 0.00005 \text{ cm}^{-2}$
U ²³⁵	0.038060	1.64	

Reflector Savings = 22 cm

b) Pb Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.056119	0.69	$\bar{\kappa}_x = 0.05633 \pm 0.00039 \text{ cm}^{-1}$
U ²³⁵	0.057116	1.26	
U ²³⁸	0.055616	1.21	$\bar{\kappa}_x^2 = 0.00317 \pm 0.00004 \text{ cm}^{-2}$

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
Np	0.037180	0.45	$\bar{\kappa}_z = 0.03672 \pm 0.00045 \text{ cm}^{-1}$
U ²³⁸	0.035854	0.25	
			$\bar{\kappa}_z^2 = 0.00135 \pm 0.00003 \text{ cm}^{-2}$

Reflector Savings = 20 cm

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Table B.I (Cont'd.)

c) Fe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.058024	0.23	
U ²³⁵	0.056126	0.12	$\bar{\kappa}_x = 0.05532 \pm 0.00064$ cm ⁻¹
U ²³⁵	0.054020	0.93	
U ²³⁸	0.053243	1.02	$\bar{\kappa}_x^2 = 0.00306 \pm 0.00007$ cm ⁻²
Np	0.056278	2.75	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.036616	3.93	
Np	0.037561	0.74	$\bar{\kappa}_z = 0.03689 \pm 0.00030$ cm ⁻¹
U ²³⁸	0.037674	0.26	
Np	0.037748	1.50	$\bar{\kappa}_z^2 = 0.00136 \pm 0.00002$ cm ⁻²
U ²³⁸	0.035529	0.68	

Reflector Savings = 30 cm

d) Results

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.067253	1.49	
Np	0.067194	1.18	
U ²³⁸	0.065143	1.16	$\bar{\kappa}_y = 0.06600 \pm 0.00047$ cm ⁻¹
U ²³⁵	0.065707	1.03	
Pu	0.065903	1.38	
U ²³⁵	0.067945	1.82	
U ²³⁵	0.063221	0.63	$\bar{\kappa}_y^2 = 0.00436 \pm 0.00007$ cm ⁻²
U ²³⁸	0.064803	0.85	
Pu	0.064412	2.34	
Np	0.068680	1.54	
U ²³⁸	0.065666	1.50	

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Table B.II

5:1 U²³⁸:U²³⁵ Atom Ratio, 0 % AlFe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04357	30.10	
U ²³⁸	0.04277	2.11	$\bar{\kappa}_x = 0.04354 \pm 0.00009$ cm ⁻¹
Np	0.04327	3.57	
U ²³⁸	0.04392	1.51	
U ²³⁸	0.04379	5.34	$\bar{\kappa}_x^2 = 0.00190 \pm 0.00001$ cm ⁻²
U ²³⁸	0.04358	2.23	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.05247	0.79	
Np	0.05092	1.59	$\bar{\kappa}_y = 0.05053 \pm 0.00036$ cm ⁻¹
U ²³⁸	0.05052	2.12	
Np	0.04971	2.08	$\bar{\kappa}_y^2 = 0.00255 \pm 0.00004$ cm ⁻²
U ²³⁸	0.04990	0.69	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.02894	6.00	$\bar{\kappa}_z = 0.02866 \pm 0.00048$ cm ⁻¹
U ²³⁸	0.02704	1.63	
U ²³⁸	0.02977	0.91	$\bar{\kappa}_z^2 = 0.00082 \pm 0.00003$ cm ⁻²

Reflector Savings = 43 ± 4 cm

Table B.III

5:1 U²³⁸:U²³⁵ Atom Ratio, 14 Vol. % Al

a) Fe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04749	1.80	
U ²³⁸	0.04580	2.57	$\bar{\kappa}_x = 0.04555 \pm 0.00040$ cm ⁻¹
Np	0.04415	5.87	
U ²³⁸	0.04564	2.32	$\bar{\kappa}_x^2 = 0.00208 \pm 0.00004$ cm ⁻²
U ²³⁵	0.04586	12.72	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.05206	0.82	
Np	0.05028	0.90	$\bar{\kappa}_y = 0.05181 \pm 0.00041$ cm ⁻¹
U ²³⁸	0.05161	1.16	
U ²³⁵	0.05304	2.43	
U ²³⁸	0.05008	0.99	$\bar{\kappa}_y^2 = 0.00268 \pm 0.00004$ cm ⁻²
U ²³⁵	0.05185	2.56	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.03187	2.39	
Pu	0.03259	1.59	$\bar{\kappa}_z = 0.03206 \pm 0.00014$ cm ⁻¹
U ²³⁸	0.03217	1.32	
Np	0.03255	2.00	
U ²³⁸	0.03238	1.65	$\bar{\kappa}_z^2 = 0.00103 \pm 0.00001$ cm ⁻²
U ²³⁵	0.03174	6.41	

Reflector Savings = 30.0 ± 3 cm

Table B.III (Cont'd.)

b) Al Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor
U ²³⁸	0.04707	26.3
U ²³⁵	0.04823	6.9
Pu	0.04750	1.7
U ²³⁸	0.04537	5.3
Np	0.04555	0.6

$$\bar{\kappa}_x = 0.04704 \pm 0.00036 \text{ cm}^{-1}$$

$$\bar{\kappa}_x^2 = 0.00221 \pm 0.00003 \text{ cm}^{-2}$$

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor
U ²³⁸	0.05157	8.18
U ²³⁵	0.05131	10.50
U ²³⁵	0.05204	12.12
Np	0.05197	3.70
U ²³⁸	0.05257	4.21
Pu	0.05103	12.01

$$\bar{\kappa}_y = 0.05161 \pm 0.00020 \text{ cm}^{-1}$$

$$\bar{\kappa}_y^2 = 0.00266 \pm 0.00002 \text{ cm}^{-2}$$

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor
Pu	0.03165	9.94
U ²³⁵	0.03237	97.06
U ²³⁸	0.03221	11.19
U ²³⁸	0.03234	27.00
U ²³⁵	0.03228	12.65

$$\bar{\kappa}_z = 0.03230 \pm 0.00004 \text{ cm}^{-1}$$

$$\bar{\kappa}_z^2 = 0.001043 \pm 0.000003 \text{ cm}^{-2}$$

Reflector Savings = $30.5 \pm 2 \text{ cm}$

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Table B.III (Cont'd.)

c) U Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
Np	0.04564	3.91	$\bar{\kappa}_x = 0.04652 \pm 0.00029$ cm ⁻¹
U ²³⁵	0.04685	7.01	
U ²³⁵	0.04615	1.09	$\bar{\kappa}_x^2 = 0.00216 \pm 0.00003$ cm ⁻²
U ²³⁸	0.04720	2.16	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁸	0.05115	2.10	$\bar{\kappa}_y = 0.05283 \pm 0.00049$ cm ⁻¹
U ²³⁵	0.05358	3.54	
U ²³⁵	0.05430	1.94	$\bar{\kappa}_y^2 = 0.00279 \pm 0.00005$ cm ⁻²
U ²³⁸	0.05165	1.02	
Np	0.05254	2.82	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.03180	4.01	$\bar{\kappa}_z = 0.03187 \pm 0.00011$ cm ⁻¹
U ²³⁸	0.03223	0.77	$\bar{\kappa}_z^2 = 0.001016 \pm 0.000007$ cm ⁻²

Reflector Savings = 27.7 ± 2 cm

Table B.III (Cont'd.)

d) Pb Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04550	.33	
U ²³⁵	0.04531	1.53	$\bar{\kappa}_x = 0.04558 \pm 0.00014$ cm ⁻¹
Pu	0.04581	.30	
U ²³⁵	0.04590	1.12	$\bar{\kappa}_x^2 = 0.00208 \pm 0.00001$ cm ⁻²

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.05272	0.88	
U ²³⁵	0.05222	1.12	$\bar{\kappa}_y = 0.05196 \pm 0.00031$ cm ⁻¹
U ²³⁸	0.05078	0.20	
U ²³⁸	0.05116	1.85	$\bar{\kappa}_y^2 = 0.00270 \pm 0.00003$ cm ⁻²
Np	0.05272	0.96	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁸	0.03151	0.10	
U ²³⁵	0.03113	0.88	$\bar{\kappa}_z = 0.03152 \pm 0.00024$ cm ⁻¹
U ²³⁵	0.03204	1.18	
Np	0.03123	0.24	$\bar{\kappa}_z^2 = 0.00099 \pm 0.00001$ cm ⁻²
Pu	0.03008	0.14	

Reflector Savings = 34 ± 3 cm

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Table B.III (Cont'd.)

e) Hetro #1 Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04529	1.46	
Np	0.04358	2.74	$\bar{\kappa}_x = 0.04557 \pm 0.00052$ cm ⁻¹
U ²³⁸	0.04534	1.61	
U ²³⁸	0.04548	1.85	
U ²³⁵	0.04337	1.96	$\bar{\kappa}_x^2 = 0.00208 \pm 0.00005$ cm ⁻²
U ²³⁵	0.04665	9.92	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.05267	3.42	
U ²³⁸	0.05219	1.33	$\bar{\kappa}_y = 0.05250 \pm 0.00007$ cm ⁻¹
U ²³⁸	0.05229	0.95	
Np	0.05250	1.46	$\bar{\kappa}_y^2 = 0.00276 \pm 0.00001$ cm ⁻²
U ²³⁵	0.05251	3.41	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁸	0.03150	1.29	$\bar{\kappa}_z = 0.03183 \pm 0.00047$ cm ⁻¹
Np	0.03185	2.36	
U ²³⁸	0.02948	0.79	$\bar{\kappa}_z^2 = 0.00101 \pm 0.00003$ cm ⁻²
U ²³⁵	0.03265	2.73	

Reflector Savings = 28 ± 4 cm

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Table B.III (Cont'd.)

f) Hetro #2 Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04592	1.48	$\bar{\kappa}_x = 0.04414 \pm 0.00053$ cm ⁻¹
U ²³⁸	0.04323	3.45	
Pu	0.04425	4.04	$\bar{\kappa}_x^2 = 0.00195 \pm 0.00005$ cm ⁻²

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
Np	0.05104	4.15	$\bar{\kappa}_y = 0.05099 \pm 0.00034$ cm ⁻¹
U ²³⁵	0.05312	.64	
U ²³⁸	0.05075	2.29	$\bar{\kappa}_y^2 = 0.00260 \pm 0.00003$ cm ⁻²
Pu	0.05021	1.29	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
Pu	0.03161	3.53	
U ²³⁸	0.03122	1.19	$\bar{\kappa}_z = 0.03132 \pm 0.00013$ cm ⁻¹
Np	0.03092	1.22	
U ²³⁸	0.03130	1.12	$\bar{\kappa}_z^2 = 0.00098 \pm 0.00001$ cm ⁻²
U ²³⁵	0.03096	1.01	

Reflector Savings 34 ± 2 cm

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Table B.IV

5:1 U²³⁸:U²³⁵ Atom Ratio, 28 Vol. % AlFe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.04545	1.30	
U ²³⁸	0.04322	2.52	$\bar{\kappa}_x = 0.04357 \pm 0.00052$ cm ⁻¹
Np	0.04268	7.90	
U ²³⁸	0.04552	2.11	$\bar{\kappa}_x^2 = 0.00190 \pm 0.00005$ cm ⁻²
U ²³⁵	0.04470	1.22	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.05500	2.75	
Np	0.05372	2.19	$\bar{\kappa}_y = 0.05460 \pm 0.00034$ cm ⁻¹
U ²³⁸	0.05290	.90	
U ²³⁸	0.05417	.89	$\bar{\kappa}_y^2 = 0.00298 \pm 0.00004$ cm ⁻²
U ²³⁵	0.05522	4.46	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.03140	.49	
U ²³⁸	0.03152	1.55	$\bar{\kappa}_z = 0.03142 \pm 0.00006$ cm ⁻¹
Np	0.03151	1.62	
Pu	0.03123	1.51	$\bar{\kappa}_z^2 = 0.000987 \pm 0.000004$ cm ⁻²

Reflector Savings = 31.5 ± 4 cm

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Table B.V

5:1 U²³⁸:U²³⁵ Atom Ratio, 55 Vol % NaFe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor
U ²³⁸	0.04583	0.31
U ²³⁵	0.04711	3.17
U ²³⁵	0.04496	2.46
U ²³⁸	0.04513	1.92
Np	0.04465	0.93
U ²³⁸	0.04524	0.79
U ²³⁵	0.04696	2.46
U ²³⁵	0.04636	3.33
U ²³⁸	0.04695	1.66

$$\bar{\kappa}_x = 0.04615 \pm 0.00030 \text{ cm}^{-1}$$

$$\bar{\kappa}_x^2 = 0.00213 \pm 0.00003 \text{ cm}^{-2}$$

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor
U ²³⁸	0.05269	1.27
U ²³⁵	0.05406	2.26
U ²³⁸	0.05236	0.84
U ²³⁸	0.05256	0.27
U ²³⁵	0.05370	0.30
U ²³⁵	0.05371	1.40
Np	0.05235	0.21
U ²³⁵	0.05234	0.56

$$\bar{\kappa}_y = 0.05328 \pm 0.00025 \text{ cm}^{-1}$$

$$\bar{\kappa}_y^2 = 0.00284 \pm 0.00003 \text{ cm}^{-2}$$

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor
Np	0.03330	5.86
U ²³⁵	0.03396	4.69
Pu	0.03458	1.55
U ²³⁸	0.03341	1.08
Np	0.03377	1.60
Pu	0.03165	1.89
U ²³⁸	0.03357	1.57
U ²³⁵	0.03331	.81

$$\bar{\kappa}_z = 0.03347 \pm 0.00025 \text{ cm}^{-1}$$

$$\bar{\kappa}_z^2 = 0.00112 \pm 0.00002 \text{ cm}^{-2}$$

Reflector Savings = 27.3 ± 3 cm

Table B.VI

6.82:1 U²³⁸:U²³⁵ Atom Ratio, 14 Vol. % Al

a) Fe Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.040715	4.54	$\bar{\kappa}_x = 0.04309 \pm 0.00110 \text{ cm}^{-1}$
U ²³⁸	0.039169	0.40	
U ²³⁵	0.045055	2.90	$\bar{\kappa}_x^2 = 0.00186 \pm 0.00010 \text{ cm}^{-2}$
Np	0.044985	3.47	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
Pu	0.041472	0.73	$\bar{\kappa}_y = 0.04240 \pm 0.00028 \text{ cm}^{-1}$
U ²³⁸	0.041830	0.39	
U ²³⁵	0.043111	1.77	$\bar{\kappa}_y^2 = 0.00180 \pm 0.00002 \text{ cm}^{-2}$
Pu	0.041605	0.88	
U ²³⁵	0.042558	1.96	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.032238	7.09	$\bar{\kappa}_z = 0.03222 \pm 0.00023 \text{ cm}^{-1}$
Np	0.032167	0.63	
Pu	0.031254	3.30	$\bar{\kappa}_z^2 = 0.00104 \pm 0.00001 \text{ cm}^{-2}$
U ²³⁸	0.030967	1.12	
U ²³⁵	0.032712	8.98	
U ²³⁸	0.032242	0.71	

Reflector Savings = 30.26 cm

Table B.VI (Cont'd.)

b) Pb Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor
Pu	0.04525	1.58
Np	0.04169	0.77
Np	0.03745	0.86
U ²³⁵	0.04019	1.21
U ²³⁸	0.04271	4.58
Np	0.03894	1.78
U ²³⁵	0.03992	0.70
Pu	0.04183	6.13
Pu	0.04171	3.26
U ²³⁵	0.04133	1.40

$$\bar{\kappa}_x = 0.04165 \pm 0.00051 \text{ cm}^{-1}$$

$$\bar{\kappa}_x^2 = 0.00174 \pm 0.00004 \text{ cm}^{-2}$$

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor
U ²³⁵	0.042708	2.24
Pu	0.041768	1.96
U ²³⁵	0.042757	8.37
Np	0.041510	1.77
U ²³⁵	0.043555	4.28
U ²³⁸	0.041467	0.64
U ²³⁸	0.040856	0.86
U ²³⁵	0.042886	3.22
U ²³⁸	0.041730	2.29
Np	0.042740	5.67

$$\bar{\kappa}_y = 0.04259 \pm 0.00020 \text{ cm}^{-1}$$

$$\bar{\kappa}_y^2 = 0.00181 \pm 0.00002 \text{ cm}^{-2}$$

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor
Np	0.031615	0.81
Pu	0.031604	9.74
U ²³⁸	0.030827	0.90
Np	0.032284	1.75
U ²³⁵	0.030033	4.86
Np	0.031144	1.16
U ²³⁵	0.031323	1.99
Pu	0.029754	1.72
Pu	0.030238	3.25

$$\bar{\kappa}_z = 0.03100 \pm 0.00026 \text{ cm}^{-1}$$

$$\bar{\kappa}_z^2 = 0.00096 \pm 0.00002 \text{ cm}^{-2}$$

Reflector Savings = 32.73 cm

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Table B.VI (Cont'd.)

c) Al Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.042717	1.17	$\bar{\kappa}_x = 0.04204 \pm 0.00029$ cm ⁻¹
U ²³⁵	0.041716	0.28	
U ²³⁸	0.041487	1.81	$\bar{\kappa}_x^2 = 0.00177 \pm 0.00002$ cm ⁻²
Np	0.042621	0.53	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.042702	1.22	$\bar{\kappa}_y = 0.04139 \pm 0.00040$ cm ⁻¹
U ²³⁵	0.040823	0.86	
U ²³⁸	0.040421	0.74	$\bar{\kappa}_y^2 = 0.00171 \pm 0.00003$ cm ⁻²
Np	0.041191	2.03	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
Pu	0.032496	5.09	$\bar{\kappa}_z = 0.03200 \pm 0.00031$ cm ⁻¹
U ²³⁸	0.032384	3.14	
Np	0.031347	0.94	$\bar{\kappa}_z^2 = 0.00102 \pm 0.00002$ cm ⁻²
U ²³⁸	0.030635	1.23	
U ²³⁵	0.031068	1.53	

Reflector Savings = 29.04 cm

Table B.VI (Cont'd.)

d) U Reflector

X Dimension (Exponential)

Detector	κ_x (cm ⁻¹)	Wt. Factor	
Pu	0.043509	5.33	
U ²³⁵	0.043818	1.02	$\bar{\kappa}_x = 0.04346 \pm 0.00023$ cm ⁻¹
U ²³⁸	0.043427	1.70	
Np	0.042181	2.33	$\bar{\kappa}_x^2 = 0.00189 \pm 0.00002$ cm ⁻²
U ²³⁵	0.043795	7.37	

Y Dimension (Transverse)

Detector	κ_y (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.042681	2.37	
U ²³⁵	0.042830	2.37	$\bar{\kappa}_y = 0.04234 \pm 0.00023$ cm ⁻¹
Pu	0.042516	1.84	
U ²³⁵	0.042718	1.95	$\bar{\kappa}_y^2 = 0.00179 \pm 0.00002$ cm ⁻²
Np	0.041718	3.10	
U ²³⁸	0.040967	0.79	

Z Dimension (Vertical)

Detector	κ_z (cm ⁻¹)	Wt. Factor	
U ²³⁵	0.034036	5.56	
Pu	0.033864	5.33	$\bar{\kappa}_z = 0.03361 \pm 0.00023$ cm ⁻¹
U ²³⁵	0.033786	2.41	
Np	0.033096	2.47	$\bar{\kappa}_z^2 = 0.00113 \pm 0.00002$ cm ⁻²
U ²³⁸	0.032124	1.02	
U ²³⁵	0.032613	1.38	

Reflector Savings = 28.11 cm

APPENDIX C

Least Squares Calculation

The parameters in the functions (5.5) are determined by a least squares calculation. The experimental data for one particular flux traverse consists of the three numbers x_i n_i t_i ($i = 1, 2, \dots, N$) which are respectively the position, the number of counts, and the counting time. From these are derived the following quantities which are more convenient: The counting rate r_i and the weight W_i are defined by:

$$r_i \equiv \frac{N_i}{t_i} \quad (C.1)$$

and

$$W_i \equiv \frac{t_i^2}{N_i}$$

The problem is to determine the three parameters α , β , and γ in an otherwise specified function of x , namely, $f(x, \alpha, \beta, \gamma)$,* such that the sum of the weighted squares of the residuals is a minimum, i.e.,

$$S = \sum_i W_i [f(X_i, \alpha, \beta, \gamma) - r_i]^2 = \text{MIN} \quad (C.2)$$

Below we shall use the abbreviations

$$\left. \begin{aligned} f(X_i, \alpha, \beta, \gamma) &\equiv f_i \\ f(X_i, \alpha_0, \beta_0, \gamma_0) &\equiv (f_i)_0 \\ \frac{\partial}{\partial \alpha} [f(\alpha, \beta_0, \gamma_0)]_{\alpha=\alpha_0} &\equiv \left(\frac{\partial f}{\partial \alpha} \right)_0, \text{etc.} \end{aligned} \right\} \quad (C.3)$$

A necessary condition that S be a minimum is

$$\frac{\partial S}{\partial \alpha} = \frac{\partial S}{\partial \beta} = \frac{\partial S}{\partial \gamma} = 0 \quad (C.4)$$

The above system is a set of three equations in three unknowns. Let $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\gamma}$ be the solution of interest. Carrying out the differentiations we obtain the set

$$\left. \begin{aligned} \sum_i W_i \frac{\partial f_i}{\partial \alpha} (f_i - N_i) &= 0 \\ \sum_i W_i \frac{\partial f_i}{\partial \beta} (f_i - N_i) &= 0 \\ \sum_i W_i \frac{\partial f_i}{\partial \gamma} (f_i - r_i) &= 0 \end{aligned} \right\} \quad (C.5)$$

* β represents any one of the components of the buckling and f either one of the functions defined in (2).

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In our experiment f is not a linear function of the parameters and a direct solution of (C.5) is not possible. However, the equations (C.5) were solved by a well known method of successive approximation as follows: Let α_0 , β_0 , and γ_0 be a first guess in the neighborhood of $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\gamma}$. We expand $f_i(X, \alpha, \beta, \gamma)$ in a Taylor series about α_0 , β_0 , and γ_0 and retain only the first order terms

$$\begin{aligned} f_i(\alpha, \beta, \gamma) &= f(\alpha_0 + a_1, \beta_0 + b_1, \gamma_0 + c_1) \\ &\cong (f_i)_0 + a_1 \left(\frac{\partial f_i}{\partial \alpha} \right)_0 + b_1 \left(\frac{\partial f_i}{\partial \beta} \right)_0 + c_1 \left(\frac{\partial f_i}{\partial \gamma} \right)_0 \end{aligned} \quad (C.6)$$

and approximate

$$\frac{\partial f_i}{\partial \alpha} \cong \left(\frac{\partial f_i}{\partial \alpha} \right)_0 \quad . \quad (C.7)$$

Substitution of (C.6) into (C.5) results in linear equations in a_1 , b_1 and c_1 as follows:

$$M_0 \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = Y_0 \quad , \quad (C.8)$$

in which M_0 is the third order symmetric matrix with the rows and columns labelled by α, β , and γ ; for example, the $\alpha\beta$ element is given by

$$M_0^{\alpha\beta} = \sum_i W_i \left(\frac{\partial f_i}{\partial \alpha} \right)_0 \left(\frac{\partial f_i}{\partial \beta} \right)_0 \quad (C.9)$$

and Y_0 is the column vector

$$Y_0 = \begin{pmatrix} \sum_i W_i \left(\frac{\partial f_i}{\partial \alpha} \right)_0 [r_i - (f_i)_0] \\ \sum_i W_i \left(\frac{\partial f_i}{\partial \beta} \right)_0 [r_i - (f_i)_0] \\ \sum_i W_i \left(\frac{\partial f_i}{\partial \gamma} \right)_0 [r_i - (f_i)_0] \end{pmatrix} \quad . \quad (C.10)$$

The subscript zero indicates that all quantities are evaluated with our first guess (zeroth approximation). The solution of (C.8) is given by

$$\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = M_0^{-1} Y_0 \quad . \quad (C.11)$$

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The values of the minimizing parameters in first approximation are given by

$$\left. \begin{aligned} \alpha_1 &= \alpha_0 + a_1 \\ \beta_1 &= \beta_0 + b_1 \\ \gamma_1 &= \gamma_0 + c_1 \end{aligned} \right\} . \quad (C.12)$$

The next approximation is obtained by expanding f about the new point α_1 , β_1 , and γ_1 and solving the equations

$$M_1 \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = Y_1 , \quad (C.13)$$

and so forth. As the number of iterations n increases the procedure converges to the correct minimizing values of the original problem (C.2), namely,

$$\alpha_n \rightarrow \bar{\alpha}, \quad \beta_n \rightarrow \bar{\beta}, \quad \gamma_n \rightarrow \bar{\gamma} .$$

The scheme as outlined above was coded for the AVIDAC, Argonne's electronic digital computer. Convergence to better than four significant figures of β , the component of the buckling, was obtained after a few iterations (4 or 5). For 30 to 40 spatial points this took about 3-5 minutes (this includes the time needed for loading the memory). The criterion which was used for terminating a calculation was that

$$|a_n| < a^* , \quad |b_n| < b^* , \quad |c_n| < c^* ,$$

where a^* , b^* and c^* are pre-assigned positive numbers which were usually taken to be 10^{-7} of a_0 , b_0 , and c_0 .